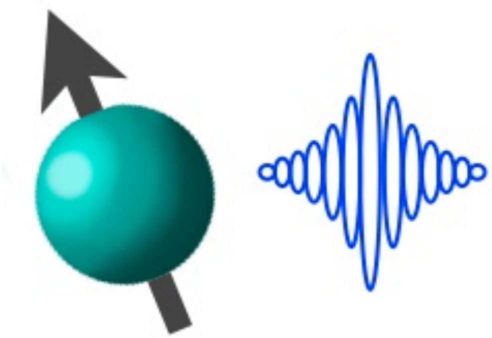
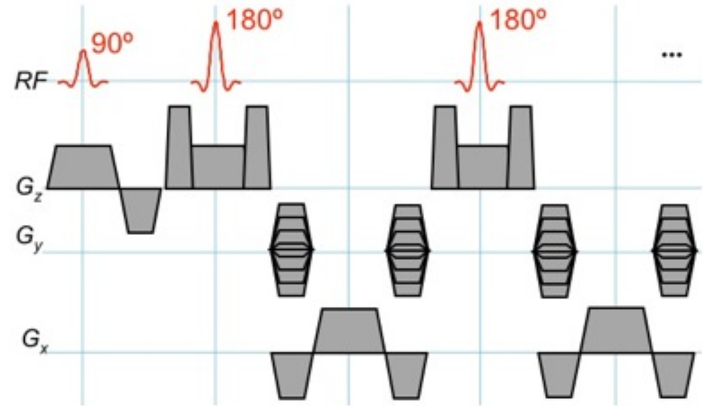




RAD 229: MRI Signals and Sequences

Brian Hargreaves

All notes are on the course website

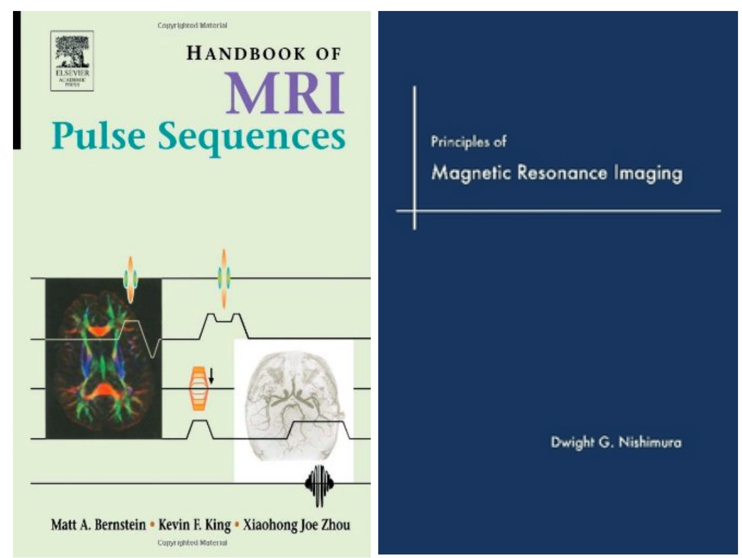
web.stanford.edu/class/rad229

Course Goals

- Develop Intuition
- Understand MRI signals
- Exposure to numerous MRI sequences and naming:
    - “gradient-echo”
    - “spiral”
    - “T_2* BOLD”
    - many, many confusing acronyms
- Expand EE369B, Complement EE369C, EE469B

General Course Logistics

- website: web.stanford.edu/class/rad229
- 3 Units, Letter or Cr/No Cr (EE300 Equivalent)
- Mon/Wed 1:30am-2:50pm
- CCSR 4107 (see calendar for changes)
- Texts (NOT required, but useful)
  - Bernstein M.
  - Nishimura D.



Prerequisites / Grading

- Prerequisite: EE369B /equivalent
  - (complements EE369C / EE469B)
- Paper / Matlab assignments / no MRI scanning
- Grading:
  - 10% Attendance / Participation
  - 10% Midterm
  - 50% Homework + Project
  - 30% Final
- Auditing:
  - Please participate, but allow for-credit students to do so first

Homework / Project Options

- **Replace** a HW Question:
  - Spend <10min explaining how you'd do a question
  - Replace it with a problem and solution that you choose, related to recent lectures
- **Project:** *(details to follow)*
  - Approximately 1-2 Homeworks
  - Simulate and present a sequence / signals / recon
    - A sequence we didn't cover or simulate
    - A novel sequence that you devise
    - A sequence/recon with EE369C

Lectures

- 75 min lectures -- Notes online at website
  - PDF, whole slide (print 4-6 per page)
  - Try to keep numbered.
  - Read ahead, but try not to ruin suspense(!)
- Please no email, texting etc in class
- I try to stay on time - please help by being on time
- Come early, I will try to entertain with questions etc!
- Class participation: questions, exercises

Homework

- Due Wednesday 11:59pm, (minus 10% per day late)
- Paper:
  - Lucas Center Rm P260 (under door)
  - Frank Chavez (nearest cubicle)
- Electronically as PDF (encouraged):
  - Email w/ subject "RAD229: HW1" or similar, <10MB please!
  - bah@stanford.edu
- *Purpose is to learn the material. Note honor code*
- ***Please do not share solutions** without permission*

Other Information

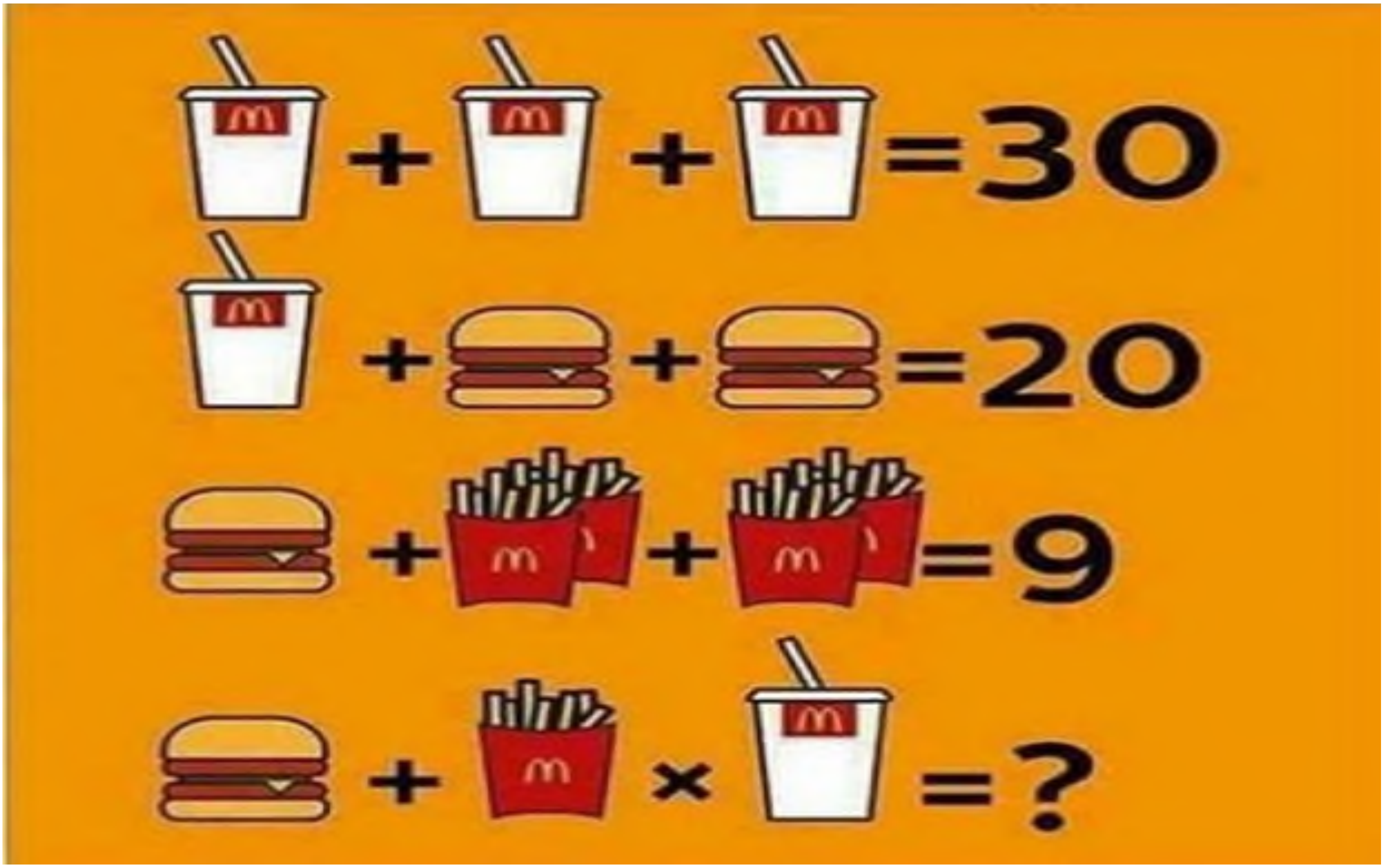
- Instructor: Brian Hargreaves
- Office Hours - See Calendar
- Other Lecturers: Jennifer McNab, Others?
- No Teaching Assistant
- Web Site: web.stanford.edu/class/rad229
  - Lecture notes, homework assignments, code
  - Schedule / Room info, Announcements

Working Together - Rules

- Follow Honor Code
  - Work together on homeworks,
    - Discuss freely, but write your own matlab code
  - Use resources, but not solutions
  - No discussion of exams with others

- In general your responsibility is to learn!
  - You should be able to explain anything you submit

Participation!



- FB: 1/10 (willing to answer, but not likely to be correct)
- ? (unlikely to answer but likely to be correct)
- Balance??!!

Introductions

- Your name?
- Who do you work with?
- Your Research?
- Comments - What you Hope to Learn?

Course Overview / Topics

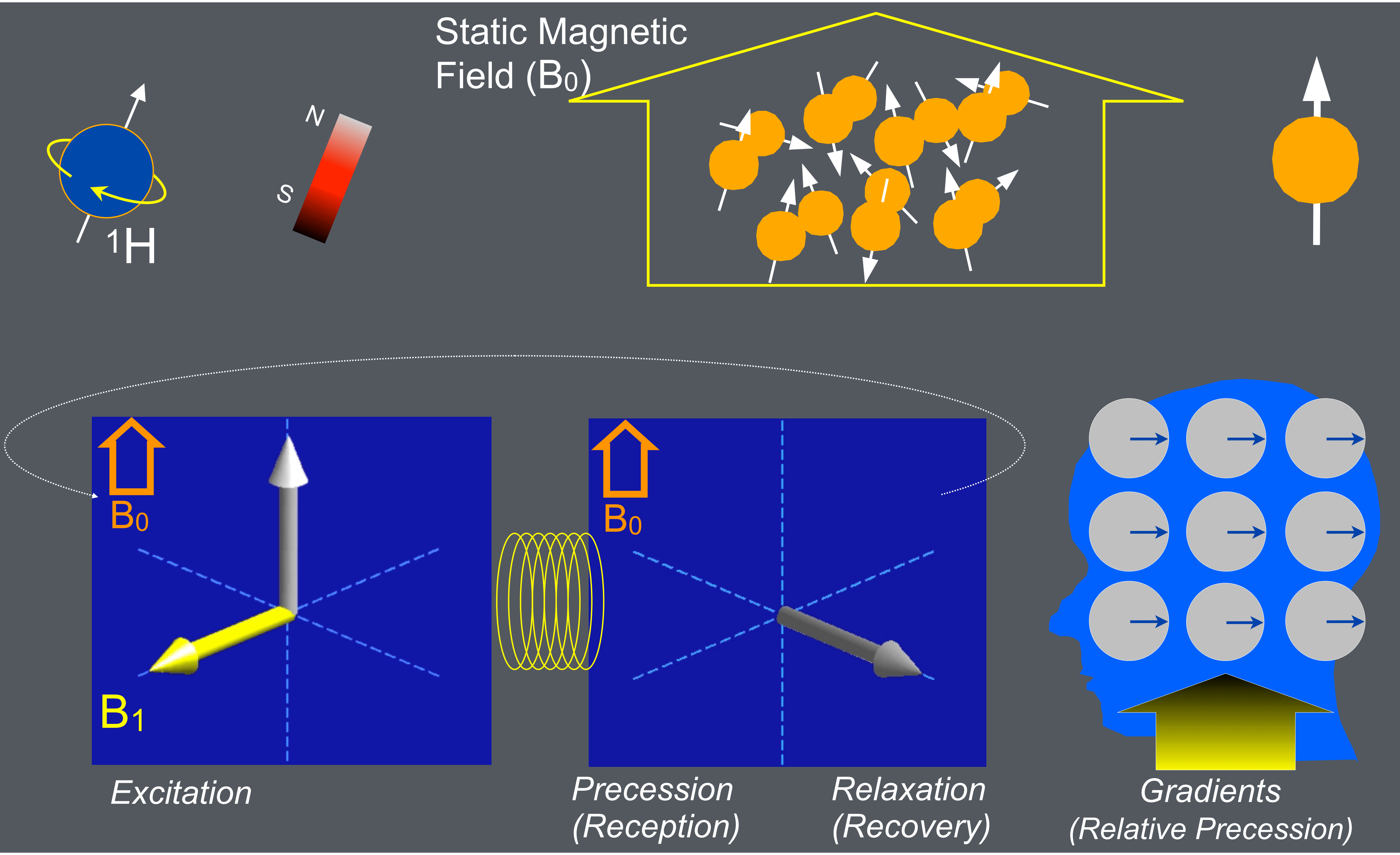
- Review of Basic MRI (EE369B)
- Signal Calculation Tools, System Imperfections
- Pulse Sequences
- Advanced Acquisition Methods

- *The RAD229 class will continue to evolve!*
  - Things might change, and your input will shape the course!
  - You may know more than me about some topics

Background (~EE369B)

- *"Magnetic Resonance Imaging" D. Nishimura*
- Overview of NMR
- Hardware
- Image formation and k-space
- Excitation k-space
- Signals and contrast
- Signal-to-Noise Ratio (SNR)
- Pulse Sequences

MRI: Basic Concepts

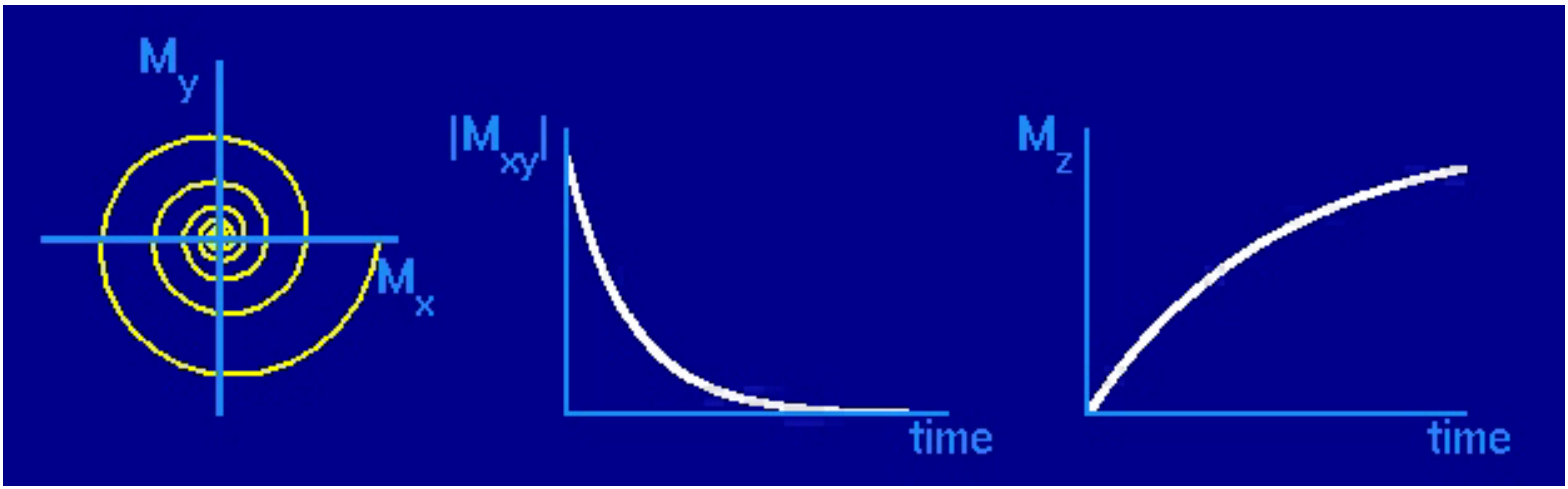


Precession and Relaxation

Relaxation and precession are independent.

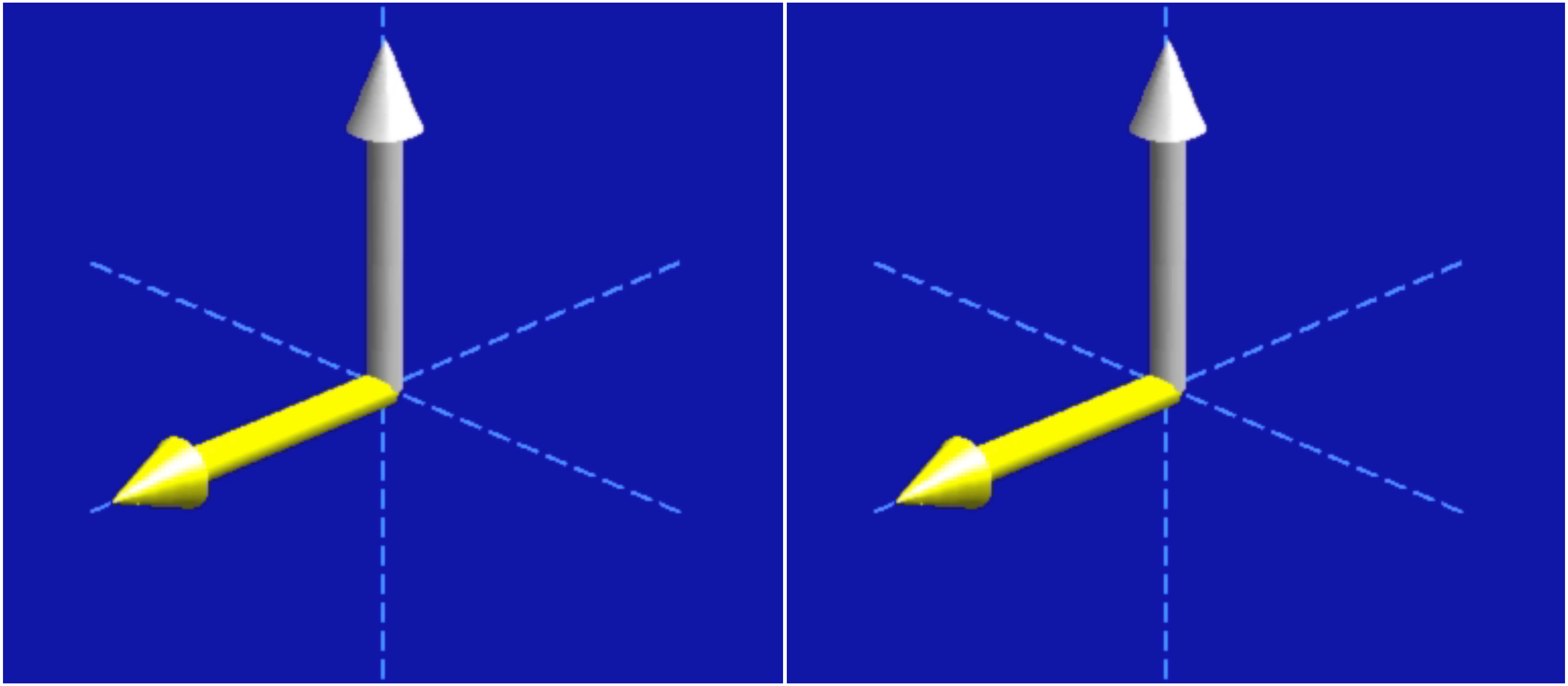
Magnetization returns exponentially to equilibrium:

- Longitudinal *recovery* time constant is T_1
- Transverse *decay* time constant is T_2

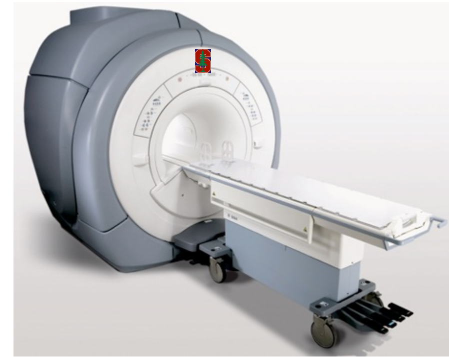


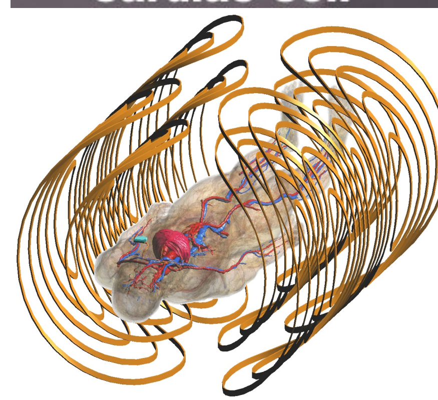
Precession Decay Recovery

Magnetic Resonance Imaging (MRI)



MRI Hardware

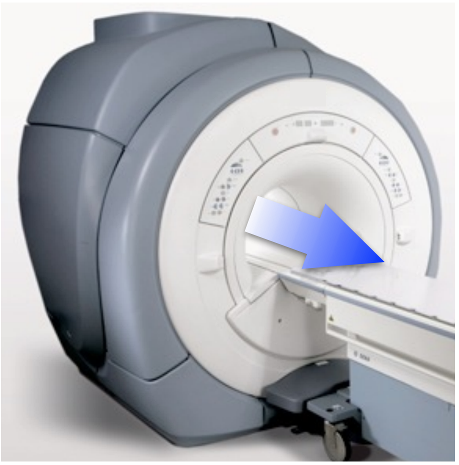




- Strong Static Field (B_0) ~ 0.5-7.0T
- Radio-frequency (RF) field (B_1) ~ 0.1uT
    - Transmit, often built-in
    - Receive, often many coils
- Gradients (G_x, G_y, G_z) ~ 50-80 mT/m

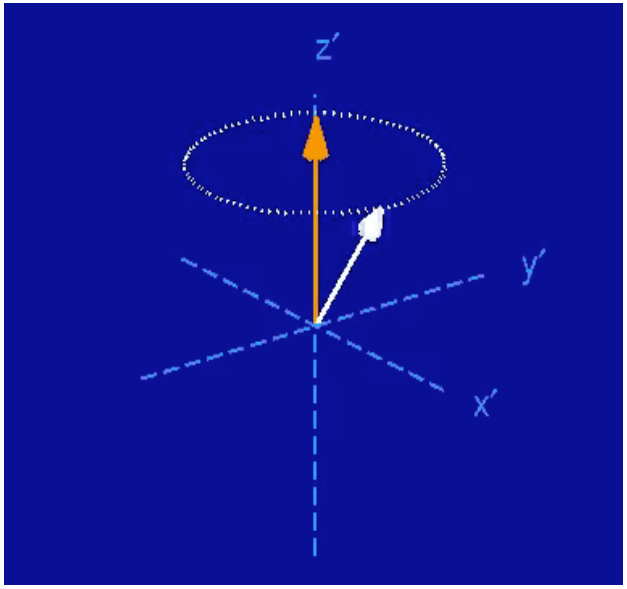
B_0: Static Magnetic Field

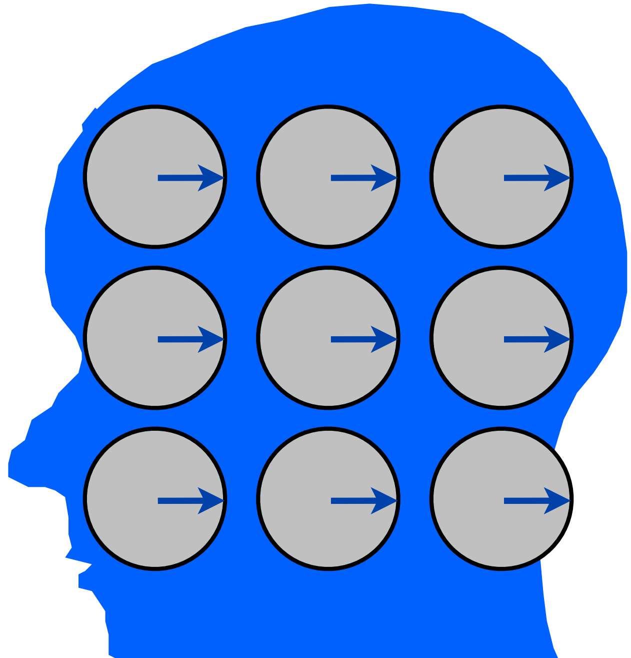
- Goal: Strong AND Homogeneous magnetic field
  - Typically 0.3 to 7.0 T
  - Resonance proportional to B_0 : $\gamma/2\pi = 42.58$ MHz/T
- Superconducting magnetic fields - always on
  - ~1000 turns, 700 A of current
  - Passively shimmed by adjusting coil locations

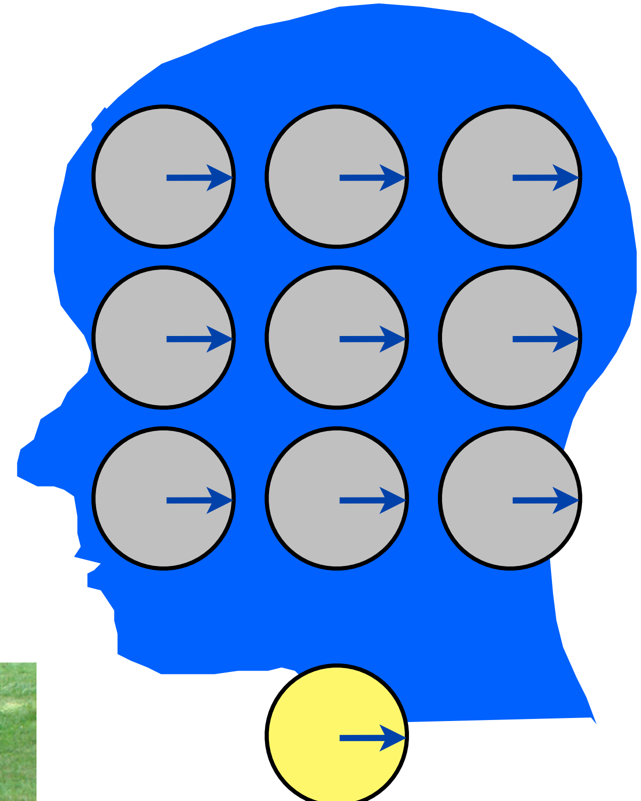


- The following increase with with B_0:
  - Polarization, Larmor Frequency, Spectral separation, T_1
  - RF power for given B_1
  - B_0 variations due to susceptibility, chemical shift

B_0: The "Rotating" Coordinate Frame







- Usually demodulate by Larmor frequency to "baseband"
- Also called the <u>rotating frame</u>

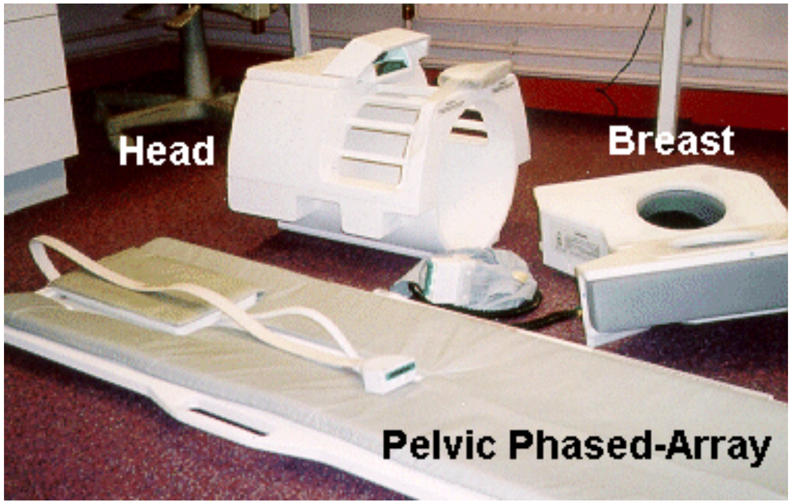
B_1^+: RF Transmit Field

- Goal: Homogeneous *rotating* magnetic field
  - Typically up to about 25 uT (Amplifier, SAR limits)
  - Requires varying power based on subject size
  - Dielectric effects cause B_1^+ variations at higher B_0
  - Amplifier power: kW to tens of kW
- Specific Absorption Rate (SAR) Limits:
  - Power proportional to B_0^2 and B_1^2
  - Goal is to limit heating to <1° C

B_1^-: RF Receive

- Goal: High sensitivity, spatially limited, low noise
- "Birdcage" coils
  - Uniform B_1^- but single channel
- Surface coils
  - Varying B_1^- but high sensitivity
- Coil arrays
  - Multiple channels with Varying B_1^-
  - Allows some spatial localization: Parallel Imaging

RF Coils



Receiver System

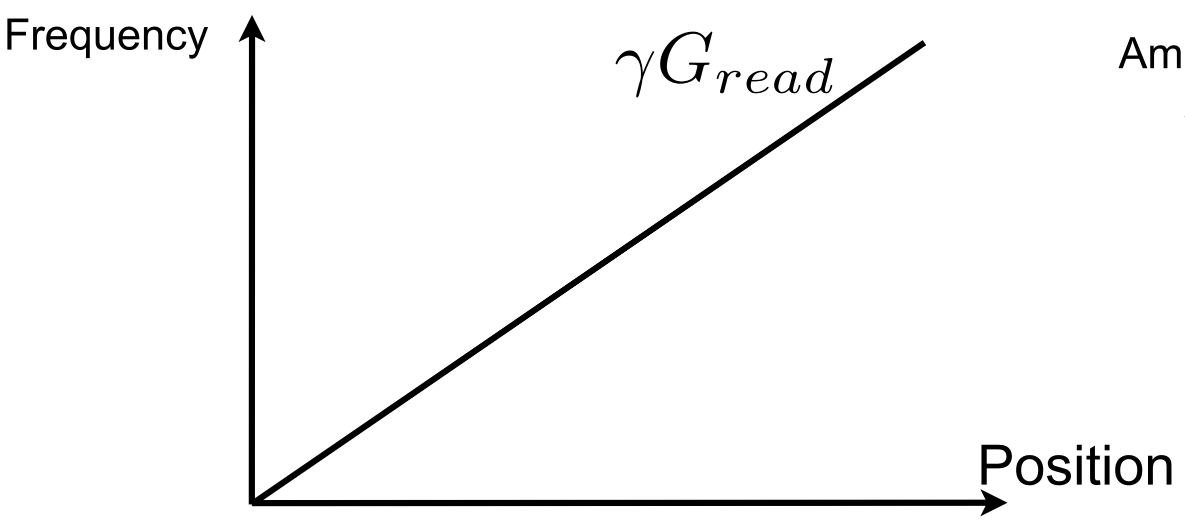
- 500 to 1000 k samples/s
- Complex sampling
- Low-pass filter capability
- Typically 32-128 channels
- Time-varying frequency and phase modulation (Typically single-channel)

Gradients

- Goal: Strong, switchable, linear B_z variation with x,y,z
  - Peak amplitude ~ 50-80 mT/m (~ 200A)
  - Switching 200 mT/m/ms (~1500 V)
- Limits:
  - Amplifier power, heating, coil heating
  - "dB/dt" limitation due to peripheral nerve stimulation
  - Switching induces Eddy Currents
  - Concomitant terms (B_x and B_y variations)
  - Non-linearities (often correctable)

Gradient Waveforms

- Mapping of position to frequency, slope = γG
- Typically waveforms are trapezoidal
  - Constant amplitude and slew-rate limits



Shims

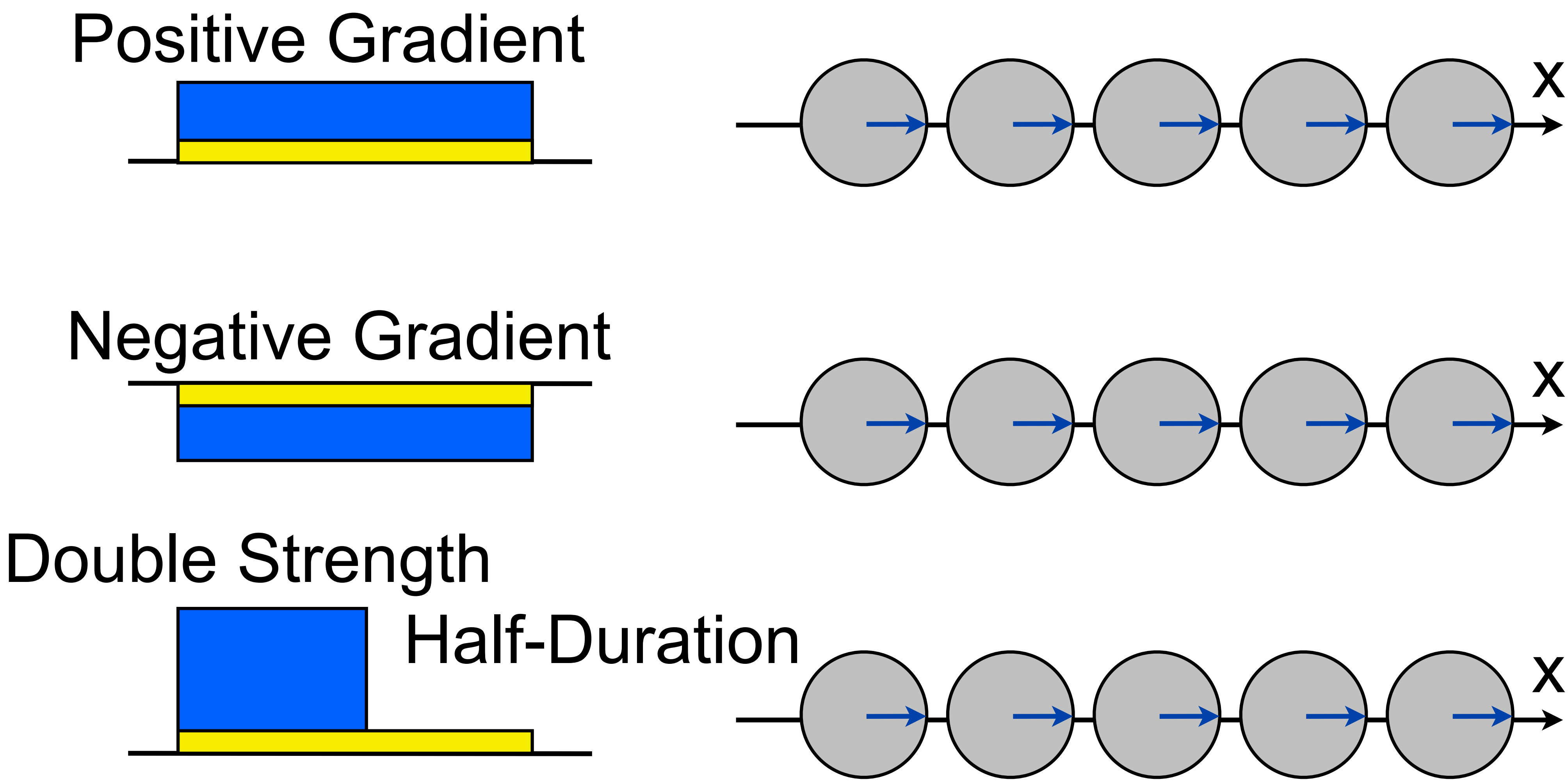
- Goal usually to make B0 more uniform with subject
  - Center frequency
  - Linear shims (Up to ~1% offset to gradients)
  - Higher-order (HO) shims (Spherical Harmonics)
  - Shim arrays, Shim+RF (Current Research)
- Usually HO shims not dynamically switchable

Review Questions

Image Formation and k-space

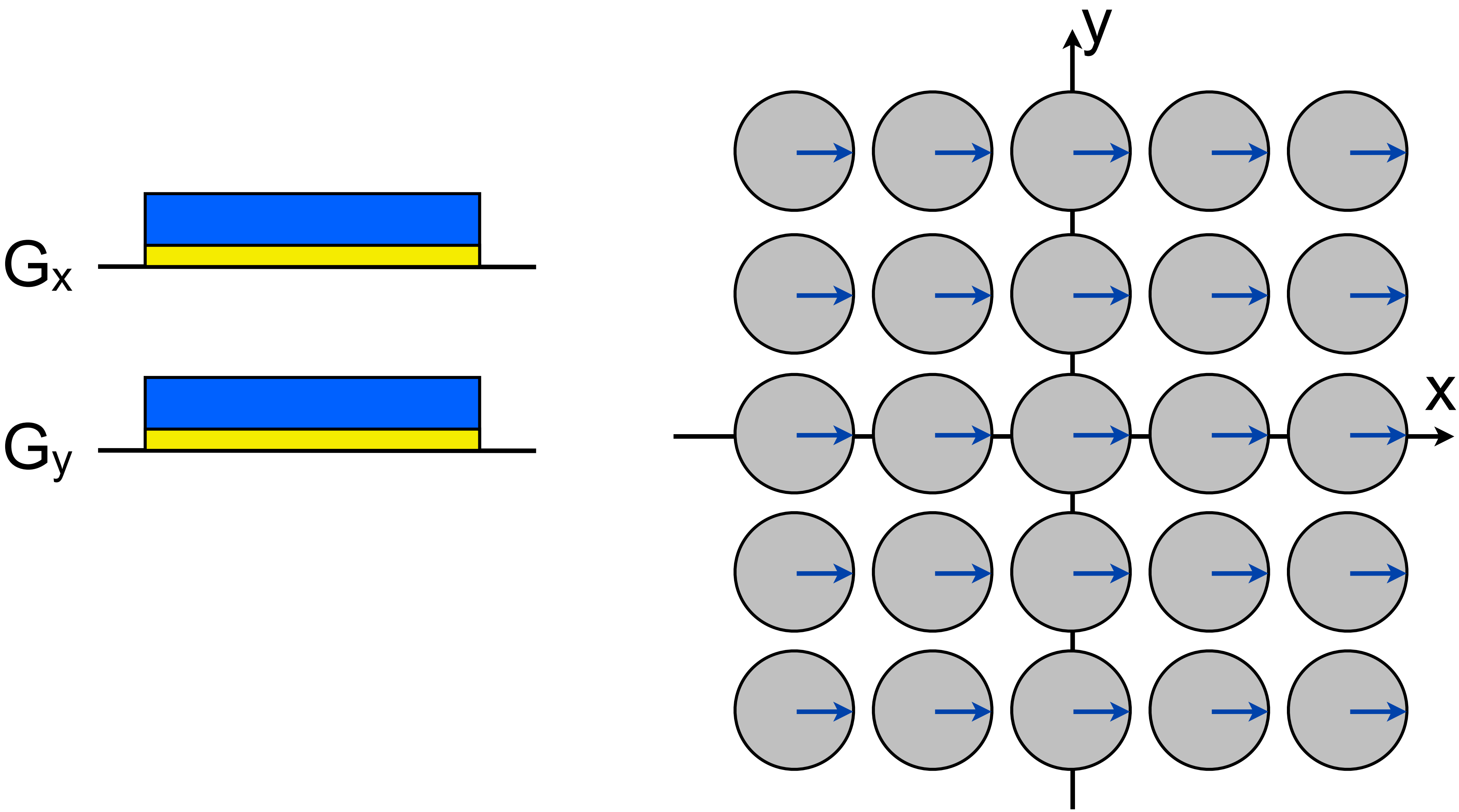
- Gradients and phase
- Signal equation
- Sampling / Aliasing
- Parallel Imaging
- *Many reconstruction methods in EE369C*

Gradient Strength and Sign



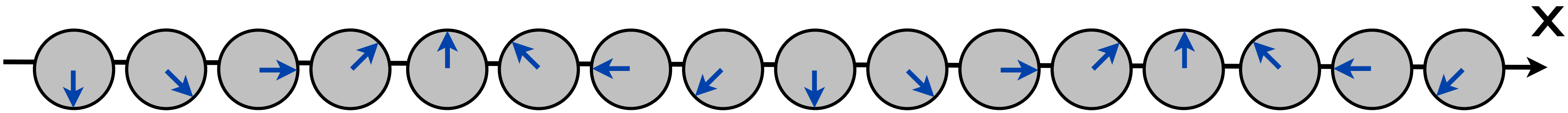
Can control both amplitude and duration

Gradient Along Both x and y



Can also vary along z

Ribbon Analogy



- Gradients induce “phase twist”
- Twist has a number of cycles and a “sign”
- Twist can be along any direction

Gradients and Phase

- Control gradient amplitude and duration
- Can control frequency:

$$Frequency = \gamma(G_x x + G_y y)$$

- Can "encode" phase over duration t

$$Angle = \gamma t\,(G_x x + G_y y + G_z z)$$

- Generally:

$$\phi = \gamma\left(x \int G_x dt + y \int G_y dt\right)$$

What are the units of Frequency and Angle (φ) here?

Signal Equations

- For a single spin:

$$\phi = \gamma(x \int G_x dt + y \int G_y dt)$$

- Represent as exponential:

$$s = e^{-i\gamma(x \int G_x dt + y \int G_y dt)}$$

- Sum over many spins:

$$s = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x,y) e^{-i\gamma(x \int G_x dt + y \int G_y dt)} dx dy$$

- Signal equation:

$$s = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x,y) e^{-2\pi i (k_x x + k_y y)} dx dy \qquad k_{x,y}(t) = \frac{\gamma}{2\pi} \int_0^t G_{x,y}(\tau) d\tau$$

$$s(t) = FT[\rho(x,y)]|_{k_x(t), k_y(t)}$$

Fourier Transform in MRI

$$s(t) = FT[\rho(x,y)]|_{k_x(t),k_y(t)}$$

$$M(\boldsymbol{k}) \Longleftrightarrow \rho(\boldsymbol{r})$$

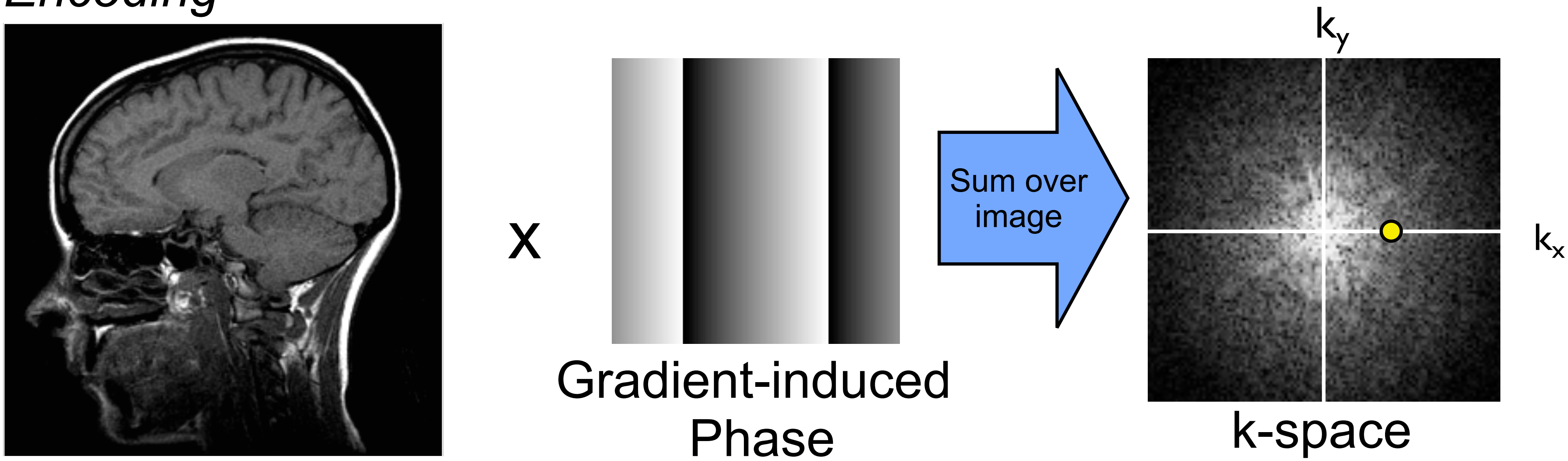
Fourier Transform

- Given $M(\boldsymbol{k})$ at enough $\boldsymbol{k}$ locations, we can find $\rho(\boldsymbol{r})$
- It does not matter how we got to $\boldsymbol{k}$!

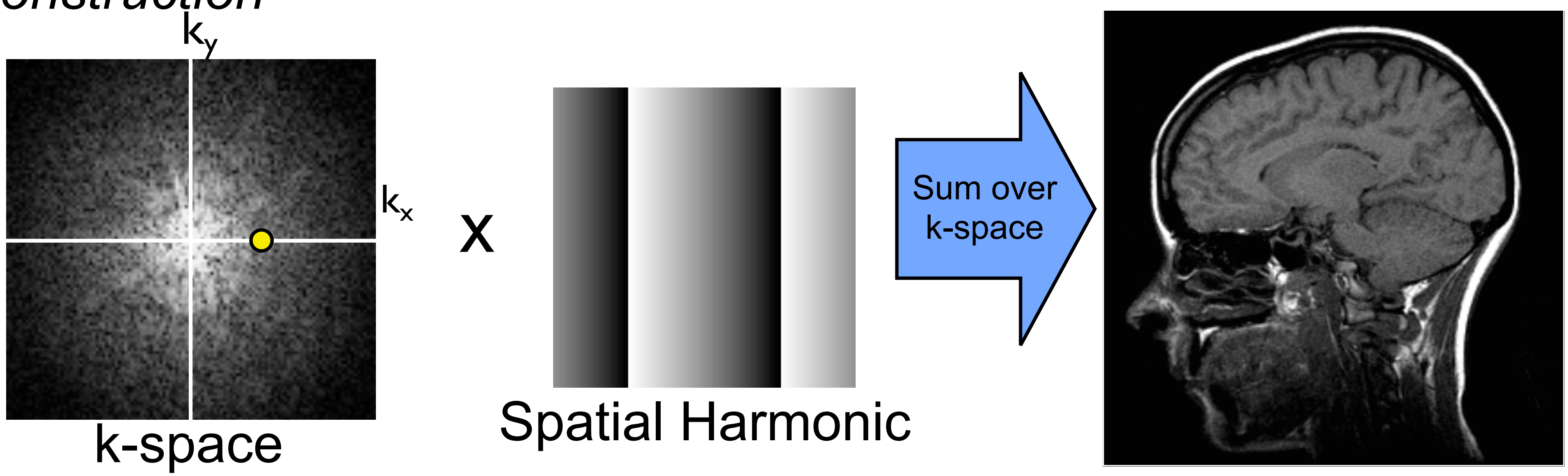
What are the units of $k_x(t)$ and $k_y(t)$?

Fourier Encoding and Reconstruction

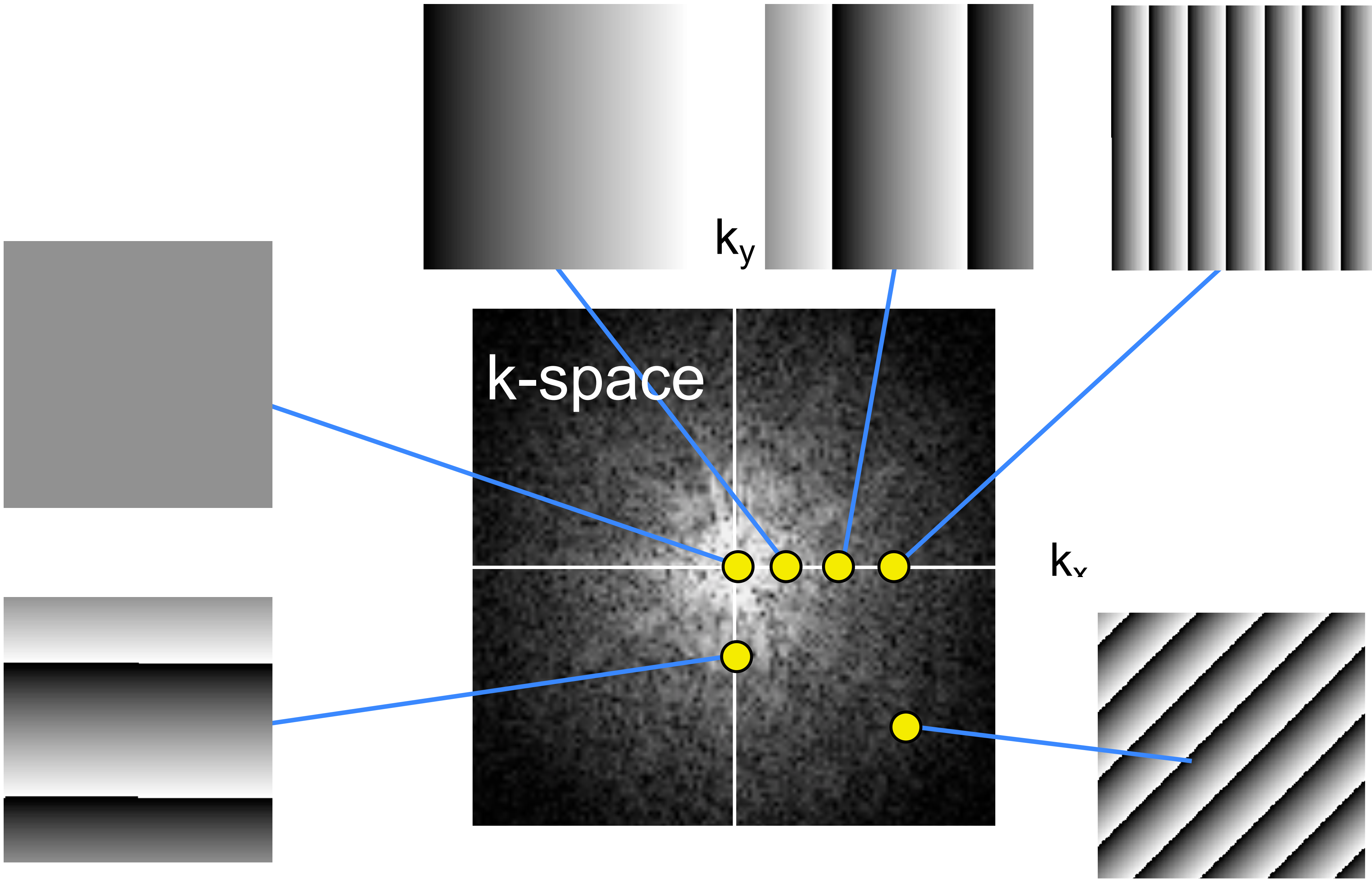
Encoding



Reconstruction



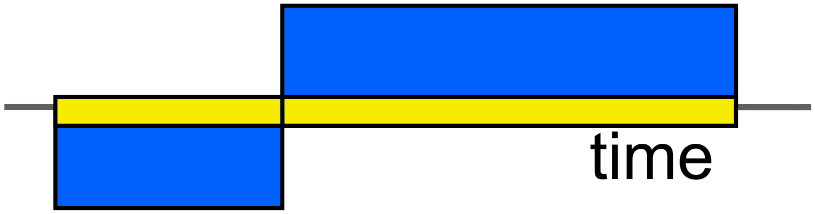
k-space: Spatial Frequency Map



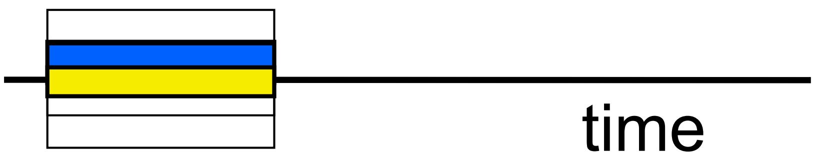
In terms of pixel-width, what is the width of k-space?

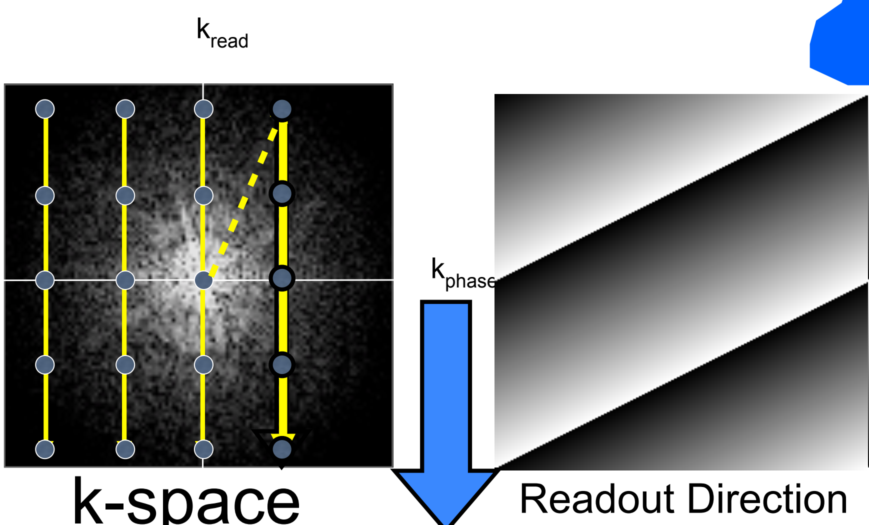
Image Formation and Sampling

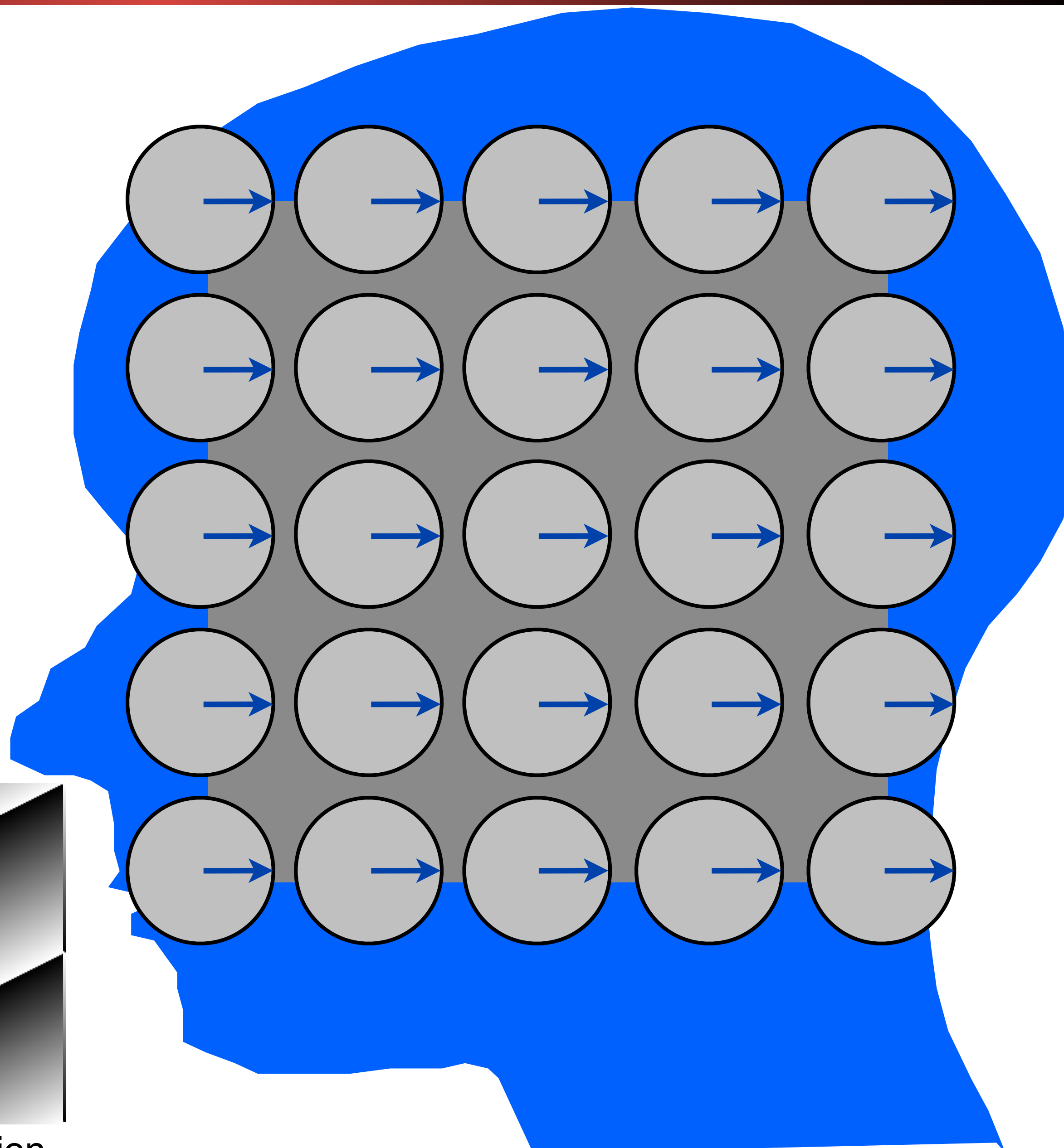
Readout Gradient



Phase-Encode Gradient

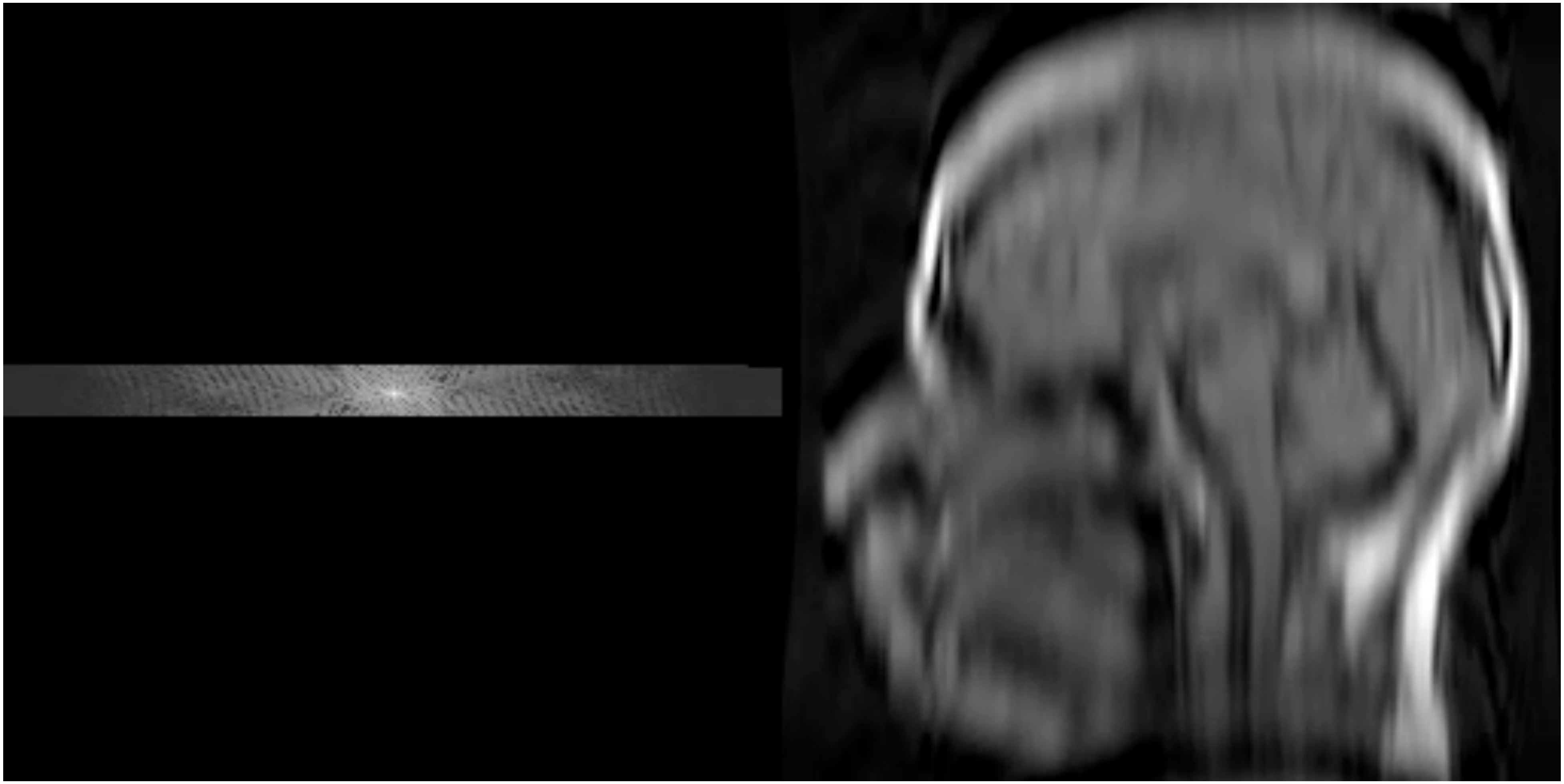






k space Extent and Image Resolution

Data Acquisition "k" space | Image Space



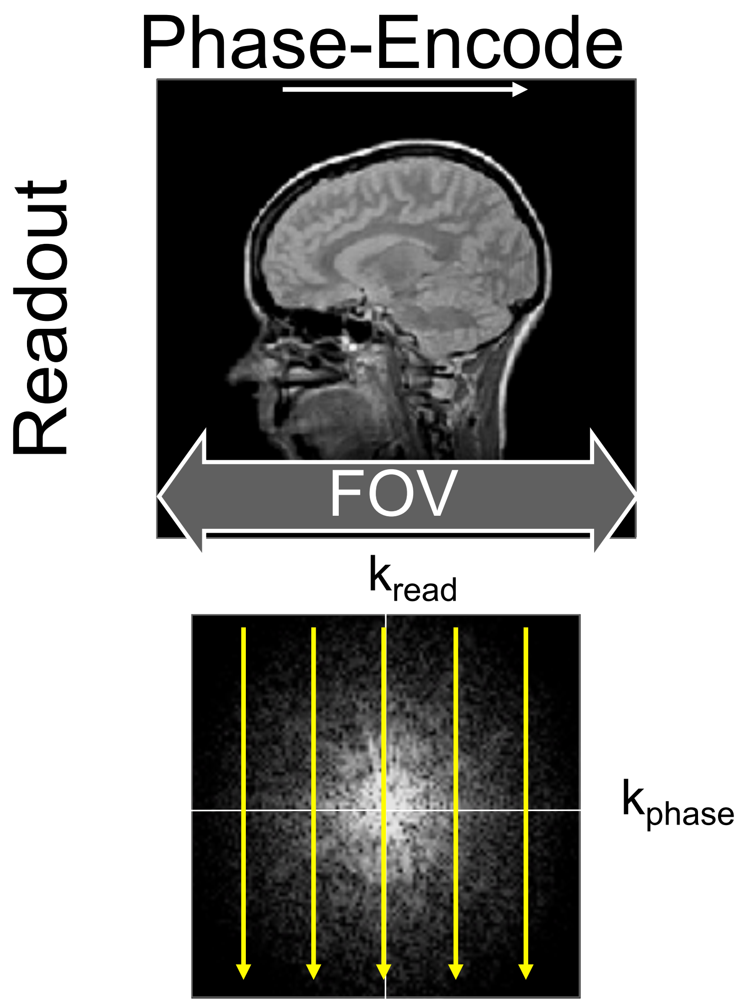
Fourier Transform

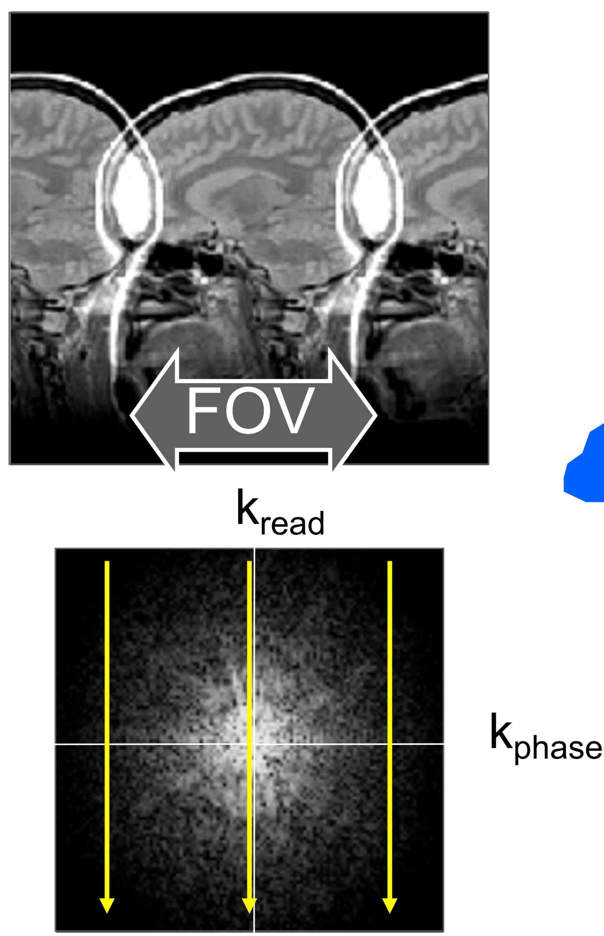
$$\Delta x = 1/(2k_{max})$$

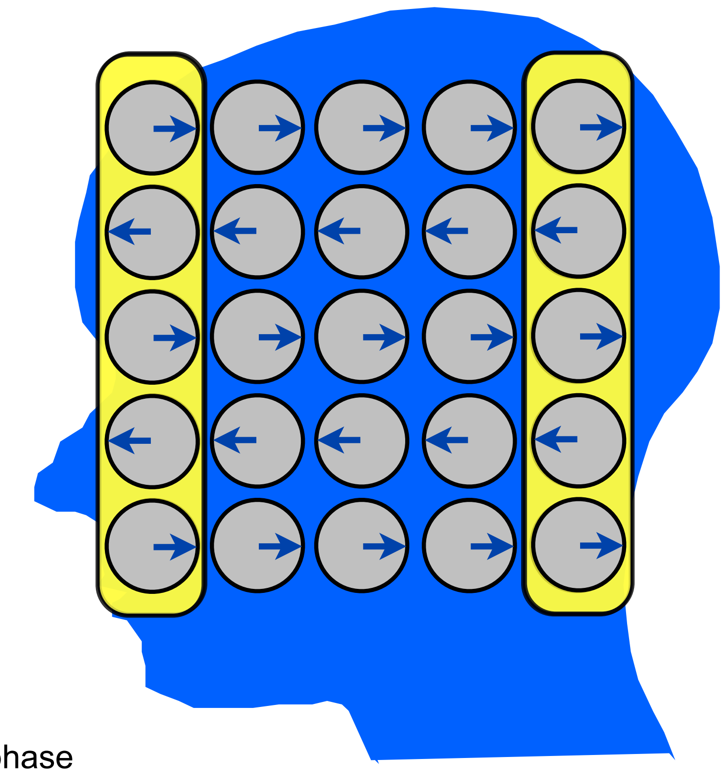
Sampling and Field of View

- Sampling density determines FOV
- Sparse sampling results in *aliasing*

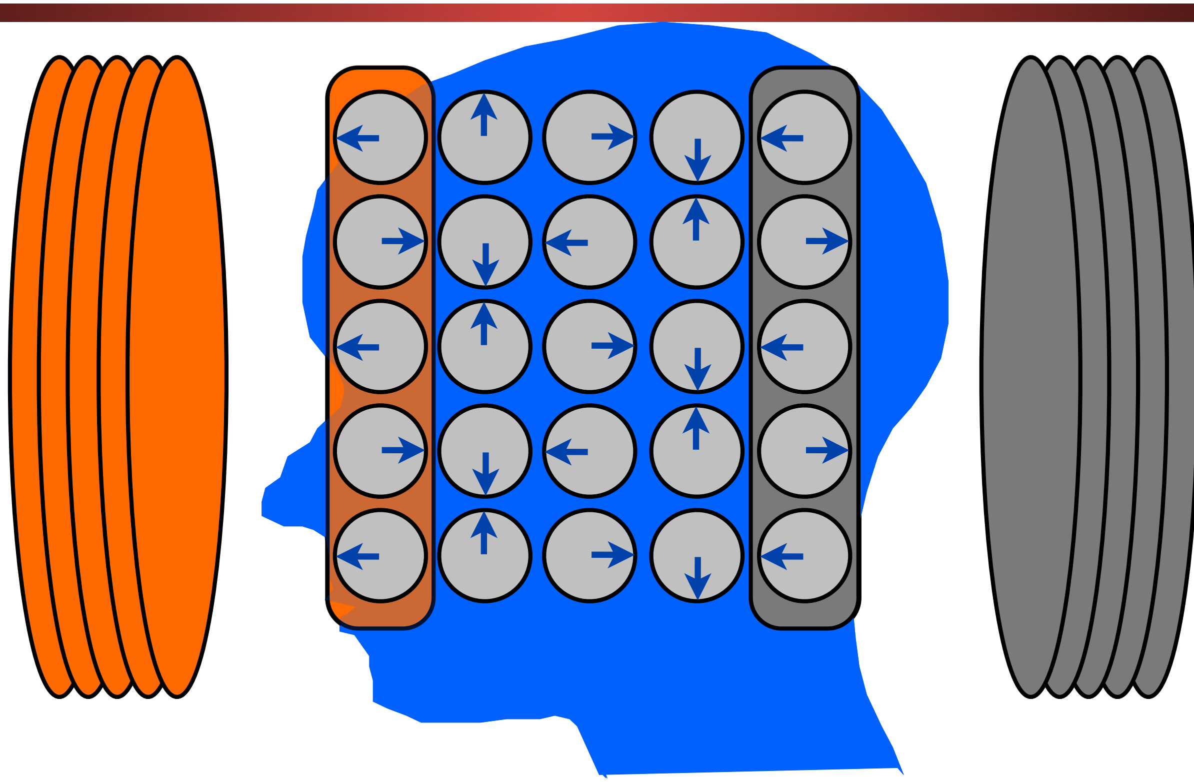
$$FOV = 1/\Delta k_y$$



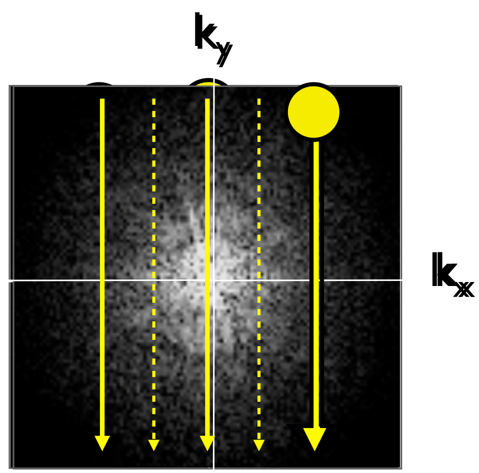




Phase-Encoding with Two Coils

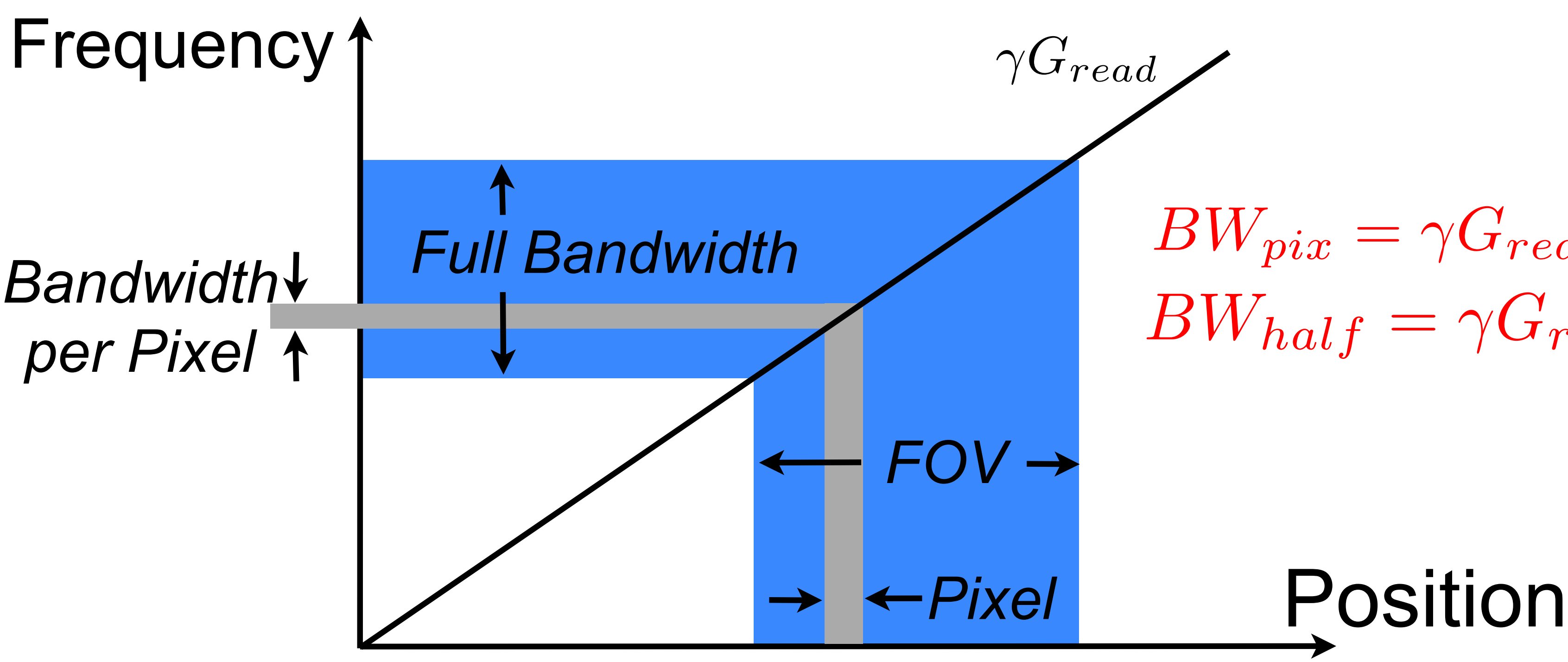


k-space



Readout Parameters

- Bandwidth linked to readout
  - “half-bandwidth” (GE) = 0.5 x sample rate
  - Same as Filter bandwidth (baseband)
- Pixel-bandwidth often useful



$$BW_{pix} = \gamma G_{read} \Delta x$$
$$BW_{half} = \gamma G_{read} FOV/2$$

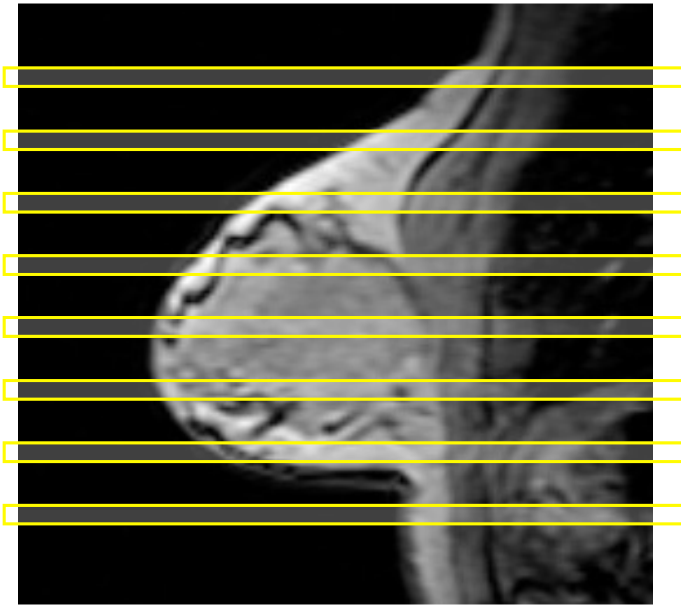
Imaging Example

- Desired Image Parameters:
  - 256 x 256, over 25cm FOV
  - (±)125 kHz bandwidth

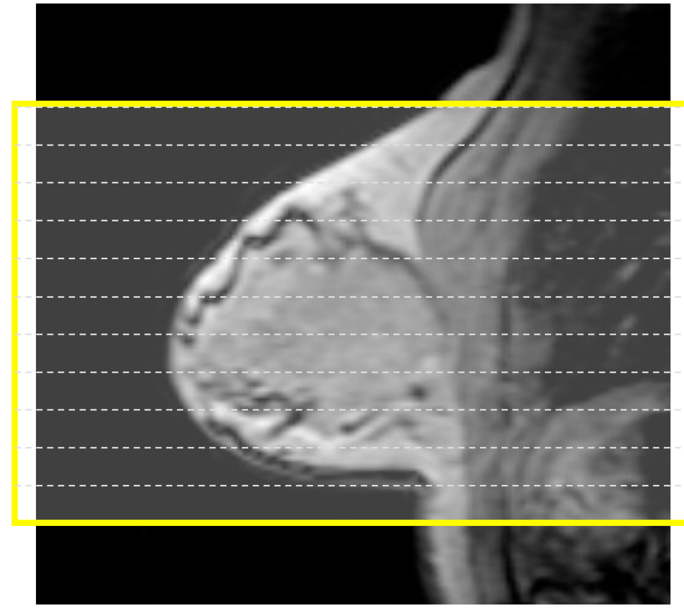
- What are the...
  - Sampling *period?*
  - Readout duration?
  - Gradient strength?
  - Bandwidth per pixel?
  - k-space extent?

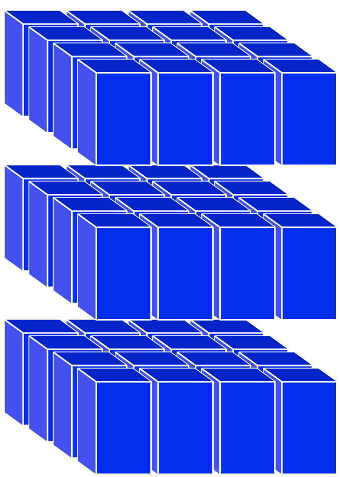
2D Multislice vs 3D Slab Imaging

2D

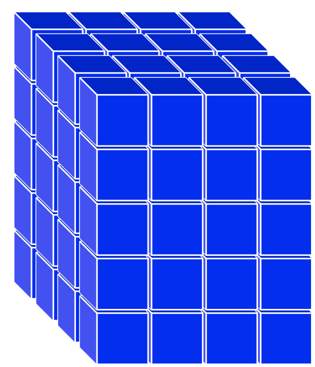


3D





Shorter scan times, reduced motion artifact



Continuous coverage

Thinner slices, reformats

Imaging Summary

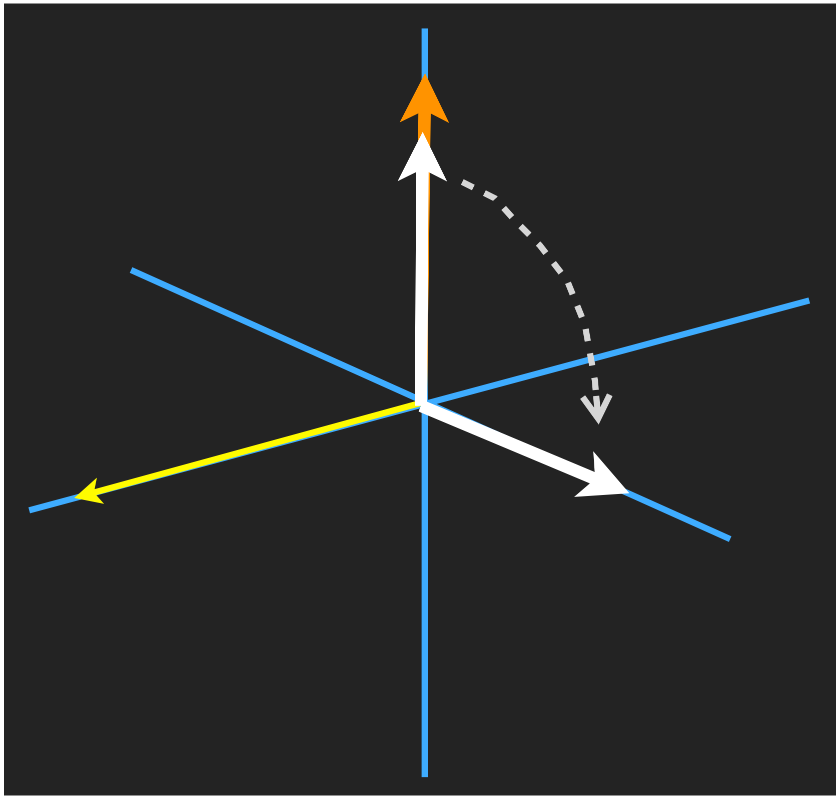
- Gradients impose time-varying linear phase
  - k-space is time-integral of gradients
- k-space samples Fourier Transform to/from image
  - Density of k-space <> FOV (image extent)
  - Extent of k-space <> Resolution (image density)
- 3D k-space is possible
- Parallel imaging uses coils to extend FOV

Excitation

- General principles of excitation
- Selective Excitation with gradients
- Relationships for slice excitation
- Excitation k-space
- *Much more covered in EE469B*

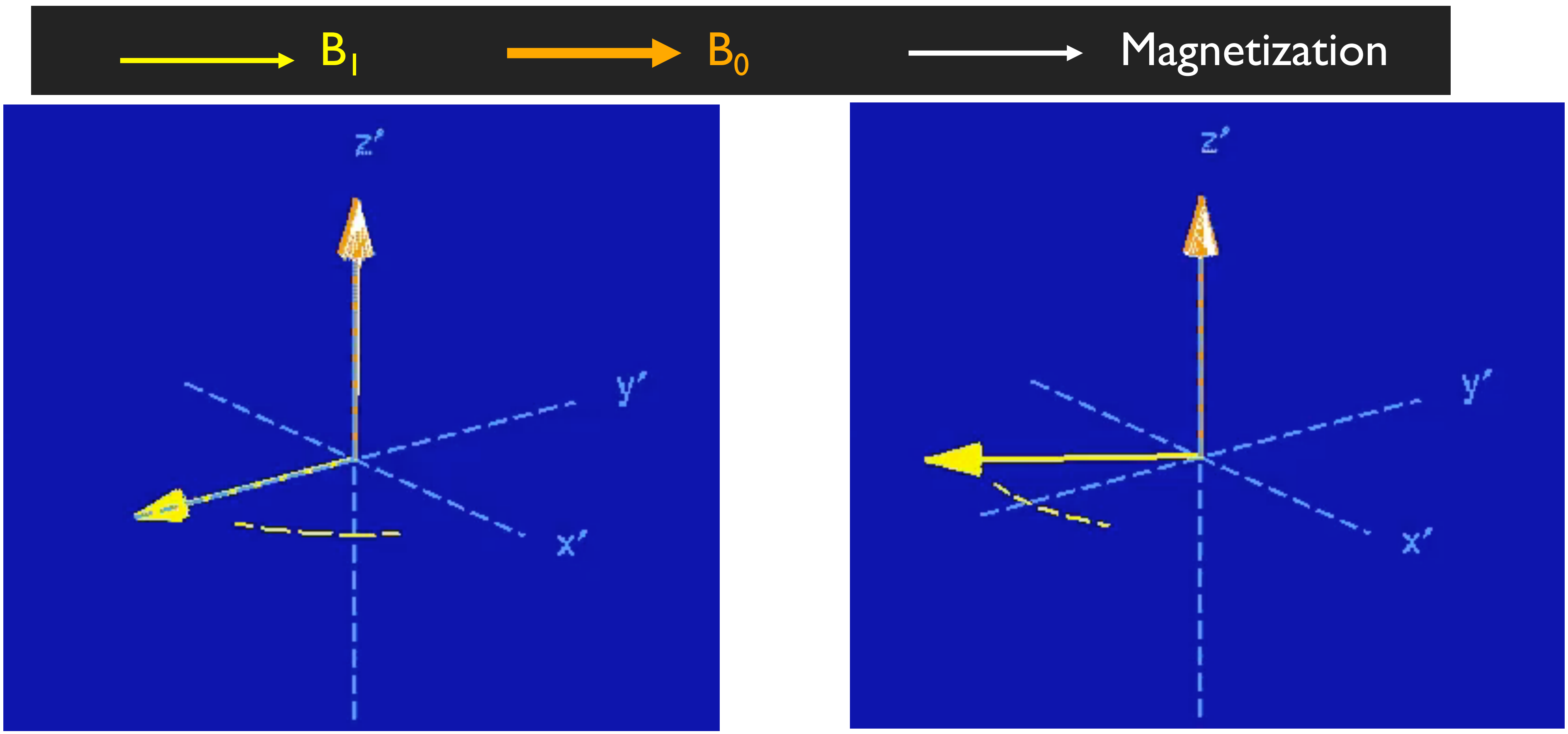
Excitation: B_1 Field

- Direction of B_1 is perpendicular to B_0
- Magnetization precesses about B_1
- Turn on and off B_1 to "tip" magnetization
- Problem: We can't turn off B_0!
- Precession still around B_0



Excitation

- Magnetization precesses about net field (B_0+B_1)
- B_1 << B_0
- Must "tune" B_1 frequency to Larmor frequency

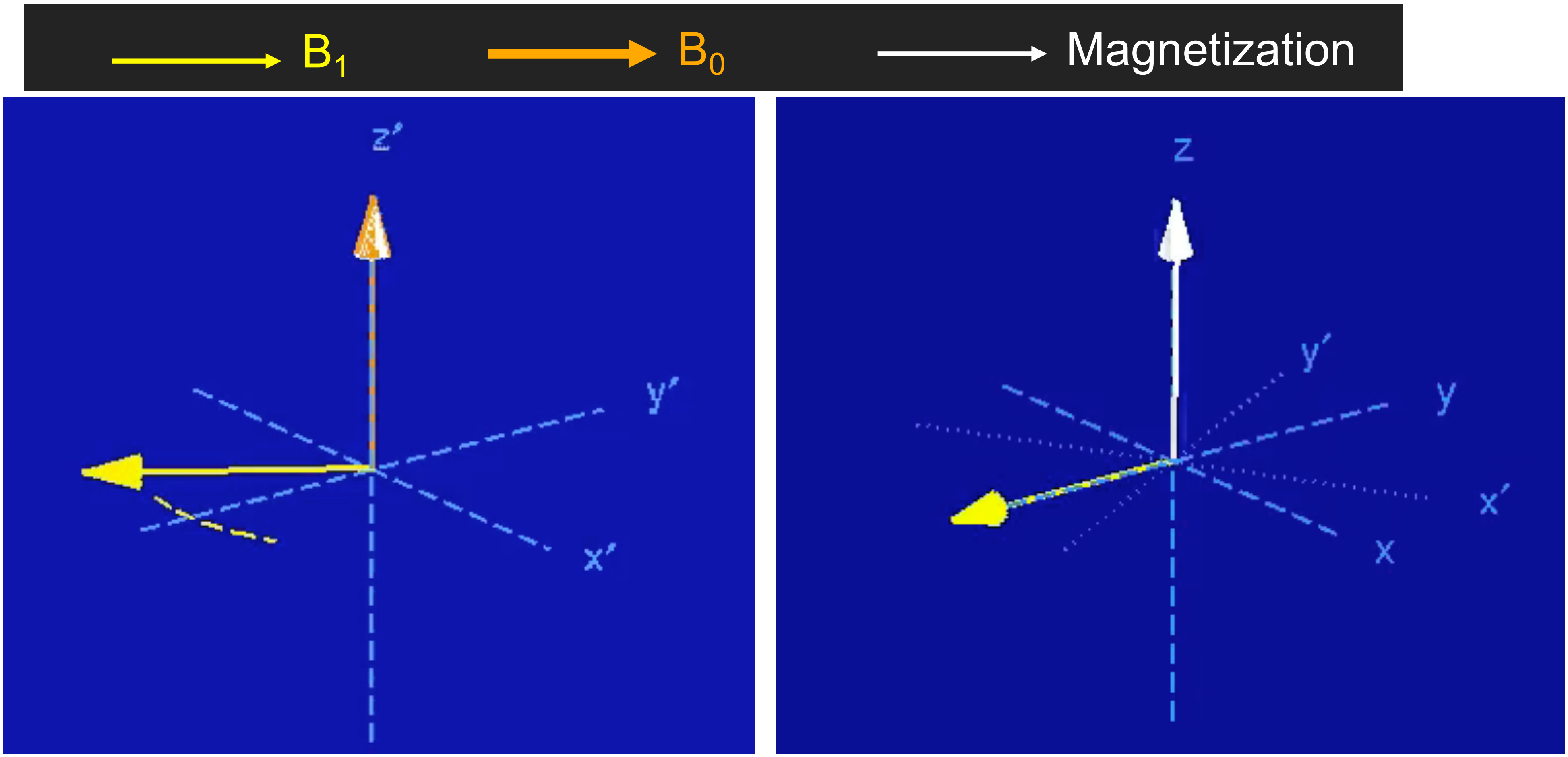


Static B_1 Field

Rotating B_1 Field

Excitation: Rotating Frame

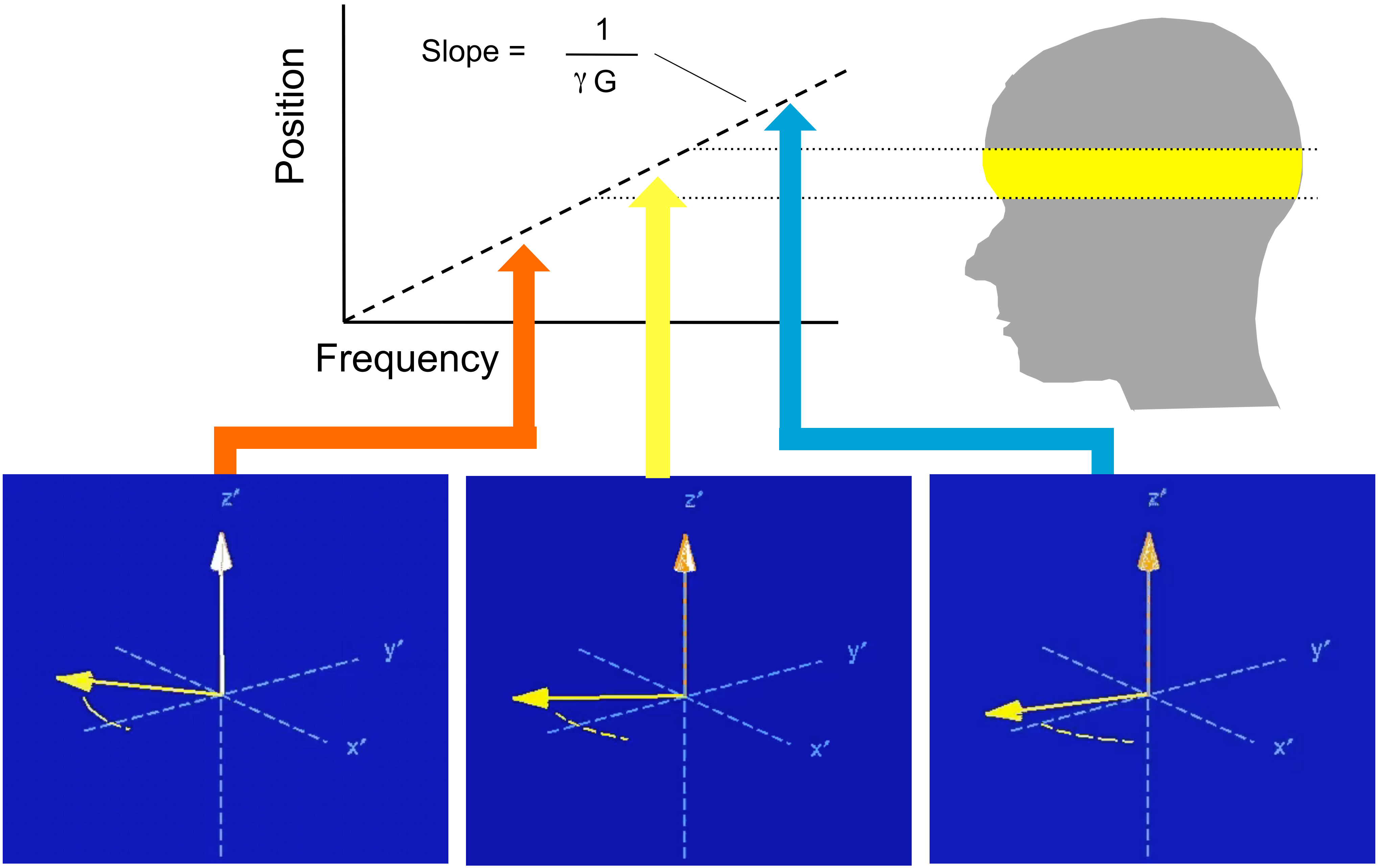
- “Excite” spins out of their equilibrium state.
- $B_1 << B_0$
- Transverse RF field (B_1) rotates at γB_0 about z-axis.



