Radial and Projection Imaging

- Sample radial spokes from $-k_{max}$ to $k_{max}$
- Trajectory design considerations (resolution, #shots)
- Reconstruction, PSF and “streak-like” aliasing
- SNR considerations
- Undersampling
- 3D Projection
Projection-Reconstruction (PR) Sequence

\[ RF \]

\[ G_z \]

\[ G_y \]

\[ G_x \]

\[ \sin(\theta) \]

\[ \cos(\theta) \]

Signal
Radial (k=0 outward)

• Similar to Full Projection, but center-out readouts
• Shortest TE (~0) of any sequence
• Low first-moments
• Fastest way to reach high-spatial frequencies
• Impact of delays
• Can do odd/even sampling
• Impact of ramp sampling
Radial (out) Sequence

Often Replace w/ Non-Selective Excitation

\[ \sin(\theta) \quad \cos(\theta) \]

\[ k_y \]
Projection (PR): Design Considerations

- Readout Resolution?
  - Same as Cartesian
- Readout FOV?
  - Same as Cartesian
- Number of angles?
  - \((\pi/2)N_{\text{read}}\) (Full Projection)
  - \(\pi N_{\text{read}}\) (Radial-Out)
- May undersample
Radial/Projection: Recon and SNR

- Usually use gridding
  - Density \((D = \frac{k_{\text{max}}}{k_r})\)
  - Compensate by multiplying by \(1/D = k_r\)
- How does this affect SNR?
  - More samples required to cover a given area
  - Noise variance is altered by gridding reconstruction
  - Noise is colored ("Speckle" or "salt and pepper")
- Efficiency:

\[
\eta = \frac{A}{\sqrt{\int_A D \int_A 1/D}}
\]

\(\sim 0.87\) for Uniform-density projections

See example later...
Projection-Reconstruction PSF / Undersampling

• PSF has a “ring” of aliasing (less coherent)
  • Intuition: No “preferred” direction for coherent peak
• Undersampling tends to result in streak artifacts

From Scheffler & Hennig, MRM 1998
Undersampled PR: Streak Artifacts

• Reduced sampling leads to streaks
• With reasonable undersampling the artifact is often benign

Radial Out - Odd vs Even #Spokes

• Odd N is a half-Fourier trajectory
• Difficult to do homodyne, but quadrature aliasing

Even, Magnitude    Odd, Magnitude

Even, Real Part    Odd, Real Part
Gradient Delay Considerations

- Full projection
  - Global Delay: Shift center
  - Inter-axis: May miss center
- Radial out:
  - Global Delay: Shifts along traj.
  - Inter-axis: Warping of trajectory

\[ G_y \]
\[ G_x \]
Off-Resonance Effects

Radial-out (0 to $\pi$ variation)

Full Projection (-$\pi$ to $\pi$ variation)
3D Projection-Reconstruction

- Encode in $k_x$, $k_y$, $k_z$ ($\theta, \phi$ end-point parameterization)
- Density $(1/k_r)^2$, compensate by multiplying by $k_r^2$
- SNR efficiency now 0.75 (See example later)
- Can undersample more though!

Temporal Radial Patterns

- Reduction R gives a fully-sampled image with R lower resolution
- Vary spatio-temporal resolution
  - “KWIC: k-space weighted image contrast”
- Field map generation by delaying odd vs even lines
- Golden-angle increment:
  - Arbitrary $N_s < N$ has “uniform” angular spacing
Radial and Projection: Summary

- Non-Cartesian, requires gridding reconstruction
- Incoherent undersampling artifact (similar to CS)
- Short TE (and UTE) imaging
- 2D and 3D options
- No phase-encoding ~ can be efficient
- Off-resonance causes blurring
- SNR efficiency loss due to high-density near center, but resampling the center can be advantageous
Spiral

- Flexible duration/coverage trade-off
  - Like radial, center-out, TE~0
  - Low first-moments
- Longer readouts maximize acq window
  - Archimedean, TWIRL, WHIRL
- Variable-density
Archimedean Spiral

- Radius proportional to angle
- Provides uniform density with \( N \) interleaves
- Extreme case: single-shot with \( N=1 \)

\[
k(t) = \frac{N\theta}{2\pi \text{ FOV}} e^{i\theta}
\]

Stopping point:
\[
\theta_{\text{max}} = \frac{2\pi}{N} k_{\text{max}} \text{ FOV}
\]

Challenge is to design \( \theta(t) \) to meet constraints
Archimeden Spiral Design

• Begin with spiral equation: \[ k(t) = \frac{N\theta}{2\pi \text{ FOV}} e^{i\theta} \]

• Differentiate to obtain \( dk/dt \) and \( d^2k/dt^2 \)

