Rad229 – MRI Signals and Sequences

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Lecture-05A — MRI Imperfections, Part I
Gradient Non-Linearity

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Learning Objectives

• Understand that we *assume* linear gradient performance.
• Explain the impact of non-linear gradients on slice selection.
• Describe the impact that non-linear gradients have on image encoding.
• Appreciate that gradient distortions can be corrected.
B-Field Assumptions in MRI

• **$B_0$-field is:**
  • Perfectly uniform over space.
    • “$B_0$ homogeneity”
  • Perfectly stable with time.

• **$B_1$-field is:**
  • Perfectly uniform over space.
    • “$B_1$ homogeneity”
  • Temporally modulated exactly as specified.

• **Gradient Fields are:**
  • **Perfectly linear over space.**
    • “Gradient linearity”
  • Temporally modulated exactly as specified.
  • Do not induce Maxwell fields (!).

Gradient fields are not, in general, generated exactly as designed.
Gradients and B-Fields

What happens with gradients are not perfectly linear?

\[ B_{G,z}(x) = G_{x}x \]

- B-field from a gradient
- Varieties with the x-direction
- Points along the z-direction
- x-gradient amplitude
- x-position relative to isocenter

Linear X-gradient

Non-Linear X-gradient
Why are gradients non-linear?

- **Design considerations**
  - Target geometry
  - Limit peripheral nerve stimulation
- **Engineering constraints**
  - Power delivery/dissipation
  - High $G_{\text{max}}$ and high $SR_{\text{max}}$
    - Compromise on linearity
Gradients and B-Fields

\[ B_{G,z}(x) = G_x x \]  
\[ B_{G,z}(y) = G_y y \]  
\[ B_{G,z}(z) = G_z z \]  

Excitation Bandwidth

\[ \Delta \omega = -\gamma (G_z \cdot \Delta z) \]

Linear gradients linearly map spatial positions to received frequencies.
Gradient Non-Linearity

\[ B_{G,z}(z) = [G_z(t) \cdot z] \hat{k} \]

\[ B_{G,z}(z) = [G_z(f(z), t) \cdot z] \hat{k} \]

\[
\begin{align*}
G_z(z) &= k_0 + k_1 z + k_2 z^2 + k_3 z^3 + \ldots \\
G_z(\tilde{r}) &= k_0 + k_1 \tilde{r} + k_2 \tilde{r}^2 + k_3 \tilde{r}^3 + \ldots
\end{align*}
\]

\[ Y_l^m(\theta, \phi) = N e^{im\phi} P_l^m(\cos \theta) \]

Gradients can be non-linear parallel or perpendicular to the applied gradient.
Gradients and B-Fields

\[ \Delta \omega = -\gamma (G_z \cdot \Delta z) \]

**Excitation Bandwidth**

Gradient non-linearity during slice-selection can lead to slice selective distortions.
What if the distortion is a more general function of space?
Gradient Non-Linearity – Distorted Slices

Gradient non-linearity leads to imperfect slice selection.

\[ \Delta \omega = -\gamma (G_z \cdot \Delta z) \]

Excitation Bandwidth

\[ \Delta \omega_{RF} \]

Gradient Non-linearity slice distortion

Slice-B

\[ \omega_0 \]

\[ z_0 \]

\[ \Delta z \]
Which of the following problems arises during slice selection if the gradients are non-linear?

A. Slice position shift  
B. Slice thickness shift  
C. Slice profile distortion  
D. Slice geometry distortion  
E. All of the above.
Gradient Non-Linearity – Distorted Images

What looks unusual in this image?

- Before…
  - Image distortion.
  - Signal pile-up.
- After…
  - Blank image pixels.

Image Courtesy of M.T. Alley & B.A. Hargreaves
Gradients move us through $k$-space…

\[ \vec{k}(t) = \frac{\gamma}{2\pi} \int_0^t G(\tau) d\tau \]
The MRI Signal Equation Maps $M_{xy}$ to $k$-points

$$\vec{k}(t) = \frac{\gamma}{2\pi} \int_0^t \vec{G}(\tau) d\tau$$

$$\int_{Object} M_{xy}^0(\vec{r}) e^{-2\pi \vec{k}(t) \cdot \vec{r}} d\vec{r} = S \left( \vec{k}(t) \right)$$
Mathematically we assume the gradients are linear…

\[ \vec{k}(t) = \frac{\gamma}{2\pi} \int_{0}^{t} \vec{G}(\tau) d\tau \]

\[ \int_{Object} M_{xy}(\vec{r}) e^{-2\pi \vec{k}(t) \cdot \vec{r}} d\vec{r} = S(\vec{k}(t)) \]
Gradient Non-linearity maps the wrong $k$-space…

\[ \vec{k}(t) = \frac{\gamma}{2\pi} \int_0^t \vec{G}(\tau) d\tau \]

\[ \vec{G}(t) = \vec{G}(f(\vec{r}), t) \]

The gradient becomes a function of space.

