Diffusion MRI: Lecture 1 of 2

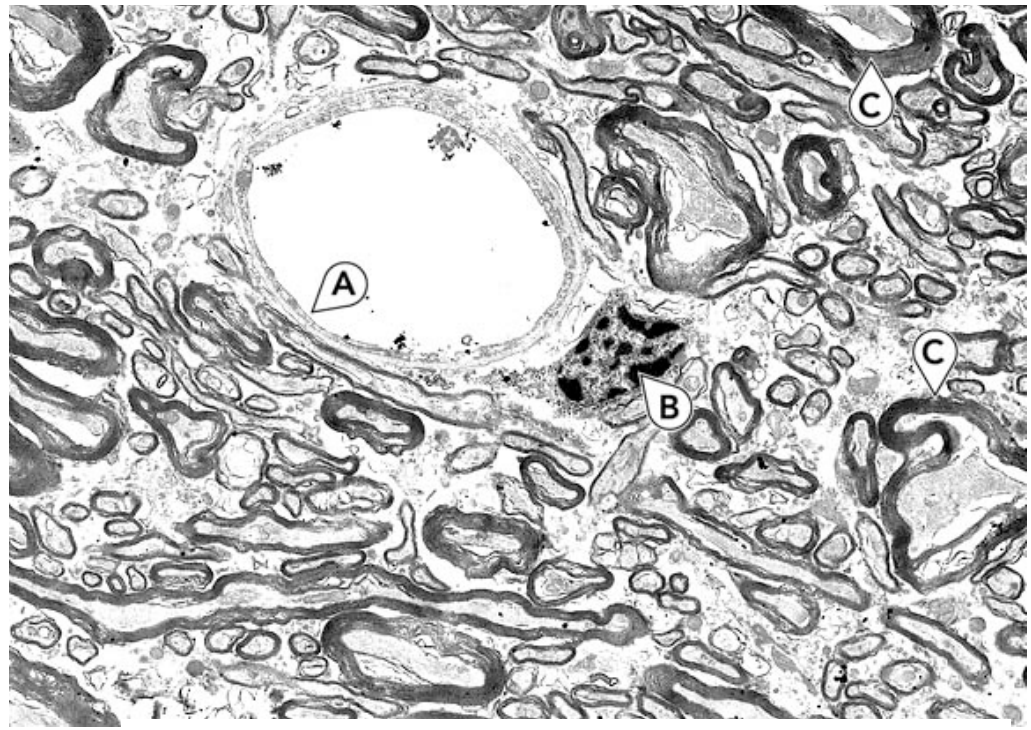
Today

- Diffusion MRI: a marker of tissue microstructure.
- What is diffusion and how do we model it?
- Sensitizing the MRI signal to diffusion.
- Diffusion MRI signal equations.
- Mapping diffusion coefficients.
- Effects of motion.
- Eddy currents.

Diffusion MRI: a marker of tissue microstructure

- Patterns of water diffusion in tissue reflect the tissue microstructure .

- membranes
- permeability of membranes
- macromolecules
- packing density
- compartment sizes



Darwin, M. et. al. 1995.

- Sensitizing the MRI signal to water diffusion is a way to indirectly get information about tissue microstructure.

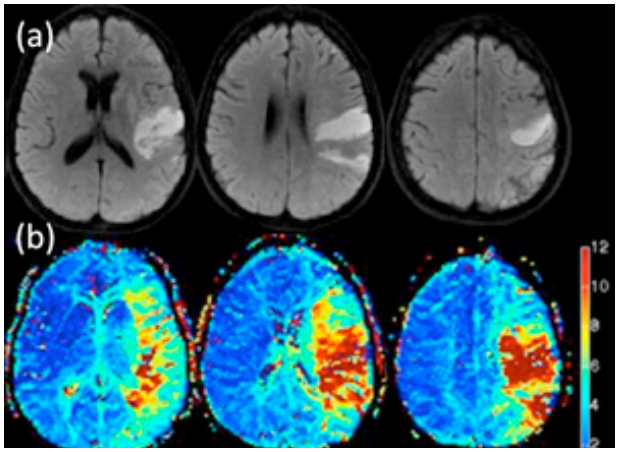
Diffusion MRI in the Brain

Clinical Applications

Stroke

Diffusion

Perfusion



Zaharachuk G, et. al. 2012.

Intracranial Infections

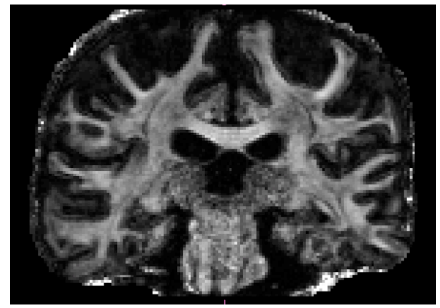
Brain Tumours

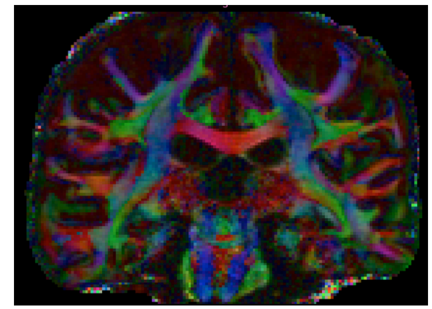
Trauma

Edema

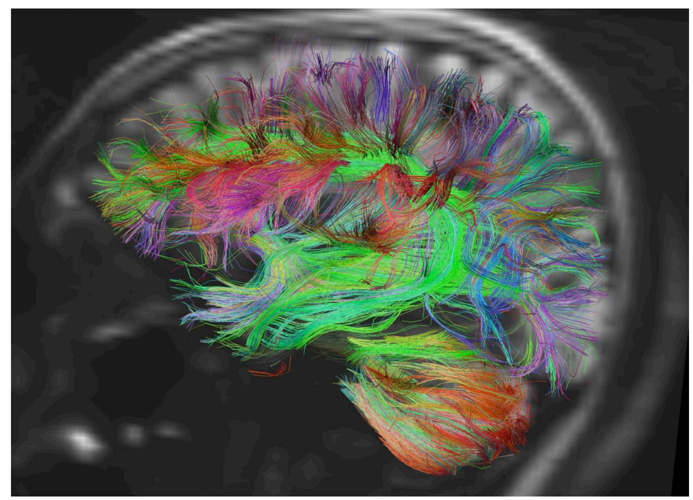
Neuroscience

White Matter Pathways



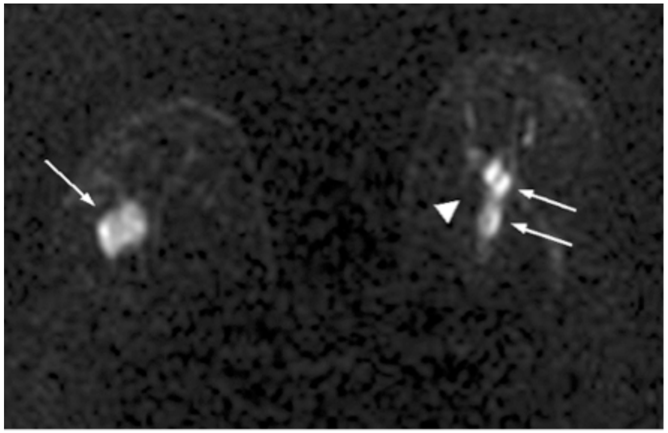


Structural Connectivity



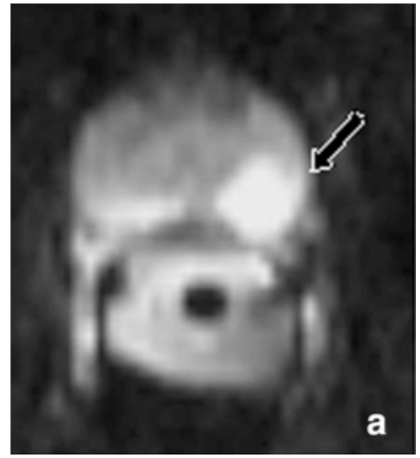
Diffusion MRI in the Body

Breast



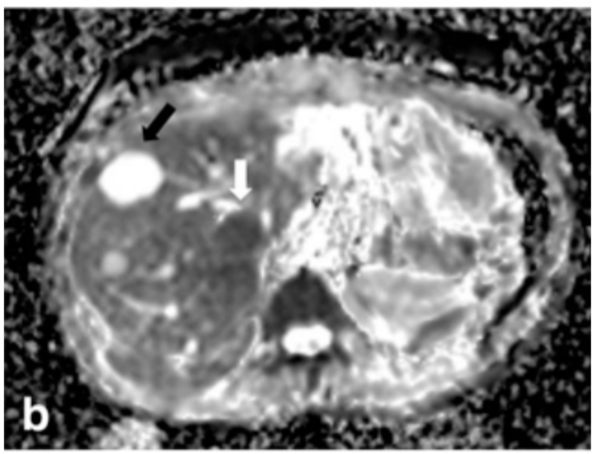
Park M-J et. al. 2007

Prostate



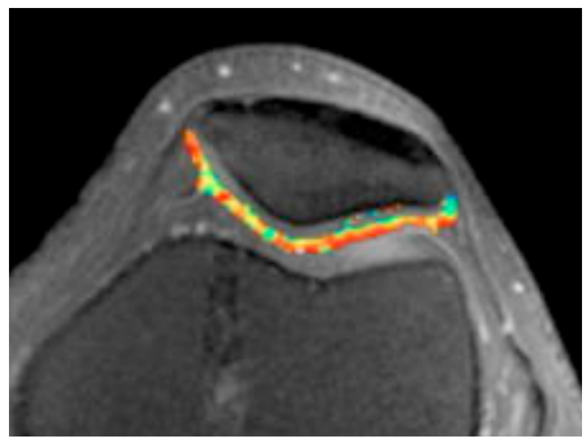
Bonekamp S, et. al. 2012

Liver



Bonekamp S, et. al. 2012

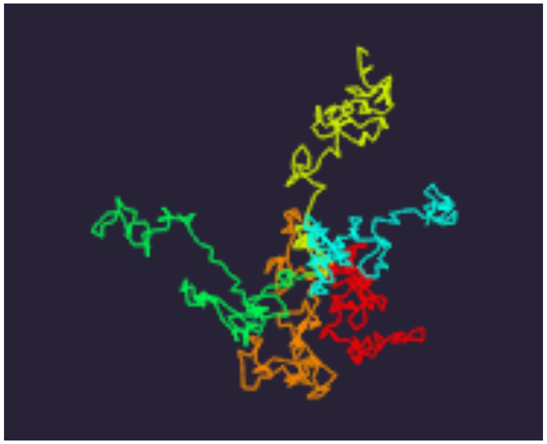
Musculoskeletal



Staroswiecki E, et. al. 2012

What is diffusion?