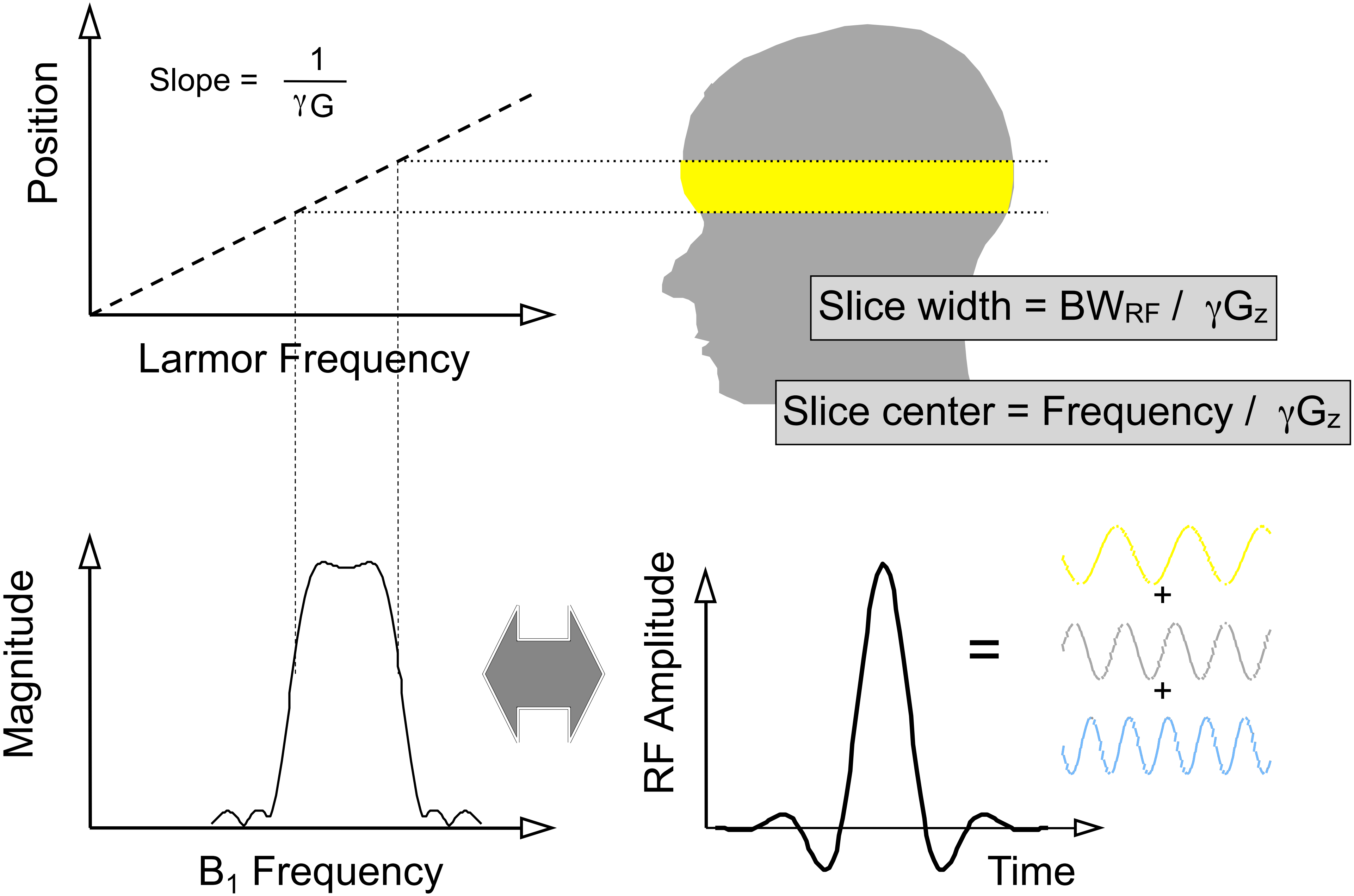
Static Frame

Rotating Frame, “On resonance”

Selective Excitation



Selective Excitation



Excitation Example

- Given a 2 kHz RF pulse bandwidth, and desired
  - 5mm thick slice
  - Slices at -2cm, 0, 2cm
- What are the...
  - Gradient strength? $(\gamma/2\pi)G_z$
  - Excitation frequencies?
  - Thinnest slice possible with 50mT/m max gradients?

Excitation k-Space

- Excitation k-space goes *backwards* from end of RF/ gradient pair:

$$k_e(t) = -\frac{\gamma}{2\pi}\int_t^T G(\tau)d\tau \qquad k_r(t) = \frac{\gamma}{2\pi}\int_0^t G(\tau)d\tau$$

- Excited profile = Fourier Transform of excitation k-space
- Central flip angle = area under pulse (may be zero!):

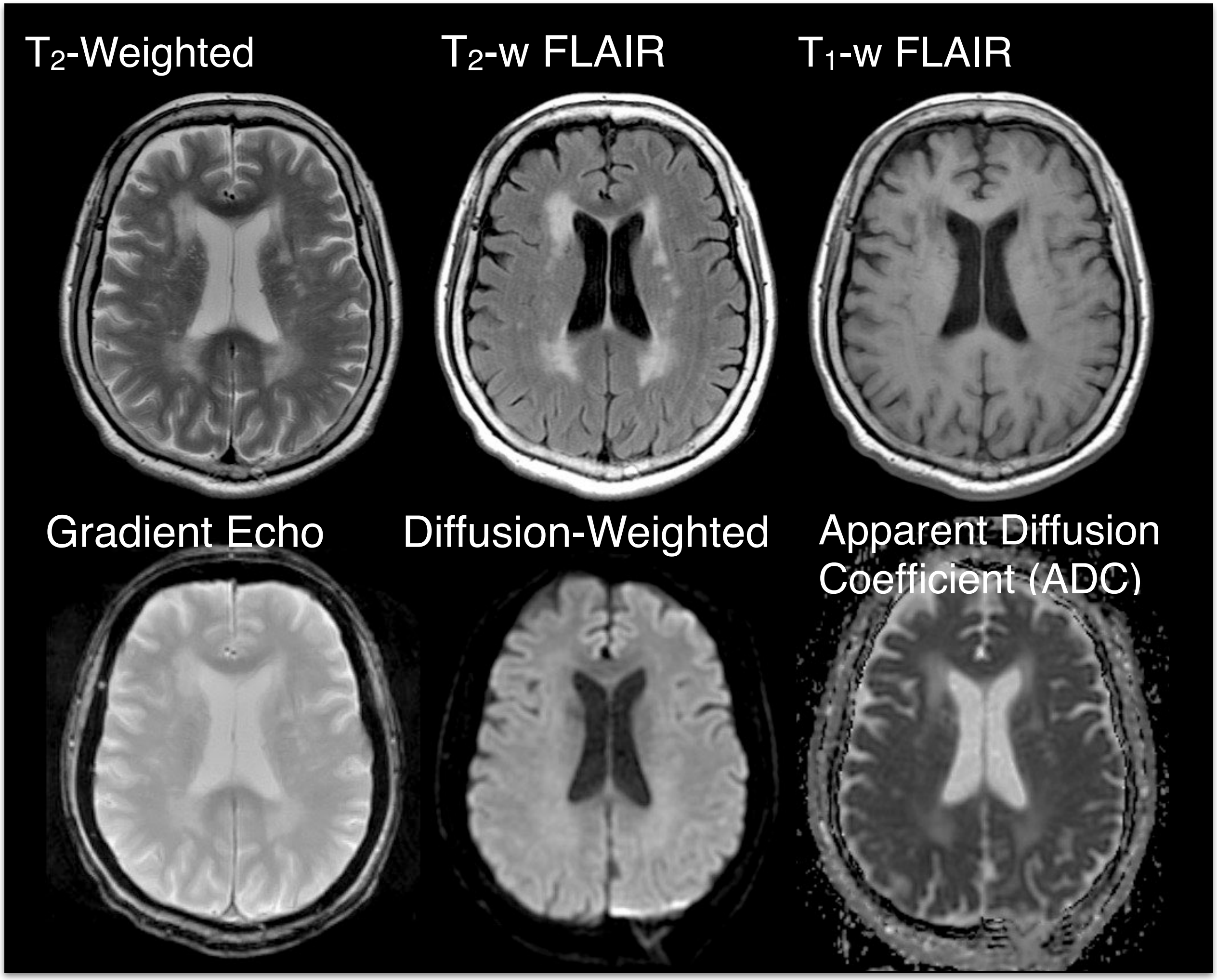
$$\alpha = \gamma \int B_1(\tau)d\tau$$

Excitation Example

- For a 1ms, constant RF pulse of amplitude 10μT …
  - What is the flip angle?
  - How does RF energy change if the duration is halved and amplitude doubled?

Signals and Contrast

- Simple Bloch Equation Solutions
- Basic contrast mechanisms: T_1, T_2, IR, Steady-State

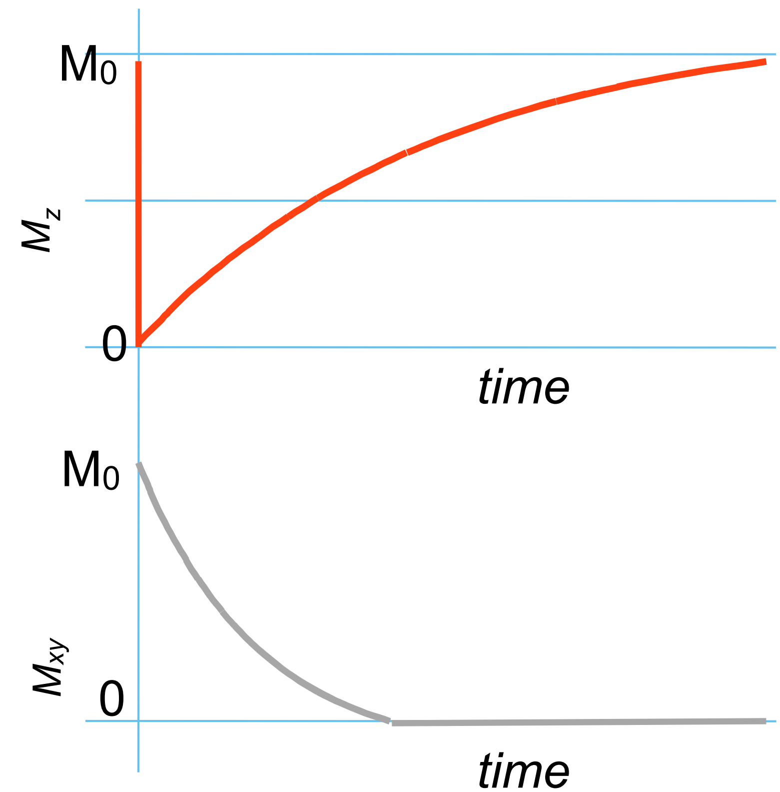


Signals and Contrast

- Bloch Equation Solutions (Relaxation):

$$M_{xy}(t) = M_{xy}(0)e^{-t/T_2}$$

$$M_z(t) = M_0 + [M_z(0) - M_0]e^{-t/T_1}$$

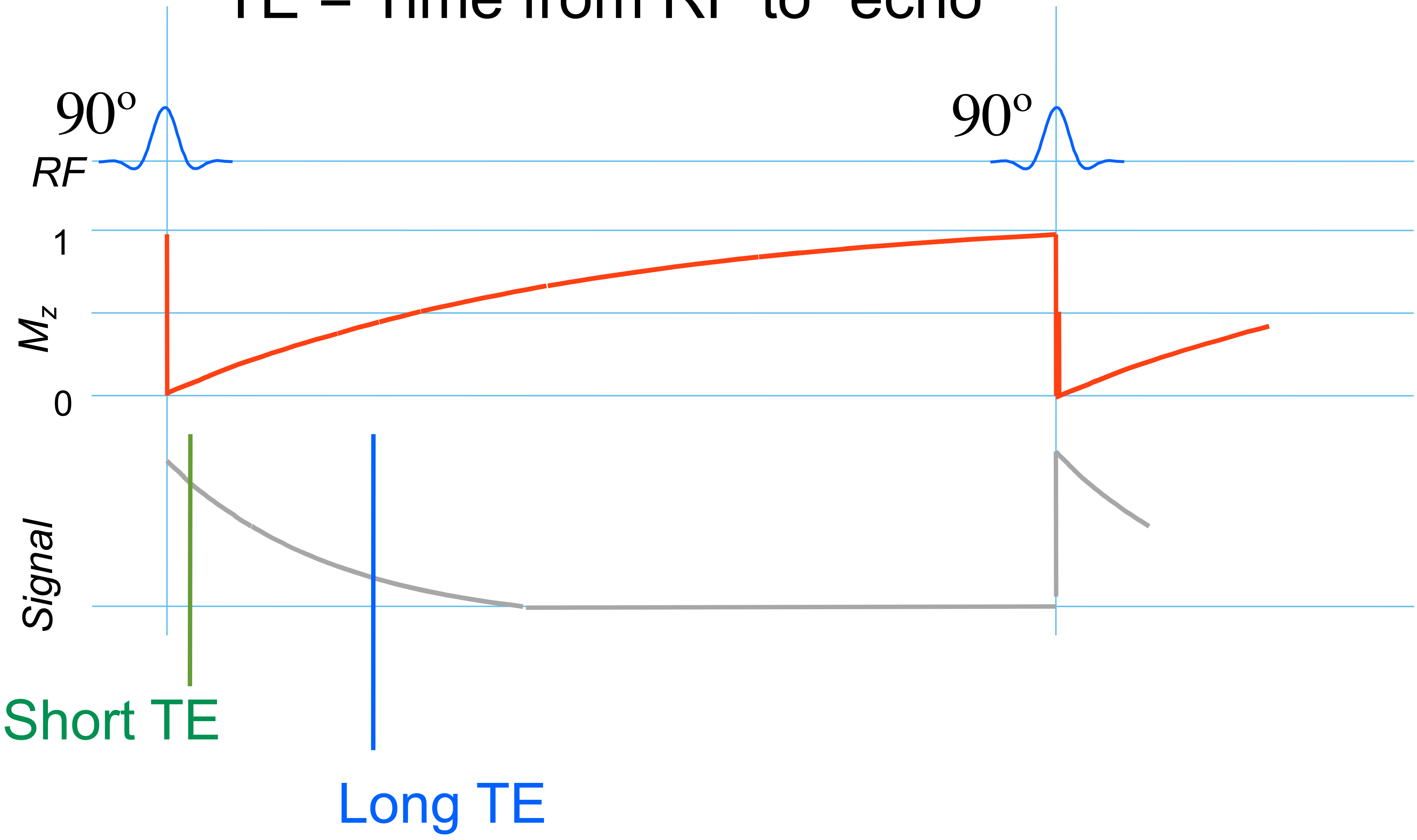


- Rotations due to excitation:

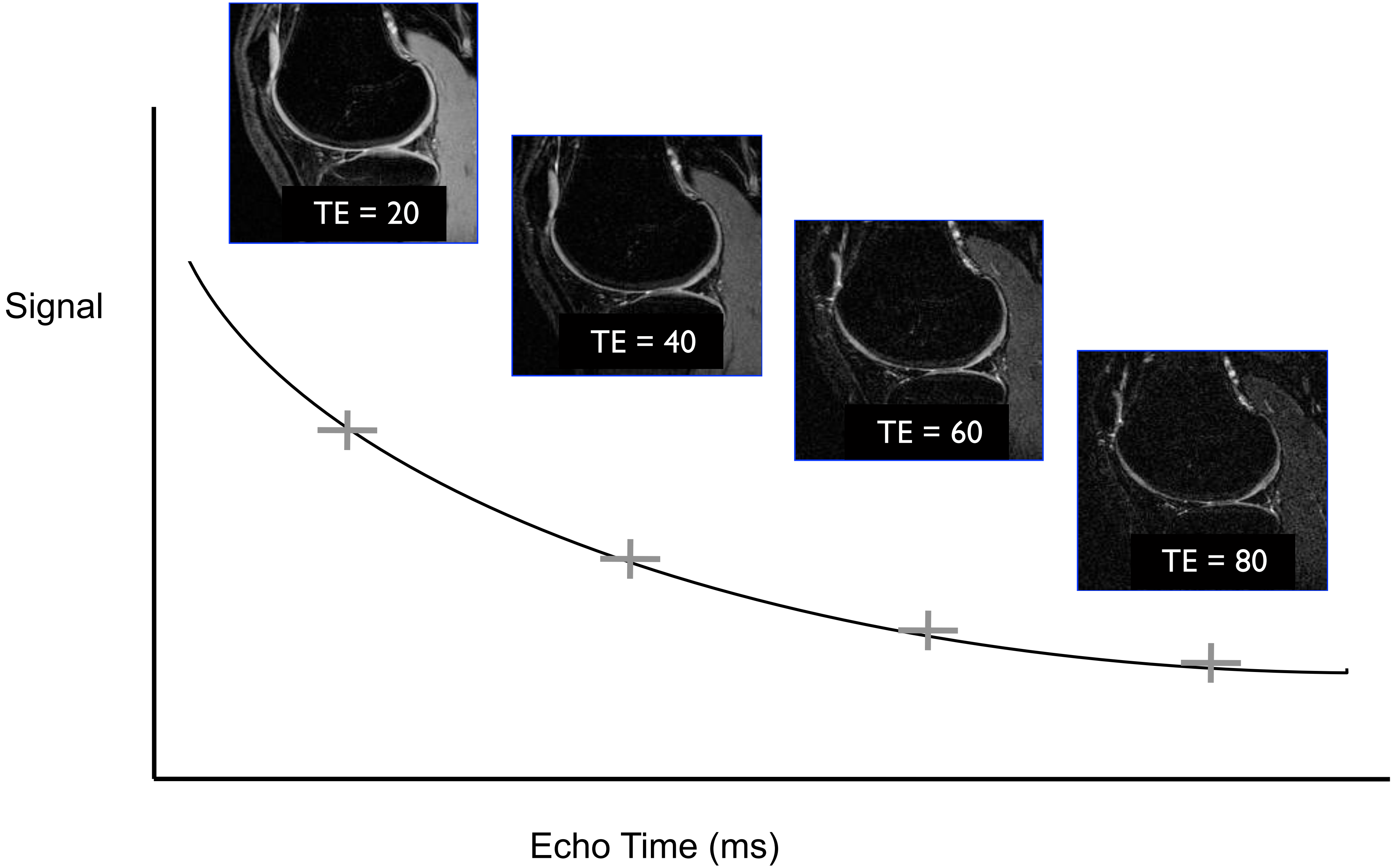
$$M'_{xy} = M_{xy}\cos\alpha + M_z\sin\alpha$$
$$M'_z = M_z\cos\alpha - M_{xy}\sin\alpha$$

Echo Time (TE): T_2 weighting

TE = Time from RF to "echo"

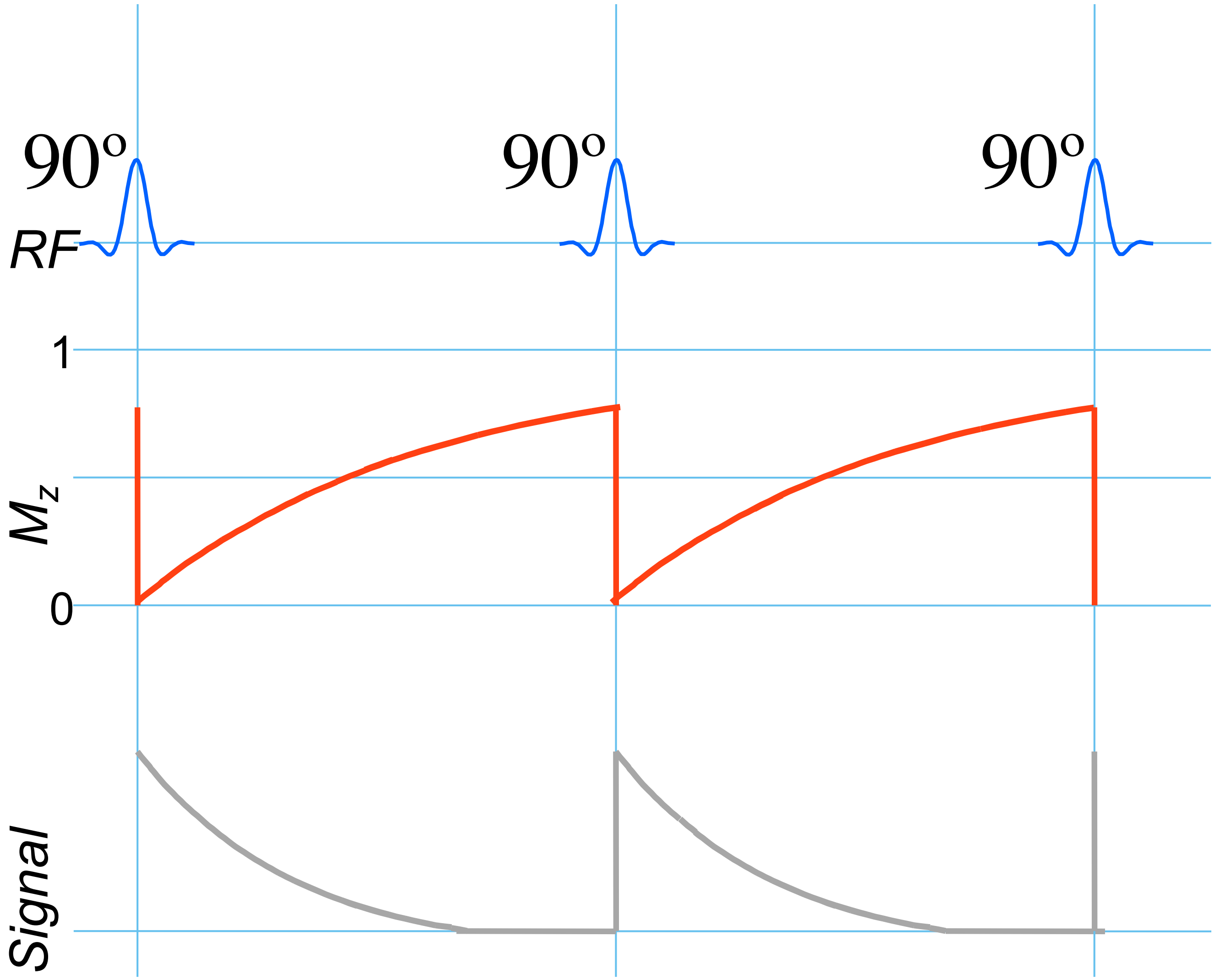


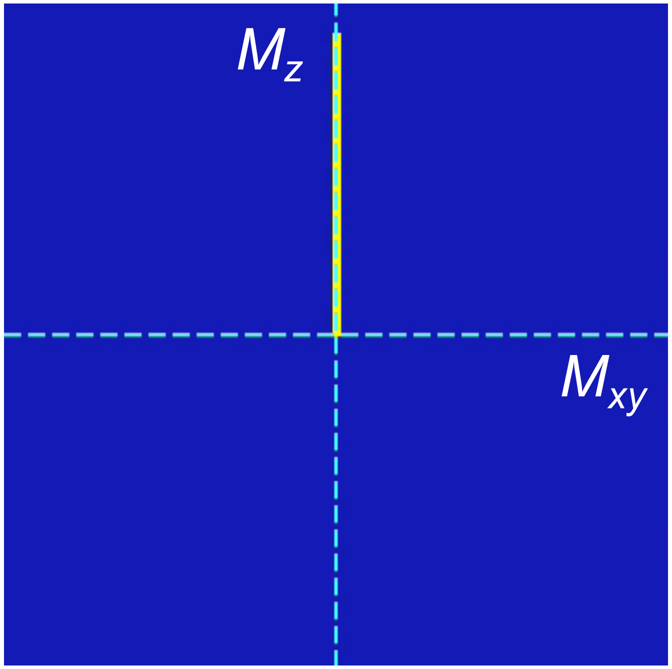
T_2 Contrast



Dardzinski BJ, et al. Radiology, 205: 546-550, 1997.

Repetition Time (TR) : T_1 Weighting



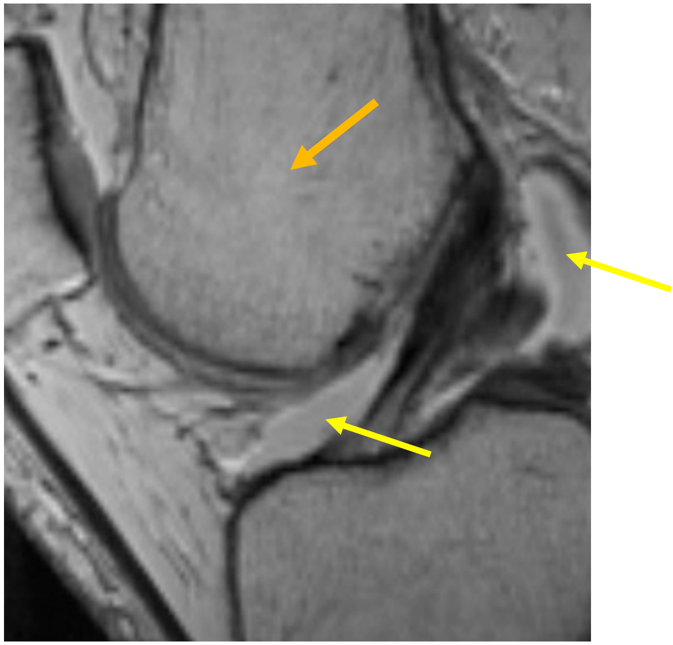


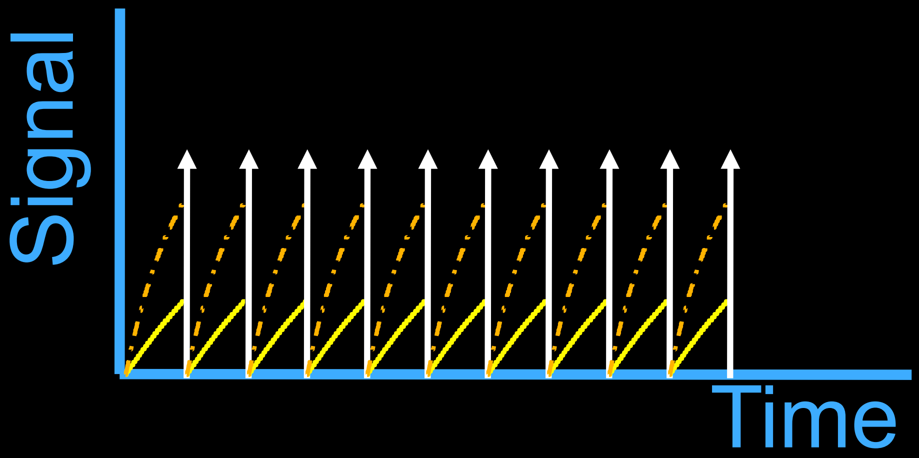
Each excitation starts with reduced M_z

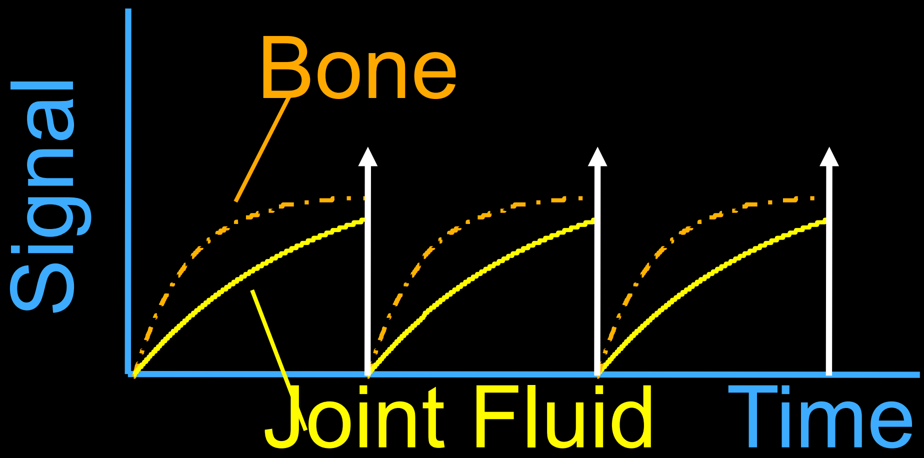
T_1-Weighted Spin Echo

Short Repetition

Long Repetition

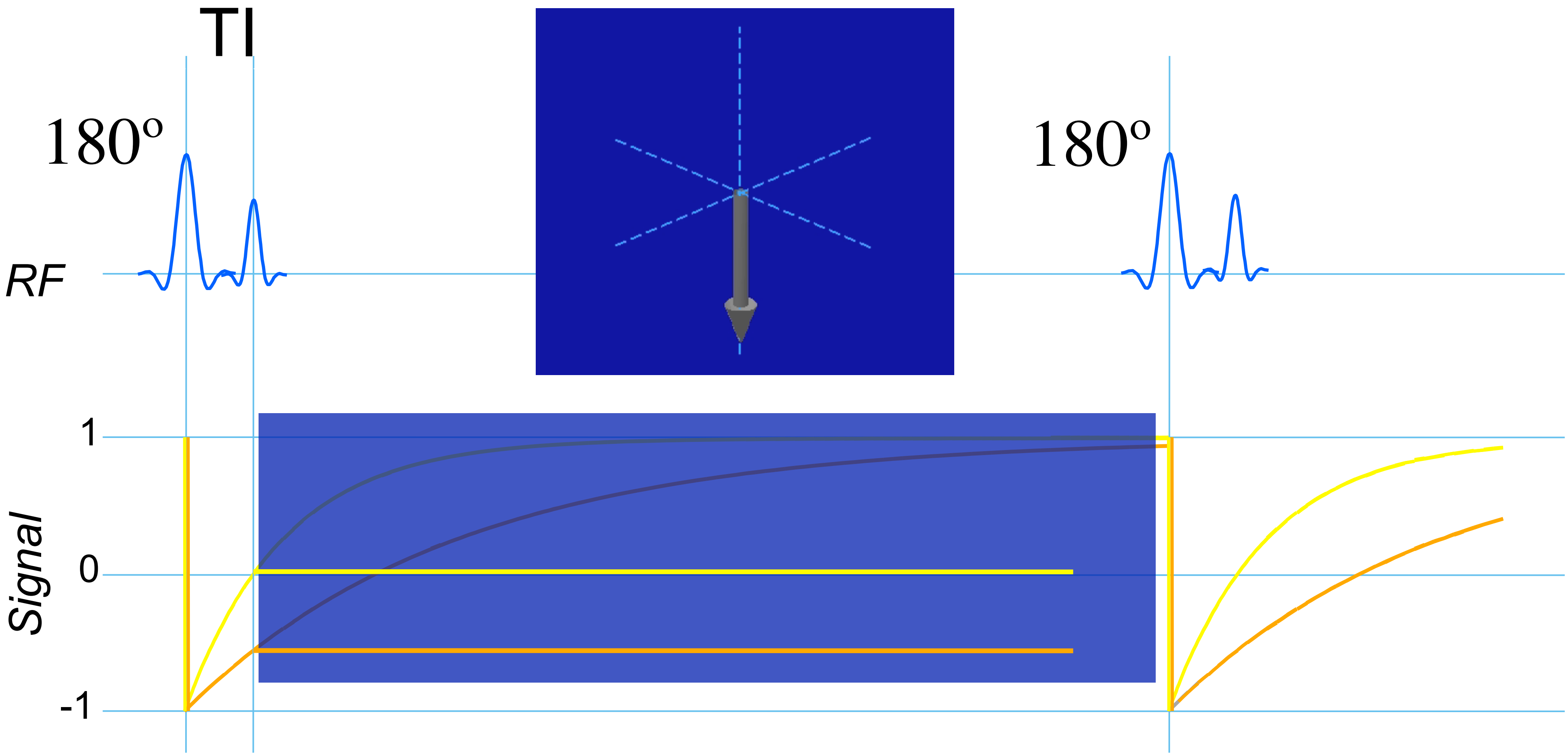






Basic Contrast Question (TE, TR)

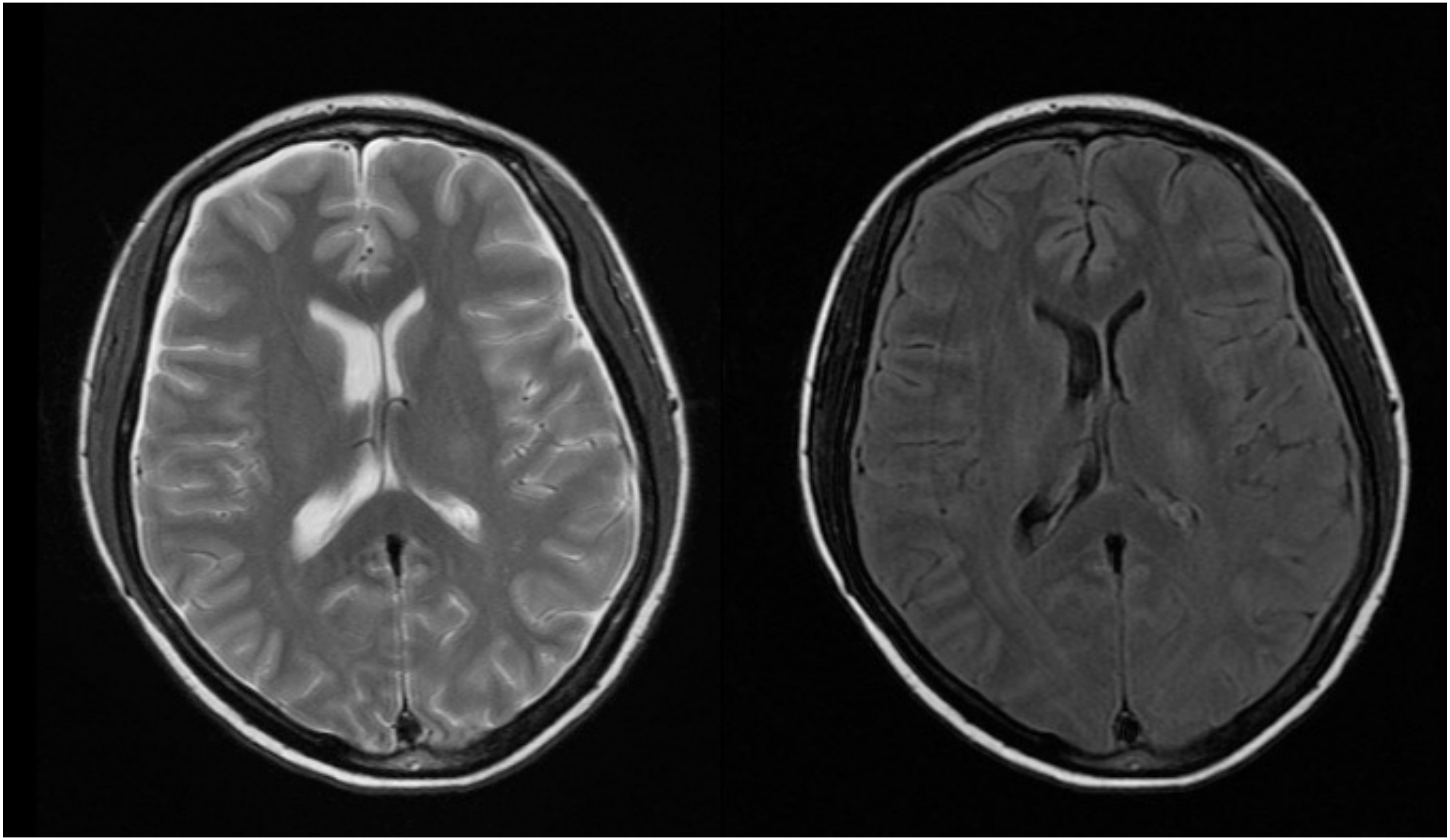
Inversion-Recovery



Fat suppression based on T_1

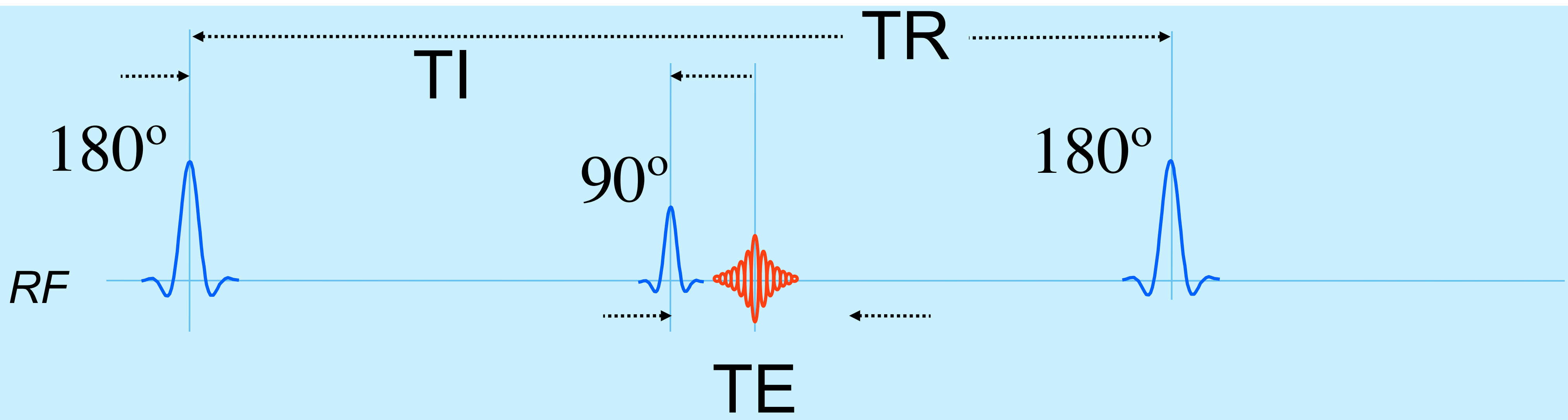
Short TI Inversion Recovery (STIR)

Long Inversion Time (TI) - FLAIR



Long TI suppresses fluid signal

Signal Question



Steady-State Sequences

- Repeated sequences always lead to a "steady state"
  - Sometimes includes equilibrium (easier)
  - Otherwise trace magnetization and solve equations
  - Example: Small-tip, TE=0

$$M_z(TE) = M_z(TR)\cos\alpha$$

$$M_z(TR) = M_0 + [M_z(TE) - M_0]e^{-TR/T_1}$$

Combining... $$M_z(TR) = M_0 \frac{1 - e^{-TR/T_1}}{1 - e^{-TR/T_1}\cos\alpha}$$

Summary ~ Background I

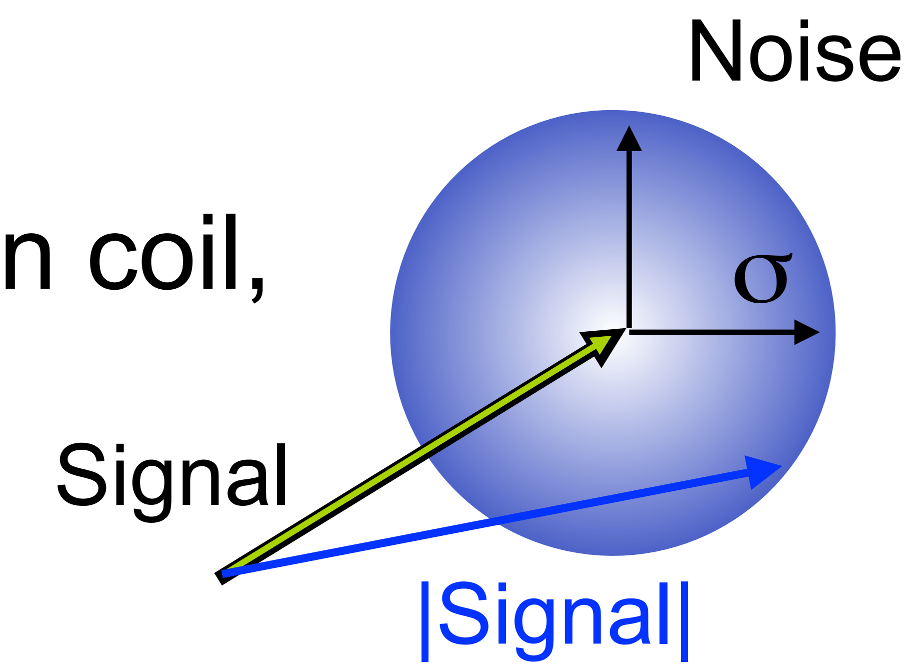
- Overview of NMR
- Hardware
- Image formation and k-space
- Excitation k-space
- Signals and contrast

Background II

- Signal-to-Noise Ratio (SNR)
- Pulse Sequences
- Sampling and Trajectories
- Parallel Imaging

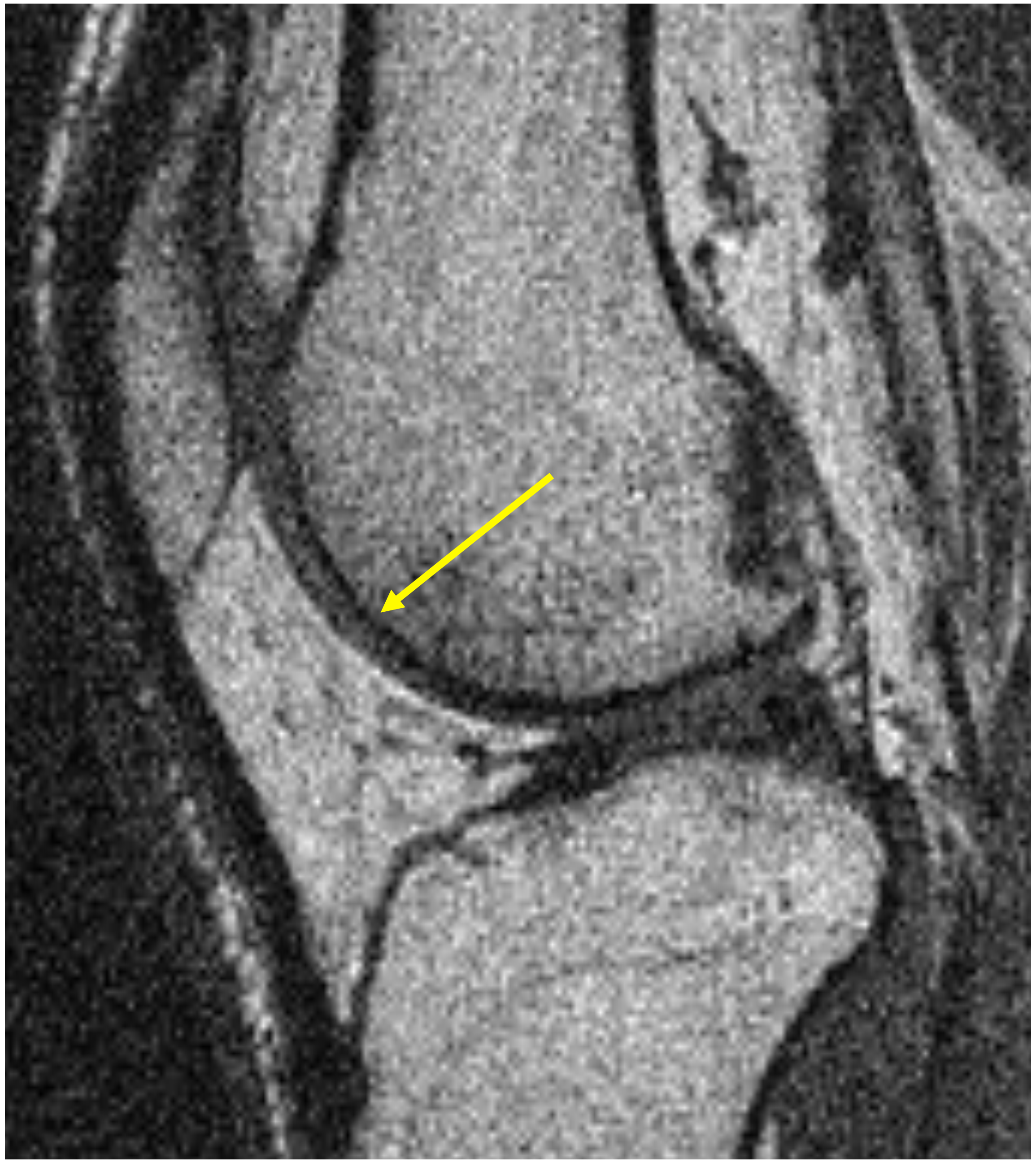
SNR: Signal-to-Noise Ratio

- Signal: Desired voltage in coil
- Noise: Thermal, electronic
  - Thermal dominates, depends on coil, patient size
- SNR = average signal / σ
  - Gaussian noise (FT is gaussian)
  - σ is for gaussian in real <u>and</u> imaginary signal components
  - N averages = sqrt(N) increase
- Magnitude noise is Rician; can obtain σ



SNR

SNR is the major limitation for MRI



Low SNR

High SNR

Averaging

- Noise is *uncorrelated*
- When adding two signals:
  - Signal portion M adds, to 2M
  - Noise variance σ^2 adds, increases to $2\sigma^2$
  - Noise σ increases by square-root of 2
  - SNR changes from M/σ to $1.4\ M/\sigma$
- SNR increases with *square-root* of #averages

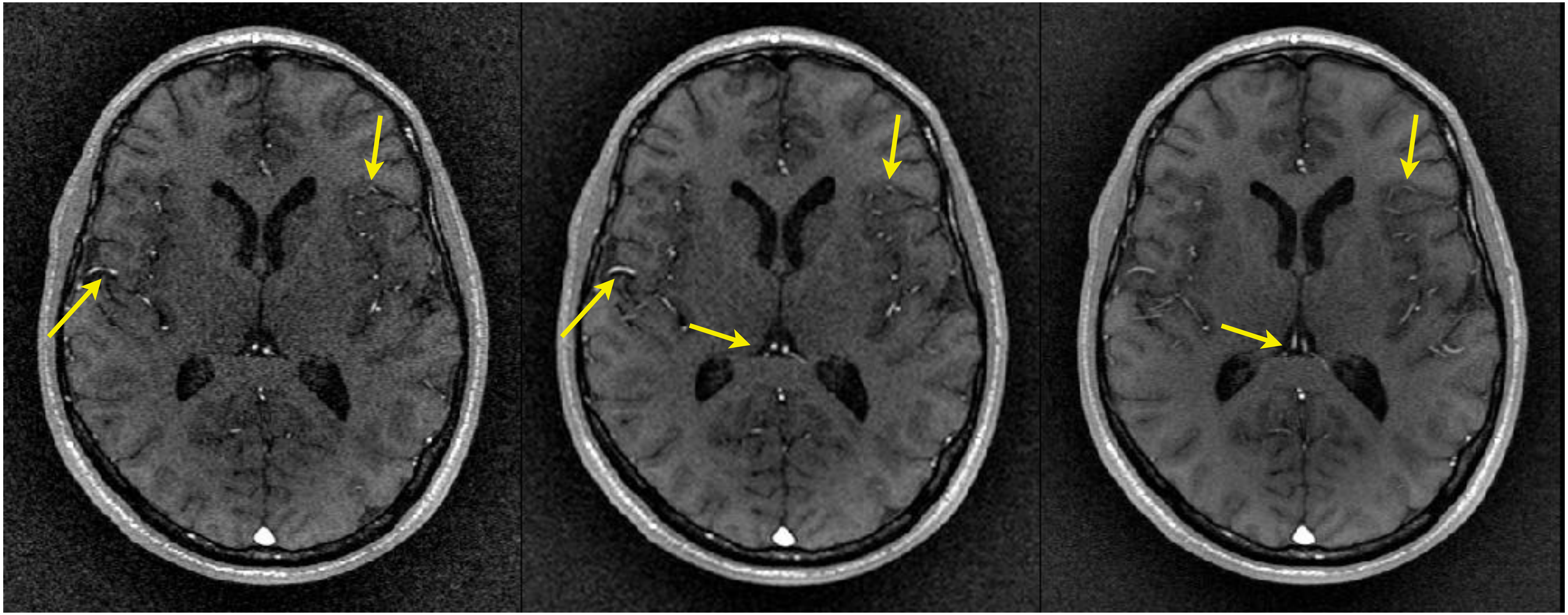
What are Examples of Averaging?

Imaging Factors Influencing SNR

- Voxel size (spatial resolution)
- Acquisition time (NEX, BW)
- Polarization or Field strength
- RF coil
- Subject size
- Pulse sequence and parameters
- Receive Electronics (Ideally insignificant)

Voxel Size Example

Full High Resolution 2x Increase (all 3 axes) 4x Increase (slice)



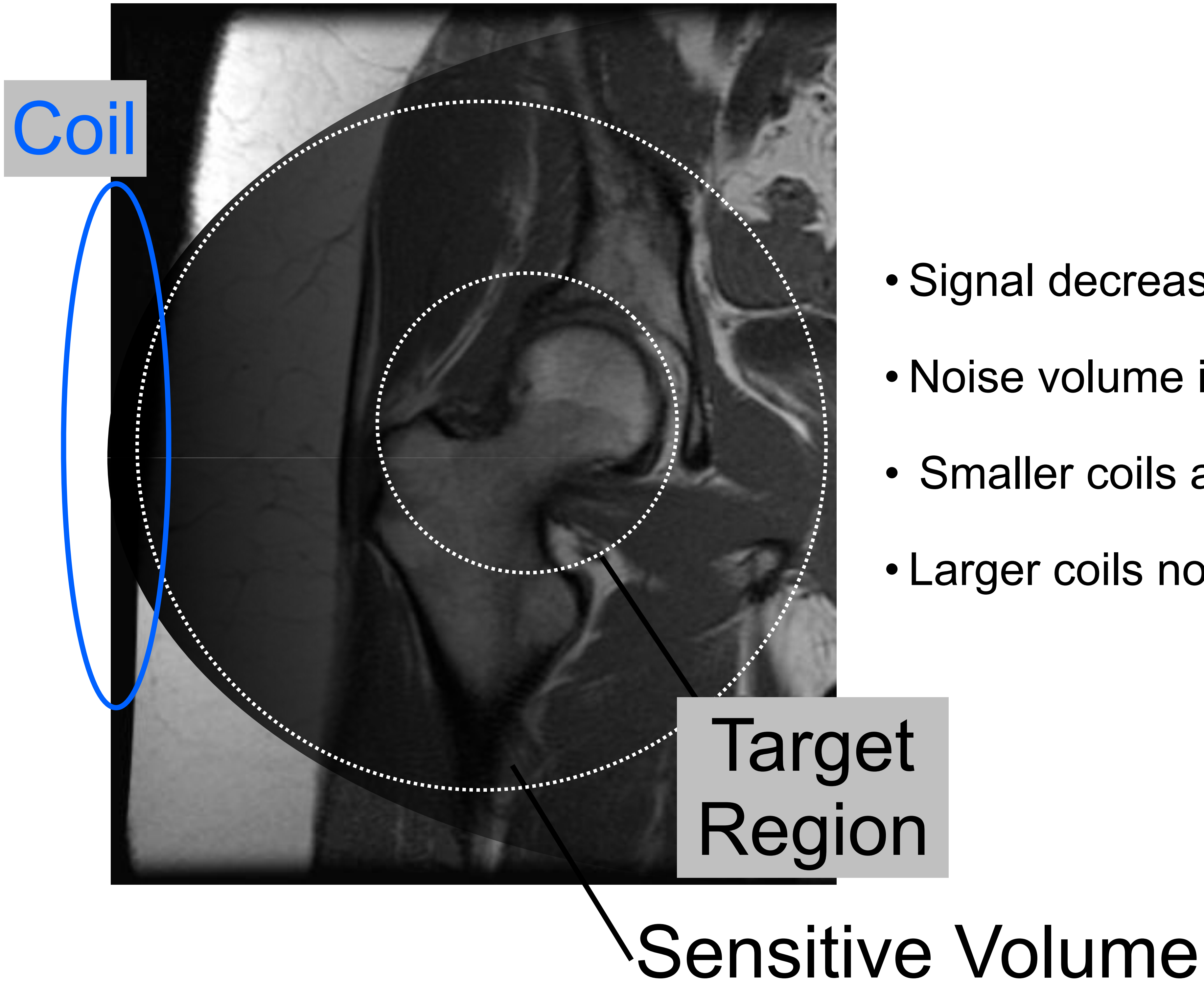
SNR and Field Strength

1.5T

3.0T

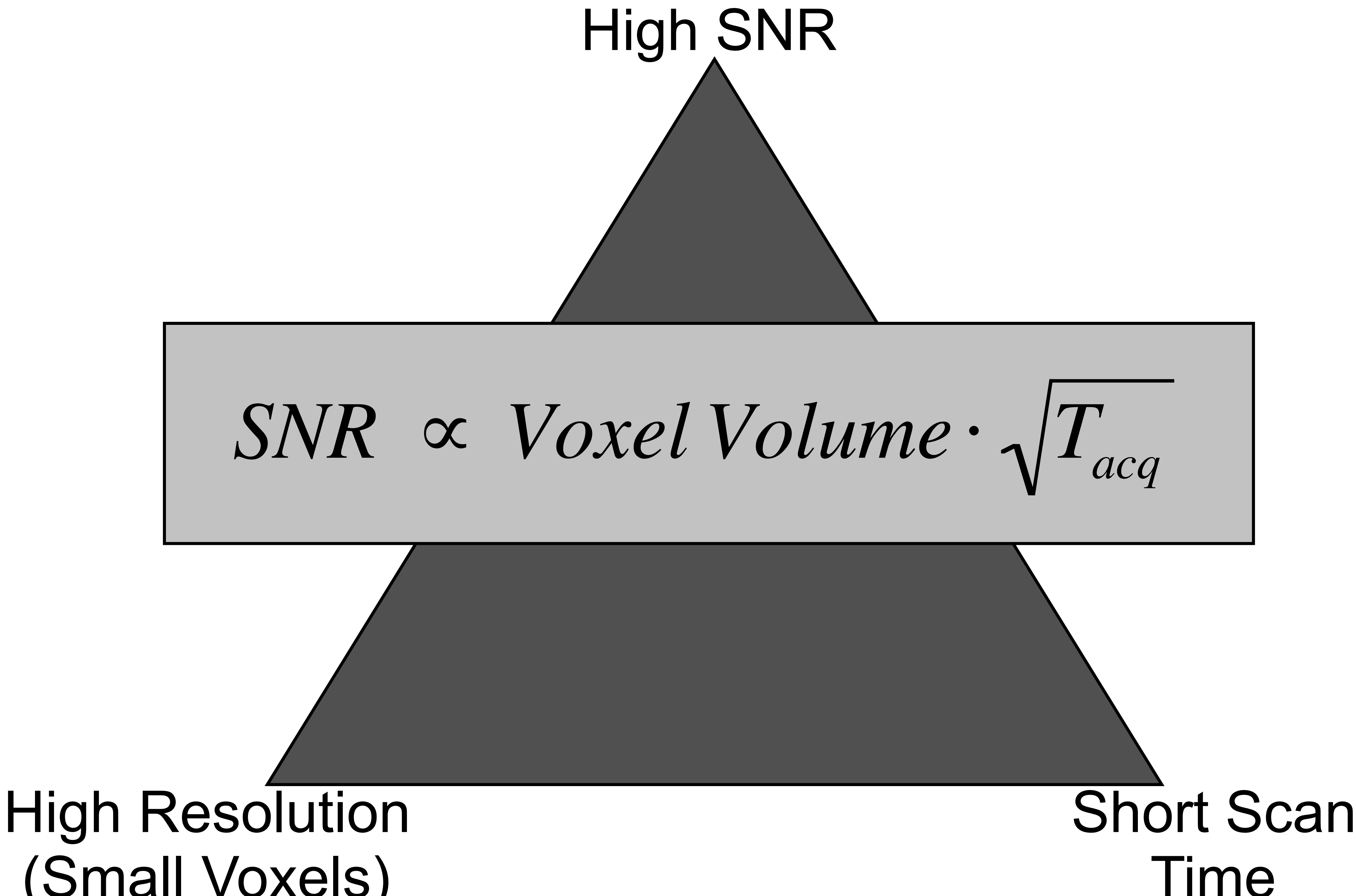
Sagittal T_2 RARE: SNR Ratio = 1.7

Coil Sensitivity



- Signal decreases further from coil
- Noise volume increases with coil size
- Smaller coils also limit FOV and aliasing
- Larger coils not ideal

SNR vs Resolution vs Scan Time



SNR Efficiency

- Often want to compare SNR of different sequences
- If times differ, comparison can be made fair by use of SNR efficiency:

$$\eta_{SNR} = \frac{SNR}{\sqrt{T_{scan}}}$$

- In many cases:

$$\eta_{SNR} = \frac{SNR}{\sqrt{TR}}$$

SNR Question

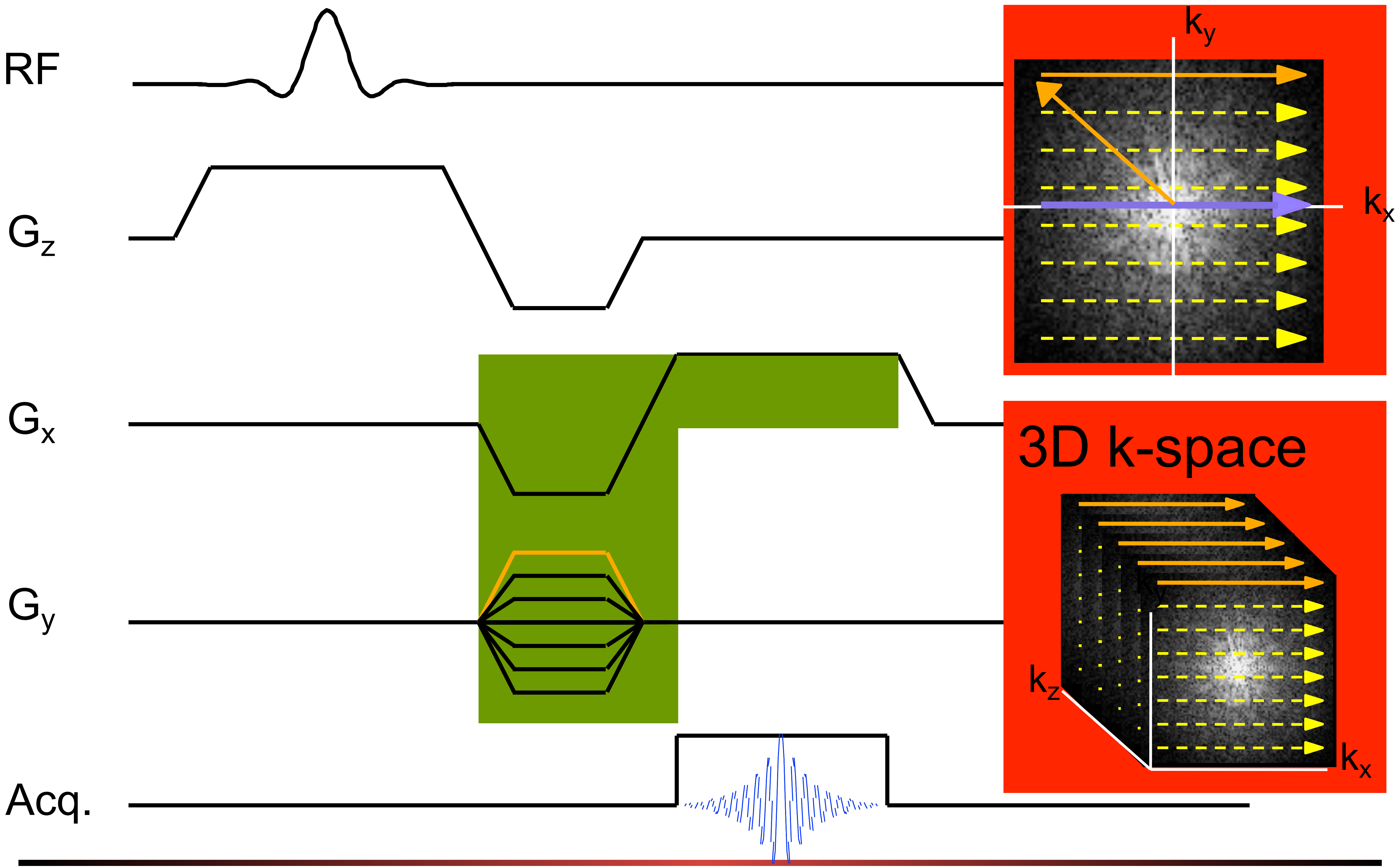
- Compare the SNR efficiency of two pulse sequences, assuming the signal level is constant:
  - Spin Echo, 8echoes, 32.25 kHz bandwidth, TR=100ms
  - Simple gradient echo, 62.5 kHz bandwidth, TR=5ms
- *Signal level would **NOT** be constant, so this is harder!*

Pulse Sequences

- Gradient Echo Sequences
- Spin Echo Sequences
- Preparation Sequences

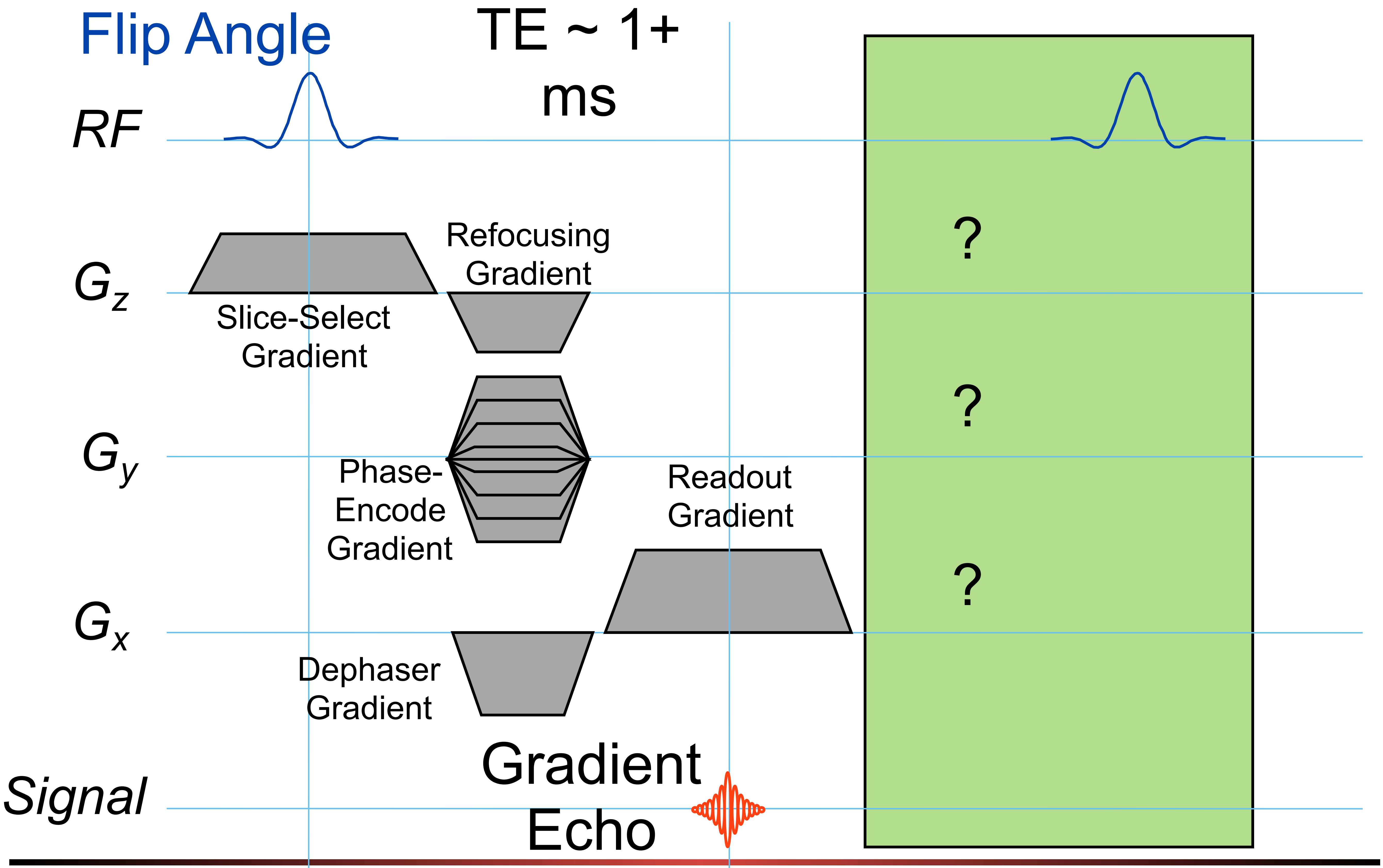
(We will expand on these a lot!)

Pulse Sequences and k-space

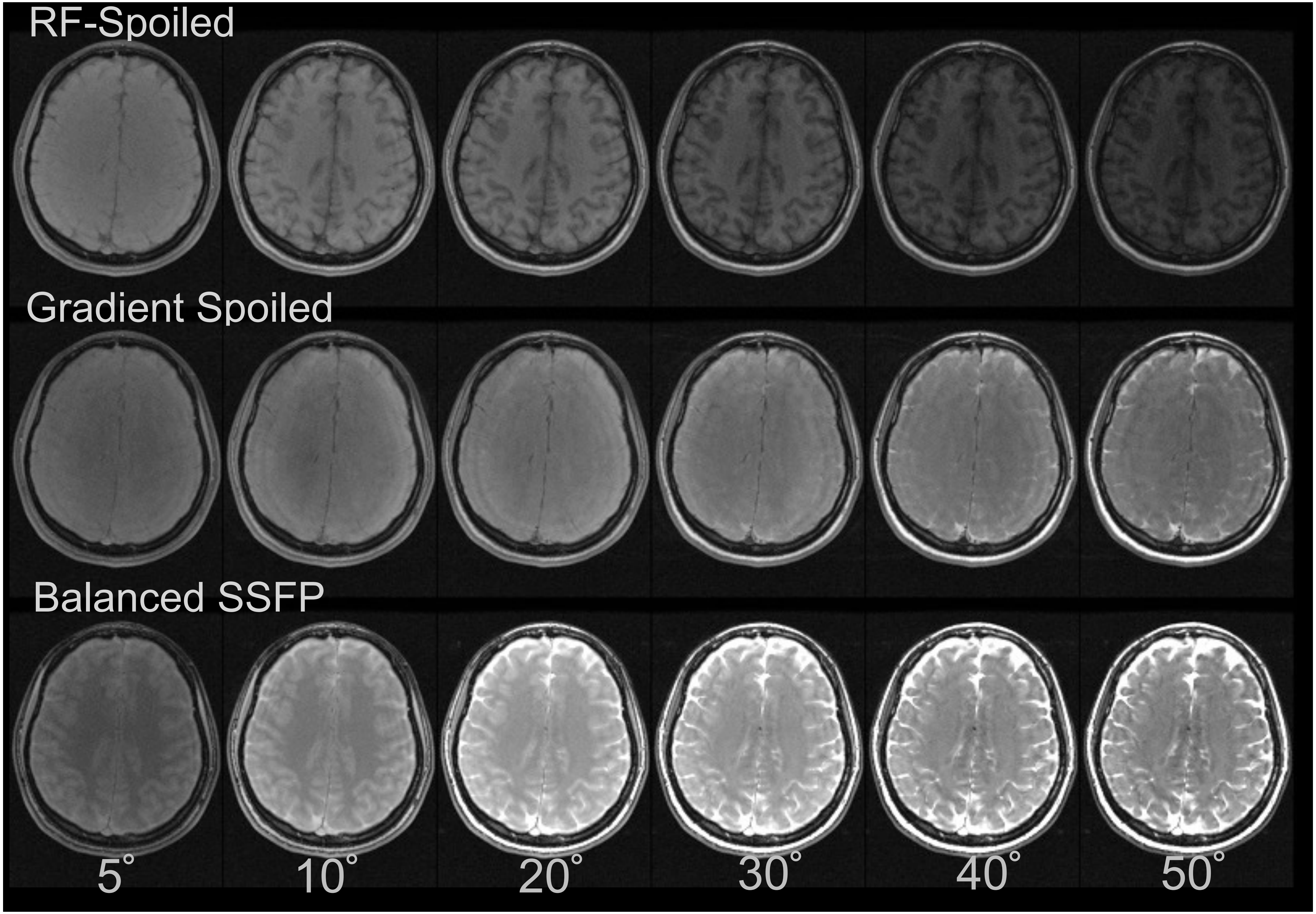


Sequence Questions

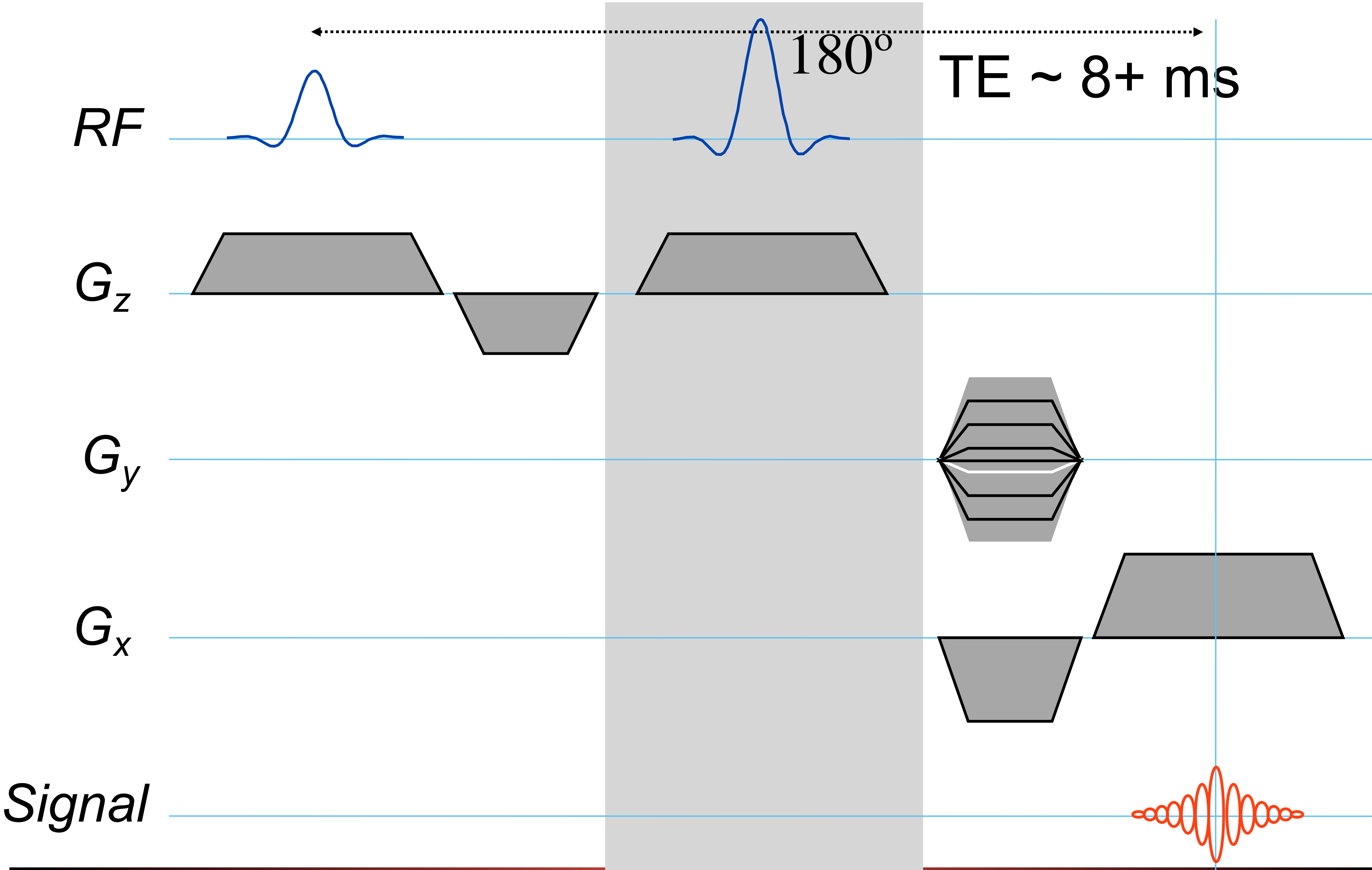
Gradient-Echo Pulse Sequence



Gradient Echo Contrasts



Spin Echo Pulse Sequence



Basic Spin Echo Considerations

Pros:

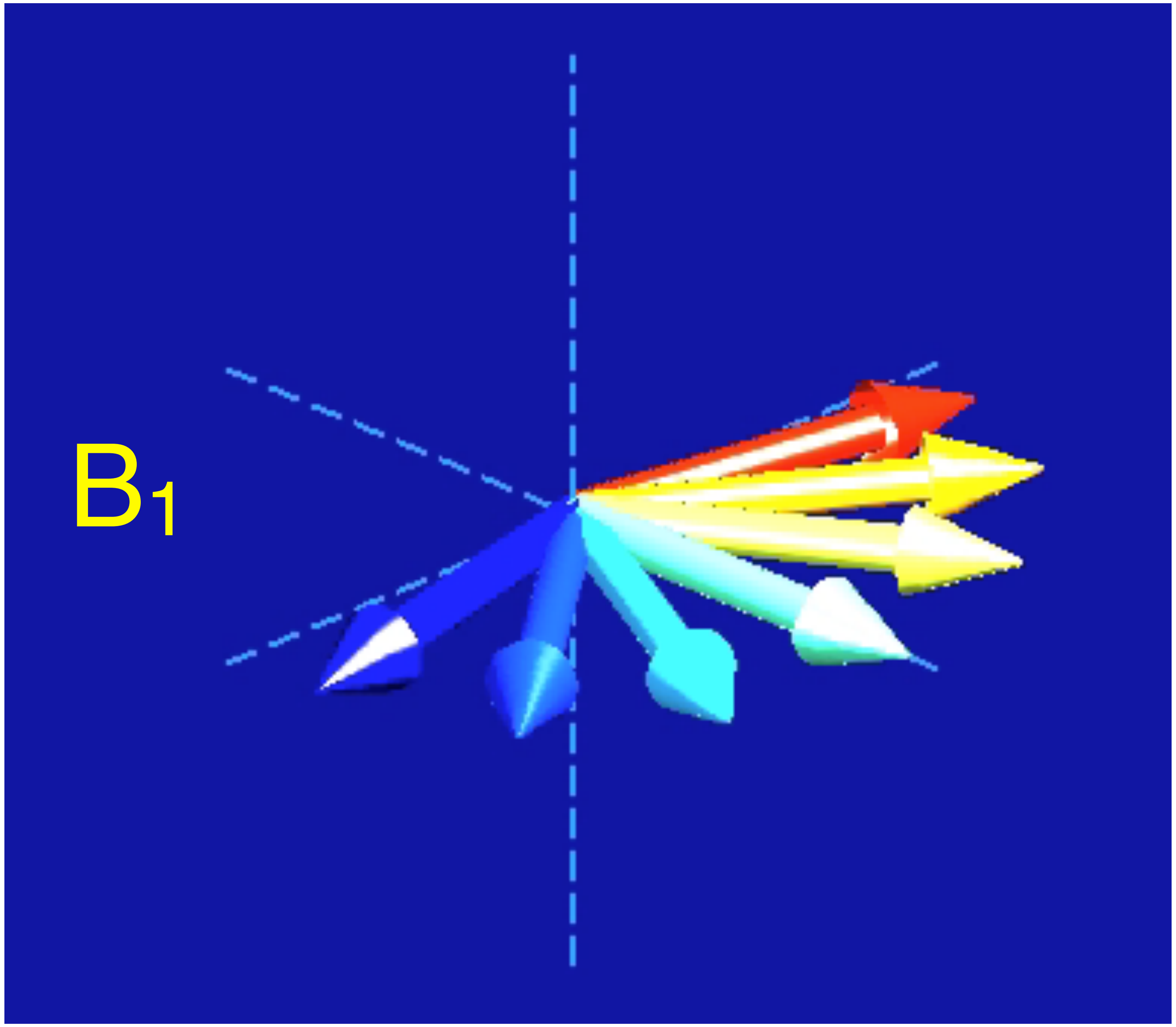
- Refocusing pulse reverses dephasing
- Image acquired at spin echo increases signal

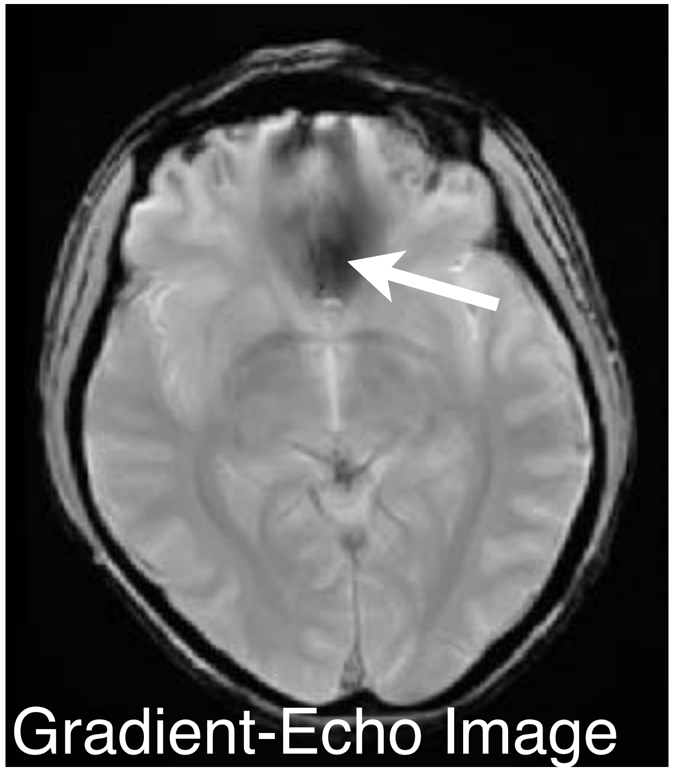
Cons:

- Increased RF power deposition (SAR)
- Longer echo times than gradient echo (GRE)

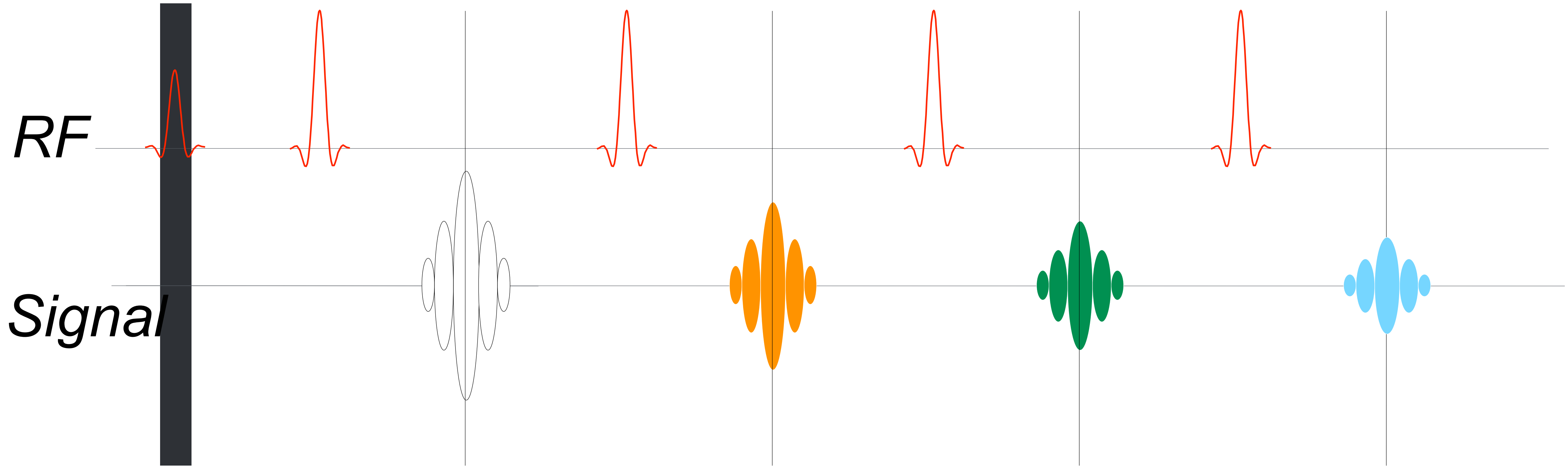
Spin Dephasing and Spin Echoes

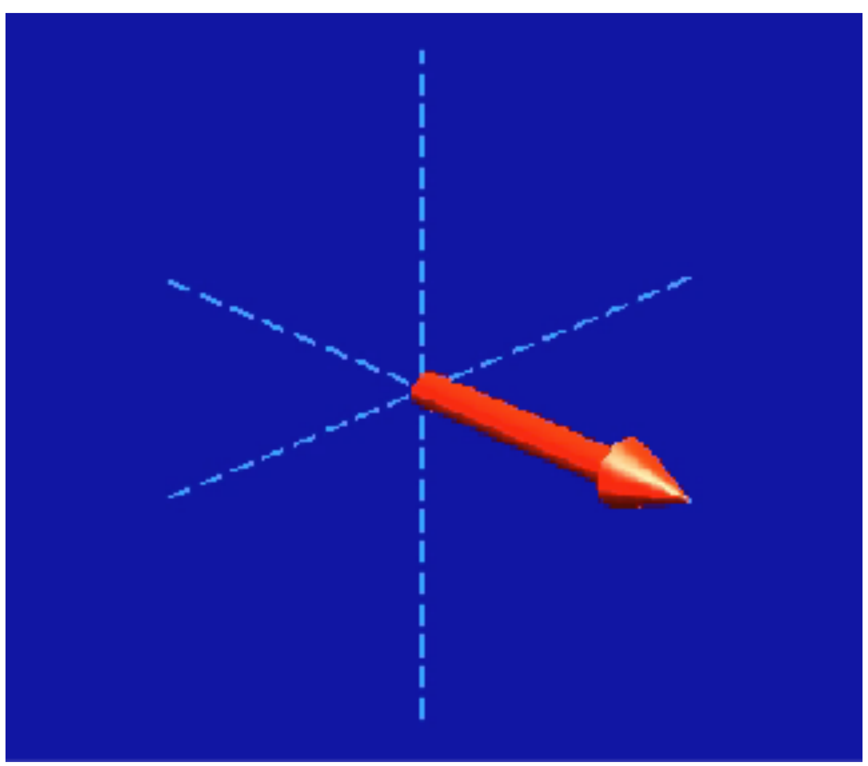
- Frequency variations cause "dephasing" (T_2')
- Results in signal loss (T_2*)
- Refocus spins to spin-echo (T_2)

