Lecture-05A — MRI Imperfections, Part I
Maxwell Fields

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Learning Objectives

• Appreciate the Maxwell fields are unavoidable.
• Understand the spatial variation and direction of Maxwell fields.
• Describe the MRI hardware specifications that impact Maxwell field amplitudes.
• List two or more methods to correct Maxwell fields.
B-Field Assumptions in MRI

• $B_0$-field is:
  • Perfectly uniform over space.
    • “$B_0$ homogeneity”
  • Perfectly stable with time.

• $B_1$-field is:
  • Perfectly uniform over space.
    • “$B_1$ homogeneity”
  • Temporally modulated exactly as specified.

• Gradient Fields are:
  • Perfectly linear over space.
    • “Gradient linearity”
  • Temporally modulated exactly as specified.
  • **Do not induce Maxwell fields (!).**
Maxwell Fields

- When gradients fields are applied, other fields are unavoidably produced as described by Maxwell’s Equations.
  - Generated concurrently with the applied gradient
  - Produce magnetic field components \( \perp B_0 \hat{k} \).
  - Introduces higher-order spatial dependence
- Maxwell fields also known as concomitant fields

\[
\nabla \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0
\]

\[
\nabla \times \vec{B} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
B_x & B_y & B_z
\end{vmatrix} = 0
\]

Gauss’s law for magnetism – There are no magnetic monopoles; magnetic flux through a closed surface is zero.

Ampère’s circuital law - Magnetic fields can be generated by changing electric fields.
Maxwell Fields

\[ \nabla \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \]

\[ \nabla \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = 0 \]

\[ \frac{\partial B_y}{\partial z} = \frac{\partial B_z}{\partial y} \quad \frac{\partial B_z}{\partial x} = \frac{\partial B_x}{\partial z} \quad \frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} \]

The active generation of the gradients \textit{necessarily} generates additional B-field gradients.
Ideally the applied gradients only generate their respective linear B-field contribution.
Concomitant fields contribute a B-field component perpendicular to the $B_0$ and gradient fields.

\[ \frac{\partial B_z}{\partial x} = \frac{\partial B_x}{\partial z} \]

$G_x \cdot x$
Concomitant Fields

- Concomitant fields are:
  A. Highest at isocenter.
  B. Lowest at isocenter.
  C. Only parallel to $B_0$.
  D. Point in all directions.
Concomitant Fields ($B_c$)

- Conventional MRI scanners use symmetric gradients
  - Each gradient’s isocenter coincides
  - Then concomitant fields reduce to the following, which approximates a second-order spatial dependence:

$$B_c(x, y, z) = B_{c,2nd}(x, y, z)$$

Inversely proportional to $B_0$!

$$= \frac{1}{2B_0} \left[ \left(G_xz - \frac{G_zx}{4}\right)^2 + \left(G_yz - \frac{G_zy}{4}\right)^2 \right]$$

Dependent on $G^2$!

Always great than zero!

Concomitant B-fields have quadratic dependence on $G$ and are inversely proportional to $B_0$. [See Bernstein’s book for more.]
Concomitant Fields ($B_c$)

- Net $\vec{B}$ deviates from $\hat{k}$.
  - Negligible effect on:
    - Polarization (too small)
    - Excitation (still $\perp$ to $\vec{B}_1$)
- Additional field components
  - Contribute to spin phase and frequency
    - Space and time dependent
    - Geometric and encoding distortion
Concomitant Field Gradients ($G_c$)

$$G_c(t) \approx \frac{1}{4B_0} \begin{bmatrix} G_z^2(t) & 0 & -2G_x(t)G_z(t) \\ 0 & G_z^2(t) & -2G_y(t)G_z(t) \\ -2G_x(t)G_z(t) & -2G_y(t)G_z(t) & 4G_x^2(t) + 4G_y^2(t) \end{bmatrix}$$

Concomitant Gradient Tensor

$$\vec{G}_c(\vec{r}, t) = G_c \cdot \vec{r}$$

$$\vec{G}(\vec{r}, t) = \vec{G}_{target}(t) + \vec{G}_c(\vec{r}, t)$$

Additional phase and frequency encoding.