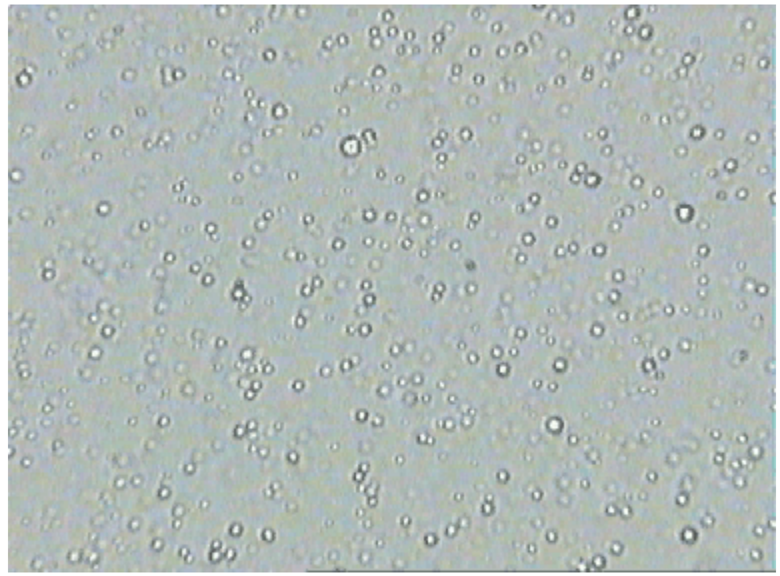
- The result of random collisions between molecules in liquids and gases.
- A form of passive transport that causes mixing but no bulk motion.
- A spontaneous, random process.



Observation of Diffusion

Robert Brown
1828

Brownian Motion



Film Clip Courtesy of
Dave Walker.

Fick's First Law

Adolf Fick

1855

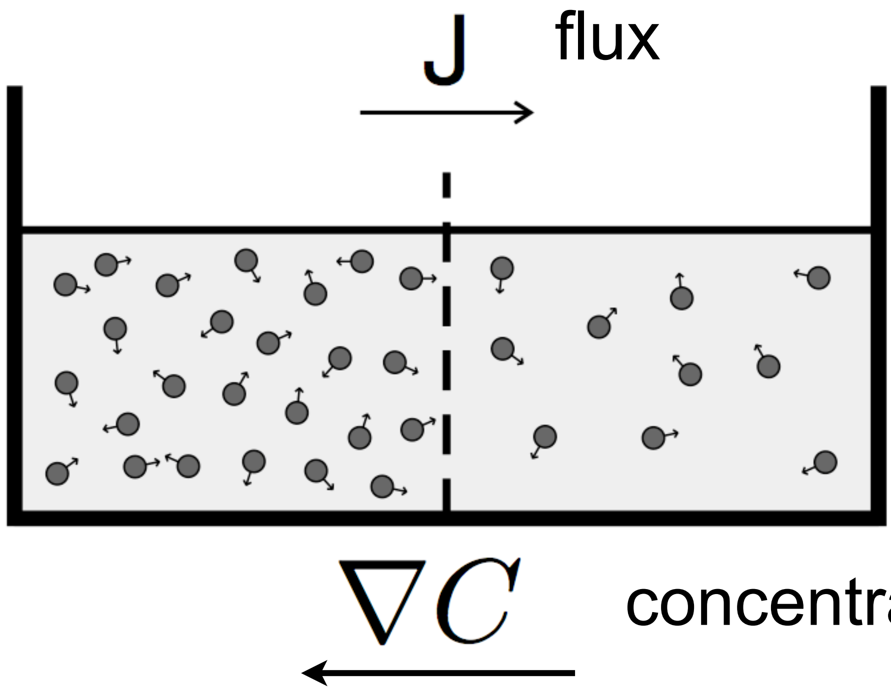


Diagram: Diffusion MRI textbook, Johansen-Berg and Behrens.

Flux of particles is proportional to concentration gradient.

$$J = -D\nabla C$$

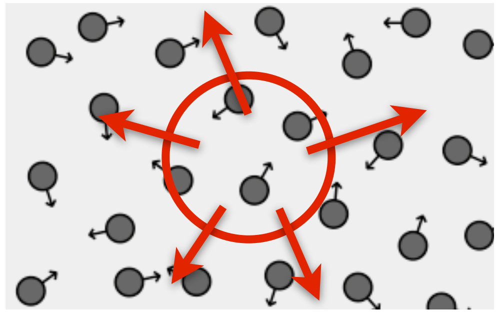
from high concentration to low

diffusion coefficient

Fick's Second Law

Adolf Fick

1855



For an elemental volume:

Rate of change in concentration is equal to the net flux across its boundaries.

$$\frac{\partial C}{\partial t} = -\nabla \cdot J$$

$$\left(J = -D\nabla C \right)$$

Diffusion Equation

Adolf Fick

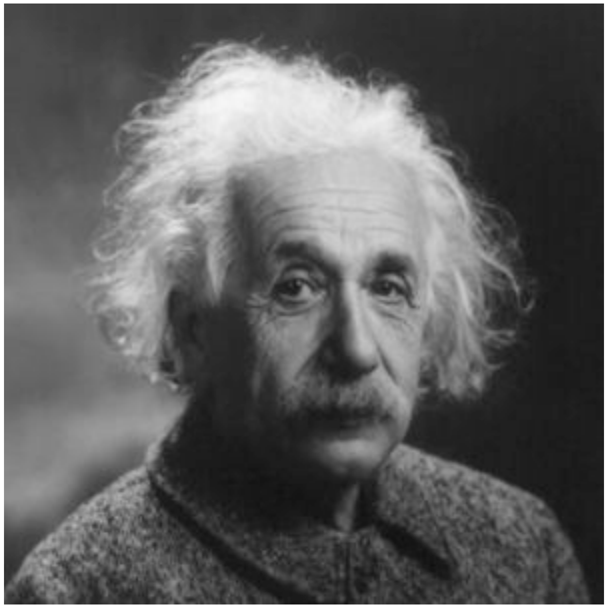
1855

$$\frac{\partial C}{\partial t} = D\nabla^2 C$$

Self-Diffusion and Probabilities

Albert Einstein

1905

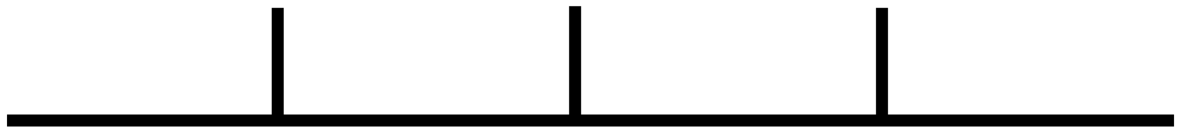


- Fick's Laws describe particles drifting from higher to lower concentration (***mutual diffusion***).
- Einstein applied the same idea to the case without a macroscopic concentration gradient (***self-diffusion***).
- Einstein ascribed Fick's Laws an interpretation based on probabilities.

Diffusion as a Random Walk

At each time-point, τ, a molecule moves a distance "s" in a random direction.

1D Example: $x - \frac{s}{2}$ $\quad x \quad$ $x + \frac{s}{2}$



Particle concentration at point $x = C(x)$

$$J^{+} = \frac{s}{\tau}\frac{C(x - \frac{s}{2})}{2} \qquad J^{-} = \frac{s}{\tau}\frac{C(x + \frac{s}{2})}{2} = \frac{s}{\tau}\frac{C(x - \frac{s}{2}) + s\frac{dC}{dx}}{2}$$

Net Flux: $J = J^{+} - J^{-} = \frac{s^2}{2\tau}\frac{dC}{dx}$

Einstein Relation

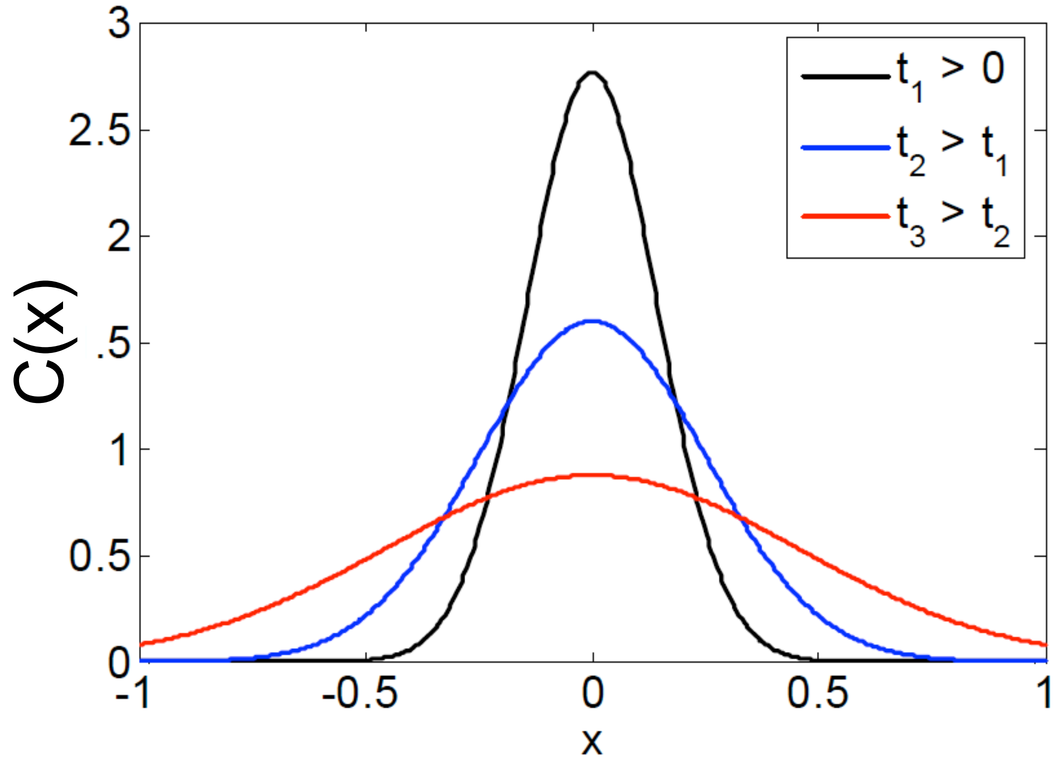
Recall Fick's Law: $J = -D\frac{dC}{dx}$

Thus, $D = \frac{s^2}{2\tau}$

$$s^2 = 2Dt$$

Gaussian Spread of Particles

Mean squared displacement given by the Einstein relation can also be interpreted as the variance of the spread of positions after a period of time.



$$\sigma^2 = 2Dt$$

For n dimensions :

$$\sigma^2 = 2nD\Delta$$

Solving the Diffusion Equation

The Diffusion Equation:

$$\frac{\partial C}{\partial t} = D\nabla^2 C$$

For boundary condition, for all times: $C(x,t) \rightarrow 0$ as $x \rightarrow \pm\infty$

initial condition: $C(x,0) = \delta(x)$

Gaussian Function:

solution:

$$C(x,t) = \frac{1}{\sqrt{2\pi\sigma^2}} exp(-\frac{x^2}{2\sigma^2})$$

with:

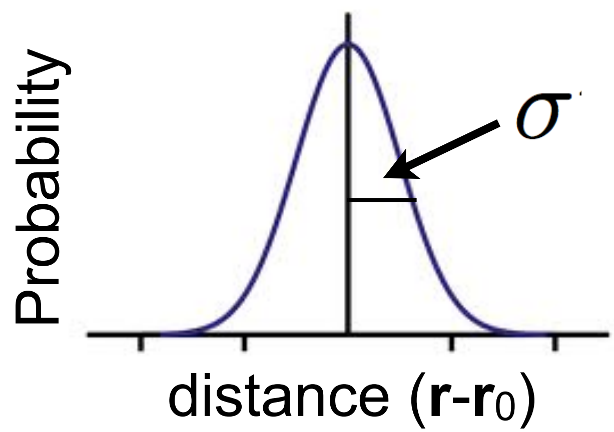
$$\sigma^2 = 2Dt$$

← Einstein Relation

Diffusion Propagator

Gaussian function can be used to determine probability of a particle being displaced from $\mathbf{r}_0$ to $\mathbf{r}$ in time t.

$$P(\mathbf{r}_0|\mathbf{r},t) = \frac{1}{\sqrt{2\pi\sigma^2}} exp(-\frac{(\mathbf{r}-\mathbf{r}_0)^2}{2\sigma^2})$$

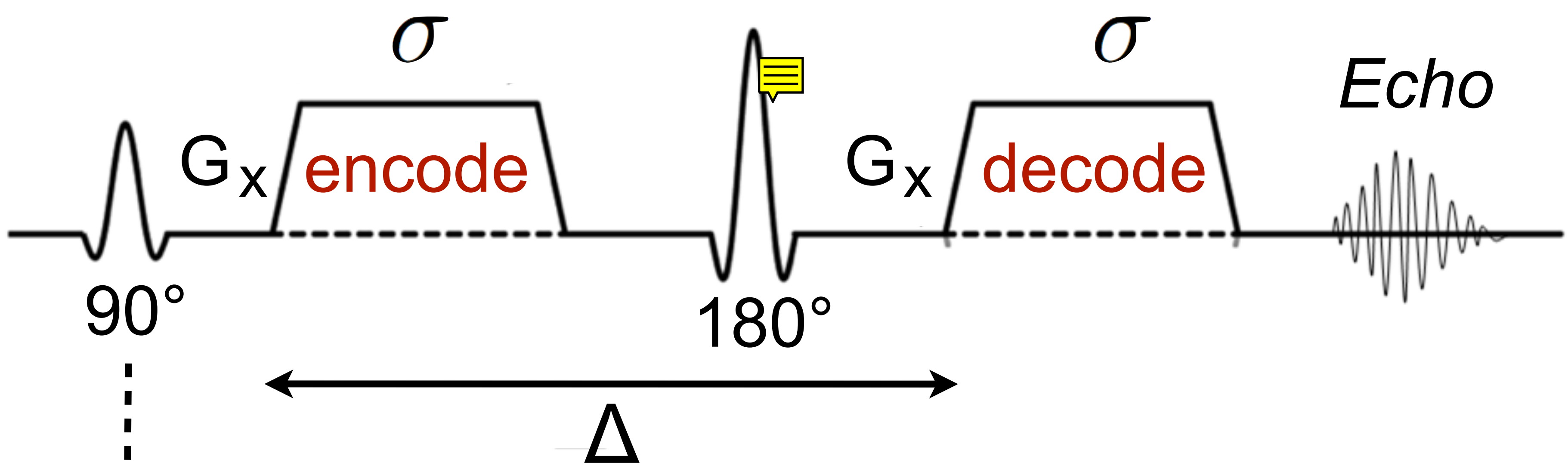


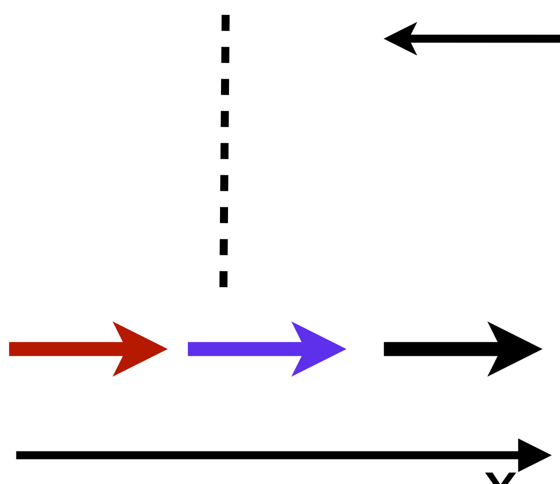
$$\sigma^2 = 2nD\Delta$$

Sensitizing the MRI Signal to Diffusion

Hahn 1950.
Carr and Purcell 1954.

Case #1: Without Diffusion Diffusion-Weighted Spin Echo

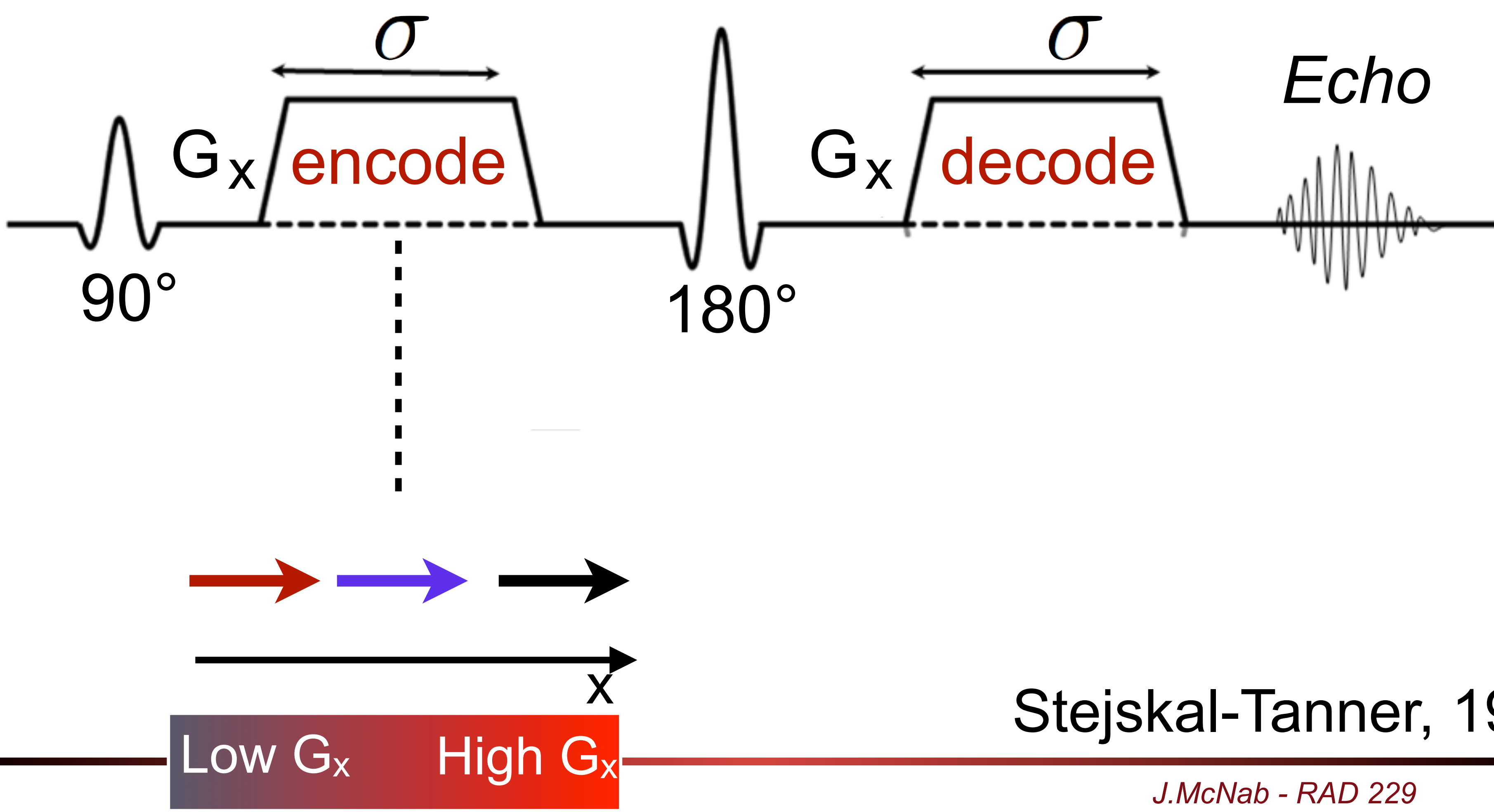




Stejskal-Tanner, 1965.

Sensitizing the MRI Signal to Diffusion

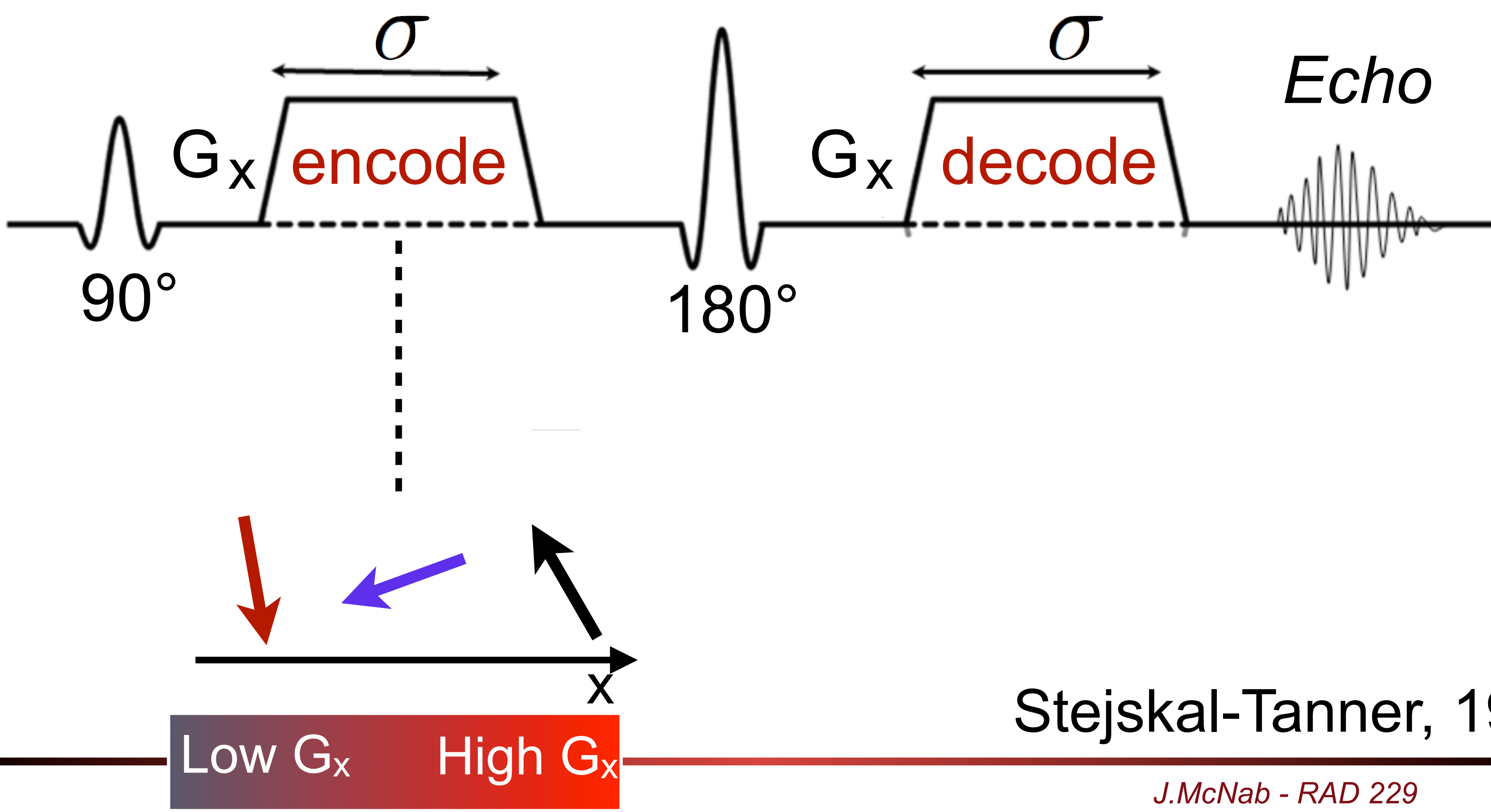
Case #1: Without Diffusion



Stejskal-Tanner, 1965.

