Rad229 – MRI Signals and Sequences

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Lecture-12C — Gradient Waveform Design
Optimal Gradient Waveform Design

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Learning Objectives

• Appreciate the basic formulation of a constrained optimization problem for gradient waveform design.
• Understand how time-optimal solutions can be obtained by successive binary searches.
• Describe the importance optimal pulse sequence design to SNR-efficient data acquisition or artifact mitigation.
• Discuss the advantages and disadvantages of optimized gradient waveform design for two applications.
Gradient Waveform Design Approaches

- **Analytic expressions**
  - Simple waveform shapes
  - Trapezoids & triangles
  - Not available for many constraint combinations
  - Precise and fast to compute
  - Single-axis

- **Optimization approaches**
  - Enable designing arbitrary shapes
    - Time-optimal/SNR-efficient
  - Can account for numerous constraints
  - Precision & comp. time challenging
  - Multi-axis is straightforward

Conventional PC-MRI Gradients

Time Optimal PC-MRI Gradients

Time-Optimal Gradient Waveform Design for Rapid Imaging

Brian A. Hargreaves, Dwight G. Nishimura, and Steven M. Conolly

Magnetic resonance imaging (MRI) is limited in many cases by long scan times and low spatial resolution. Recent advances in gradient systems hardware allow very rapid imaging sequences, such as steady-state free precession (SSFP), fast imaging employing steady-state acquisition (FESTA), and balanced-FFE sequences (3,4), which produce high signal and contrast, are becoming common as improved gradients allow imaging with minimal artifacts from off-resonance. All of these sequences demand efficient gradient waveform design. Efficient acquisition methods include echo-planar imaging (EPI) (5) and spiral imaging (6). Aside from imaging topography, gradient waveform design includes preparatory waveforms such as phase-encoding, proton, and re-encoding gradients. In rapid sequences with short repetition times (TRs), the design of these latter gradients is an important consideration for improving imaging efficiency, because their duration reduces the image acquisition duty cycle.

In particular, the design challenge is to minimize gradient waveform durations subject to both hardware constraints and sequence constraints, such as the desired gradient slew rates. Numerous previous works have presented different methods to optimize gradients in different situations (7–14). Many of these methods are limited to the focused steady-state free precession (SSFP), fast imaging with steady precession (True-FISP), fast imaging employing steady-state acquisition (FESTA), and balanced-FFE sequences (3,4), which produce high signal and contrast, are becoming common as improved gradients allow imaging with minimal artifacts from off-resonance. All of these sequences demand efficient gradient waveform design. Efficient acquisition methods include echo-planar imaging (EPI) (5) and spiral imaging (6). Aside from imaging topography, gradient waveform design includes preparatory waveforms such as phase-encoding, proton, and re-encoding gradients. In rapid sequences with short repetition times (TRs), the design of these latter gradients is an important consideration for improving imaging efficiency, because their duration reduces the image acquisition duty cycle.

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Optimized Gradient Waveform Design

- Formulate gradient waveform design as an optimization problem.

\[ \arg \min_{\vec{G}(t)} f \left( \vec{G}(t) \right) \]

subject to \( \|c_i \vec{G}(t)\| \leq b_i \) for \( i = 1, \ldots, n \)

- Solve for time-optimal gradient waveforms with successive binary searches.

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**Table:** Constraints and Optimization

<table>
<thead>
<tr>
<th>Category</th>
<th>Constraint</th>
<th>Abbreviation</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardware</td>
<td>Maximum gradient amplitude</td>
<td>( G_{\text{max}} )</td>
<td>mT/m</td>
</tr>
<tr>
<td></td>
<td>Maximum gradient slew rate</td>
<td>( S_{\text{max}} )</td>
<td>mT/m/ms</td>
</tr>
<tr>
<td></td>
<td>Gradient heating(^\text{18})</td>
<td></td>
<td>J</td>
</tr>
<tr>
<td>Pulse Sequence</td>
<td>Gradient moment target(s)(^{13})</td>
<td>( M_0 )</td>
<td>mT/m x ms(^1)</td>
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<td></td>
<td>Diffusion b-value target(^{13})</td>
<td>( b )</td>
<td>mT/mm(^2)</td>
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<td>G(t)=0 during RF and ADC(^{13})</td>
<td></td>
<td>mT/mm</td>
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<tr>
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<td>Peripheral Nerve Stimulation(^{13})</td>
<td>PNS</td>
<td>%</td>
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<tr>
<td></td>
<td>Acoustic noise(^{16})</td>
<td></td>
<td>dB</td>
</tr>
<tr>
<td>Correction</td>
<td>Eddy currents(^{16})</td>
<td>( \epsilon )</td>
<td>mT/m x ms</td>
</tr>
<tr>
<td></td>
<td>Concomitant fields(^{18-20})</td>
<td>( R_c )</td>
<td>mT</td>
</tr>
</tbody>
</table>

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Optimized Gradient Waveform Design

- **Time-optimal flow encoding:**
  - Conventional approaches are not time-optimal
  - Constraints for $G_{\text{Max}}$, $S_{\text{Max}}$, $M_0$, and $M_1$
  - Minimize ramping
  - Can be arbitrarily shaped

Examples from: Rad229_Lecture12B_Flow_Encoding_Gradients.mlx
Optimized Gradient Waveform Design

• Time-optimal flow encoding:

\[ \arg \min_{\tilde{G}(t)} 0 \left( \tilde{G}(t) \right) \]

Feasibility Problem

subject to \( \|c_i \tilde{G}(t)\| \leq b_i \) for \( i = 1, \ldots, n \)

\[
M_{0,1} = M_{0,2} = \int_{t_0}^{TE} G(t) dt = 0
\]

\[
M_{1,1} = \int_{t_0}^{TE} G(t) t dt = \alpha \Delta M_1
\]

\[
M_{1,2} = \int_{t_0}^{TE} G(t) t dt = (1 - \alpha) \Delta M_1
\]

VENC = 150cm/s

How do we solve this?

https://github.com/mloecher/gropt
Diffusion Weighted Spin Echo EPI

Monopolar SE-EPI
- Widely available
- Very fast
- Wasted time before 180°
- Moderate SNR
- Moderate spatial resolution

Can the “wasted time” be more well used?

