Lecture-02B — $k$-space and Imaging

Slice Selection

Daniel Ennis
dbe@stanford.edu
Learning Objectives

• List the three steps required for spatial encoding.
• Explain which B-fields are active during slice selection.
• Describe how gradient fields produce a spatial distribution of the Larmor frequency.
• Appreciate the steps needed to design a slice selective gradient and RF pulse.
Spatial Encoding

- Three key steps:
  - Slice selection
    - You have to pick slice!
  - Phase Encoding
    - You have to encode 1 of 2 dimensions within the slice.
  - Frequency Encoding (aka readout)
    - You have to encode the other dimension within the slice.
Slice Selection

1. RF ($B_1$) Pulse
   • Frequencies matched to slice

2. Slice selection gradient
   • Create spatial gradient of frequency
   • Constant magnitude

3. Slice-select re-phasing gradient
   • Re-phases spins within slice
   • Increases SNR
   • AKA “slice refocusing gradient”
Slice Selection Gradient

\[ \omega = \gamma (B_0 + G_z \cdot z) \]

This frequency excites a slice at position \( z \) when \( G_z \) is turned on.
A shaped RF pulse plus a gradient excites specific spins. Negating the gradient can refocus/rephease.
Slice Selective Excitation

Slice selection requires a simultaneous RF pulse and gradient.

- **x-slice** (yz-plane)
- **y-slice** (xz-plane)
- **z-slice** (xy-plane)
Selective Excitation

- What factors control slice selection?

\[ B_1^e(t) \]  

Pulse envelope function (e.g. \( B_{1,\text{max}} \) and \( \Delta \omega \))

\[ \omega_{\text{RF}} \]  

Excitation carrier frequency

\[ \vec{G} \]  

Gradient amplitude

\[ B_1 \quad t \]

\[ G_x \quad t \]
Gradients Map Frequency to Position

- Gradients produce a spatial distribution of frequencies.

\[ \vec{B}(z) = (B_0 + G_z \cdot z) \hat{k} \quad \bar{\omega}(z) = -\gamma \vec{B}(z) = -\gamma (B_0 + G_z \cdot z) \hat{k} \]

Gradients create a direct correspondence between frequency and spatial position.
Slice Selective Excitation

\[ \Delta \omega = -\gamma (G_z \cdot \Delta z) \]

Excitation Bandwidth

How do you move the slice along ±z?
How does \( \Delta \omega \) and \( \omega_{RF} \) change for a different slice?
Do we usually use \( \omega_{RF} > \omega_0 \)?
Slice Selective Excitation - Example

\[ \Delta \omega = -\gamma (G_z \cdot \Delta z) \]

**Excitation Bandwidth**

\[ \Delta \omega_{RF} = \frac{\text{TBW}}{\tau_{RF}} \]

\[ \text{TBW} = \tau_{RF} \cdot \Delta \omega_{RF} \]

\[ \Delta \omega_{RF} = \frac{4}{1 \text{ms}} = 4 \text{kHz} \]

\[ G_z = \frac{\Delta \omega_{RF}}{\gamma \Delta z} \]

\[ = \frac{4000 \text{Hz}}{42.57 \times 10^6 \text{Hz/T} \cdot \frac{1}{10000} \text{G} \cdot 10 \text{mm}} \]

\[ = 0.94 \frac{\text{G}}{\text{cm}} \]
How do we pick the envelop function?

