PERFORMANCE ENHANCEMENT OF A STOCK INDEX INVESTMENT UTILIZING A COLLAR OPTIONS STRATEGY

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INTRODUCTION

Collar options strategies are frequently used to improve the performance of an investment by protecting an investor from downside risk at the expense of a portion of the upside gains. This project explored the protection of an underlying index fund investment by purchasing out-of-the-money (OTM) puts, funding this purchase with the sale of OTM calls. The analysis was made up of two major components: (1) the implementation of passive and active collars as detailed by Ed Szado and Thomas Schneeweis in their study Loosening Your Collar: Alternative Implementations of QQQQ Collars and (2) optimizing expected utility in each time period (monthly) as a function of puts and calls selected. The analysis of how a strategy would have performed based on 5 and 10 year historical datasets was performed using MATLAB. In the optimization case, GAMS was used to determine the optimal strategy and this was fed back into MATLAB to determine historical performance. Results of Szado and Schneeweis implementation on the NASDAQ 100 (QQQQ) were verified and results from the passive collar strategy, active collar strategy, and the GAMS optimization were compared.

COLLAR OPTIONS STRATEGIES

In general, collar strategies consist of purchasing a put option and selling a call option on an underlying asset (e.g. QQQQ) while also holding shares of the same underlying asset. Passive strategies follow a set of trading rules that do not change regardless of market conditions. These rules dictate the width of the collar (i.e. percentage OTM), as well as the time to expiration of puts and calls. Active collars, on the other hand, take into account market/economic signals when selecting which puts to purchase and calls to sell so the width of the collar and the ratio of the asset to calls may change from month to month (ratio of shares of underlying index to puts is always equal to one). In addition, the time to expiration of the puts and calls may not be the same.

PASSIVE STRATEGY

The passive collar strategy was implemented using the QQQQ as the underlying asset beginning on March 19, 1999. On this date, and every third Friday of the month thereafter through December 18, 2009 a 1-month call was sold and a 1, 3, or 6-month put purchased. Implementations were completed with initial moneyness of the calls and puts set at 1%, 2%, 3%, 4%, or 5% OTM. Although implementations for each collar width were explored, the passive results included here focus on the 2% OTM strategy, since it represented a generally conservative approach.

ACTIVE STRATEGY

For the active strategy implementation, three market signals were used to inform the options purchased and sold (as outlined in Szado and Schneeweis). These three signals were based on momentum, volatility, and macroeconomic indicators for the short, medium, and long-term.

The momentum signal is a binary indicator based on a simple moving average crossover (SMACO) of the underlying asset. If the trend was upward the signal was set at +1. Similarly it was downward the signal was set at -1. For short, medium, and long-term signals, a 1/50, 5/150, and 1/200 day SMACO rule, respectively, was utilized.
The *volatility signal* is based on the daily VIX close using 50, 150, and 250 day moving average windows to determine the short, medium, and long-term volatility signals. On roll dates if the VIX close is more than one standard deviation above the moving average, a ratio of 0.75 calls per index position determined the number of calls sold. If the previous day’s close is more than one standard deviation below the moving average a ratio of 1.25 calls per underlying asset shares determines calls sold. Otherwise, 1 call per share was sold of underlying index held.

The *macroeconomic signal* is a binary indicator based on unemployment claims and the state of the economy (expansionary or contractionary) reported by the US National Bureau of Economic Research (NBER). In an expansionary economy, rising unemployment claims are a bullish sign, so the signal is set to +1. Accordingly, falling unemployment claims would set the signal to -1. In a contractionary period, rising unemployment would set the signal to -1, and falling unemployment would set the signal to +1.

Each of these signals was assessed on the trading day and the percent OTM adjusted according to the following trading rules:

Call % OTM = 2 + Momentum signal + Macroeconomic signal

Put % OTM = 3 + Momentum signal – Macroeconomic signal

The ratio of the underlying asset to puts to calls is adjusted according to the volatility signal. These two strategies were compared with an optimization strategy which maximized expected utility.