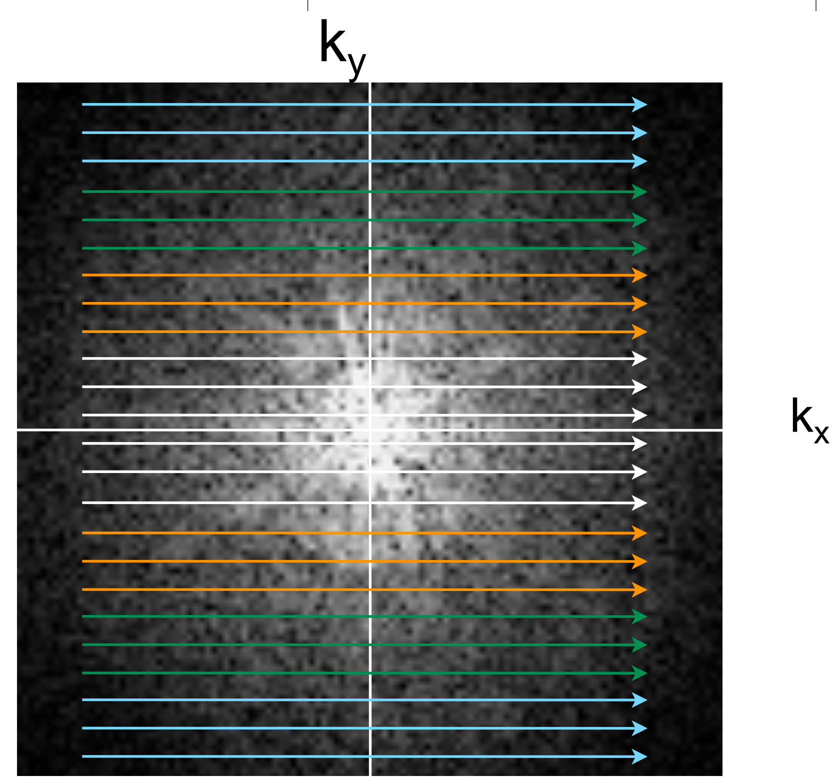




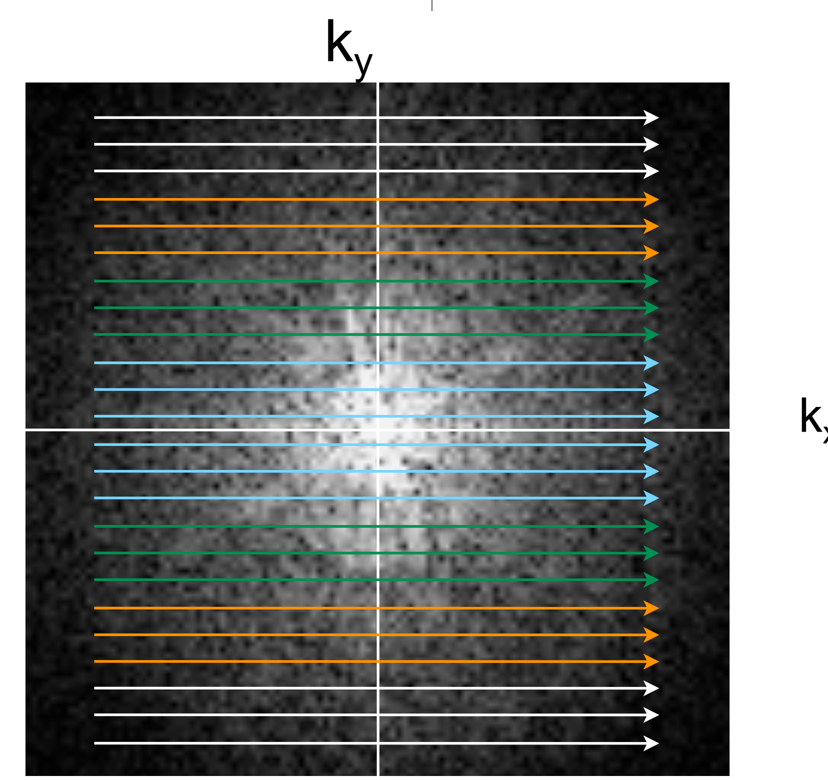
Spin-Echo-Train Imaging







PD-weighted k-space

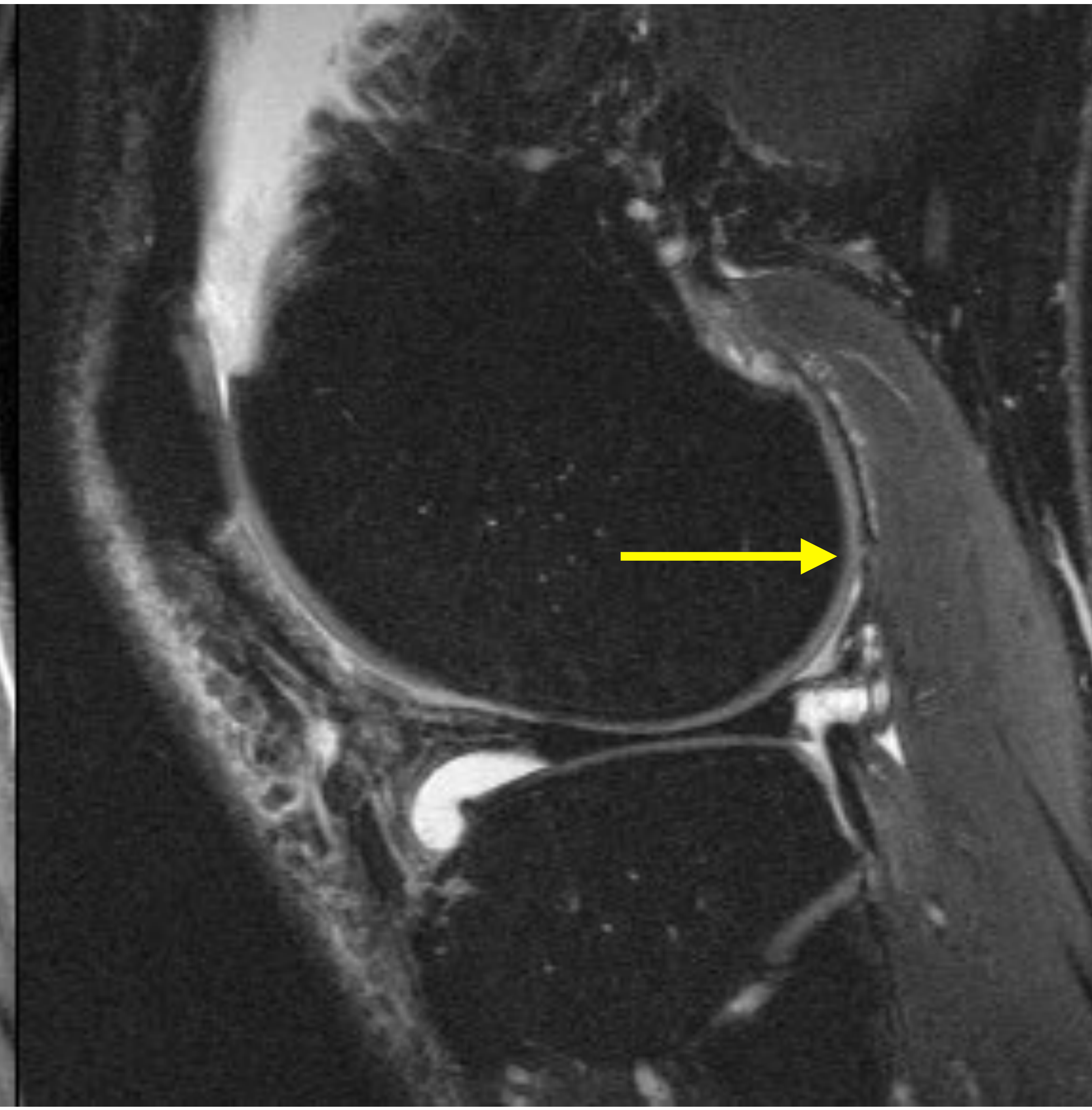


T_2-weighted k-space

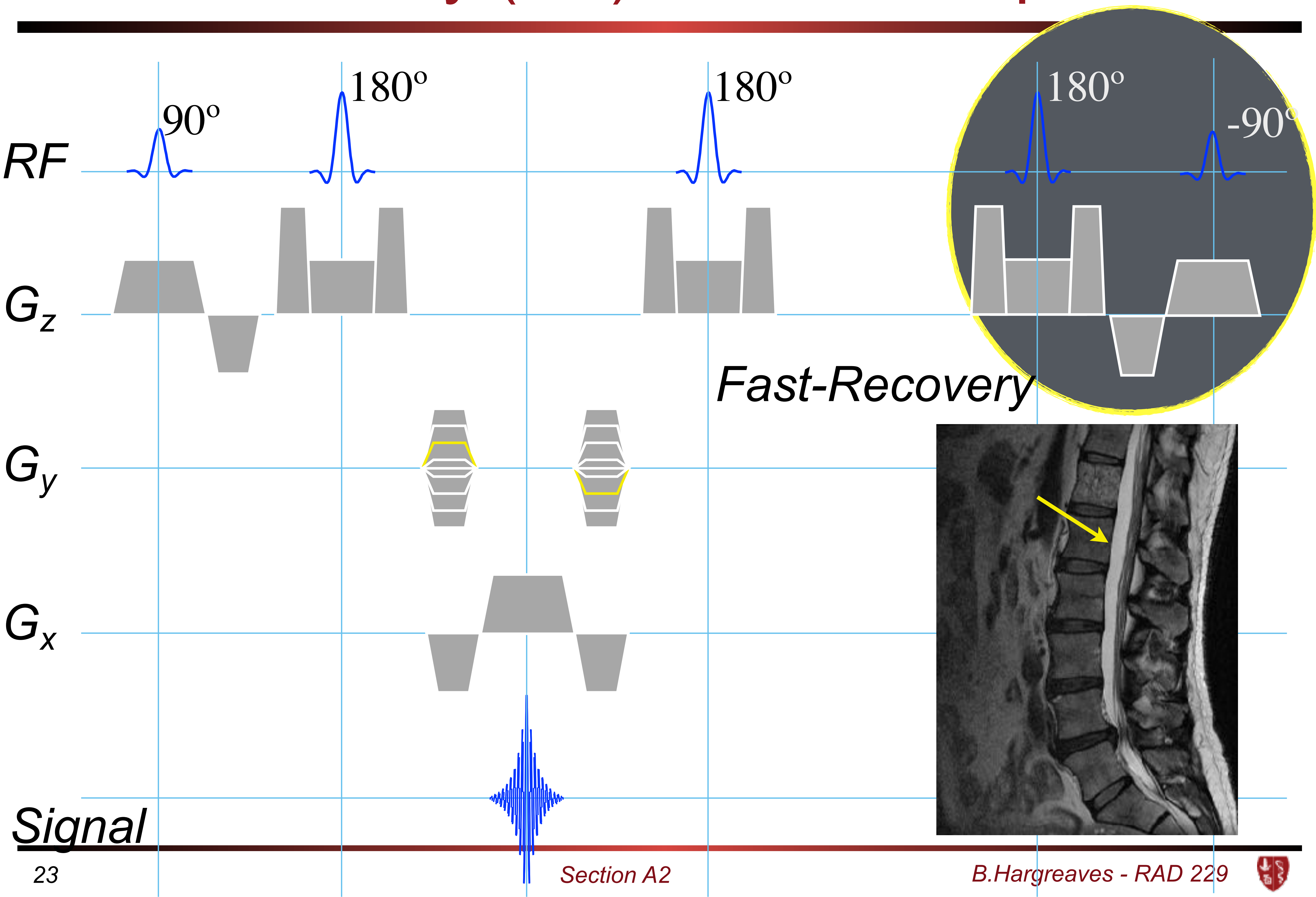
Proton-Density and T_2-weighted Spin Echo

Proton Density Weighted

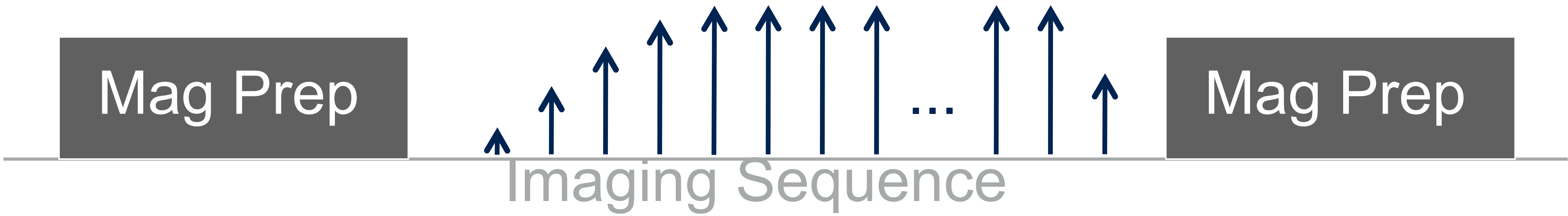
T_2 Weighted



Fast Recovery (FR) or Driven Equilibrium



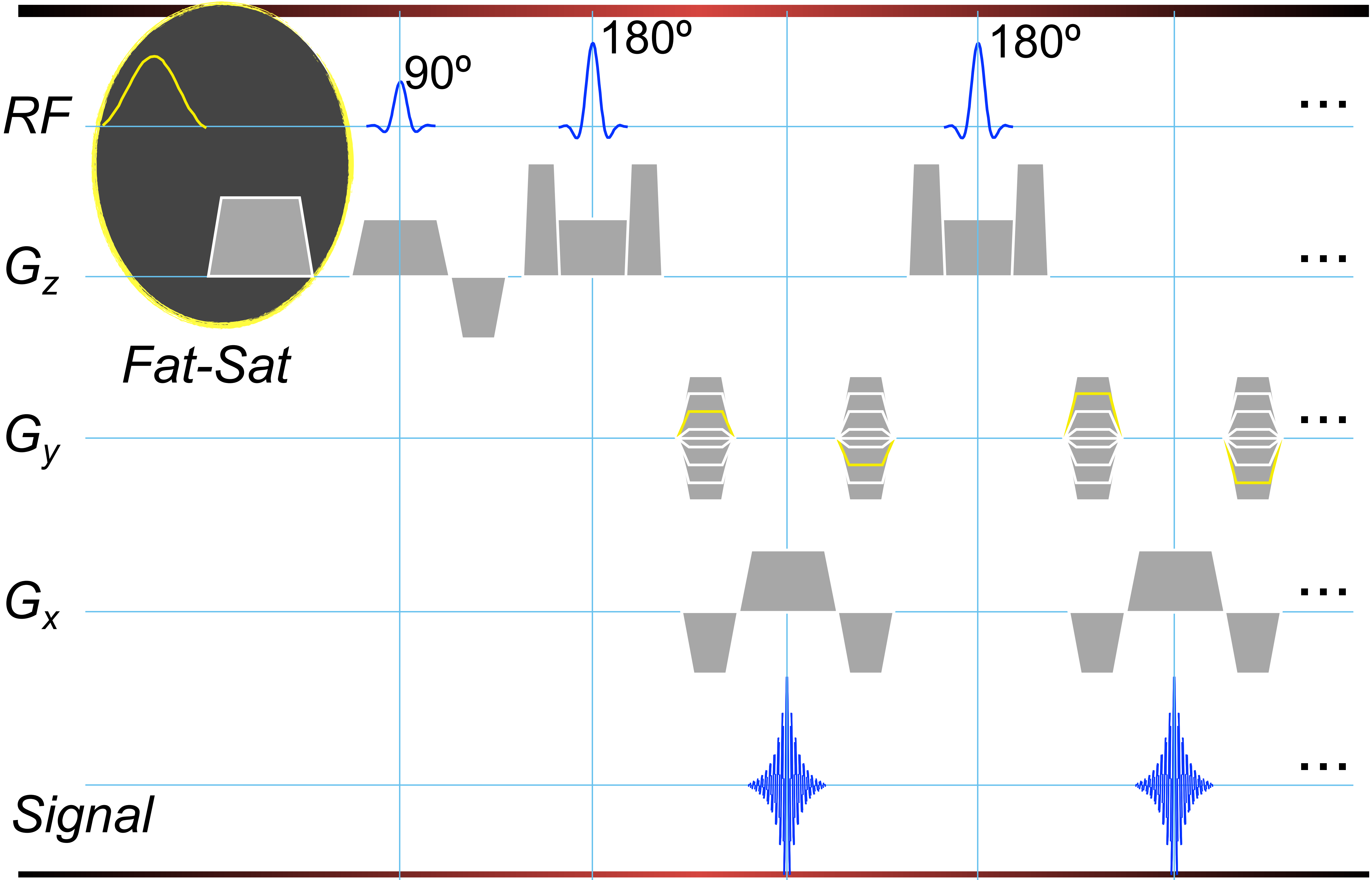
Magnetization Preparation



- "Prepare" contrast
- Image rapidly before steady-state evolves

- Examples:
  - Fat Saturation
  - Inversion - Recovery
  - Myocardial Tagging
  - T2-prep
  - Magnetization Transfer

Fat-Saturated FSE

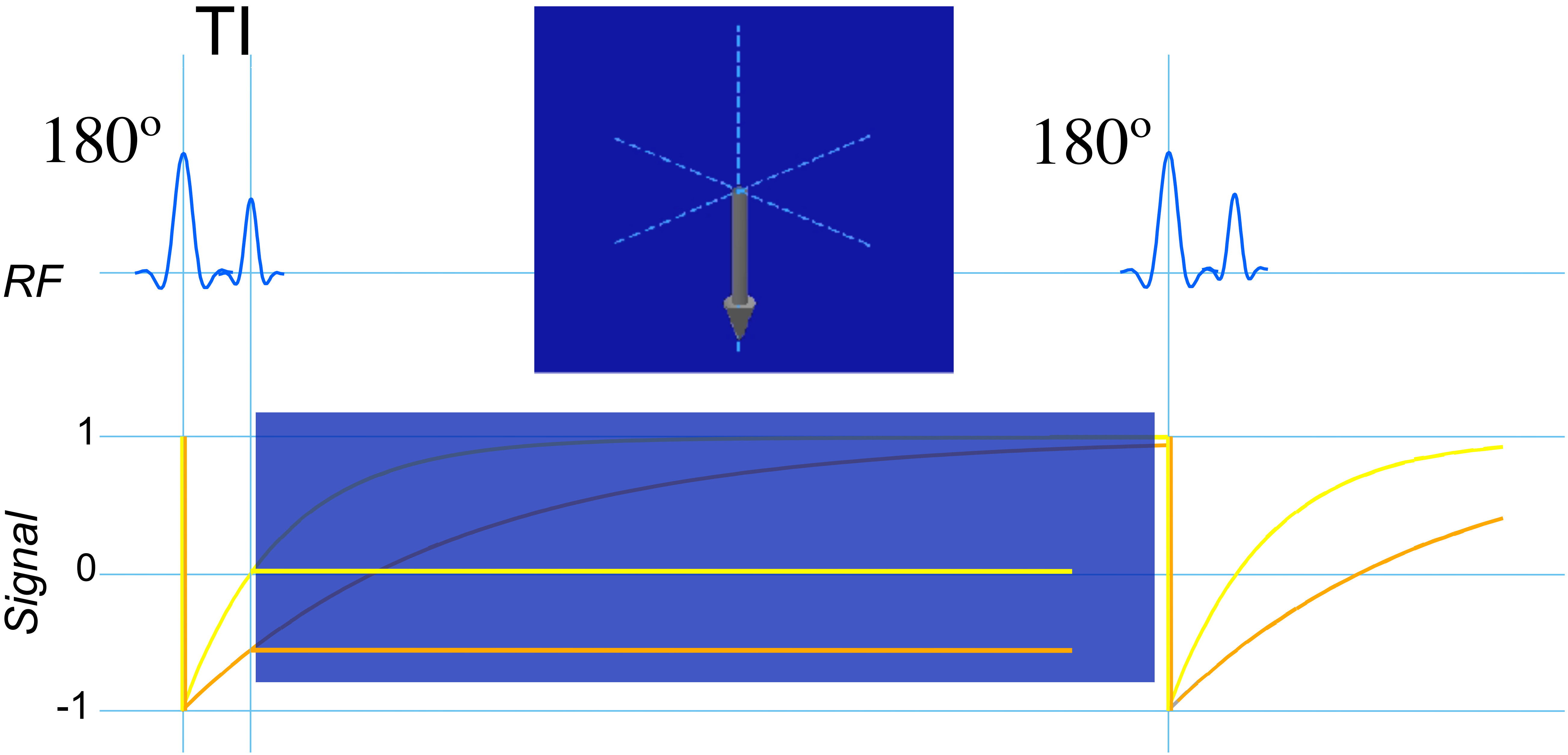


Fat Saturation (Magnetization Preparation)

Fat Saturated

T1w FSE

Inversion-Recovery

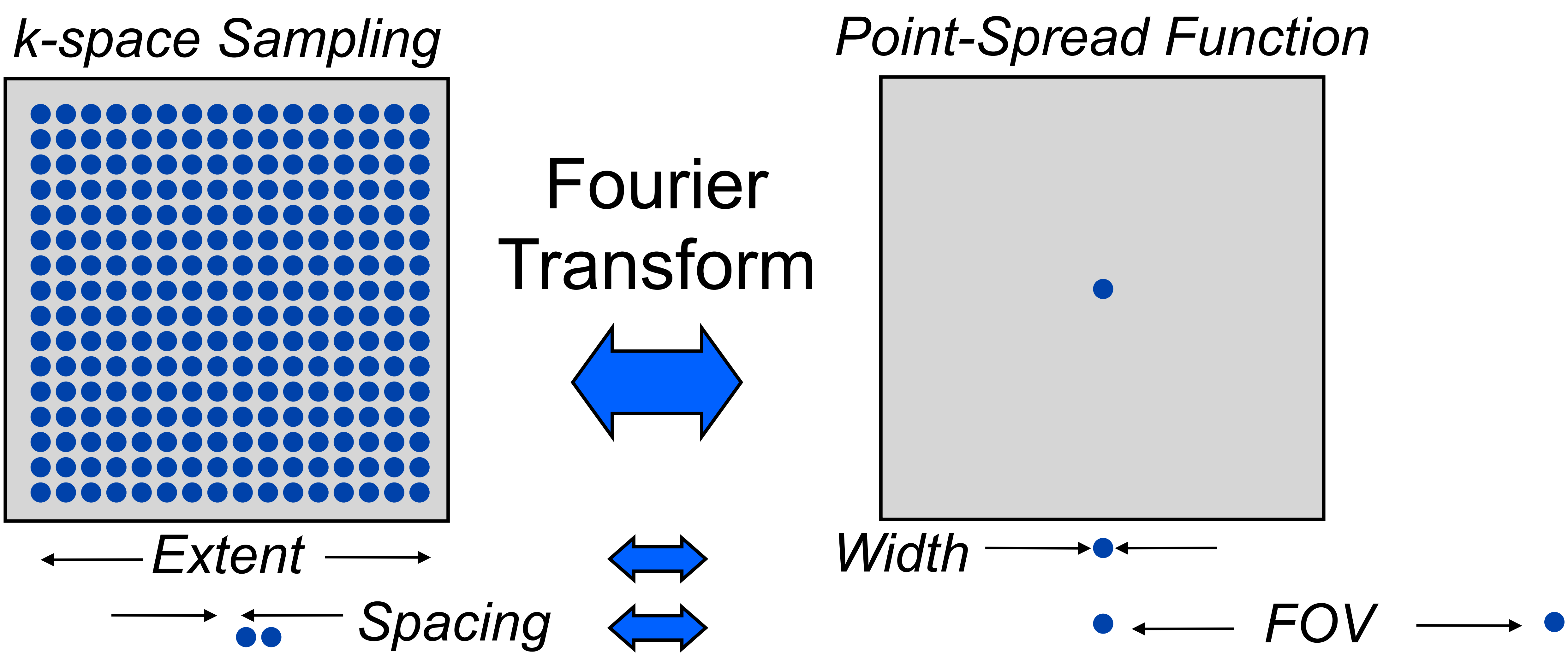


Fat suppression based on T_1

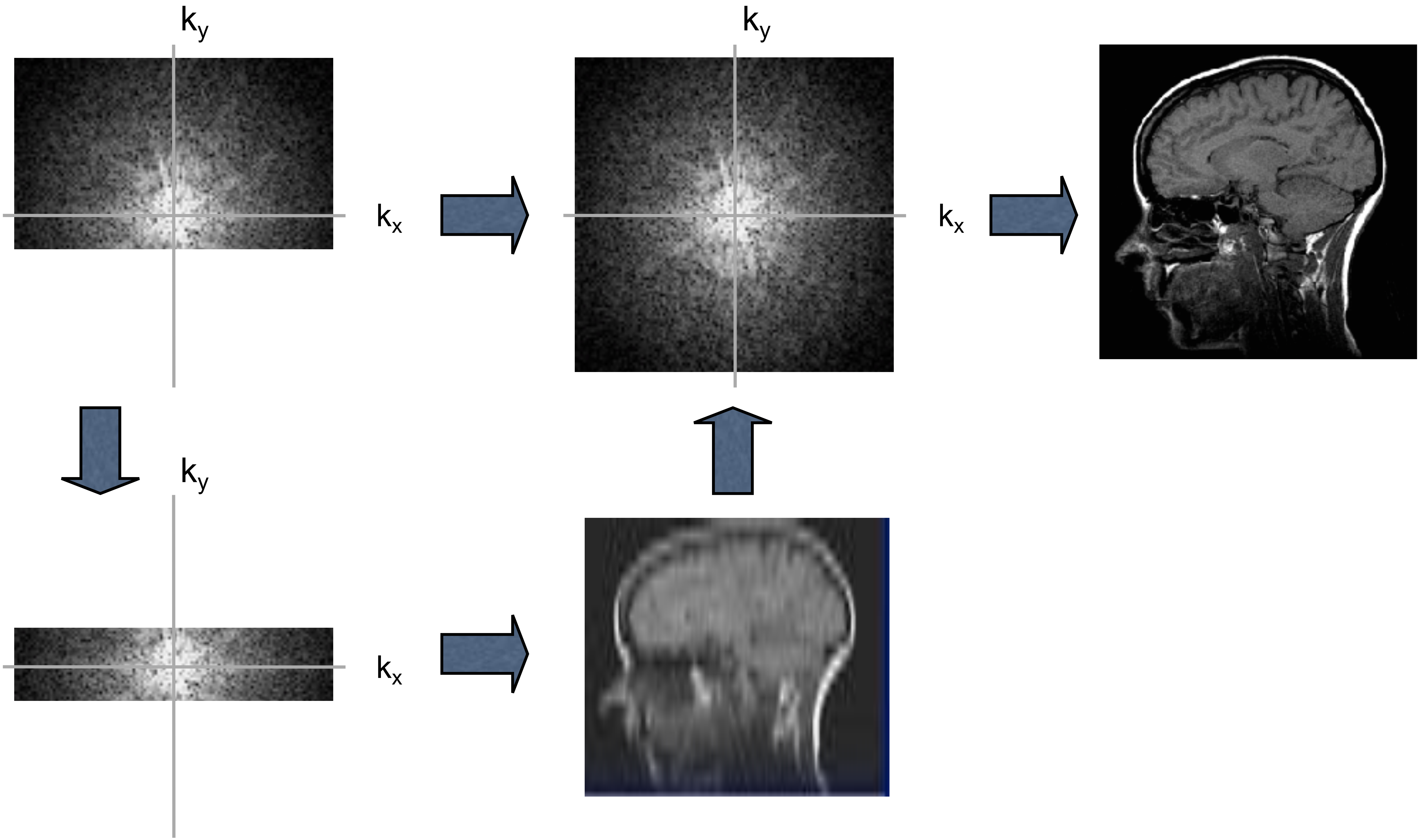
Short TI Inversion Recovery (STIR)

Sampling & Point-Spread Functions

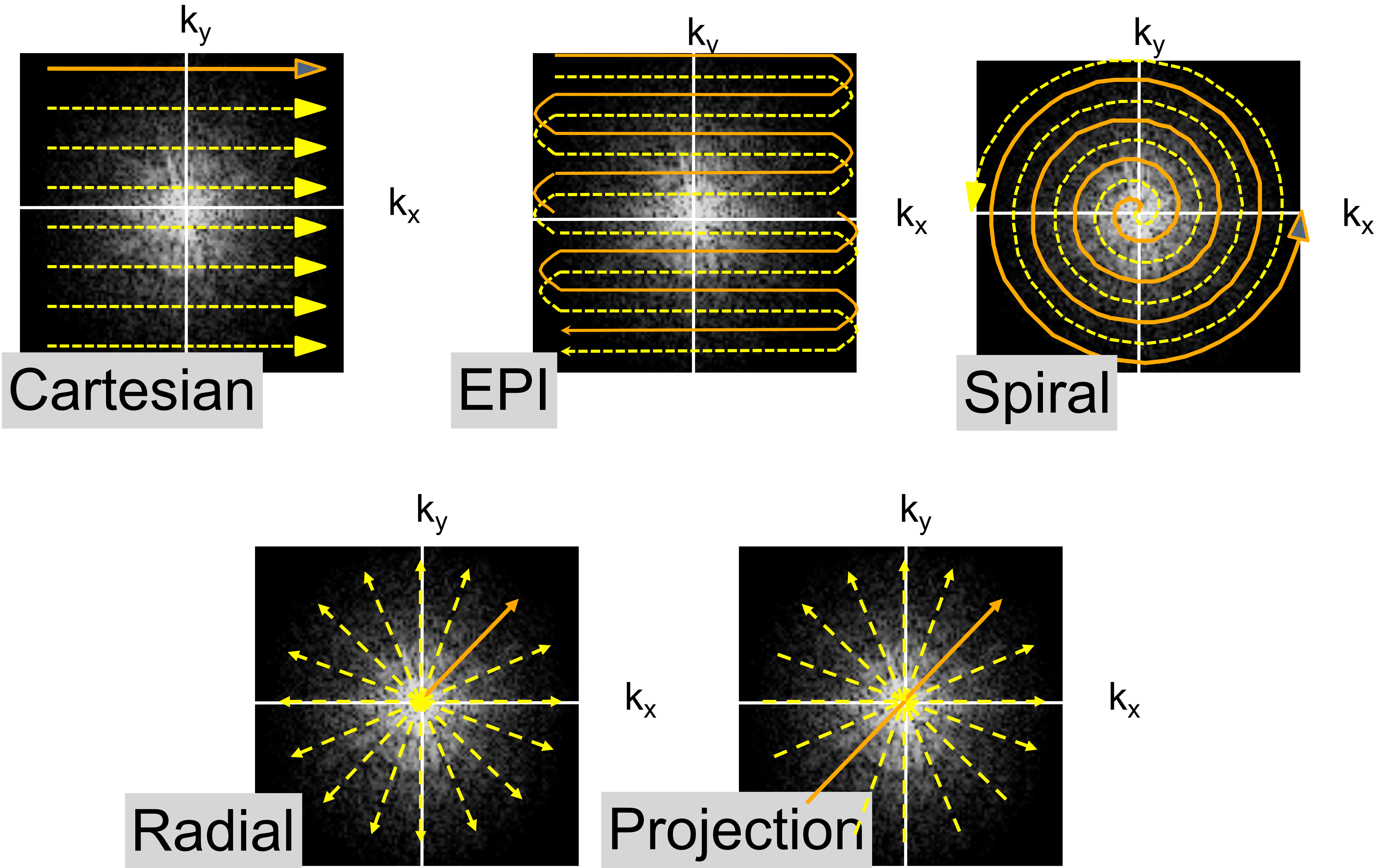
- PSF = Fourier transform of sampling pattern
  - Just 1's as samples, mostly a matter of scaling
- Lots more you can do with this...!



Partial Fourier Acquisition/Reconstruction

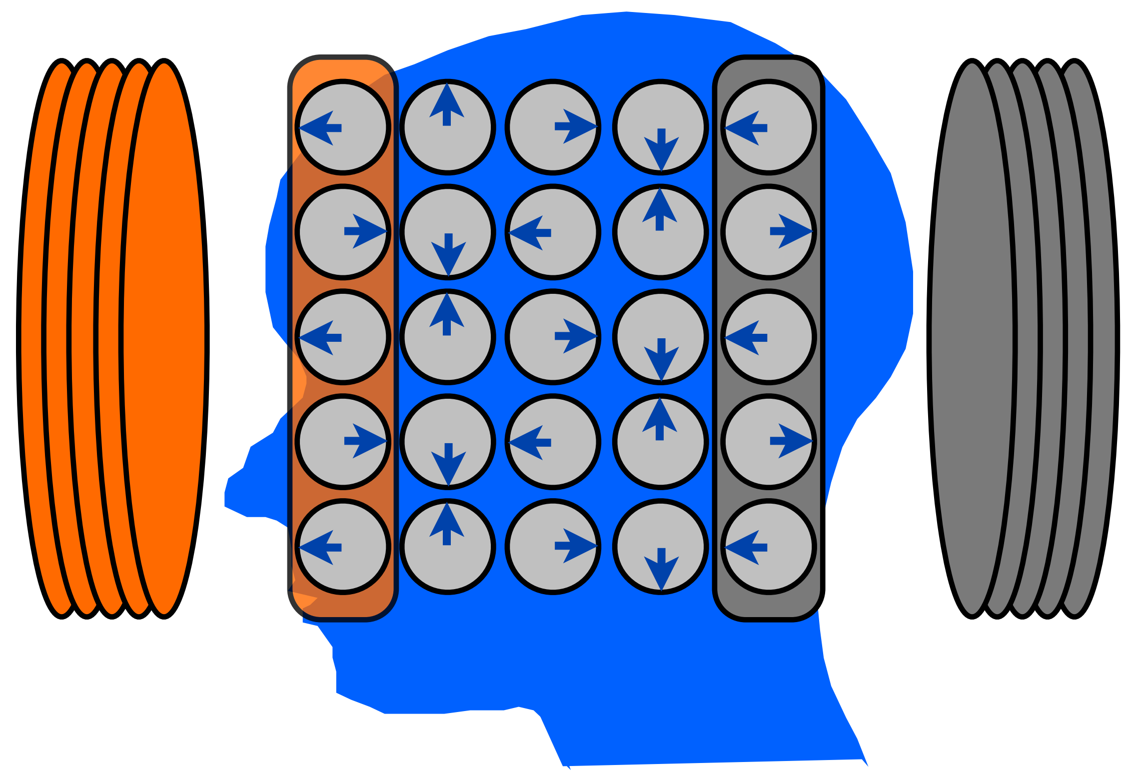


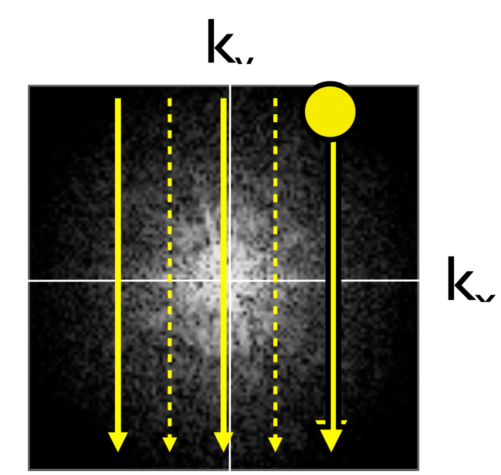
Alternate k-Space Trajectories



Parallel Imaging

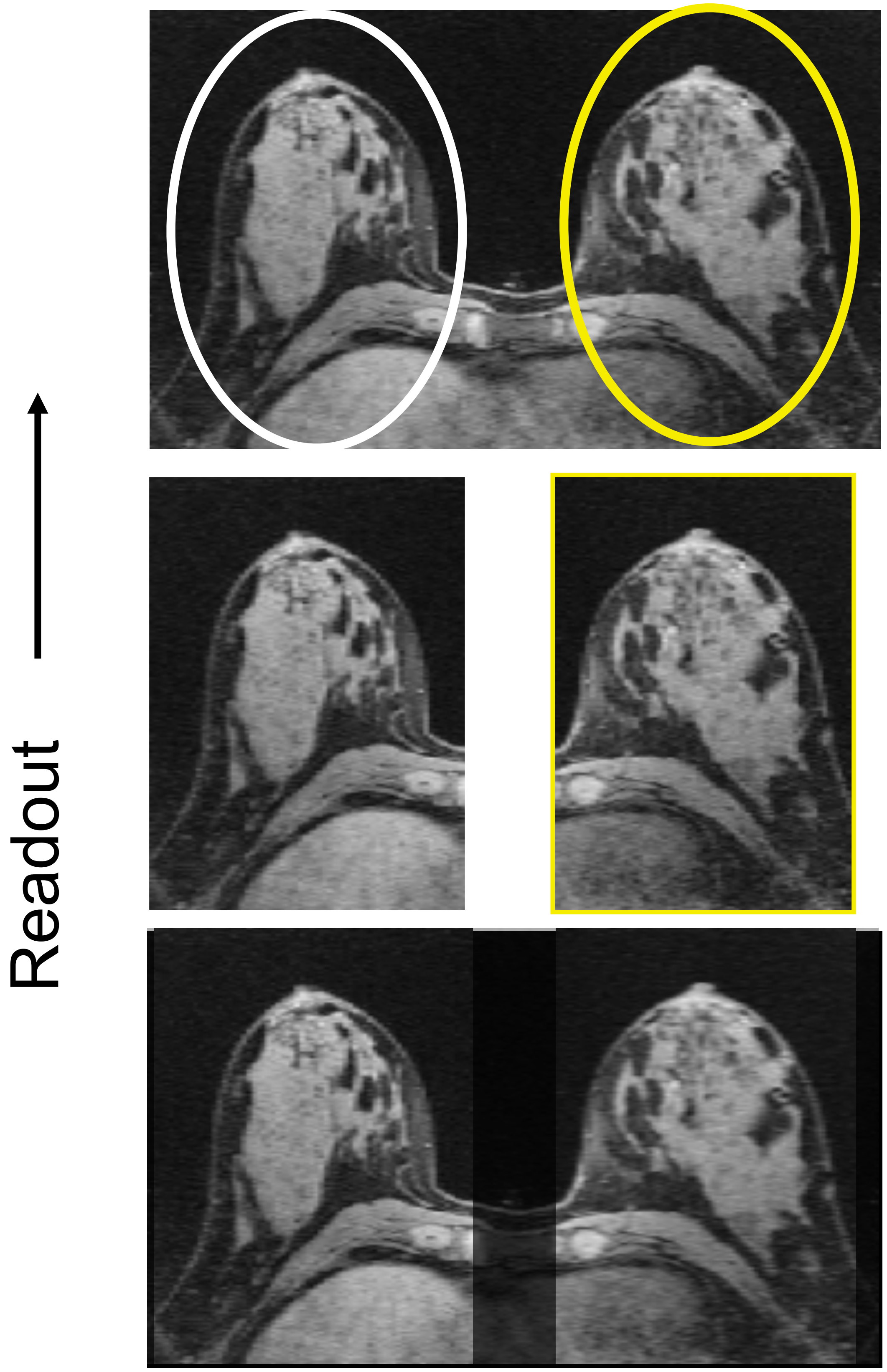
- Coils have limited sensitivity
  - Unalias based on known sensitivities (SENSE)
- Limited sensitivity results in k-space correlations
  - Fill in missing k-space (GRAPPA)
- Build up FOV with coil arrays





Basic Parallel Imaging: PILS (Parallel Imaging with Localized Sensitivities)

Griswold 2000

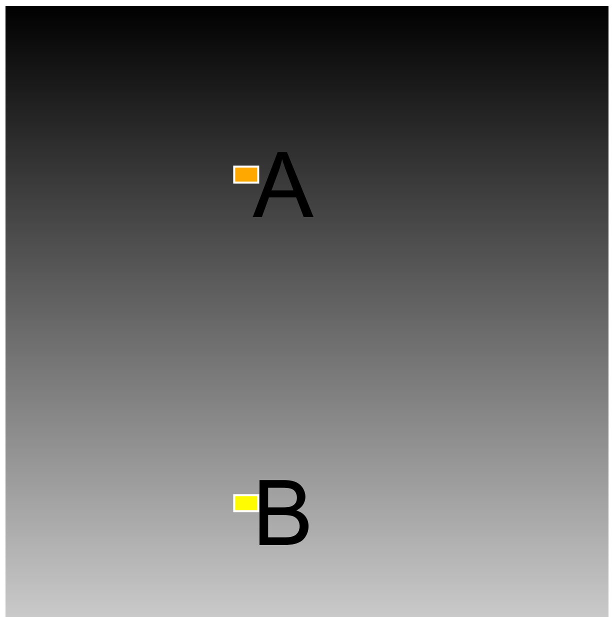


- Consider 2 coils
- Each sensitive to exactly 1 breast
- Each coil uses a reduced FOV
- …but simultaneous acquisition
- Combination allows full image *in less time*

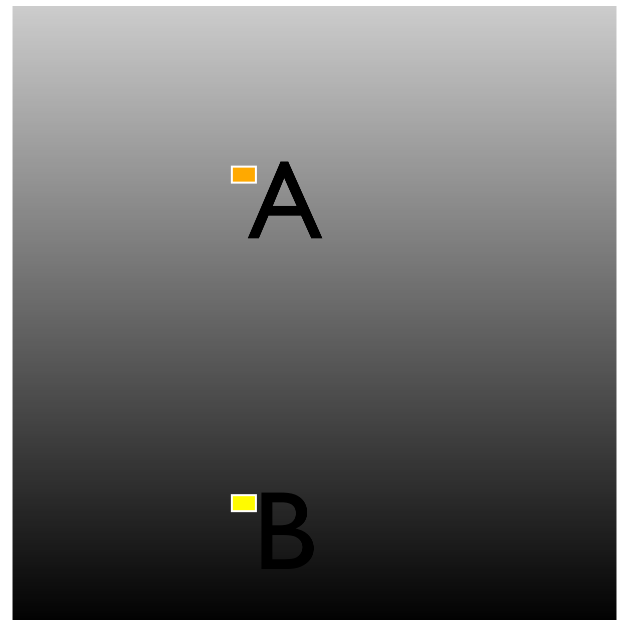
SENSE: Unalias Image

Pruessmann 1999

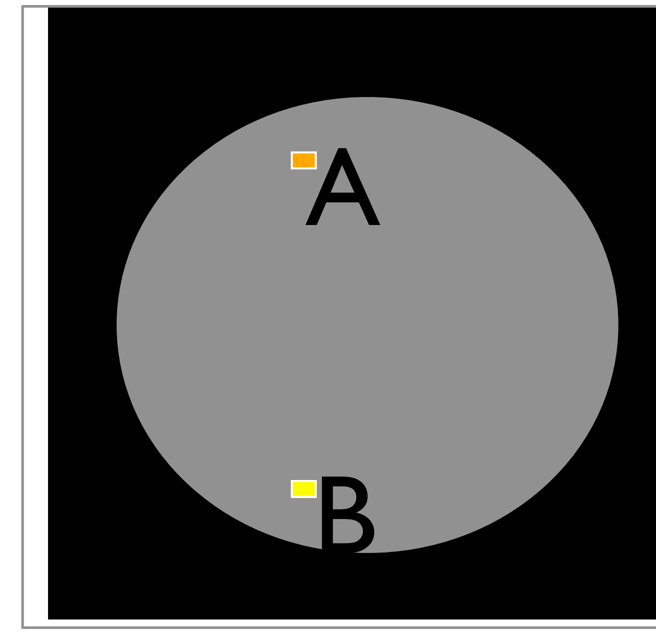
Sensitivity 1 (S_1)



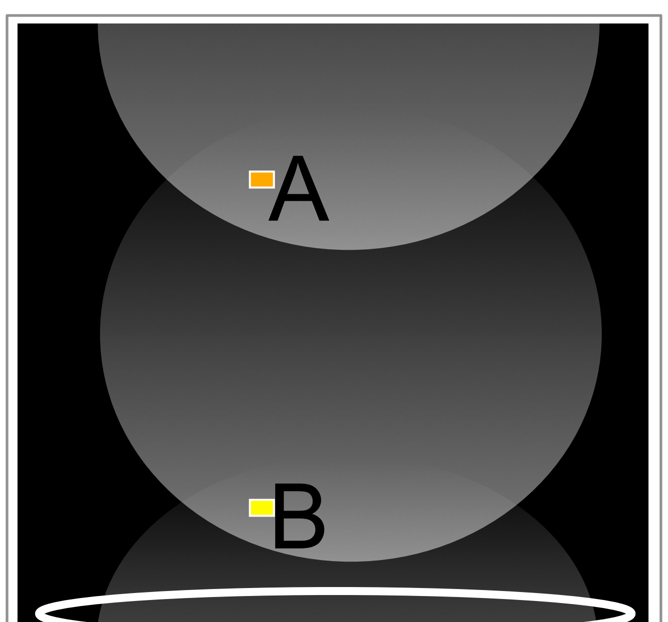
Sensitivitiy 2 (S_2)



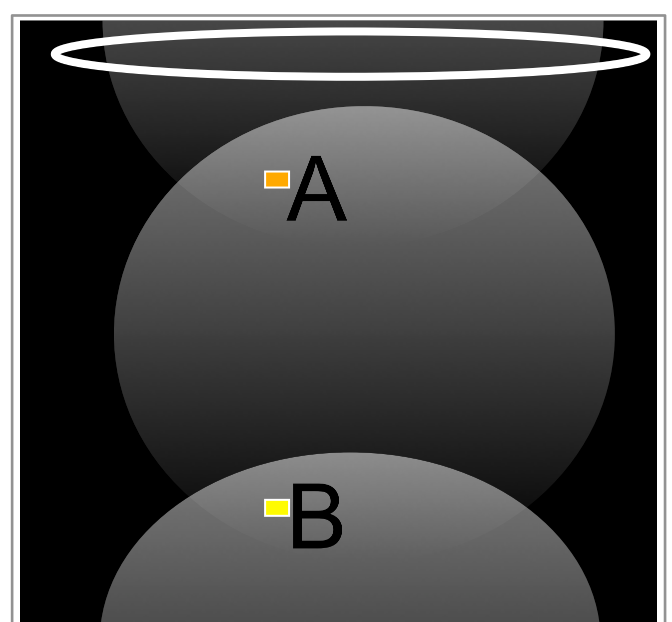
SENSE Image



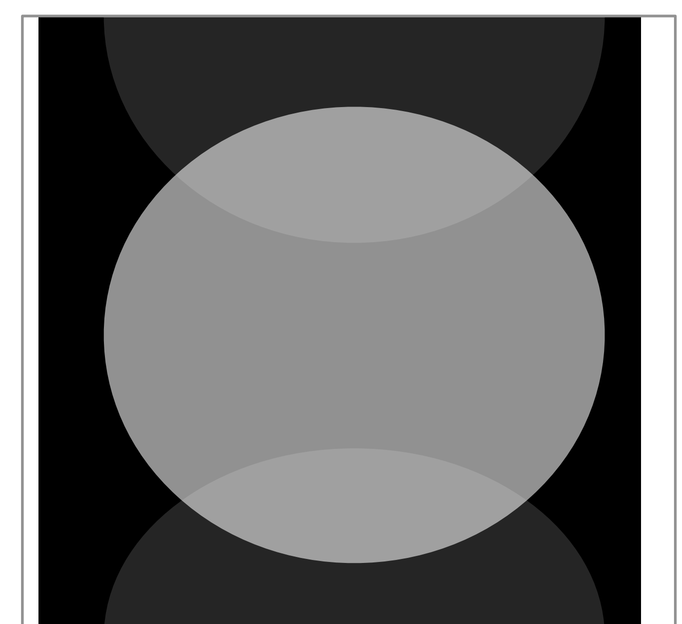
Coil 1 Signal (C_1)



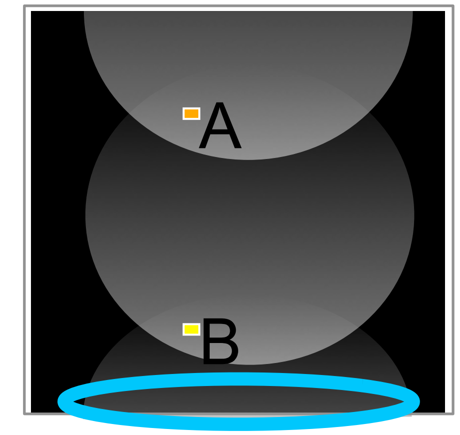
Coil 2 Signal (C_2)

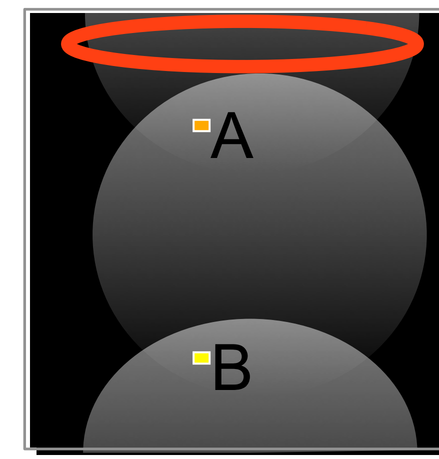


When it fails…



SENSE: Brief Mathematics





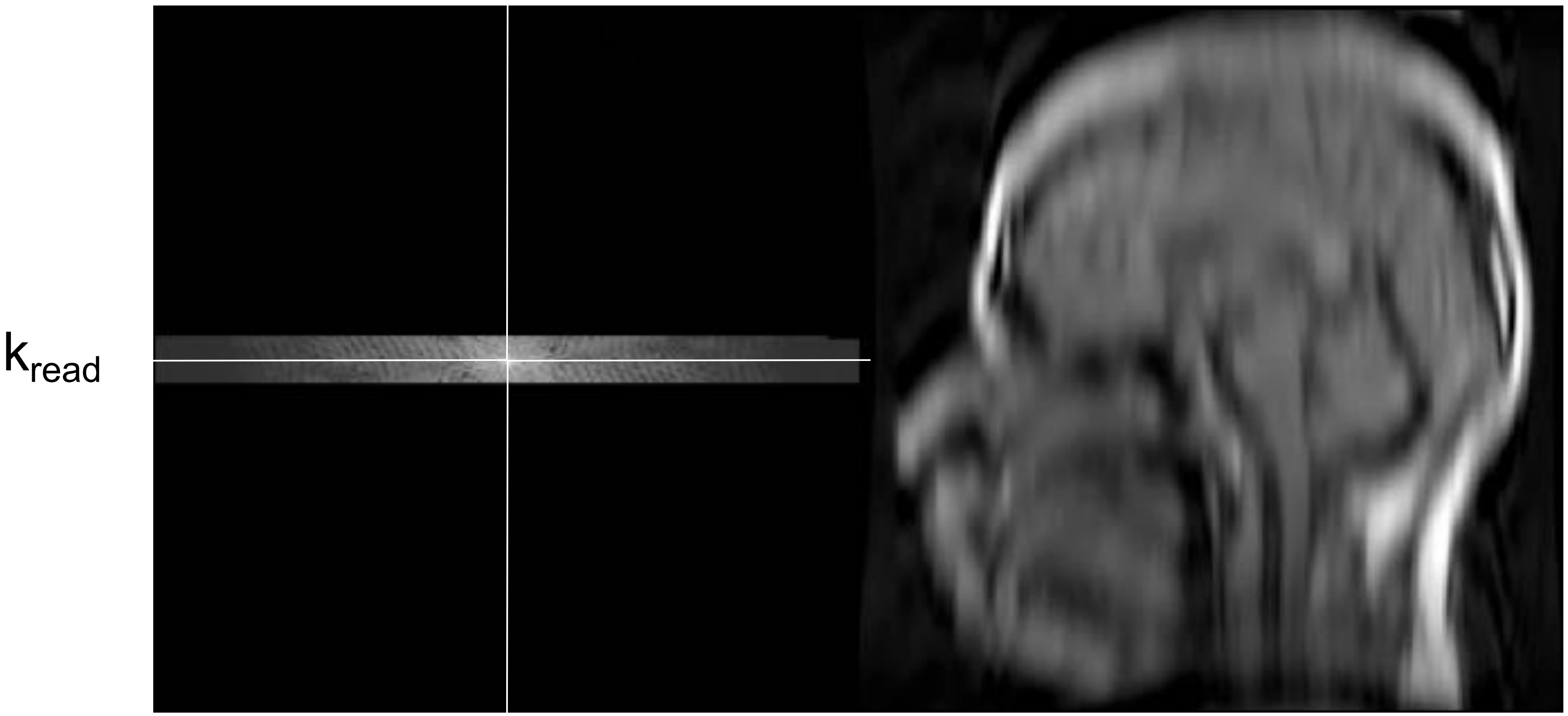
- At each pixel
  - Using Coil 1: $C_1 = S_{1A} \times A + S_{1B} \times B$
  - Using Coil 2: $C_2 = S_{2A} \times A + S_{2B} \times B$
- If we know S_1 and S_2 at A,B and signals C_1 and C_2,

$$A = \frac{S_{1B} C_2 - S_{2B} C_1}{S_{2A} S_{1B} - S_{2B} S_{1A}} \qquad B = \frac{S_{2A} C_1 - S_{1A} C_2}{S_{2A} S_{1B} - S_{2B} S_{1A}}$$

  - More complicated with more than 2 coils
  - If denominator is small, noise amplification
  - Just a matrix inversion or pseudoinverse

SENSE Calibration

- Low-resolution images from each coil
- Divide images by RMS image or body coil image
- Challenge: coil sensitivity in area of low signal

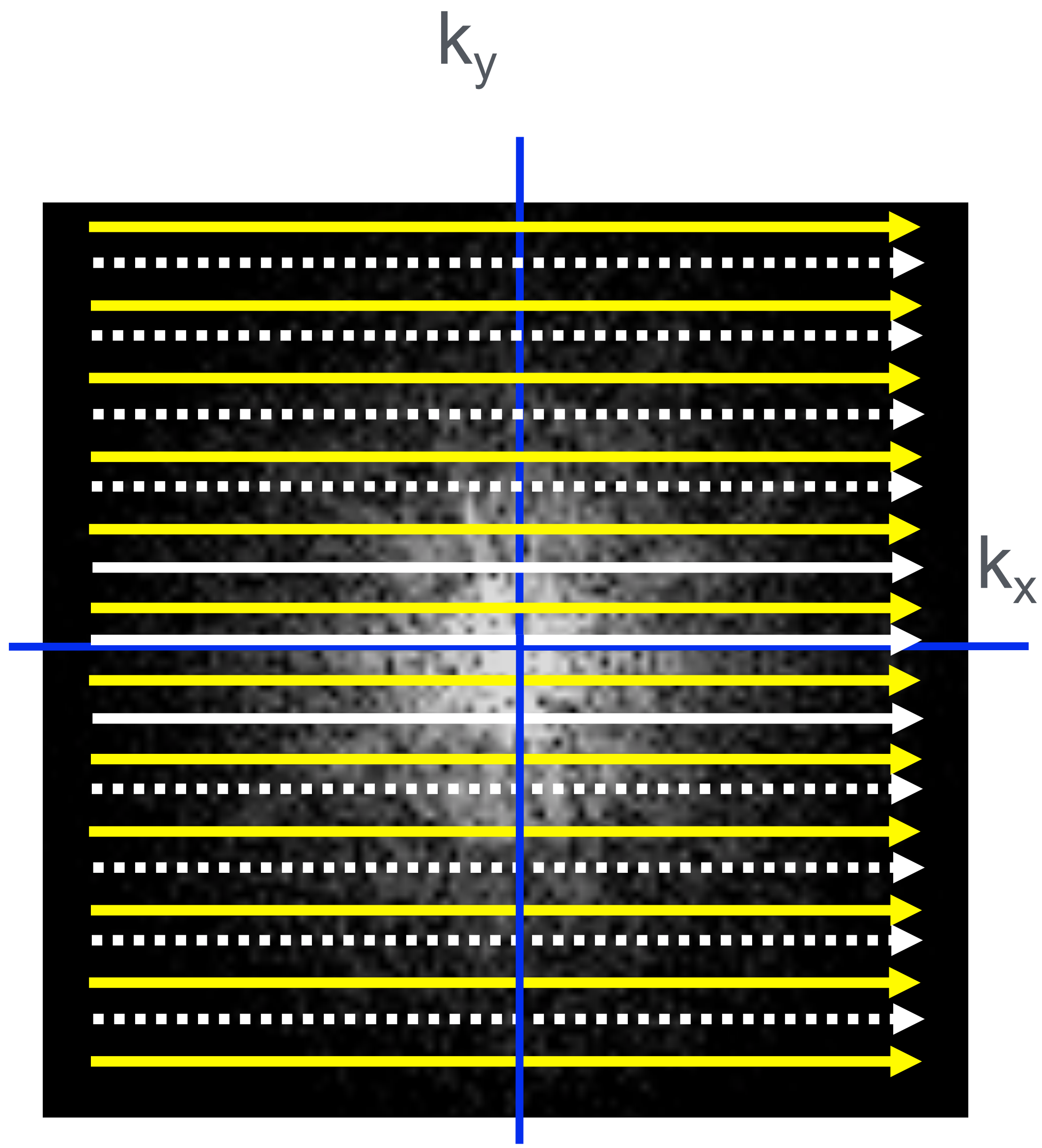


Low Resolution Image

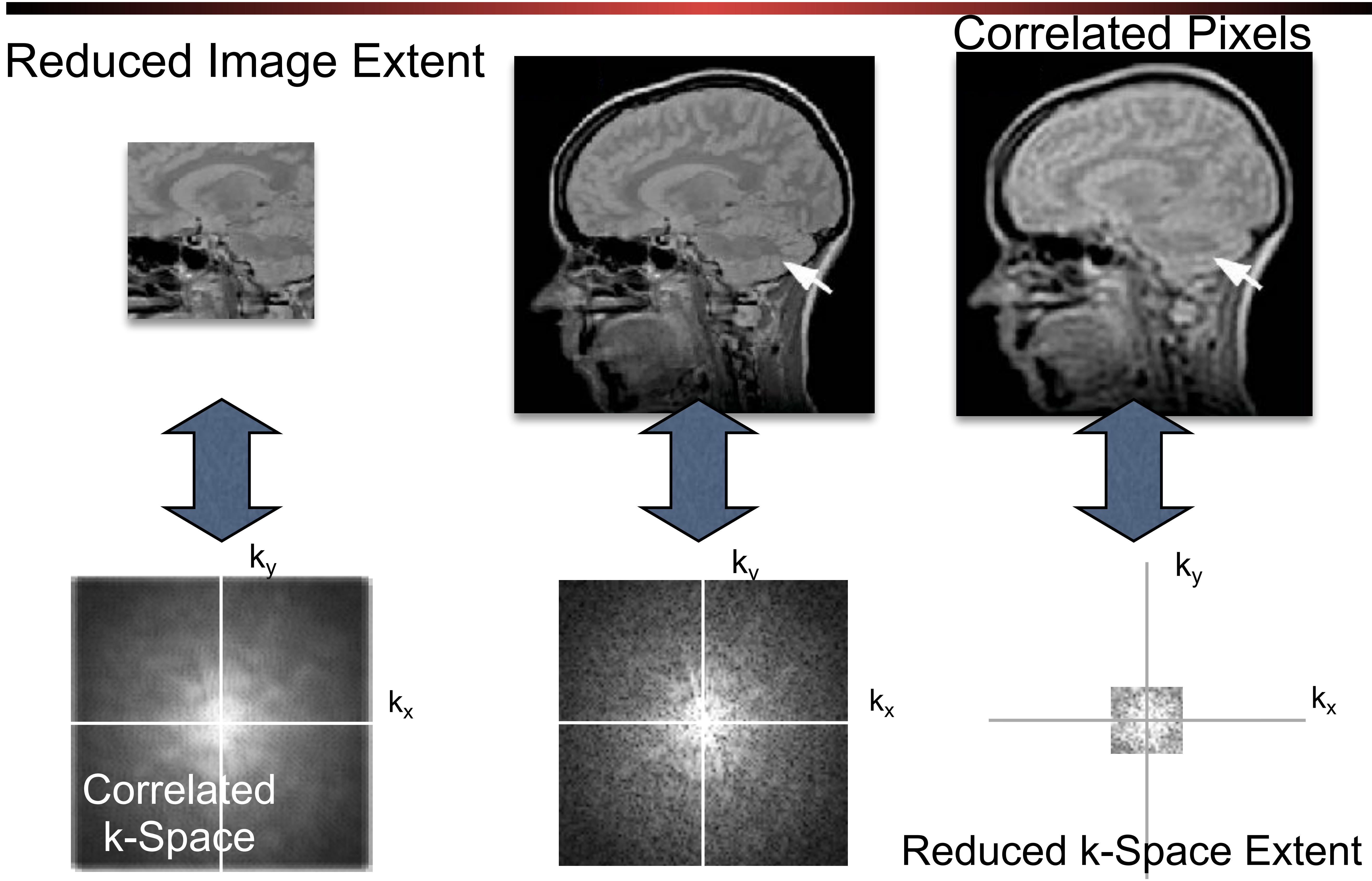
Parallel Imaging: k-space Approaches

Sodickson 1997 (SMASH), Griswold 2002 (GRAPPA)

- Acquire reduced FOV, and some "calibration" lines
- Fill in missing lines to extend the FOV



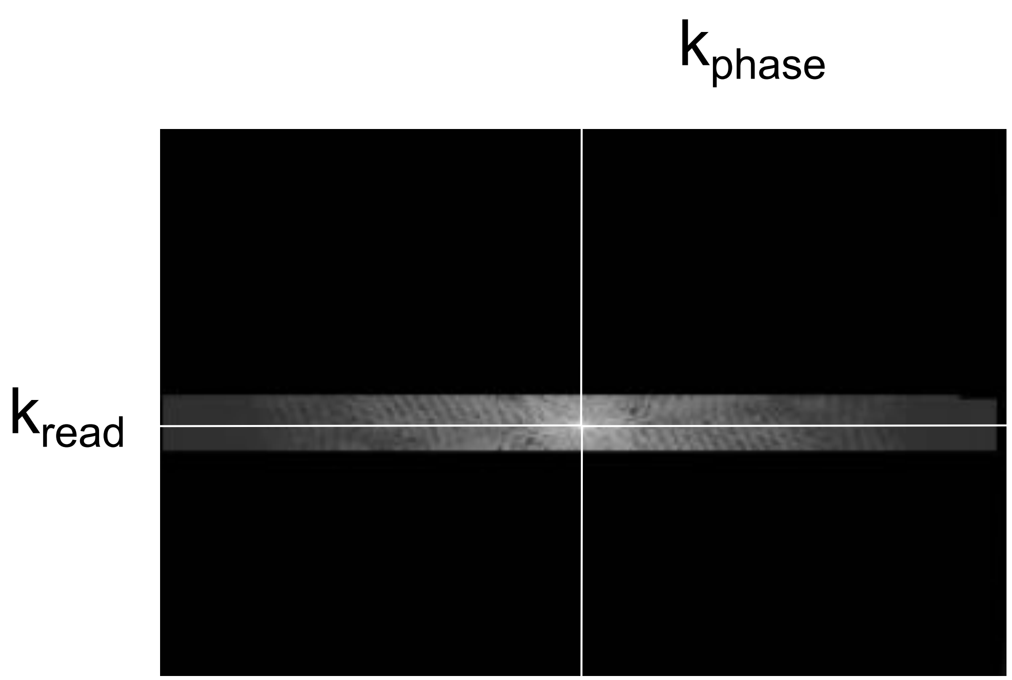
GRAPPA: Coil Sensitivities and k-space

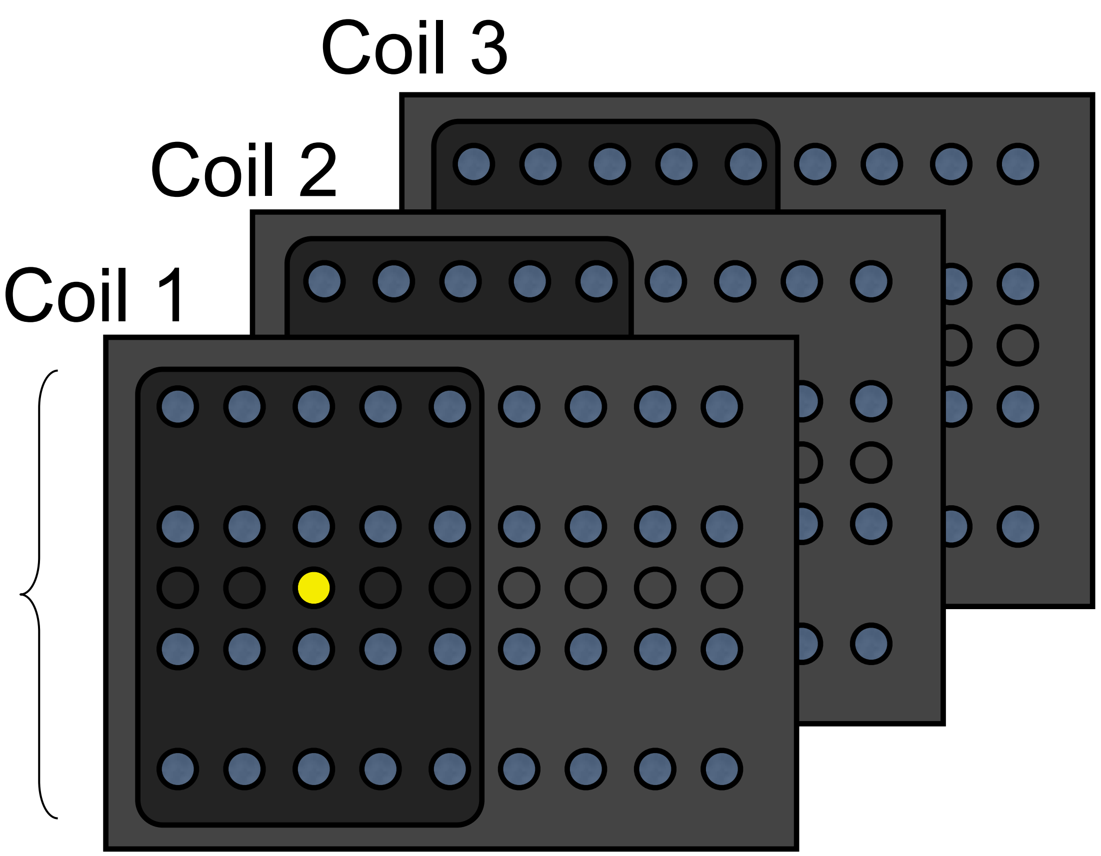


GRAPPA Calibration

Griswold 2002

- Fully-sampled central k-space
- Find “data correlation” between lines/coils
  - Note: data-driven vs model (SENSE)
  - Not just image vs k-space!



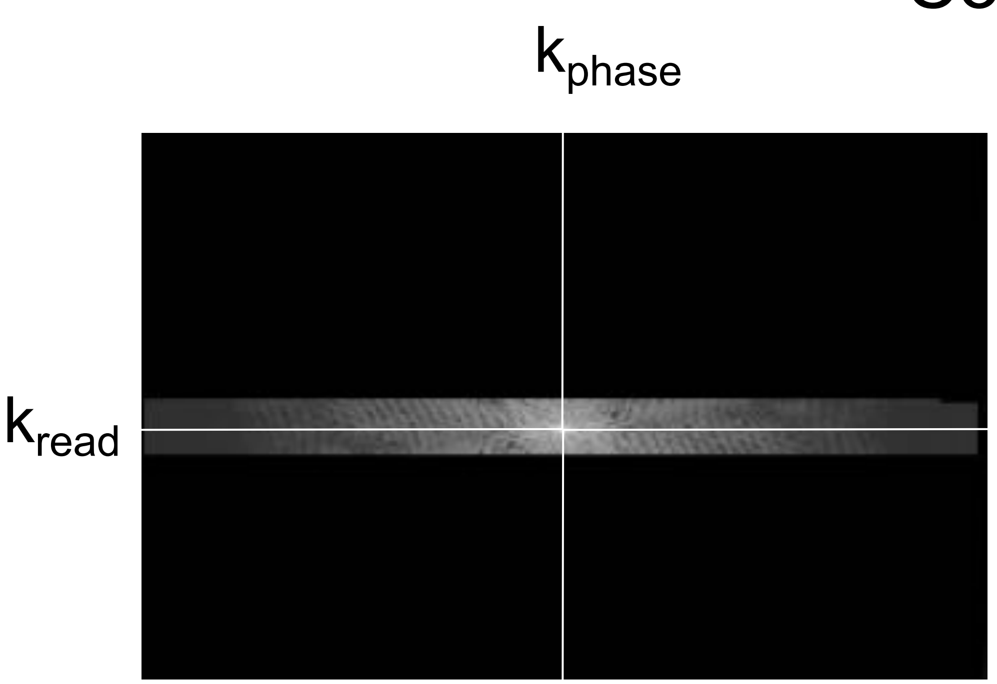


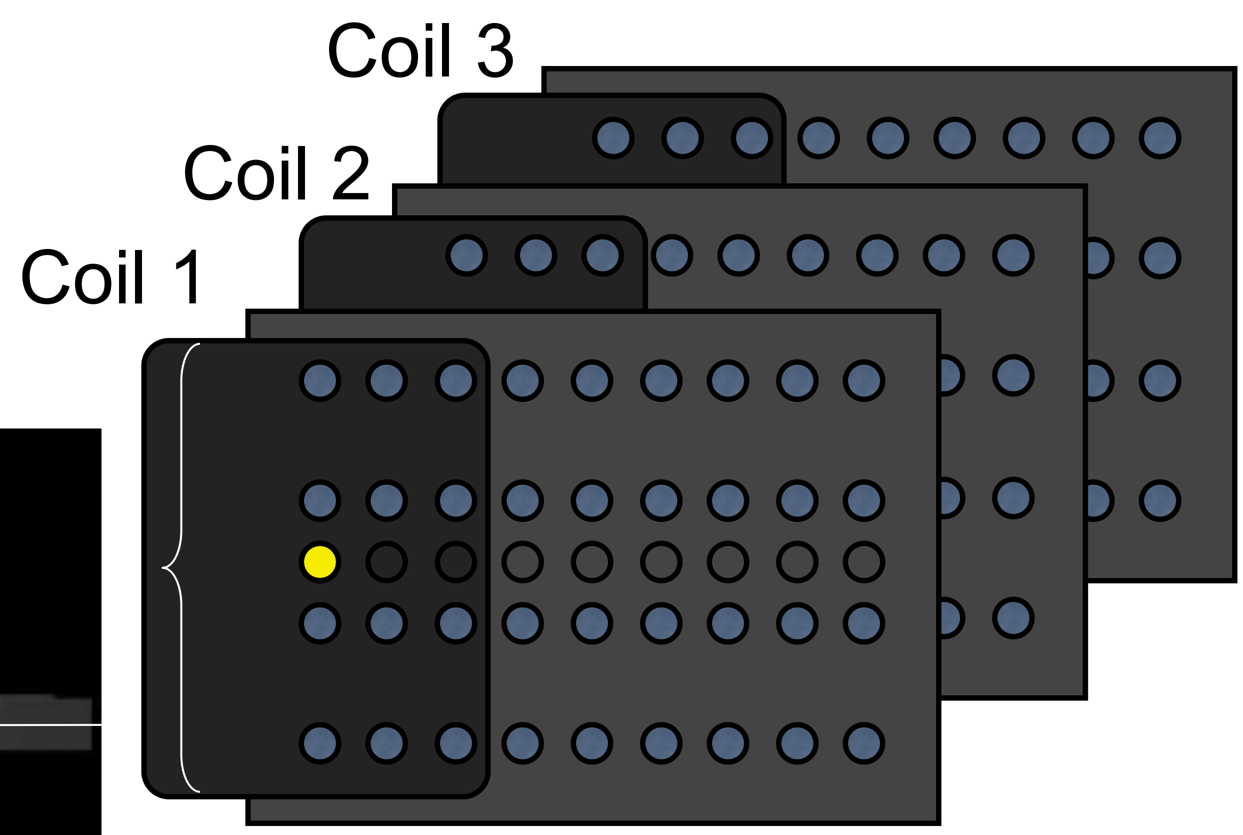
Repeat for all calibration points and all coils

GRAPPA Synthesis

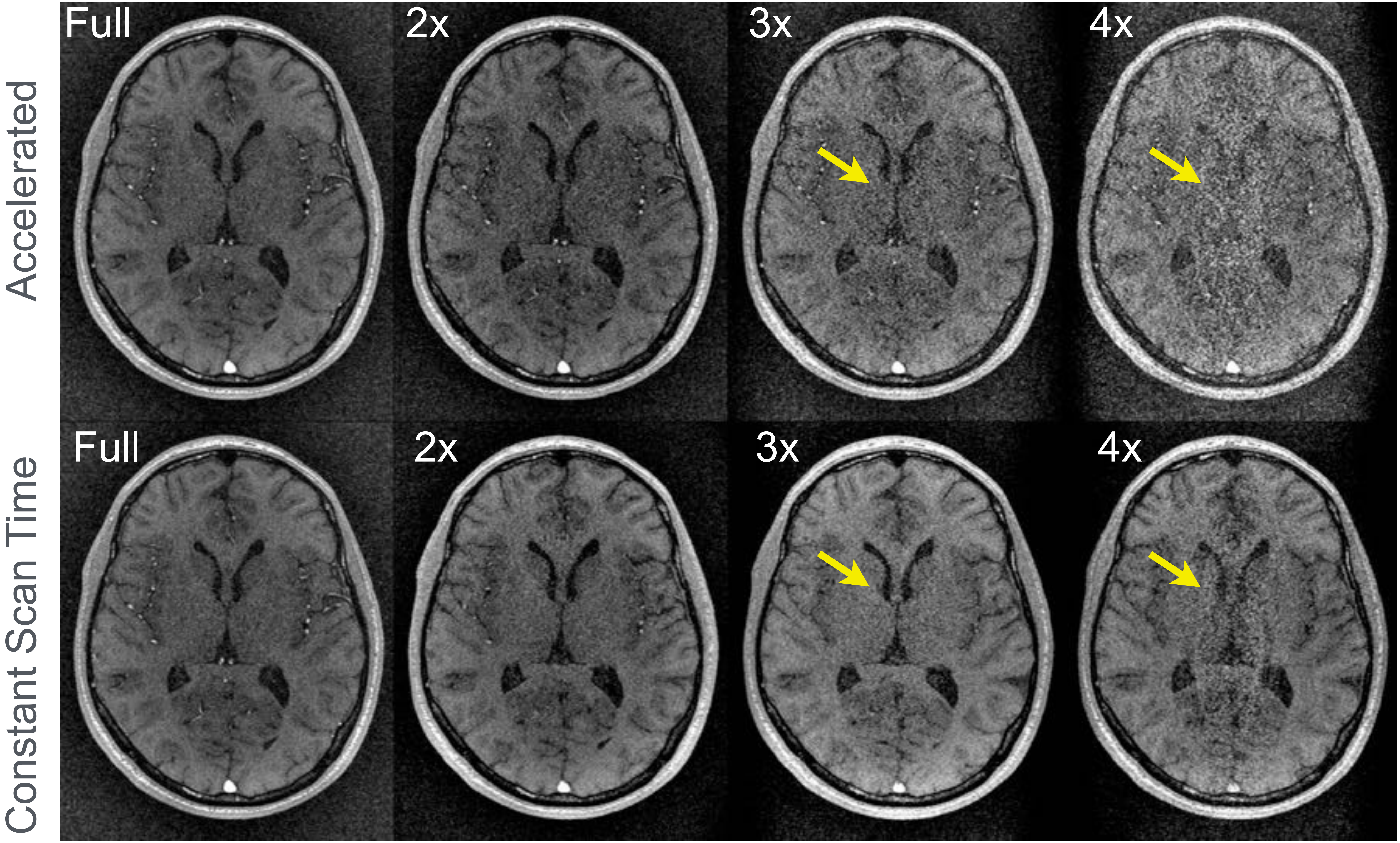
Griswold 2002

- Use kernel information to synthesize data
- Repeat for all coils
- Combine coils and reconstruct





Parallel Imaging & Noise

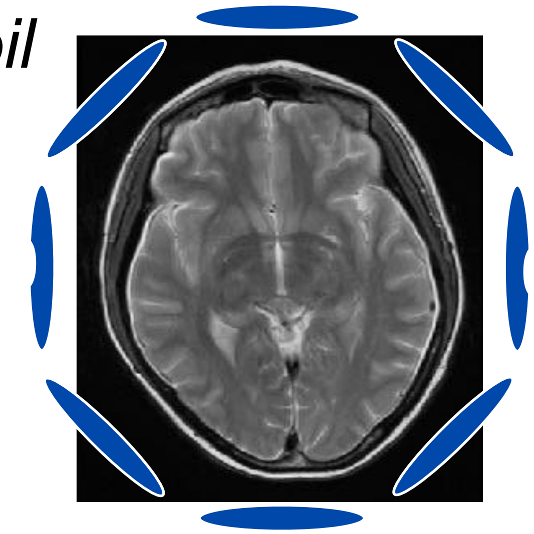


Acceleration is in Left-Right Direction in Images

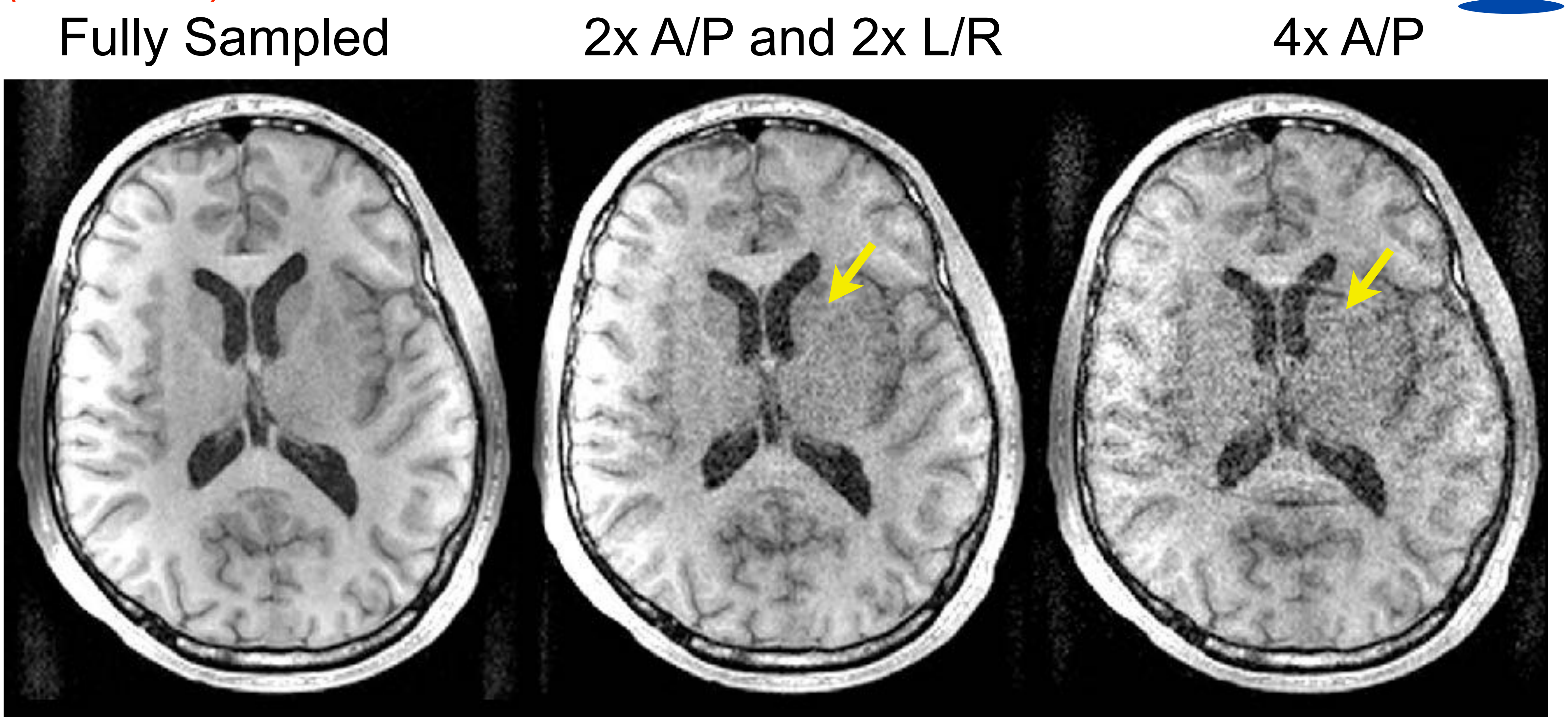
2D Parallel Imaging (for 3D Acquisitions)

- 3D imaging uses 2 phase-encode directions
- Can apply parallel imaging in 2 directions

8-Channel Phased-Array Coil



Note: Readout is in S/I (head-foot) direction!



Parallel Imaging Questions

Summary ~ Background

- Overview of NMR
- Hardware
- Image formation and k-space
- Excitation k-space
- Signals and contrast
- Signal-to-Noise Ratio (SNR)
- Pulse Sequences
- Sampling and Trajectories
- Parallel Imaging

Signal Calculations

- Bloch Equations and Matrix Calculations
- Extended Phase Graphs
- Examples (both)

Bloch Equation Matrix Simulations

- Basic Bloch Equation
- Bloch Equation with B_1 / rotating frame
- Basic matrix simulations / Hard Pulse Approx.
- Many-spin simulations: Brute force
- Bloch-McConnell Equation with Exchange
- Bloch-Torrey Equations (McNab?)

Bloch Equation

- Basic Bloch Equation:

$$\frac{dM}{dt} = M \times \gamma B - \frac{M_{xy}}{T_2} + \frac{M_0 - M_z}{T_1}$$

- In Matrix form with:

$$M = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

- Becomes:

$$\frac{dM}{dt} = \begin{bmatrix} -1/T_2 & \gamma B_z & -\gamma B_y \\ -\gamma B_z & -1/T_2 & \gamma B_x \\ \gamma B_y & -\gamma B_x & -1/T_1 \end{bmatrix} M + \begin{bmatrix} 0 \\ 0 \\ M_0/T_1 \end{bmatrix}$$

Standard right-handed cross product

Relaxation

- Over time period τ

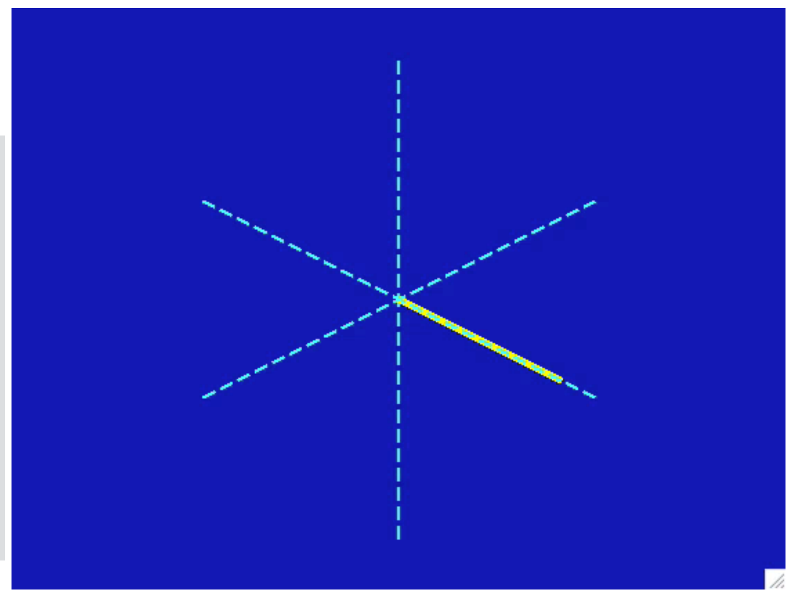
$$E_1 = e^{-\tau/T_1}$$

$$E_2 = e^{-\tau/T_2}$$

$$M' = \underbrace{\begin{bmatrix} E_2 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_1 \end{bmatrix}}_{A} M + \underbrace{\begin{bmatrix} 0 \\ 0 \\ M_0(1-E_1) \end{bmatrix}}_{B}$$

(See relax.m)

```
>> [AB] = relax(0.5,0.5,0.1,1)

  AB =
     0.0067         0         0         0
          0    0.0067         0         0
          0         0    0.3679    0.6321
```

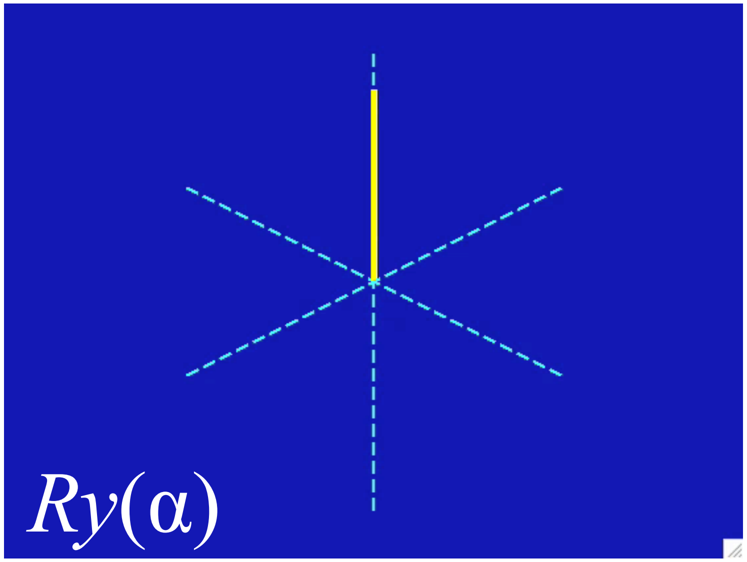


RF Rotations

- For a flip angle α

$$M' = R_x M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} M$$

$$M' = R_y M = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} M$$



$$\alpha = \gamma B_1 \tau$$

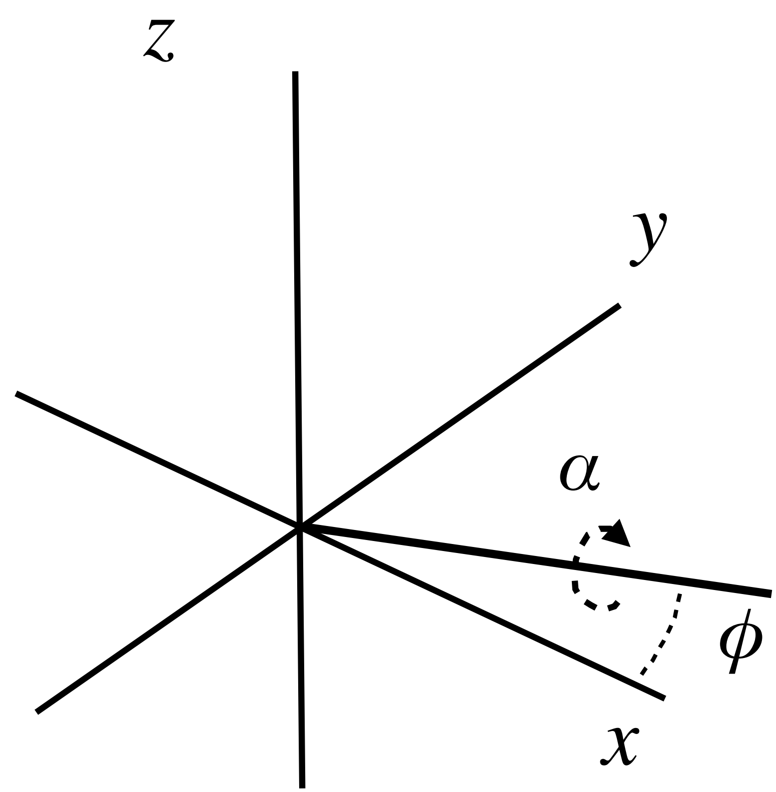
- *Rotations are left-handed*
- *(also achieved with negative γ)*
- *See xrot.m and yrot.m*

```
>> A = xrot(90)

A =

     1.00         0         0
        0         0      1.00
        0     -1.00         0
```

RF Rotations (Arbitrary B_1 phase)

$$R_\phi = \begin{bmatrix} \cos^2\phi + \sin^2\phi\cos\alpha & \cos\phi\sin\phi(1-\cos\alpha) & -\sin\phi\sin\alpha \\ \cos\phi\sin\phi(1-\cos\alpha) & \sin^2\phi + \cos^2\phi\cos\alpha & \cos\phi\sin\alpha \\ \sin\phi\sin\alpha & -\sin\alpha\cos\phi & \cos\alpha \end{bmatrix}$$

$$\phi = \tan^{-1}(B_y/B_x)$$



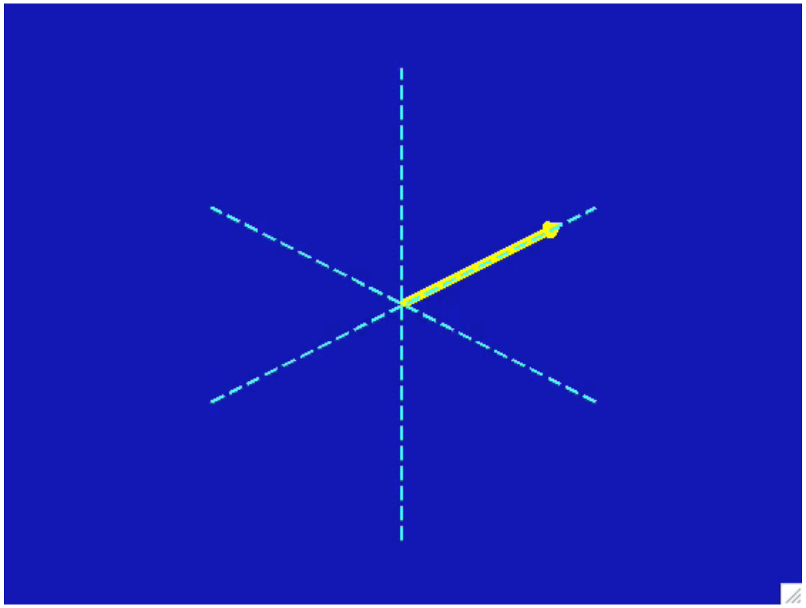
- Rotation axis in x-y plane
- Note R_ϕ is just $R_z(-\phi)R_x(\alpha)\,R_z(\phi)$

(See throt.m)

Gradient / ΔB_0 Rotations

$$M' = R_z M = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} M$$

$$\theta = \gamma(G \cdot \overrightarrow{r} + \Delta B_0)\tau$$

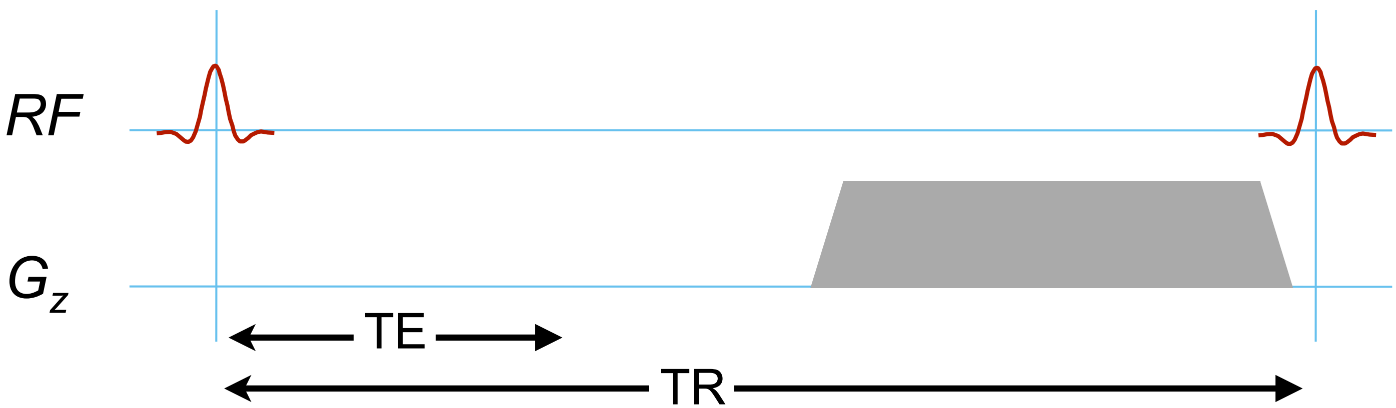


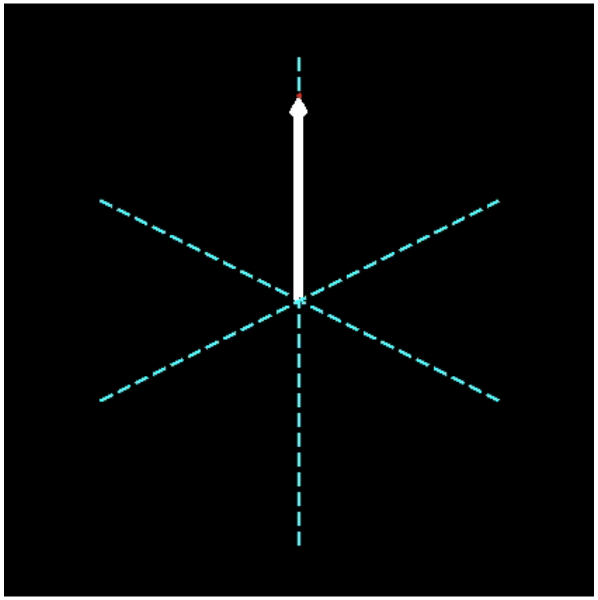
- *Rotations are left-handed*
- *(also achieved with negative γ)*
- *See zrot.m*

Perfect Spoiling

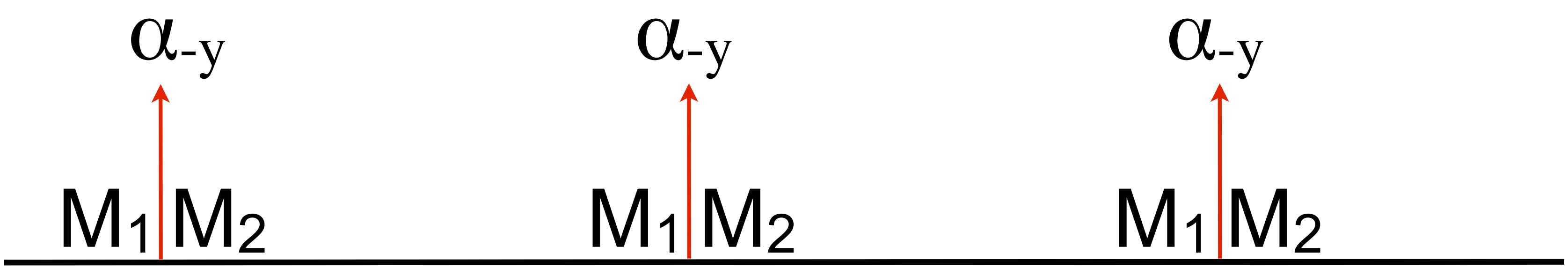
- Explicitly set transverse magnetization to zero

$$M' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} M$$





Example: Excitation/Recovery



$$M_2 = \begin{bmatrix} cos\alpha & 0 & sin\alpha \\ 0 & 1 & 0 \\ -sin\alpha & 0 & cos\alpha \end{bmatrix} M_1$$

Neglect residual transverse magnetization

$$M_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & E_1 \end{bmatrix} M_2 + \begin{bmatrix} 0 \\ 0 \\ m_0(1 - E_1) \end{bmatrix}$$

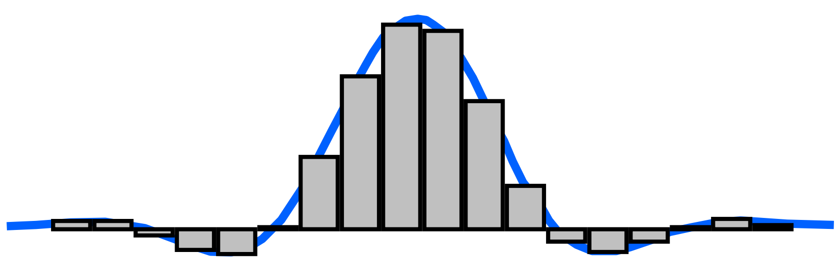
$$M_1 = E_1 cos\alpha\, M_1 + m_0(1 - E_1)$$

$$M_1 = \frac{m_0(1 - E_1)}{1 - E_1 cos\alpha}$$

Recall Prior Example!