Sensitizing the MRI Signal to Diffusion

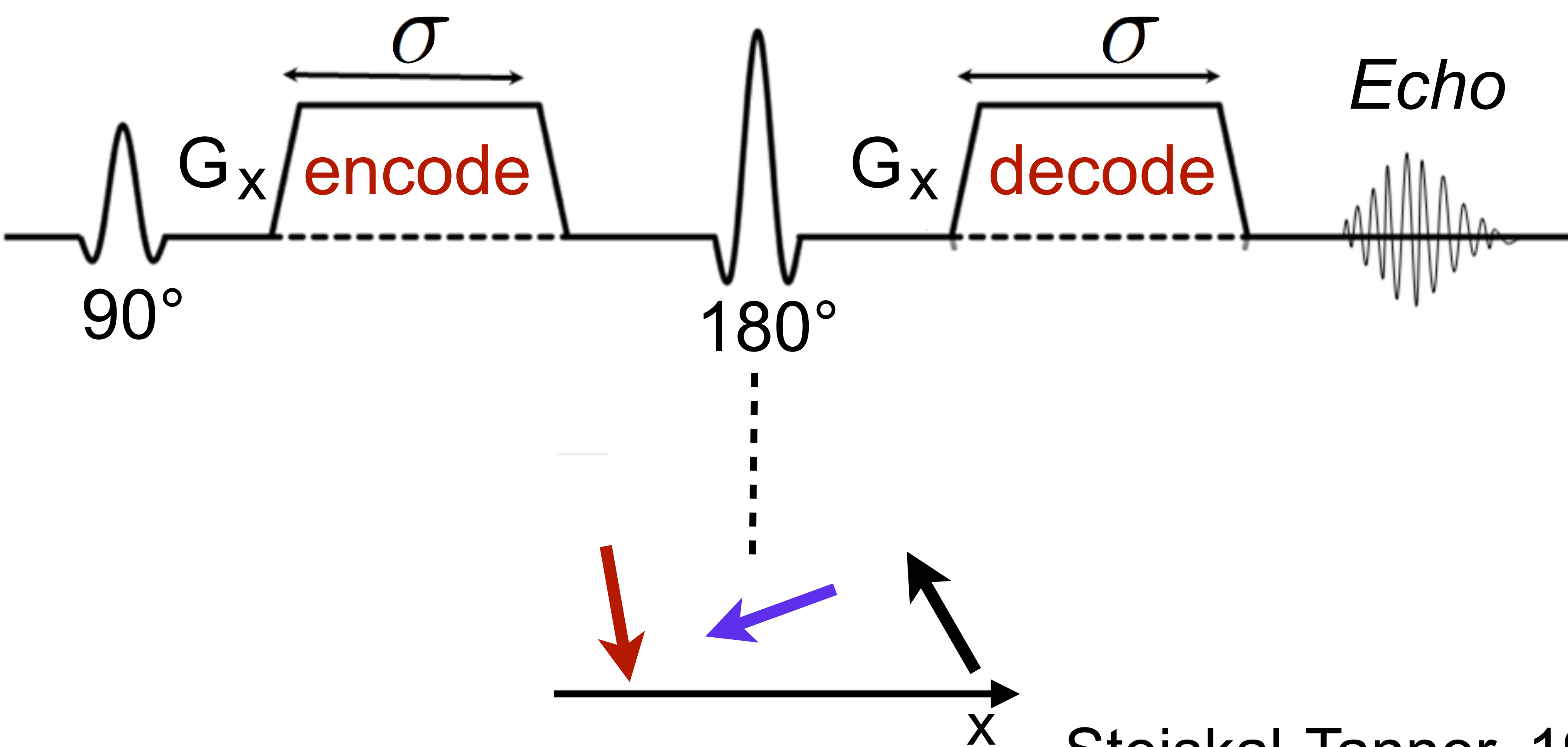
Case #1: Without Diffusion



Stejskal-Tanner, 1965.

Sensitizing the MRI Signal to Diffusion

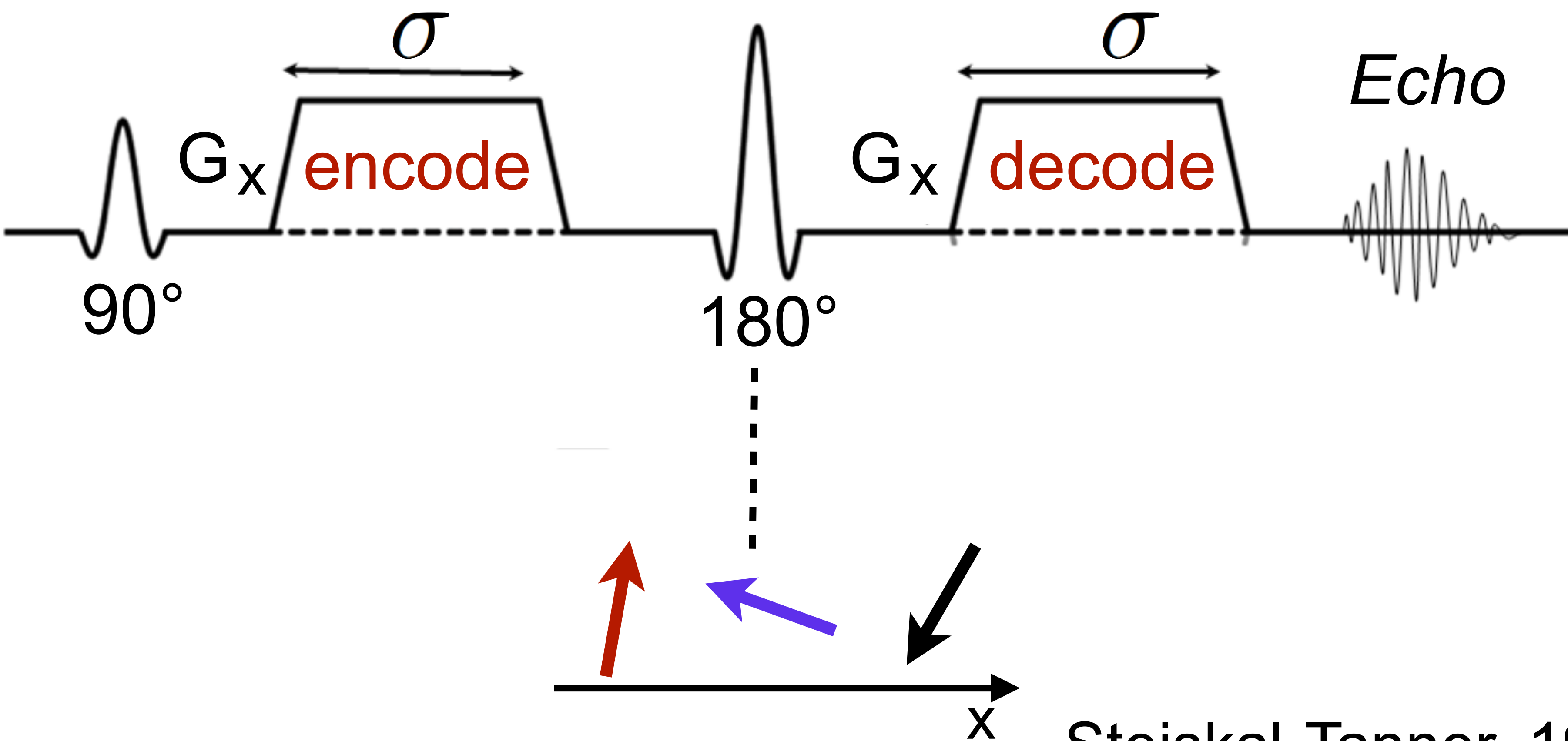
Case #1: Without Diffusion



Stejskal-Tanner, 1965.

Sensitizing the MRI Signal to Diffusion

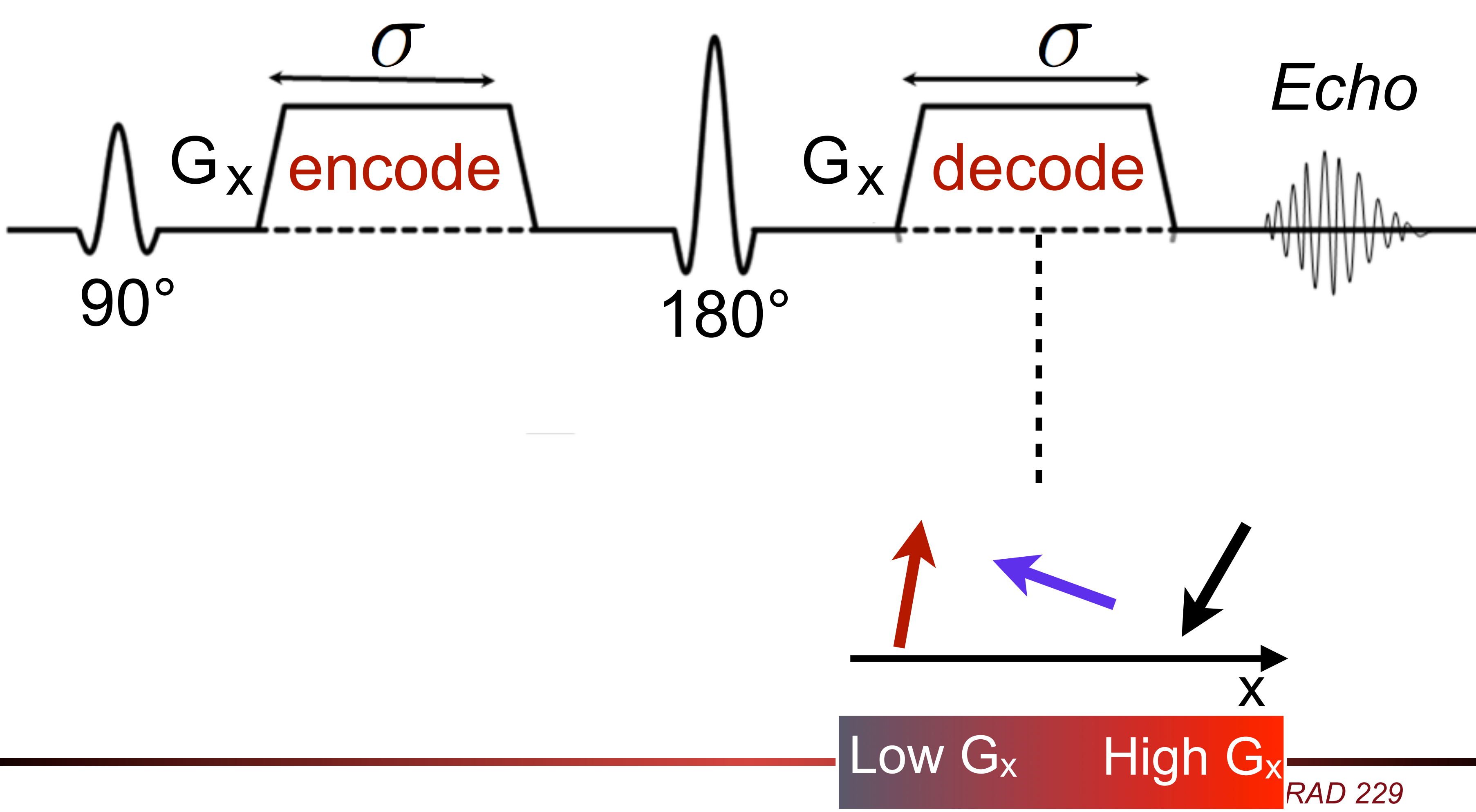
Case #1: Without Diffusion



Stejskal-Tanner, 1965.