**GAMS OPTIMIZATION**

The optimized collar strategy utilized stochastic optimization techniques to maximize the expected utility of the investor. The values of inputs of this optimization problem were determined using MATLAB then fed into GAMS. GAMS performed the optimization which produced the optimal amount of wealth to spend on puts and collect by selling calls. MATLAB then applies these outputs for the current time period. This allowed the performance of the optimization through time to be analyzed.

Based on historical data (the previous sixty months), a probabilistic distribution of expected returns on the underlying asset was created. This distribution was created using two methods:

- Assume future returns are normally distributed (only maintains 1st and 2nd moments)
- Use distribution of previous sixty months (sixty data points) to preserve higher moments (see regression section for details of how z-score is calculated)

In both of the above mentioned distributions the forward looking mean was calculated using the following two methods:

- Use mean and variance of historical window
- Update forward looking mean and variance based on linear regression of economic indicators (see section on regression for more details)
The distribution of index fund returns, \( r^\omega \), was used to determine the one month returns for call and put options, a deterministic relationship.

\[
r_{i}^{c,\omega} = \max(0, CP \times (1 + r^\omega) - s^c_i) - p^c_i \quad (\$/share return of call under scenario \omega)
\]

\[
r_{i}^{p,\omega} = \max \left( 0, s^p_i - CP \times (1 + r^\omega) \right) - p^p_i \quad (\$/share return of put under scenario \omega)
\]

\( X \) is defined as the share of wealth used to sell call options. \( Y \) is defined as the share of wealth used to purchase put options. The following optimization problem was then solved:

\[
\max E [u(w^\omega)] = E [\log(\alpha + w^\omega) + c] \quad \text{such that}
\]

\[
\begin{align*}
w^\omega &= w_0 \left[ (1 + \sum_i x_i + \sum_i y_i) \times (1 + r^\omega) + \sum_i \frac{y_i r_{i}^{p,\omega}}{p^p_i} - \sum_i \frac{x_i r_{i}^{c,\omega}}{p^c_i} \right] \\
\sum_i \frac{x_i}{p^c_i} &- \frac{(1 + \sum_i x_i + \sum_i y_i)}{CP} \leq 0 \quad (*) \\
\sum_i \frac{y_i}{p^p_i} &- \frac{(1 + \sum_i x_i + \sum_i y_i)}{CP} \leq 0 \quad (*) \\
\sum_i \frac{x_i}{p^c_i} - \sum_i \frac{y_i}{p^p_i} &= 0 \\
x_i, y_i &\geq 0 \text{ and } \alpha \leq 0
\end{align*}
\]

Both forced collar and free collar strategies were developed. In the forced strategy, the optimization formulation required that a put was purchased and a call sold for each unit of the underlying asset owned. This is done by making the constraints above equality, in effect making the last constraint redundant. The unforced strategy only mandates that for every put purchased, a call is sold. The number of puts must be less than or equal to the shares of the underlying asset owned.

To simulate the six month put strategy, the returns were instead calculated assuming 6 month returns for calls and puts instead of one month. This was done using the following two formulas:

\[
r_{i}^{c,\omega} = 6 \times \max(0, CP \times (1 + r_{1M}^\omega) - s^c_i) - p^c_i \quad (\$/share return of call under scenario \omega)
\]

\[
r_{i}^{p,\omega} = \max \left( 0, s^p_i - CP \times (1 + r_{6M}^\omega) \right) - p^p_i \quad (\$/share return of put under scenario \omega)
\]

Where the six month return of the underlying stock is determined as following:

\[
\tilde{r}_{1M} = \text{Normal} \left( \tilde{\mu}, \tilde{\sigma}^2 \right)
\]

\[
\tilde{r}_{6M} = \text{Normal} \left( 6\tilde{\mu}, 6\tilde{\sigma}^2 \right)
\]

The same optimization problem was then solved using these returns. The strategy remained the same for each month of the six months. See Appendix C for optimization strategy charts.
LINEAR REGRESSION OF ECONOMIC INDICATORS

Multiple linear regression (MLR) was used to estimate the forward-looking mean and variance. The inputs to the MLR were the three market signals (indicators) used in the active collar strategy, namely: momentum, volatility and macroeconomics. With this MLR, it is assumed that the relationship between the return and each indicator (or combination of indicators) is linear.