Overlapping RF/Gradients?

- z rotations and relaxation commute
- RF rotations do not commute with others
- **Hard-Pulse Approximation:**
  - Small rotations/relaxation can be applied sequentially
  - Break pulses into very short segments
  - Aside: Basis for Variable Rate Selective Excitation
- **Alternatively, calculate arbitrary rotations**



Matrix Propagation

- Rotations / Relaxation: $M' = AM+B$
  - $M_1 = A_1 M_0 + B_1$ *(M_0 is starting M, not equilibrium M!)*
  - $M_2 = A_2 M_1 + B_2$ ..... M_n *(n "operations")*
  - *Propagation: multiply A's, sum B's after multiplying by all successive A's*
  - $M_2 = (A_2 A_1) M_0 + A_2 B_1 + B_2$

$$A = \prod_{i=n}^{1} A_i$$

$$B = \sum_{i=1}^{n} \left(\prod_{j=n}^{i+1} Aj \right) B_i$$

Example (n=3)

$$A = A_3 A_2 A_1$$
$$B = A_3 A_2 B_1 + A_3 B_2 + B_3$$

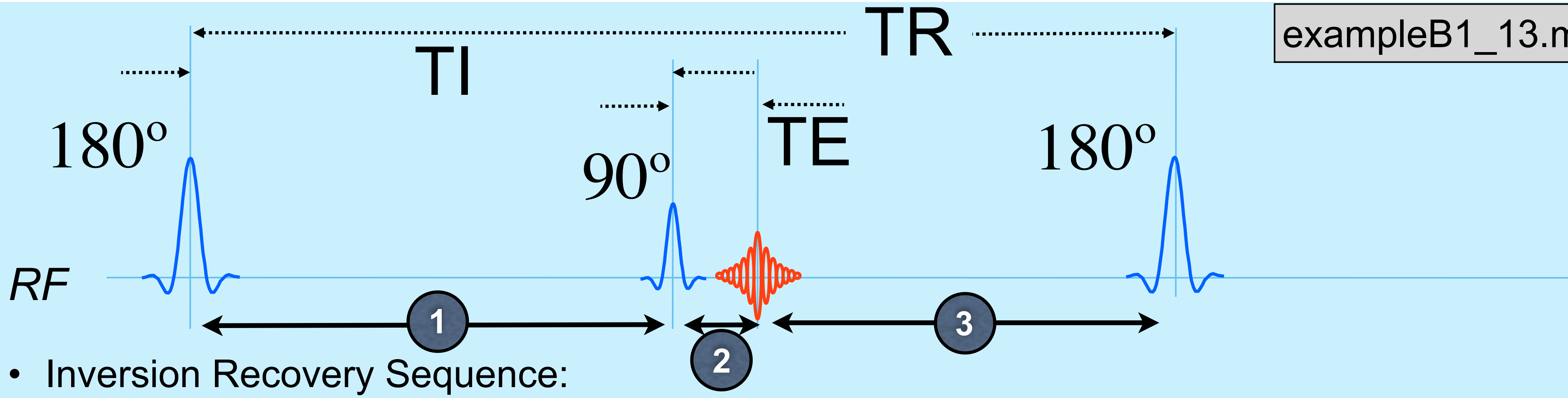
(See abprop.m)

Steady States

- Propagation over 1 TR: $M_{n+1} = AM_n + B$
- Steady State: $M_{n+1} = M_n$
- Combine: $M_{ss} = AM_{ss} + B \quad = (I-A)^{-1}B$
  - As long as there is relaxation, there is a steady state, since eigenvalues of A are less than one in magnitude

Short-TR IR Signal (Again!)

exampleB1_13.m



- Inversion Recovery Sequence:
  - TR = 1s, TI = 0.5s, TE=50ms
- What is the signal for T_1=0.5s, T_2=100ms?
  - "Operations"
  - $M_{180} = R_x(180)M_{TR}$
  - $M_{90} = R_x(90)E(0.5s)M_{180}$
  - $M_{TE} = E(0.05s)M_{90}$
  - $M_{TR} = E(0.45s = 1\text{-}0.5\text{-}0.05s)M_{TE}$

```
>> A1 = diag([E2a E2a E1a]) * Rx(180);
>> B1 = [0;0;1-E1a];
>> A2 = diag([E2b E2b E1b]) * Rx(90);
>> B2 = [0;0;1-E1b];
>> A3 = diag([E2c E2c E1c]);
>> B3 = [0;0;1-E1c];

>> A = A2*A1*A3;
>> B = B2+A2*(B1+A1*B3);

>> M = inv(eye(3)-A)*B
     [ 0;  0.2424;  0.0952 ]
```

Compact Simulation: abprop.m

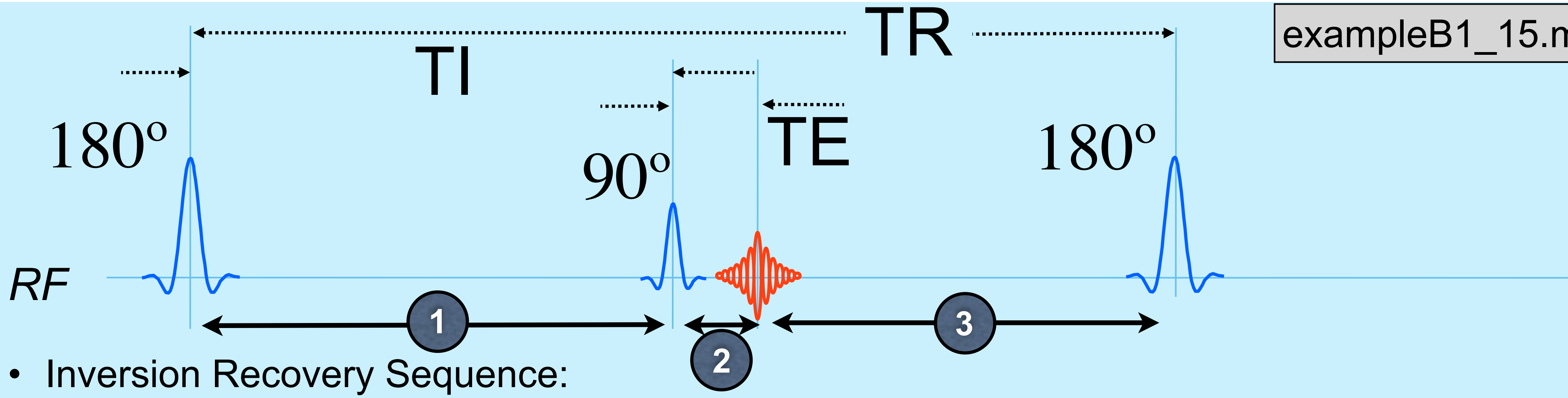
- Propagate spins through series of A,B matrices
- Compact way to simulate sequences

```
function [A,B,mss] = abprop(A1,B1,A2,B2,A3,B3,...)

  If mss is provided, the steady-state is calculated.

  If an Ai is 3x4, then it is assumed to be [Ai Bi]

  If a Bi vector is omitted (the next argument is 3x3
  or 3x4, it is assumed to be zero.
```

- abprop(A_1,B_1, [A_2 B_2], A_3, A_4) *(Here $B_3 = B_4 = 0$)*

Short-TR IR Signal (Compact)

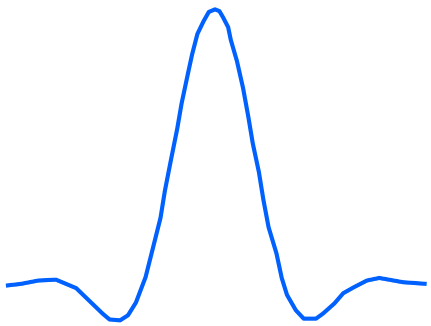
exampleB1_15.m

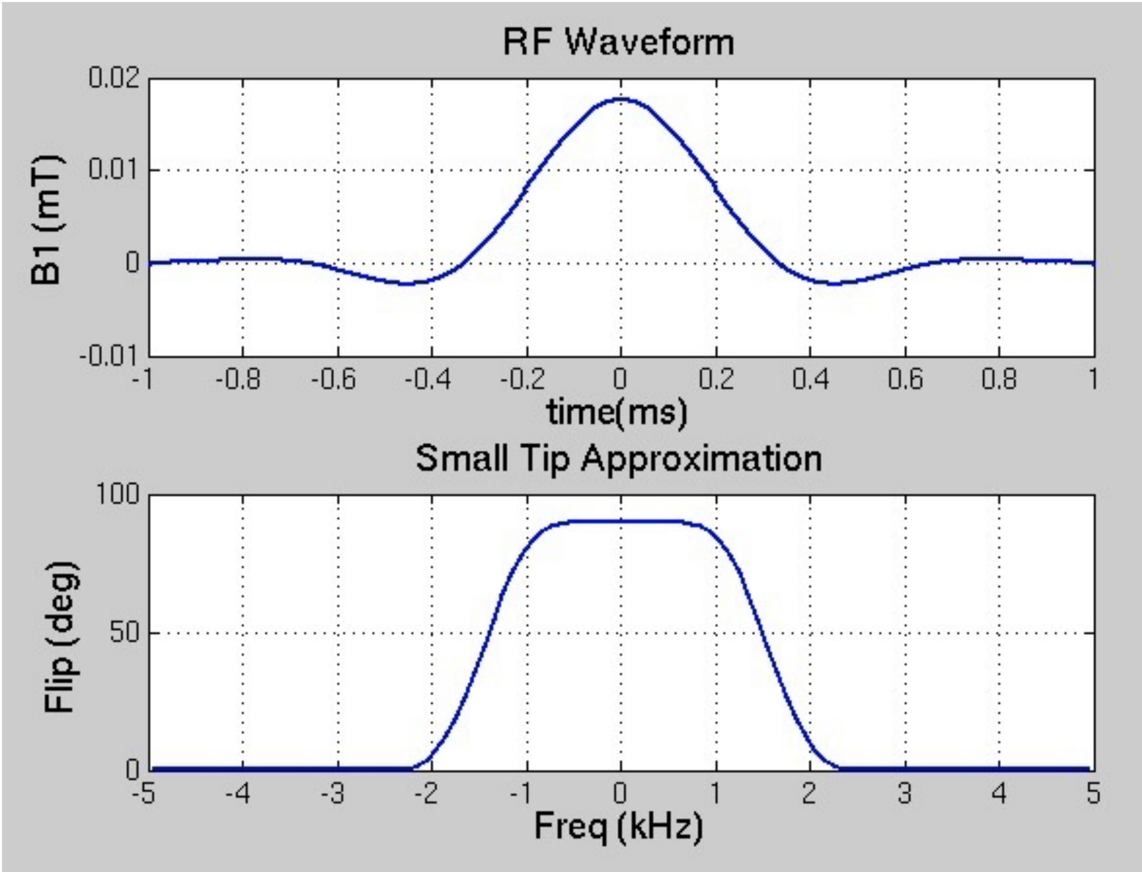


- Inversion Recovery Sequence:
  - TR = 1s, TI = 0.5s, TE=50ms
- What is the signal for T_1=0.5s, T_2=100ms?
  - "Operations"
  - $M_{180} = R_x(180)M_{TR}$
  - $M_{90} = R_x(90)E(0.5s)M_{180}$
  - $M_{TE} = E(0.05s)M_{90}$
  - $M_{TR} = E(0.45s = 1\text{-}0.5\text{-}0.05s)M_{TE}$

```
>> [A,B,Mss] = abprop(
      relax(TR-TE-TI,T1,T2,1),  …
      xrot(180), ...
      relax(TI,T1,T2,1),  …
      xrot(90), …
      relax(TE,T1,T2,1));

>> Mss
      [ 0;  0.2424;  0.0952 ]
```

Example

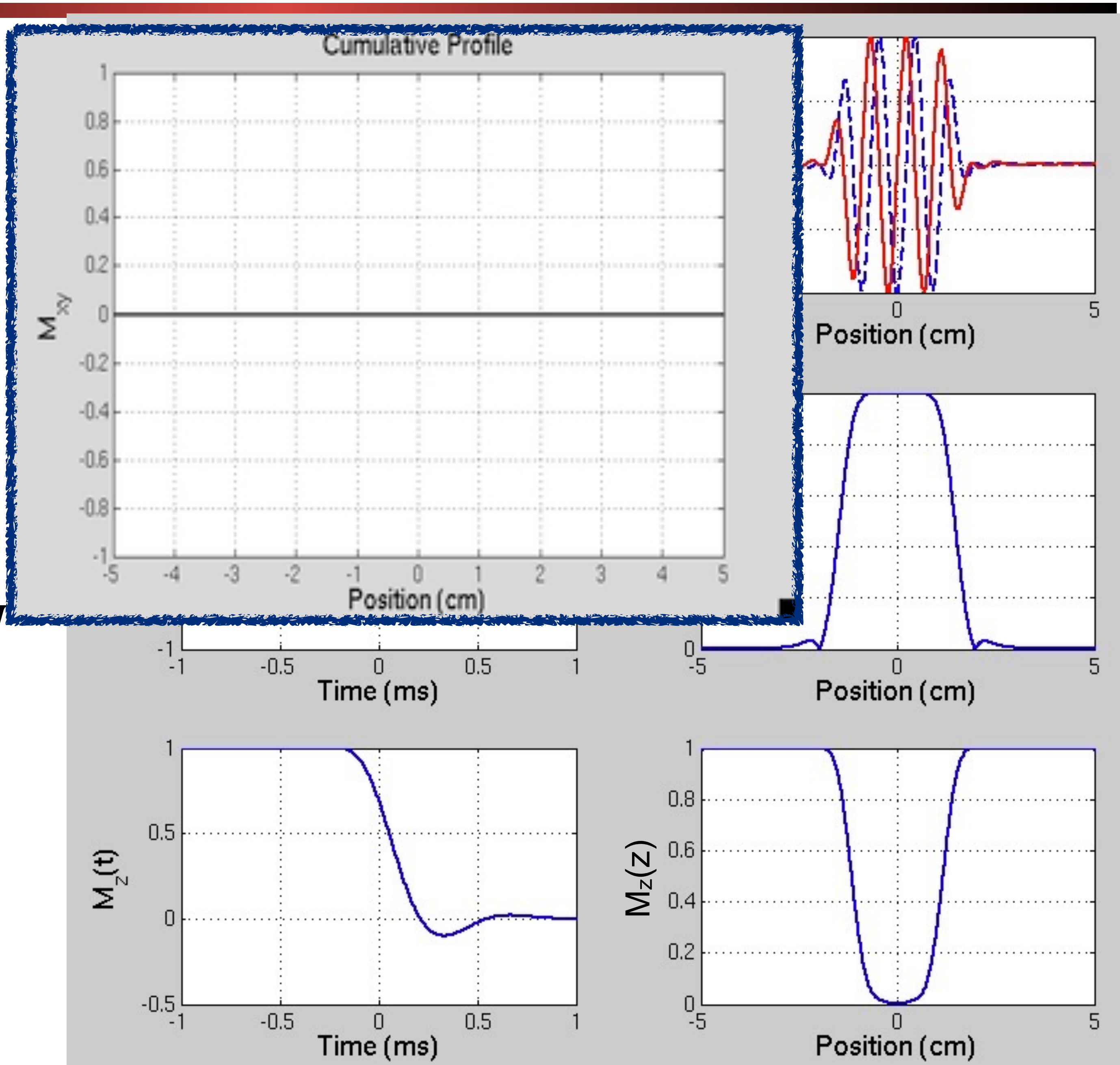




- 90_{-y} Excitation pulse
  - Time samples of 4μs
  - 3 sinc cycles
  - 2ms duration
  - Area of 5.9 μT*ms
  - BW ~ 3 kHz
  - 2.3 mT/m gradient (1kHz/cm)

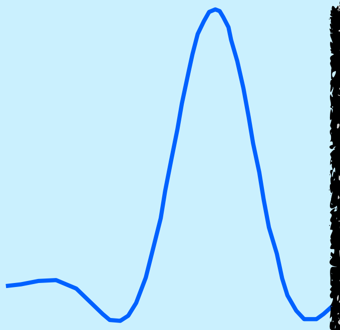
Simulation

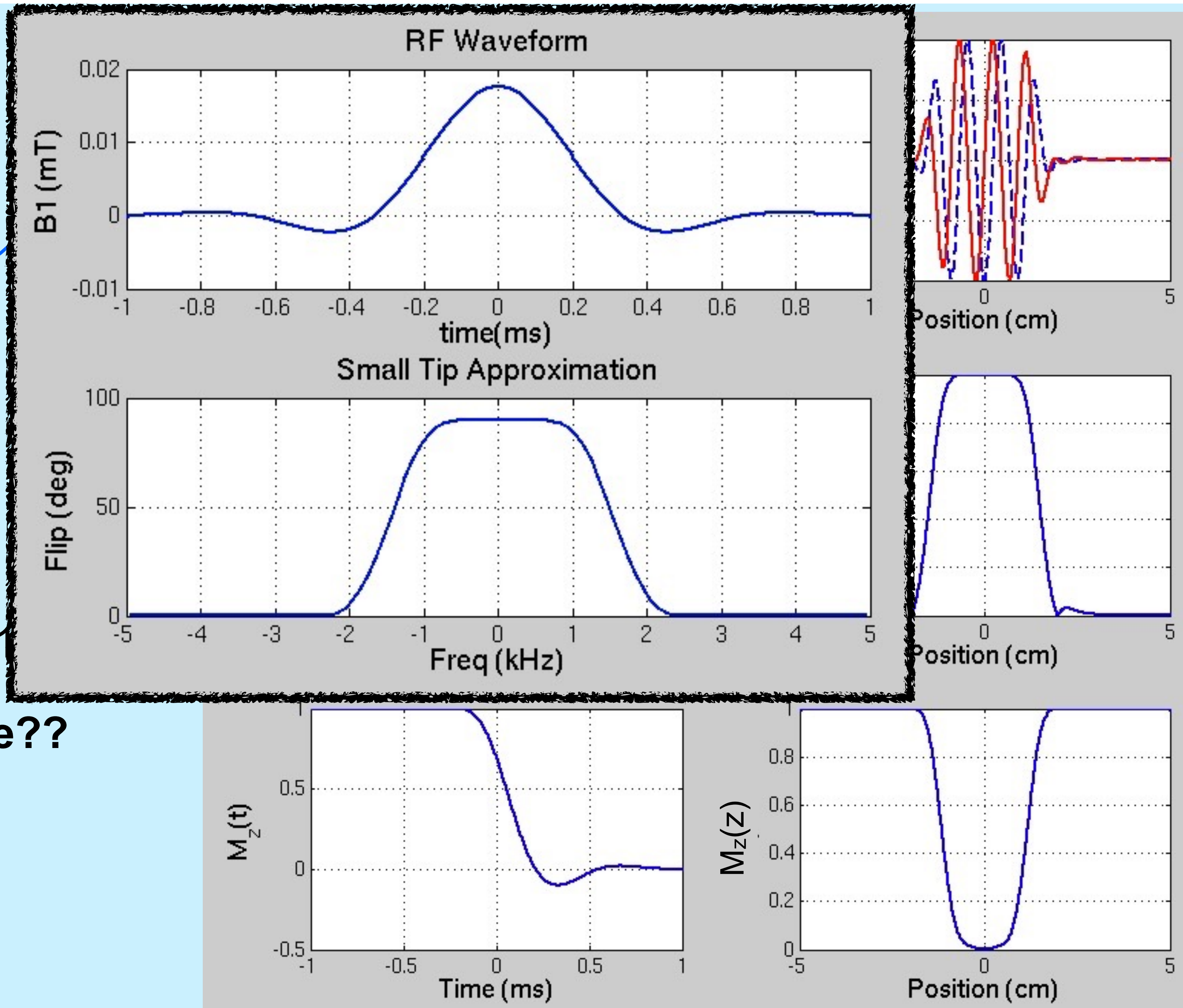
- Loop over z
  - Define R_z
  - Loop over t
    - $M' = R_z R_{-y}(t) M$
- Plot M over time and space



exampleB1_17.m

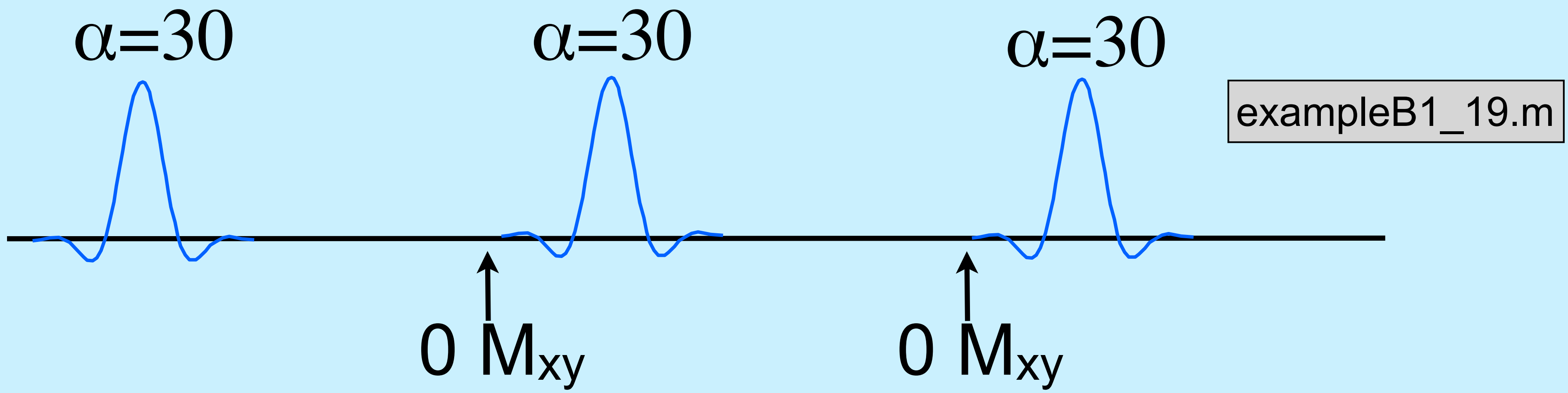
Example (With Off-Resonance)





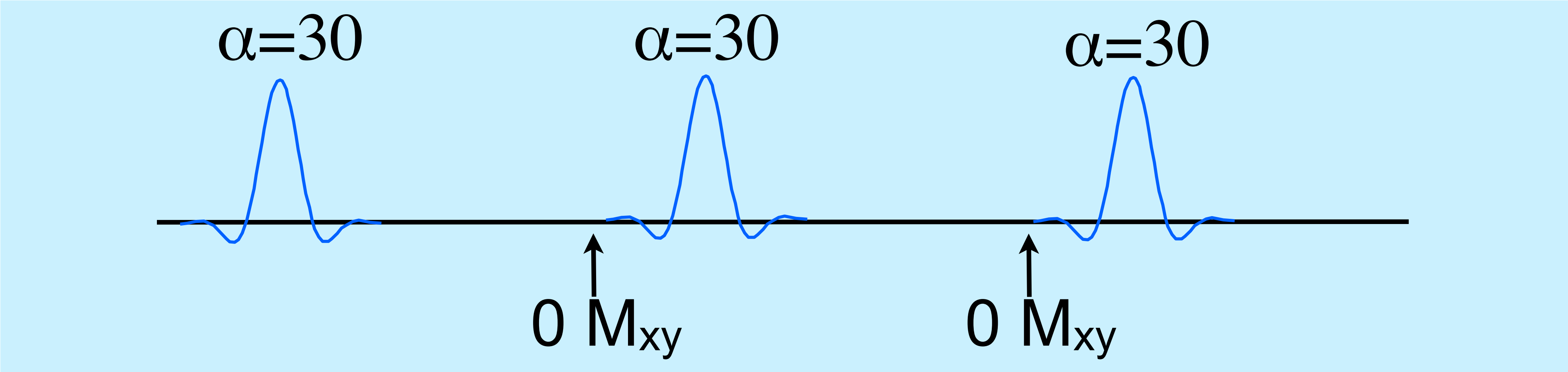
- 90 Excitation pulse
- BW ~ 3 kHz
- 2.3 mT/m gradient (1
- **2kHz off-resonance??**

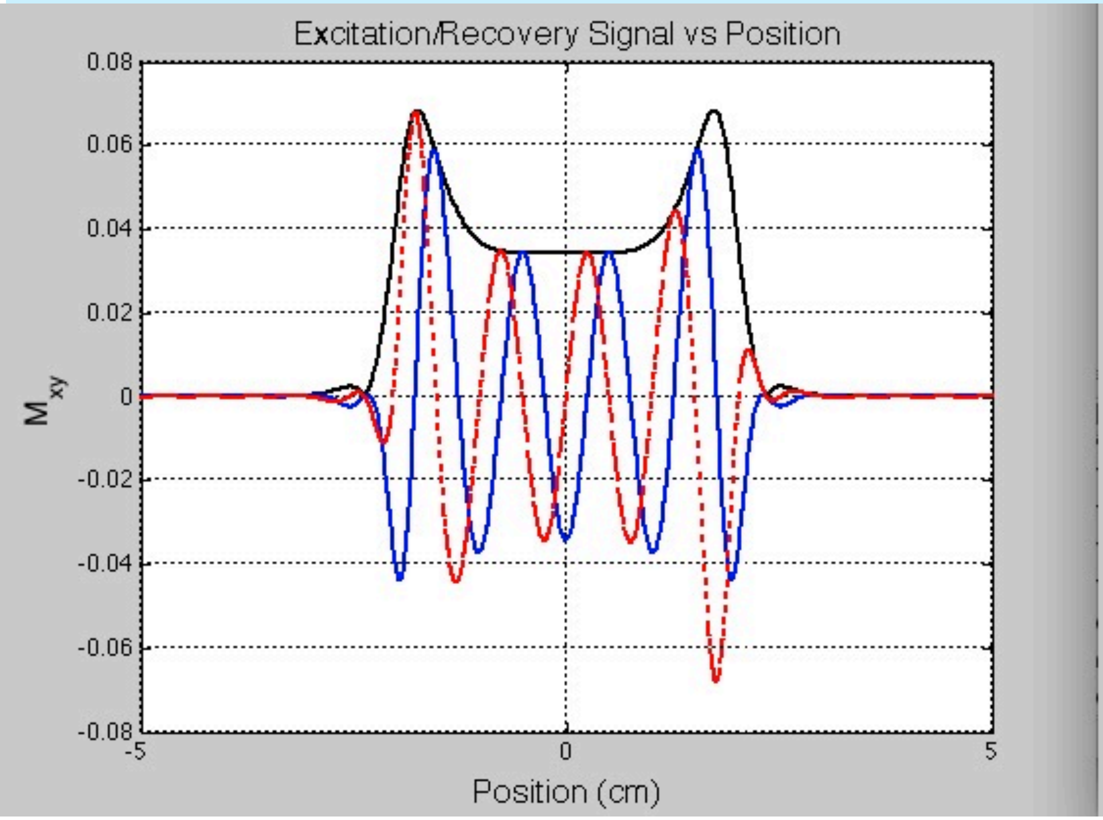
Excitation Recovery (Real Pulse)

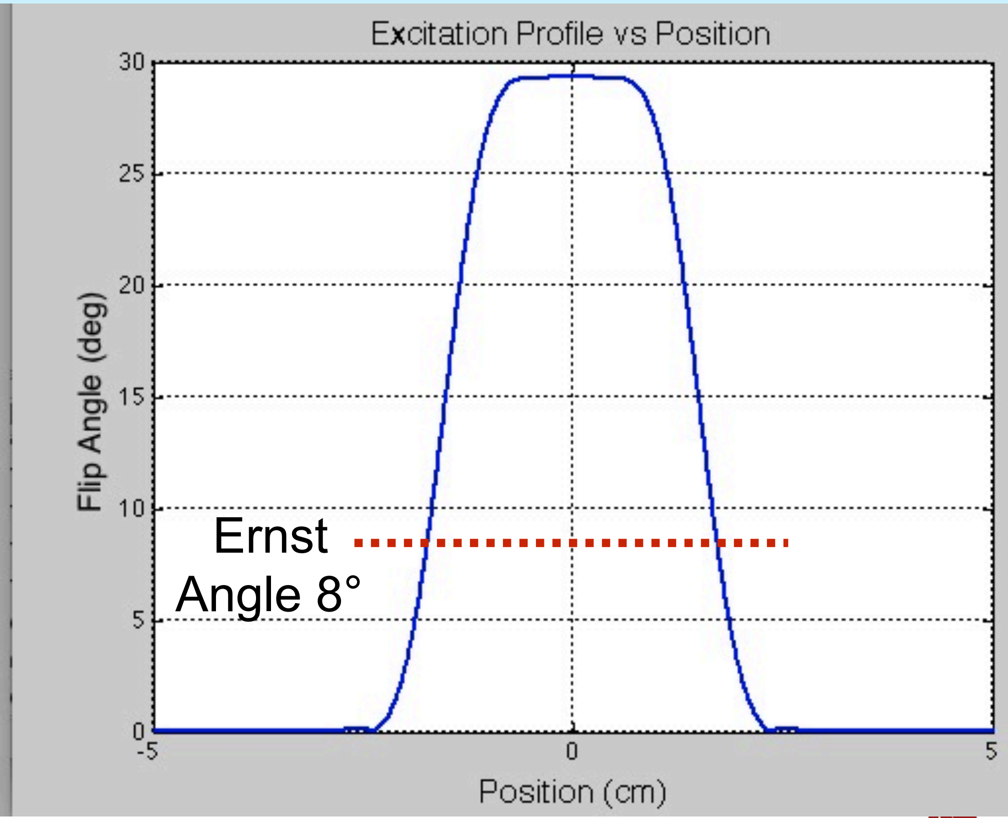


- Simulate full pulse and position
- Perfect spoiling ("keep only M_z" matrix)
- Matrix propagation to calculate steady-state at each position: E() = relaxation
  - End of RF to TR: Spoil, $E(TR-T_{RF})$
  - Over RF: [$E(\tau)R_z(\gamma Gz\tau)\ R_{-y}(t,\tau)$] at each interval

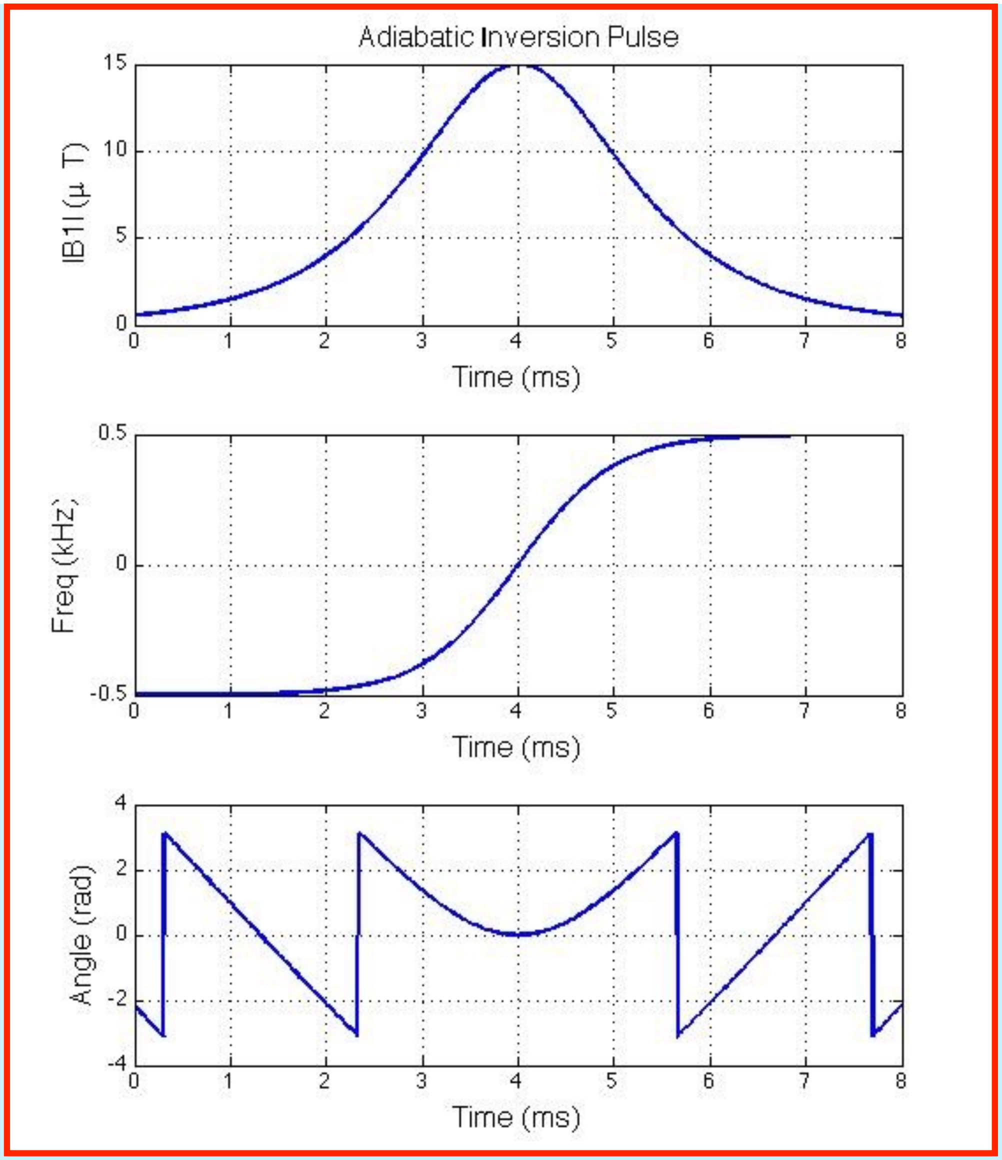
Excitation Recovery (Real Pulse)

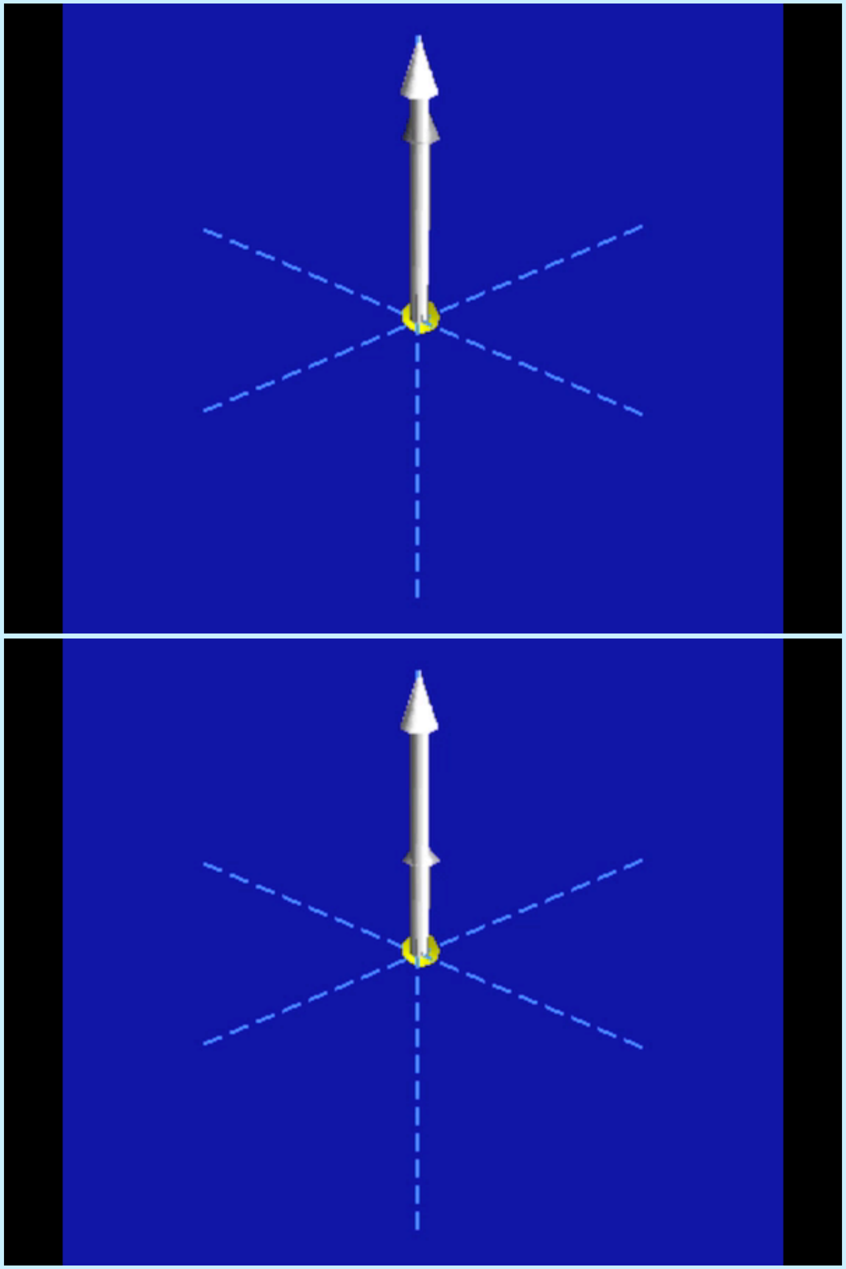






Example: Adiabatic Pulse





(See adiabatic.m,
[b1,freq,phase] = adiabatic(.15,1000,1000,0.008,.000004))

Exchange: Bloch-McConnell Equation

- M^a and M^b are magnetizations in "exchanging" pools:
  - τ_a and τ_b are resident times

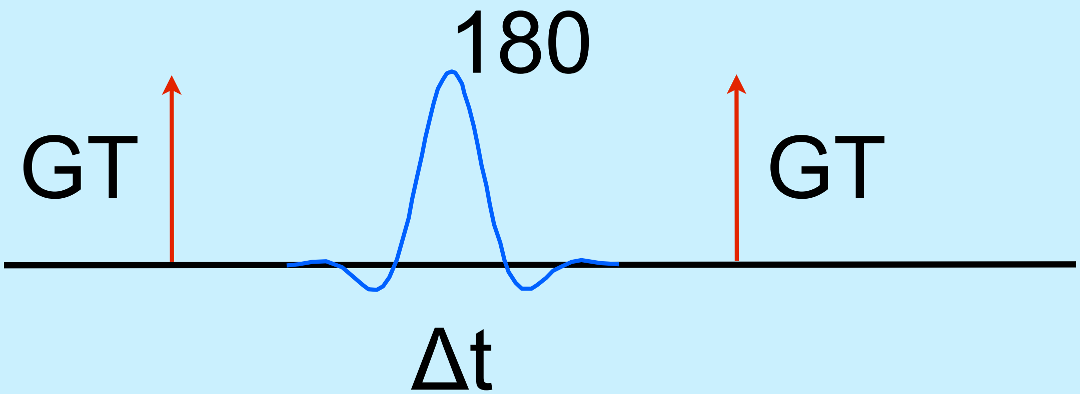
$$\frac{dM}{dt} = \begin{bmatrix} -1/T_2^a - 1/\tau_a & \gamma_a B_z & -\gamma_a B_y & 1/\tau_b & 0 & 0 \\ -\gamma_a B_z & -1/T_2^a - 1/\tau_a & \gamma_a B_x & 0 & 1/\tau_b & 0 \\ \gamma_a B_y & -\gamma_a B_x & -1/T_1^a - 1/\tau_a & 0 & 0 & 1/\tau_b \\ 1/\tau_a & 0 & 0 & -1/T_2^b - 1/\tau_b & \gamma_b B_z & -\gamma_b B_y \\ 0 & 1/\tau_a & 0 & -\gamma_b B_z & -1/T_2^b - 1/\tau_b & \gamma_b B_x \\ 0 & 0 & 1/\tau_a & \gamma_b B_y & -\gamma_b B_x & -1/T_1^b - 1/\tau_b \end{bmatrix} M + \begin{bmatrix} 0 \\ 0 \\ M_0^a/T_1^a \\ 0 \\ 0 \\ M_0^b/T_1^b \end{bmatrix}$$

$$M = \begin{bmatrix} M_x^a \\ M_y^a \\ M_z^a \\ M_x^b \\ M_y^b \\ M_z^b \end{bmatrix}$$

Single-Pool Bloch Equation:

$$\frac{dM}{dt} = \begin{bmatrix} -1/T_2 & \gamma B_z & -\gamma B_y \\ -\gamma B_z & -1/T_2 & \gamma B_x \\ \gamma B_y & -\gamma B_x & -1/T_1 \end{bmatrix} M + \begin{bmatrix} 0 \\ 0 \\ M_0/T_1 \end{bmatrix}$$

Challenge: Diffusion



- 1D Gaussian Diffusion: $\Delta l = \sqrt{2D\Delta t}$
- Imagine a sequence with 2 gradients of area GT, with a 180 refocusing pulse between.
- What is the expected value of the spin echo signal as a function of D, Δt, GT, ignoring T_2?

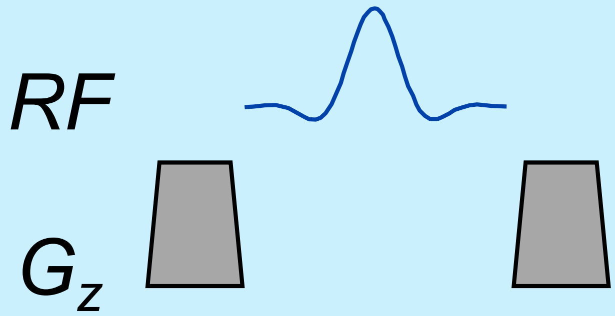
Summary

- Bloch equation: Rotations and Relaxation
- Consider M as a 3x1 vector
  - Rotations ~ Simple multiplier
  - Relaxation ~ $M' = AM+B$
  - Propagate Effects like "Operators"
- Brute force simulations by looping:
  - Time, Position, Frequency, etc

Extended Phase Graphs (EPG)

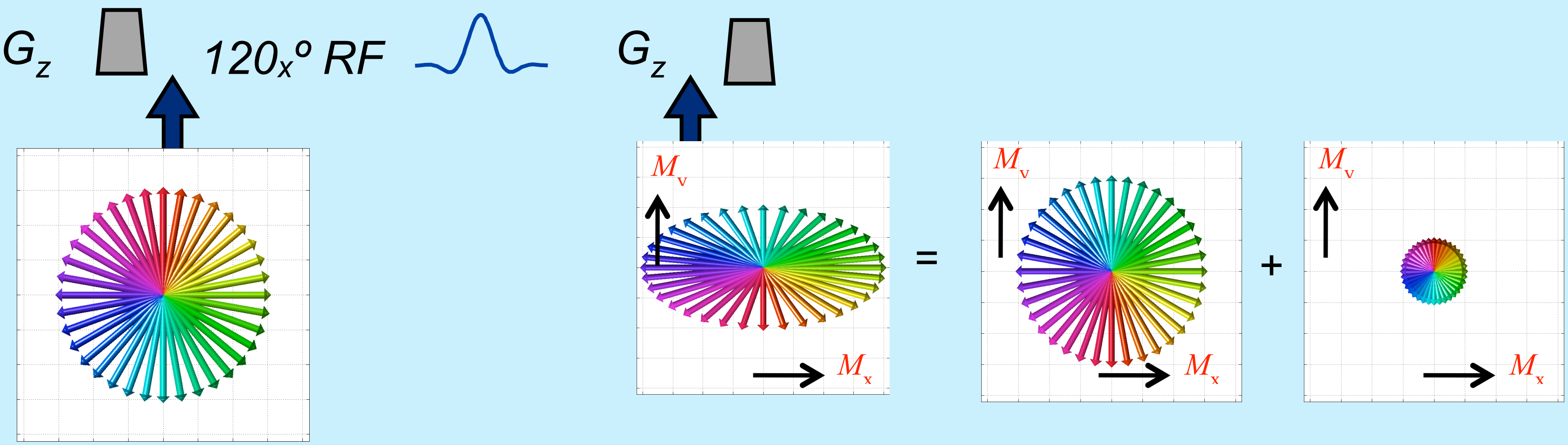
- Purpose / Definition
- Propagation
  - Gradients, Relaxation, RF
  - Diffusion
- Examples

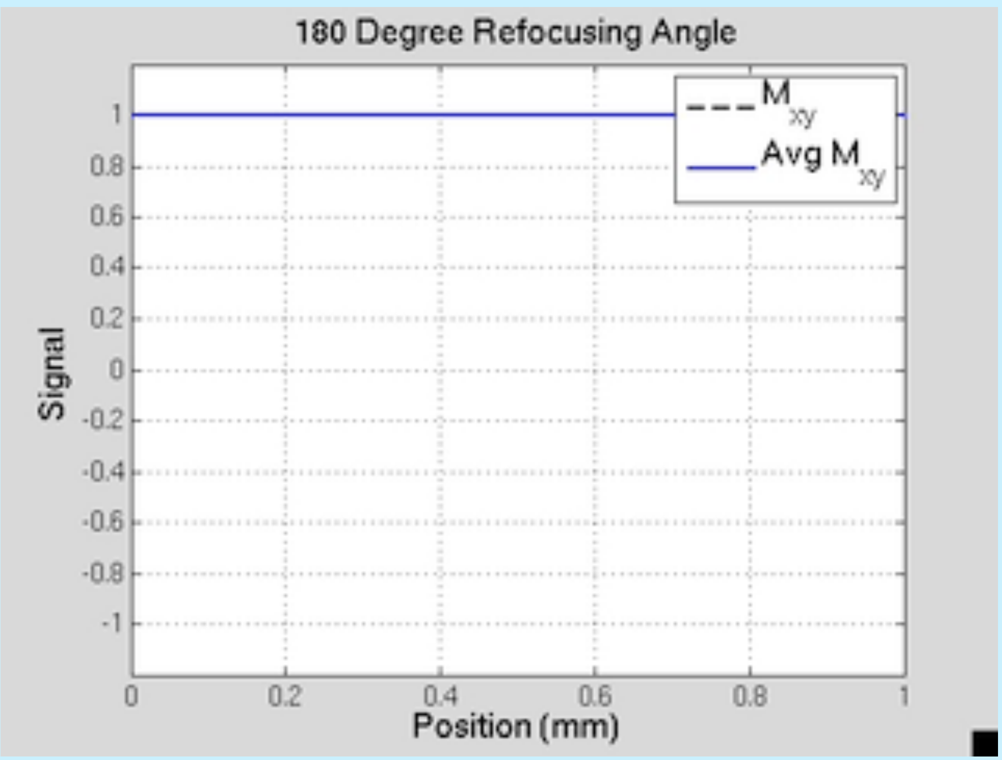
EPG Motivating Example: RF with Crushers

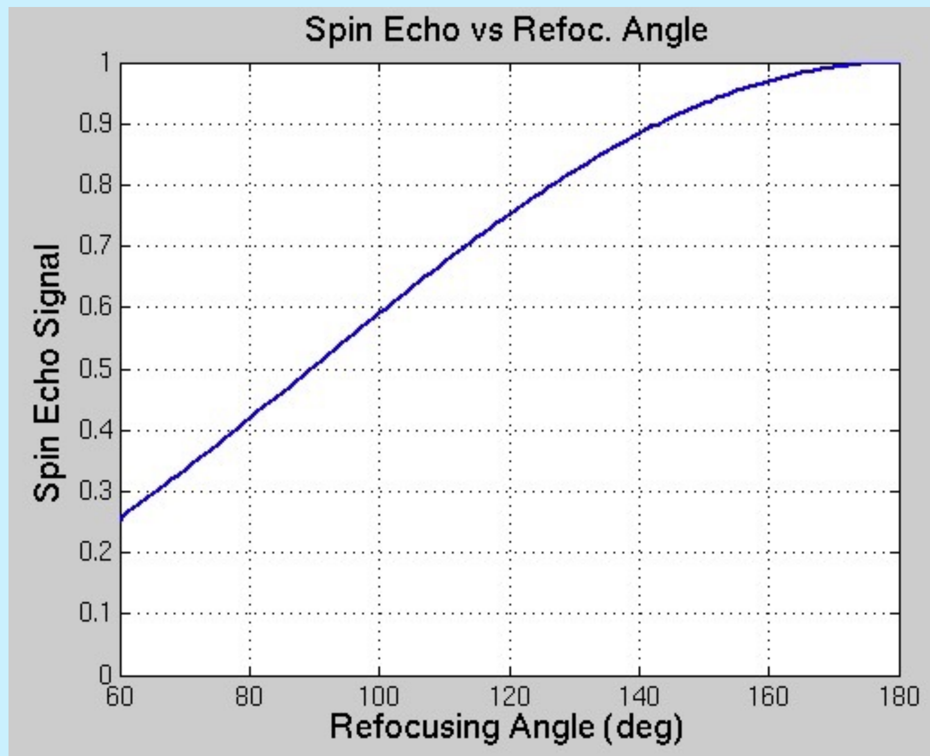


- Crushers are used to suppress spins that do not experience a 180º rotation
- Dephasing/Rephasing work if 180º is perfect

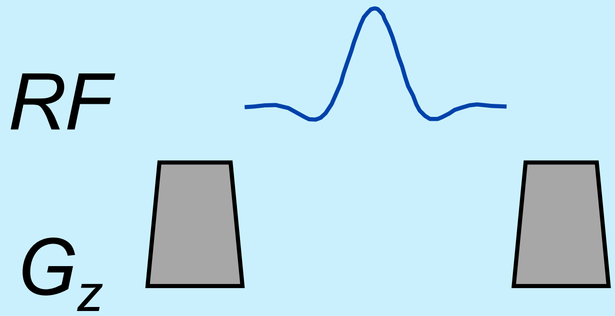
EPG Motivating Example: RF with Crushers







EPG Motivating Example: RF with Crushers

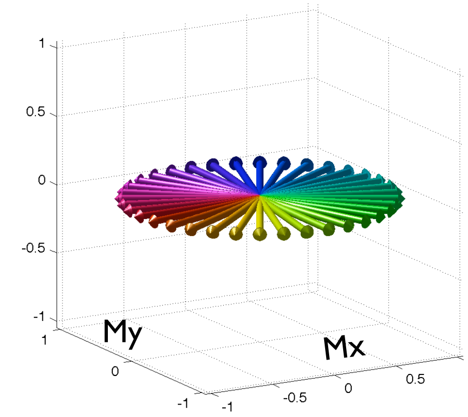


- Brute-force simulation works, but little intuition
- Quantized gradients produce “dephased cycles”
- Decompose elliptical distributions to +/- circular

Extended Phase Graphs: Purpose

If we assume…

Gradient areas are quantized into units that give a phase twist of one cycle (2π) across a voxel



One cycle of phase twist from a spoiler gradient (for example)

… then we can easily represent large groups of spins with a simple Fourier basis, and accurately calculate MR signals

Hennig J. Multiecho imaging sequences with low refocusing flip angles. J Magn Reson 1988; 78:397–407.

Hennig J. Echoes – How to Generate, Recognize, Use or Avoid Them in MR-Imaging Sequences, Part I. Concepts in Magnetic Resonance 1991; 3:125-143.

Hennig J. et al. Calculation of flip angles or echo trains with predefined amplitudes with the extended phase graph (EPG)-algorithm: principles and applications to hyperecho and TRAPS sequences. MRM 2004; 51:68-80

Weigel M. Extended Phase Graphs: Dephasing, RF Pulses, and Echoes - Pure and Simple. JMRI 2014;

Basic Magnetization Vector Definitions

- Common representations for magnetization:

$$M_{xy} = M_x + iM_y \qquad M_{xy}^* = M_x - iM_y$$

$$\begin{bmatrix} M_x \\ M_y \\ Mz \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0 \\ -0.5i & 0.5i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_{xy} \\ M_{xy}^* \\ M_z \end{bmatrix}$$

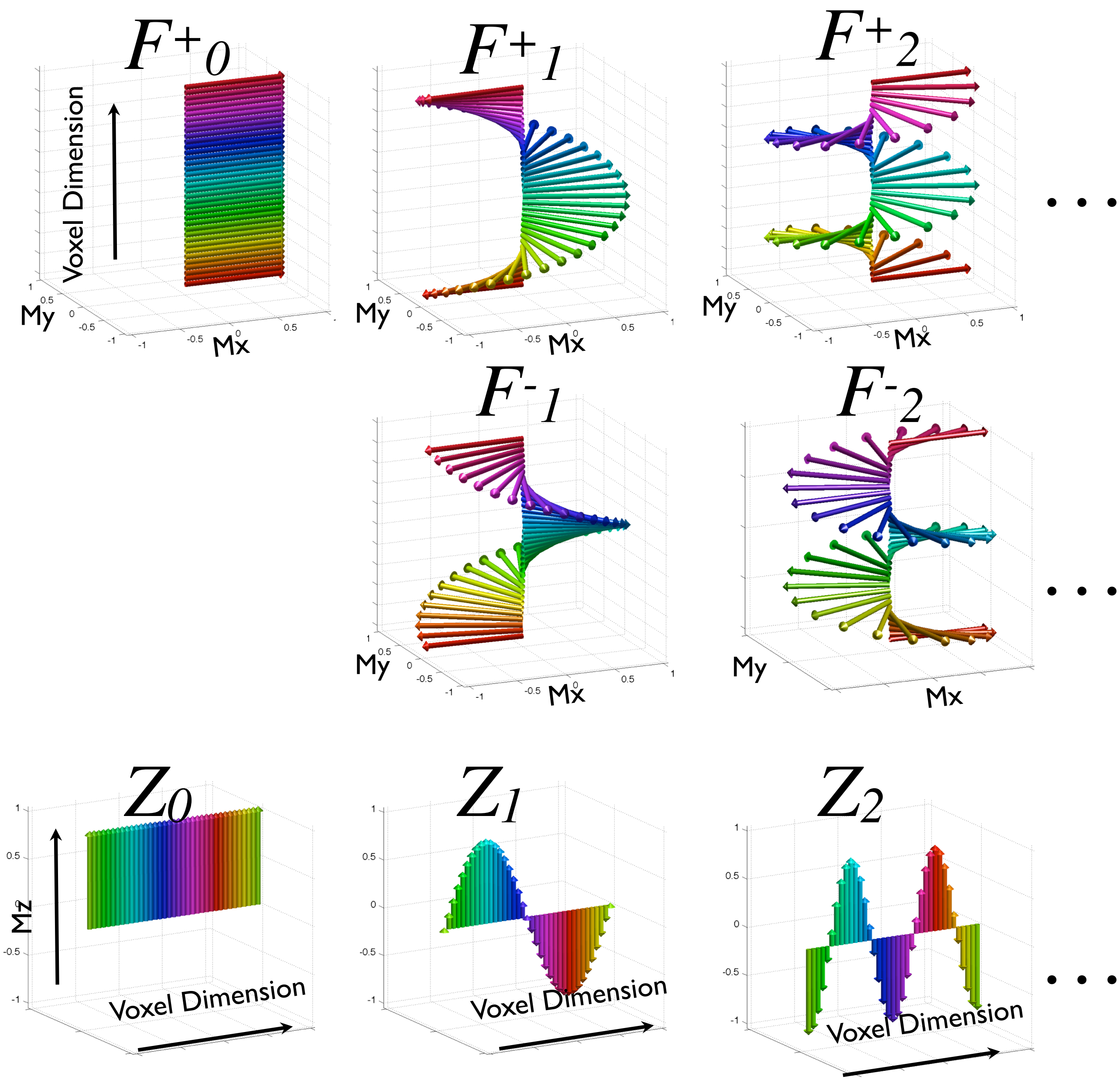
$$\begin{bmatrix} M_{xy} \\ M_{xy}^* \\ M_z \end{bmatrix} = \begin{bmatrix} 1 & i & 0 \\ 1 & -i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ Mz \end{bmatrix}$$

The EPG Basis

- Consider spins in a voxel:
  - z is the location (0 to 1) across the voxel
  - $M_{xy}(z)$ and $M_{xy}^{*}(z)$ are the transverse magnetization
  - $M_z(z)$ is the longitudinal magnetization
- Represent the (huge) number of spins using a compact basis set of F_n and Z_n coefficients
- Motivation: Propagation of the basis functions through RF, gradients, relaxation is fairly simple

EPG Basis: Graphical

- F_n and Z_n are basis <u>*coefficients*</u>
- Transverse basis are simple phase twists *(sign of n indicates direction)*
- Longitudinal basis are sinusoids

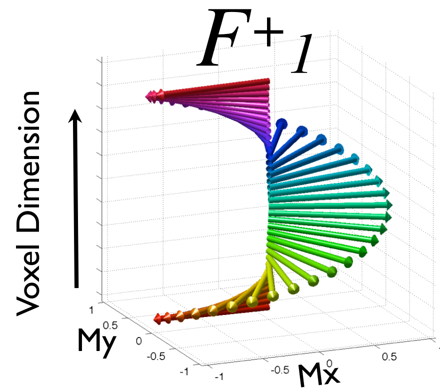


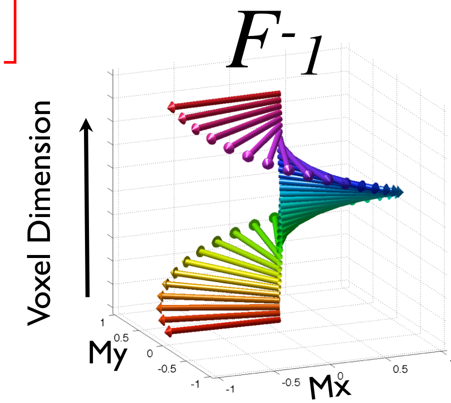
EPG Basis: Mathematically

- Transverse basis functions (F_n) are just phase twists:

$$M_{xy}(z) = \sum_{n=-\infty}^{\infty} F_n^+ e^{2\pi i n z}$$

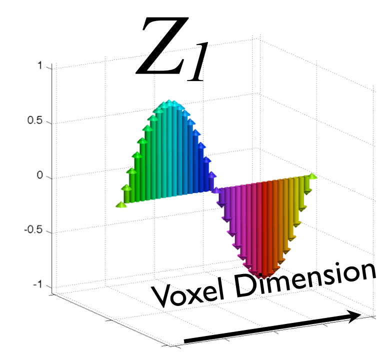
$$M_{xy}(z) = F_0^+ + \sum_{n=1}^{\infty} \left[F_n^+ e^{2\pi i n z} + (F_n^-)^* e^{-2\pi i n z}\right]$$





- Longitudinal basis functions (Z_n) are sinusoids:

$$M_z(z) = Real\left\{Z_0 + 2\sum_{n=1}^{N} Z_n e^{2\pi i n z}\right\}$$



F_n and Z_n are the coefficients, but we sometimes use them to refer to the basis functions ("twists") they multiply

Although there are other basis definitions, this is consistent with that of Weigel et al. J Magn Reson 2010; 205:276-285

Magnetization to EPG Basis

- F^+ states:

$$F^+{}_n = \int_0^1 M_{xy}(z) e^{-2\pi i n z} dz$$

- F^- states:

$$F^-{}_n = F^*{}_{-n} = \int_0^1 M_{xy}^*(z) e^{-2\pi i n z} dz$$

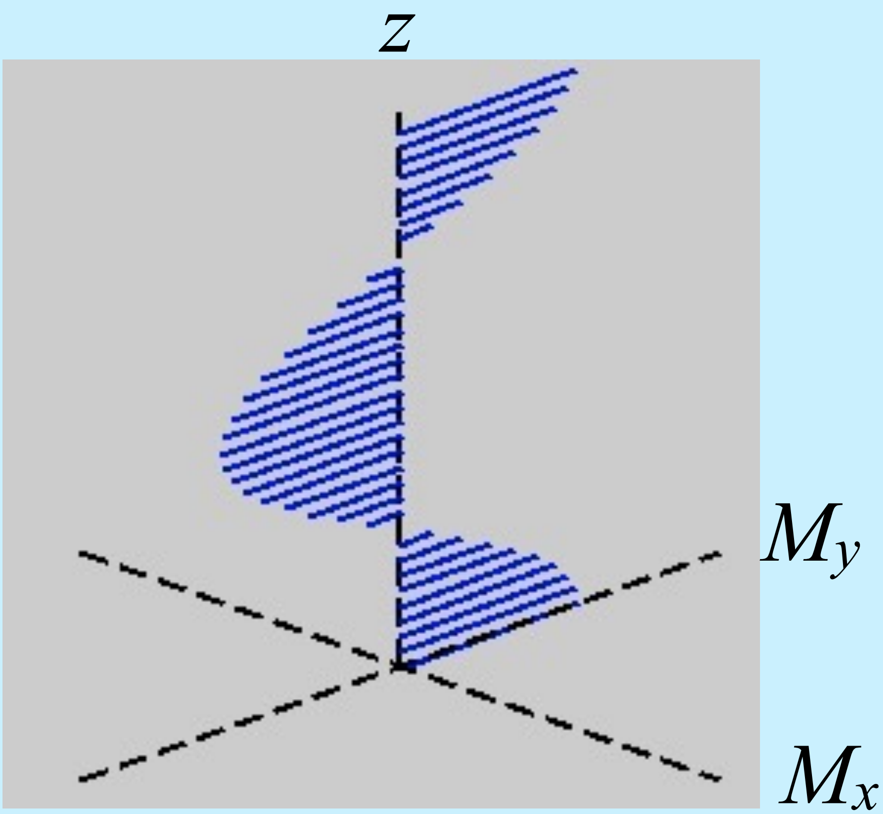
- Z states:

$$Z_n = \int_0^1 M_z(z) e^{-2\pi i n z} dz$$

Note redundancy $F^-{}_n = (F^+{}_{-n})^*$
(Can use $F^+{}_n$ and $F^-{}_n$ for n>0, or just $F^+{}_{-n}$ (all n)

Review Question

- What are the F^+ and F^- states that represent this magnetization (entirely along M_y)?



Basis Functions in MR Sequences

Key Point: Basis is easily propagated in MR sequences:

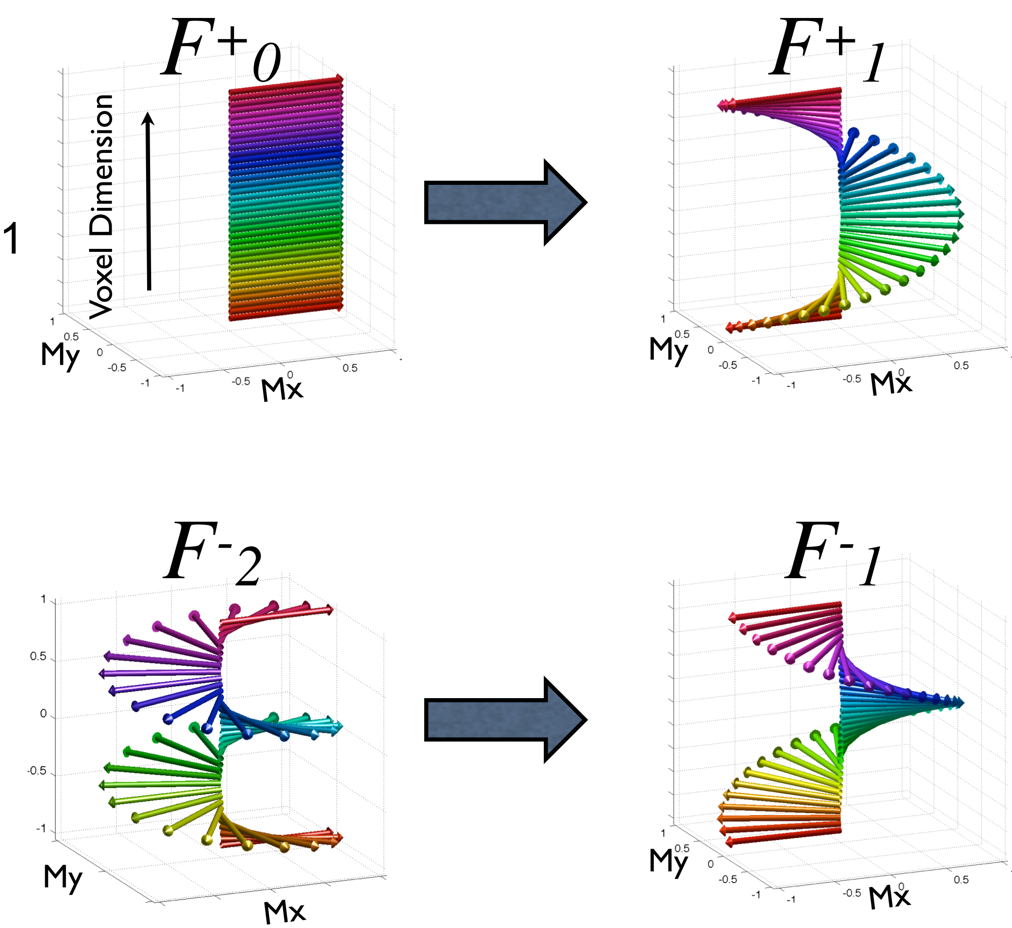
- Gradient (one cycle over voxel):
  - Increase/decrease F^+_n or F^-_n state number (n), e.g. $F^+_{n+1}=F^+_n$
- RF Pulse
  - Mixes coefficients between Z_n, F^+_n, and F^-_n [or $(F^+_{-n})^*$]
- Relaxation
  - T_2 decay attenuates F_n coefficients
  - T_1 recovery attenuates Z_n coefficients, and enhances Z_0
- Diffusion
  - Increasing attenuation with n (described later)

EPG Propagation: Gradient

Gradients induce one cycle (2π) of phase across a voxel

$$F^{+}_{n+1} = F^{+}_{n}$$

- Magnetization in F^{+}_{n} moves to F^{+}_{n+1}
  - Dephasing for n>=0
  - Rephasing for n<0 (or F^{-})
    - Generally can apply p cycles (increase n by p)
- Z states are unaffected

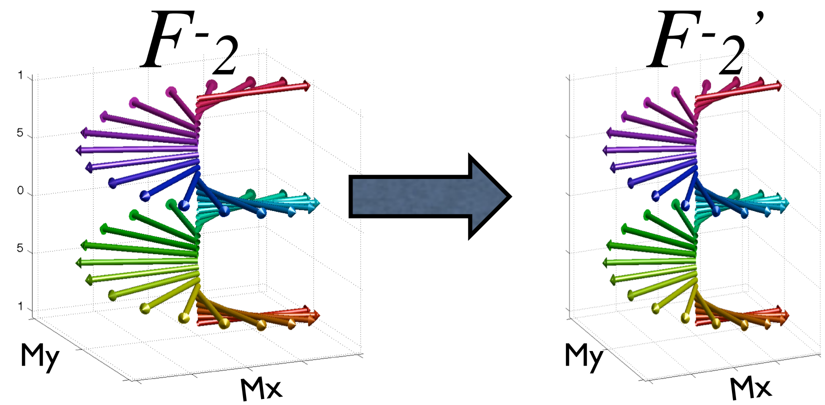


Assume the "gradient" such as a spoiler or crusher induces one twist cycle per voxel

EPG Relaxation over Period T

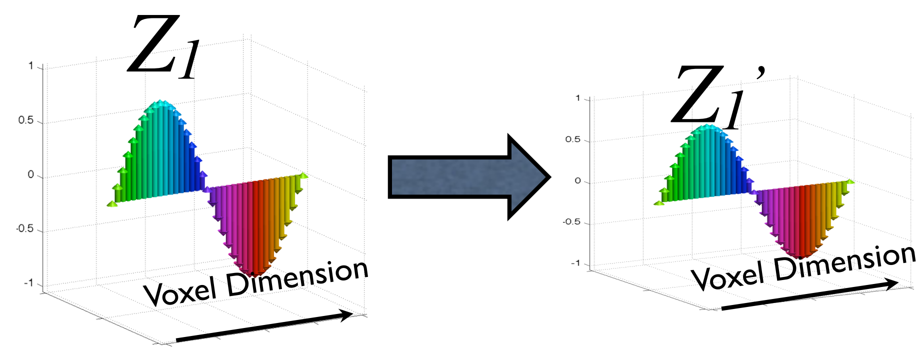
- Transverse states:

$$F_n{}' = F_n\, e^{-T/T2}$$



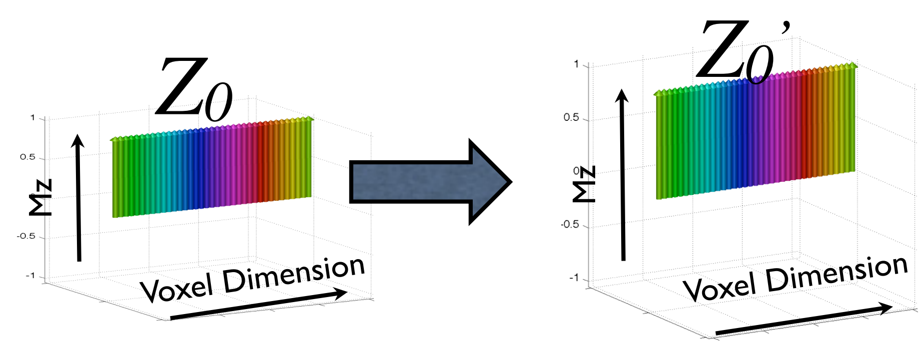
- Z_n states attenuated:

$$Z_n{}' = Z_n\, e^{-T/T1} \qquad (n>0)$$



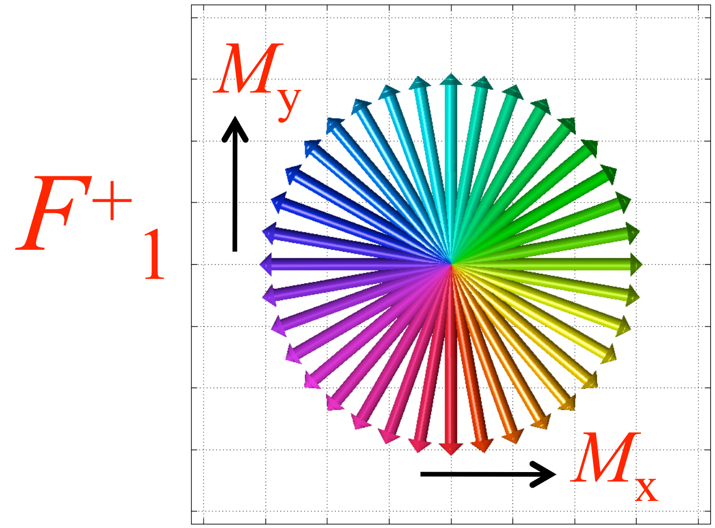
- Z_0 state also experiences recovery:

$$Z_0{}' = M_0\,(1-e^{-T/T1}) + Z_0\, e^{-T/T1}$$

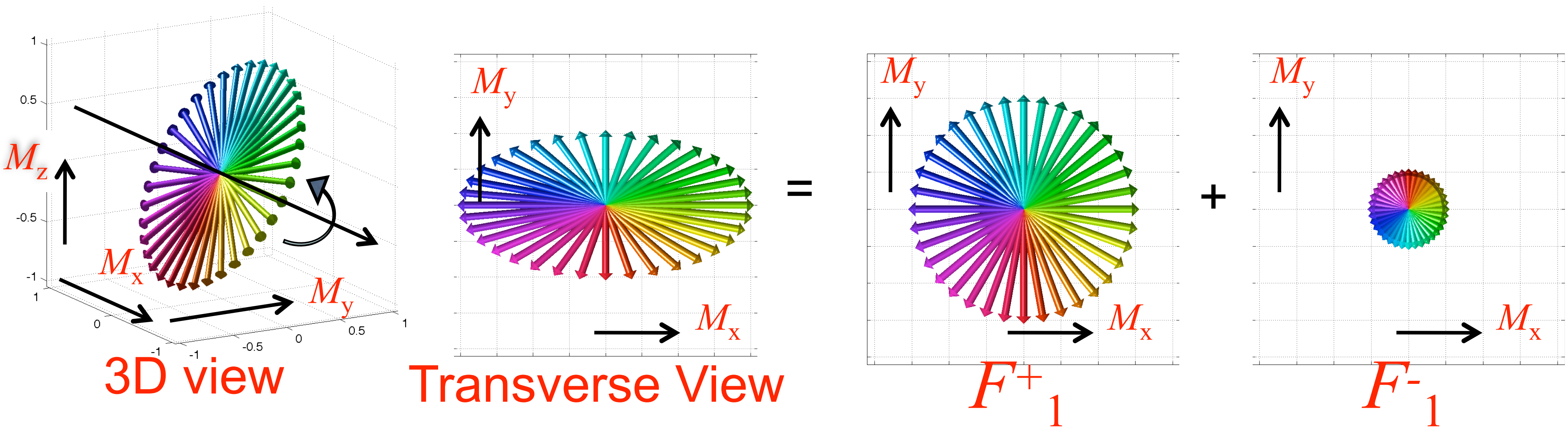


EPG RF: Transverse Effects

- First consider transverse spins after one cycle of dephasing:
- After a 60_x^o tip, the transverse distribution is elliptical, but can be decomposed into opposite circular twists (F^+_1 and F^-_1)



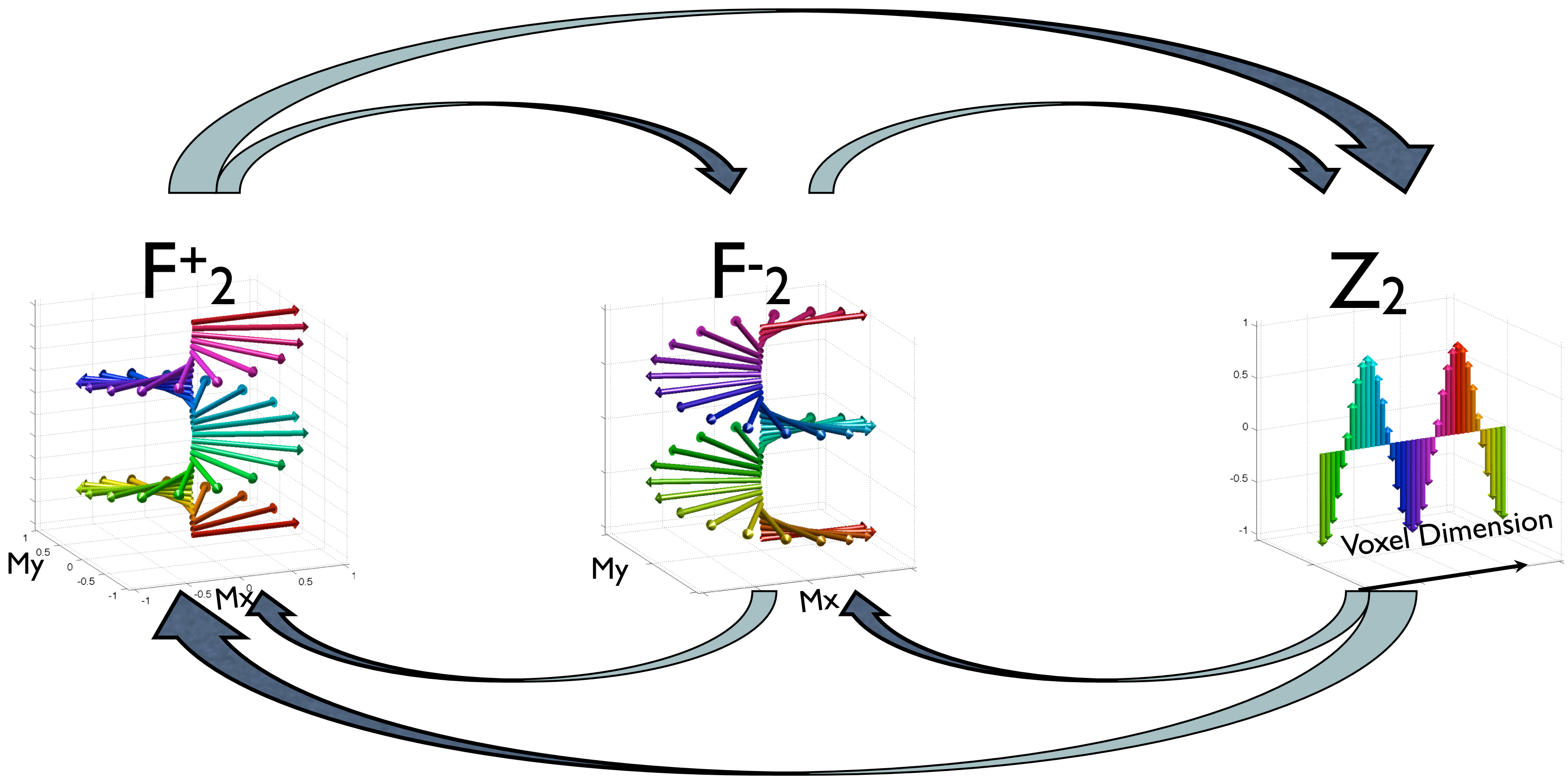
Transverse View



Longitudinal (Zn) states are also affected – described shortly

EPG RF Rotations

- An RF pulse cannot change number of cycles
- "Mixes" F^+_n, F^-_n and Z_n *(details next…)*



EPG RF Rotations

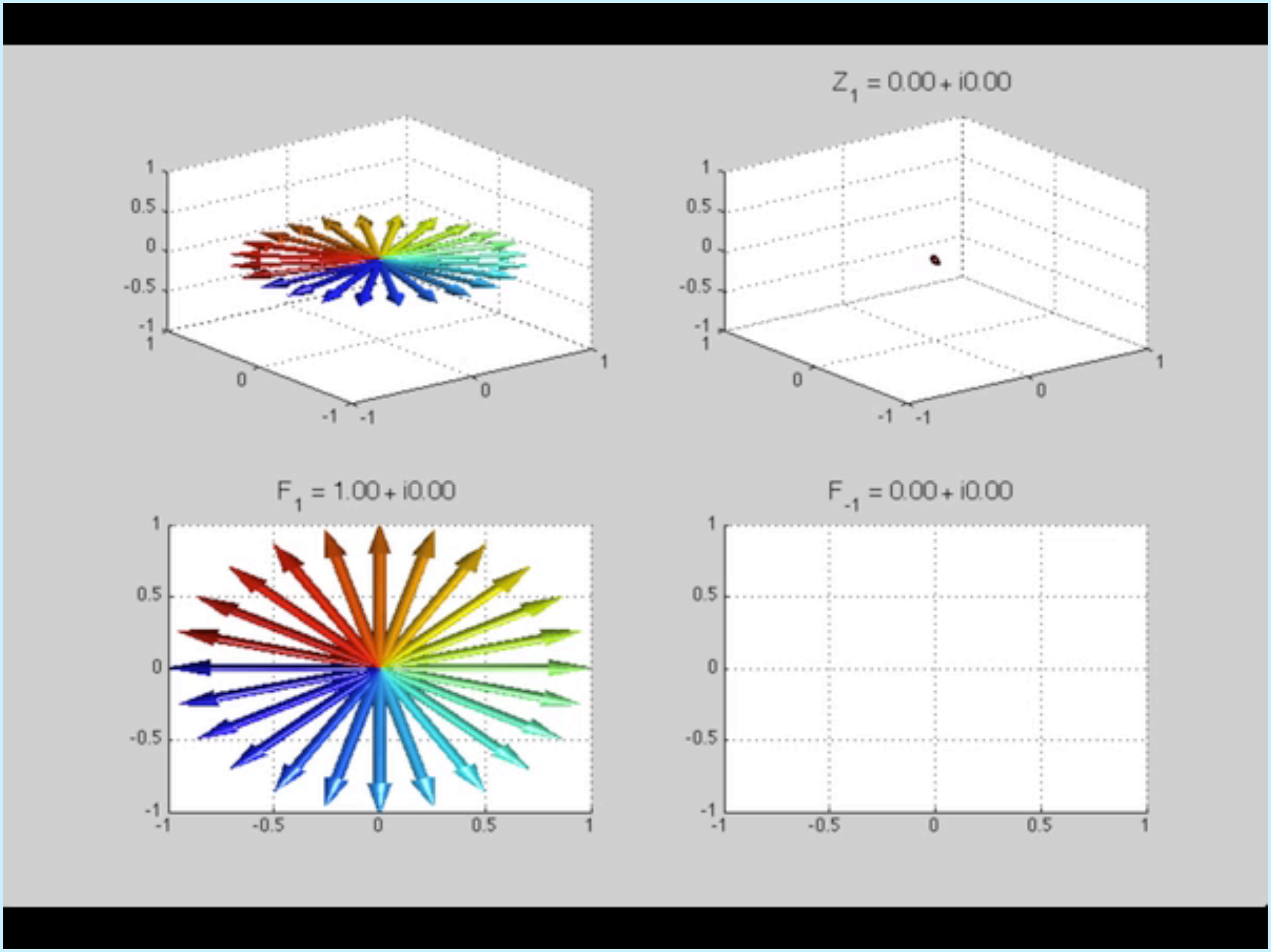
- Dephased magnetization generally has an elliptical distribution, even after additional nutations (flips)
- Can decompose into a sum of opposite circular twists (previous slide) and cosines along M_z
- Can derive (trigonometrically) for flip angle α about a transverse axis with angle ϕ from M_x:

$$\begin{bmatrix} {F^+}_n \\ {F^-}_n \\ Z_n \end{bmatrix}' = \begin{bmatrix} \cos^2(\alpha/2) & e^{2i\phi}\sin^2(\alpha/2) & -ie^{i\phi}\sin\alpha \\ e^{-2i\phi}\sin^2(\alpha/2) & \cos^2(\alpha/2) & ie^{-i\phi}\sin\alpha \\ -i/2e^{-i\phi}\sin\alpha & i/2e^{i\phi}\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} {F^+}_n \\ {F^-}_n \\ Z_n \end{bmatrix}$$

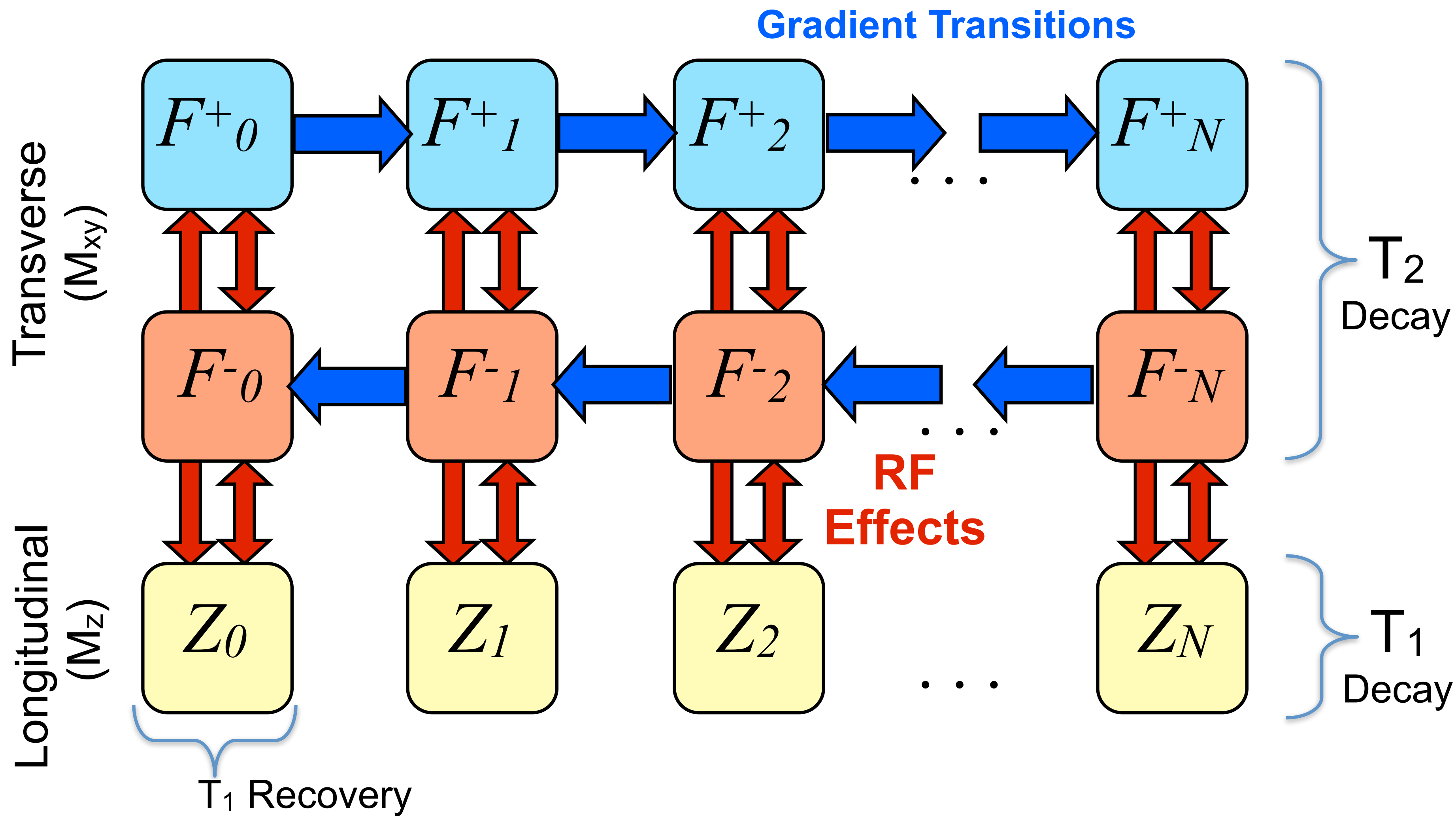
Same R_ϕ RF rotation as before, in $[M_{xy}, {M_{xy}}^*, M_z]^T$ frame

Example: 180_x^o Refocusing Pulse

Magnetization Starting as $F^+_1=1$



Phase Graph "States" (Flow Chart)



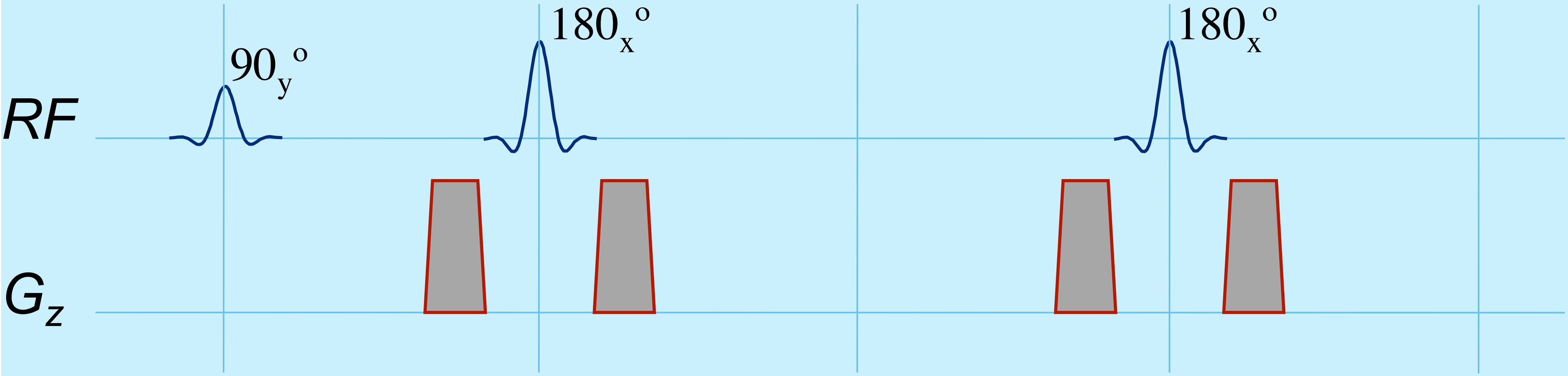
Examples:

- For a 90_x rotation:
- (Right-handed!)