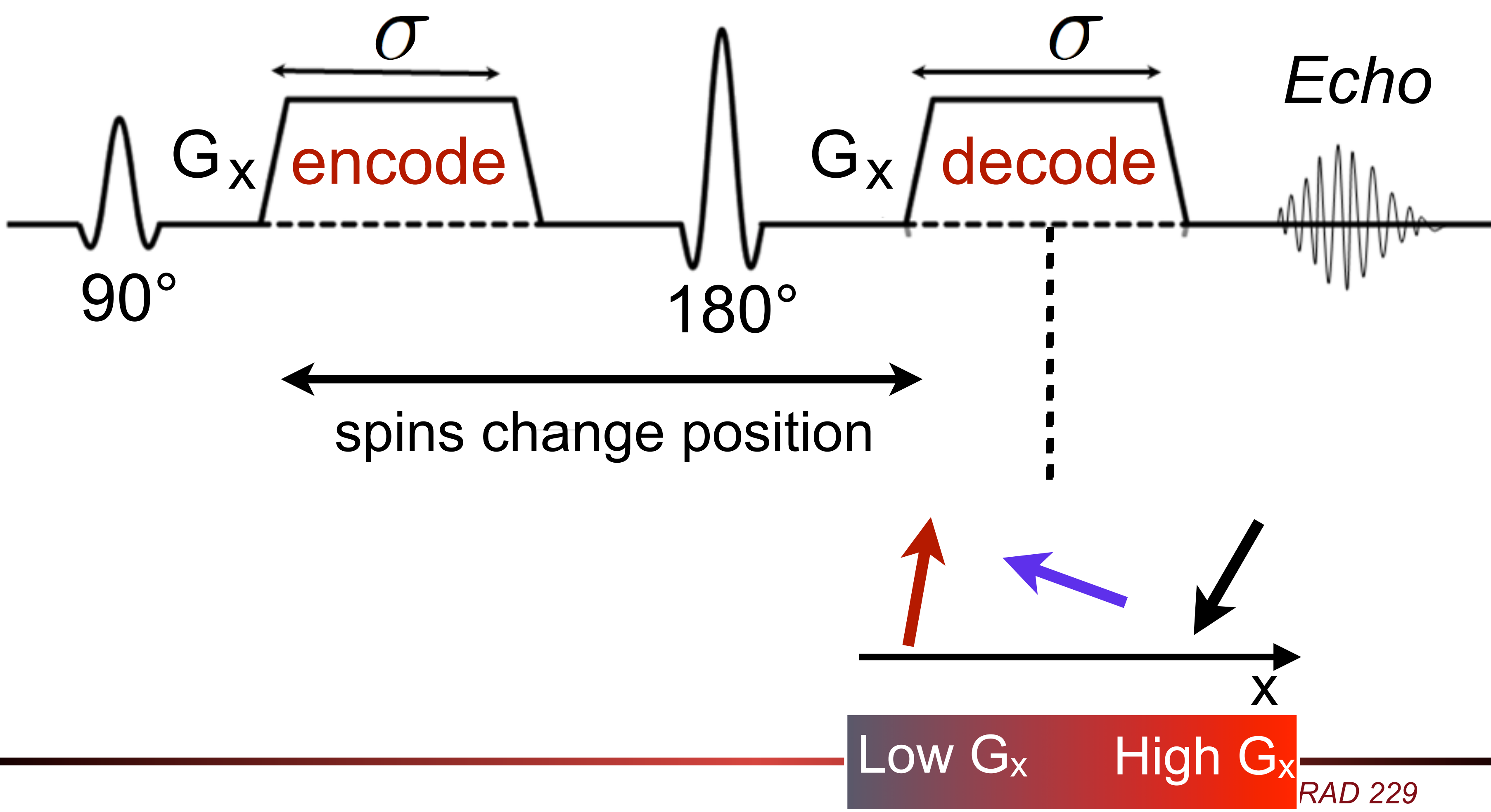
Sensitizing the MRI Signal to Diffusion

Case #1: Without Diffusion



Sensitizing the MRI Signal to Diffusion

Case #1: With Diffusion



The Bloch Equation

$$\frac{d\mathbf{M}}{dt} = \gamma(\mathbf{M} \times \mathbf{B_0}) + \begin{pmatrix} -\frac{M_x}{T_2} \\ -\frac{M_y}{T_2} \\ \frac{M_0 - M_z}{T_1} \end{pmatrix}$$

torque due to B_0

relaxation

The Bloch-Torrey Equation

Torrey H.C. *Physical Review* 1956.

$$\frac{d\mathbf{M}}{dt} = \gamma(\mathbf{M} \times \mathbf{B_0}) + \begin{pmatrix} -\frac{M_x}{T_2} \\ -\frac{M_y}{T_2} \\ \frac{M_0 - M_z}{T_1} \end{pmatrix} + \boxed{D\nabla^2\mathbf{M}}$$

Recall the diffusion equation:

$$\frac{\partial C}{\partial t} = \boxed{D\nabla^2 C}$$

The Bloch-Torrey Equation

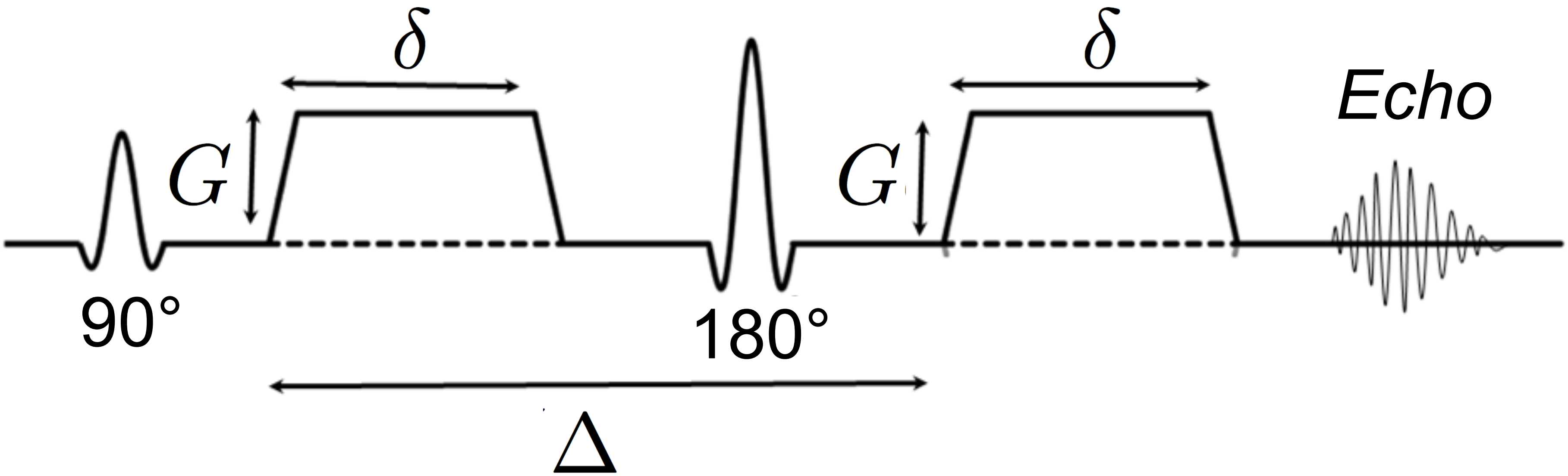
Torrey H.C. *Physical Review* 1956.

$$\frac{d\mathbf{M}}{dt} = \gamma(\mathbf{M} \times \mathbf{B_0}) + \begin{pmatrix} -\frac{M_x}{T_2} \\ -\frac{M_y}{T_2} \\ \frac{M_0 - M_z}{T_1} \end{pmatrix} + D\nabla^2\mathbf{M}$$

$$\mathbf{M}_+ = M_0 e^{\frac{-t}{T_2}} e^{-bD}$$

with $b = \gamma^2 \int_0^{TE} (\int_0^t G(t')dt')^2 dt$

b-value



$$b = \gamma^2 \int_0^{TE} \left(\int_0^t G(t')dt' \right)^2 dt$$

$$b = (\gamma G \delta)^2 (\Delta - \frac{\delta}{3})$$

Measuring the Diffusion Coefficient

b-value: $$b = (\gamma G \delta)^2 (\Delta - \frac{\delta}{3})$$

Non-Diffusion-Weighted Signal: S_0

Diffusion-Weighted Signal: $S(b) = S_0 exp(-bD)$

Diffusion Coefficient: $$D = \frac{ln \frac{S(b)}{S_0}}{-b}$$

Apparent Diffusion Coefficient

- For Diffusion MRI, the diffusion coefficient is referred to as the “Apparent” Diffusion Coefficient (ADC). Why?

