<u>Problem Set – Linear Algebra II</u>

1. Show that for an orthogonal matrix X, $X^T = X^{-1}$. If X is nxn, what is the rank?
   a. This means that the determinant is definitely not ____. Fill in the blank.

2. What are the eigenvectors and eigenvalues of a 2x2 identity matrix?

3. Show that the sum of the eigenvalues of a matrix is equal to the trace, and that the product of the eigenvalues is equal to the determinant. You can show this just for a 2x2 matrix, but this property holds for larger matrices.

4. Diagonalizable matrices can be written as: $A = V\Lambda V^{-1}$. This form of A makes for a convenient computation if you need to compute A^n (i.e. A*A*…*A, n times). In particular, if A is a 2x2 matrix, then A^n can be computed by raising only 2 scalar numbers (in the entire matrix product $V\Lambda V^{-1}$) to the n^{th} power. Which two numbers are they? What do these numbers correspond to in eigenvector-eigenvalue terms? Why is this so convenient?

5. Show that if A is diagonalizable, then A^{-1} (assuming that A is invertible) is also diagonalizable. Make sure you justify every step!

6. Playing around with rank and determinant and the handy-dandy invertible matrix theorem
   a. Let's play around with linear dependence and determinant = 0. Let's say you have a matrix: $W = \begin{bmatrix} a & k*a \\ b & k*b \end{bmatrix}$. What is the rank of this matrix? How many linearly independent columns are there? What is the determinant? Just for fun, what is the trace?
   b. Let's say you have matrix A, and det(A) = 0. Using the fact that det(AB) = det(A)*det(B) for any matrix A and B, show that if det(A) = 0, then A cannot have an inverse. Do this proof by contradiction – start by assuming that A^{-1} does exist and show why $A^{-1}*A = I$ cannot be.
   c. What is the determinant if one of the eigenvalues is 0? What interesting things can we say about a matrix if it has at least one eigenvalue = 0?
   d. Go here: https://en.wikipedia.org/wiki/Invertible_matrix and meditate on the all the properties in the "properties" tab. Some properties might have words we haven't used yet; you can ignore those if you want.