- For a 90_y rotation:
- (Right-handed!)

$$\begin{bmatrix} {F^+}_n \\ {F^-}_n \\ Z_n \end{bmatrix}' = \begin{bmatrix} \cos^2(\alpha/2) & e^{2i\phi}\sin^2(\alpha/2) & -ie^{i\phi}\sin\alpha \\ e^{-2i\phi}\sin^2(\alpha/2) & \cos^2(\alpha/2) & ie^{-i\phi}\sin\alpha \\ -i/2e^{-i\phi}\sin\alpha & i/2e^{i\phi}\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} {F^+}_n \\ {F^-}_n \\ Z_n \end{bmatrix}$$

Example 1: Ideal Spin Echo Train



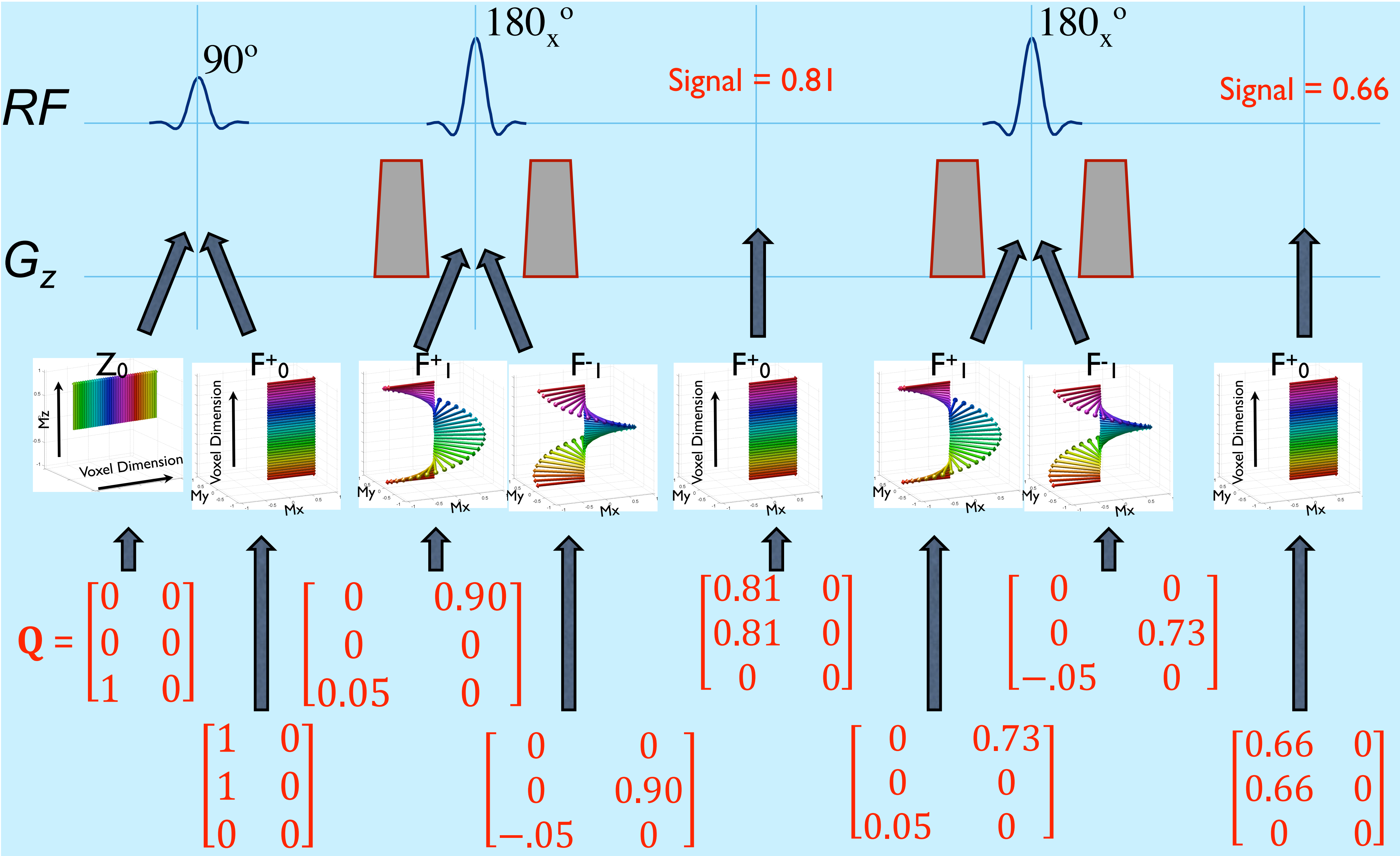
- 90 excitation transfers Z_0 to F_0

exampleB2_23.m

- Crusher gradients cause twist cycle
- Relaxation between RF pulses (Here $e^{-TE/T2}=0.81$, $e^{-TE/T1}=0.9$)
- 180_x pulse rotation:
  - Swap F_n and F_{-n}
  - Invert Z_n

$$\begin{bmatrix} F^+{}_n \\ F^-{}_n \\ Z_n \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} F^+{}_n \\ F^-{}_n \\ Z_n \end{bmatrix}$$

Example 1: Ideal Spin Echo Train



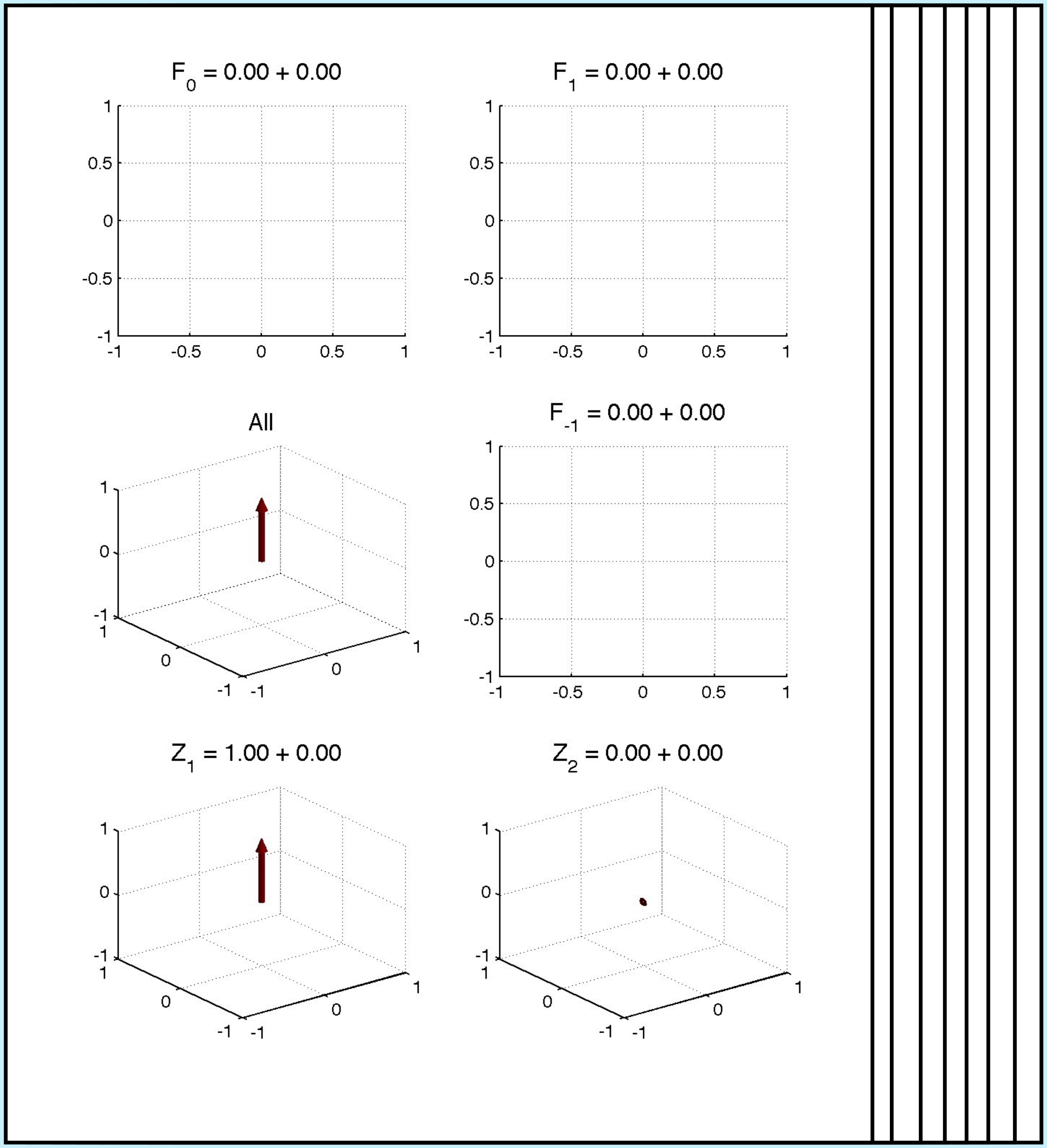
Example 1: Summarized

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

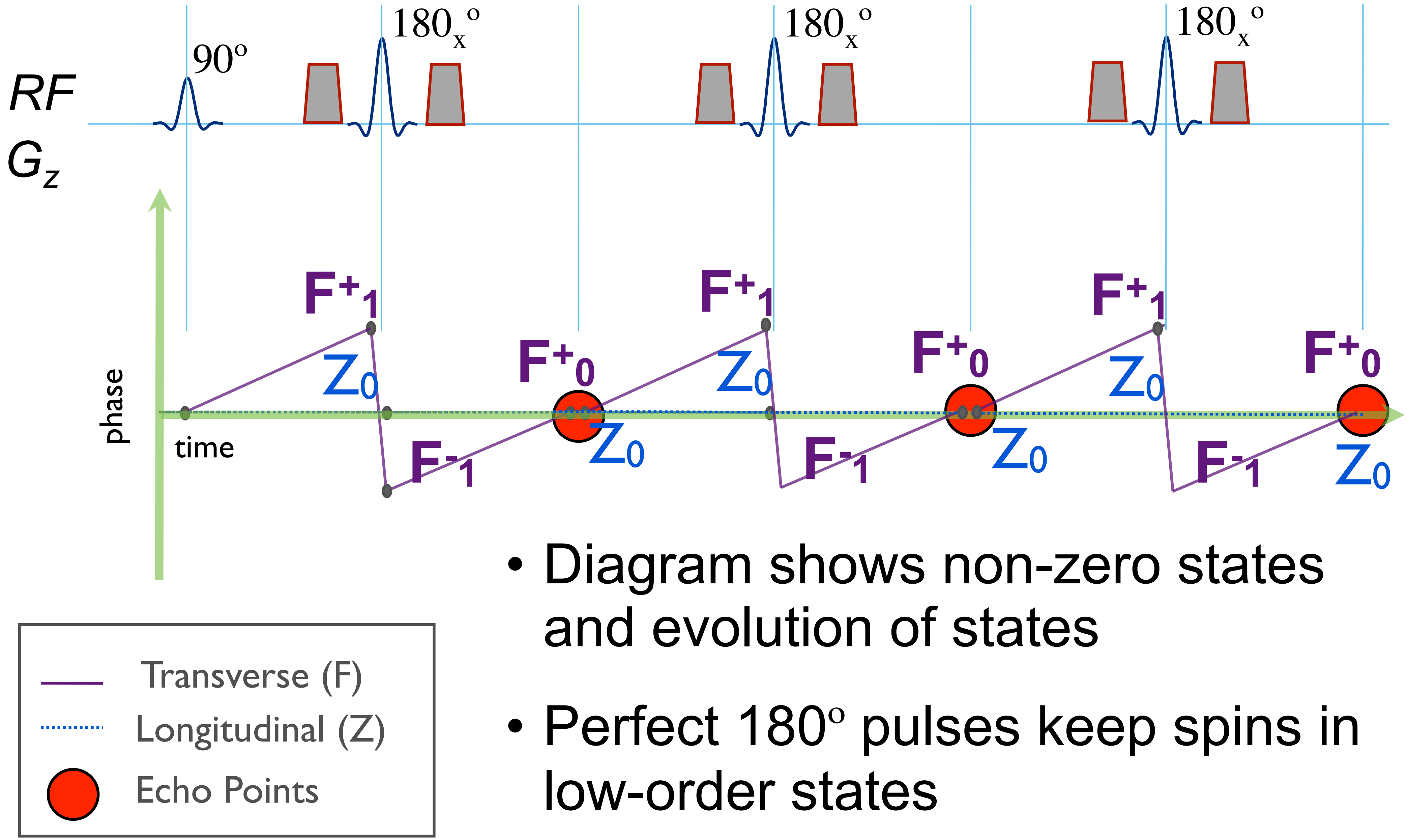
$$\begin{bmatrix} 0 & 0.90 \\ 0 & 0 \\ 0.05 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 0.90 \\ -.05 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.81 & 0 \\ 0.81 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0.73 \\ 0 & 0 \\ 0.05 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0.73 \\ -.05 & 0 \end{bmatrix} \quad \begin{bmatrix} 0.66 & 0 \\ 0.66 & 0 \\ 0 & 0 \end{bmatrix}$$



Coherence Pathways: Spin Echo



- Diagram shows non-zero states and evolution of states
- Perfect 180^o pulses keep spins in low-order states

Example 2: Non-180° Spin Echo

- Ideal spin echo train gives simple RF rotations
- Now assume refocusing flip angle of 130°
- Compare RF rotations:

180_x^o

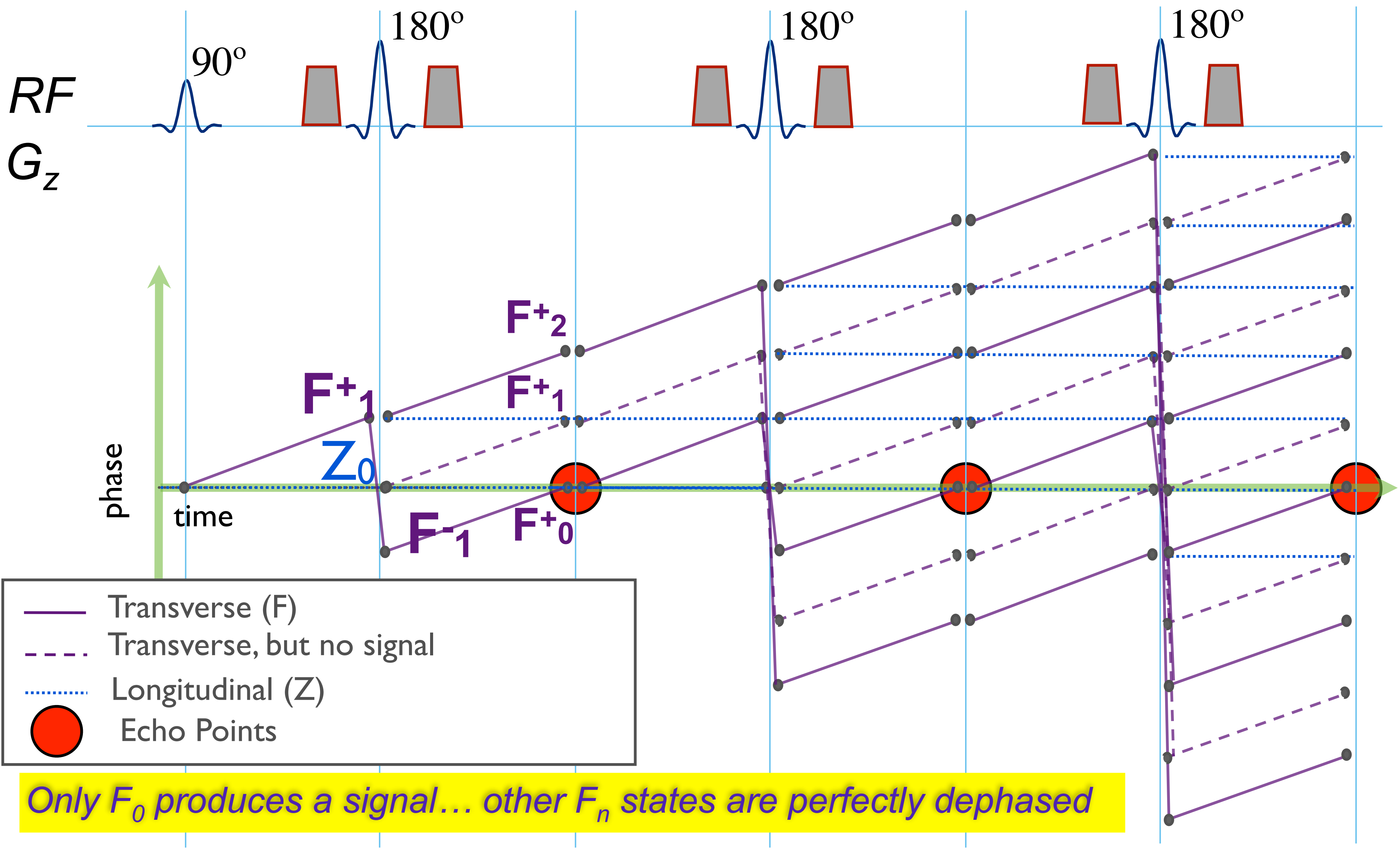
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

130_x^o

$$\begin{bmatrix} 0.18 & 0.82 & -0.77i \\ 0.82 & 0.18 & 0.77i \\ 0.38i & 0.38i & -0.64 \end{bmatrix}$$

- Positive F^+_n states remain (from F^+_n) because magnetization is not perfectly reversed, generating higher order states
- Many more coherence pathways (see next slide…)

Coherences: Non-180° Spin Echo

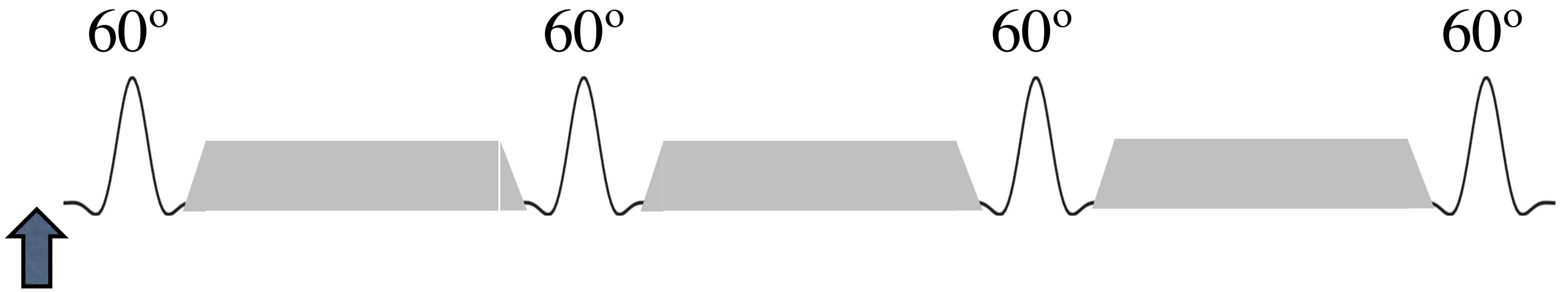


Only F_0 produces a signal… other F_n states are perfectly dephased

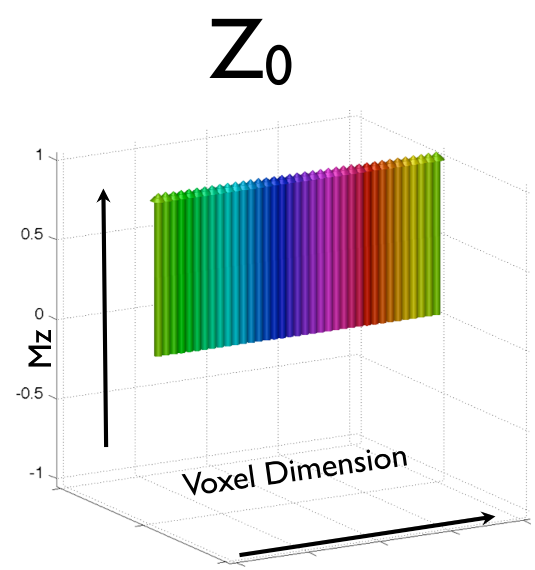
Example 3: Stimulated Echo Sequence

- We will follow this sequence through time
- Show which states are populated at each point

Example 3: Stimulated Echo Sequence

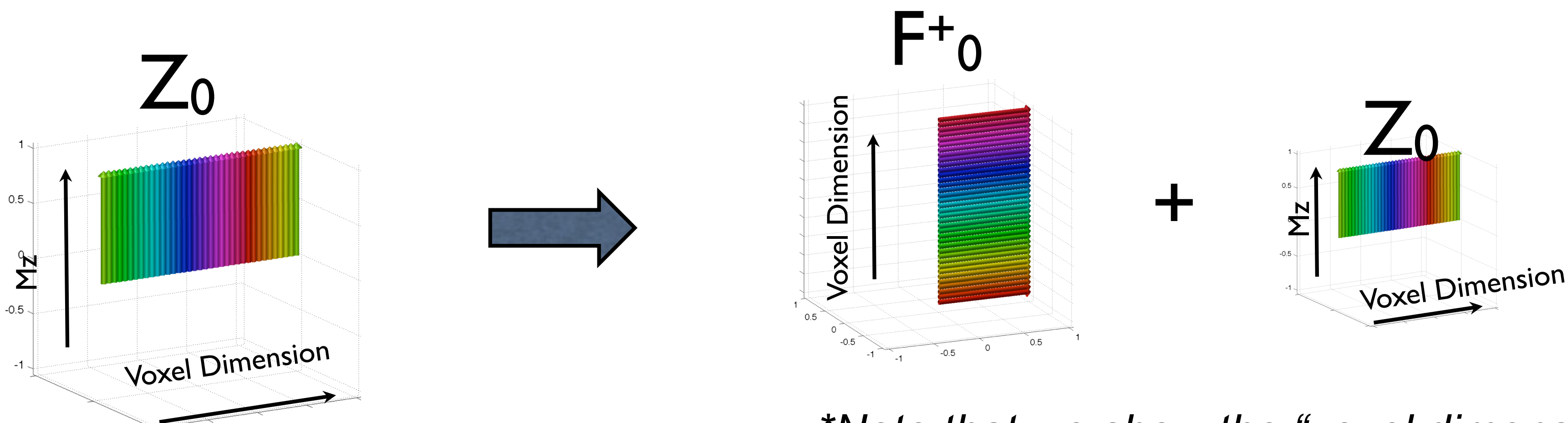


- Prior to the first pulse, all spins lie along M_z
- This is Z_0=1



Stimulated Echo: Excitation: F^+_0

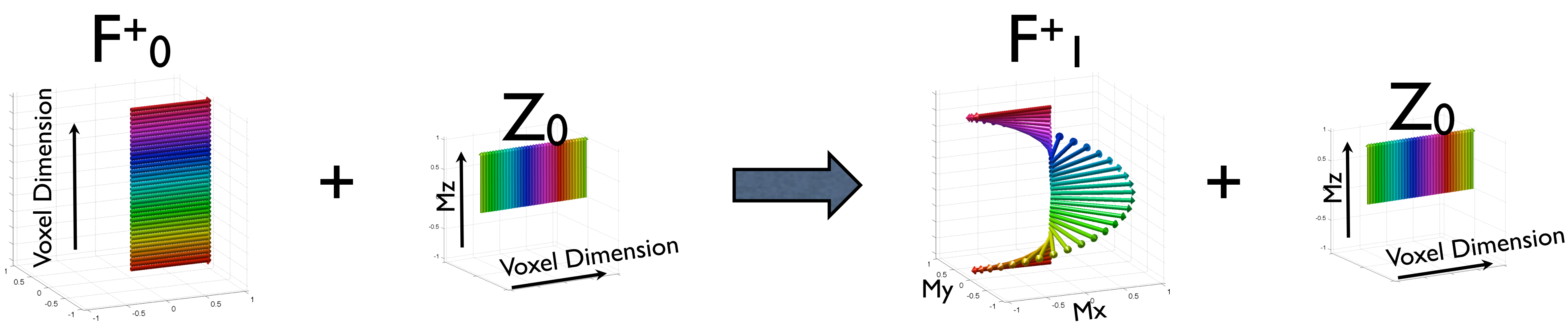
- After a 60º pulse, transverse spins are aligned along M_y (F^+_0 = sin60º)
- One half (cos60º) of the magnetization is still represented by Z_0



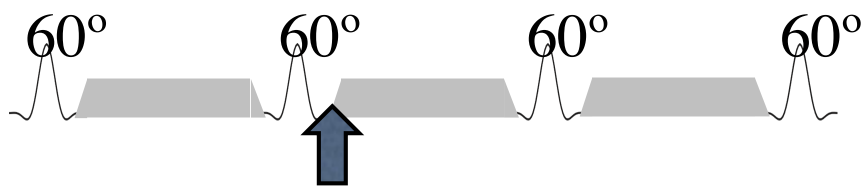
**Note that we show the "voxel dimension" along different axis for Z and F states*

One Gradient Cycle

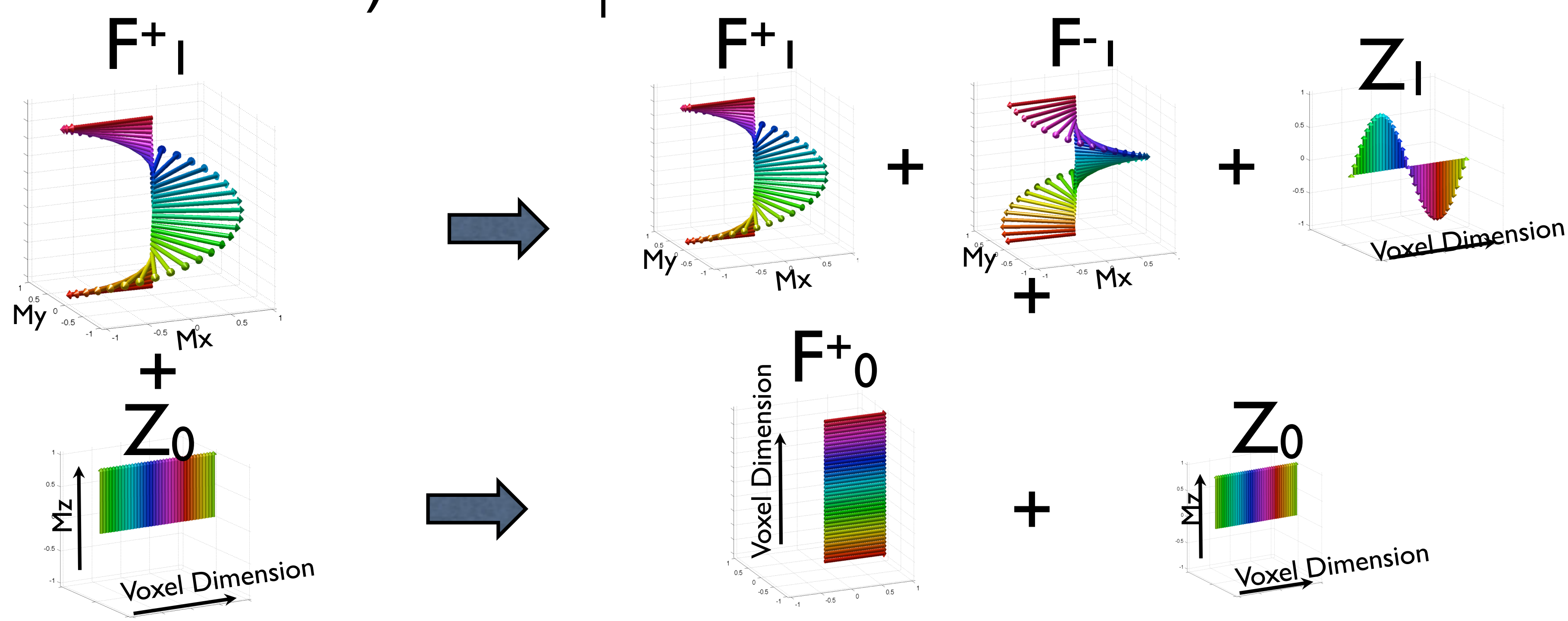
- The gradient “twists” the spins represented by F^+_0
- We call this state F^+_1, where the 1 indicates one cycle of phase (F^+_1=0.86)
- The spins represented by Z_0 are unaffected



Another Excitation (60°)

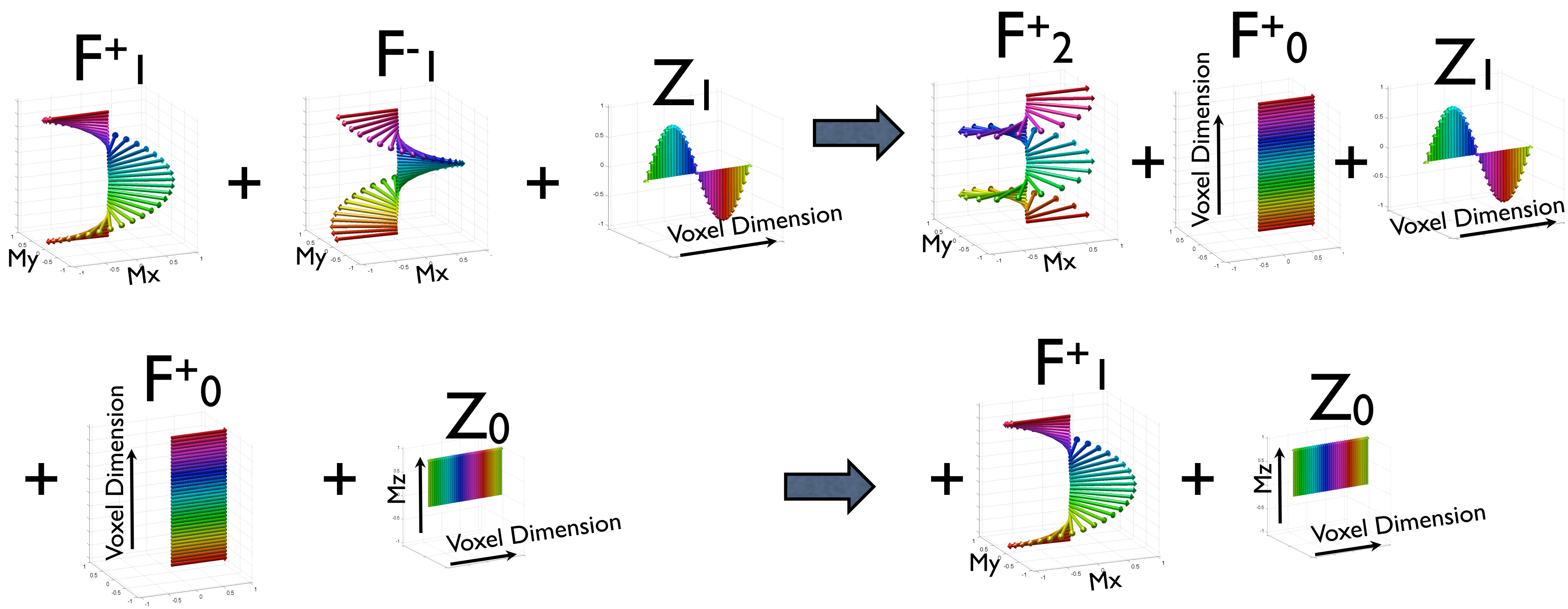


- The Z_0 magnetization is again split to F^+_0 and Z_0
- The F^+_1 magnetization is split *three* ways, to F^+_1, F^-_1 (reverse twisted) and Z_1



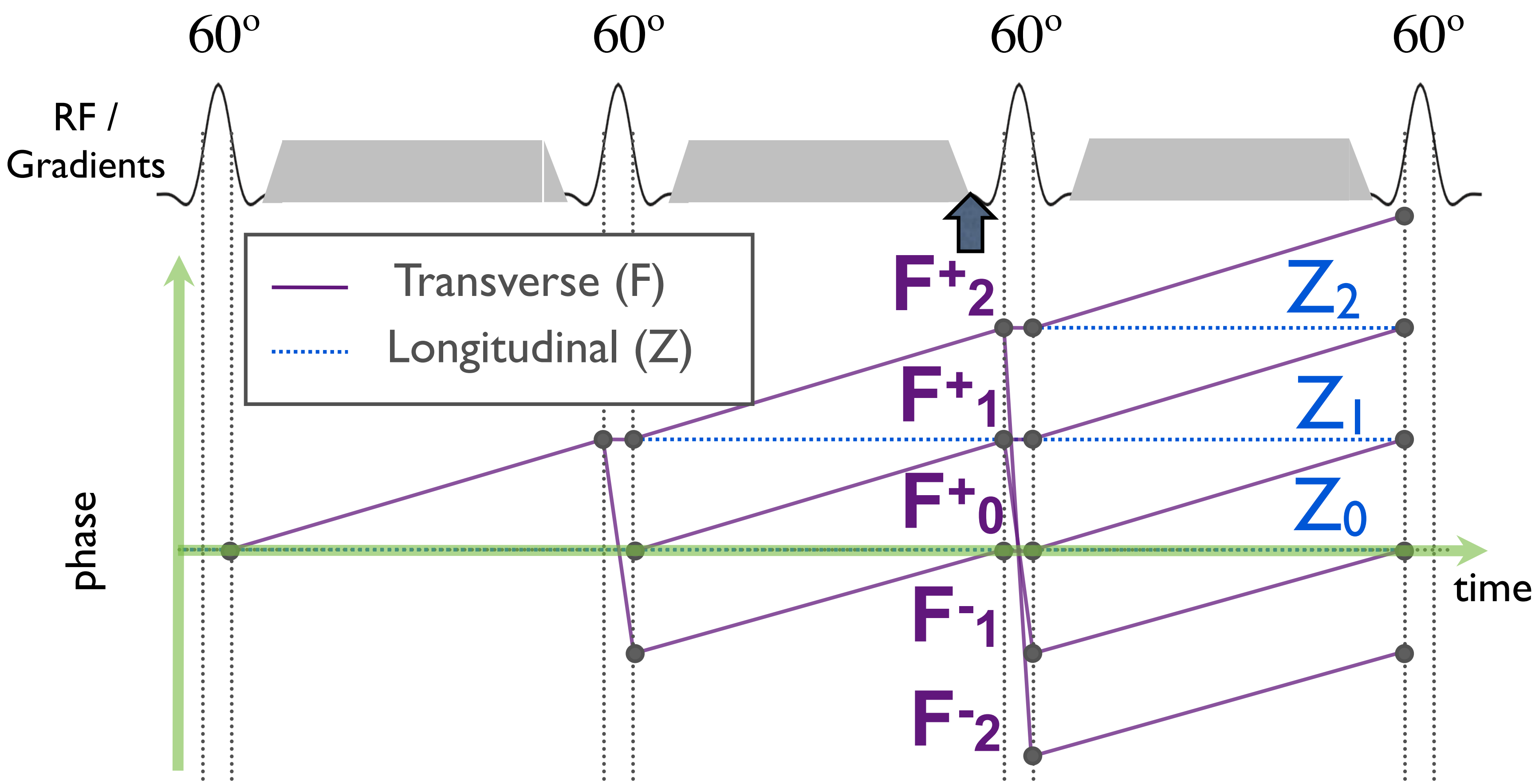
Another Gradient Cycle

- The F^-_1 state is refocused to F^-_0 or F^+_0
- The F^+_0 and F^+_1 states become F^+_1 and F^+_2
- The Z states are *all* unaffected
- The process continues…!



Example 3: Coherence Pathways

- The stimulated echo sequence coherence diagram is shown below
- Compare with F and Z states on prior slide (location of arrow)

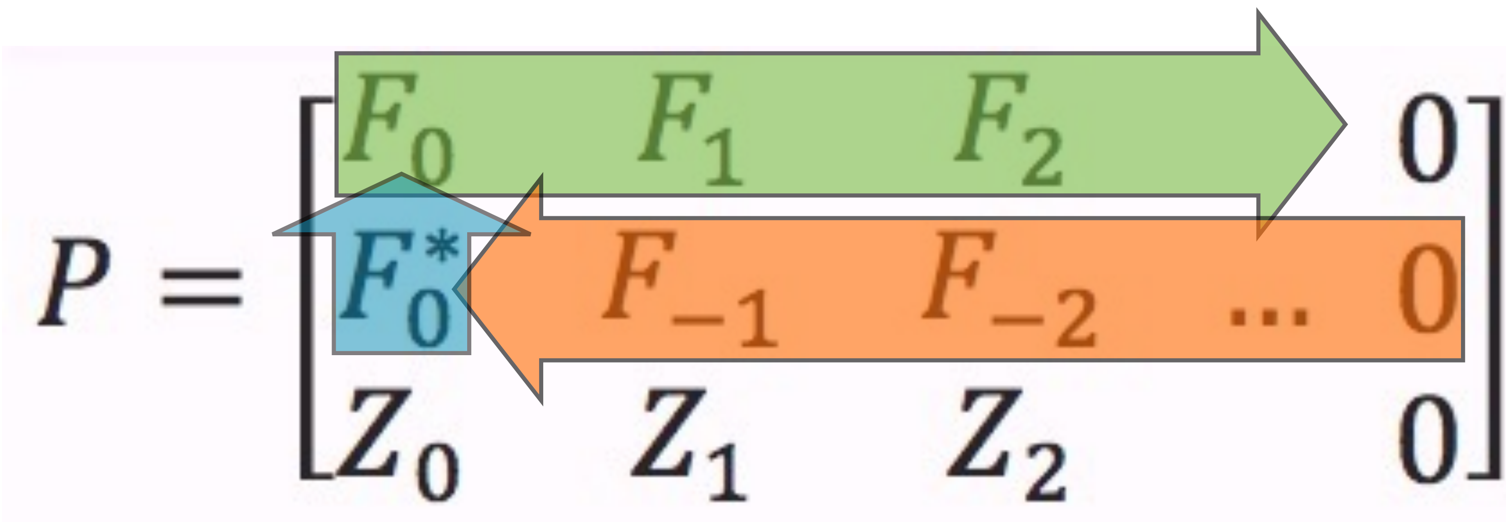


Summary of Sequence Examples

- 90^o and 180^o RF pulses "swap" states
- Generally RF pulses "mix" states of order n
- Gradient pulses transition F^+_n to F^+_{n+p} and F^-_n to F^-_{n-p}
- Coherence diagrams show progression through F and/or Z states to echo formation
- Signal calculation examples actually *quantify* the population of each state

Matlab Formulations

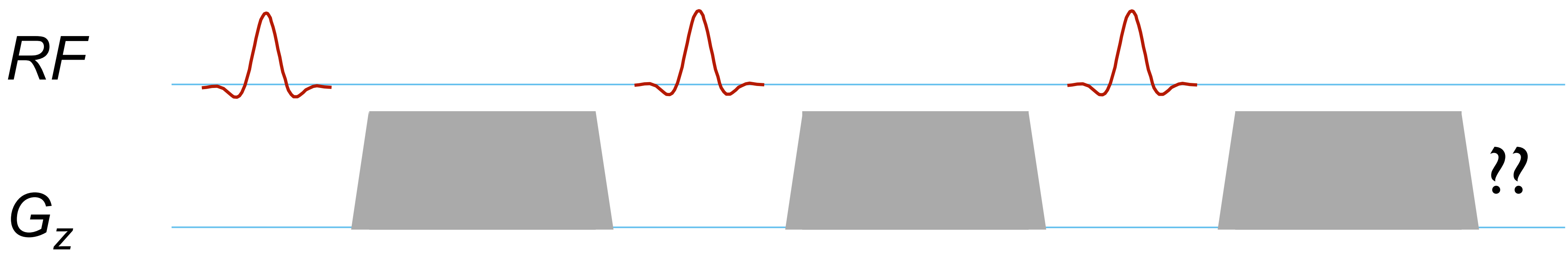
- Single matrix called “P” or “FZ”   (for example)
- Rows are F^{+}_{n}, F^{-}_{n} and Z_n coefficients, Column each n
- RF, Relaxation are just matrix multiplications
- Gradients are shifts



Matlab Formulations

- EPG simulations can be easily built-up using modular functions: *(bmr.stanford.edu/epg)*
  - Transition functions:
    - epg_RF.m Applies RF to Q matrix
    - epg_grad.m Applies gradient to Q matrix
    - epg_grelax.m Gradient, relaxation and diffusion
  - Transformation to/from $(M_{x,} M_{y,} M_z)$:
    - epg_spins2FZ.m Convert M vectors to F,Z state matrix Q
    - epg_FZ2spins.m Convert F,Z state matrix Q to M vectors

Stimulated-Echo Example



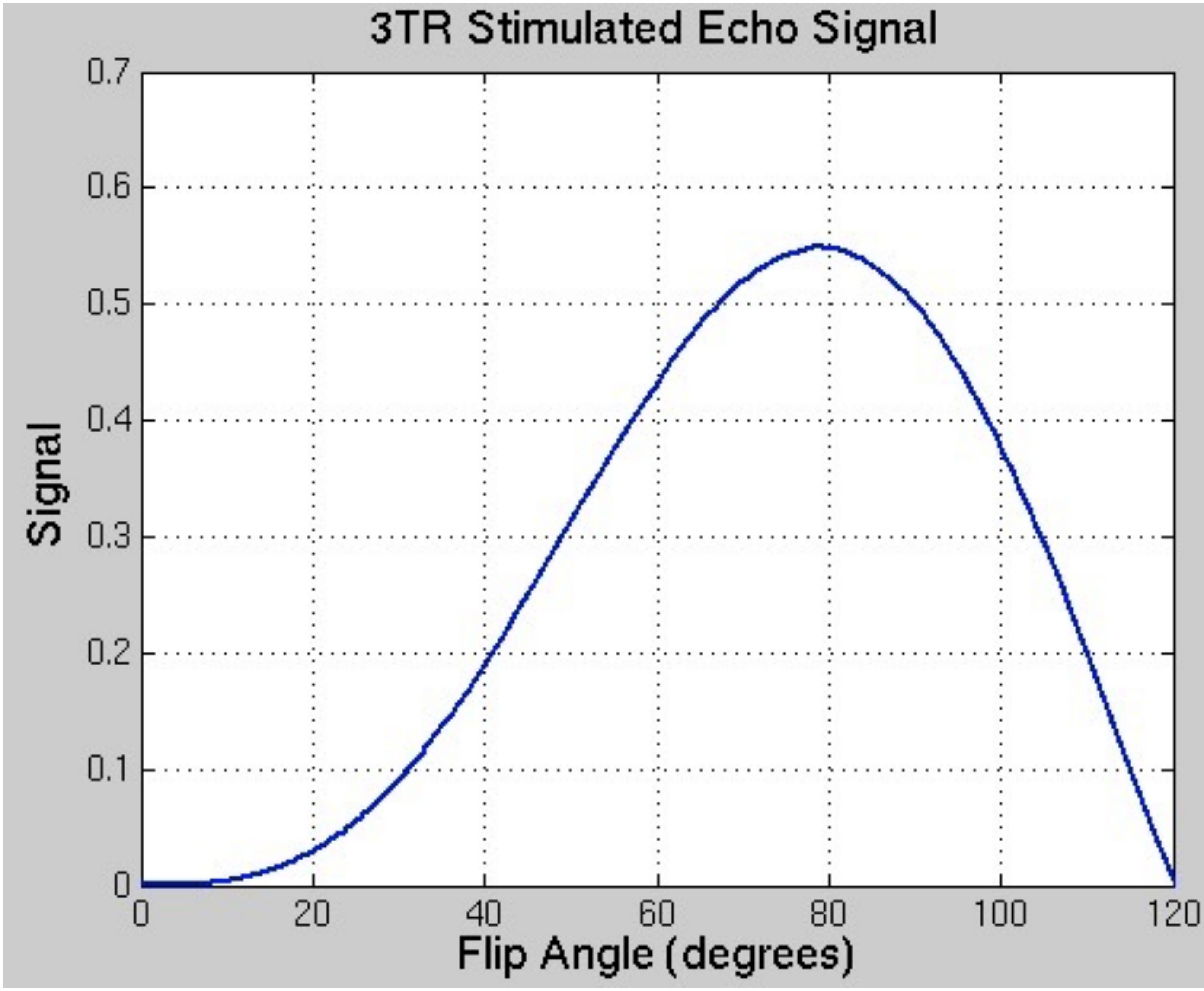
- Simulate 3 Steps: RF, gradient and relaxation
- Sample Matlab code:

(See epg_stim.m)

```
function [S,Q] = epg_stim(flips)
Q = [0 0 1]';                                   % Z0=1 (Equilibrium)
for k=1:length(flips)
  Q = epg_rf(Q,flips(k)*pi/180,pi/2);           % RF pulse
  Q = epg_grelax(Q,1,.2,0,1,0,1);               % Gradient/Relax
end;
S = Q(1,1);                                     % Signal from F0
```

Stimulated Echo Example (Cont)

Calculate signal vs flip angle: fplot(epg_stim([x x x],[0,120])



Diffusion

- Diffusion weighting can easily be applied
  - *Details in Weigel et al. J Magn Reson 2010; 205:276-285*
- Attenuation greater with n for both F_n and Z_n
- Requires physical "2π" gradient twist k (m^{-1})
- If gradient is played, $\Delta k = k$, otherwise $\Delta k = 0$

$$b_n(k, \Delta k) = \left[\left(nk + {}^{\Delta k}/_{2}\right)^2 + \left({}^{\Delta k^2}/_{12}\right)\right] T$$

$$F_n{}' = F_n e^{-b_n(k,\Delta k)D}$$

$$Z_n{}' = Z_n e^{-b_n(k,0)D}$$

Summary

- F_n, Z_n basis represents <u>many</u> spins in a voxel
- MR operations on F_n, Z_n states are simple
- Coherence diagrams show which states are non-zero
- Signal at any time is $F_{0.}$ Other states are dephased

- Matrix formulation allows easy Matlab simulations:
  - Construct sequence of RF, gradient, relaxation, diffusion
  - Transient and steady state signals by looping
  - Can simulate multiple gradient directions
  - Can also simulate multiple diffusion directions (Weigel 2010)

System/Imaging Imperfections

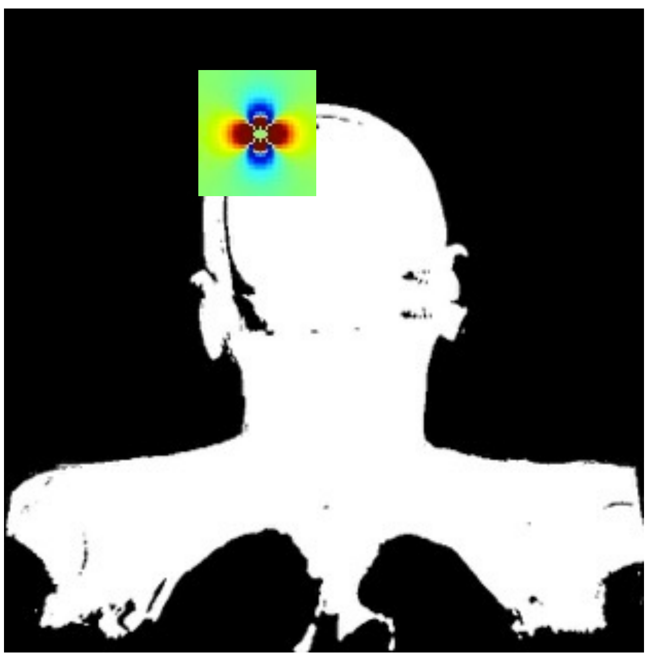
- B_0 variations: Shim, Susceptibility
- B_1 variations: Transmit, Receive
- Gradient Imperfections:
  - Non-linearities
  - Delays and Eddy currents
  - Concomitant terms

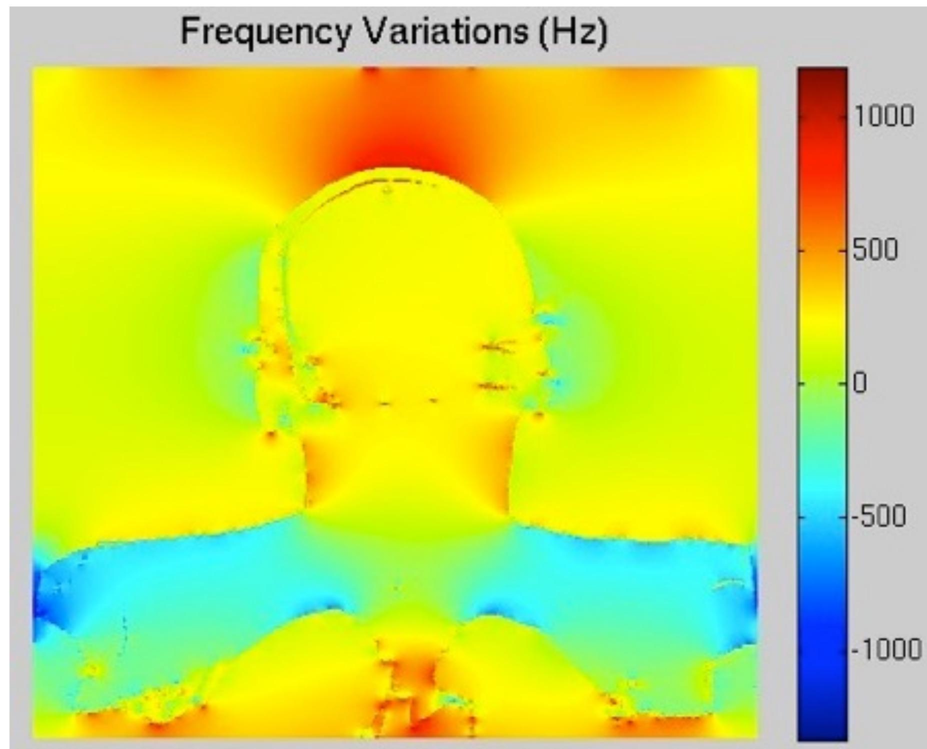
B_0 Variations - “Off-Resonance”

- Imperfections in the B_0 field (~1ppm)
  - Coil design
  - Imperfections in coil placement, currents
- Susceptibility variations (~1-5ppm, normal body)
- Chemical shift (-3.5ppm for fat, most common)

Susceptibility

- Tendency to be magnetized
- H field is continuous H = χB
- B_0 is convolution of χ with a dipole field





Susceptibility

- Air-tissue interfaces and complex shapes
- Field can be improved by shimming
- Measured B_0 may still vary considerably (500Hz here)

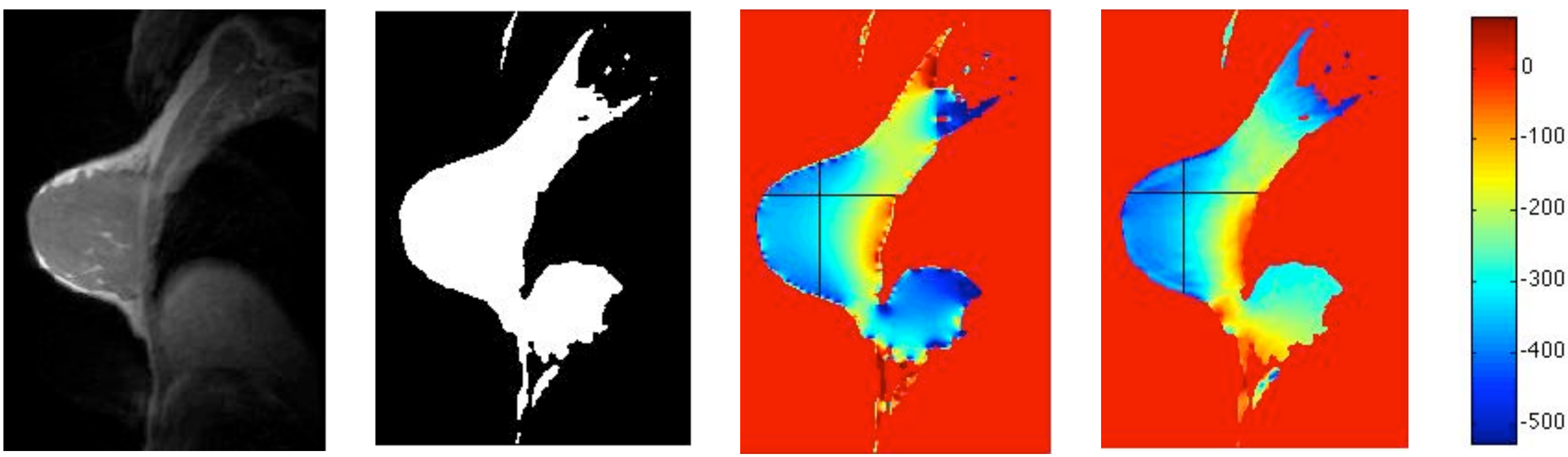
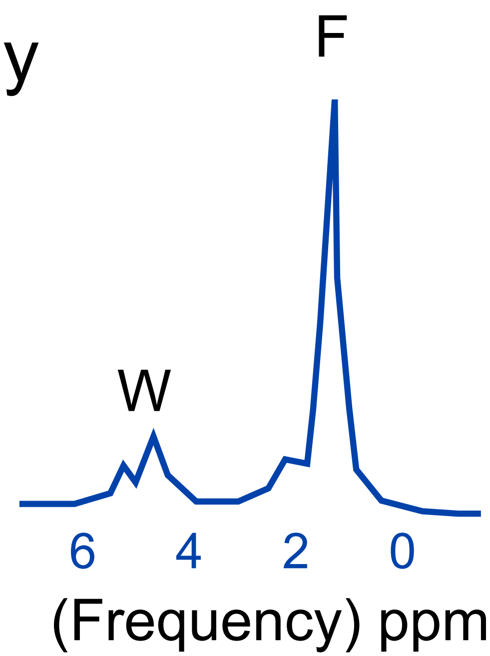


Image "Mask" B_0 Shimmed B_0

Chemical Shift

- Refers to the ***frequency*** shift due to electron shielding
- Reduces the resonance frequency
- Most common: Fat-Water:
    - -3.5ppm (220Hz at 1.5T / 440Hz at 3T)
    - Actually multiple peaks in fat (more complicated)
- Many Others ~ Spectroscopy



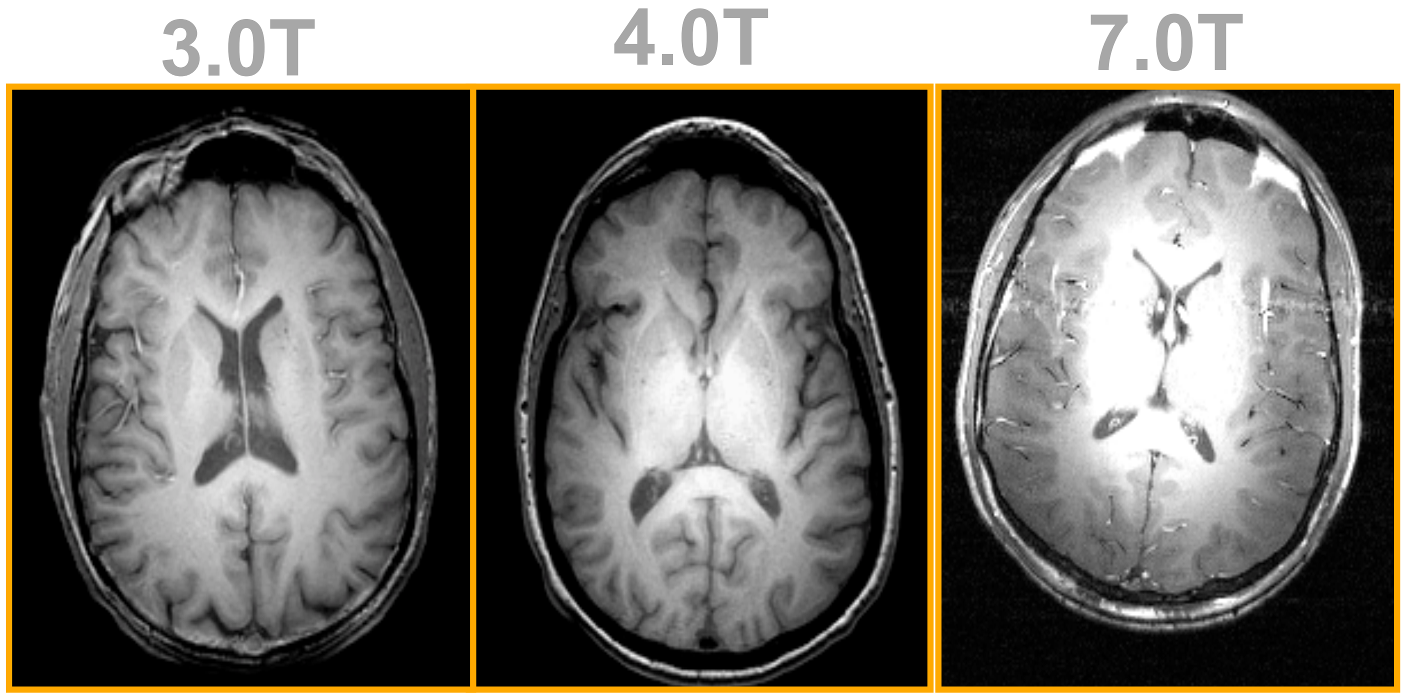
Shimming

- Passive Shimming:
  - Add small materials to correct field
- Active Shims:
  - Coils with adjustable currents to correct field
- Linear:
  - Small current usually added to imaging gradient
- High-Order:
- $z^2-(x^2+y^2)/2$, $3zx$, $3zy$, $3(x^2-y^2)$, $6xy$

Minimizing Effects of B_0 Variations

B_1 Transmit (B_1^+) Variations

- Coil Inhomogeneities (minimal with Birdcage)
- Dielectric effects (worse at high fields - shorter wavelengths, standing waves)
- RF amplifier non-linearity (small, harmonics)

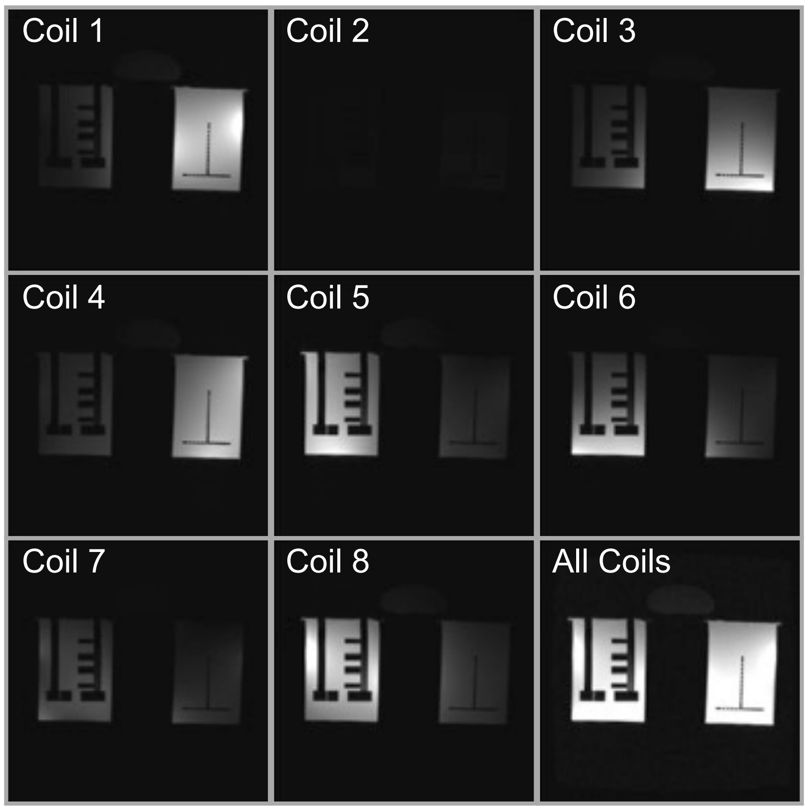


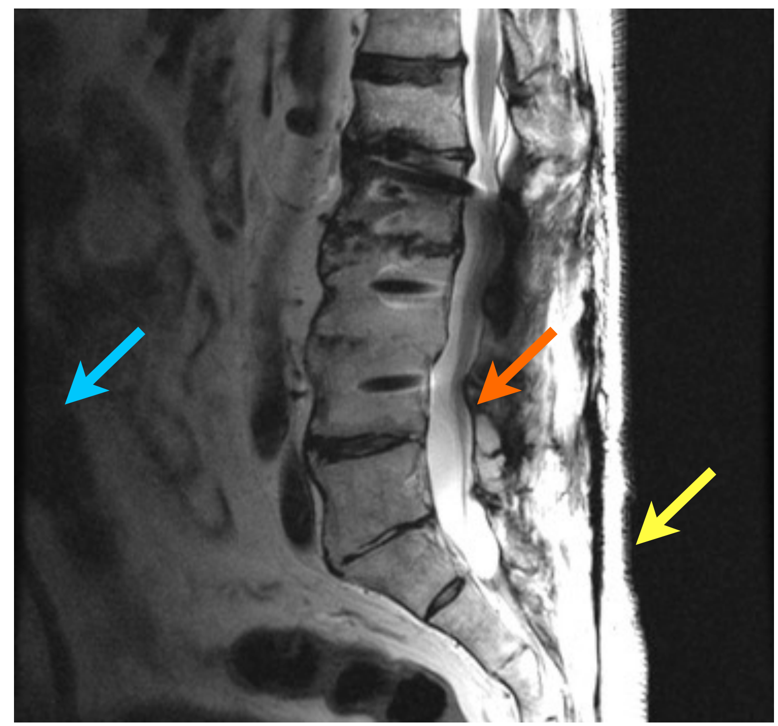
3T Images courtesy G. Glover, Stanford Univ
4T Images courtesy C. Charles, Duke Univ. 9/2000
7T Images courtesy T. Vaughan, M. Garwood, Univ. Minn. 6/2000

Minimizing Effects of B_1^+ Variations

B_1^- Receive

- Primarily coil sensitivity variations
- Somewhat fixable in reconstruction
  - Measure sensitivities (SENSE)
  - “Surface-coil intensity correction”

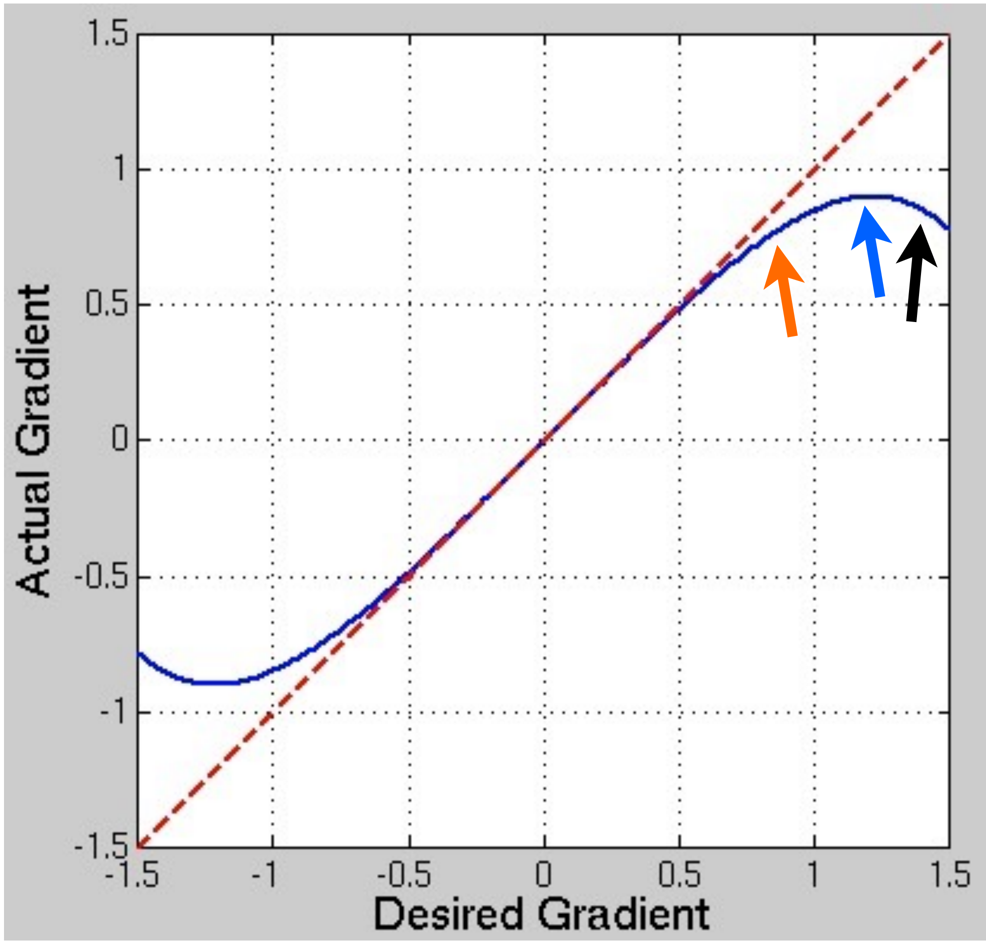




Gradient nonlinearity

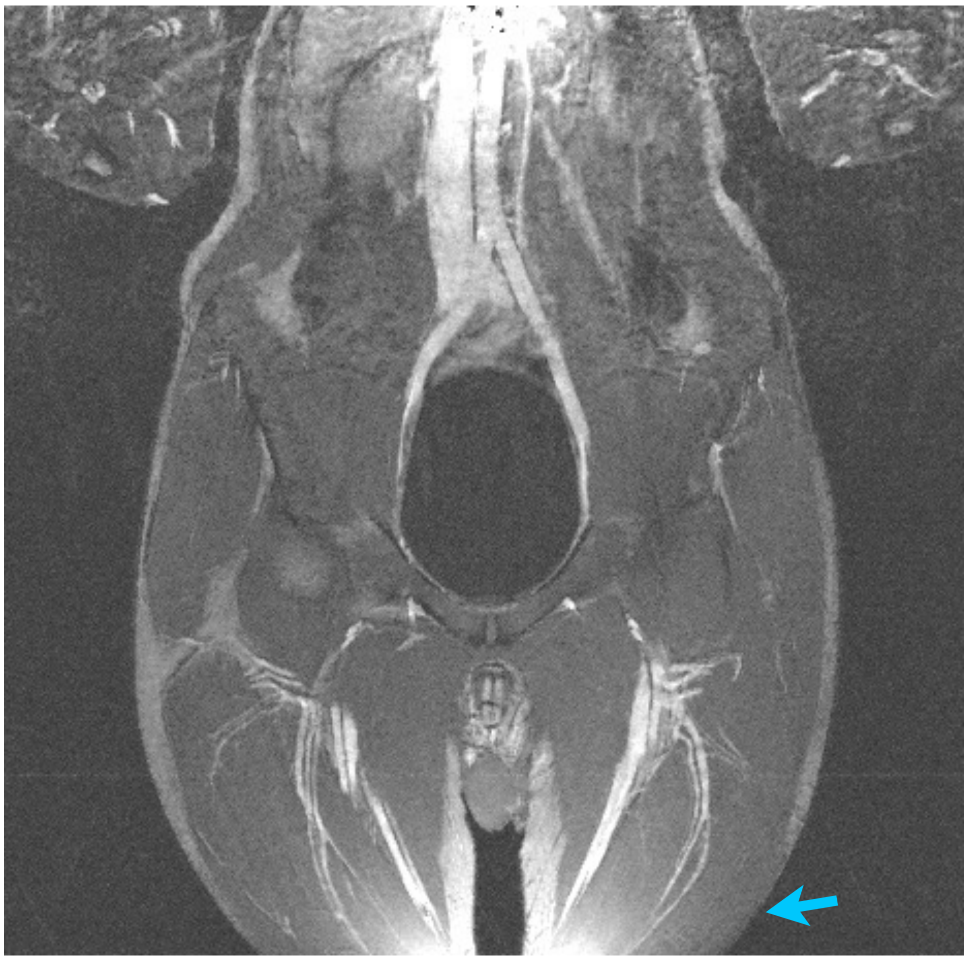
- Ideally, linear mapping of position to B_z
- Must end somewhere(!)
- dB/dt limited too

- Distortion of image
- Loss of resolution
- Aliasing



Gradient Non-linearity Correction

- Apply “grad-warp” warping to correct distortion
- Note image boundaries are curved



Gradient Non-linearity and B_0 Variation

$$B(z) = \Delta B_{G_z}(r) + \Delta B_0(r)$$

- If $B(r_1) = B(r_2)$, positions are indistinguishable
- Aliasing or "Annefact" (if RF coil sensitive in region)
- Applies to slice selection too:
  - Can correct slice location
  - Warps slice (harder to fix)

Marc Alley

Example: B_0 and Gradient Nonlinearity

RF / Gradient Delays

- Amplifier delays, circuit delays
- Can vary between scanners but also calibrated
- Cartesian imaging insensitive, but other methods much more affected
- Gradient delays can be axis-dependent
- RF transmit and receive delays can vary

Delay Questions

How do these delays affect standard imaging:

- Slice select gradient delay (general)?
  -
- Readout gradient delay of 2 samples?
  -
- Phase-encode gradient delay of 2 readout samples?
  -

Eddy Currents

- Generated by gradient switching, subject independent
- Linear system models (dG/dt):

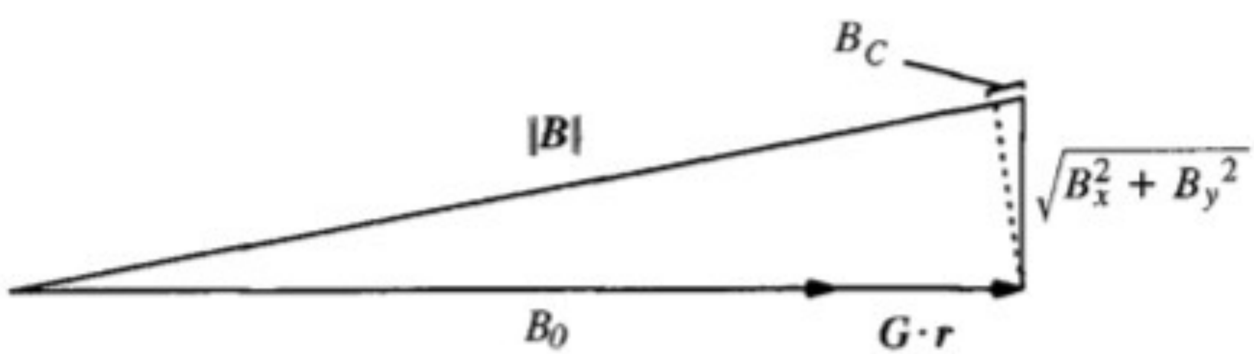
$$\Delta B_0(\overrightarrow{r}, t) = \sum_{i=x,y,z} h_i(\overrightarrow{r}, t) * G_i(t)$$

  - Spatially-independent terms (global phase/rotation)
  - Linear terms ~ gradient errors (self, cross axis)
  - Higher-order terms - hardest to correct

Concomitant Gradients

$$B_c(x,y,z) = \frac{1}{2B_0}\left(G_x^2 z^2 + G_y^2 z^2 + G_z^2 \frac{x^2+y^2}{4} - G_x G_z xz - G_y G_z yz\right)$$

- "Maxwell" terms - impossible to create B_z variation without some B_x and B_y variation
- Bigger problem at lower field strengths
- Some correction schemes



(From Bernstein, MRM 39:300 (1998)

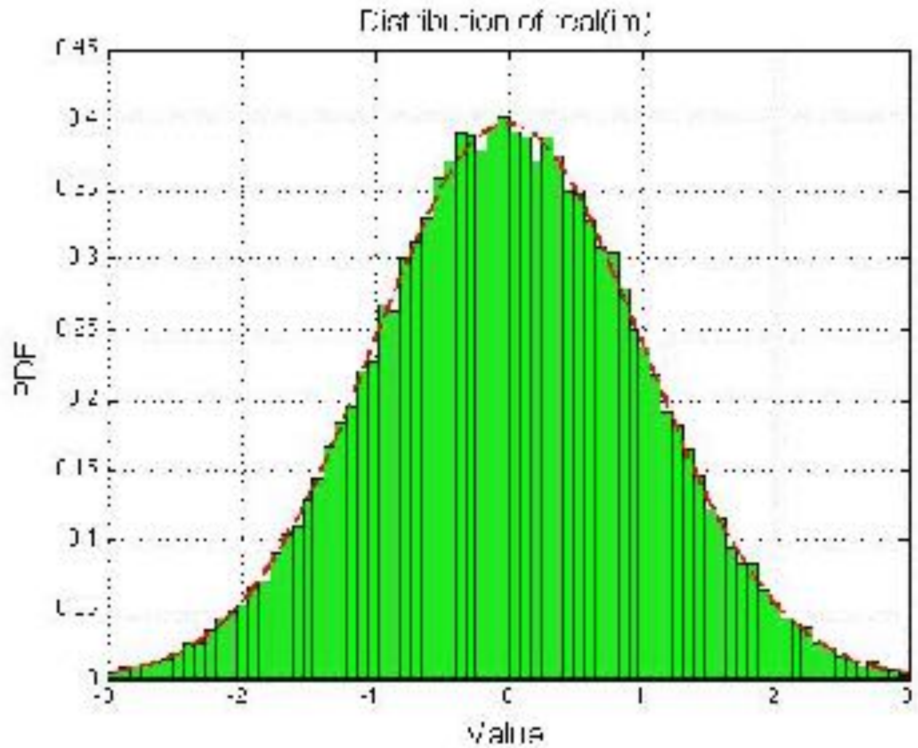
Summary: System Imperfections

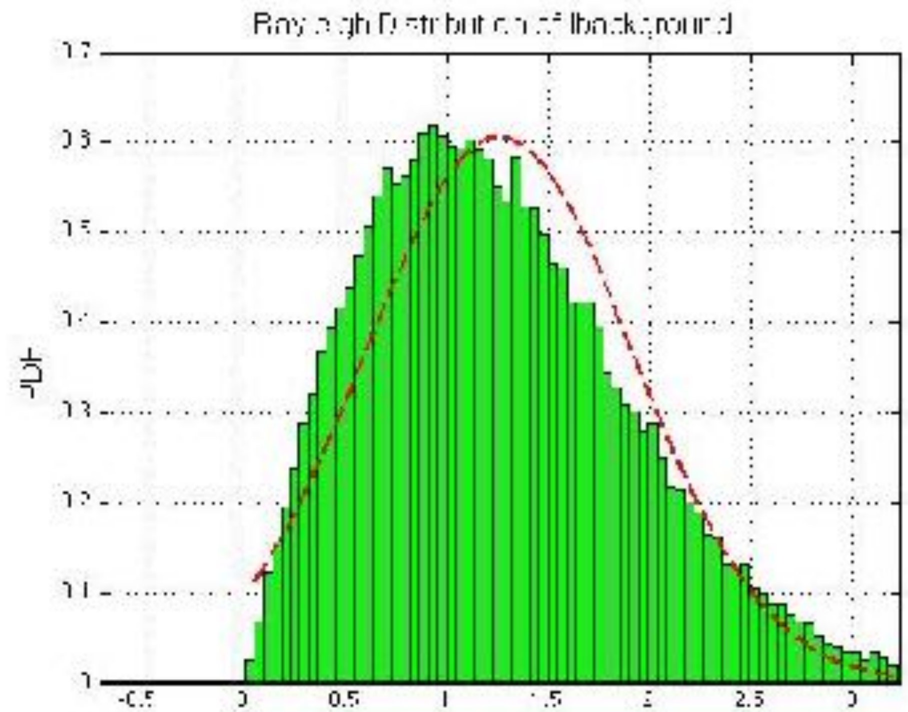
- B_0 variations: Shim, Susceptibility
- B_1 variations: Transmit, Receive
- Gradient Imperfections:
  - Non-linearities
  - Delays and Eddy currents
  - Concomitant terms

Signal-to-Noise in MRI

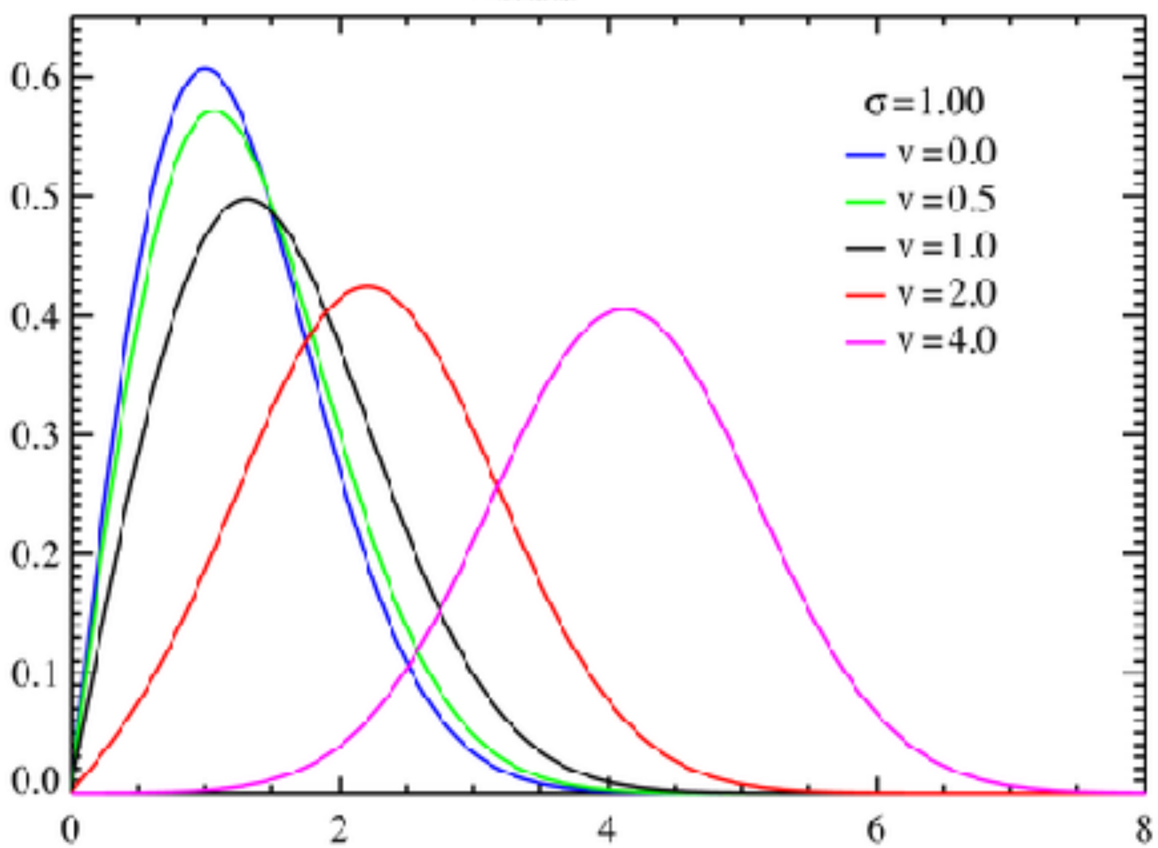
- What is SNR?
- Noise statistics & Measurement
- Multichannel noise