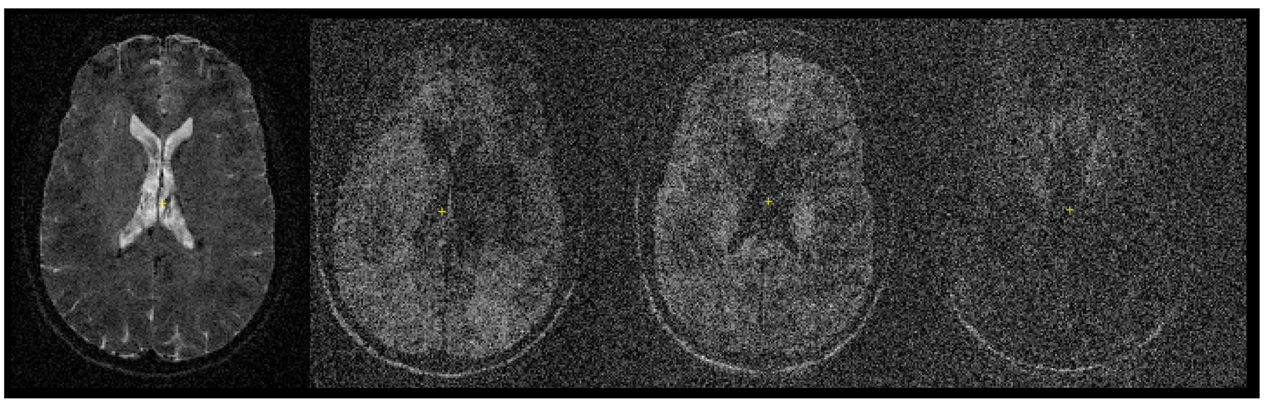
Motion Sensitivity for Diffusion MRI

- Motion effects on diffusion MRI are different compared to other types of MRI pulse sequences.
- For diffusion MRI we are purposely sensitizing the MRI signal to motion of only a few microns.
- Therefore, any motion (bulk or physiological) during the time between the diffusion gradients will cause phase-offsets between read-outs.
    - Phase-encoding across multiple shots/read-outs does not work well.
    - This is why single-shot read-outs such as EPI or spiral are the preferred for diffusion MRI.

Motion Sensitivity for Diffusion MRI

- Phase-encoding across multiple shots/read-outs does not work well.

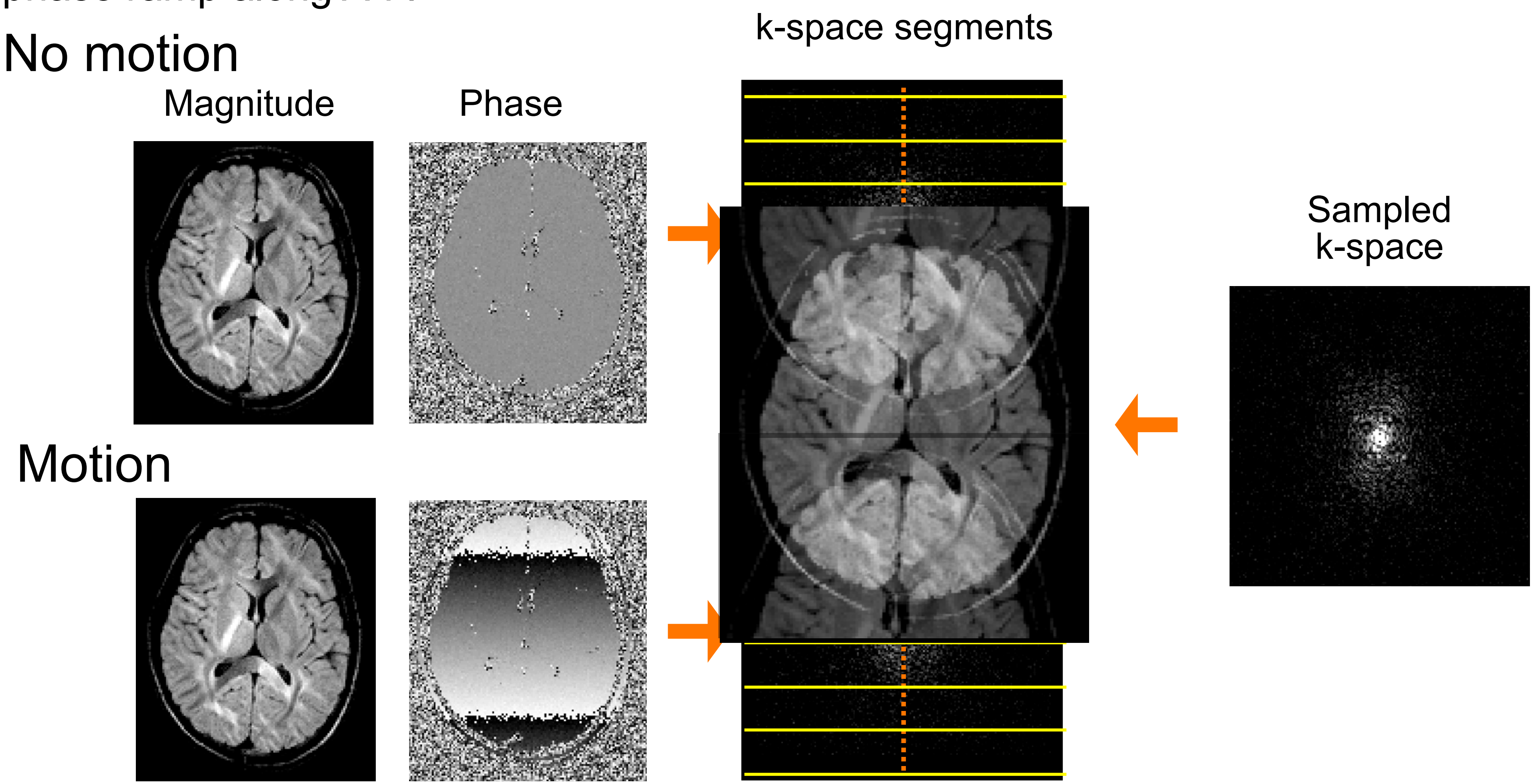
Diffusion-Weighted MRI with a segmented EPI read-out



- This is why single-shot read-outs are the preferred for diffusion MRI.

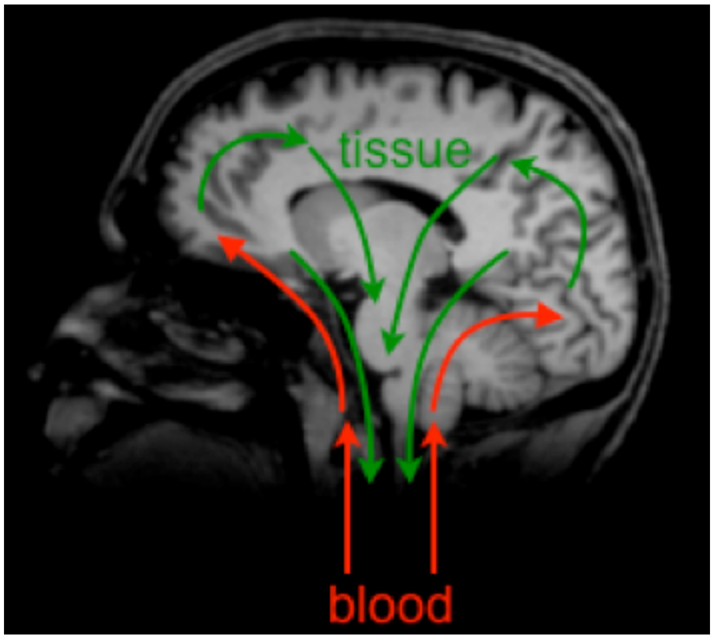
Motion Artifacts for Diffusion MRI

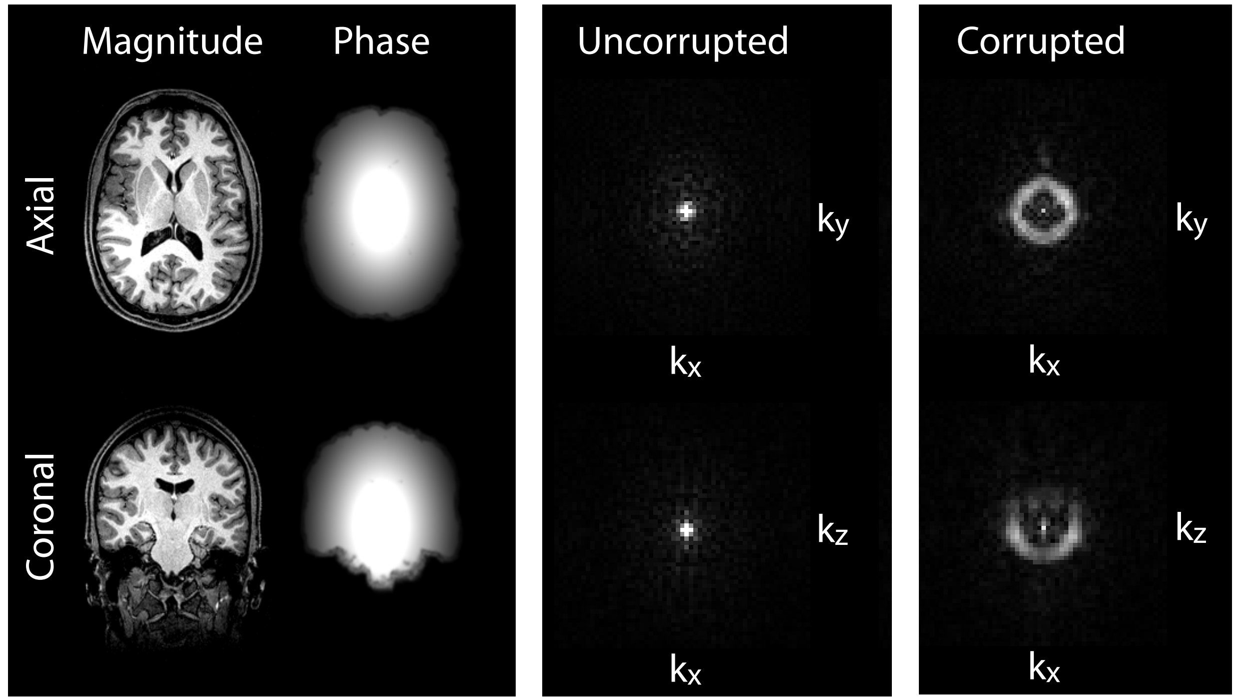
A simple example: Head rotation about through-plane (S-I) orientation (i.e. nodding motion) while diffusion encoding along L-R orientation causes a linear phase ramp along A-P.