Aliotta E et al. MRM 2017.
Convex Optimized Diffusion Encoding (CODE)

- Time-optimal gradient design is a convex-optimization problem, for which efficient solution methods exist [1,2].

Convex Optimized Diffusion Encoding (CODE)

\[ F(t) = \int_0^t G(\tau) d\tau \]

\[ b = \gamma^2 \int_0^{T_{Diff}} F(t)^2 dt \]

- 2nd Variation is (+)-definite
- Non-unique function of \( G(t) \)

\[ \beta = \int_0^{T_{Diff}} F(t) dt \]

- 2nd Variation is (-)-definite
- Unique function of \( G(t) \)

Aliotta E et al. MRM 2017.
Monopolar vs. CODE DWI

Monopolar

“Monopolar” CODE

ΔTE

1.5mm x 1.5mm, GRAPPA 2x

TE~65ms

90° 180°

Tₑ

“Monopolar” CODE SE-EPI
- Very fast
- Short diffusion prep
- Shortest possible TE
- Improved SNR

Aliotta E et al. MRM 2017.
CODE Turns 45mT/m into 80mT/m

2x2mm for 300x300mm FOV (40ms EPI readout), 2x GRAPPA
CODE DWI in the Brain

MONO

CODE

Repeated 10x

N=10 Volunteers
3.0T Siemens Prisma
$G_{\text{MAX}}=80\text{mT/m}$
1.6x1.6x3mm Resolution

CODE SNR increased 31%.

Can we add additional constraints?

Aliotta E et al. MRM 2017.
Eddy Currents - What’s the problem?

- Diffusion encoding gradients sensitize the image spin diffusion.
- Large diffusion encoding gradients induce eddy currents.
  - Eddy currents produce B-fields during readout.
  - Eddy current B-fields are diffusion gradient direction dependent.
- Results in image \textit{distortions} and \textit{mis-registration}

Eddy current induced fields persist and distort the intended gradients during imaging (readout).
Eddy Current Induced Distortions

- Eddy currents are produced during gradient ramps and generate fields that decay exponentially\(^1\):
  \[ \varepsilon(\lambda, t) = \frac{dG}{dt} * e^{-t/\lambda} \]

**Goal:** Minimize eddy currents with a small TE penalty.

EN-CODE Simulations

The eddy current spectrum can be estimated by convolving the gradient waveforms with a series of different time constants.

$E(t)$

$\varepsilon(\lambda,t)$

$\lambda = 1 \text{ ms}$

$\lambda_{null}=40\text{ms}$

Comparing Eddy Current Sensitivity

\[ \epsilon(\lambda, t) = \frac{dG(t)}{dt} * e^{-\frac{t}{\lambda}} \]

\[ \epsilon(\lambda, T_{\text{Diff}}) \Rightarrow \text{Eddy Current Spectrum} \]

EN-CODE Framework

**Timing Initialization**

\[ TE_L = TE' \]

\[ TE_U = TE' \]

**Constraints**

<table>
<thead>
<tr>
<th>CODE</th>
<th>EN-CODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulse Sequence</td>
<td>Eddy Currents</td>
</tr>
<tr>
<td>( G(t) = 0 )</td>
<td>( \varepsilon(\lambda, T_{DIFF}) = 0 )</td>
</tr>
<tr>
<td>During RF and EPI</td>
<td></td>
</tr>
</tbody>
</table>

\[ M_n = \int_0^{T_{DIFF}} t^n G(t) dt = 0 \]

\[ n = 0, 1, 2, \ldots \]

**Convex Optimization**

\[ G(t) = \arg\max_G \beta(G) \]

**b-value Check**

\[ b_{\text{max}} < b_{\text{target}}? \]

Yes: \( TE_L = TE' \)

No: \( TE_U = TE' \)

**Termination Condition**

\[ TE_U - TE_L \leq \Delta t? \]

Yes

No

**Solution**

\[ G(t) \]

\[ T_{E_{\text{min}}} = TE_U \]

\[ b = b_{\text{target}} \]

EN-CODE DWI Gradient Waveforms

EN-CODE Waveforms

Eddy Current Spectra

EN-CODE Echo Times

EN-CODE DWI in the Brain

<table>
<thead>
<tr>
<th>MONO</th>
<th>CODE</th>
<th>TRSE</th>
<th>EN-CODE</th>
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</thead>
<tbody>
<tr>
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<td><img src="image2" alt="Image" /></td>
<td><img src="image3" alt="Image" /></td>
<td><img src="image4" alt="Image" /></td>
</tr>
</tbody>
</table>

- DWI b=1000
- TE=80ms
- TE=71ms
- TE=96ms
- TE=78ms

300mm x 300mm FOV, 1.7 x 1.7 x 5.0mm, TR=2300, 2x Parallel Imaging, 15-slices, 5 averages.

Summary

• Understanding the hardware is very important.
• Gradient waveform design is complex.
  – Simple, SNR-inefficient solutions exist.
  – Complex, SNR-efficient solutions exist.
  – Defining the equations is key.
  – Optimization methods can handle complex problems.