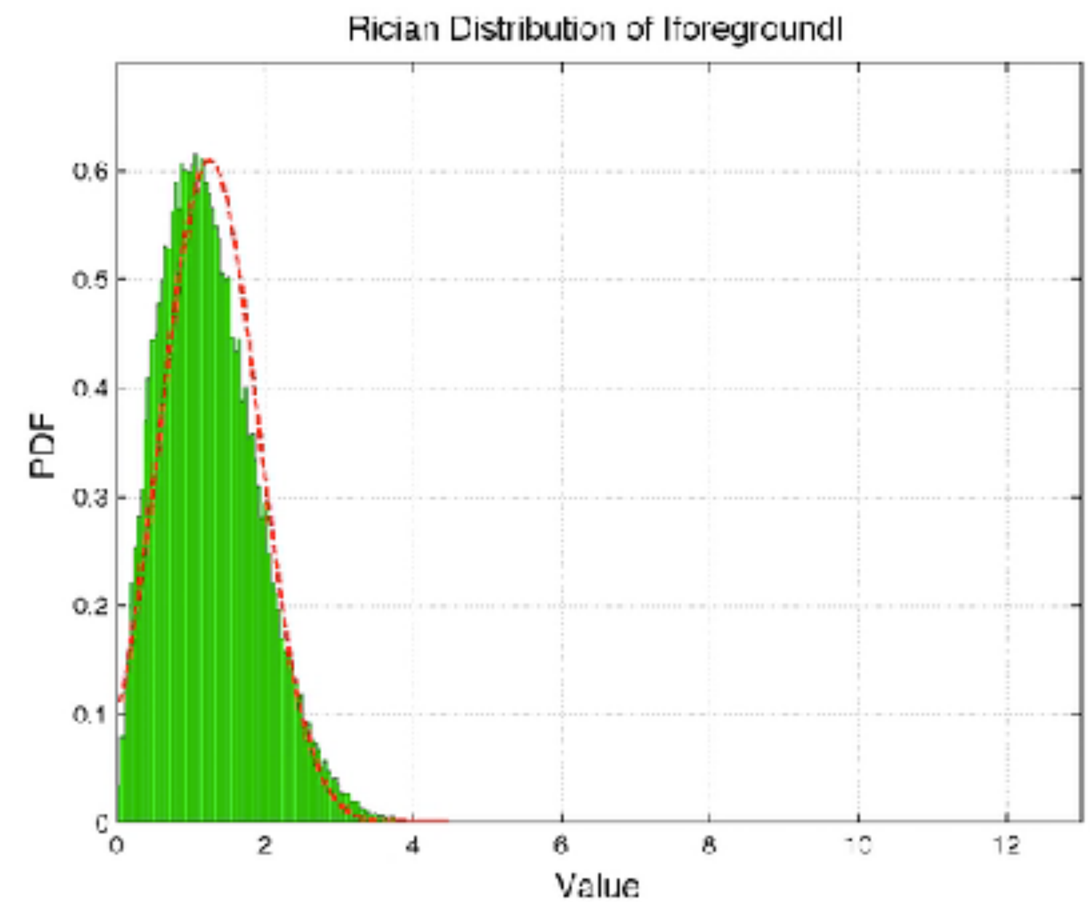
Basic Noise Statistics





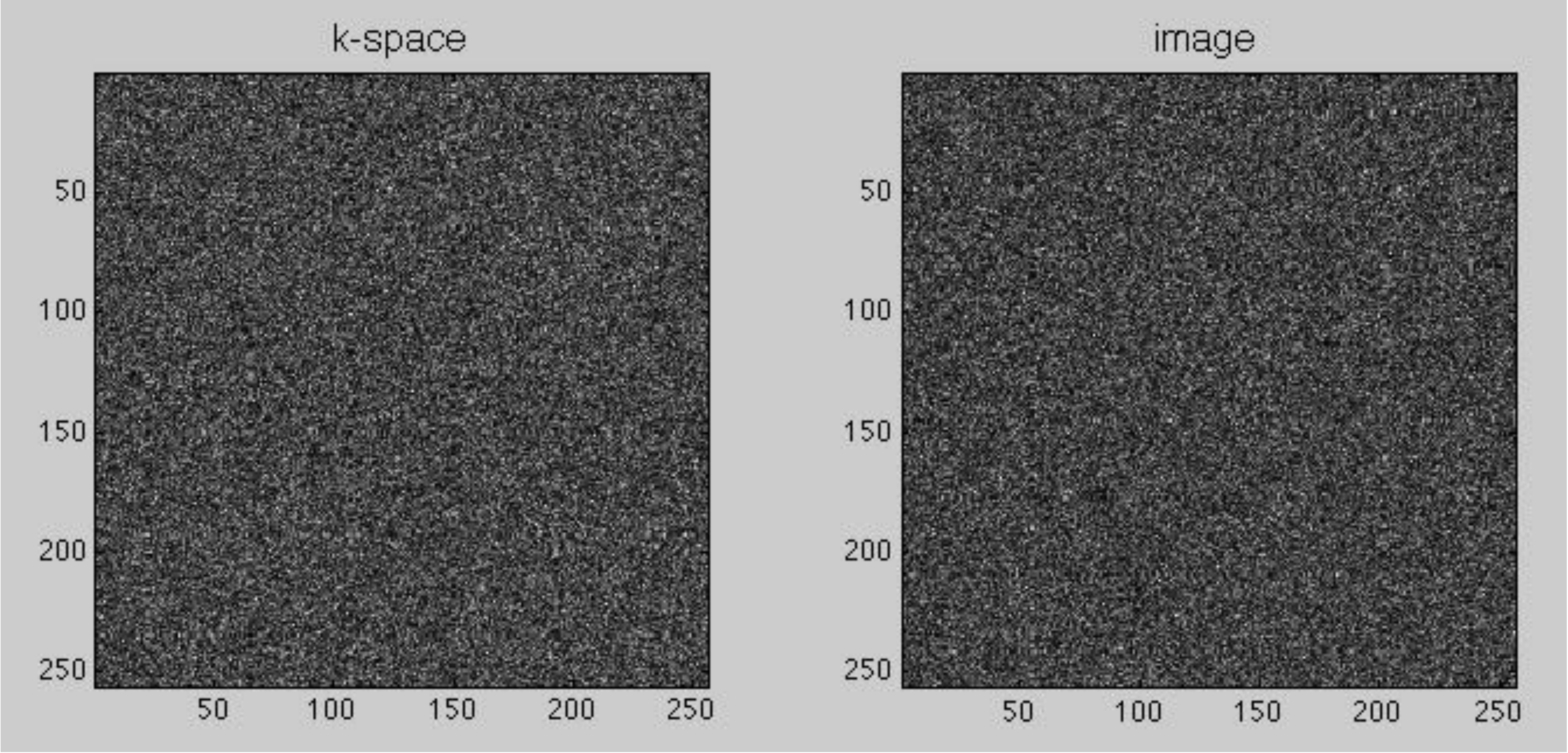
v=0
(mean)





(See conceptB4_2.m)

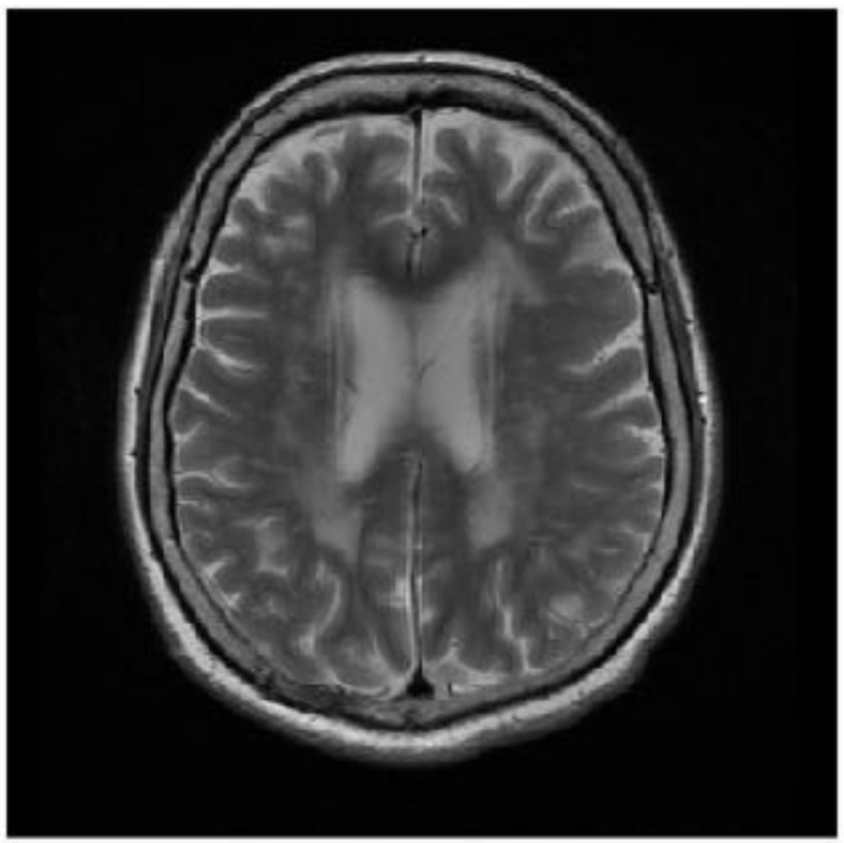
FFT of Gaussian Noise

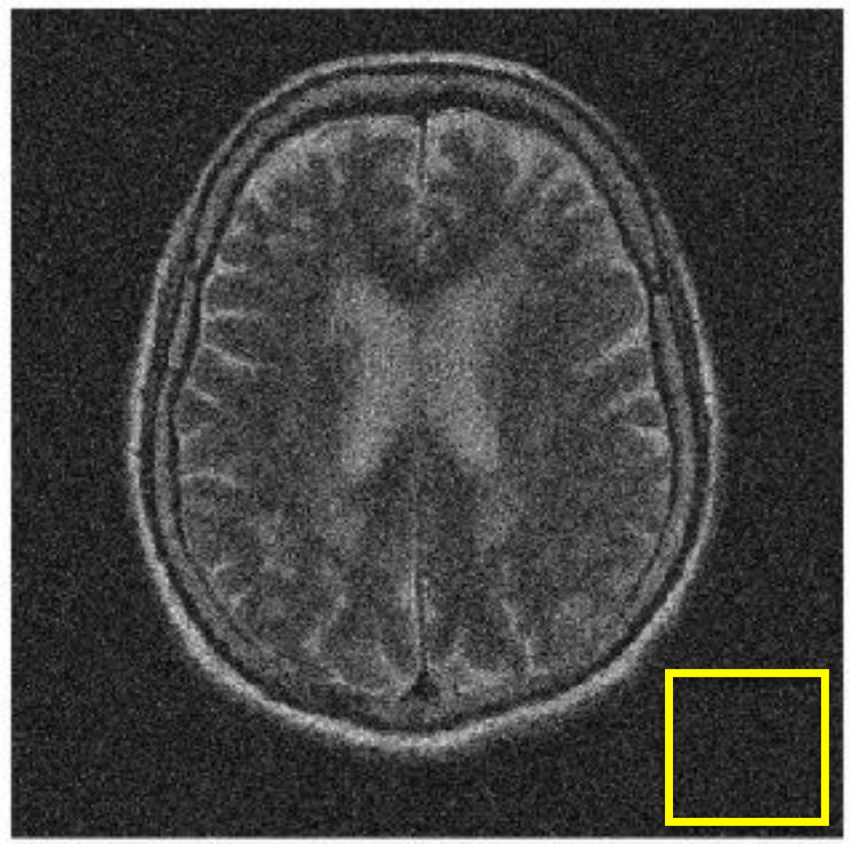


- Note sqrt(N) scaling preserves noise energy

Basic SNR Measurement (1 coil)

- Measure mean in signal area ROI
- Measure std-deviation in magnitude background ROI
- Correct for Rayleigh distribution in background





$\text{mean}_{\text{Rayleigh}} = 1.26$

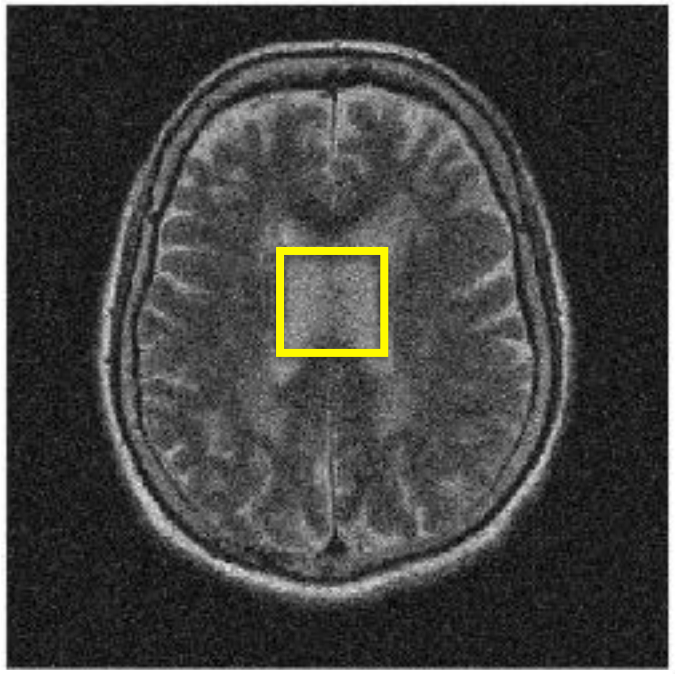
$\sigma_{\text{Rayleigh}} = 0.65$

$\sigma_{\text{gaussian}} = \text{mean}_{\text{Rayleigh}} / \text{sqrt}(\pi/2) = 1.008$

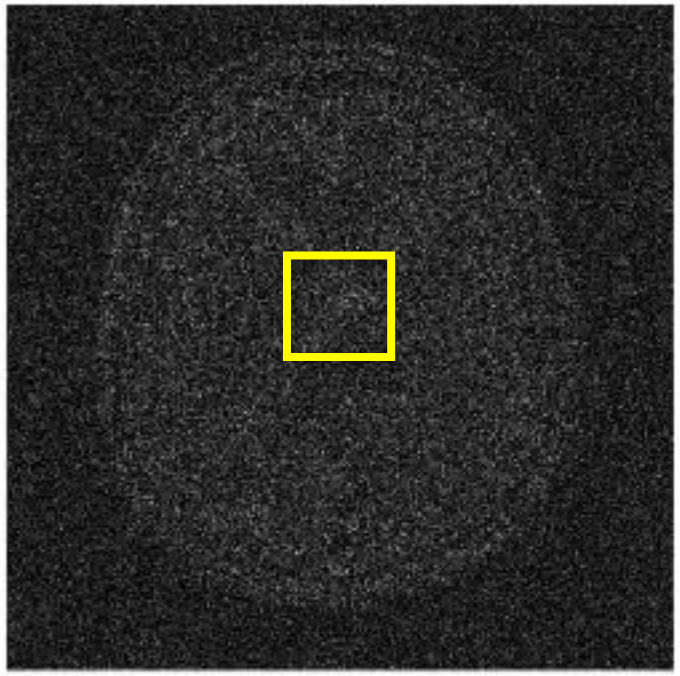
$\sigma_{\text{gaussian}} = \sigma_{\text{Rayleigh}} / \text{sqrt}(2-\pi/2) = 0.997$

Difference Method SNR

- In theory, N measurements should give you a population, and at each pixel you get a (roughly gaussian) distribution
- With 2 measurements you can still estimate mean and standard deviation (Reeder et al)



Sum



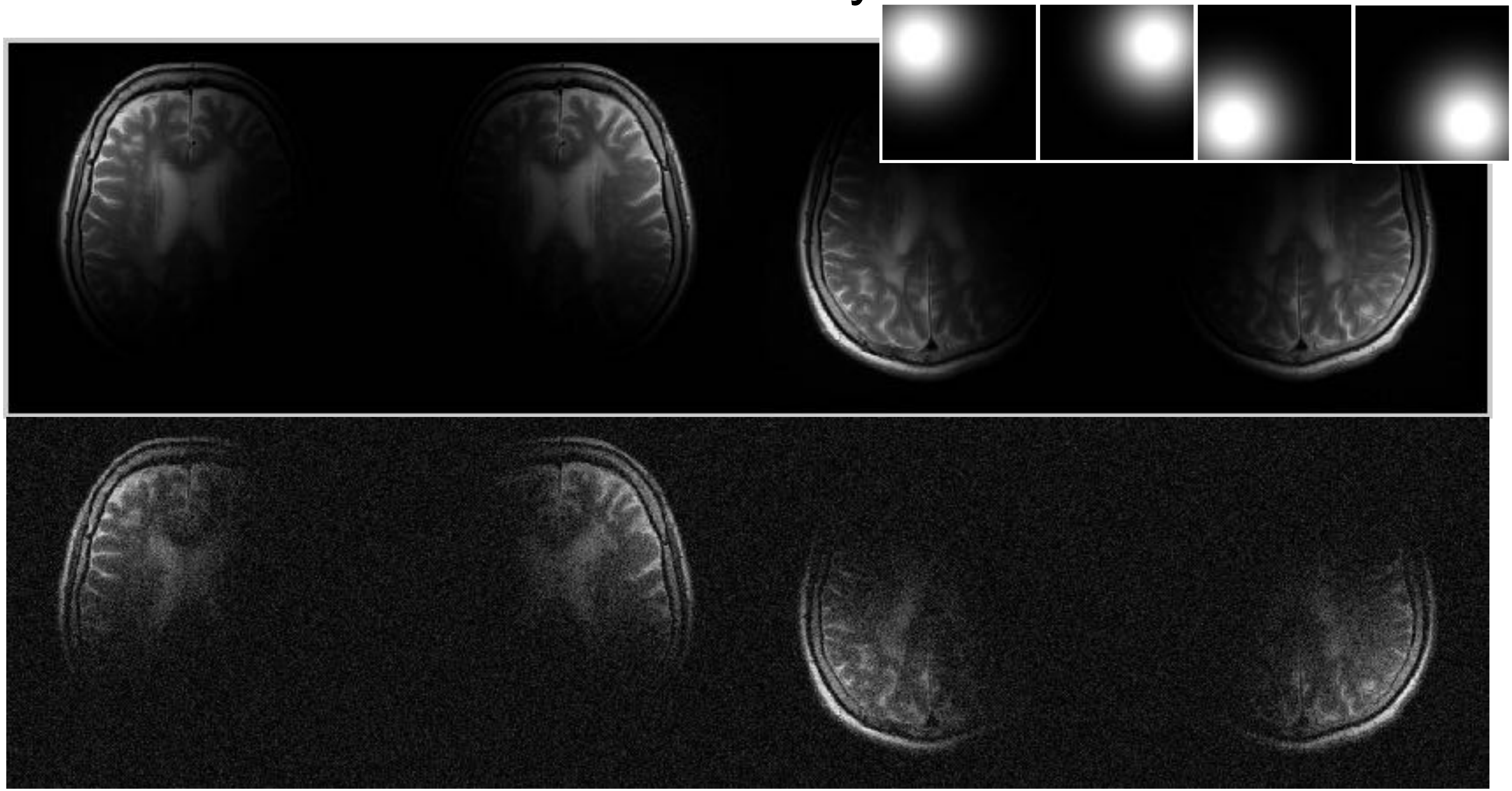
Difference of Magnitude Images

$\sigma_{Mag\text{-}Diff} = 1.394$

$\sigma_{gaussian} = \sigma_{Mag\text{-}Diff} / \mathrm{sqrt}(2)$

Multiple Coils

- Multiple images, ideally with uncorrelated noise
- Combine with RMS or sensitivity-based methods



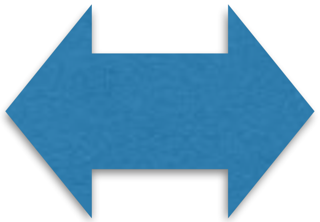
Multi-Coil Combinations

- S_i = signal from coil i, C_i = sensitivity of coil i
- Root-mean-square (RMS)

$$S_{RMS} = \sqrt{\sum_{i-1}^{N_{coils}} S_i^2}$$

- SENSE (Each pixel, for any reduction factor R)

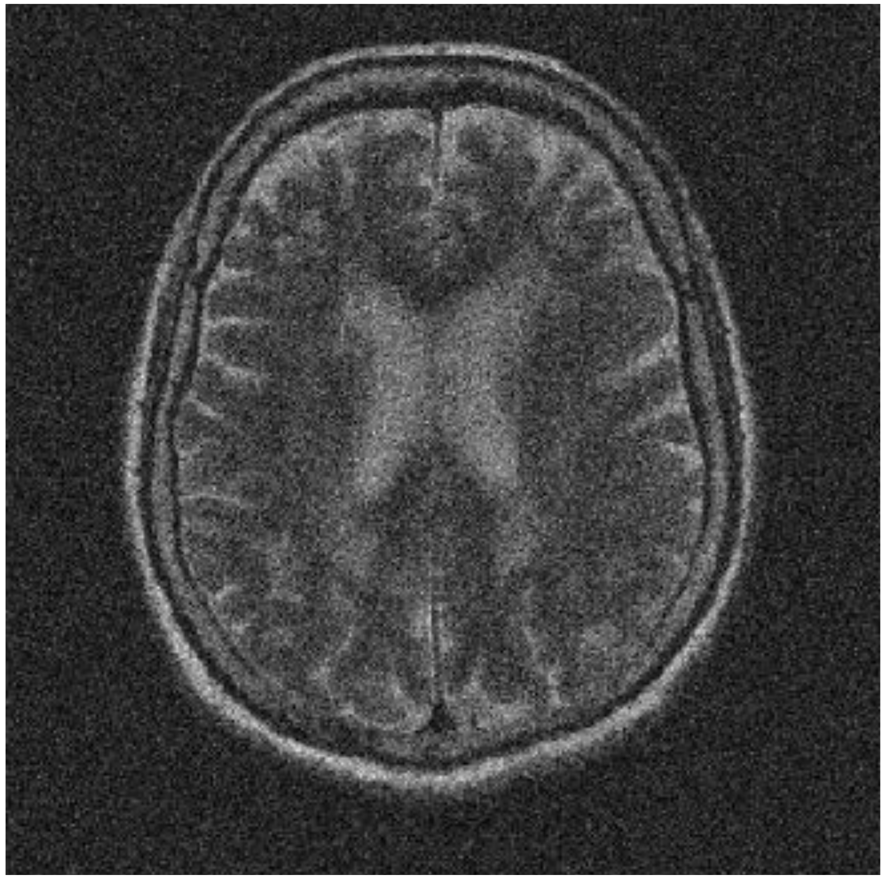
$$\mathrm{S} = m\mathrm{C}$$

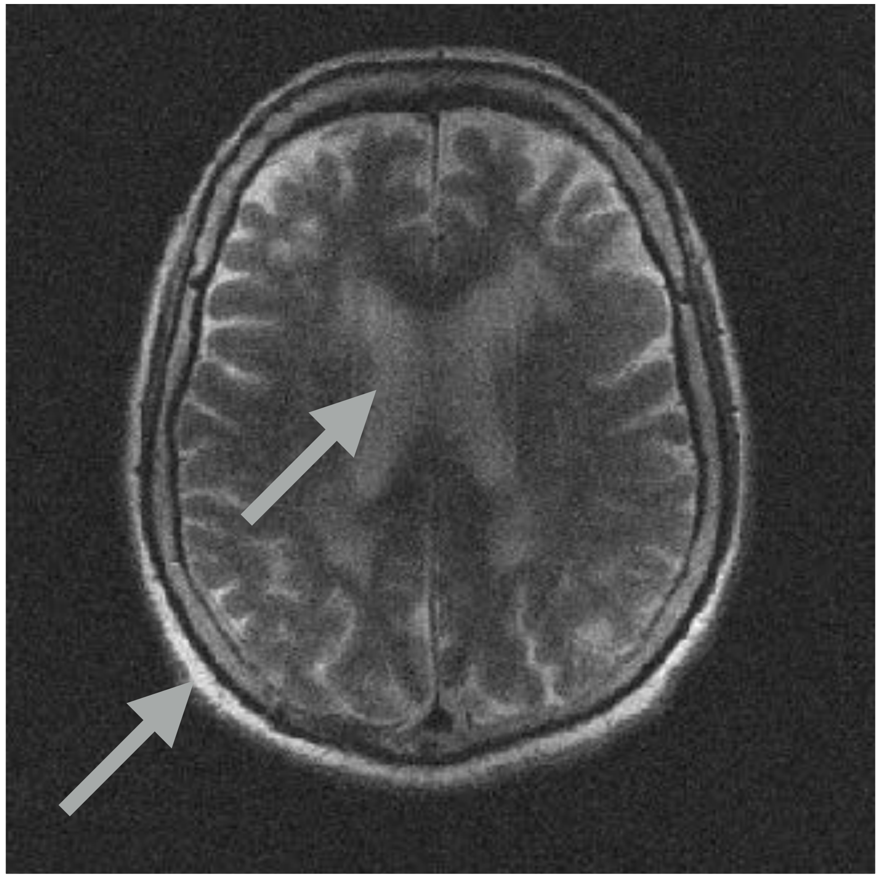


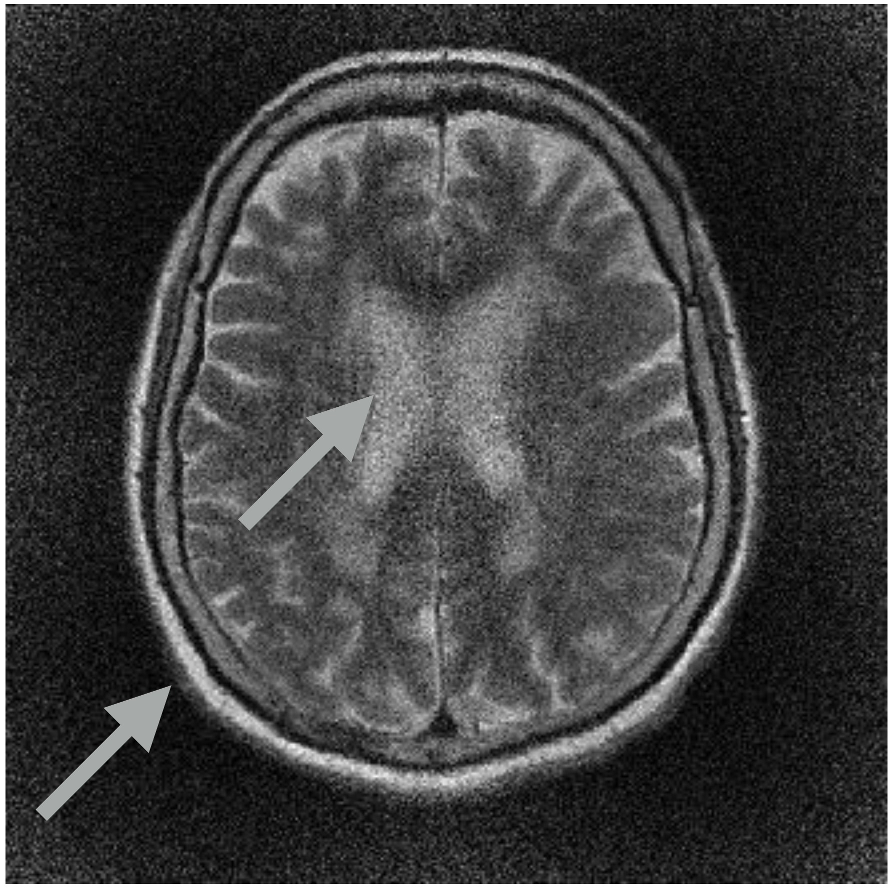
$$S_{SENSE} = \left(C^H \Psi^{-1} C\right)^{-1} C^H \Psi^{-1} S$$

$$S_{SENSE} = \sum_{i=1}^{N_{coils}} \alpha_i S_i$$

Multiple Coils



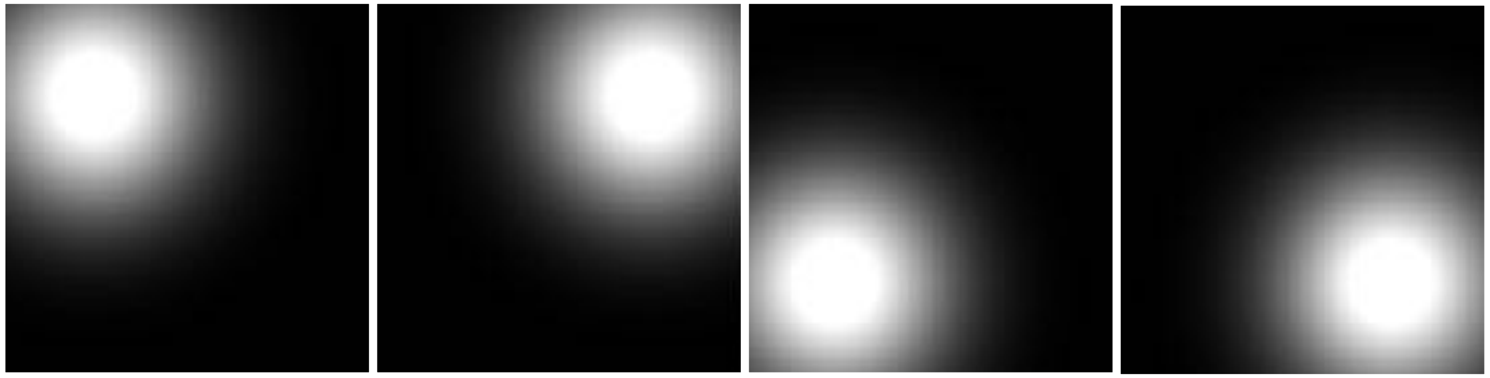




Single-Channel (uniform noise)

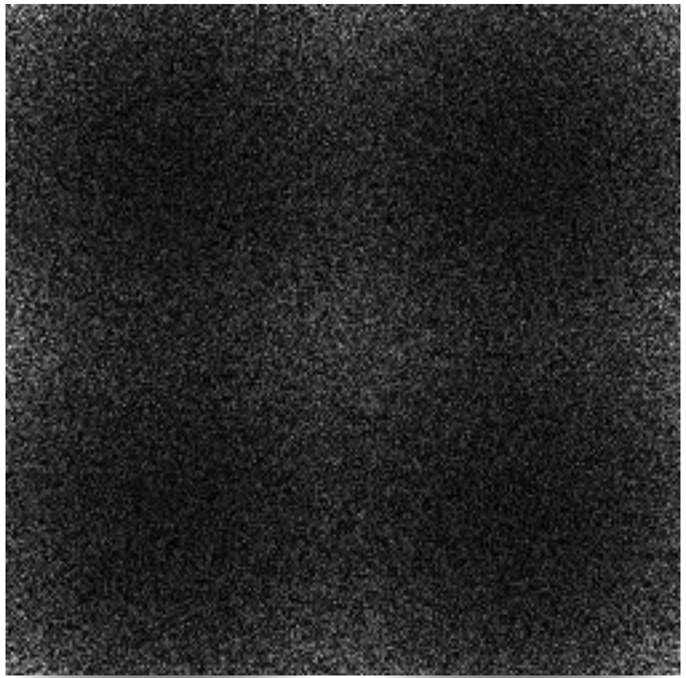
RMS (Signal shading)

SENSE (Noise Varies)



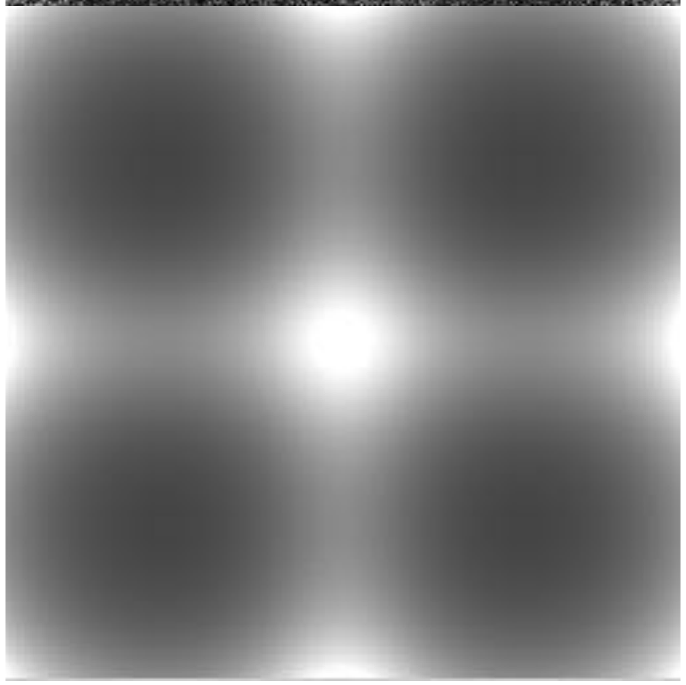
Multiple Coils - Noise Statistics

- RMS combination: C_i=coil image, S_i = sensitivity
  - Image = sqrt($C_1^2 + C_2^2 \ldots$)
    - = sqrt[$(mS_1+n_1)^2 + (mS_2+n_2)^2 + \ldots$]
  - Noise will not be gaussian



- R=1 SENSE combination
  - (linear)
  - Noise remains gaussian
  - Can calculate amplification

$$S_{SENSE} = \sum_{i=1}^{N_{coils}} \alpha_i S_i$$



SENSE Reconstruction

- $S_i(k)$ = signal from coil i,
- R = reduction factor
- C_{ij} = sensitivity of coil i, to aliased pixels $j=1\ldots R$
- m = aliased image for coils (N_c x 1)
- $m = \mathcal{F}\{ S(k) + n \}$ (includes all channels)
- Ψ is noise covariance matrix

$$\hat{M} = \left(C^H \Psi^{-1} C\right)^{-1} C^H \Psi^{-1} \mathrm{m}$$

$$SNR = \frac{SNR_0}{g\sqrt{(R)}}$$

$$g = \sqrt{\left[\left(C^H \Psi^{-1} C\right)^{-1}\right]_{x,x} \left[C^H \Psi^{-1} C\right]_{x,x}}$$

SNR Measurement

- (correlated) Noise is additive in k-space
- (correlated) Noise is added to channel images
- Linear combination - noise is a function of combination coefficients and covariance
- Good reconstruction should give something:
  - coefficients or noise maps

$$\hat{M} = \left(C^H \Psi^{-1} C\right)^{-1} C^H \Psi^{-1} \mathrm{m}$$

$$g = \sqrt{\left[\left(C^H \Psi^{-1} C\right)^{-1}\right]_{x,x} \left[C^H \Psi^{-1} C\right]_{x,x}}$$

$$SNR = \frac{SNR_0}{g\sqrt(R)}$$

Pseudo-Multiple Replicas (Numerical)

Robson PM, et al. MRM 2008; 60(4):895-907

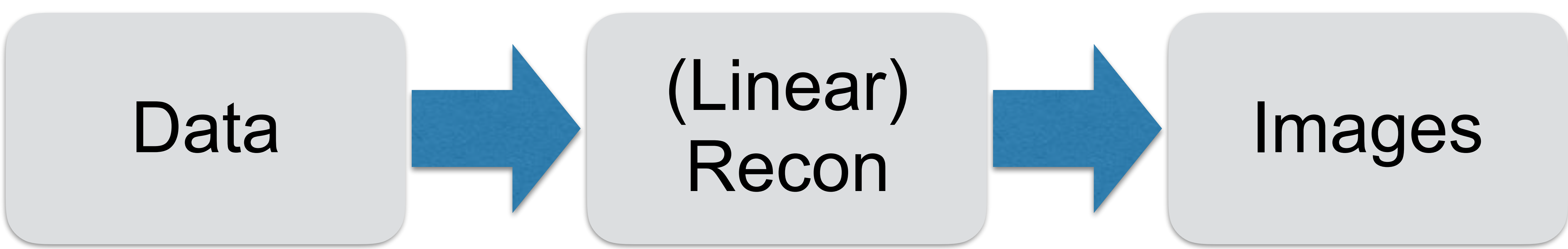
- Like difference method, but don't want N scans??
  - Motion, scan time, contrast-enhancement
- Measure noise covariance

$$\Psi_{ij} = \frac{1}{2N} \sum_{k=1}^{N} \mathrm{n}_{ik}\mathrm{n}_{jk}^{*}$$

- Neglect existing noise
- Generate and add noise to raw data
- <u>Linear</u> reconstruction (ex. R=1 SENSE)
- Measure stats at each location

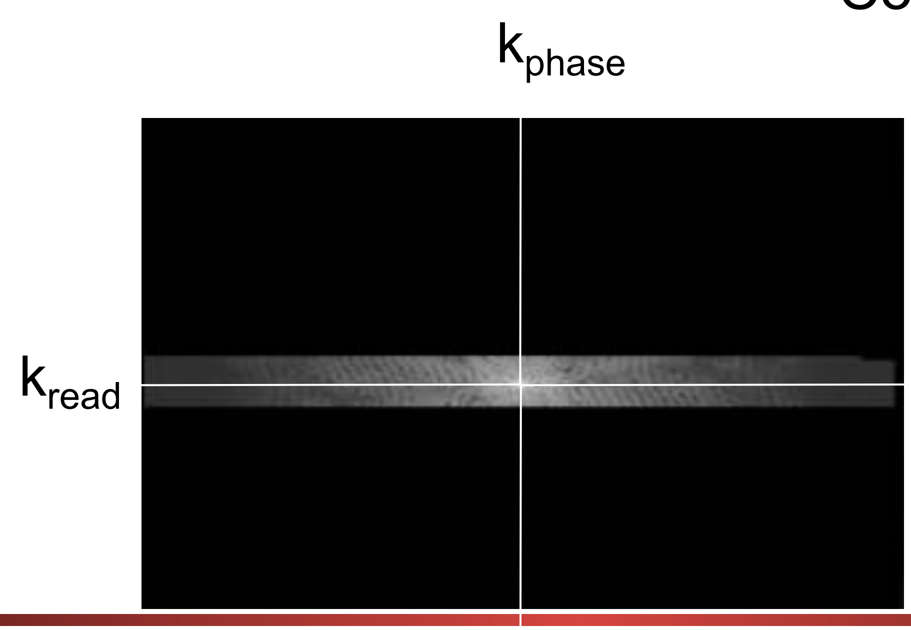
SNR Measurement - Pipeline

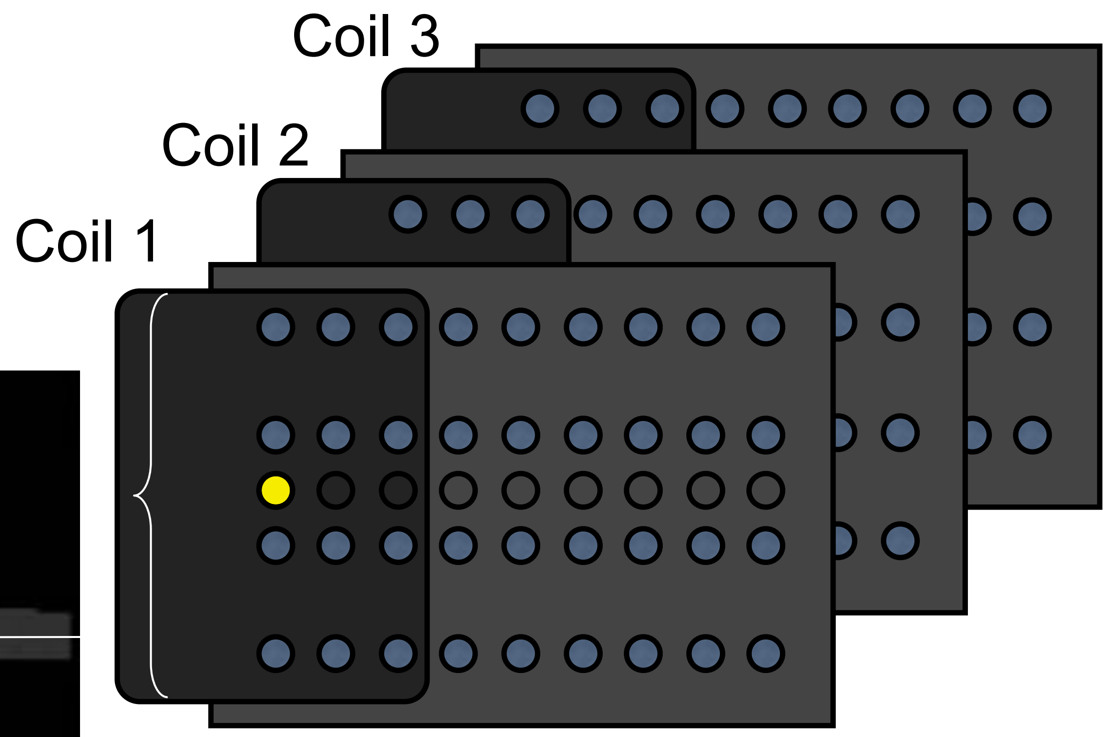
- What if we have a “pipeline” for reconstruction
- No simple coefficients
- Generate noise-only, given Ψ
- Reconstruct only noise, multiple times
- Measure statistics



Other Reconstructions

- GRAPPA uses linear combination to synthesize coils
- Self-calibrating (“pipeline” is data-driven)
- Coil combination can be RMS or SENSE
- Many variations, most linear
- Numeric and analytic g-factor methods





Noise Measurement

$$\Psi_{ij} = \frac{1}{2N} \sum_{k=1}^{N} \mathrm{n}_{ik} \mathrm{n}_{jk}^{*}$$

- Ideally measure noise-only
  - Sequence with no RF, but same receive BW
  - Turn off RF amplifier (some scanners)
  - Outer k-space
  - Ideally a few thousand samples
- Background noise in separate channel images?

Noise Example

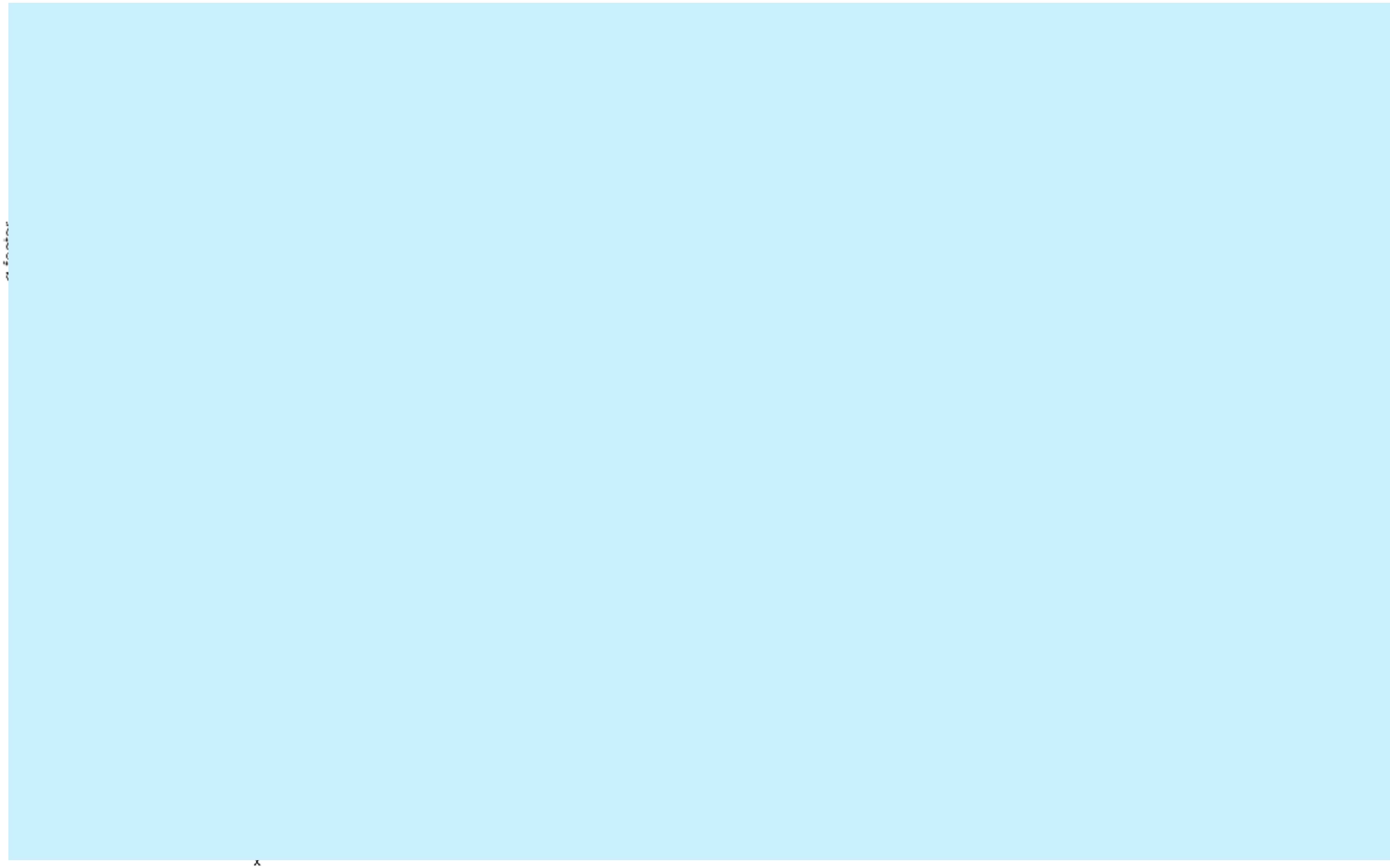
exampleB4_14.m

- Code for you to play with (modify, improve!)
- Parallel Imaging with up to 4 coils
- 1D objects for visualization / speed
- g-factor calculation
- Noise calculation (direct)
- Noise calculation (propagated)

SENSE Example (R=2)

SENSE Example (R=2)

SENSE Example (R=2)



Noise Example (Try!)

exampleB4_14.m

- Change acceleration (R)
- Change coil sensitivities (width, etc)
- Change noise covariance
- Complex sensitivities

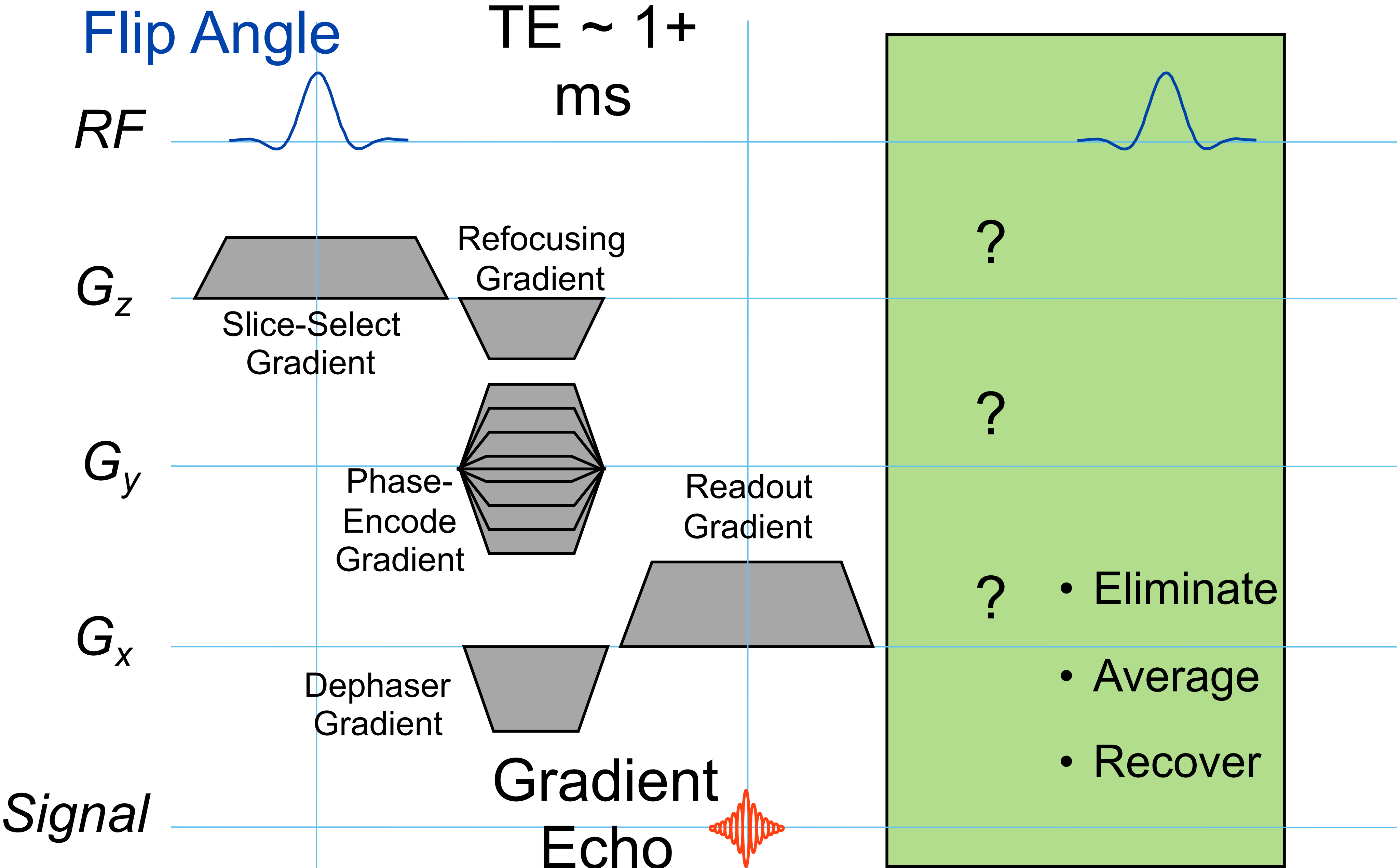
Noise ~ Summary

- Noise is additive, complex, gaussian
- Reconstruction steps affect statistics:
  - Magnitude ~ Rayleigh, Rician
  - RMS combination ~ complicated
  - (R=1) SENSE combination ~ linear, space varying
  - Similar for parallel imaging
- Pseudo-multiple replica / difference / propagation methods
- *Ideally scanners/recon would do this for you!*

Gradient Echo Sequences

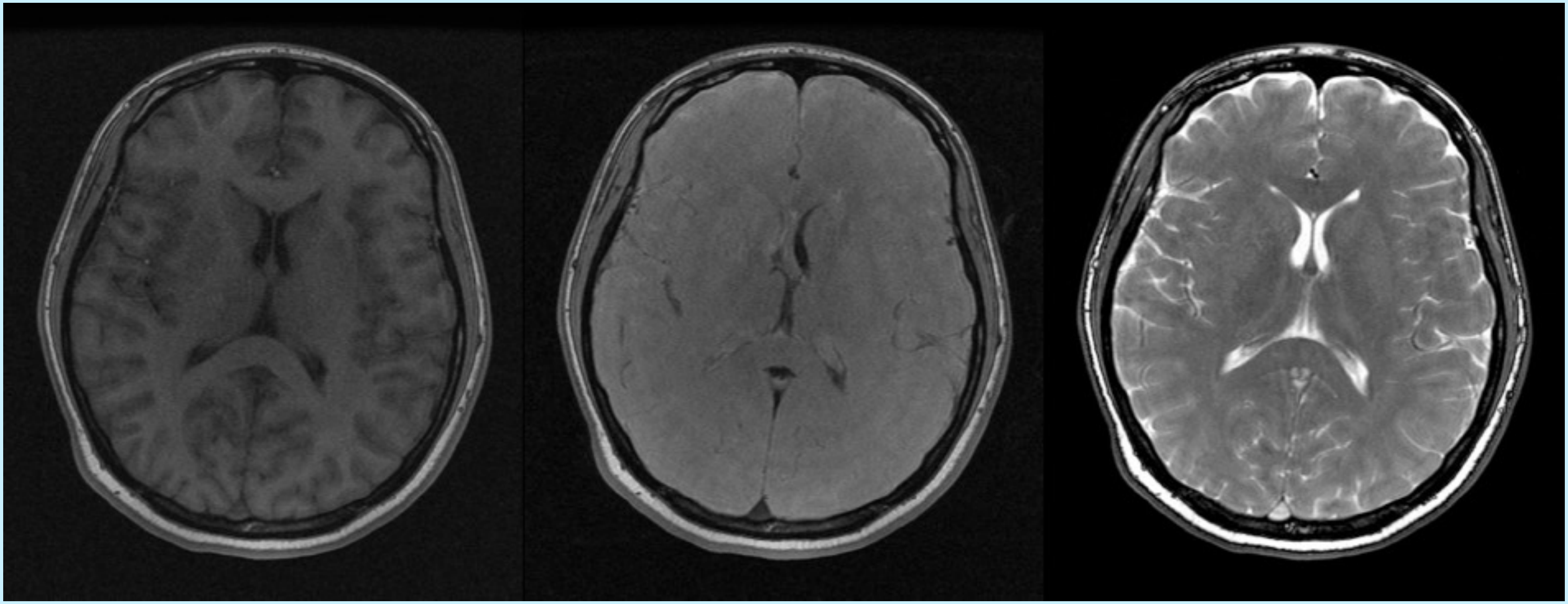
- Balanced SSFP
- Gradient Spoiled Sequences
- RF Spoiled Sequences
- Variations
  - Double-echo, Reversed
  - UTE, BOLD, T2*

Gradient Echo Pulse Sequence



Contrast Example

- Contrast based <u>solely</u> on end-of-TR action



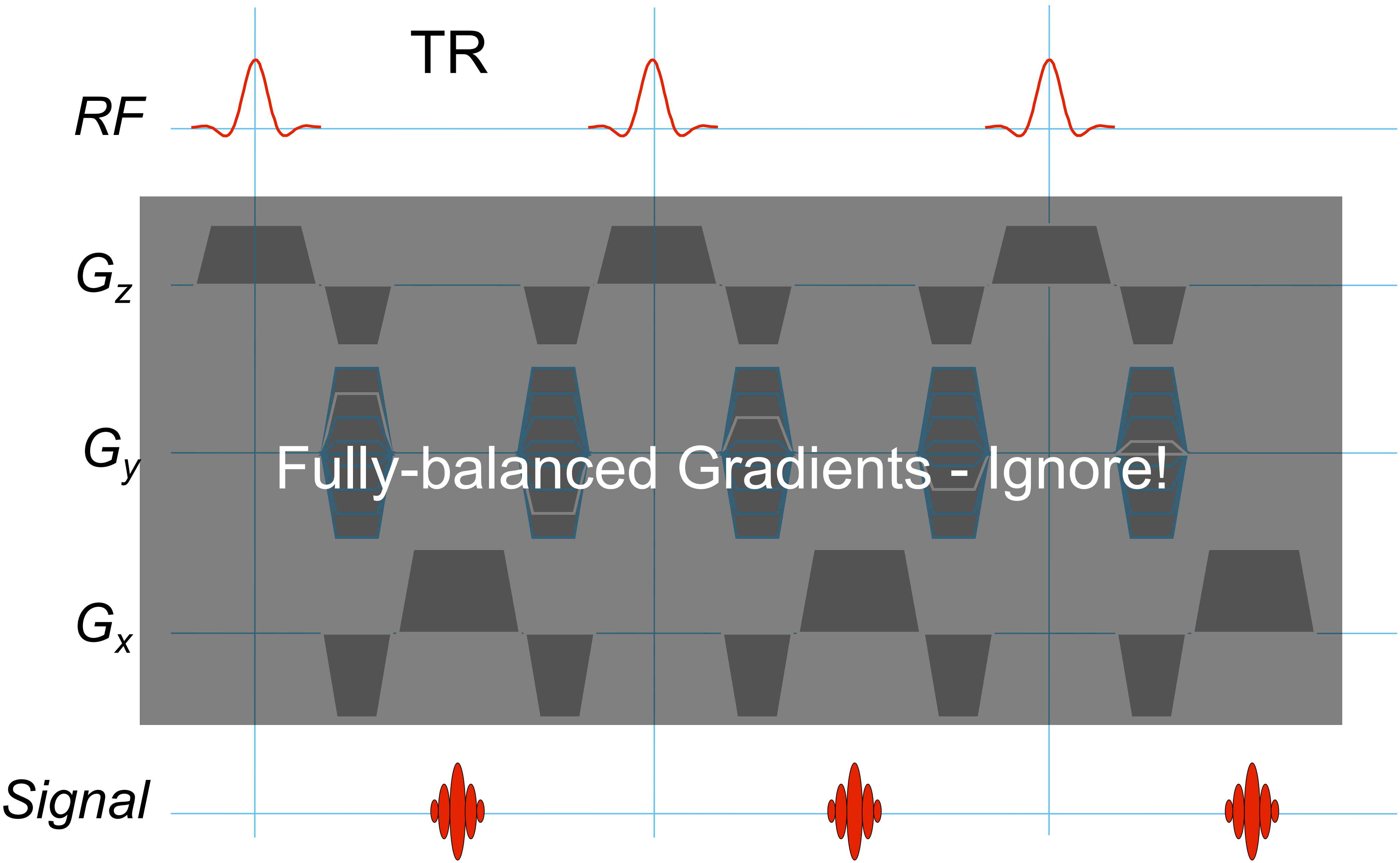
Balanced SSFP

- Goal: Maximize signal
  - Steady-state on-resonance
  - Steady-state off-resonance
  - Flip-angle effects
  - Transients

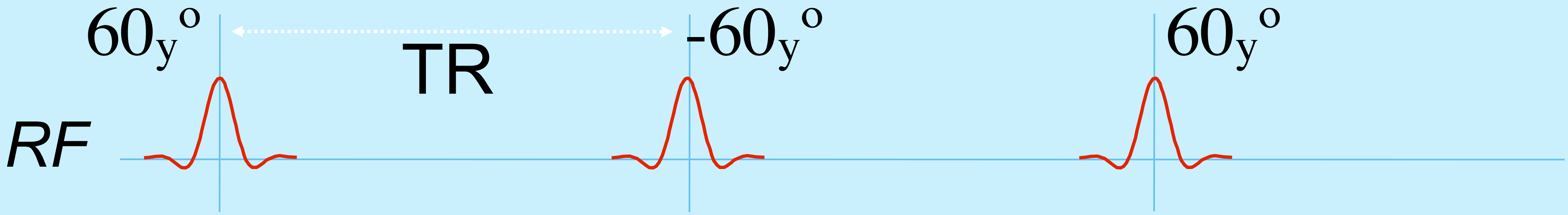
True-FISP, FIESTA, balanced FFE, BASG

Oppelt 1986, Duerk 1997

Balanced SSFP

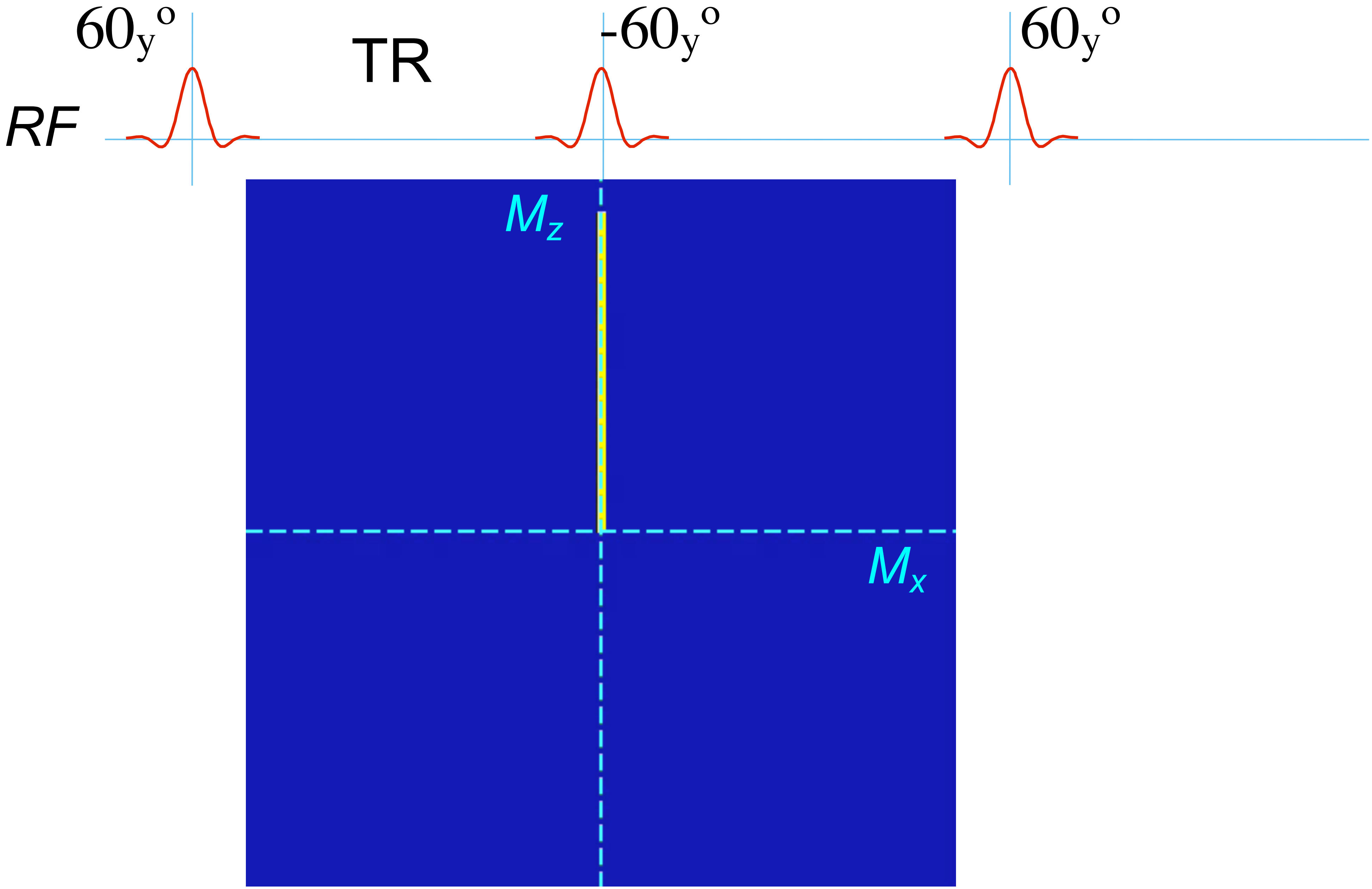


Balanced SSFP

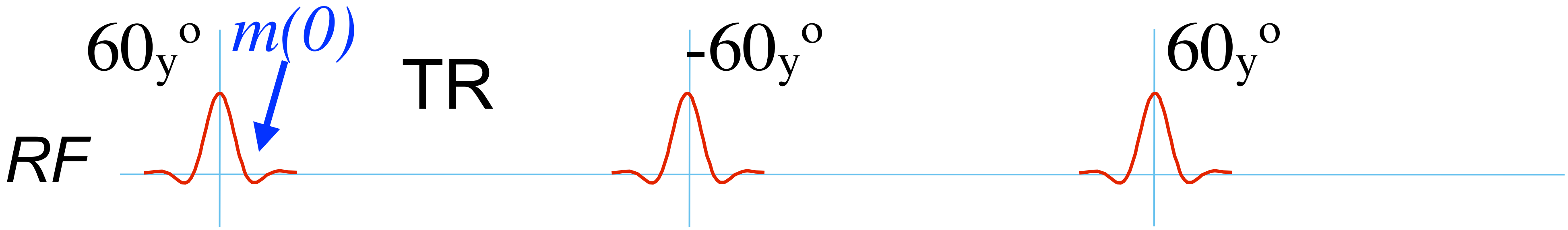


- So what happens here?

Balanced SSFP



Matrix Derivation



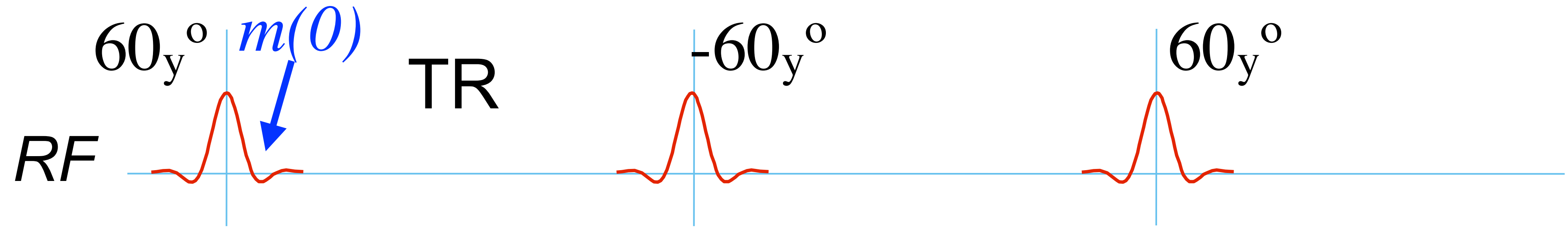
- Sequence: $E_{Relaxation}$ — $R_z(180^o)$ — $R_{-y}(\alpha)$
  - Note $R_z(180^o)$ lets us do this over only 1TR
- $m(0) = R_{-y}(\alpha)\, R_z(180^o)\, E\, m(0) + R_{-y}(\alpha)\, [0;0;1-E_1]$
  - Note: Could do as 2x2 formulation (no m_y)

$$\begin{bmatrix} 1+E_2\cos\alpha & 0 & -E_1\sin\alpha \\ 0 & 1+E_2 & 0 \\ -E_2\sin\alpha & 0 & 1-E_1\cos\alpha \end{bmatrix} m(0) = \begin{bmatrix} \sin\alpha(1-E_1) \\ 0 \\ \cos\alpha(1-E_1) \end{bmatrix}$$

Some algebra…

$$m(0) = \frac{1-E_1}{1+\cos\alpha(E_2-E_1)-E_1E_2} \begin{bmatrix} \sin\alpha \\ 0 \\ E_2+\cos\alpha \end{bmatrix}$$

Matrix Derivation (cont)

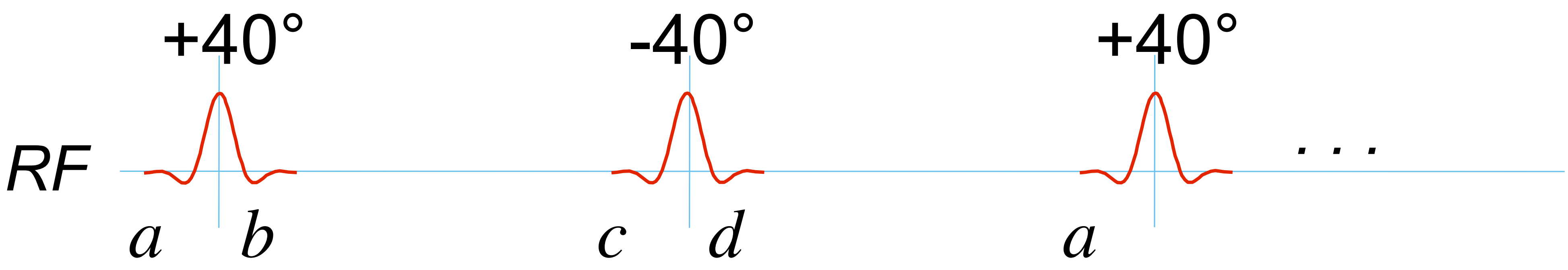


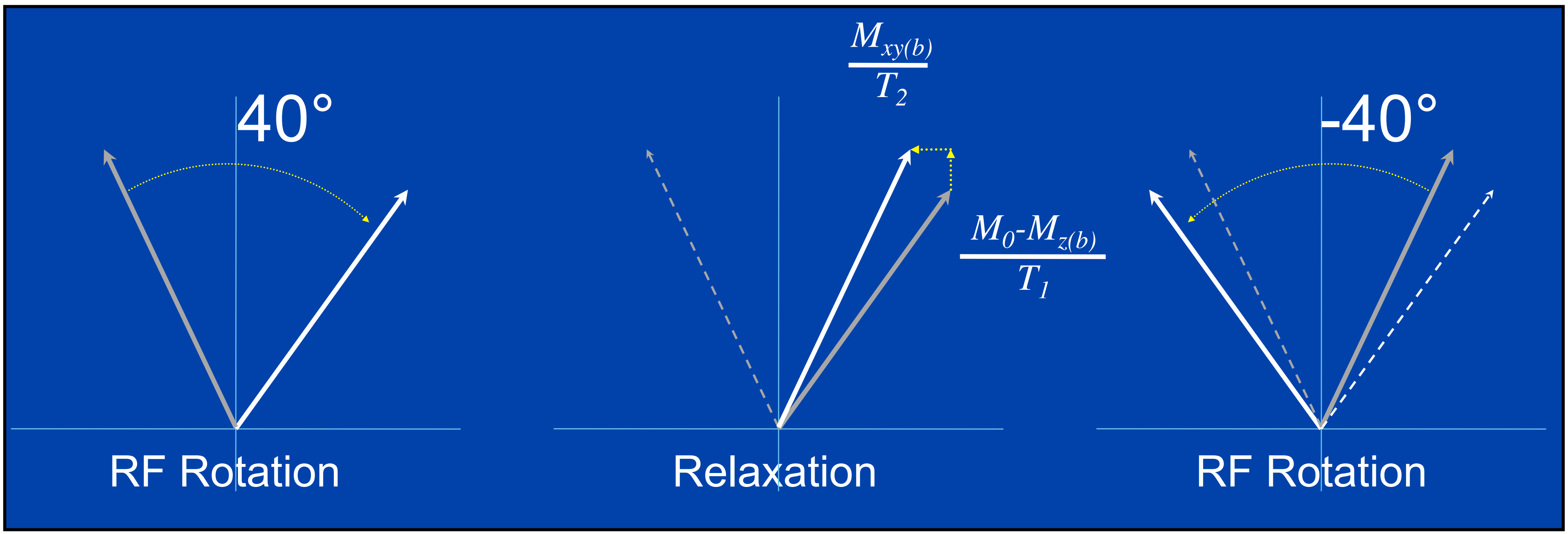
$$m(0) = \frac{1 - E_1}{1 + \cos\alpha(E_2 - E_1) - E_1 E_2} \begin{bmatrix} \sin\alpha \\ 0 \\ E_2 + \cos\alpha \end{bmatrix}$$

$$(E_2 \approx 1) \qquad \frac{\sin\alpha}{E_2 + \cos\alpha} = \tan(\alpha/2) = \frac{m_x}{m_z}$$

- Steady-state $m(0)$ is tilted by α/2 (good!)
- If E_2=0, same as excitation-recovery (good!)

Example: Alternating RF, No precession



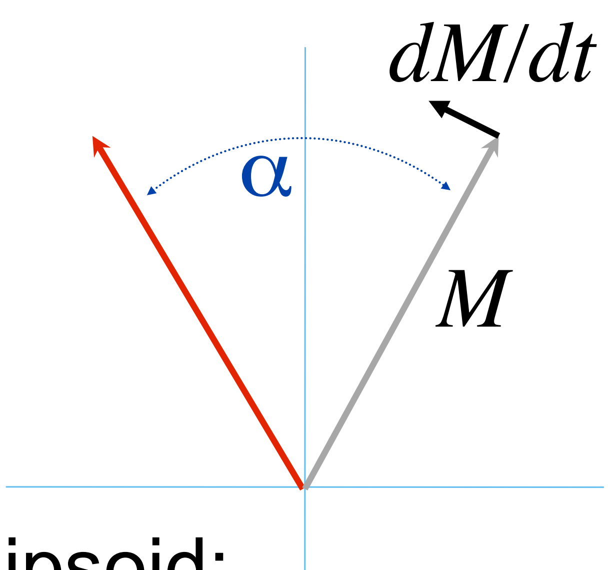


- Intuition: Relaxation does not change length much over TR

Length Solution

- If Relaxation does not change length:

$$dM/dt \cdot M = 0$$



$$\frac{-M_x}{T_2}M_x + \frac{-M_y}{T_2}M_y + \frac{M_0 - M_z}{T_1}M_z = 0$$

x T_1 & Rearrange...

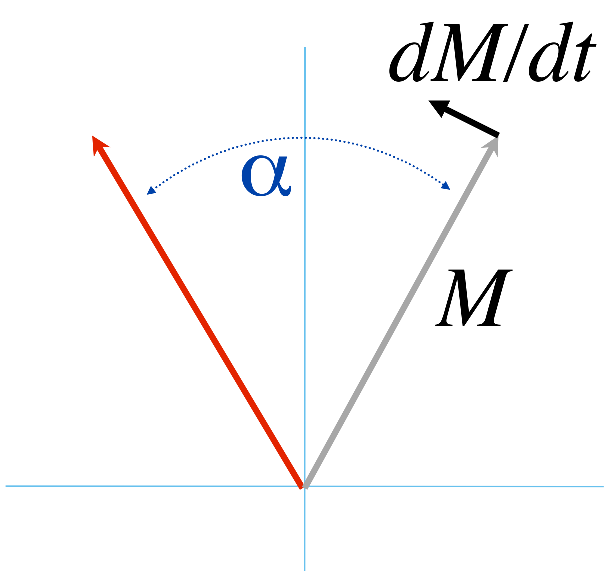
$$\left(M_z - \frac{M_0}{2}\right)^2 + \frac{M_x^2 + M_y^2}{T_2/T_1} = \left(\frac{M_0}{2}\right)^2$$

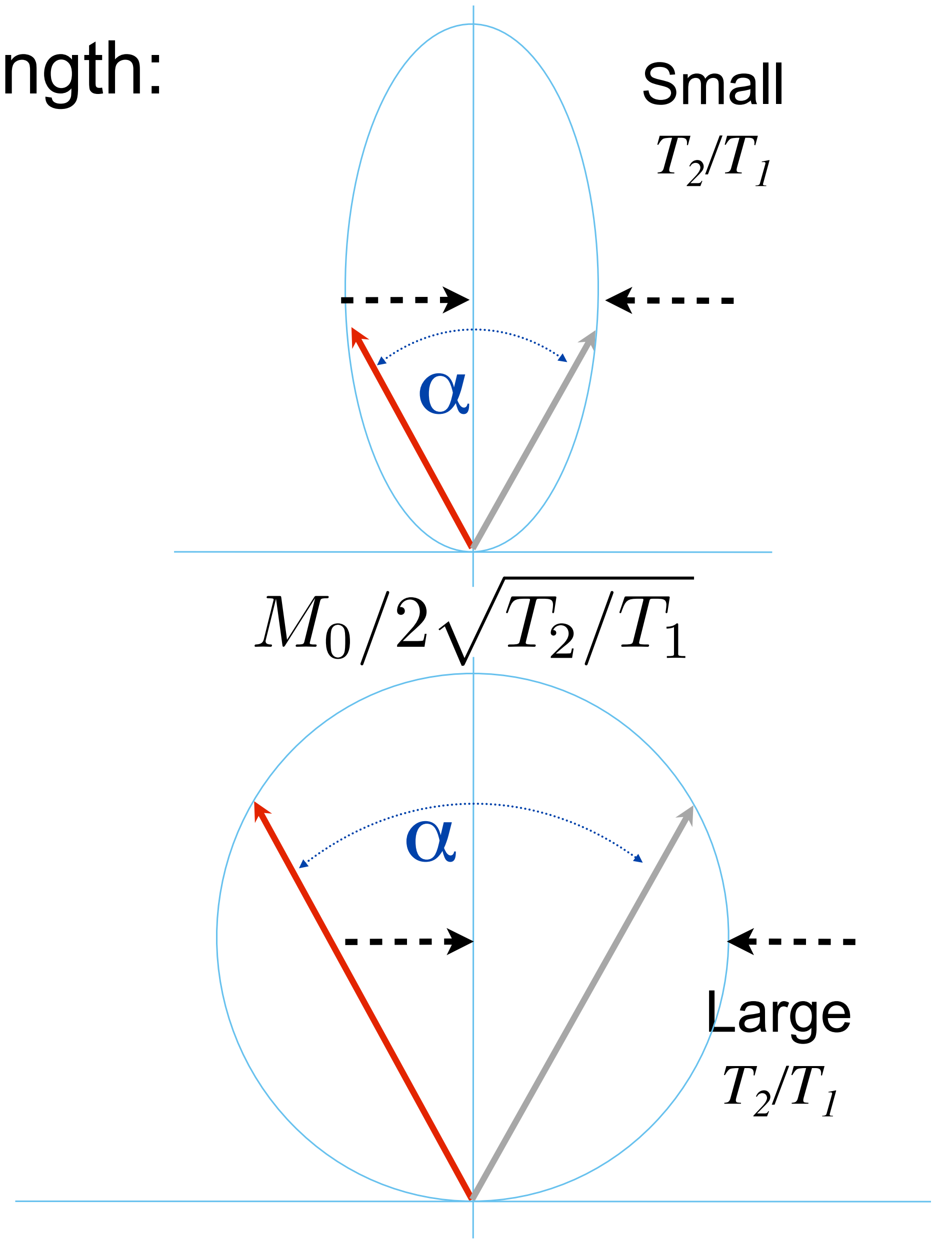
- Ellipsoid:
  - height M_0, half-width (signal) $M_0/2\sqrt{T_2/T_1}$
- T_2/T_1 contrast!

Full Solution

- If Relaxation does not change length:

$$dM/dt \cdot M = 0$$

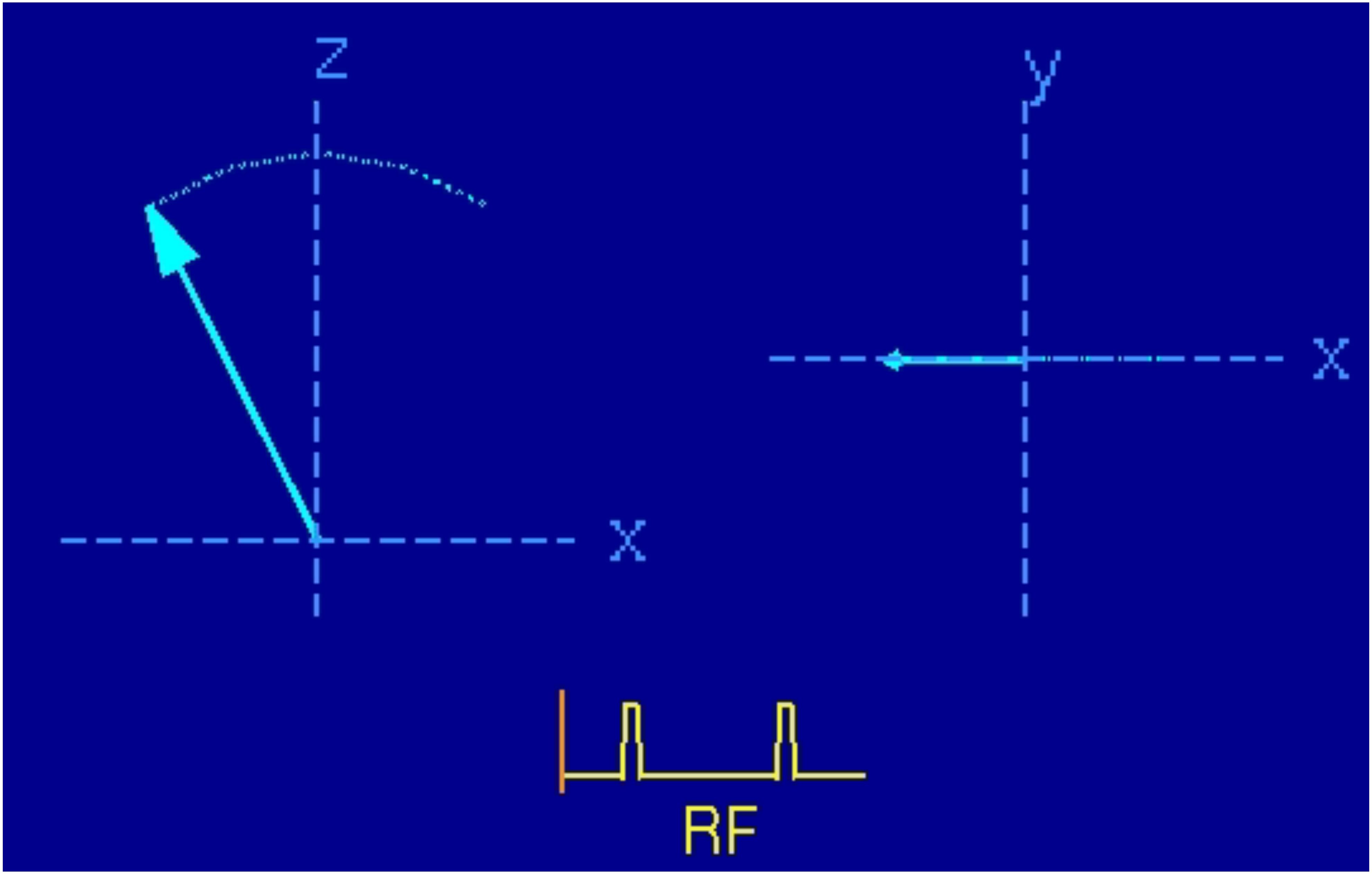




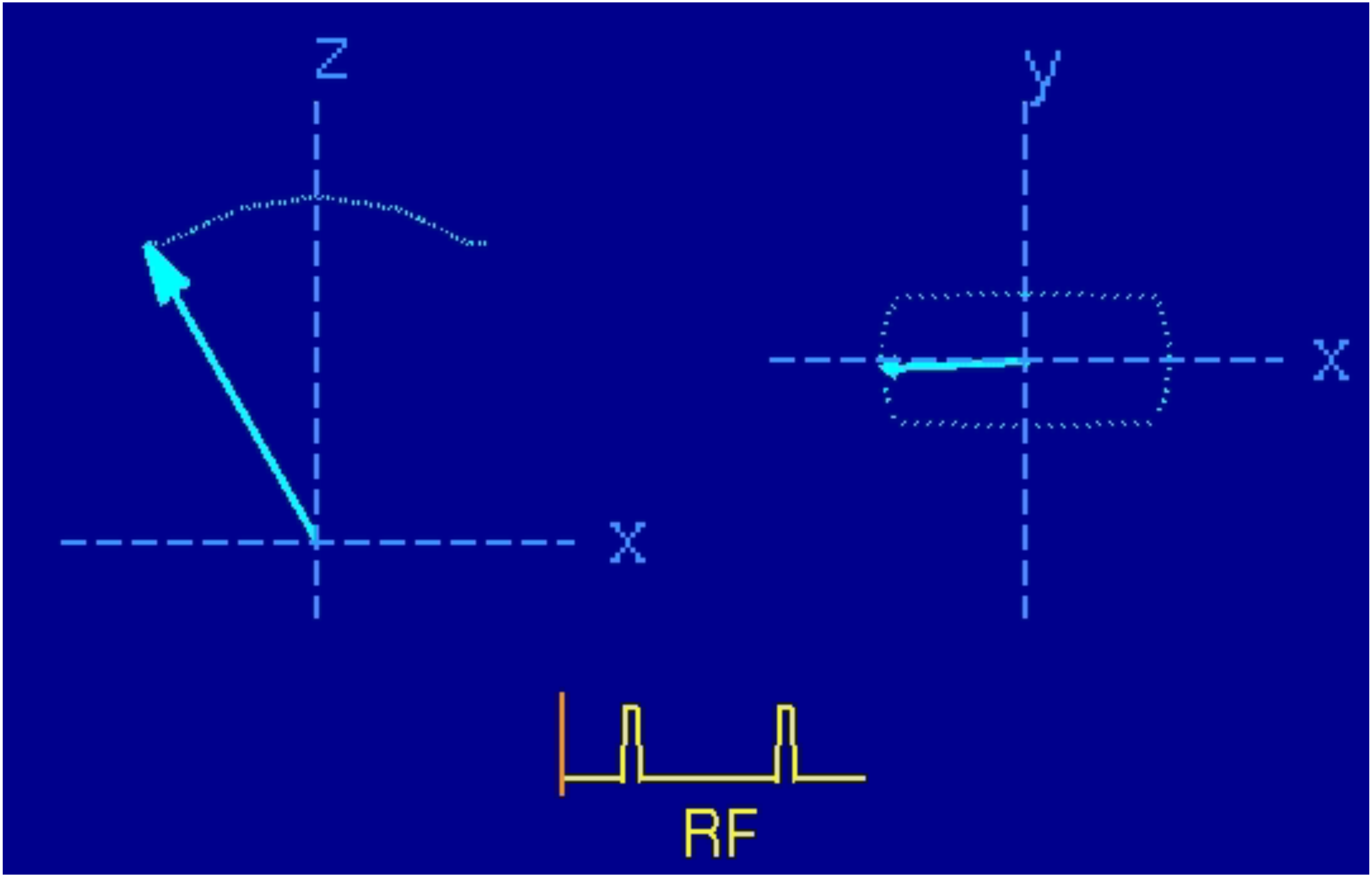
- Length: Ellipsoid intersection
- Direction: $\alpha/2$ angle from M_z

Signal Question

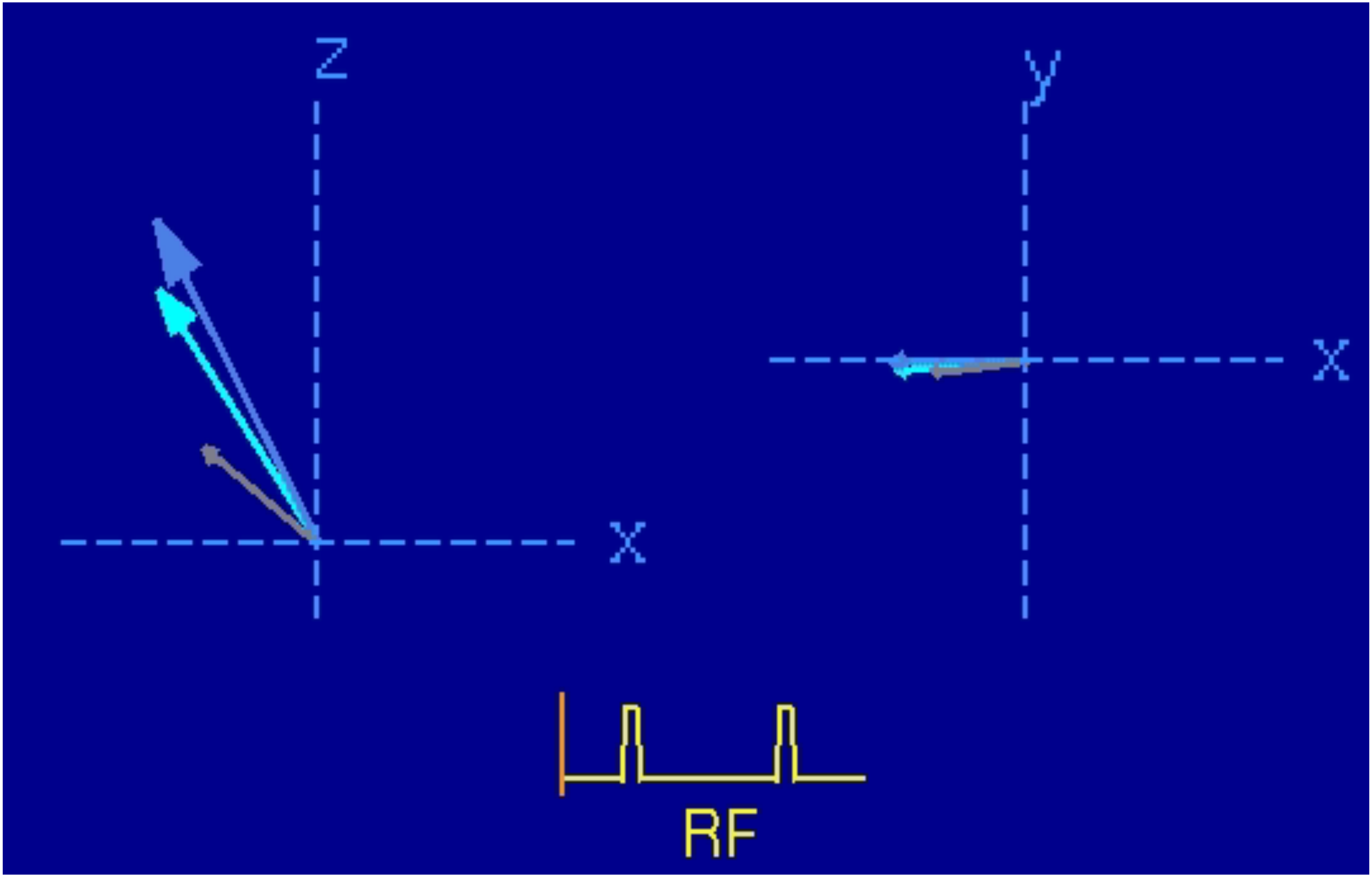
Steady State: No Precession



Off-Resonance: Precession

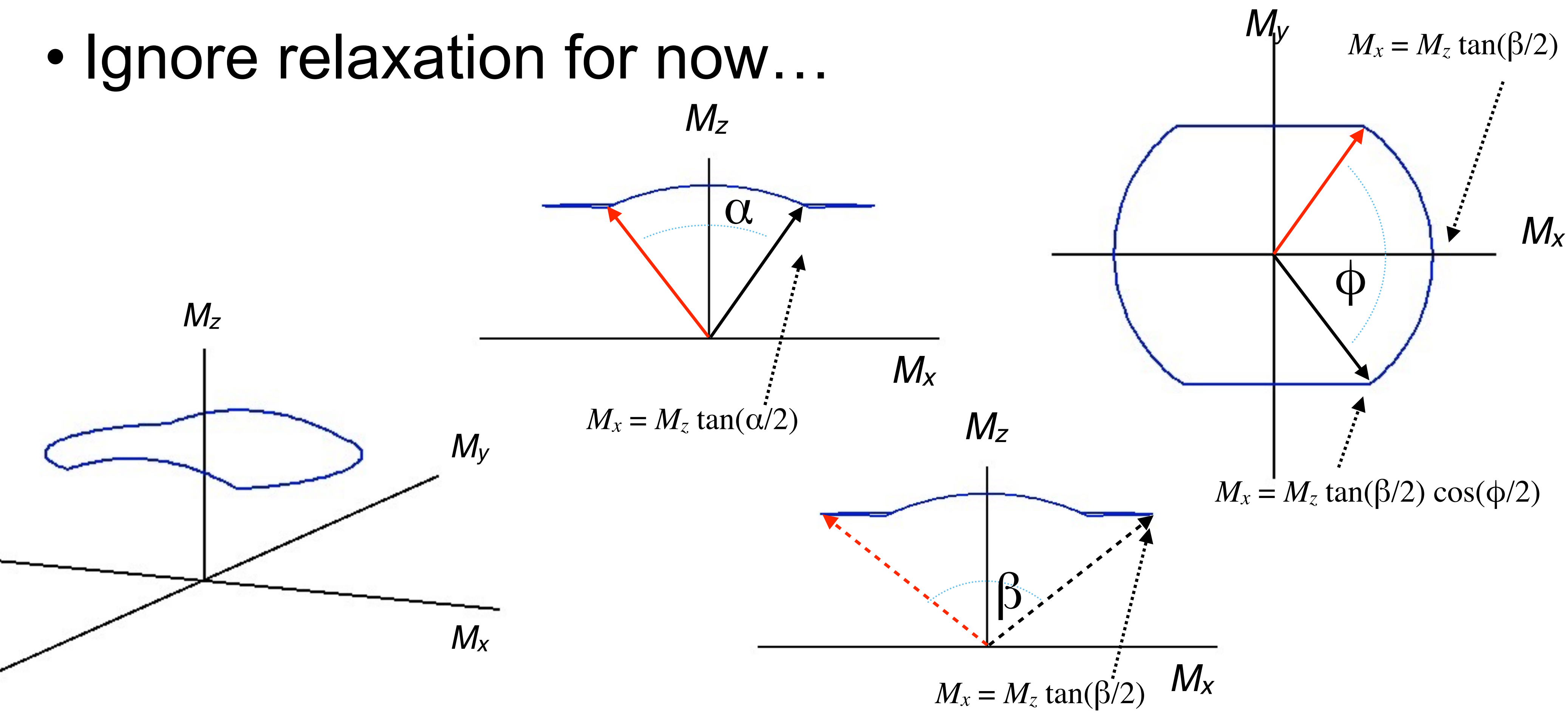


Increasing Precession



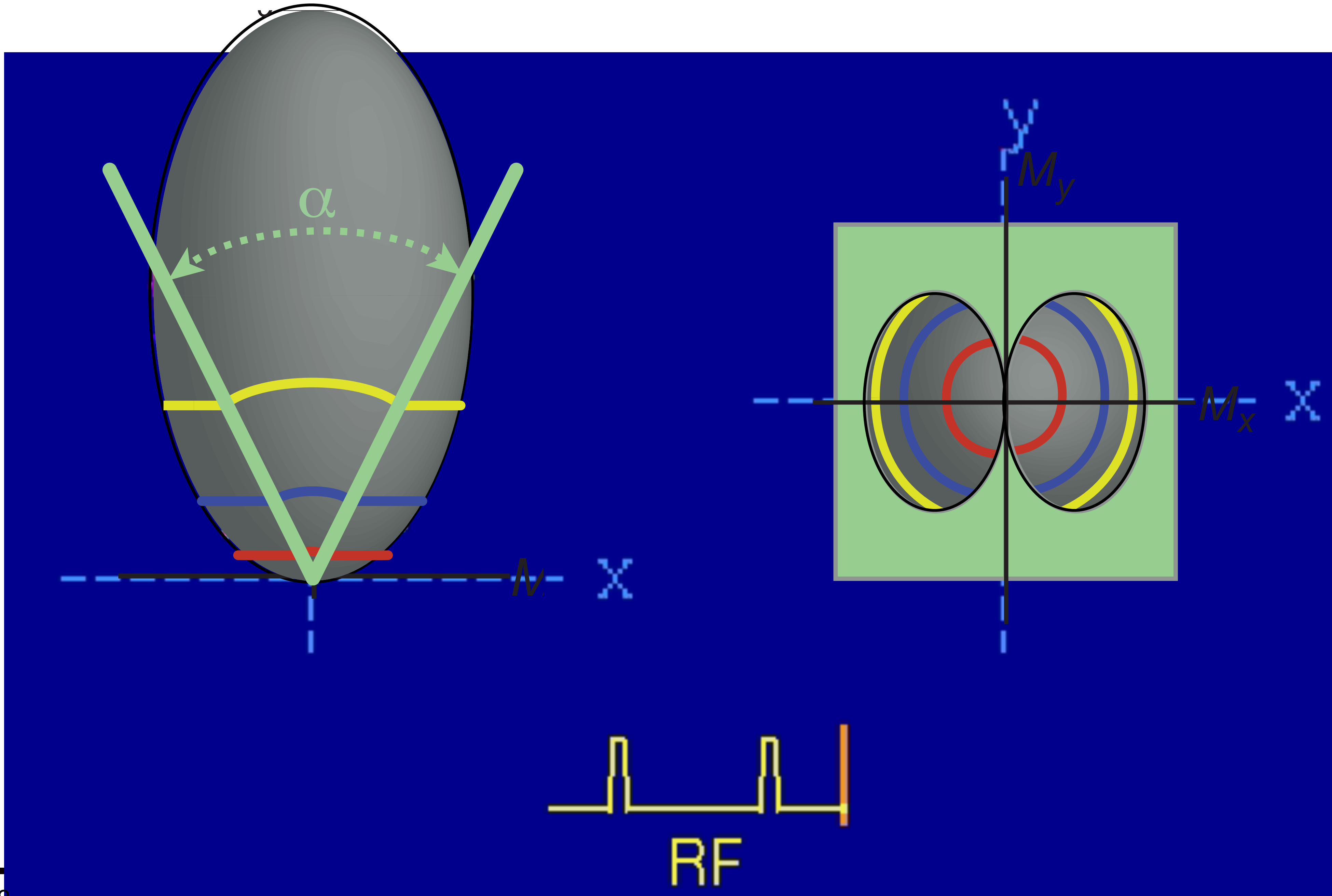
bSSFP: RF, Relaxation and Precession

- RF is balanced by relaxation and precession
- Length is still relatively unchanged over TR
- Ignore relaxation for now…

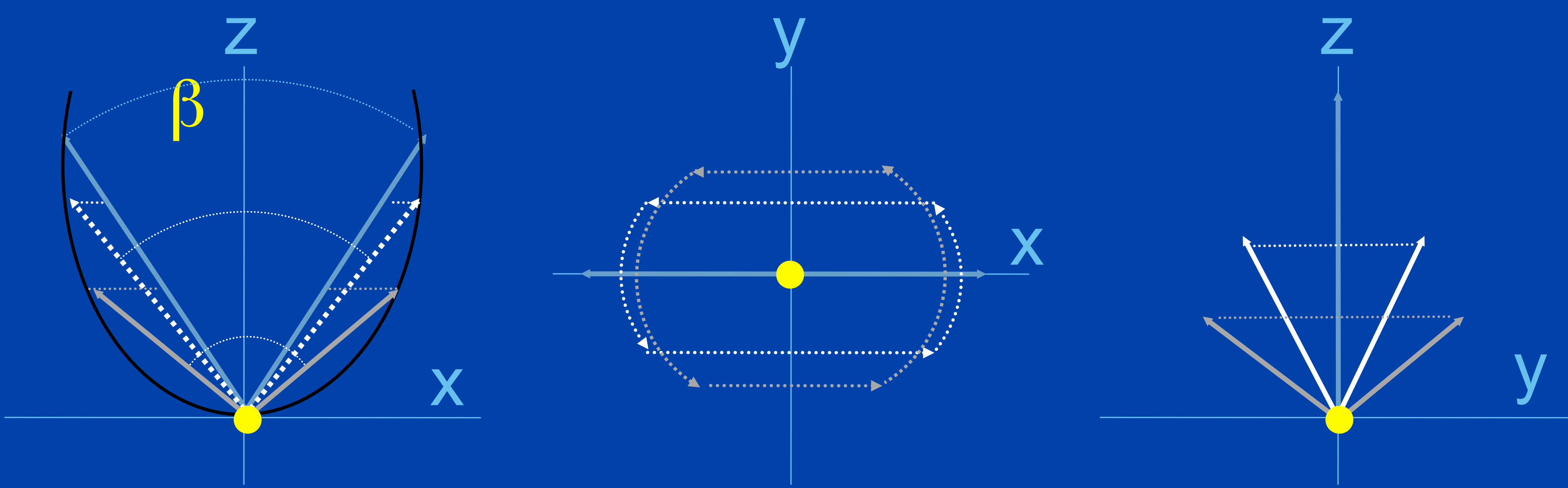


$$\tan(\alpha/2) = \tan(\beta/2)\cos(\phi/2)$$

Balanced SSFP (FIESTA)



Precession and "Effective flip angle"



- Larger precession gives a larger "effective flip," β
- $\tan(\alpha/2) = \tan(\beta/2)\cos(\phi/2)$
- $\beta >= \alpha$
- Can replace flip (α) with effective flip (β) for all calculations
- Limiting case (•) where $\beta = 180°$

(Schmitt MRM 2006, Zun, ISMRM 2006)

"Ellipsoid" Derivation of Signal

- Starting with ellipsoid: $$\left(M_z - \frac{M_0}{2}\right)^2 + \frac{M_x^2 + M_y^2}{T_2/T_1} = \left(\frac{M_0}{2}\right)^2$$

$M_{xy} = M \sin(\beta/2)$

$M_z = M \cos(\beta/2)$

$$\left[M\cos(\beta/2) - \frac{M_0}{2}\right]^2 + \frac{M^2\sin^2(\beta/2)}{T_2/T_1} = \left[\frac{M_0}{2}\right]^2$$

$$M^2\cos^2(\beta/2) + (T_1/T_2)M^2\sin^2(\beta/2) = M M_0 \cos(\beta/2)$$

$$M\sin(\beta/2) = \frac{M_0\cos(\beta/2)\sin(\beta/2)}{\cos^2(\beta/2) + (T_1/T_2)\sin^2(\beta/2)}$$

$$S = \frac{M_0}{\cot(\beta/2) + (T_1/T_2)\tan(\beta/2)}$$

- Signal drops with increasing T_1/T_2
- At β=180° signal is 0. At β=90°, $T_1=T_2$, $S=M_0/2$

Ellipsoid vs Matrix Derivation

Ellipsoid:

$$S = \frac{M_0}{\cot(\beta/2) + (T_1/T_2)\tan(\beta/2)}$$

Matrix:

$$m(0) = \frac{1 - E_1}{1 + \cos\alpha(E_2 - E_1) - E_1 E_2} \begin{bmatrix} \sin\alpha \\ 0 \\ E_2 + \cos\alpha \end{bmatrix}$$

Approximate $E_{1,2} = \exp(-TR/T_{1,2}) \sim (1-TR/T_{1,2})$, neglect $TR^2/(T_1 T_2)$

=> If $\beta=\alpha$ then the two equations (for $S=M_{xy}$) give the same result.