Nonlinear Phase Offsets

Non-Rigid Motion

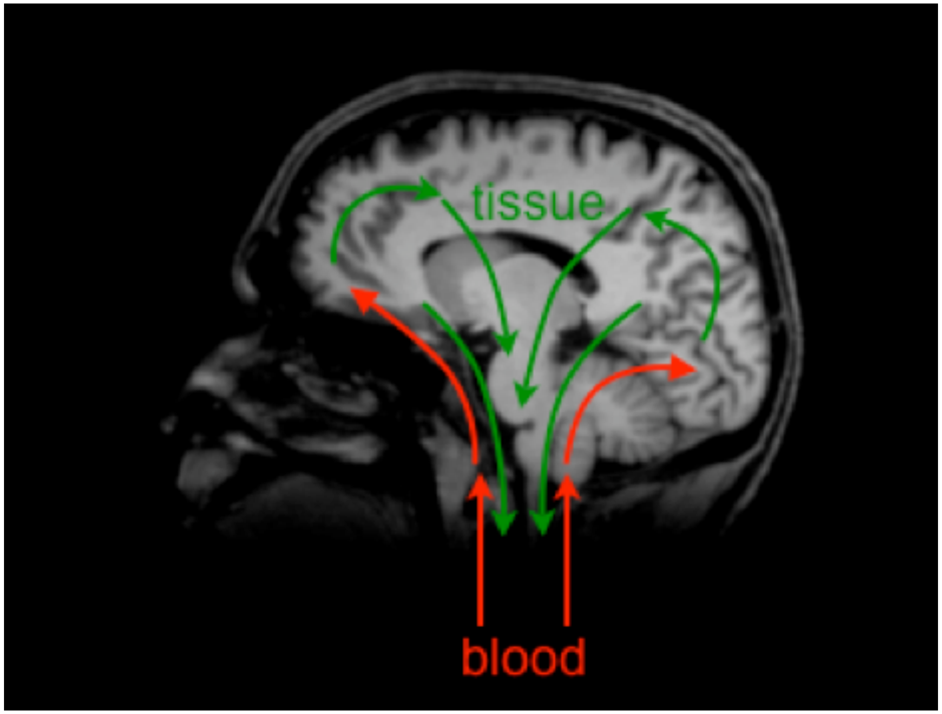




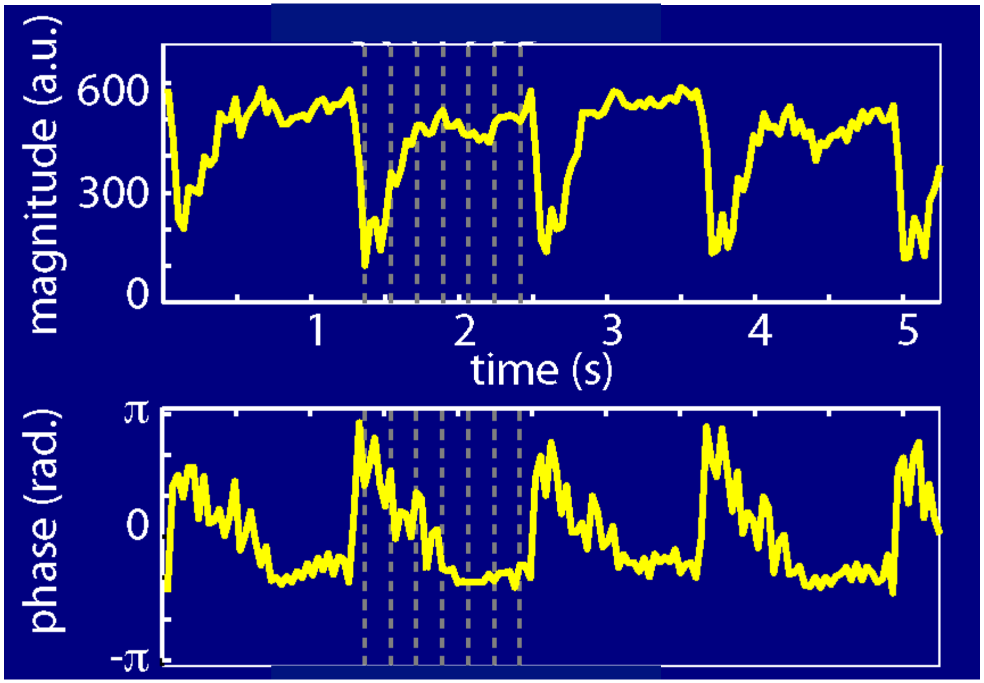
Subject restraints can reduce bulk motion, but
....in the brain, there is significant non-rigid motion from cardiac pulsatility that causes nonlinear phase-offsets.

The Brain is Never Still

Non-Rigid Motion



Time Course of
Diffusion-Weighted Signal



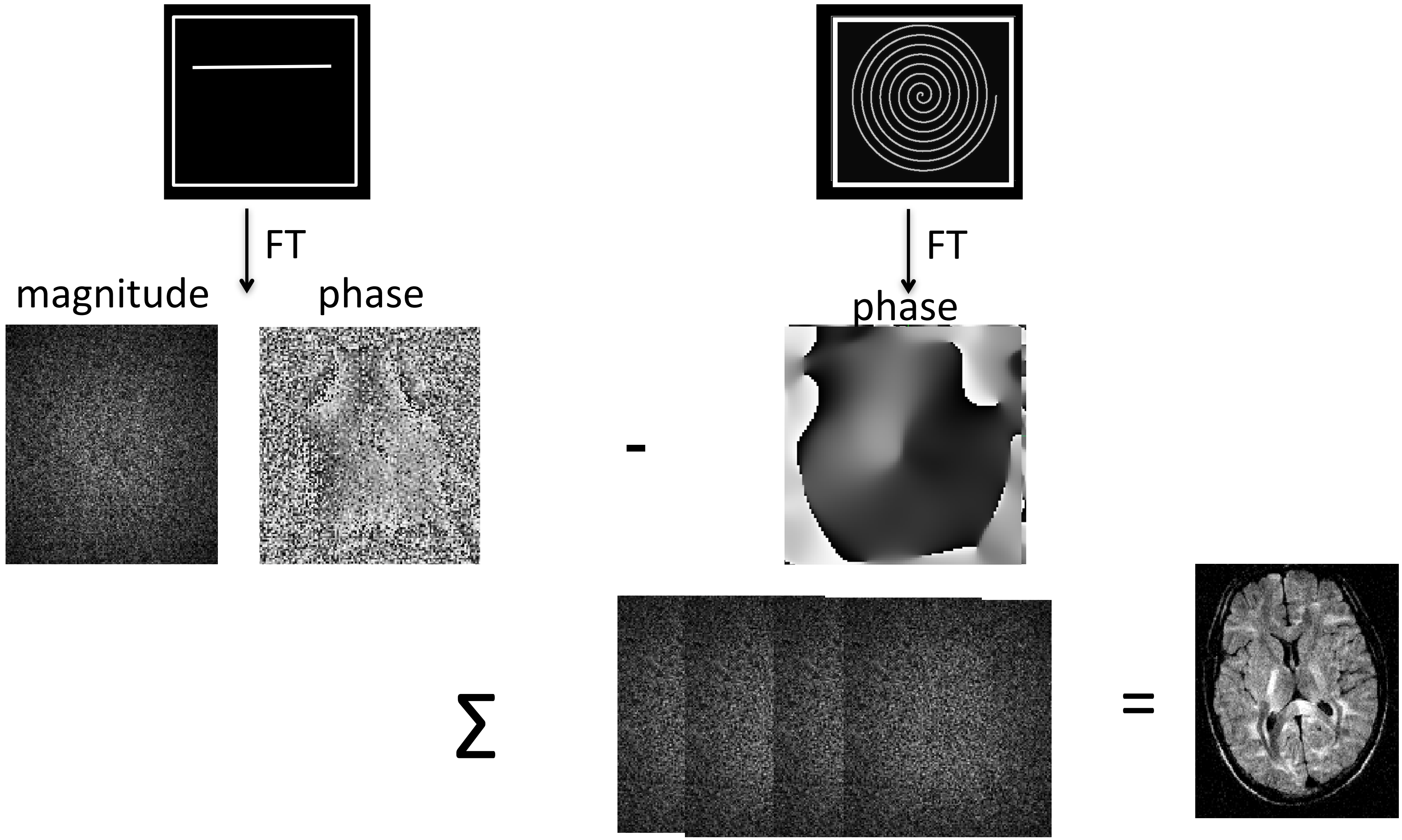
Miller KL. MRM 2003.

Cardiac gating helps, but the brain is never still!

Navigation for Multi-Shot Diffusion

- Navigator: External or Internal (Self-Navigated)
- Examples:
    - ➡ PROPELLER
    - ➡ read-out segmented EPI
    - ➡ variable density spiral
    - ➡ variable density EPI (EPI with Keyhole)
- 0th, 1st or 2nd Order Linear and/or Non-Linear Corrections

Nonlinear Phase Correction



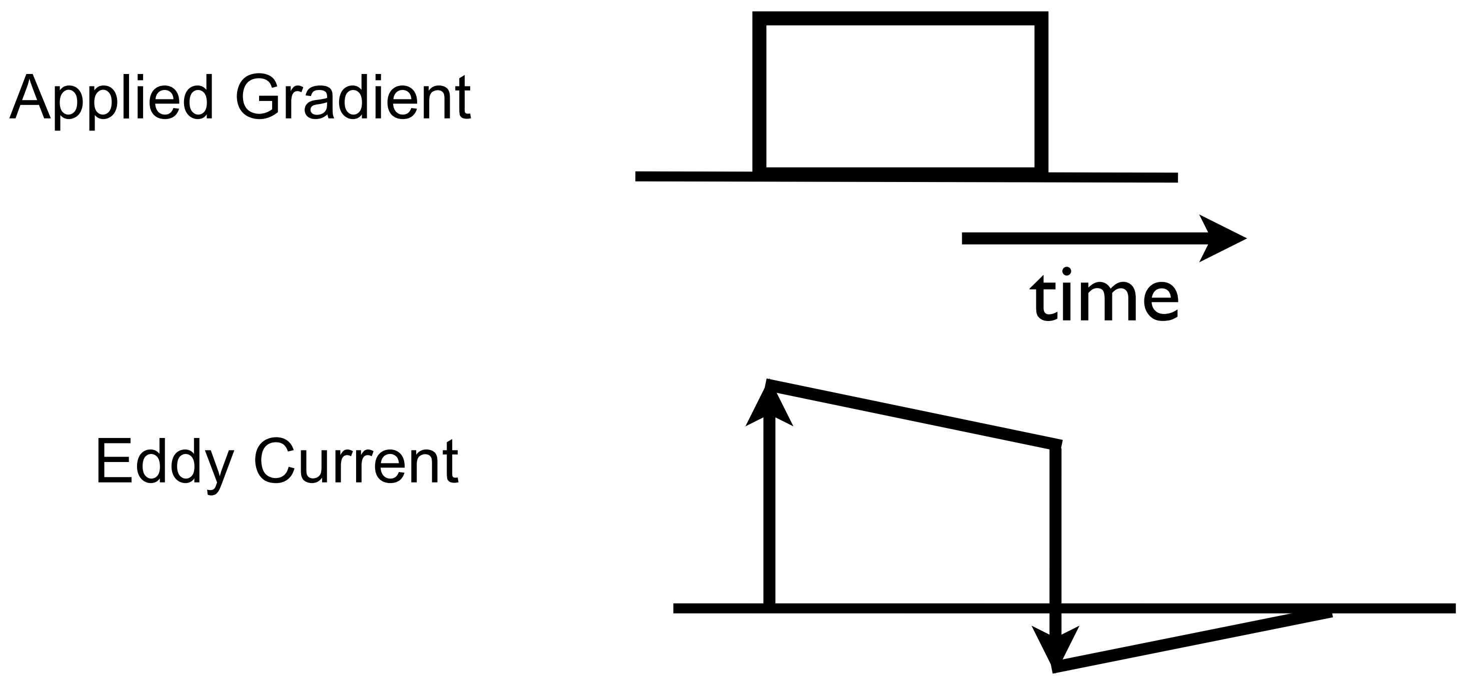
Miller and Pauly, MRM 2003.

Eddy Currents

- The time-varying magnetic fields created by the magnetic gradient pulses in MRI sequences induce currents in the conducting structures within the magnet.
- These induced currents are called eddy currents and create unwanted magnetic fields that are detrimental to image quality.