The first step in the MLR was to standardize the monthly return using the z-score formulation:

$$z_t = \frac{r_t - \hat{\mu}}{\hat{\sigma}}$$

This z-score is able to accommodate the “fat-tails” distribution of returns (i.e. kurtosis >3) which is fairly assumed within widely traded assets. It also allows the higher moments of the historical distribution to be maintained in the forward looking distributions.

The historical mean and standard deviation are given by:

$$\hat{\mu} = \frac{1}{T} \sum_{t=1}^{T} r_t \quad \hat{\sigma} = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} (r_t - \hat{\mu})^2}$$

The monthly return of QQQQ (or the any of the underlying indices/asset) is denoted by $r_t$ where $t = 1, \ldots, T$ are historical time periods and $T + 1$ is the next month (time period). The forward looking mean, a “conditional” term, is predicated on historical returns and the beliefs regarding the relationship between future returns and observable market signals. This forward looking mean is represented by:

$$\hat{\mu}_{T+1} | \bar{r}_{T+1} = \hat{\alpha} + \sum_k \hat{\beta}_k I^k_{T+1}$$

Where $\alpha$ and $\beta$ are the regression coefficients (regressors) used to fit the linear relationship and $k$ denotes the index of the market indicators, $I_k$. Notice that each indicator has its own unique $\beta_k$ coefficient. To be clear, given that 3 indicators are used in this model, there are 3 $\beta$-coefficients.

The forward looking standard deviation is estimated from the residuals of the forward looking mean. The standard deviation formulation is given by:

$$\hat{\sigma}_{T+1} = \sqrt{\frac{\sum e_t^2}{n - 1}}$$

This relationship is modeled through a so-called “disturbance term” $e_t$, which an unobserved random noise term that represents the error in fit of the linear relationship between the variables (return vs. indicator). Thus the model takes form:

$$\bar{R}_{T+1} = \hat{\mu}_{T+1} | \bar{r}_{T+1} + \hat{\sigma}_{T+1} z_t$$

The forecasted next-month return is thus a combination of:
The market signal conditioned next-month mean return

- \( \hat{\mu}_{t+1} | \hat{I}_{t+1} \)

The next-month standard deviation-scaled difference between the current month’s return and the historical mean measured to the current month

- \( \hat{\sigma}_{t+1} z_t \)

The results of this multiple regression were relatively poor. While the plots shown are for only one time period, it can be generalized for all time periods and is representative of the effectiveness of an MLR.

![Figure 1: Linear Regression of Market Signal Indicators to Monthly Index Returns](image)

**RESULTS**

Over five years, the passive collar, active collar and optimization strategies all succeeded in outperforming the strategy of simply purchasing and holding the QQQQ index. The free collar optimization strategy was consistently the best strategy, followed in order of success by the forced collar optimization then the active and passive collar strategies. This discussion compares optimized results to passive and active collars with 6-month puts, and 2% OTM puts and calls. A comparison of the best of these cases can be found in Figure 2. Additional passive and active results can be found in the appendix.

In all cases, the active strategy outperformed the passive strategy. Furthermore, both strategies outperformed simply holding the underlying asset. Longer put durations performed better than shorter durations since they are less sensitive to fluctuations and market events which send options prices out of equilibrium in the short term. Although transaction costs were not accounted for in this analysis, the 6-month puts will also incur fewer transaction costs since they turn over less frequently.
Among the optimization strategies, the free collar performed better than the forced collar. The free collar allows GAMS to optimize for the price paid for protection. If put prices are high for the value of protection they provide GAMS will simply neglect to purchase a put. In the forced collar, puts must always be purchased, even when they do not have good value.