• Amplitude limit: \( dk/dt < \gamma/2\pi G_{max} \)

• Slew limit: \( d^2k/dt^2 < \gamma/2\pi S_{max} \)
Solution Options

Approximations for $\theta(t)$ (Glover 1999)

Consider slew-limited and amplitude-limited regions

Solve numerically at each point

Find all limits, use active limit

Can include circuit model easily

(Meyer, King methods ~ 1995)

Both methods allow variable density design (Kim 2003)
Example Spiral Waveforms

K-space Trajectory

Archimedean

|k| linearly increasing

Slew Limit at First

Moments are small at low |k|

Gradient/FOV Limit late-stage

Slew Rates vs time

First Moment vs time

Second Moment vs time
Variable-Density Spiral

- Undersample outer k-space
- Vary spacing (1/FOV) with $|k/k_{\text{max}}|$ or $\theta$
- Increase spacing along trajectory

$$k(t) = \frac{N\theta}{2\pi \text{FOV}(\theta)} e^{i\theta}$$
Spiral Point-Spread Functions

- “Swirl” artifacts from undersampling
- Again, odd/even selection applies
- Variable Density: Less coherent

Variable Density (Brightened)

Even, Magnitude
Odd, Magnitude
Even, Real Part
Odd, Real Part
Spiral Imaging Sequence

RF

$G_z$

$G_y$

$G_x$

Signal
Off-Resonance Sensitivity

- Uniform-density - $\varphi \sim A|k|^2$
- Spiral usually longer than radial
- PSF broadening
- Off-resonance correction in recon
Spiral Design Trade-offs

1. Choose tolerable readout duration
2. Trade scan time (#interleaves) for spatial resolution
TWIRL / WHIRL

• Faster start, with radial segment

• TWIRL:  Jackson 1990
  • Radial, then Archimedean spiral

• WHIRL:  Pipe 1999
  • Non-archimedean spiral
  • constrained by trajectory spacing
  • *Faster spiral*, particularly for many interleaves
  • whirl.m on website
WHIRL vs Archimedean Spiral

**WHIRL**

K-space Trajectory

K-space vs time

Gradient vs time

**Archimedean Spiral**

K-space Trajectory

K-space vs time

Gradient vs time

25 interleaves, 24cm FOV, 1mm resolution

10% Shorter
Spiral in - Spiral out

- Useful for delayed TE
- Perhaps also for spin echo
- Simply add time-reversed gradient

Glover 2000
3D Methods: Spiral Stack, TPI, Cones

Stack of Spirals

(Irarrazabal, 1995)

Cones, Twisted-Projections

(Irarrazabal 1995, Boada 1997)

• Many variations (spherical stack of spirals)
• Density-compensated cones, TPI
• 3D design algorithms get very complicated
Rewinders and Prewinders

• “Preparatory gradients”
  • Spiral rewinders
  • Phase-encode/rewind
  • spoilers
• Consider 3D rotation again
• Arbitrary path
• Speed always helps efficiency
Approaches to “Rewinder” Design

- Goal: Bring G and k to zero quickly
- Just use trapezoids
- Problem:
  - How much “power” to use on each axis
  - Finite segment method solutions (Meyer 2001)
  - Convex Optimization
• Delays can have a few effects:
  • Mis-mapping of center of k-space, causes a low-frequency “cloud”
  • In outer k-space, delays cause rotation of the image
  • Other effects can be similar to radial
  • Often measure actual gradient waveforms (later I hope)
  • Study in homework!
Spiral Summary

• Flexible duration/coverage trade-off
  • Center-out: TE~0, Low first-moments
• Archimedean, TWIRL, WHIRL, variable-density
• PSF with circular aliasing, swirl-artifact outside
• Off-resonance sensitivity, correct in reconstruction
• Variations: Spiral in/out, 3D TPI, 3D Cones
• Rewinder design
Radial Examples / Corrections

- Corrections to radial SNR efficiency
- Gridding code
- More Examples
Radial Sampling Efficiency

- Number of samples per-unit-area
  - Cartesian requires $N \times N$ samples, Area $4k_{\text{max}}^2$
  - Radial requires $(\pi/2)N \times N$ samples, Area $\pi k_{\text{max}}^2$
  - Radial requires 2x number of samples
- Variance averaged over k-space = average of $k_r$
  - “pill-box minus cone” = 2/3
- Efficiency is $1 / \sqrt{2 \times 2/3} = \sqrt{3}/2 = 0.87$
Radial/Projection: SNR Efficiency ($\eta$)