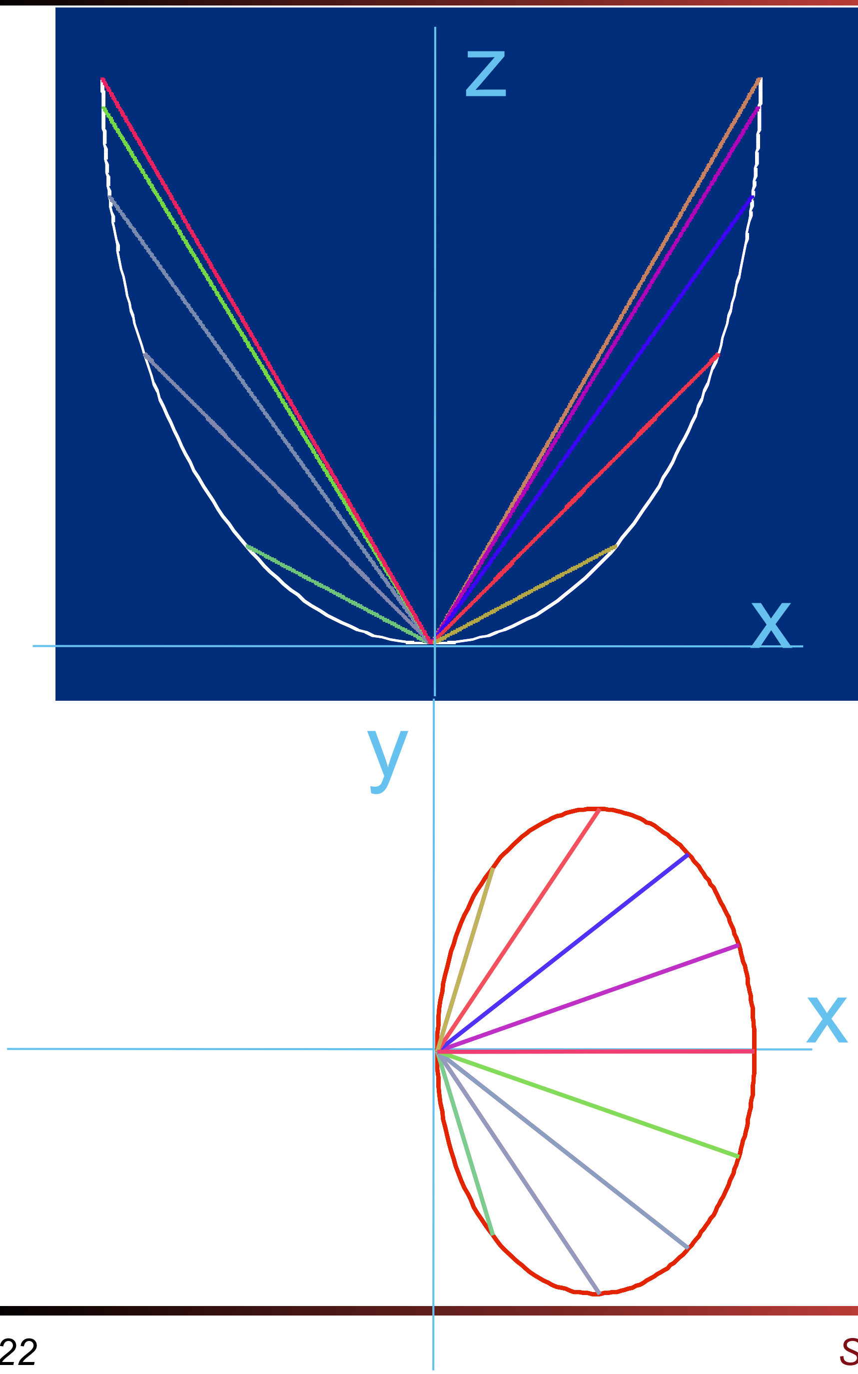
Substitute $E_{1,2}$, Divide out TR/T_1

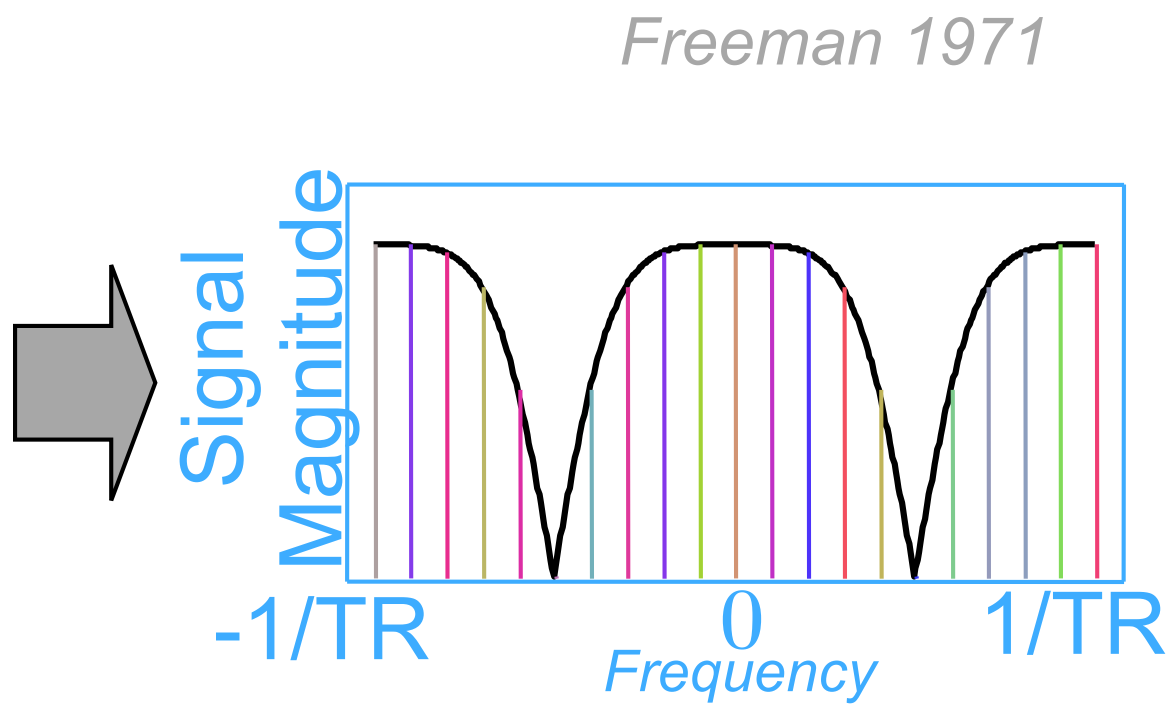
$$m(0) = \frac{\sin\alpha}{(1+\cos\alpha) + (T_1/T_2)(1-\cos\alpha)}$$

Divide out $\sin(\alpha/2)\cos(\alpha/2)$:

$$m(0) = \frac{2\sin(\alpha/2)\cos(\alpha/2)}{2\cos^2(\alpha/2) + (T_1/T_2)2\sin^2(\alpha/2)}$$

Signal vs Precession/Frequency

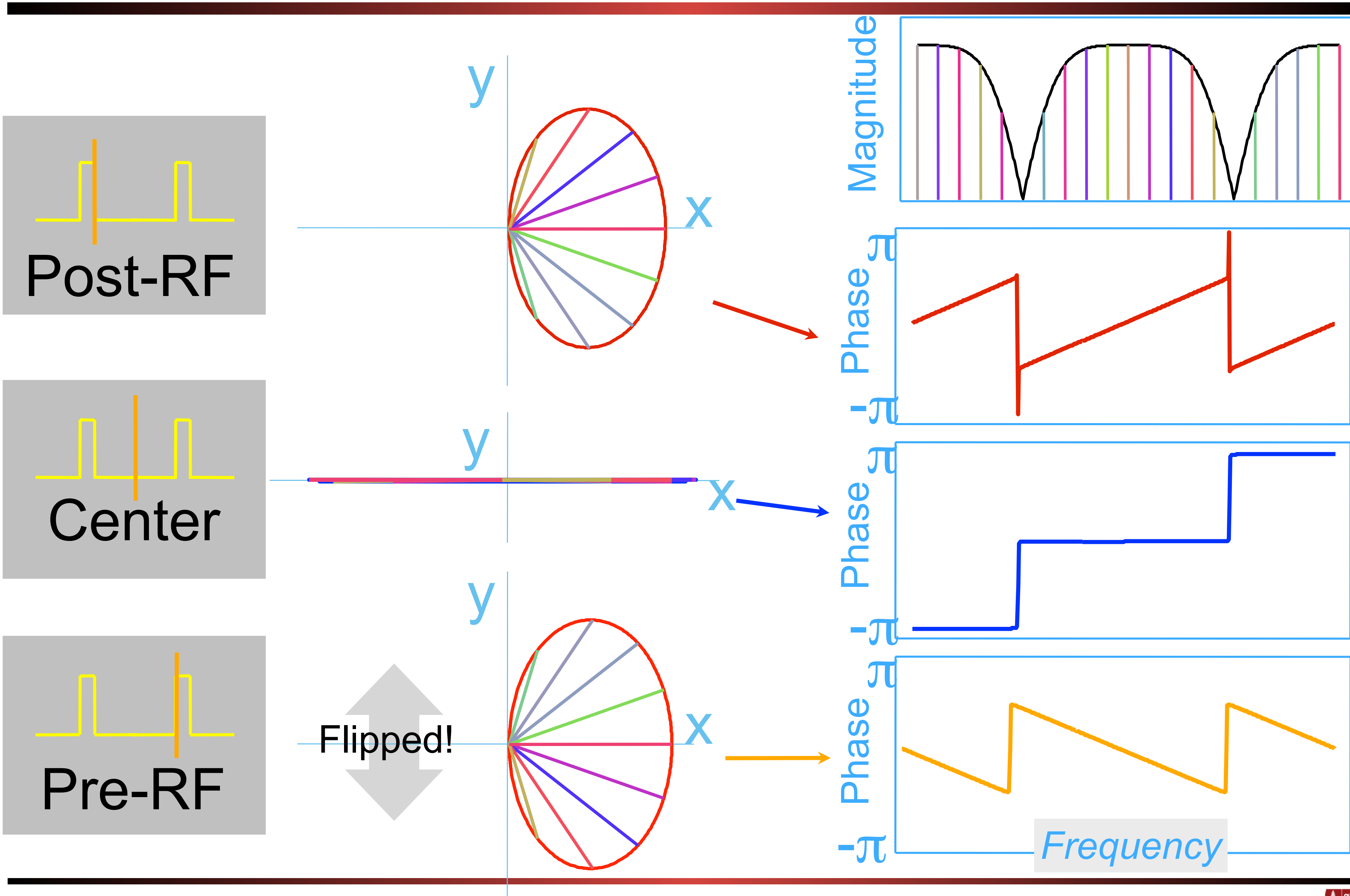




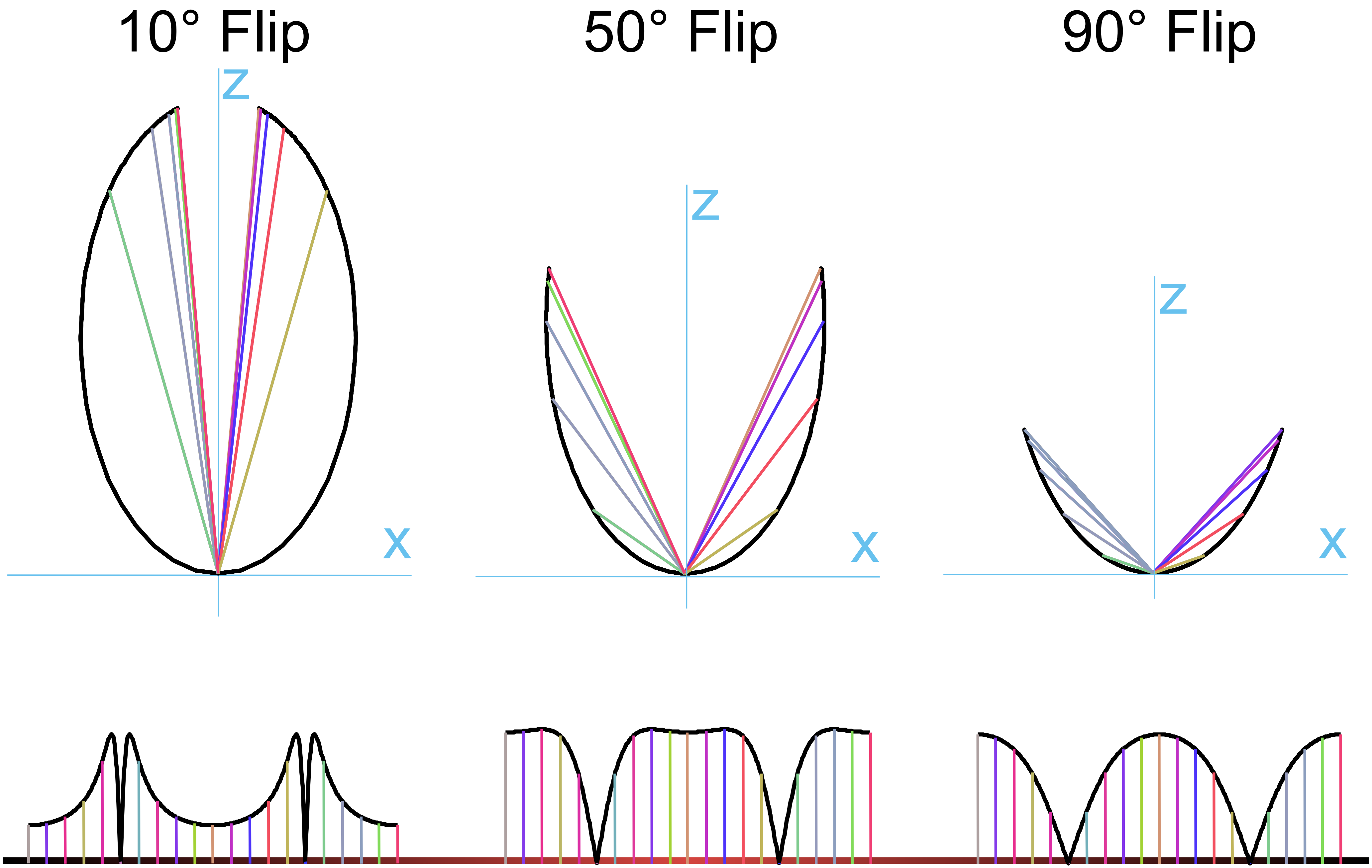
Signal depends on many factors:

- *Resonant frequency*
- T1, T2 (contrast)
- TR, TE
- RF flip / phase

Signal vs Frequency: Phase



Flip angle effects

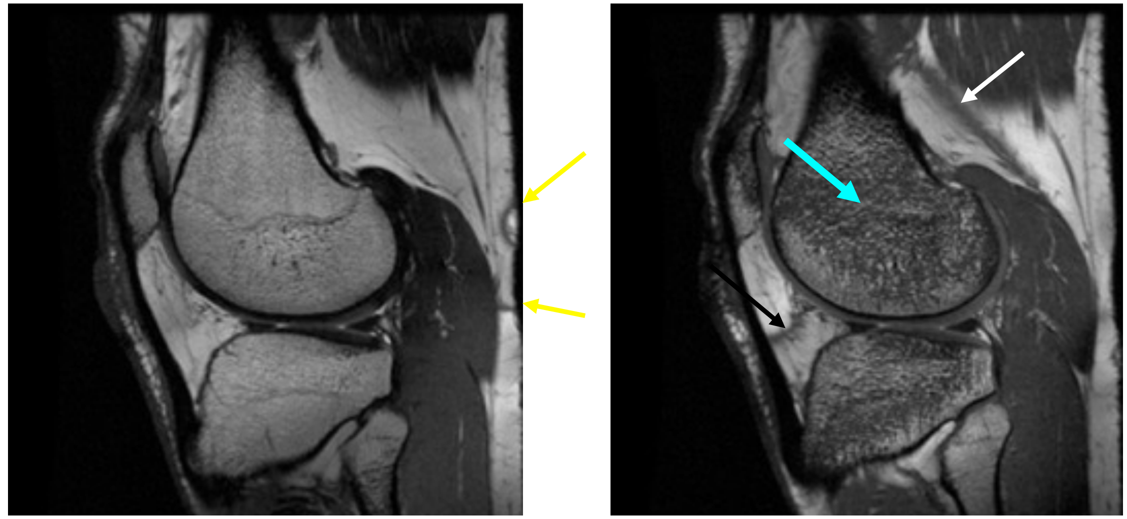


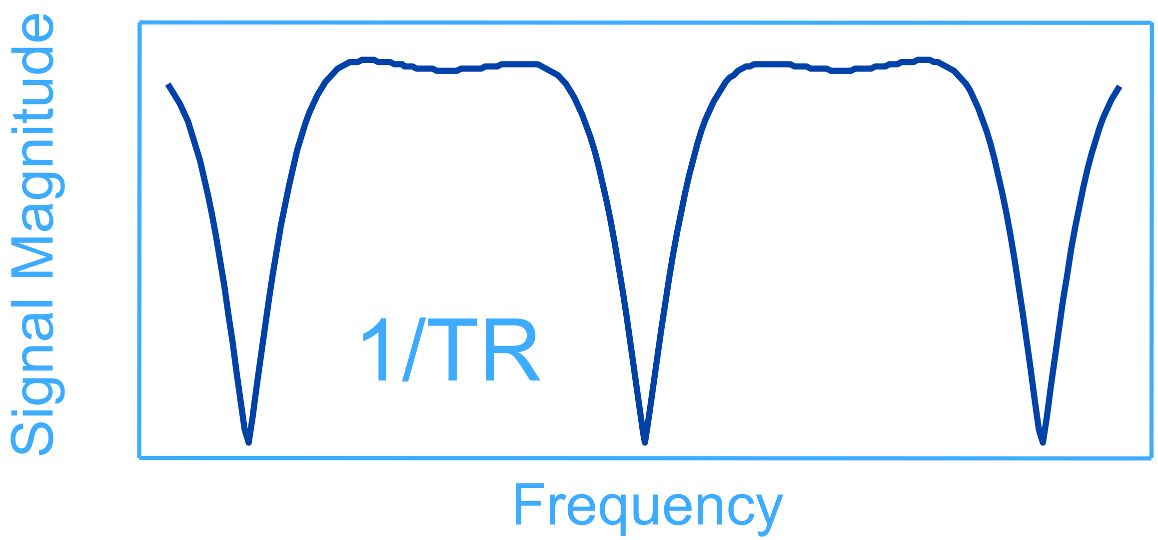
bSSFP Dark Bands

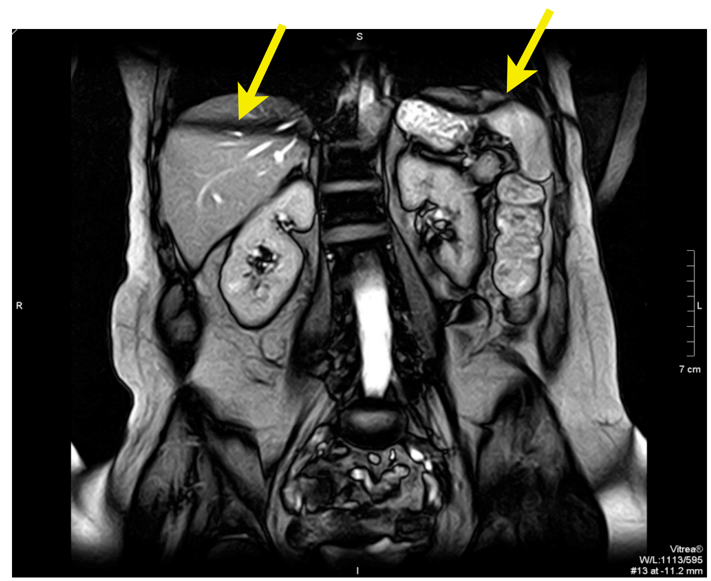
Must limit precession:

Short TR

Limits resolution

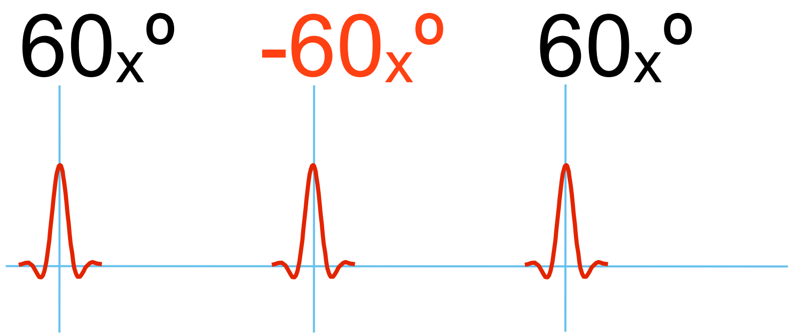


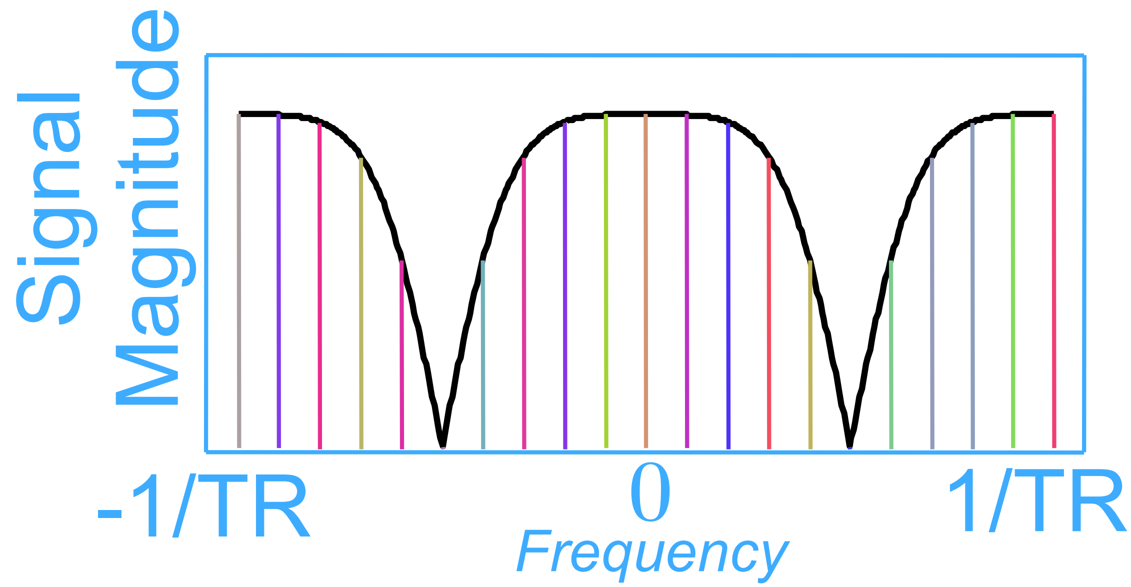


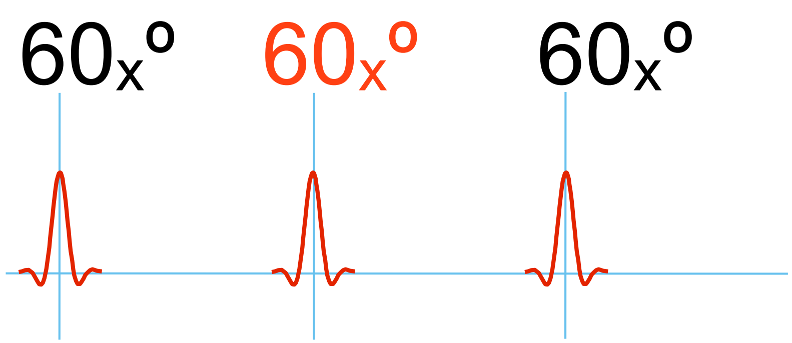


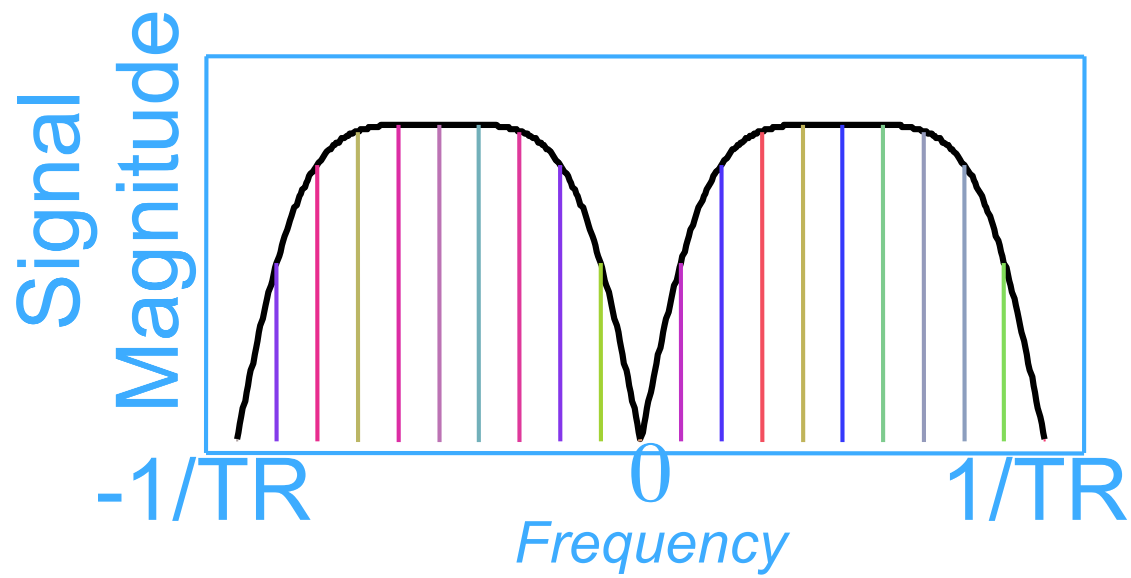
Phase Cycling

Hinshaw 1976







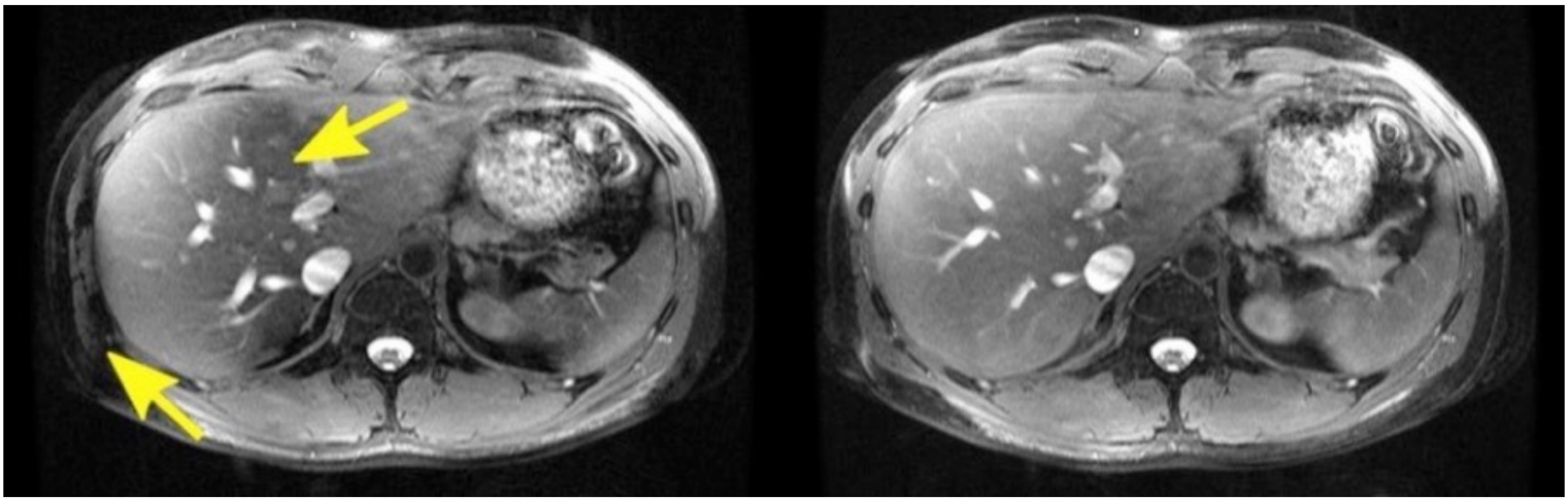


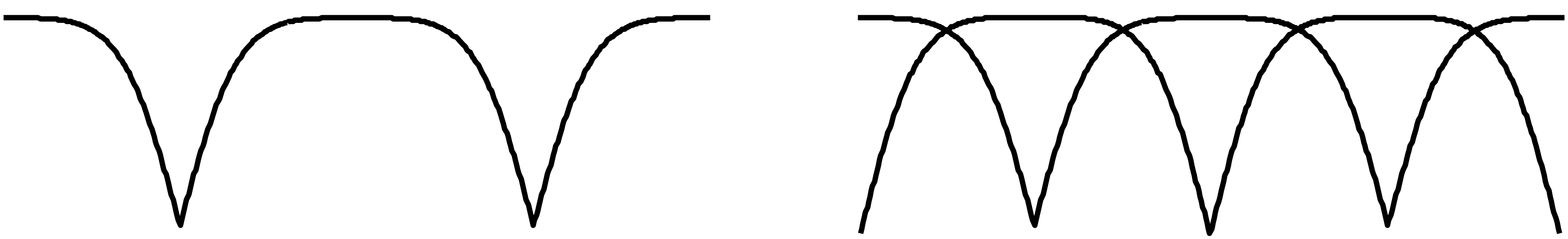
Review Question: Phase Cycling

Balanced SSFP Abdomen Example

Alternating RF

Combined Acquisition



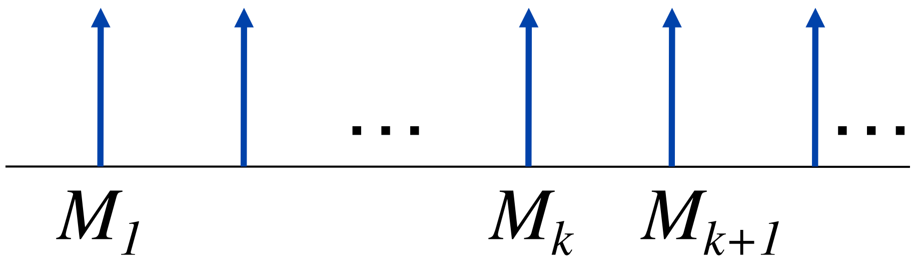


Matrix Solutions

(Jaynes 1955)

Apply 3x3 matrix scheme:

$$M_{k+1} = A\,M_k + B \qquad [1]$$



In steady-state:

$$M_{ss} = AM_{ss} + B \qquad [2]$$

$$M_{k+1} - M_{ss} = A(M_k - M_{ss}) \qquad [1\text{-}2]$$

Consider Eigenvector Decomposition:

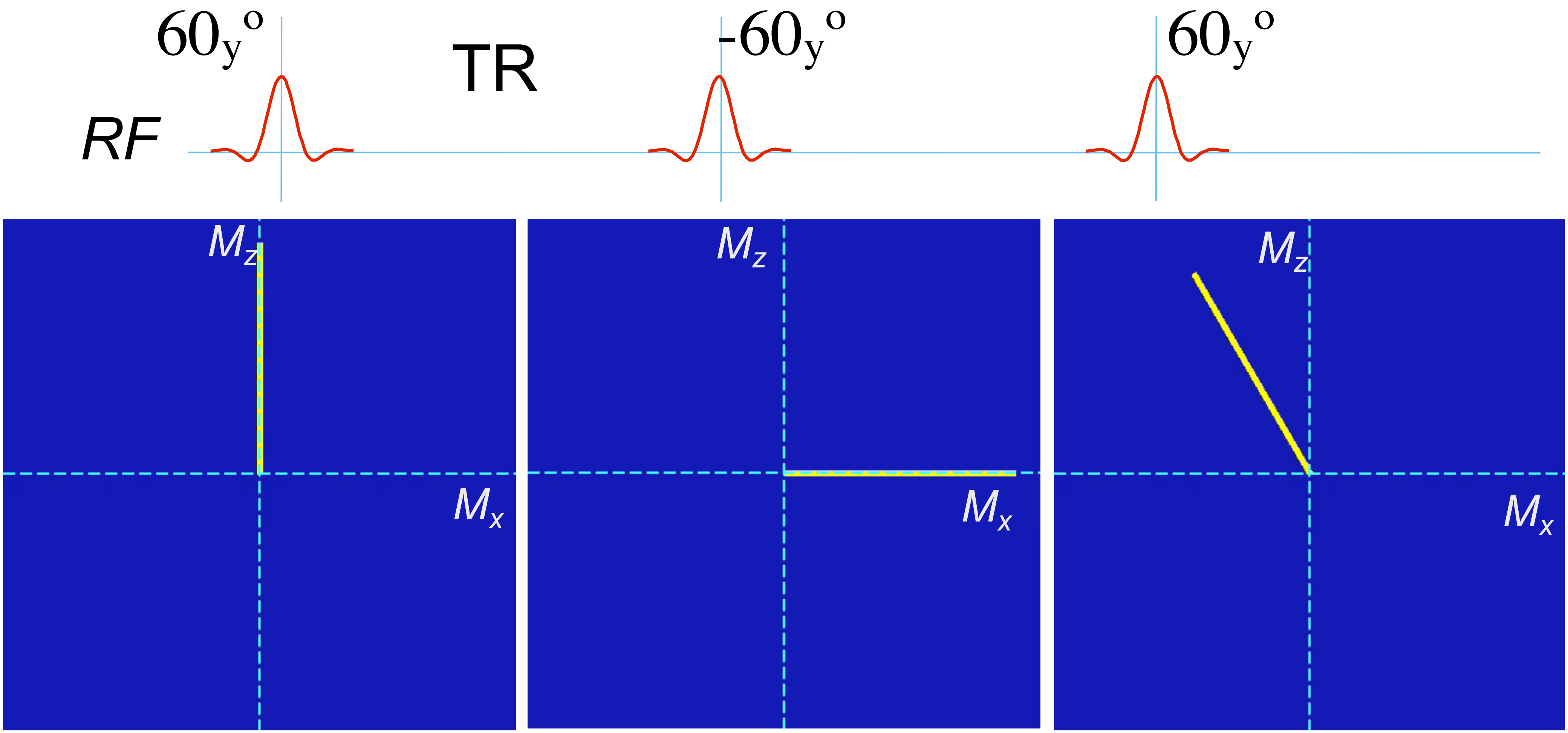
$$A = V\Lambda V^{-1} \qquad M_{k+1} - M_{ss} = (V\Lambda)\; V^{-1}(M_k - M_{ss})$$

At least one eigenvector/value is real.

Others often oscillatory and die out in steady state

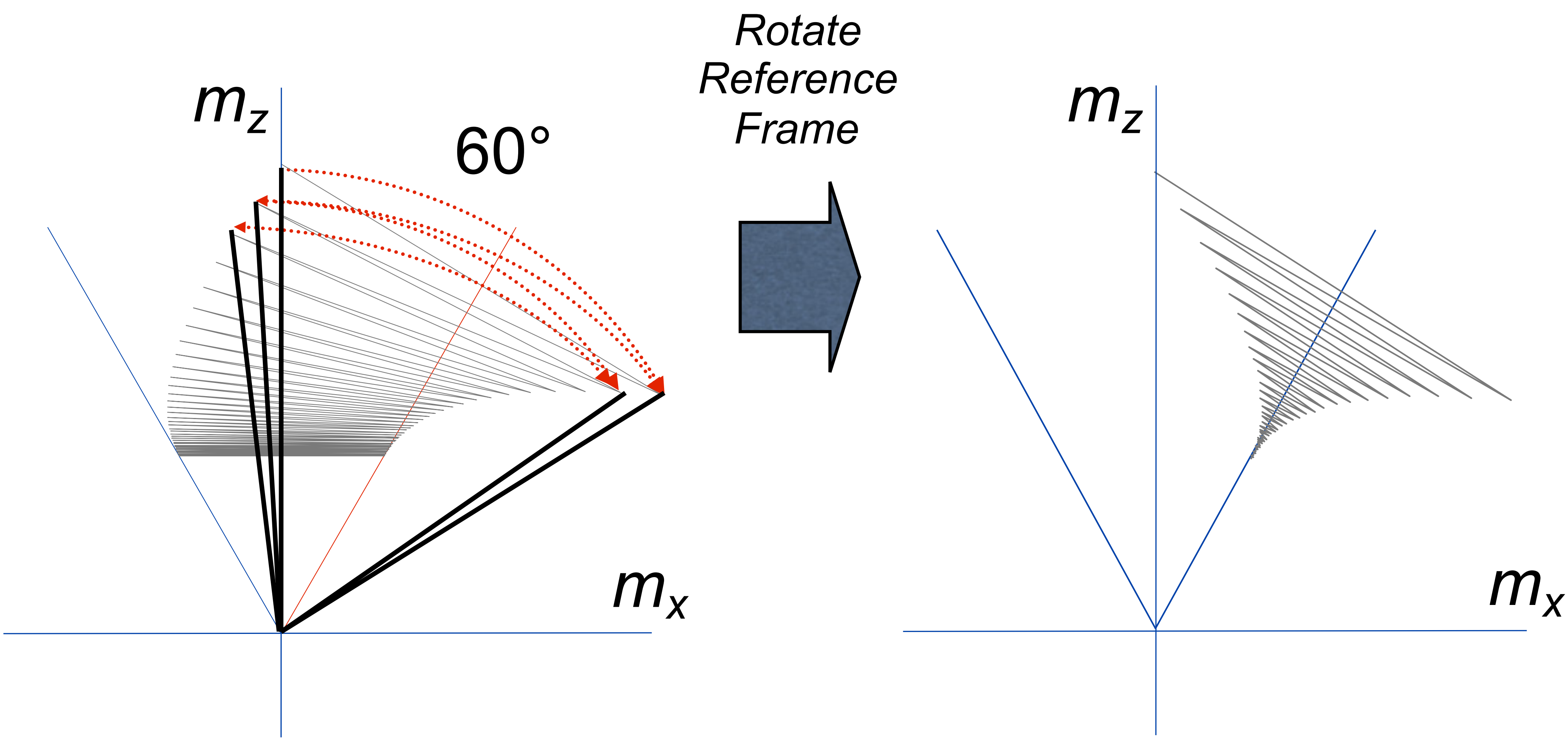
Note in [2] A is mostly rotation, B is small, so M_{ss} lies almost along the real-valued eigenvector

bSSFP: Transients and Steady States



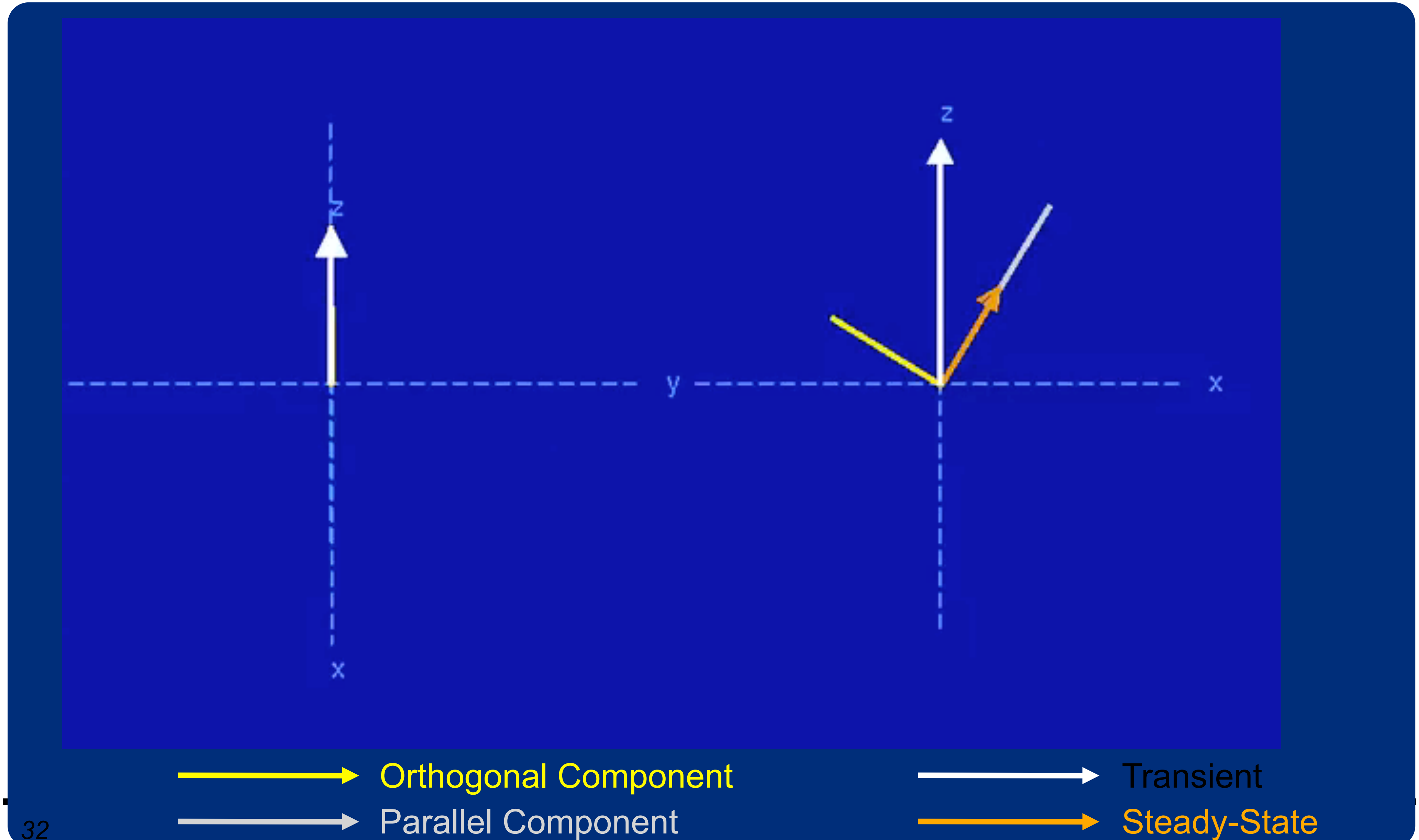
- Same periodic steady state
- Transient paths differ based on initial state

Balanced SSFP: Transient



Transient: Off-resonance

Viewed along Steady-State Vector   Orthogonal View



Transients (General)

- Generally include two components:
  - Smooth exponential (useful!)
  - Oscillatory (problematic)

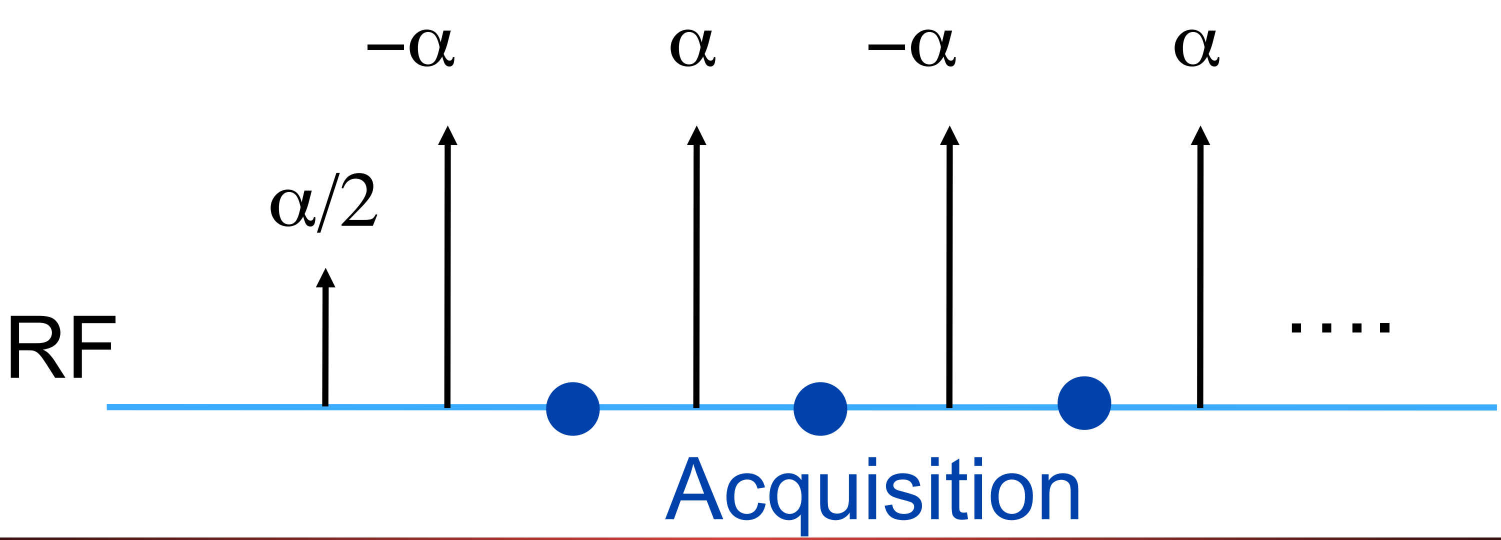
- Smooth transient is along steady-state direction
- Manipulate to steady-state direction to avoid oscillations

Review Question: Transients

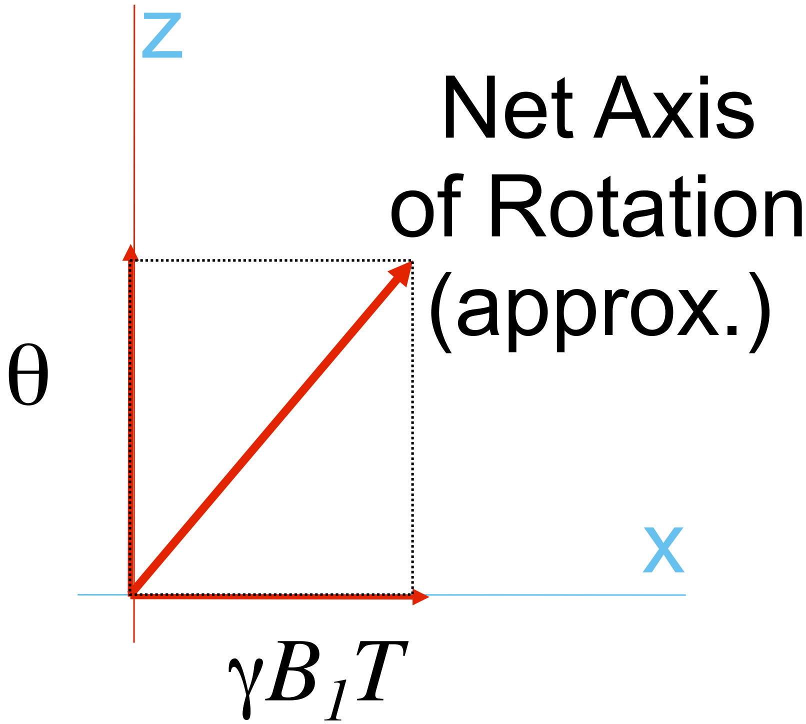
Half-TR, α/2 Setup

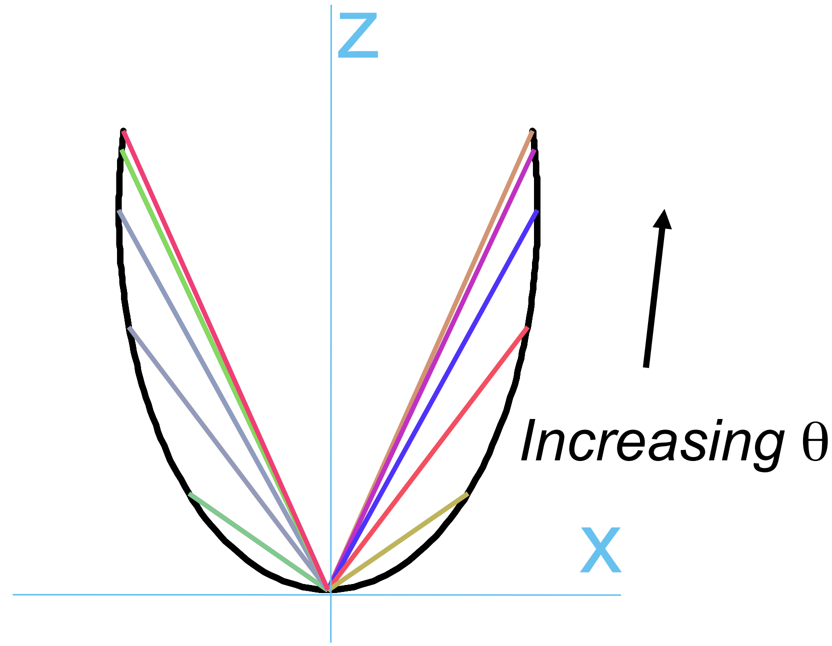
Deimling and Heid, 1994

- First RF pulse has half-amplitude
- Pulse applied TR/2 before next RF pulse
- (More complicated schemes exist)



bSSFP Direction: Some Intuition

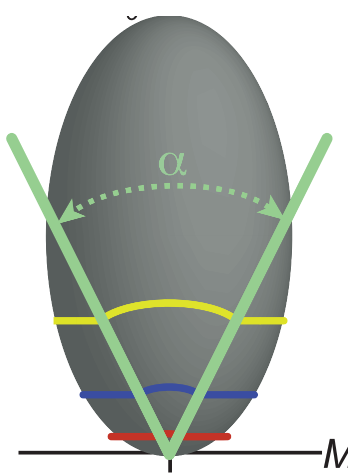


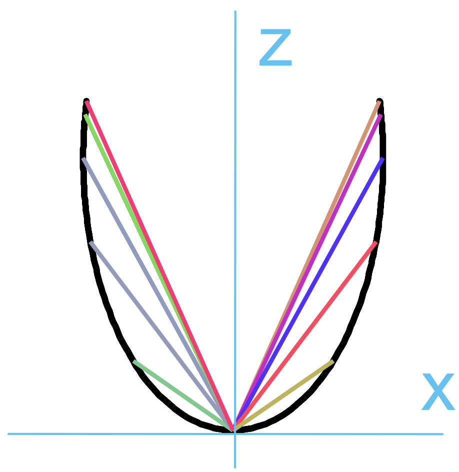


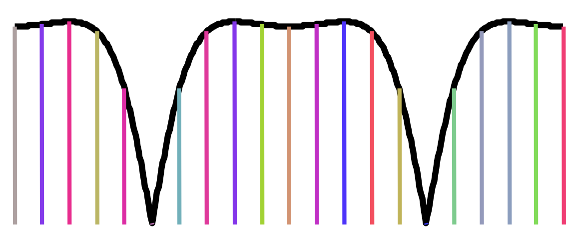
- Magnetization aligns to rotation axis
- Rotation θ includes RF phase-increment
- As θ approaches 0°, axis is transverse, signal dies out
- Negative θ means steady state on -x

bSSFP Steady-State: Summary

- Ellipsoidal distribution: shape given by T_2/T_1
- Path depends on flip angle and precession
- Signal very sensitive to resonant frequency





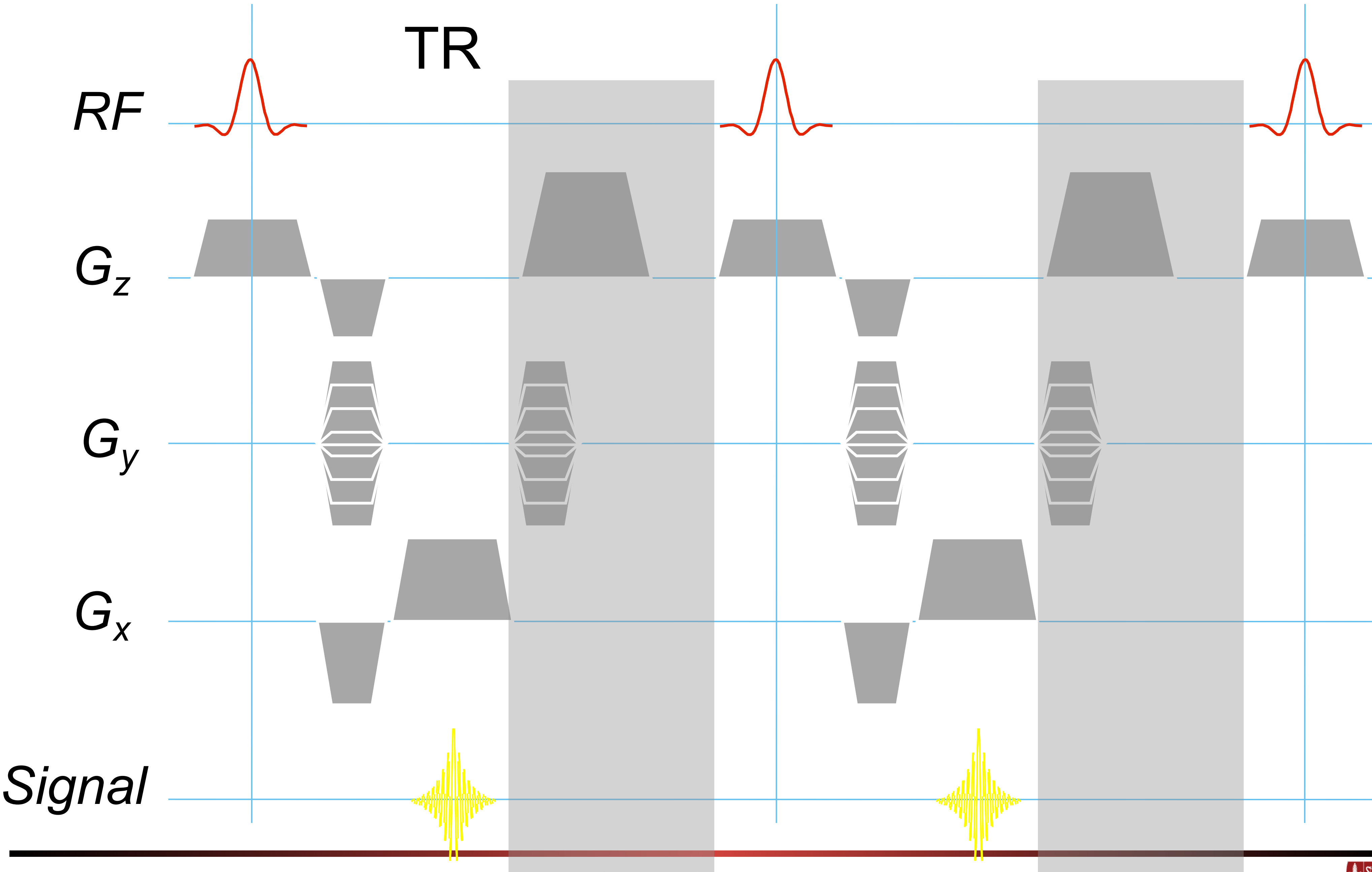


Gradient Spoiling

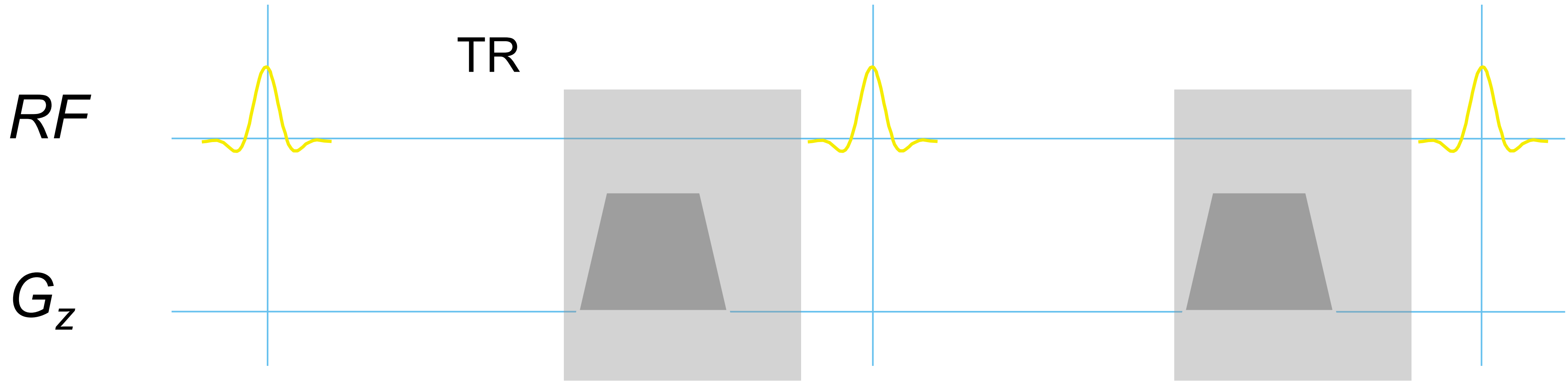
- Average balanced SSFP magnetization
- Reduce sensitivity to off-resonance

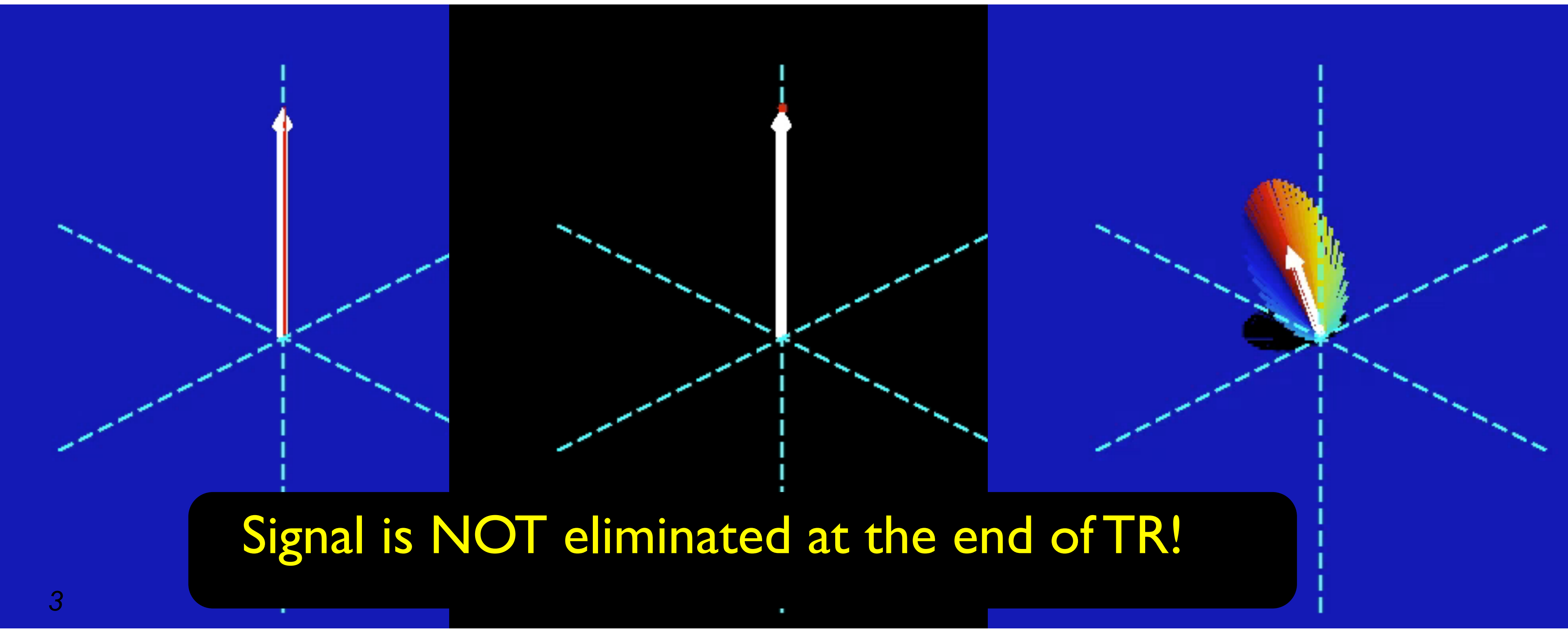
FFE, FISP, GRASS, GRE, FAST, Field Echo

Gradient-Spoiled Sequence (GRE, FFE, FISP, GRASS)

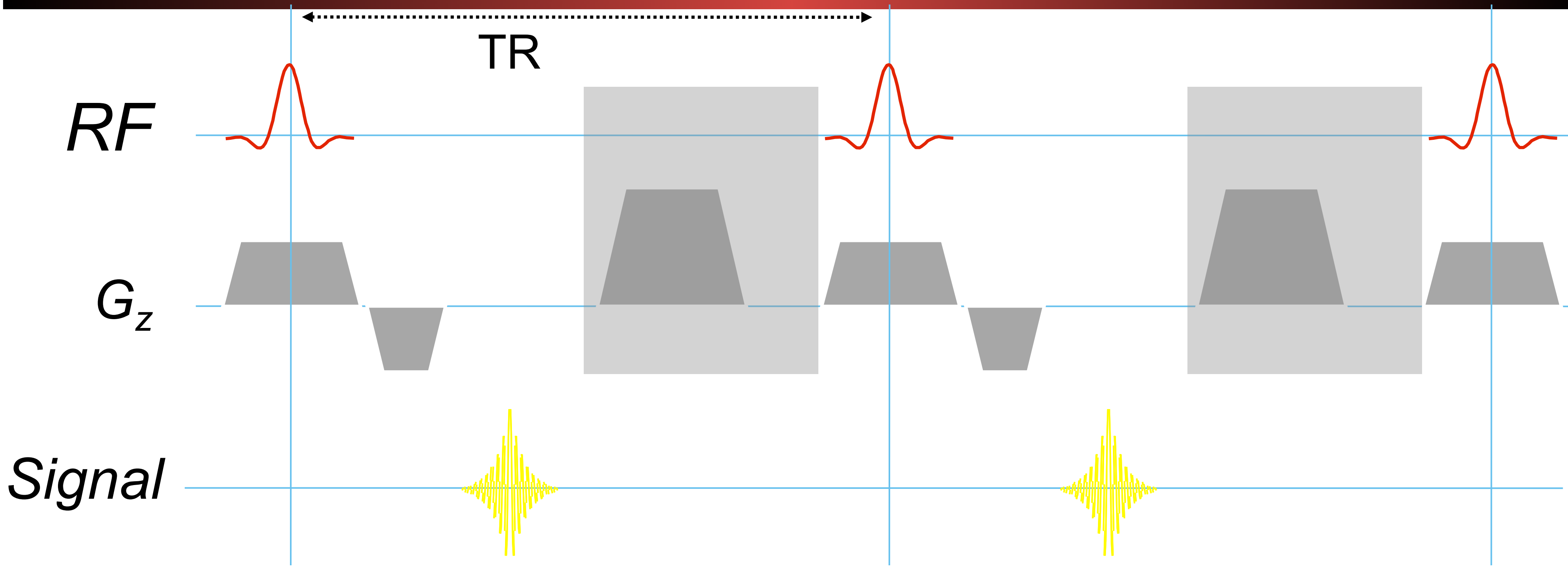


Gradient Spoiling



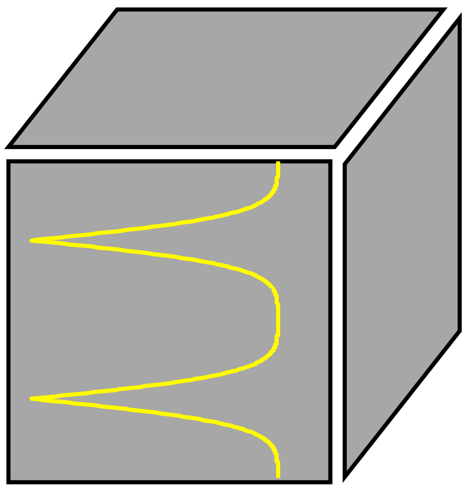


Gradient Spoiling



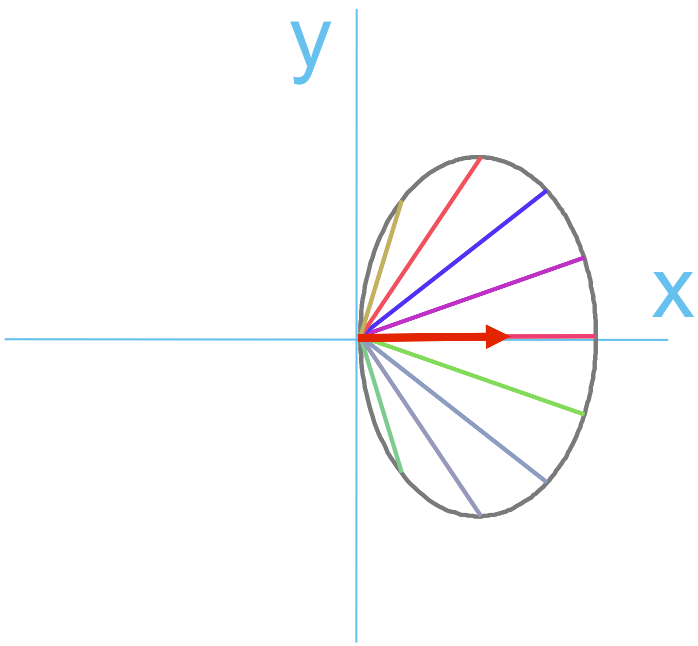
Precession across a voxel dominated by spoiler:

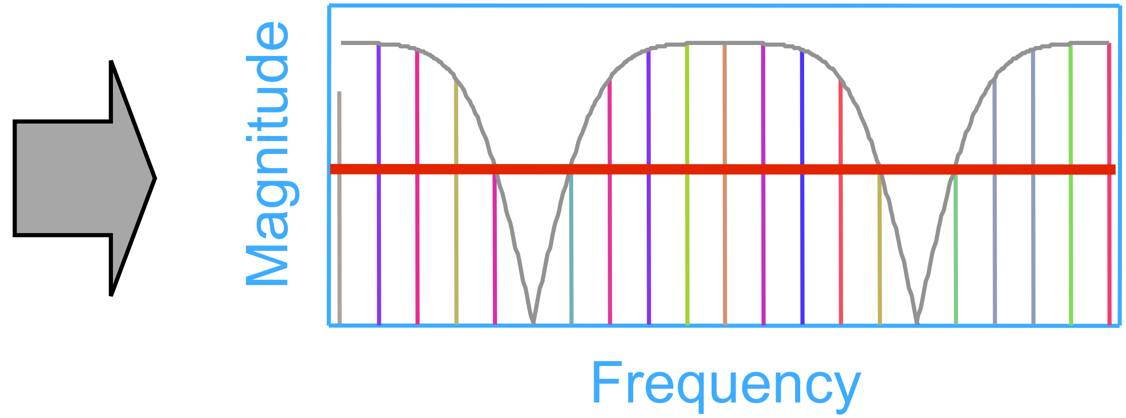
- Each spin has a different precession
- Average of balanced SSFP



Gradient Spoiled Signal

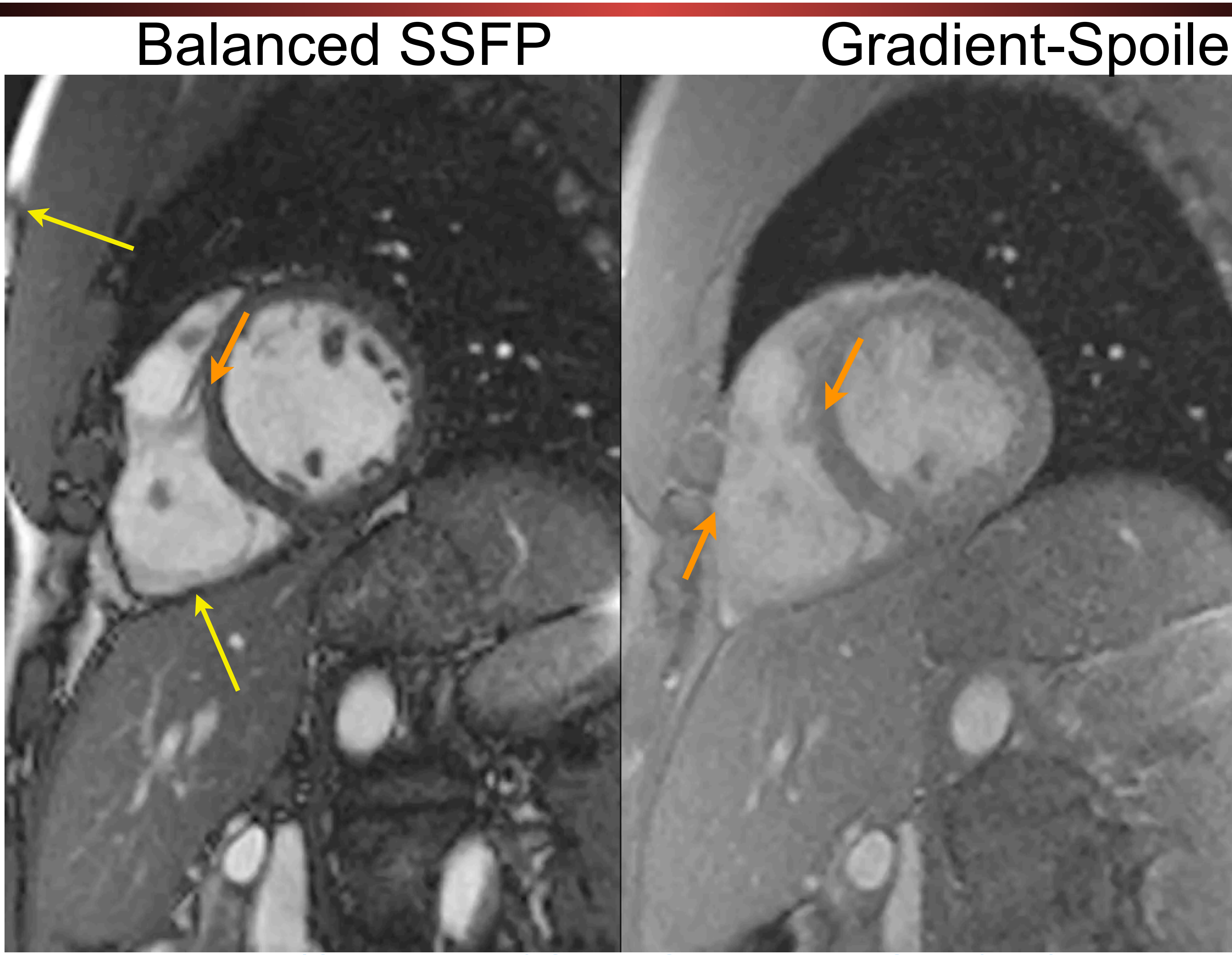
Spin Distribution





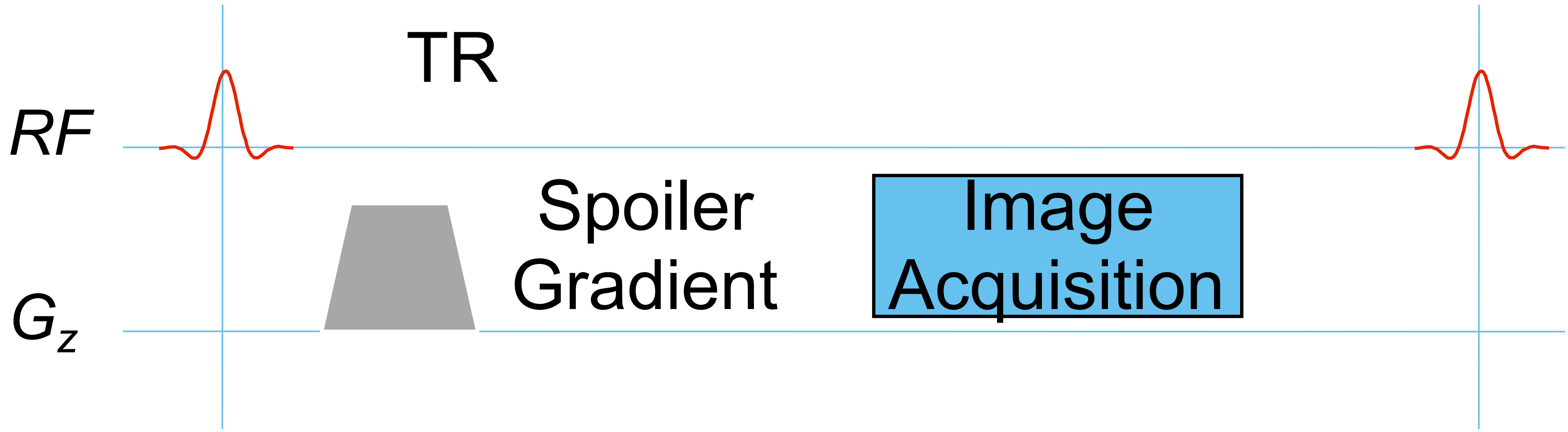
- Lower signal than balanced SSFP
- Flat signal vs. frequency profile
- No dark band artifacts!