In all cases, the free collar performed better than the forced collar. Both optimization strategies also outperformed the active and passive collar strategies when using 6-month puts. In Figure 3, one can see a comparison of using the actual (z-score) distribution verses a normal distribution with the forced and free collar. The 60 month free collar strategy, based on a normal distribution, returned 130% over the 5 year period while the forced collar returned 110%. The 60 month free collar strategy, with an actual distribution, returned 90%, while the forced collar returned only 70%. One would expect the actual distribution to perform better than the normal distribution; however, with so few data points (one data point for each of the previous 60 monthly returns) this is not the case. Limitations on number of data points available restrict sampling, but future studies could explore different distributions with fatter tails than a normal distribution.
FIGURE 3: COMPARISON OF USING ACTUAL (Z-SCORE) DISTRIBUTION AND NORMAL DISTRIBUTION ON 1 MONTH PUT OPTIMIZATION

FIGURE 4: OPTIMIZATION OF ONE MONTH PUT WITH REGRESSION ANALYSIS ON FORWARD LOOKING MEANS
Figure 4 shows the optimized one month put analysis when regression is used to update the forward looking means and variances. In this case, if a normal distribution is used to estimate the returns of the index funds, the regression analysis does improve performance by about 20% over five years. If the actual distribution is used, the regression does not influence the output. This is likely due to the actual distribution not being a good fit for future returns as a result of the limited data points. One can find a similar analysis for the optimized six month put strategy in the Appendix. In this case, the regression has almost no change in the strategy results. Since the linear regression is such a poor fit, only limited information is added by using it. In the six month case, one is already trying to make a long term (six month) decision based on very limited information. The small addition added by the regression then has little impact. This result could change if a six stage optimization model was used to make decision throughout the time period, instead of the current static model which makes all decisions at the beginning of each six months.

An attempt was made to test the active, passive and optimized strategies against other index funds. Unfortunately, the availability of long term options data is limited (less than two years in most cases), restricting our ability of testing these strategies on other funds. Future studies should include additional funds or stocks depending on data availability.

FUTURE WORK

It is worth noting that although this strategy performs well over a five year period, in extremely strong markets, such as the past year, it may not be ideal. Further study would determine whether this is a result of the strong market or if a collar options strategy is simply not intended as a short term (less than a year) strategy. Overall, the optimized strategy appears to be a strong trading strategy warranting additional study.

Additional studies could include investigating the different types of probability distributions which place more emphasis on extreme event, potentially better predicting future returns. In addition, different forms of regression or Bayesian methods could improve the use of market signal indicators for updating forward looking return distributions.

In addition, the current six month optimization model makes all decisions at the beginning of the six month period without updating ones actions based on what actually occurs through months one through five. Future research could improve this decision making strategy by creating a six stage (one stage for each month) for the six month put strategies. One could also come up with a method for optimizing the choice mechanism for put duration, as certain market conditions may make different put expirations lengths preferred.

Finally, further improvements to the model should take into account the transaction costs involved in purchasing call and put contracts. While some of these are considered by purchasing at the ask price and selling at the bid price, additional call and put contract writing cost may diminish overall returns. In addition, there are liquidity constraints when operating in the options markets. Options contracts can only be written in blocks of 100 shares. As the current analysis allows fractions of shares to be bought and sold, further study should investigate what levels of wealth this strategy would be appropriate for.
BIBLIOGRAPHY


QQQQ Options data (closed source) provided by Optionmetrics via Wharton Research Data Services.

VIX data (closed source) provided by MetaStock/Equis International via Reuters Datalink.
Passive Strategy: 6-Month Puts, 1-Month Calls

Trading Date (Expiration Day; 3rd Friday of Month)

Passive % OTM

Trading Date (Expiration Day; 3rd Friday of Month)
APPENDIX B – ACTIVE STRATEGY

Active Strategy: 1-Month Puts, 1-Month Calls

Active Strategy: 3-Month Puts, 1-Month Calls
Active Strategy: 6-Month Puts, 1-Month Calls

Trading Date (Expiration Day; 3rd Friday of Month)
APPENDIX C - COMPARISON OF PASSIVE AND ACTIVE STRATEGIES

10 Year: Passive 2% OTM and Active Strategies

5 Year: Passive 2% OTM and Active Strategies
APPENDIX D – OPTIMIZED STRATEGY

Optimization Forced Normal -vs Actual Distribution

6 Month Put Optimization with Regression