- We assume the gradients perform linearly.
- We record data and assign it to a $k$-point.
- We didn’t actually sample a spatial frequency.
- We don’t even assign the data to the right $k$-point.

\[ \int_{Object} M_{xy}^0(\vec{r}) e^{-2\pi \vec{k}(t) \cdot \vec{r}} d\vec{r} = \mathcal{S}(\vec{k}(t)) \]
Gradient Non-Linearity and Image Encoding

Which of the following problems arises during image encoding if the gradients are non-linear?

A. In-plane distortion
B. In-plane shifts
C. Intensity modulations
D. Phase errors
E. All of the above.
Gradient Non-Linearity Corrections

\[ Y^m_l(\theta, \phi) = N e^{im\phi} P^m_l(\cos \theta) \]

3D displacement field corrections can be applied to correct for 3D image distortions.
Summary – Gradient Non-linearity

• Basic assumption in MRI is that the z-component of the B-field created by the gradient coils varies linearly with x, y, or z over the FOV.

• Gradient non-linearity causes:
  – Slice distortions, in-plane geometric distortion, and intensity distortions.

• Gradient non-linearity can be measured, modeled, and used for image magnitude and phase correction.
Further Learning...

**Generalized Reconstruction of Phase Contrast MRI:**


To characterize gradient field nonuniformity and its effect on velocity encoding in phase contrast (PC) MRI, a generalized method for simulation and quantification of nonuniform gradient field distortion is presented. In addition to radiation divergence, bending, and cone beam (cone shape), the generalization of the field distortions includes also the effects of the magnet's permanent magnets and superconducting magnets. The method presented here is based on a solution of the free-space Maxwell equations in the presence of the magnetic field gradients. The phase differences of finite-wavelength fields are computed with the help of a recording tape controller. The phase differences are then corrected by a digital signal processor, which requires the acquisition of full three-dimensional information. The generalized reconstruction of gradient fields allows the determination of the magnitude and direction of the distortions and the optimization of the observability of contrast agents. Furthermore, the generalization of gradient field distortions allows the treatment of the nonuniform gradient fields from the Stanford whole-body MR system. (Image reproduced from Magn Reson Med 50:560-569, 2003.)

**Analysis and Generalized Correction of the Effect of Spatial Gradient Field Distortions in Diffusion-Weighted Imaging:**

R. Bammer, M. Markl, A. Barnett, B. Anc, M.T. Alley, N.J. Pelc, G.H. Glover, and M.E. Moseley

Diffusional gradients in MRI is a continues process, and it is important to understand the effects of gradient field distortions on PC-MRI can be corrected by a generalized method. The method presented here allows the determination of the magnitude and direction of the distortions and the optimization of the observability of contrast agents. Furthermore, the generalized reconstruction of gradient fields allows the treatment of the nonuniform gradient fields from the Stanford whole-body MR system. (Image reproduced from Magn Reson Med 50:560-569, 2003.)

**Use of Spherical Harmonic Deconvolution Methods to Compensate for Nonlinear Gradient Effects on MRI Images:**

Andrew Jude, Huiwei Zhao, Gary J. Corn, Graham J. Galloway, and David M. Doddrell

Nonuniform noise in MRI techniques is caused by sampling over a nonlinear phase of the image. If we assume that the field is linear, then the Fast Fourier Transform (FFT) is applicable for noise. However, if we assume that the field is nonlinear, then the FFT is not applicable. In order to achieve this, we need to use a technique that is applicable for nonlinear fields. The technique presented here is based on a method for compensating for nonlinearities in the field. The method presented here allows the determination of the magnitude and direction of the distortions and the optimization of the observability of contrast agents. Furthermore, the generalized reconstruction of gradient fields allows the treatment of the nonuniform gradient fields from the Stanford whole-body MR system. (Image reproduced from Magn Reson Med 50:560-569, 2003.)
How else can gradients perform imperfectly?