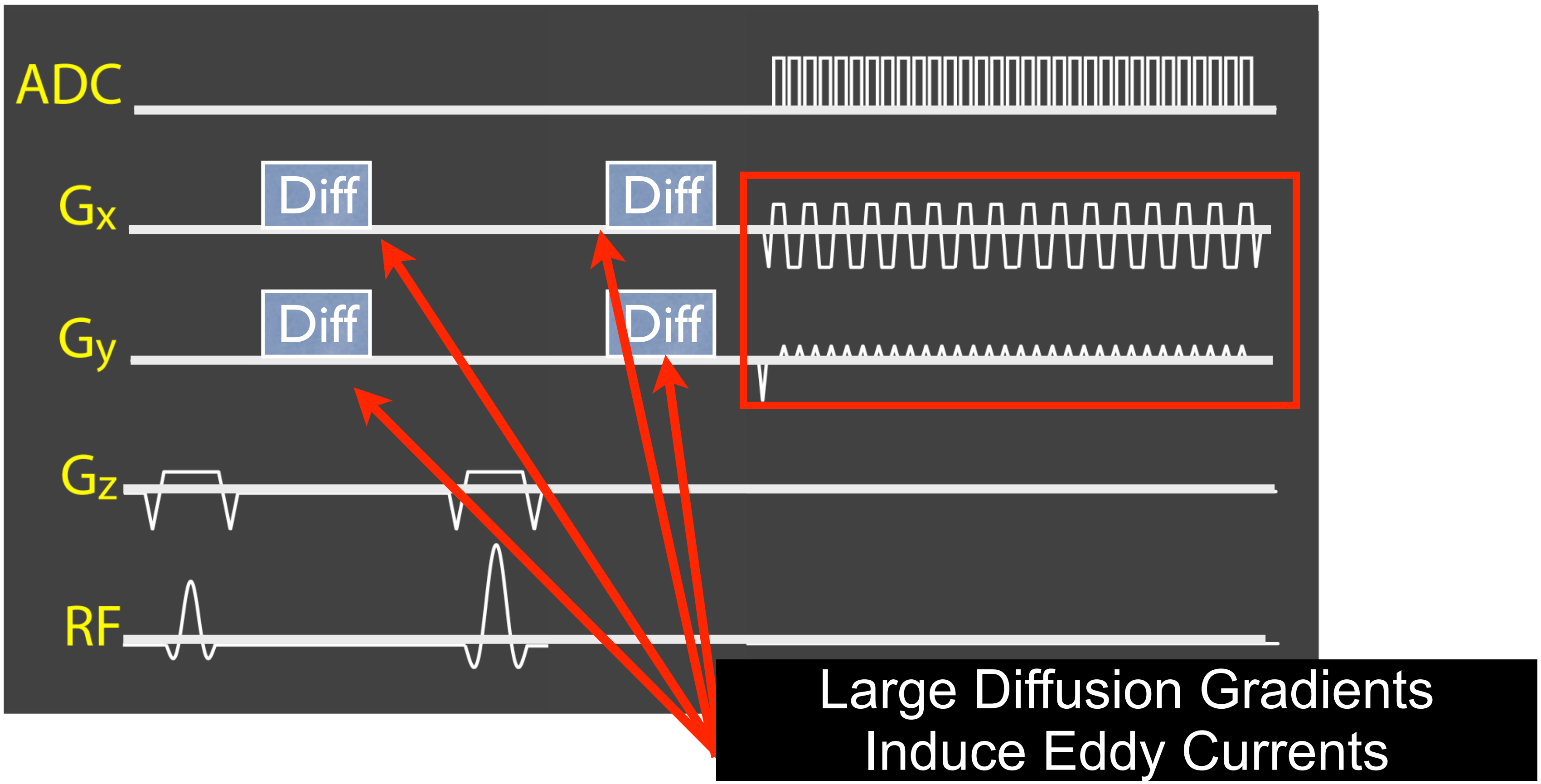
Eddy Currents

- Eddy currents build up during the time varying portion of the gradient waveforms and decay during the constant portions.

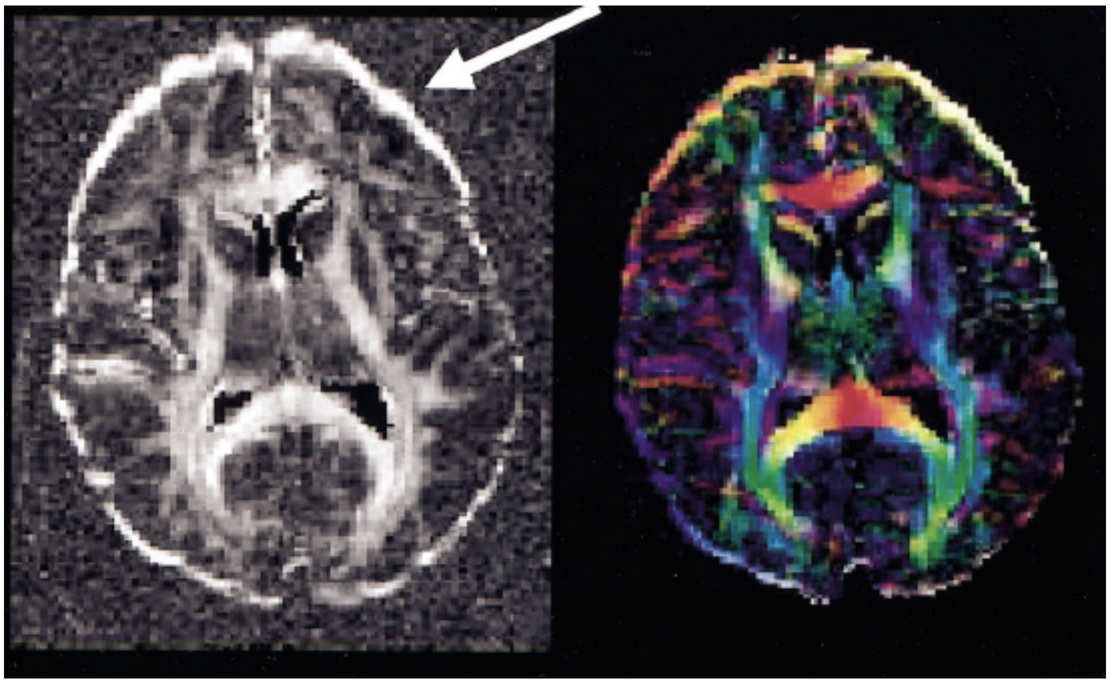


Eddy Currents

- Eddy currents are more significant for diffusion imaging because the diffusion encoding gradients have high amplitude and can cause eddy currents that are still decaying during the read-out.

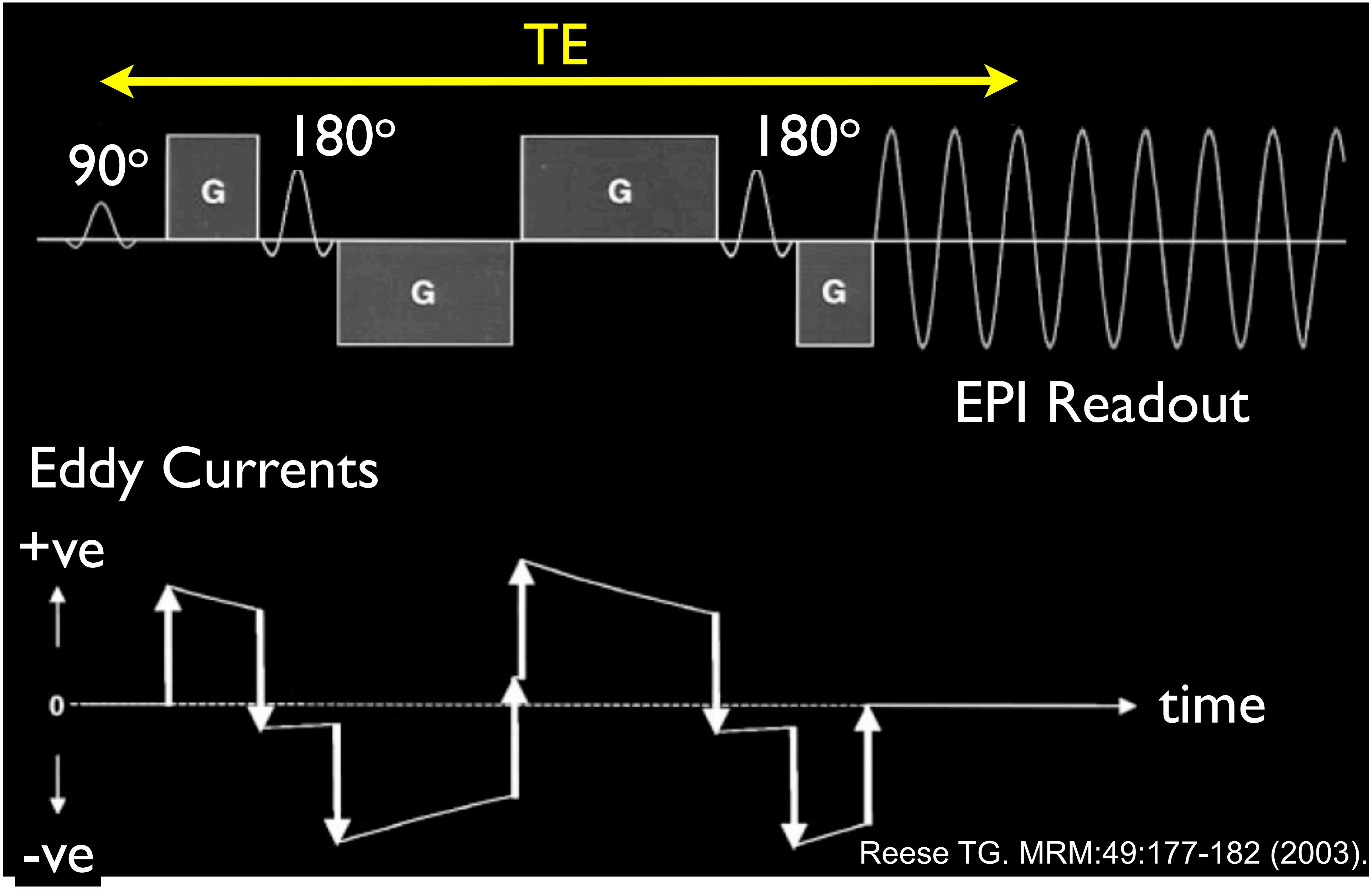


Eddy Currents



Movie courtesy R. Nunes

Twice Refocused Spin Echo



Eddy Current Correction in PostProcessing

Image registration of raw DWIs using FLIRT to correct for stretches and shears in your images.

Can be difficult for images with very low SNR.

Check your results carefully!

Diffusion MRI: Lecture 1 of 2

Today

- Diffusion MRI: a marker of tissue microstructure.
- What is diffusion and how do we model it?
- Sensitizing the MRI signal to diffusion.
- Diffusion MRI signal equations.
- Mapping diffusion coefficients.
- Effects of motion.
- Eddy currents.