- Radial density $D = k_{max}/k_r$, extent $[-1,1]$

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Cartesian</th>
<th>2D Radial</th>
</tr>
</thead>
<tbody>
<tr>
<td>k-space area</td>
<td>$(A)$</td>
<td>4</td>
<td>$\pi$</td>
</tr>
<tr>
<td>#samples needed to cover area</td>
<td>$\int_A D$</td>
<td>4</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>Integrated density</td>
<td>$\int_A 1/D$</td>
<td>4</td>
<td>$2\pi/3$</td>
</tr>
<tr>
<td>Efficiency</td>
<td>$\eta = \frac{A}{\sqrt{\int_A D \int_A 1/D}}$</td>
<td>1</td>
<td>$\sqrt{3}/2 = 0.87$</td>
</tr>
</tbody>
</table>
3D Radial/Projection: SNR Efficiency ($\eta$)

- Radial density $D = (k_{max}/k_r)^2$, extent [-1,1]

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<th>3D Radial</th>
</tr>
</thead>
<tbody>
<tr>
<td>k-space volume</td>
<td>$(V)$</td>
<td>8</td>
<td>4/3$\pi$</td>
</tr>
<tr>
<td>#samples needed to cover volume</td>
<td>$\int_V D$</td>
<td>8</td>
<td>$4\pi$</td>
</tr>
<tr>
<td>Integrated density compensation</td>
<td>$\int_V 1/D$</td>
<td>8</td>
<td>$4\pi/5$</td>
</tr>
<tr>
<td>Efficiency</td>
<td>$\eta = \frac{V}{\sqrt{\int_V D \int_V 1/D}}$</td>
<td>1</td>
<td>$\sqrt{5}/3 = 0.75$</td>
</tr>
</tbody>
</table>
Radial-Outward Design Example

- Desire 0.5mm resolution, 20cm FOV
- How many radial spokes do we need (fully sampled)?
- What is the minimum readout duration?

- $k_{\text{max}} = \frac{1}{2 \times 0.5\text{mm}} = 1\text{mm}^{-1}$
- $\Delta k = \frac{1}{\text{FOV}} = 0.05\text{cm}^{-1}$ or $5\text{m}^{-1}$
- Circumference is $6283\text{m}^{-1}$.

- $\frac{6283}{5} = 1257$ spokes
- $\left(\gamma/2 \pi\right)(G_{\text{area}}) = 1000\text{m}^{-1} = 10\text{cm}^{-1}$
- HW1 plot gives ~0.8ms.
Gridding Code: gridmat.m

- Designed to be reasonably fast, but Matlab (readable)
- Uses Kaiser-Bessel interpolation kernel (precalculated)
- For each k-space sample $M(k)$:
  - Build a “neighborhood” of affected grid points $k_{grid}$
  - Calculate contribution at each grid point:
    - $M(k) \times \text{kernel}(k-k_{grid})$
  - Add the values to a full-size grid
  - No deapodization
gridmat.m

• Inputs:
  • ksp = list of k-space locations, $k_x + ik_y$
  • kdat = data samples, ie $M(k_x, k_y)$
  • dcf = density compensation factors at each k-space location
  • gridsize = size of grid

• Convention:
  • $k$ is in “inverse reconstructed pixels”
  • $|k| < 0.5$
  • Larger gridsize zero-pads image (reduce apodization)
  • Scale ksp smaller to “fill” FOV and interpolate pixels
Density Calculation Options

- Radial Density: $1/k_r$ (2D) or $1/k_r^2$ (3D)
- Spiral: Approximate $DCF = \vec{k} \times \vec{G}$
- Gridding: Can grid 1s to k-space points, then sum.
- Iterative (Pipe): $W_{k+1} = W_k / (W_k ** C)$
  - $W = \text{weights (DCFs)}, \ C = \text{convolution kernel}$
- Voronoi regions (areas/volumes around each point, and calculate area/volume)
Gridding Example (FOV / Res)

- 19-interleaves
- 20 cm FOV
- 1mm Res
- 200x200 matrix

Normalized for gridding to 256x256

Gridded data

Gridded to 512x512 to show FOV

100x PSF showing FOV from 512x512

Scale ksp 1/8

Grid x 8 (pad)

Main lobe: ksp=ksp/8, 4096 grid
Spiral Design Example

• Desire maximum 2π cycles between fat/water at 3T, and 1 mm resolution over 30cm FOV

• What is the maximum spiral duration?

• What is the minimum number of interleaves to achieve this?

\[
\frac{1}{440\text{Hz}} = 2.2 \text{ ms}
\]

• Loop: Design whirl (N, resolution, FOV)

• If duration of g < 2.2ms, decrease N, otherwise increase N (using Bisection)
Spiral Design Example

- For resolution of 1mm, 24cm FOV and 12 interleaves
- How many turns in each interleaf?
- How far does each interleaf “travel?”
- Estimate the duration of each interleaf