Gradient Echo Sequences

- Balanced SSFP
- Gradient Spoiled Sequences
- RF Spoiled Sequences
- Variations
  - Double-echo, Reversed
  - UTE, BOLD, T2*
Gradient Echo Pulse Sequence

Flip Angle

RF

TE ~ 1+ ms

$G_z$
Slice-Select Gradient

Refocusing Gradient

$G_y$
Phase-Encode Gradient

Readout Gradient

$G_x$
Dephaser Gradient

Signal

Gradient Echo

• Eliminate
• Average
• Recover
Contrast Example

- Contrast based **solely** on end-of-TR action
Balanced SSFP

• Goal: Maximize signal
  • Steady-state on-resonance
  • Steady-state off-resonance
  • Flip-angle effects
  • Transients

True-FISP, FIESTA, balanced FFE, BASG

Oppelt 1986, Duerk 1997
Balanced SSFP

Fully-balanced Gradients - Ignore!
Example: Alternating RF, No precession

Intuition: Relaxation does not affect direction much
Length Solution

• If Relaxation does not change length:

\[ \frac{dM}{dt} \cdot M = 0 \]

\[ \frac{-M_x}{T_2} M_x + \frac{-M_y}{T_2} M_y + \frac{M_0 - M_z}{T_1} M_z = 0 \]

\[ M_x T_2 M_x + M_y T_2 M_y + M_0 T_2 M_0 = 0 \]

x \ T_1 & Rearrange...

\[ \left( M_z - \frac{M_0}{2} \right)^2 + \frac{M_x^2 + M_y^2}{T_2/T_1} = \left( \frac{M_0}{2} \right)^2 \]

• Ellipsoid:

• height \( M_0 \), half-width (signal) \( M_0/2 \sqrt{T_2/T_1} \)

• \( T_2/T_1 \) contrast!
Full Solution

• If Relaxation does not change length:
  \[ \frac{dM}{dt} \cdot M = 0 \]

• Length: Ellipsoid intersection

• Direction: \( \alpha/2 \) angle from \( M_z \)
Signal Question

• With no off-resonance, $T_1 = T_2$, what is the transverse magnetization (signal) for $\alpha = 90^\circ$?

  - $T_1 = T_2$: Circular distribution!
  - $90^\circ$: Full extent

• Signal = $M_0/2$

• What if $\alpha = 60^\circ$?

  - Signal = $M_0/2 \sin(120^\circ/2) \approx 0.43M_0$
Steady State: No Precession
Off-Resonance: Precession
Increasing Precession
Balanced SSFP (FIESTA)
Precession and “Effective flip angle”

- Larger precession gives a larger “effective flip,” $\beta$
- $\tan(\alpha/2) = \tan(\beta/2) \cos(\phi/2)$
- $\beta \geq \alpha$
- Can replace flip ($\alpha$) with effective flip ($\beta$) for all calculations
- Limiting case ($\bullet$) where $\beta = 180^\circ$

(Schmitt MRM 2006, Zun, ISMRM 2006)
“Ellipsoid” Derivation of Signal

• Starting with ellipsoid:

\[
\left( M_z - \frac{M_0}{2} \right)^2 + \frac{M_x^2 + M_y^2}{T_2/T_1} = \left( \frac{M_0}{2} \right)^2
\]

\[
\left[ M \cos(\beta/2) - \frac{M_0}{2} \right]^2 + \frac{M^2 \sin^2(\beta/2)}{T_2/T_1} = \left[ \frac{M_0}{2} \right]^2
\]

\[
S = \frac{M_0}{\cot(\beta/2) + \left( \frac{T_1}{T_2} \right) \tan(\beta/2)}
\]

• Signal drops with increasing \( T_1/T_2 \)
• At \( \beta=180^\circ \) signal is 0
Signal vs Precession/Frequency

Signal depends on many factors:

- Resonant frequency
- T1, T2 (contrast)
- TR, TE
- RF flip / phase

 Freeman 1971
Signal vs Frequency: Phase

- Post-RF
- Center
- Pre-RF

Flipped!
Flip angle effects

10° Flip

50° Flip

90° Flip
bSSFP Dark Bands

Must limit precession:

Short TR

Limits resolution
Phase Cycling

Hinshaw 1976

$60^\circ$ $-60^\circ$ $60^\circ$

Signal

Magnitude

Frequency

$-1/TR$ $0$ $1/TR$

$60^\circ$ $60^\circ$ $60^\circ$

Signal

Magnitude

Frequency

$-1/TR$ $0$ $1/TR$
Review Question: Phase Cycling

• With TR=5ms, where are signal nulls if $\alpha$ sign is alternated?
  - 100 Hz, -100 Hz

• What if $\alpha$ sign is constant?
  - 0 Hz, -200 Hz, 200 Hz
Balanced SSFP Abdomen Example

Alternating RF

Combined Acquisition
Matrix Solutions

Apply 3x3 matrix scheme:
\[ M_{k+1} = AM_k + B \]  \[1\]

In steady-state:
\[ M_{ss} = AM_{ss} + B \]  \[2\]
\[ M_{k+1} - M_{ss} = A(M_k - M_{ss}) \]  \[1-2\]

Consider Eigenvector Decomposition:
\[ A = V\Lambda V^{-1} \]
\[ M_{k+1} - M_{ss} = (V\Lambda) V^{-1}(M_k - M_{ss}) \]

At least one eigenvector/value is real.
Others often oscillatory and die out in steady state

Note in [2] \(A\) is mostly rotation, \(B\) is small, so \(M_{ss}\) lies almost along the real-valued eigenvector
bSSFP: Transients and Steady States

- Same periodic steady state
- Transient paths differ based on initial state
Balanced SSFP: Transient

Rotate Reference Frame

\[ m_z \]

\[ m_x \]
Transient: Off-resonance

Viewed along Steady-State Vector

Orthogonal Component
Parallel Component

Orthogonal View

Steady-State
Transient
Steady-State
Transients (General)

- Generally include two components:
  - **Smooth** exponential (useful!)
  - **Oscillatory** (problematic)

- Smooth transient is along steady-state direction
- Manipulate to steady-state direction to avoid oscillations
Review Question: Transients

• If we set the bSSFP flip angle to 0, what are the magnitudes of eigenvalues of the A matrix?

• What is the real-valued eigenvector?

• What is this situation (only precession/relaxation)?

This is a free-induction decay (FID)!

Eigenvalues are $e^{-TR/T2}$, $e^{-TR/T2}$, $e^{-TR/T1}$
Half-TR, $\alpha/2$ Setup

- First RF pulse has half-amplitude
- Pulse applied TR/2 before next RF pulse
- (More complicated schemes exist)

Deimling and Heid, 1994
bSSFP Direction: Some Intuition

- Magnetization aligns to rotation axis
- Rotation $\theta$ includes RF phase-increment
- As $\theta$ approaches $0^\circ$, axis is transverse, signal dies out
- Negative $\theta$ means steady state on -x
bSSFP Steady-State: Summary

• Ellipsoidal distribution: shape given by $T_2/T_1$
• Path depends on flip angle and precession
• Signal very sensitive to resonant frequency