\texttt{fov=0.256; \texttt{\textcolor{red}{\% Field-of-view \ [m]}}} \\
\texttt{B0=0.5; \texttt{\textcolor{red}{\% Field strength \ [T]}}} \\
\texttt{gMax=100e-3; \texttt{\textcolor{red}{\% Gradient maximum \ [T/m]}}} \\
\texttt{G_dir=[1 -1 1]; \texttt{\textcolor{red}{\% Gradient direction vector}}} \\
\texttt{pos=[-fov/2 fov/2 fov/2]; \texttt{\textcolor{red}{\% Distance from isocenter \ [m]}}}
Concomitant Fields

- Concomitant fields are least important for:
  A. High-field, high $G_{\text{max}}$ systems.
  B. Low-field, high $G_{\text{max}}$ systems.
  C. High-field, low $G_{\text{max}}$ systems.
  D. Low-field, low $G_{\text{max}}$ systems.
Concomitant Fields in Diffusion Imaging

Concomitant fields produce b-value errors that are direction dependent.

```matlab
% Field-of-view [m]
fov=0.256;

% Field strength [T]
B0=0.5;

gMax=100e-3; % Gradient maximum [T/m]
G_dir=[1 -1 1]; % Gradient direction vector
pos=[-fov/2 fov/2 fov/2]; % Distance from isocenter [m]

% Gyromagnetic ratio [rad/s/T]
gamma=267.52e6;

deltaDiff=12.8e-3; % Diffusion gradient lobe duration [s]
G_dir=[1 -1 1]; % Gradient direction vector

% Calculate the b-value (target~1000s/mm^2)
b_val=2*gamma^2.*(gMax.*G_dir).^2.*((deltaDiff)^3/3)*1e-6; % [s/mm^2]
```

![Graphs showing desired, concomitant, and actual gradients along with b-value plots for different diffusion gradient axes.](image-url)
Concomitant Fields in Diffusion Imaging

Conditions:
0.5T and 100mT/m
b-value=1000s/mm²
Axial slice @ isocenter
Z-position=[0, 100, 200mm]
Concomitant Fields in Diffusion Imaging

Conditions:
0.5T and 100mT/m
b-value=1000s/mm²
Axial slice @ isocenter
Z-position=[200mm]

ADC_{fat}=0.2e-3 mm²/s
ADC_{WM}=1.0e-3 mm²/s
ADC_{GM}=2.0e-3 mm²/s
ADC_{CSF}=3e-3 mm²/s

Concomitant fields create offsets and spatial gradients in estimated ADC if not corrected.

\[ \Delta \text{ADC maps are } \text{ADC(Concomitant)} - \text{ADC(True)} \]
Concomitant Field Correction Methods

- Waveform symmetry about 180° refocusing pulse
  - $B_c$ phase will be refocused
- Phase subtraction
  - Repeated acquisition with gradients off or toggled
- Waveform design
  - Shape the gradients to limit effect
**Further Reading…**

**The Effect of Concomitant Gradient Fields on Diffusion Tensor Imaging**

C. A. Basser, R. M. LeBihan, A. H. Teruel, and G. Buxton

Concomitant gradient fields are transverse magnetic fields that arise from concatenation of slice-select and readout gradients. When these fields are large, they can significantly affect the measured diffusion tensor. To understand their effect, we performed an MRI experiment in which the diffusion tensor was measured in both a normal and a concomitant gradient field. The results showed that the concomitant gradient field has a significant effect on the diffusion tensor, and that this effect can be minimized by careful design of the MRI sequence. The results also suggest that the concomitant gradient field can be used as a tool for studying the effects of transverse magnetic fields on diffusion tensor imaging.

**Concomitant Gradient Terms for Asymmetric Gradient Coils: Consequences for Diffusion, Flow, and Echo-Planar Imaging**


The presence of concomitant gradient terms in asymmetric gradient coils can significantly impact the performance of diffusion, flow, and echo-planar imaging. In this paper, we describe the effects of concomitant gradient terms and provide guidelines for designing gradient coils that minimize these effects. The results show that concomitant gradient terms can cause significant artifacts in diffusion imaging, and that these artifacts can be reduced by careful design of the gradient coil. The results also suggest that concomitant gradient terms can have a significant impact on the performance of echo-planar imaging, and that these effects can be minimized by careful design of the MRI sequence.
What about imperfections that arise from object motion or flow?
Rad229 – MRI Signals and Sequences

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