Rad229 – MRI Signals and Sequences

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Lecture-13A — Sampling and Timing
Sampling and Point-Spread Functions

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Learning Objectives

• Describe the relationship between k-space sampling and the point-spread function (PSF)
• Explain resolution, FOV and k-space apodization using a PSF
• Understand half-Fourier imaging effects and reconstruction
• Describe k-space modulation effects on the PSF
Sampling & Point-Spread Functions

- PSF = Fourier transform of sampling pattern
  - k-space: Extent, Density, Windowing
  - PSF: Width, Replication, Ripple (side-lobes)

The PSF is the Fourier Transform of the k-space sampling function, with standard Fourier relationships applying.
Basic Cartesian Sampling and PSF

- Finite-extent k-space lines
- Readout filtered
- PSF Features:
  - Non-zero width
  - Ringing
  - Replication in y (phase)
  - No replication in x (readout)

Standard sampling leads to a PSF with replicas in the phase-encode direction — lec13_01.m
Question 1: PSF and Cartesian Sampling

Why is there no PSF duplication in the readout direction?

A. Samples are adequately spaced
B. A/D filter suppresses duplicates
C. Sampling is continuous

B. (Note that any discrete sampling without a filter would lead to replication, and sampling is NOT continuous.)
The “Discrete” sinc function

\[ h(x) = \Delta k \frac{\sin(\pi N \Delta k x)}{\sin(\pi \Delta k x)} \]

- Function of extent
- Shows challenge of low N

At low N, the discrete sinc function has increased ringing compared to the continuous sinc.
Windowing in k-space

- Reduce ringing in PSF
- Slight broadening of PSF
- Fermi filter is common:
  \[
  \frac{1}{e^{x/a} + 1}
  \]
Variable Density Sampling Example

- 2x undersampling
- $\Delta k$ linear with $|k|
- Minor Aliasing
- PSF broadens
Variable Density Sampling: Density Compensated

- Multiply by $\Delta k$
- No PSF Broadening
- Higher ringing (center less dominant)
- Need to apodize
Question 2 - SNR?

What happens to the noise in this sampling/reconstruction scenario?

Higher density at center = averaging
Density compensation at outer = amplification

$$\sigma(k) = w(k)$$
Half Fourier and PSF

• Full k-space is $S_f(k)$, PSF is $\delta(r)$
• Half-k-space trajectory is $S_h(k)$
• $S_h(k)$ is zero for either $k$ or $-k$
  – PSF is $s_h(r)$
  – $S_h(k)$ is real, with even part $0.5 \ S_f(k)$
  – $\text{Real}\{s_h(r)\} = 0.5 \ \delta(r)$

$M(k) \ S_h(k) \iff \ m(r) *[0.5 \ \delta(r) + \text{Imag}\{s_h(r)\}]$

• If $m(r)$ is real, the image is the real-part of $m(r) * s_h(r)$
• How can we remove phase when $m(r)$ is complex?

Half-Fourier sampling leads to a PSF that has a real-valued delta function and an imaginary part
Partial k-space PSF - Contiguous

- Odd component is a step function
- Imaginary PSF is “localized”
  - Assume similar phase across region of significant energy
- Low-resolution phase can be demodulated from image

Contiguous half-Fourier sampling produces a localized imaginary part of the PSF.
Partial Fourier Acquisition and Homodyne Reconstruction

Homodyne reconstruction demodulates low-resolution phase to reconstruct partial-Fourier data — Noll 1991
Question 3: Half k-space Alternatives?

What if we sample even lines on one side and odd on the other?

- Odd component is a step function modulated by
- Imaginary PSF is localized but shifted a half-FOV
- Challenge to demodulate phase
Partial k-space PSF - Random Selection

- Randomly select $S_f(k)$ or $S_f(-k)$
- Odd $S_h(k)$ part is random ±1
- Imaginary PSF is spread out, incoherently
- May or may not help(!)
k-space Modulation

- Many sequences acquire multiple lines with different signal levels:
  - Echo trains: $T_2$ and $T_2^*$ decay over k-space
  - Magnetization-prepped bSSFP, RF-spoiled transients
  - Off-resonance (EPI, Spiral primarily)
  - Temporal signal effects (non-motion):
    - Contrast uptake, inflow, varying $B_0$,
  - PSF is a function of order and signal change
  - Eg: Proton-density-weighted Spin-Echo Train

$k$-space modulation leads to a convolution of the PSF with the Fourier transform of the modulation
View Ordering / Grouping

Sequential

Interleaved

Centric / Center-out

Segmented

Each color is a different “modulation” (echo, time, etc)
Example: Echo-Train + CS + Half-Fourier + Elliptic

- 2D k-space sampling variation ($k_y$-$k_z$ phase encodes)
- “smooth” modulation with echo train
- Random sampling for CS
- Choose trajectories through regions to minimize change (eddy-current)

Sampling patterns can combine goals of half-Fourier, incoherence and smooth modulation — Worters 2011
Summary

• The PSF is the Fourier transform of the k-space sampling pattern
  – k-space extent determines PSF main-lobe width
  – k-space density determines PSF replica spacing
  – k-space apodization determines PSF side-lobe height
  – k-space modulation is a convolution of the PSF with some function

• Variable-density and density compensation affect PSF
• Half-Fourier sampling gives real and imaginary PSF
How do we extend PSF understanding to different dimensions of sampling?