- $B_1^e(t)$ determines the “slice profile”.
- What is the ideal slice profile?
- Changing the shape (envelope function) of the pulse affects the excitation bandwidth.
- How do we know which shape to use?
  - Small Tip Angle Approximation
    ➡ Slice profile depends on the FT of the envelope function.
Forced Precession with an Applied Gradient

\[ \vec{B} = B^e_1(t) \left( \cos (\omega_{RF}t + \theta) \hat{i}' - \sin (\omega_{RF}t + \theta) \hat{j}' \right) + \left( B_0 + \vec{G} \cdot \vec{r} \right) \hat{k}' \]

Laboratory Frame  
B-field (RF+B_0+G)

\[ \vec{\omega}_{rot} = \vec{\omega}_{RF} = \vec{\omega}_0 = -\gamma B_0 \hat{k} \]  
On-resonance Condition

\[ \vec{B}_{eff} = \vec{B}^e_1(t) \cos (\theta) \hat{i} - \vec{B}^e_1(t) \sin (\theta) \hat{j} + \left( \vec{G} \cdot \vec{r} \right) \hat{k} \]  
Effective B-field

\[ \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff} \]
Equation of Motion

\[ \frac{d\vec{M}_{rot}}{dt} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ \gamma B^e_1(t) \cos (\theta) & -\gamma B^e_1(t) \sin (\theta) & \gamma \vec{G} \cdot \vec{r} \end{vmatrix} \]

Complex system of coupled ordinary differential equations
Forced Precession with an Applied Gradient

\[
\frac{d\vec{M}_{rot}}{dt} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
M_x & M_y & M_z \\
\gamma B_1^e(t) \cos(\theta) & -\gamma B_1^e(t) \sin(\theta) & \gamma \vec{G} \cdot \vec{r}
\end{vmatrix}
\]

Let, \( \omega_1(t) = \gamma B_1^e(t) \) and \( \omega_\vec{G}(\vec{r}) = \gamma \vec{G} \cdot \vec{r} \)

\[
\frac{dM_x}{dt} = M_y \cdot \vec{\omega}_\vec{G}(\vec{r}) - M_z \omega_1(t) \sin(\theta)
\]

\[
\frac{dM_y}{dt} = -M_x \cdot \vec{\omega}_\vec{G}(\vec{r}) + M_z \omega_1(t) \cos(\theta)
\]

\[
\frac{dM_z}{dt} = -M_x \omega_1(t) \sin(\theta) - M_y \omega_1(t) \cos(\theta)
\]

Complex system of coupled ordinary differential equations
Small Tip Angle Approximation

\[ M_z(t) \approx M_0 \]

This alone uncouples the ODEs.

\[ \frac{dM_x}{dt} = M_y \cdot \vec{\omega}_G(\vec{r}) - M_0 \omega_1(t) \sin(\theta) \]
\[ \frac{dM_y}{dt} = -M_x \cdot \vec{\omega}_G(\vec{r}) + M_0 \omega_1(t) \cos(\theta) \]
\[ \frac{dM_z}{dt} = 0 \]

System of coupled ordinary differential equations

\[ M_{xy}(\vec{r}, t) = iM_0 e^{-i\omega(\vec{r})\tau_{RF}/2} \int_{-\tau_{RF}/2}^{+\tau_{RF}/2} \omega_1(t + \tau_{RF}/2) e^{-i\omega(\vec{r})t} dt \]

The small tip angle approximation shows that the excitation profile is the FT of the pulse envelop function.
Small Tip Angle Approximation

\[ M_{xy}(\vec{r}, t) = iM_0 e^{-i\omega(\vec{r})\tau_{RF}/2} \int_{-\tau_{RF}/2}^{+\tau_{RF}/2} \omega_1(t + \tau_{RF}/2) e^{-i\omega(\vec{r})t} dt \]

Slice Profile  Through-plane de-phasing  Fourier transform of the pulse envelop function.

Given a target slice profile (e.g. a rect-function), then the inverse FT informs us about the envelope function.
Post-Excitation Refocusing

\[ M_{xy}(\vec{r}, t) = iM_0 e^{-i\omega(\vec{r})\tau_{RF}/2} \int_{-\tau_{RF}/2}^{+\tau_{RF}/2} \omega_1 (t + \tau_{RF}/2) e^{-i\omega(\vec{r})t} dt \]

Post-excitation refocusing re-phases spins through the slice, thereby increasing signal levels.
How do we use slice selection to generate signals and $k$-space?