Gradient Spoiled vs Balanced SSFP



(Courtesy of Suba Srinivasan, Stanford)

Reversed Gradient Spoiling

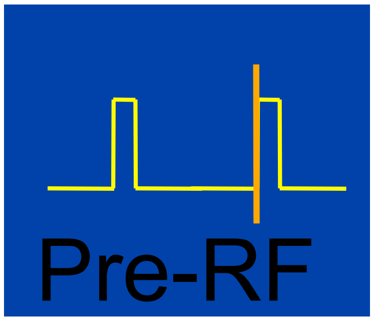


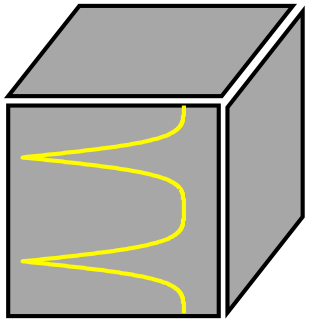
Same as gradient-spoiling, but

- Precession before imaging

  (SSFP Signal at t=TR)

- Some T_2 contrast

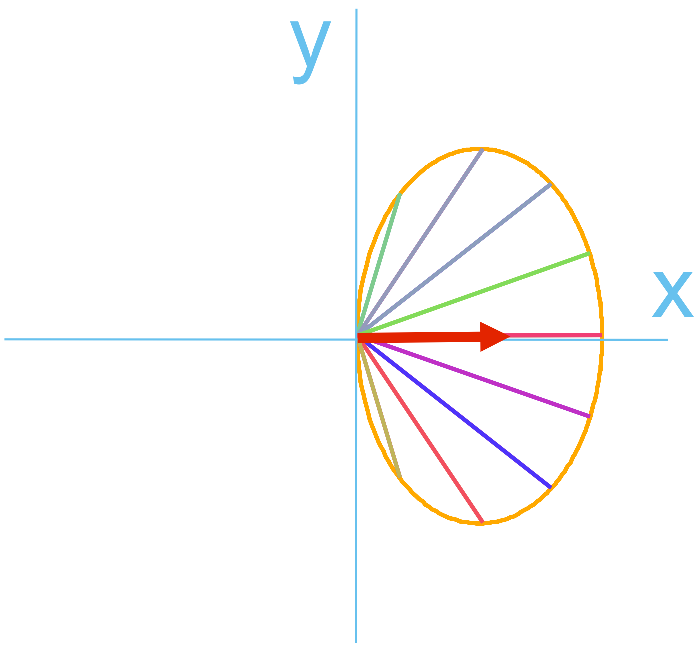


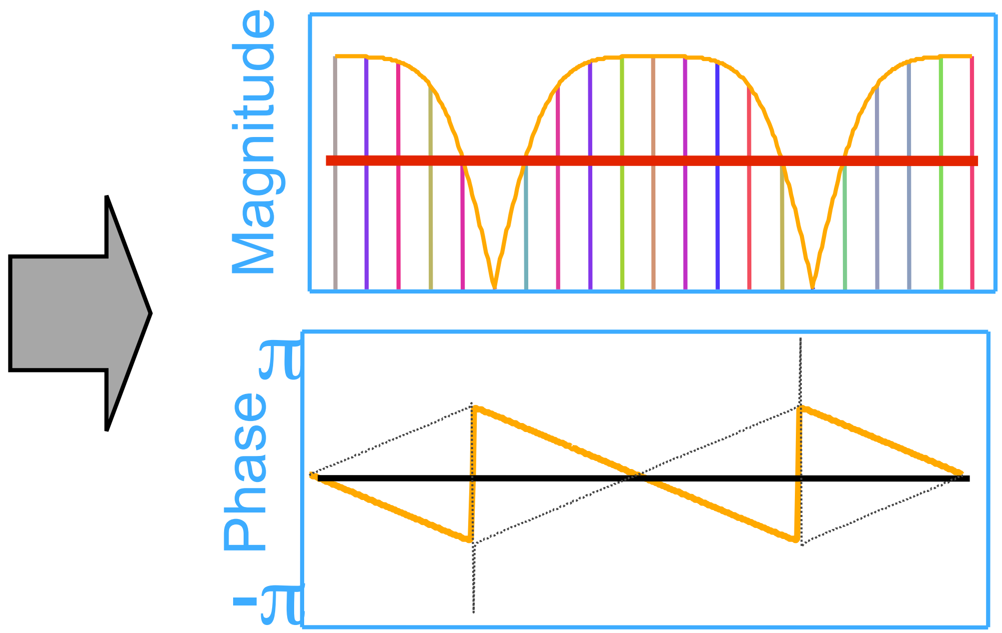


Reversed Gradient Spoiled Signal

Spin Distribution

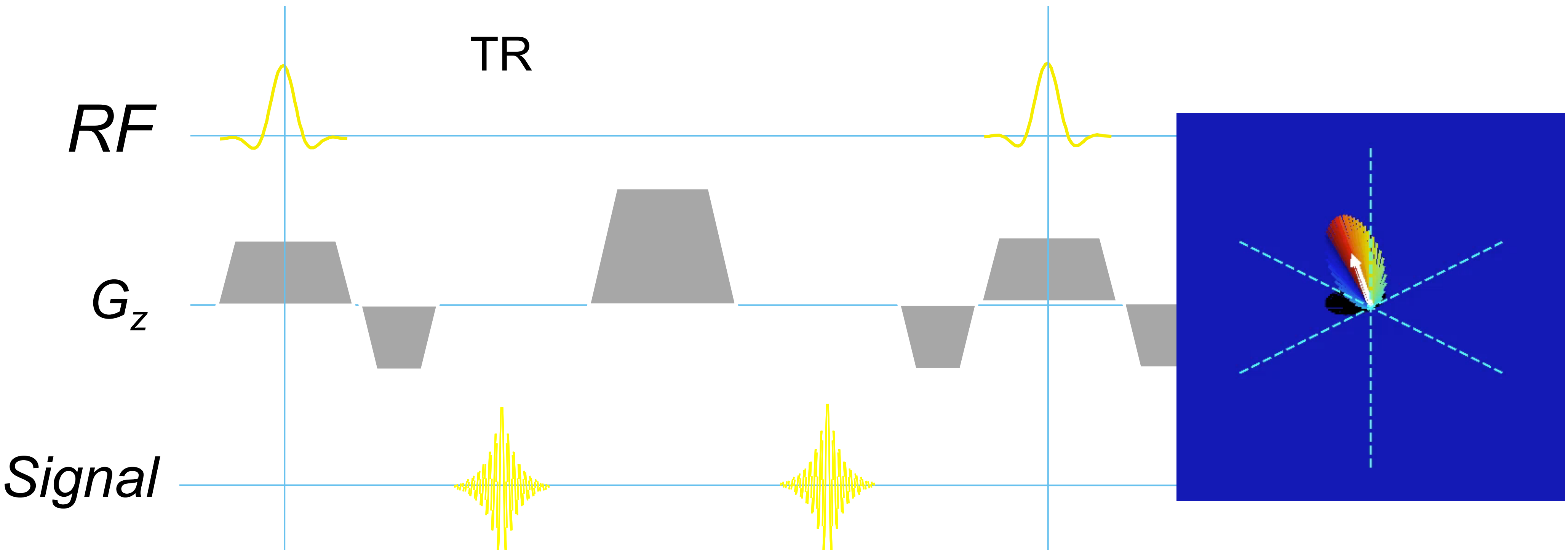
Signal vs Balanced SSFP

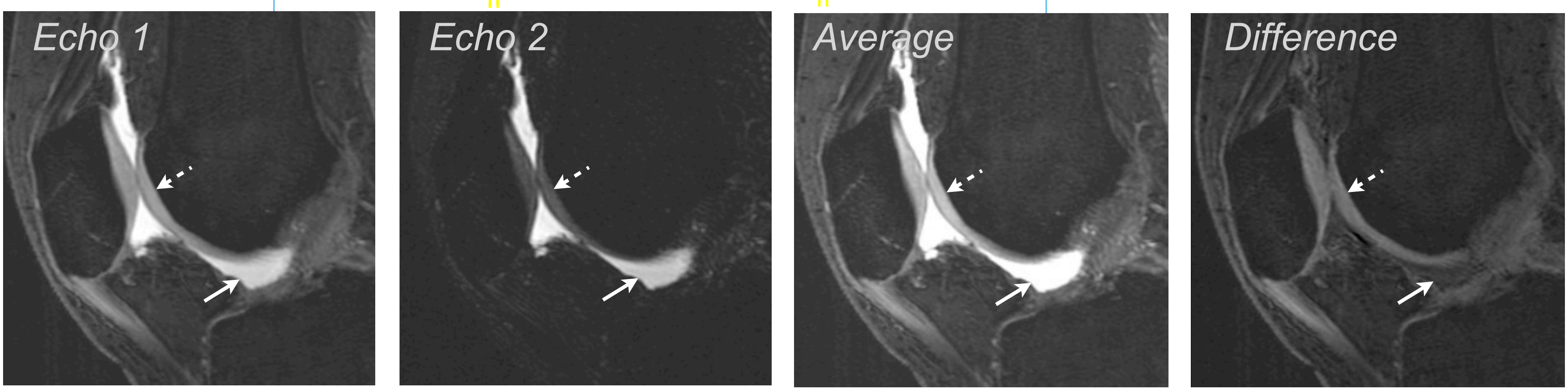




- Almost identical signal to gradient-spoiled imaging
- Flat signal vs. frequency profile

Double Echo Imaging: DESS/FADE





Gradient-Spoiler Question

Gradient-Spoiler Question (cont)

Summary: Gradient Spoiling

- Average balanced SSFP magnetization
- Reduce sensitivity to off-resonance
- Unbalanced gradients - more motion sensitivity

FFE, FISP, GRASS, GRE, FAST, Field Echo,

T2-FFE, PSIF, CE-FAST

FADE, DESS

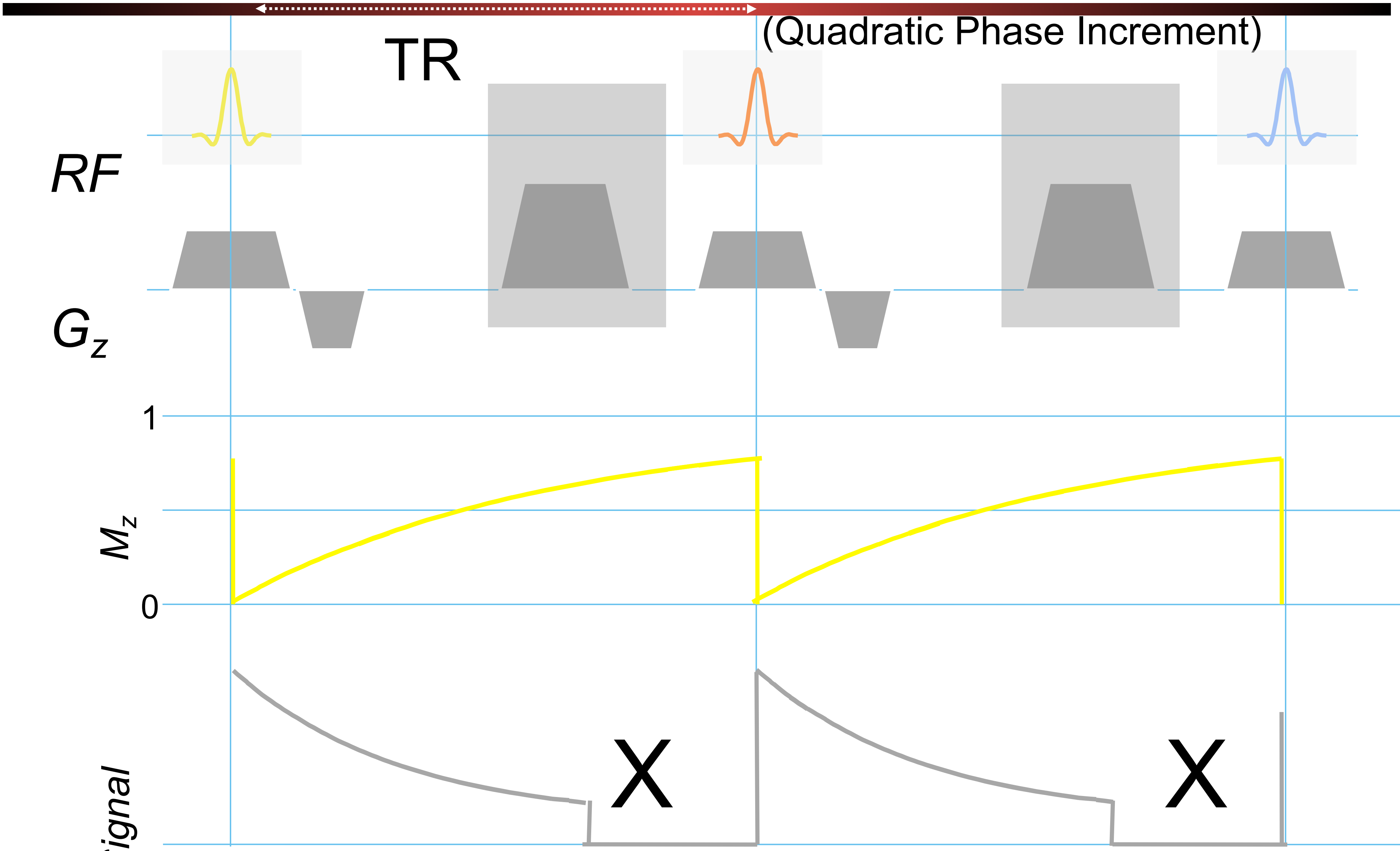
RF Spoiled Imaging

- Goal: Pure T_1 contrast with short TR
- Fast, 3D T_1-weighted imaging
- Need to "zero" M_{xy} at end of TR

SPGR, FLASH, T1-FFE, RF-spoiled FAST

Frahm 1987, Zur 1991

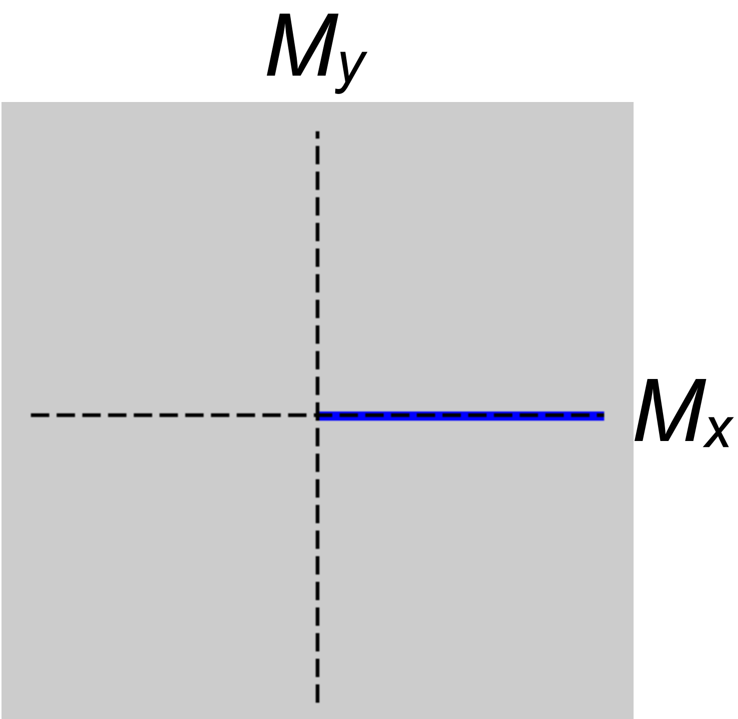
RF-Spoiled Gradient Echo

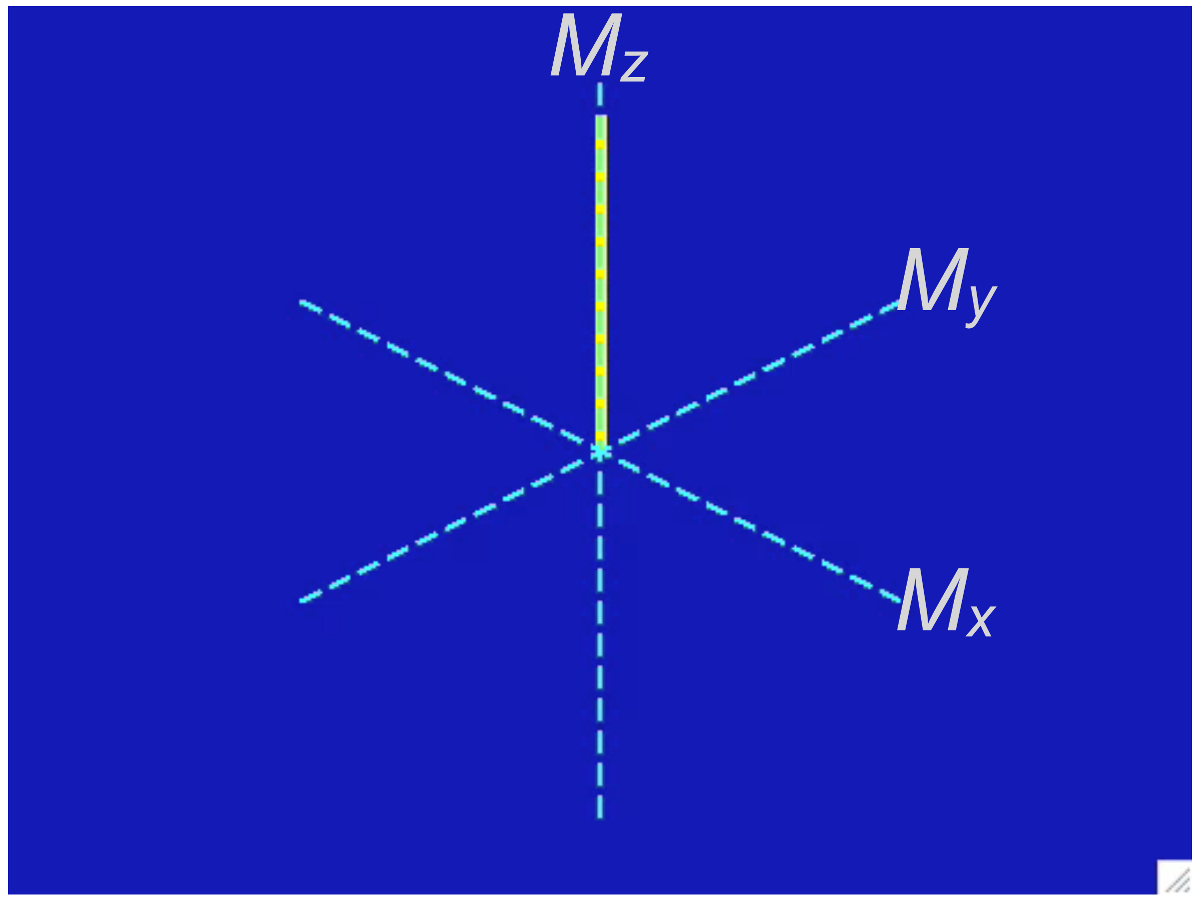


Eliminate transverse magnetization: T1 contrast

Quadratic Phase RF

- RF axis of rotation, not flip angle!
- RF phase is “randomized”





RF Spoiling

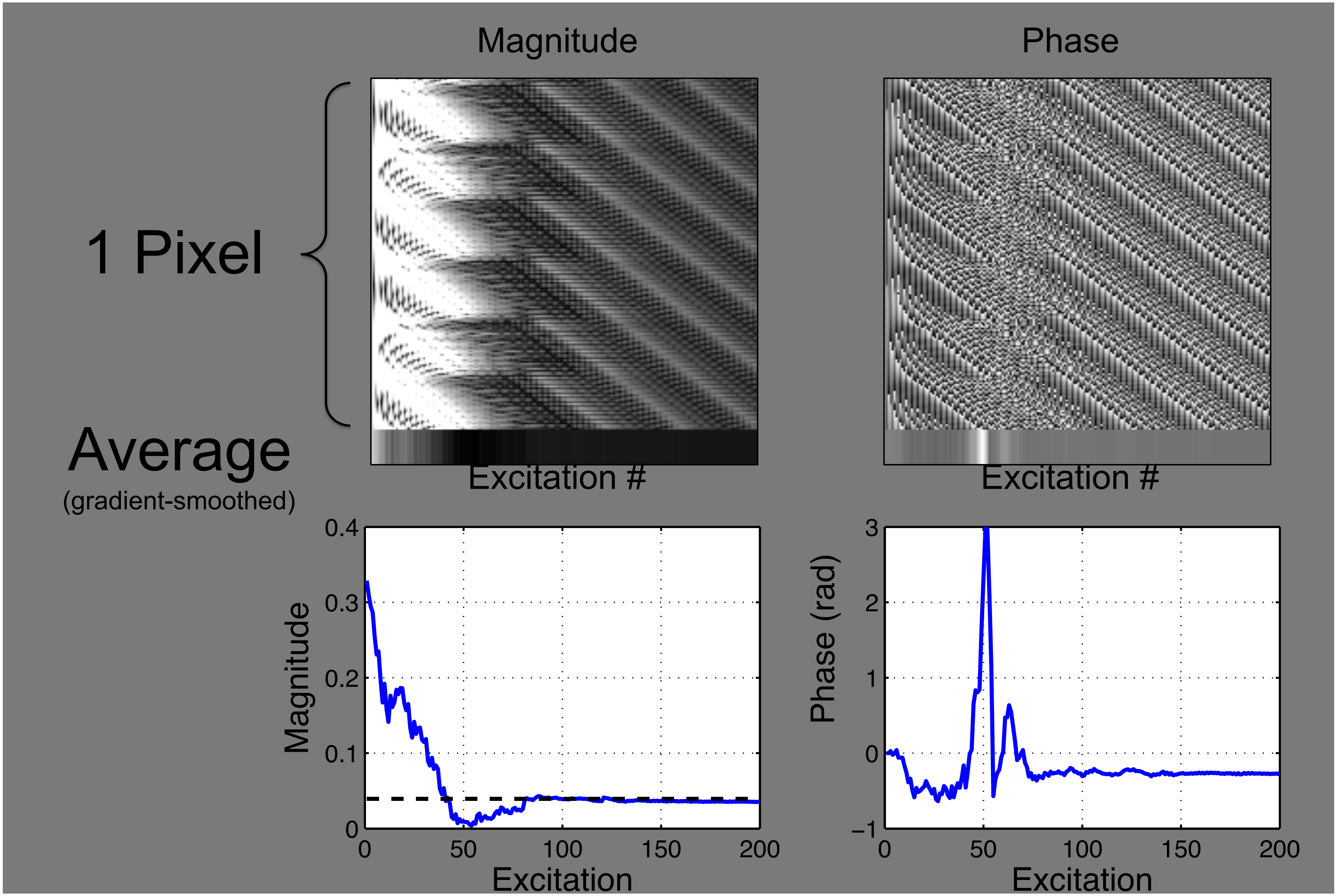
Frahm 1987, Zur 1991

- Goal: *Eliminate* transverse magnetization
- Quadratic phase increment with gradient spoiling:

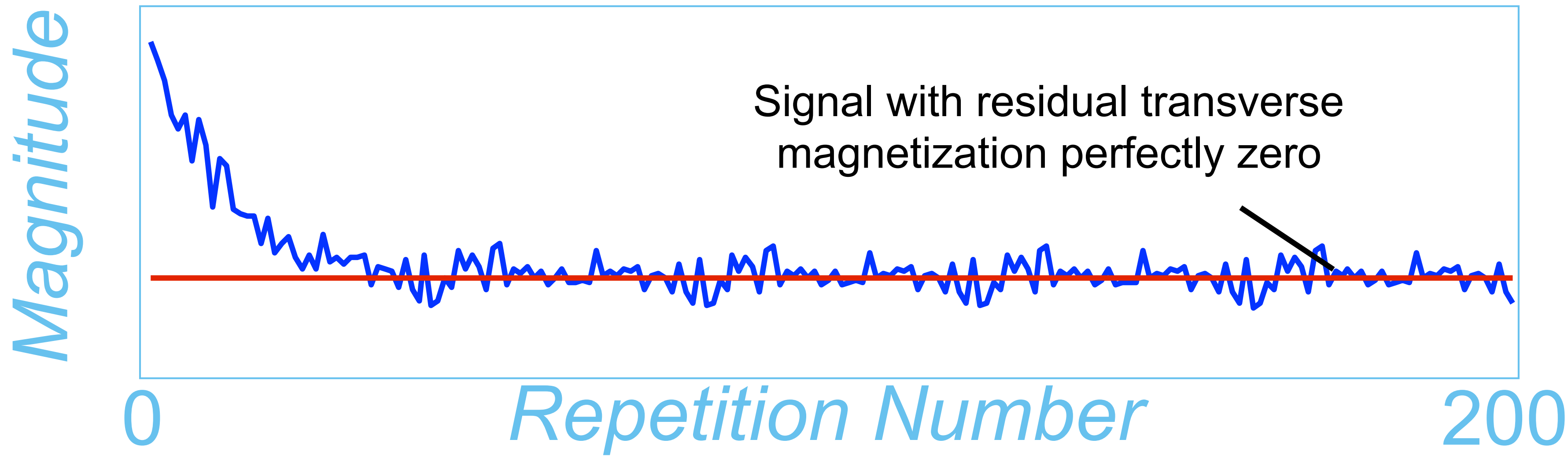
$$\phi_k = (0.5)117^\circ\, k^2$$

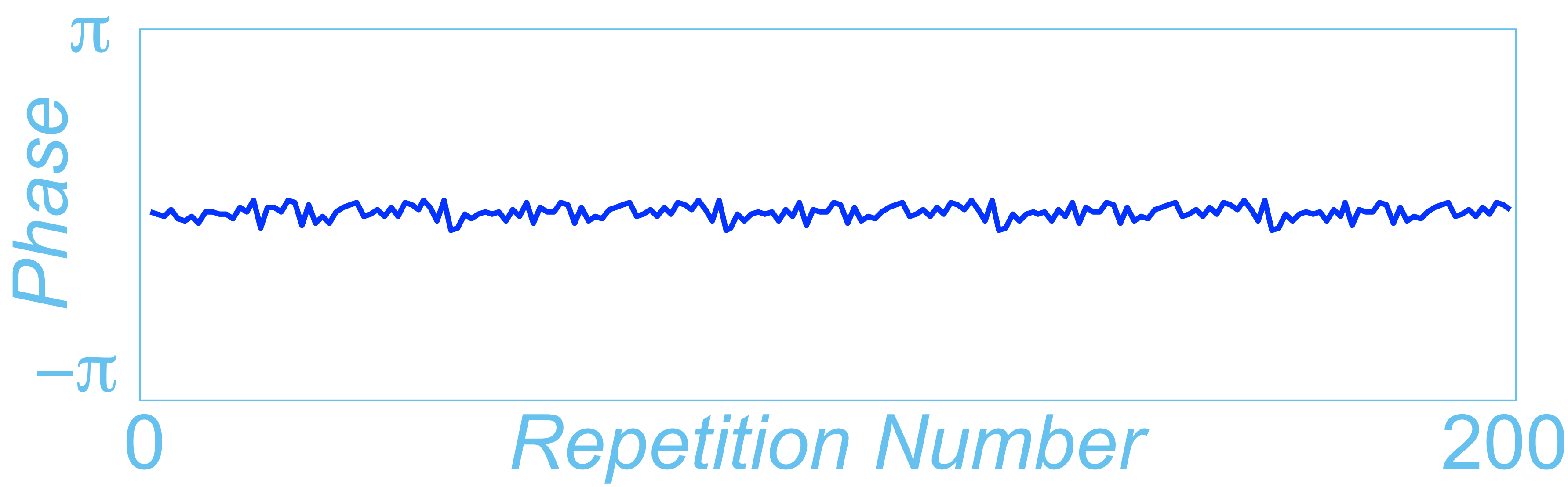
- Shifting, spoiled profile
- Transverse magnetization “cancels”
- T_1 contrast

SPGR Signal Evolution (Bloch)

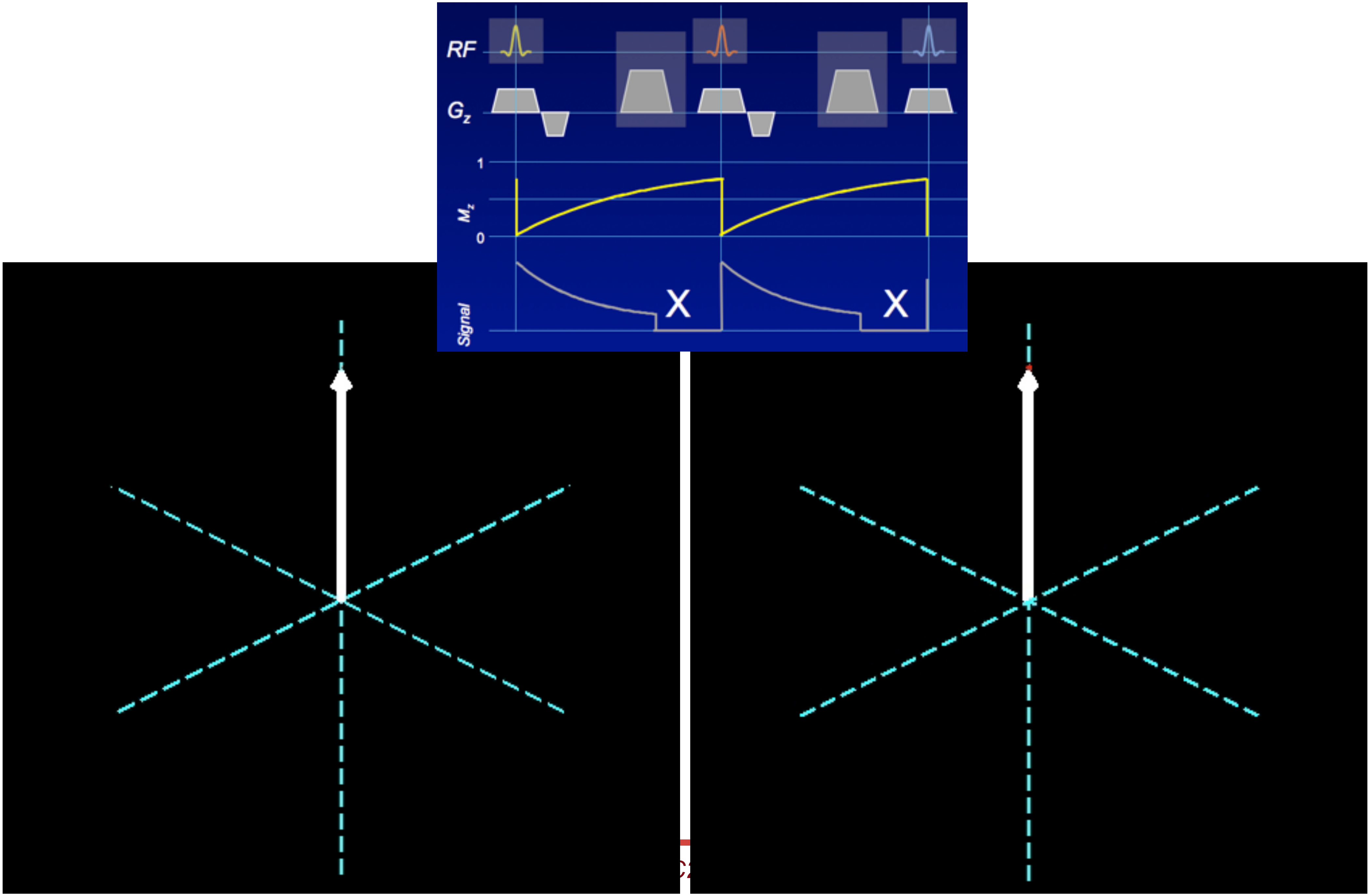


RF Spoiled Signal





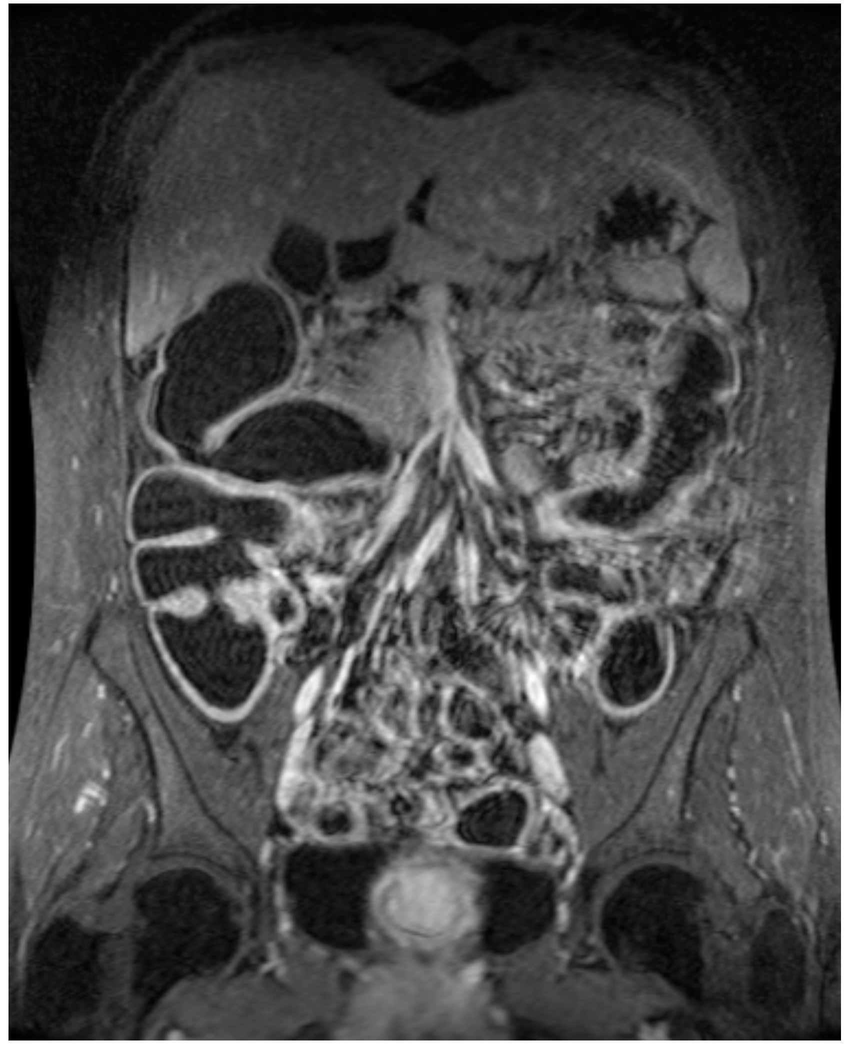
RF-Spoiled Gradient Echo



RF-Spoiled Contrast-Enhanced MR

Pre-Contrast SPGR

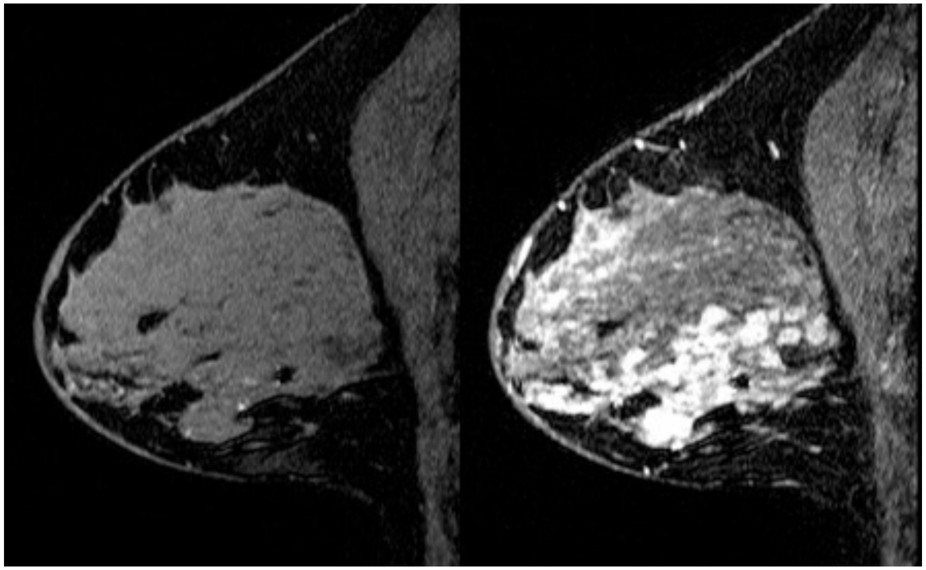
Post-Contrast SPGR



Courtesy Lewis Shin

RF Spoiling: Summary

- Gradient spoiling + Quadratic phase RF
- "Eliminates" transverse magnetization
  - Lower signal than GRE or balanced SSFP
  - Pure T_1 contrast

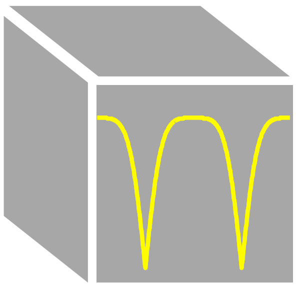


Gradient Echo Sequence Comparison

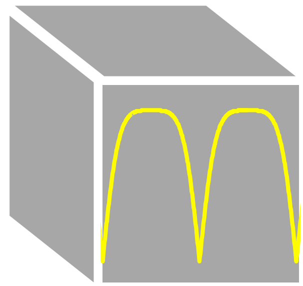
Sequence	Balanced SSFP	Gradient Spoiled	RF-Spoiled
Spoiling	None	Gradient	RF + Gradient
Transverse Magnetization	Retained	Averaged	Cancelled
Contrast	T_2/T_1	T_2/T_1	T_1
SNR	High (but Banding)	Moderate	Lower

Just for Fun… Putting it Together!

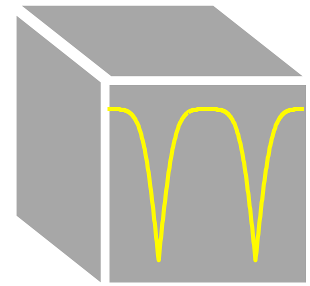
Gradient-
Spoiled

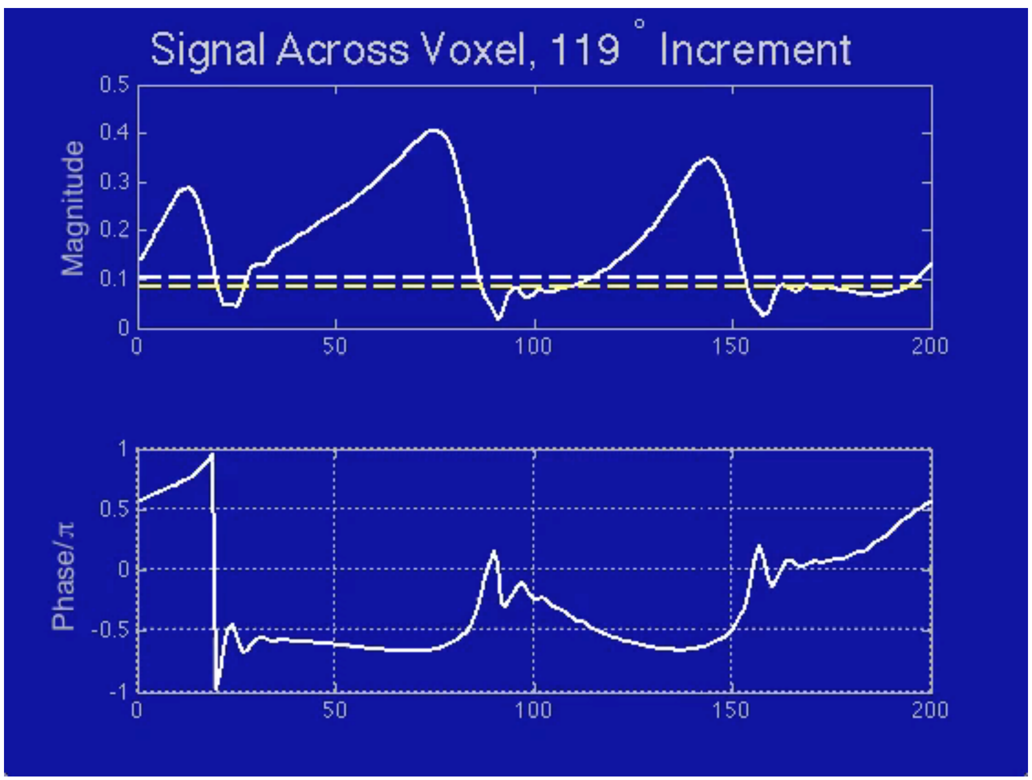


Constant RF
Phase Increment



Quadratic Phase
(Increasing Phase
Increment)





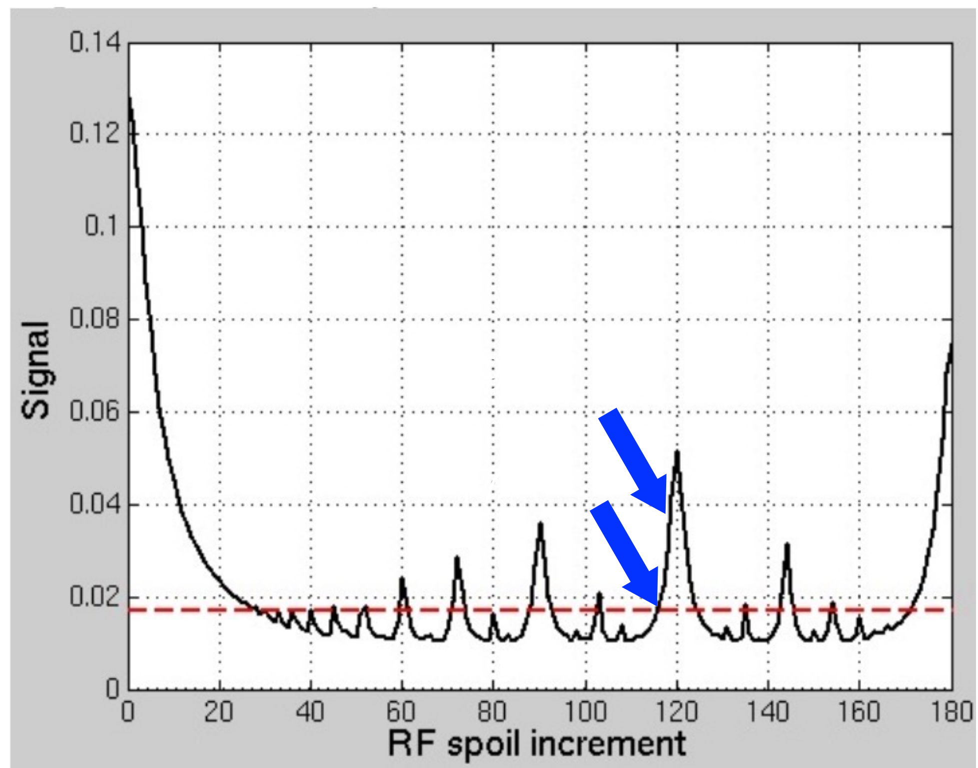
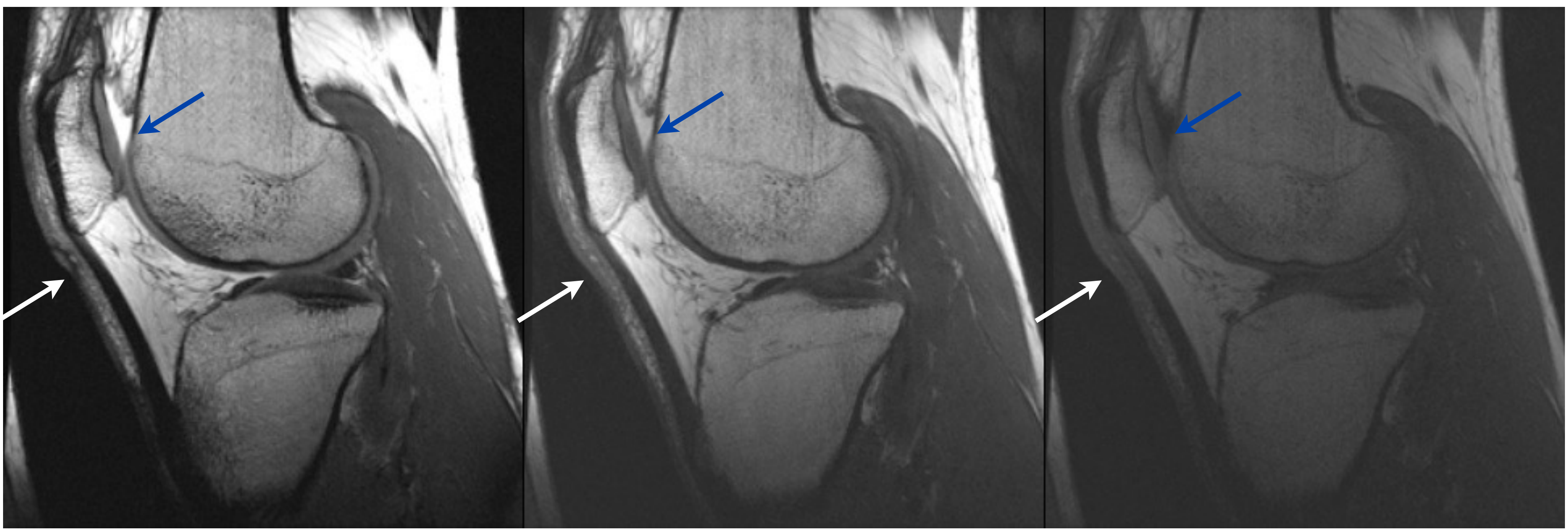
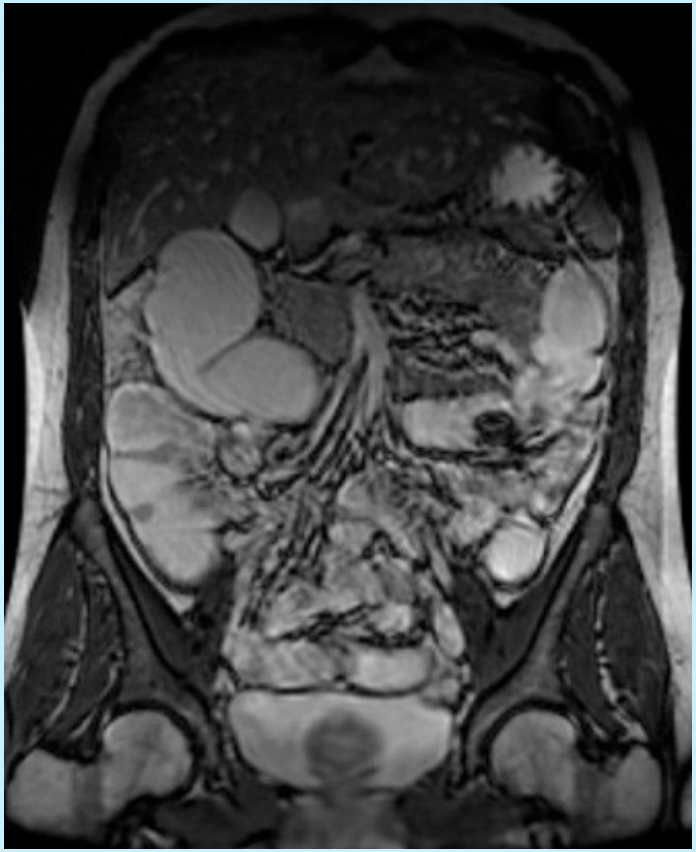


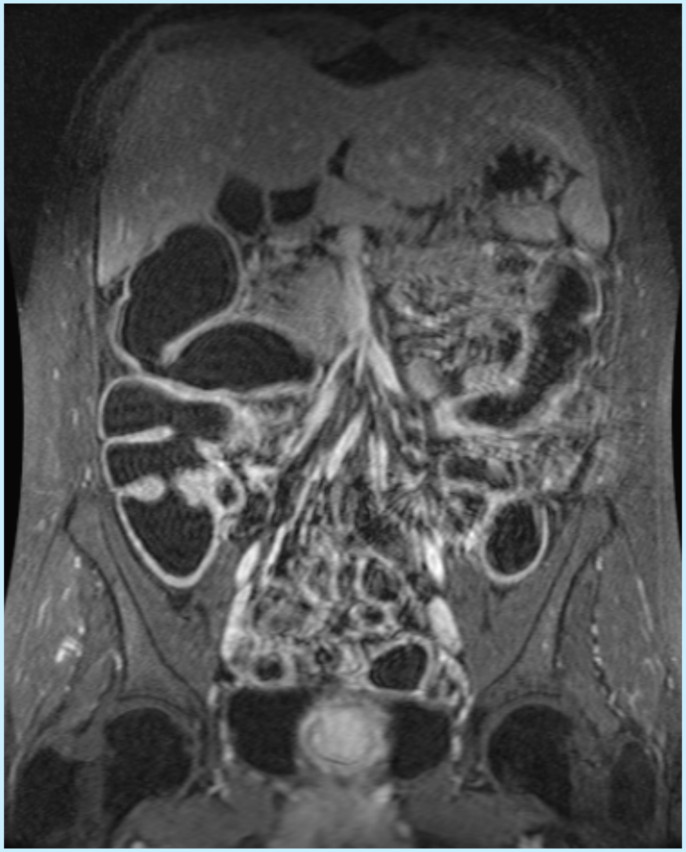
Image Comparison

Balanced SSFP Gradient Spoiled RF Spoiled



Your Turn...

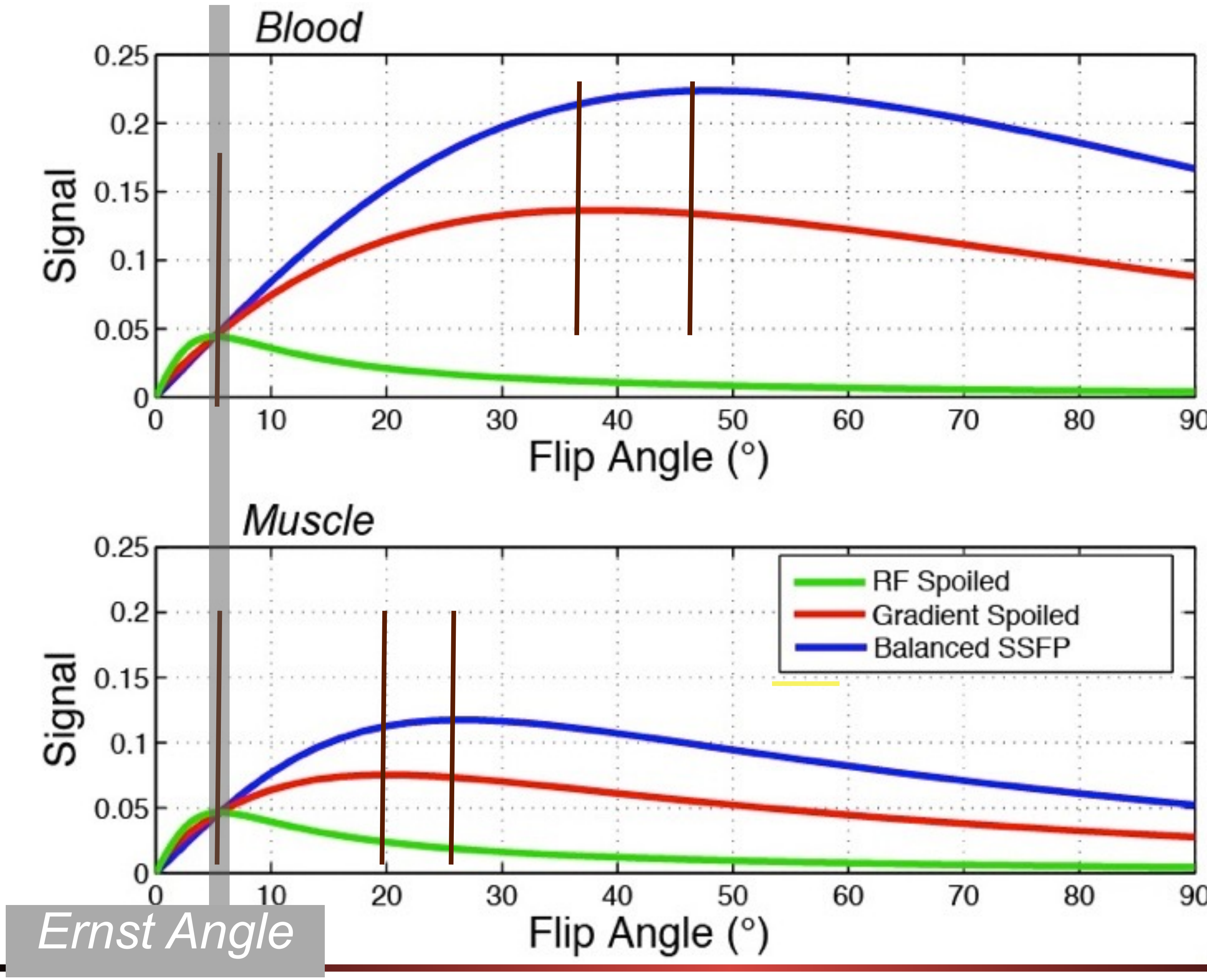




Your Turn...

An RF-Spoiled and balanced SSFP image are shown. Which is on the right?

Flip Angle Selection

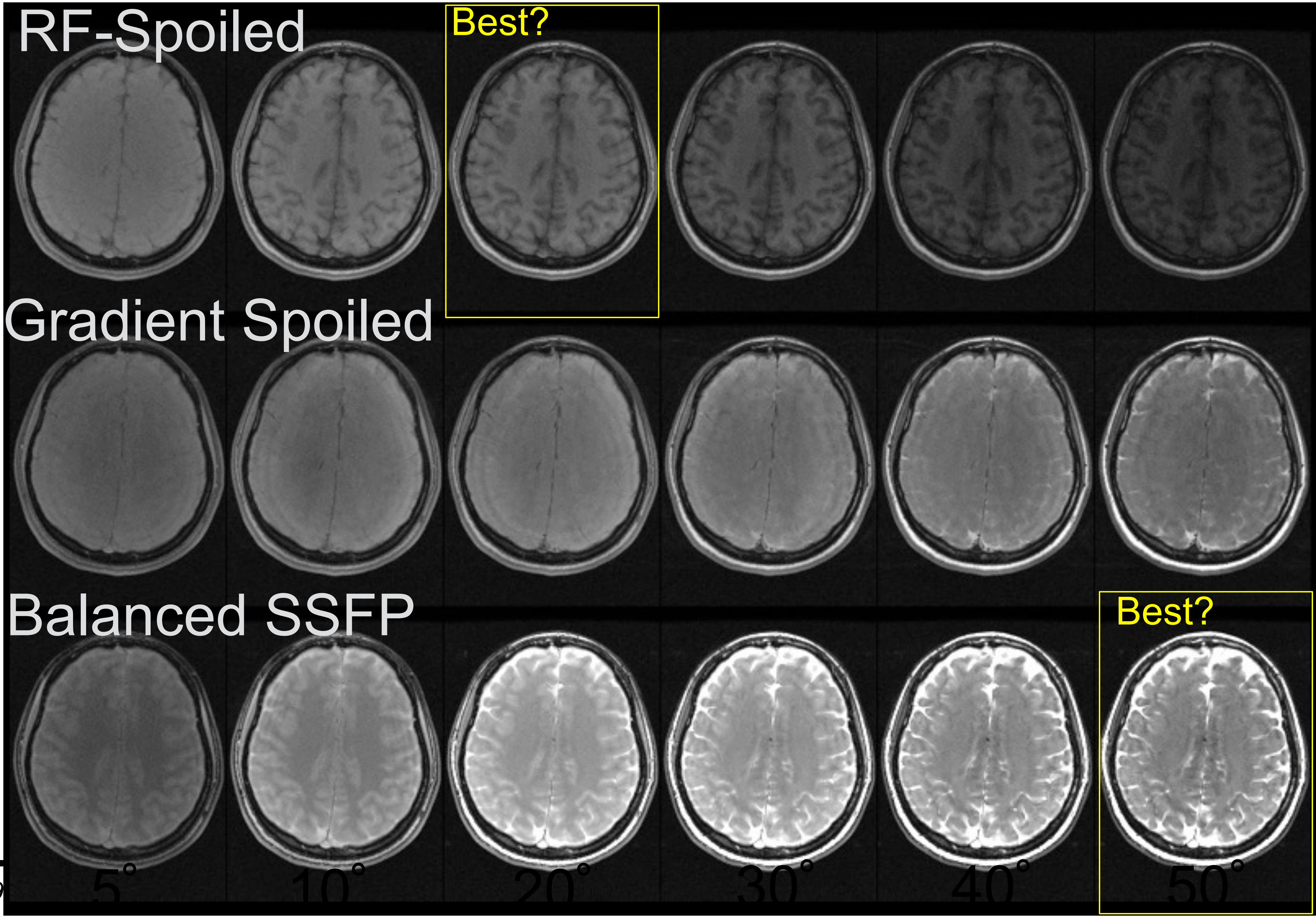


Buxton 1990

Flip Angle Selection?

The best flip angle to use is found by:

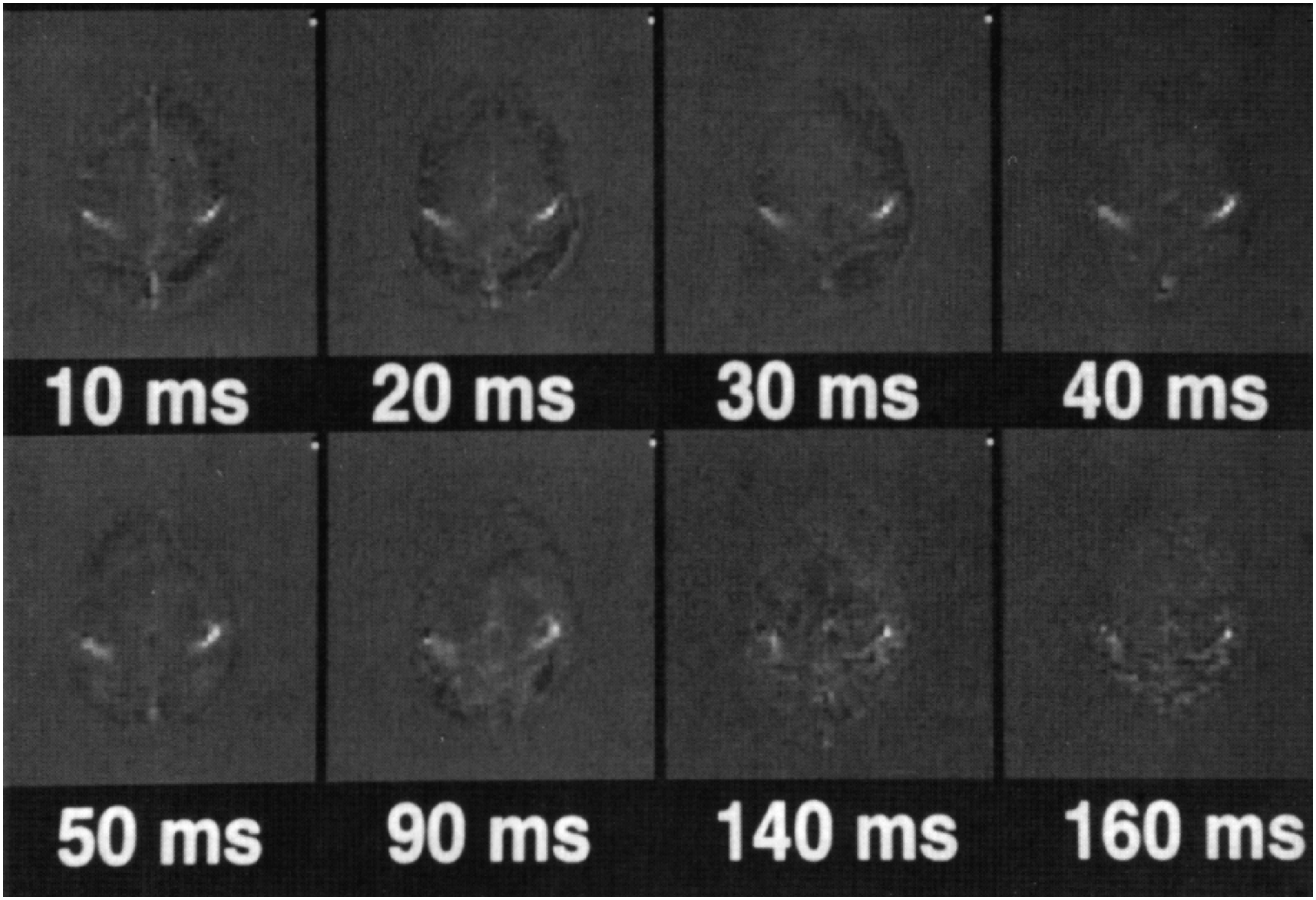
Flip Angle Examples



Echo Time (TE) Considerations

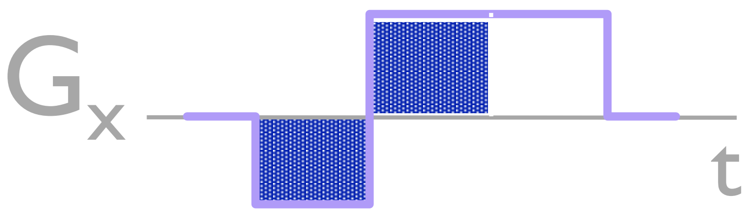
- Longer TE: T2* weighting (BOLD, Perfusion)
  - BOLD Imaging for fMRI
  - T2*-weighted perfusion
- Short TE
  - Reduced flow/motion sensitivity
  - Reduced T2* weighting
- In-phase and Out-of-phase TE
  - Water/Fat cancellation, Dixon Imaging

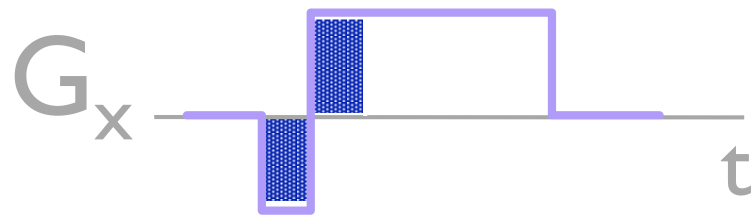
Blood-Oxygen Level Dependent (BOLD) Imaging

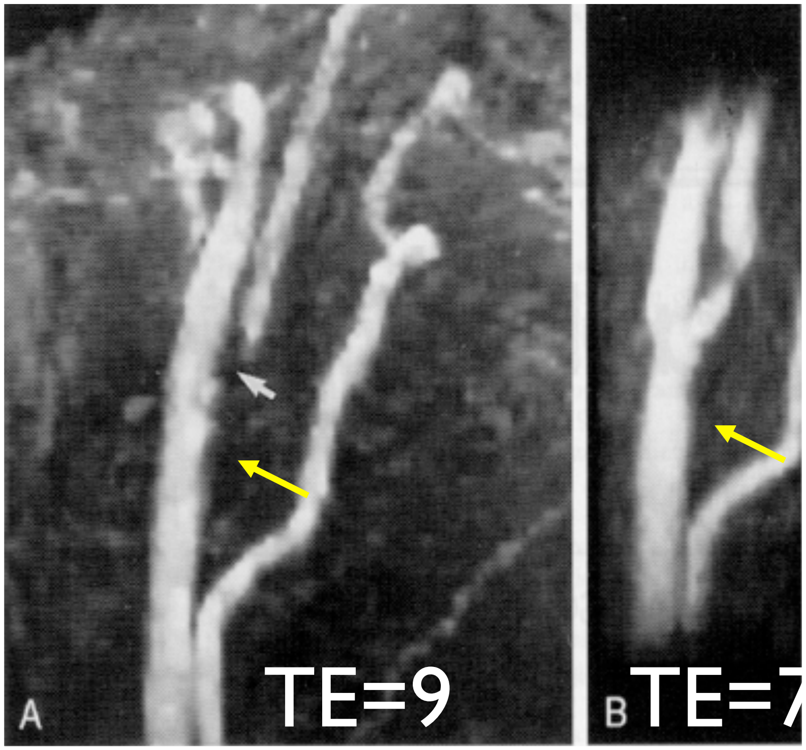


Bandettini, et al. NMR Biomed (1994)

Echo Time Reduction



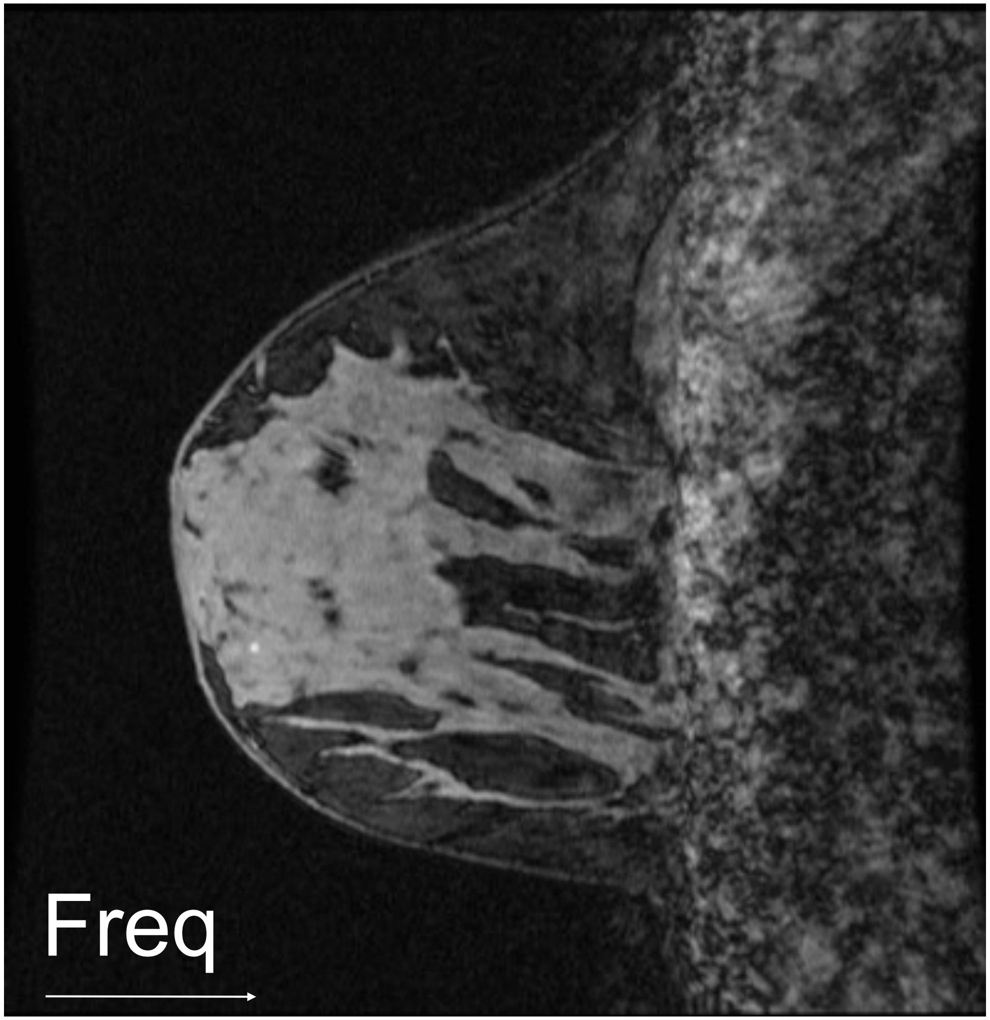




Clinical MRI, ed. Edelman, R et al., W.B. Saunders Co., 1996

Short TE to Reduce Artifacts

- Fractional TE reduces motion and flow sensitivity

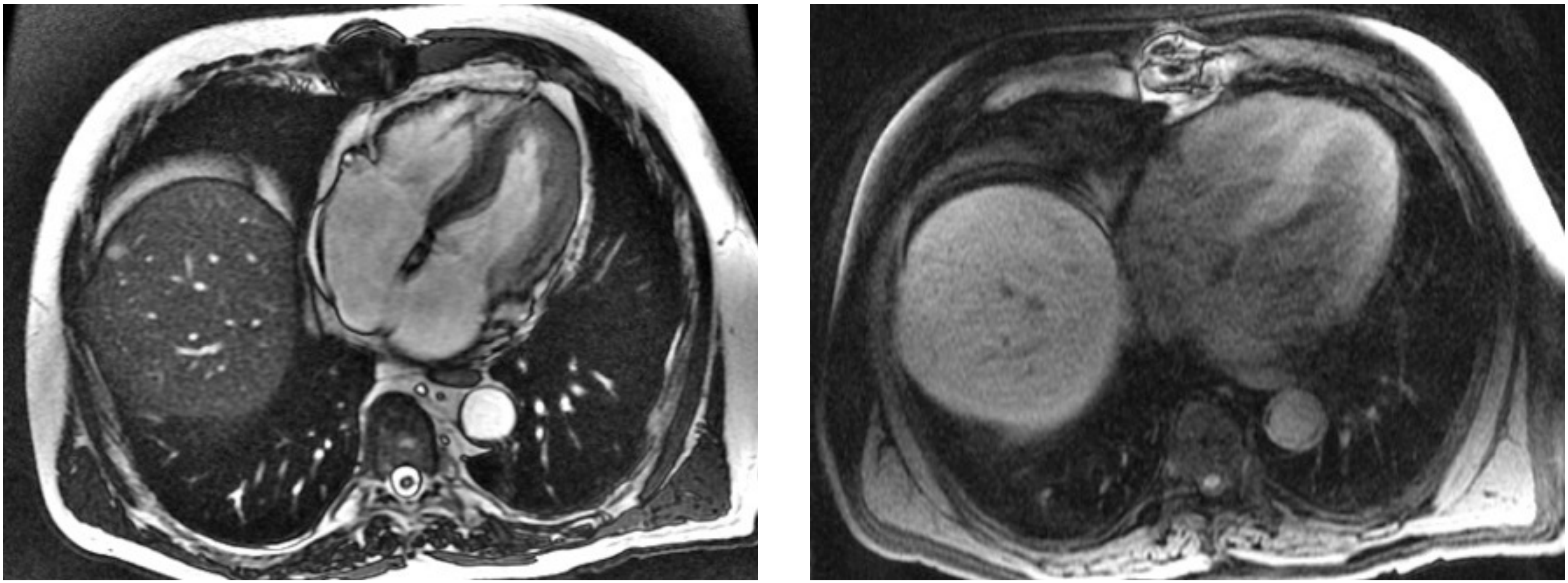


Full TE

Fractional TE

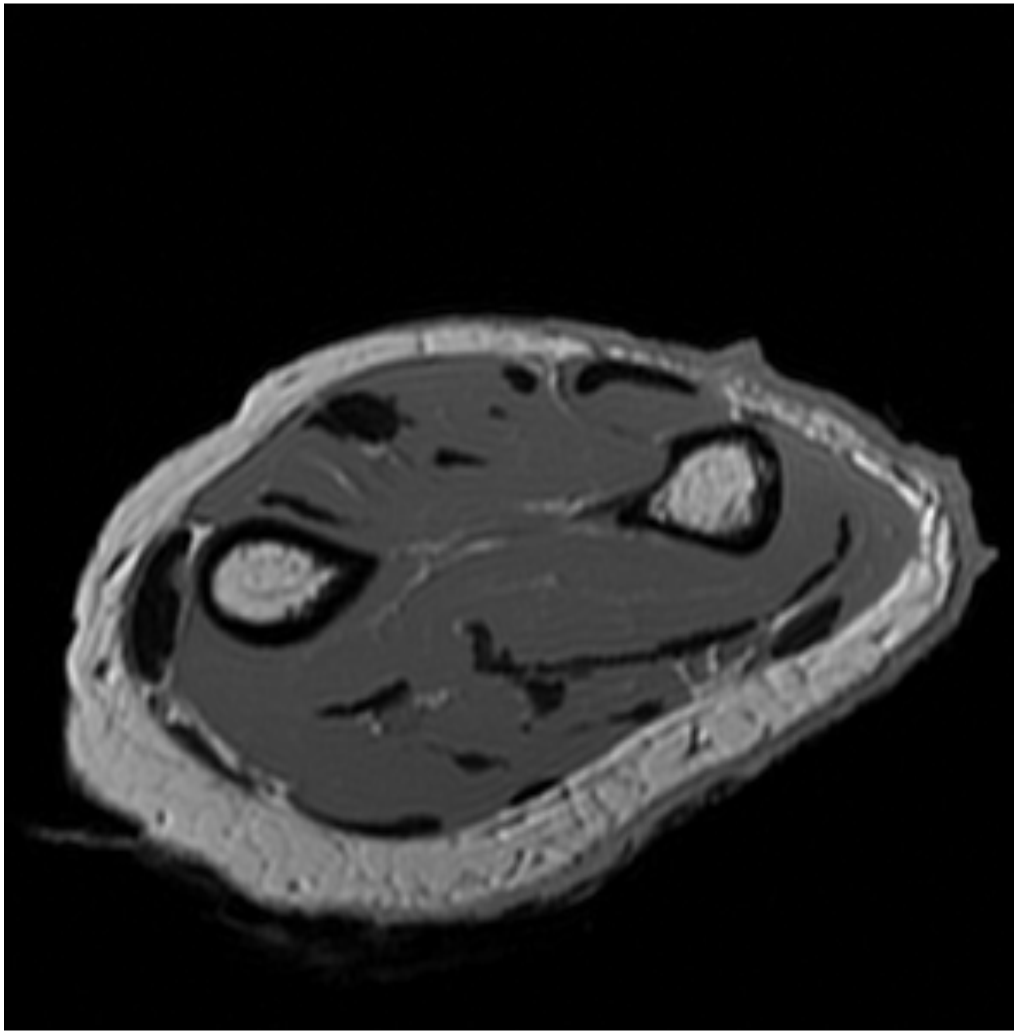
TE and balanced SSFP

- SSFP signal is refocused at TE = TR/2
- Fat / water are often opposed-phase
- SSFP is naturally flow-compensated

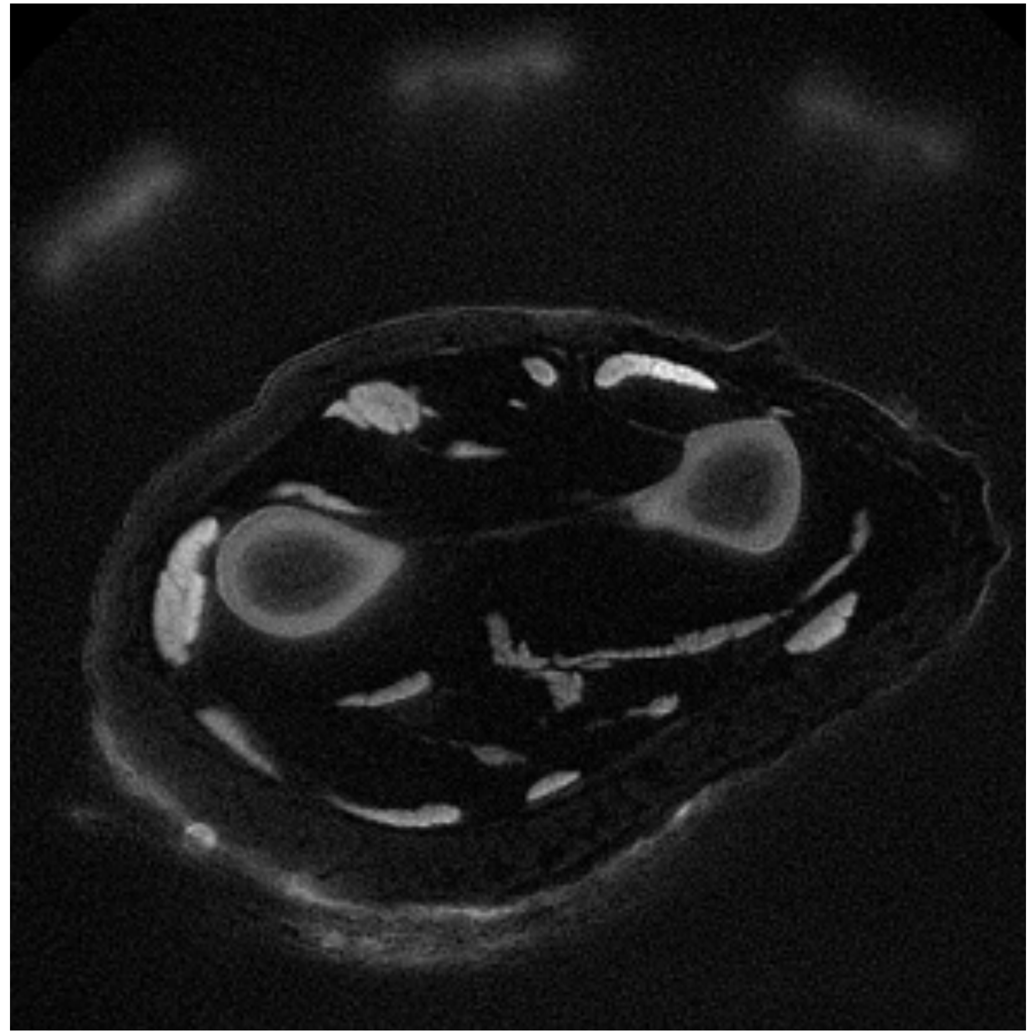


Balanced SSFP RF-Spoiled

Ultrashort TE Imaging



Fast Spin Echo

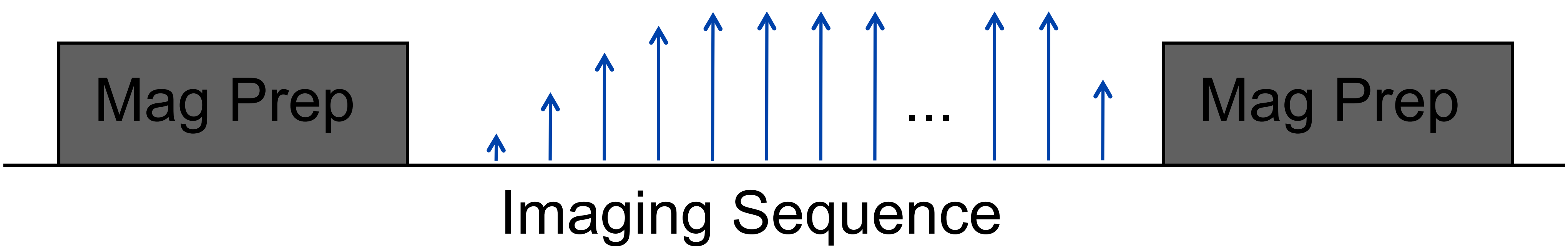


UTE

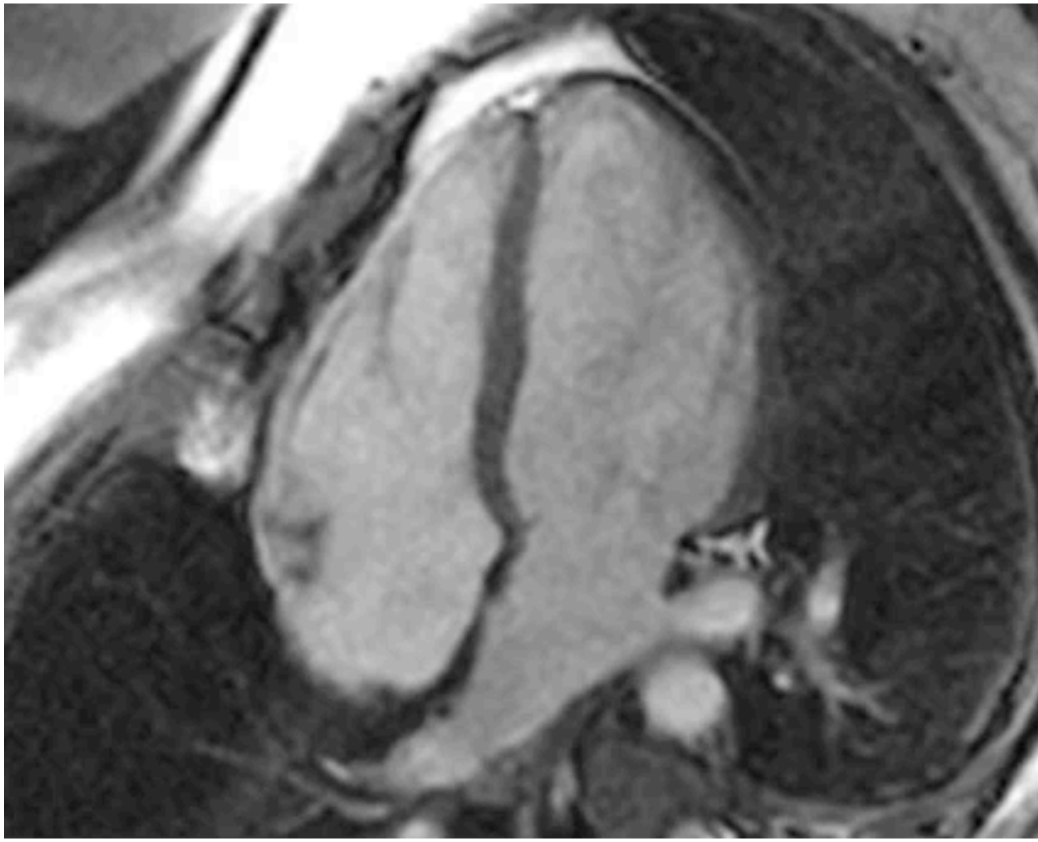
(Courtesy of Jiang Du, Graeme Bydder)

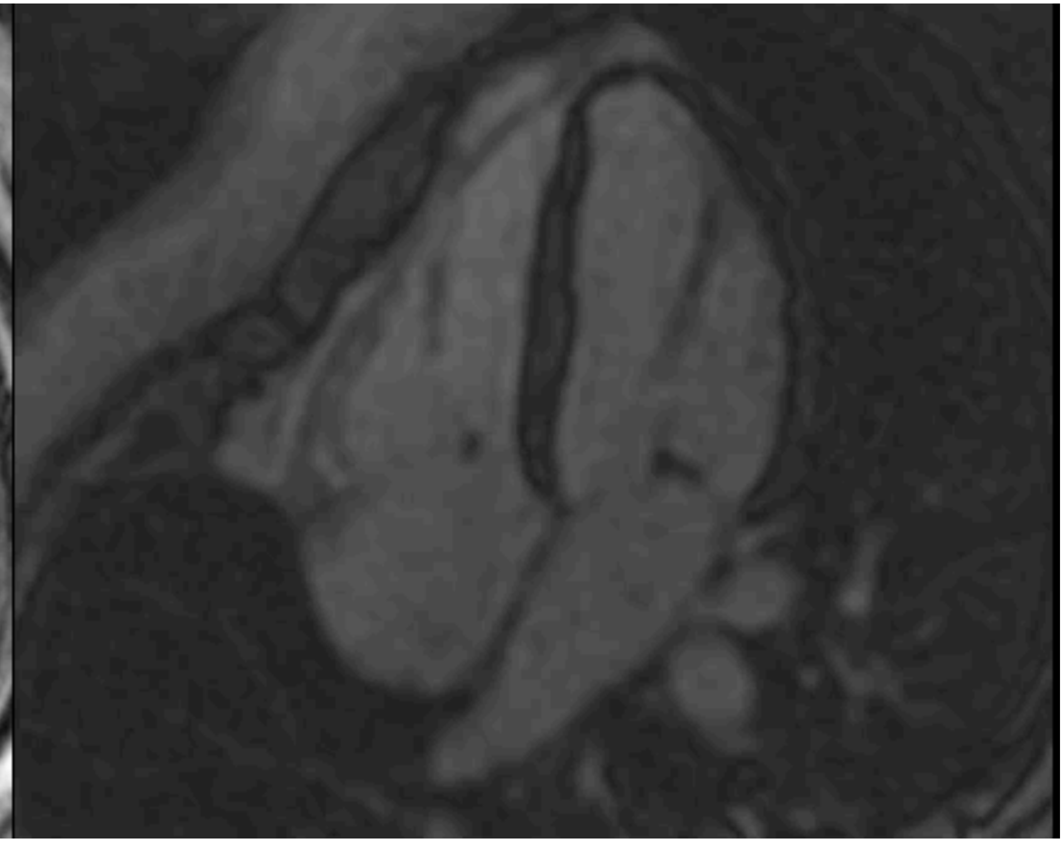
Magnetization Preparation Options

- Fat Saturation
- Inversion - Recovery
- Myocardial Tagging
- T2-prep
- Magnetization Transfer



Cardiac: bSSFP and IR-RF-Spoiled





Balanced SSFP

IR-Prep RF-Spoiled

Gradient Echo Summary

- Spoiling Variations
- Flip Angle Selection
- Echo Time (TE) Selection
- Magnetization Preparation

Spin-Echo Sequences

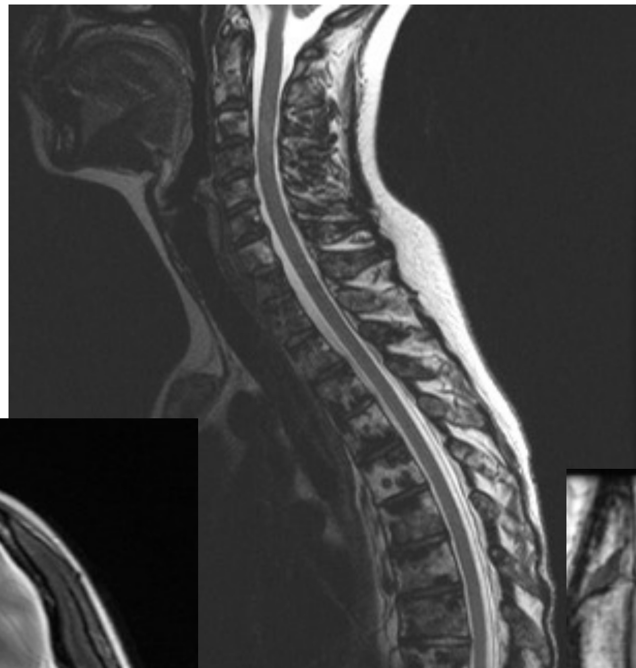
- Spin Echo Review
- Echo Trains
  - Applications: RARE, Single-shot, 3D
  - Signal and SAR considerations
- Hyperechoes

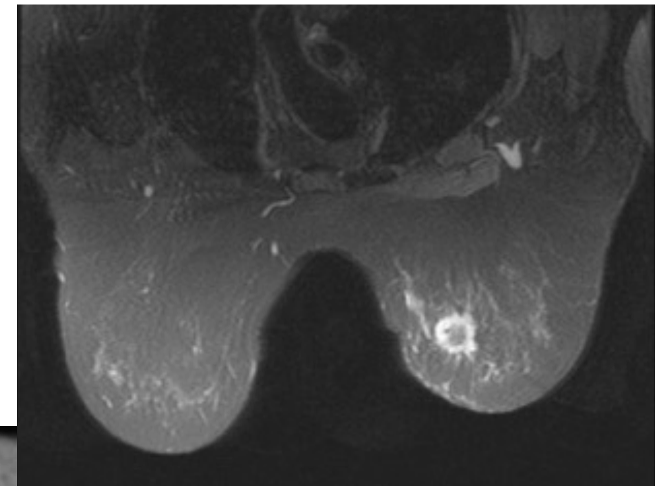
Spin Echo Review

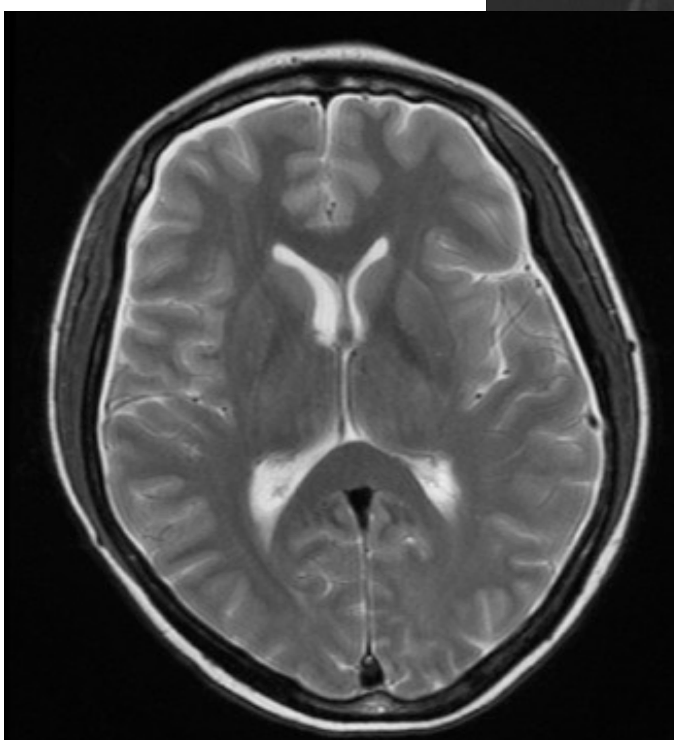
- Static Dephasing: $1/T_2^* = 1/T2 + 1/T2'$
- Spin echo "rephases" magnetization
- Spin echoes can be repeated

Motivation: Spin Echo Imaging

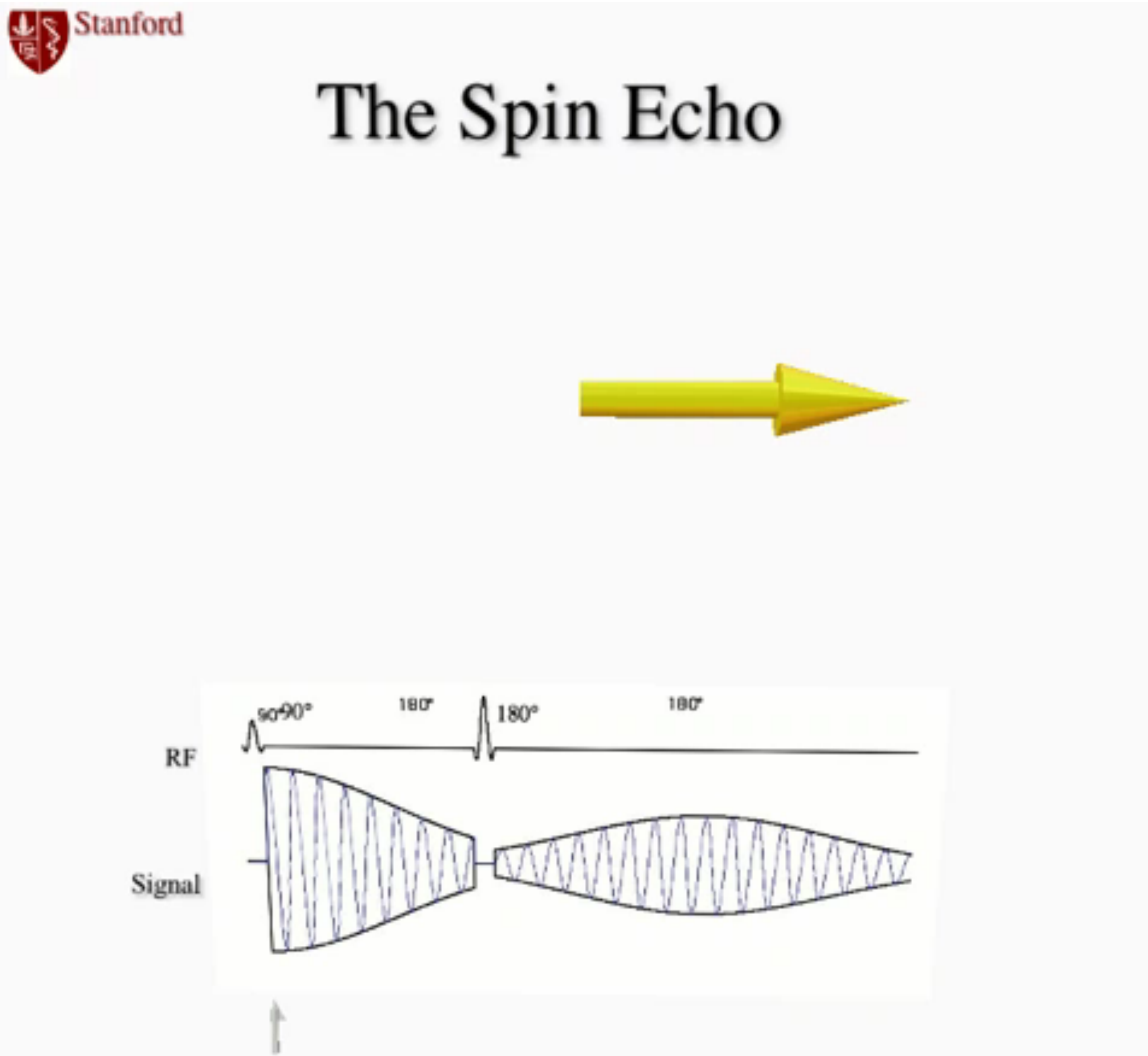
- Probably over 75% of clinical MRI



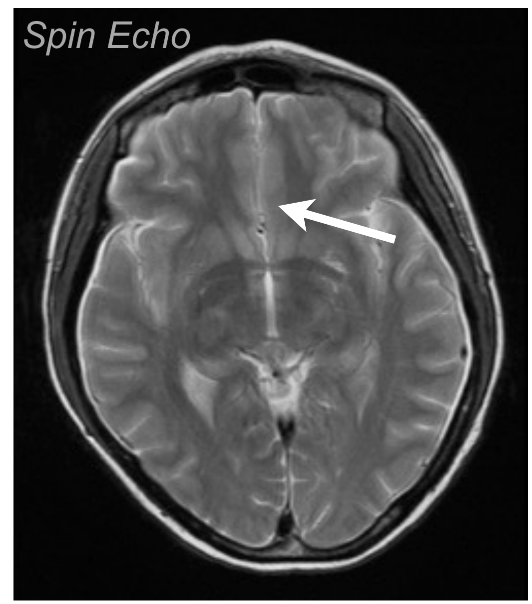




Spin Echo: T_2 and T_2^* Decay

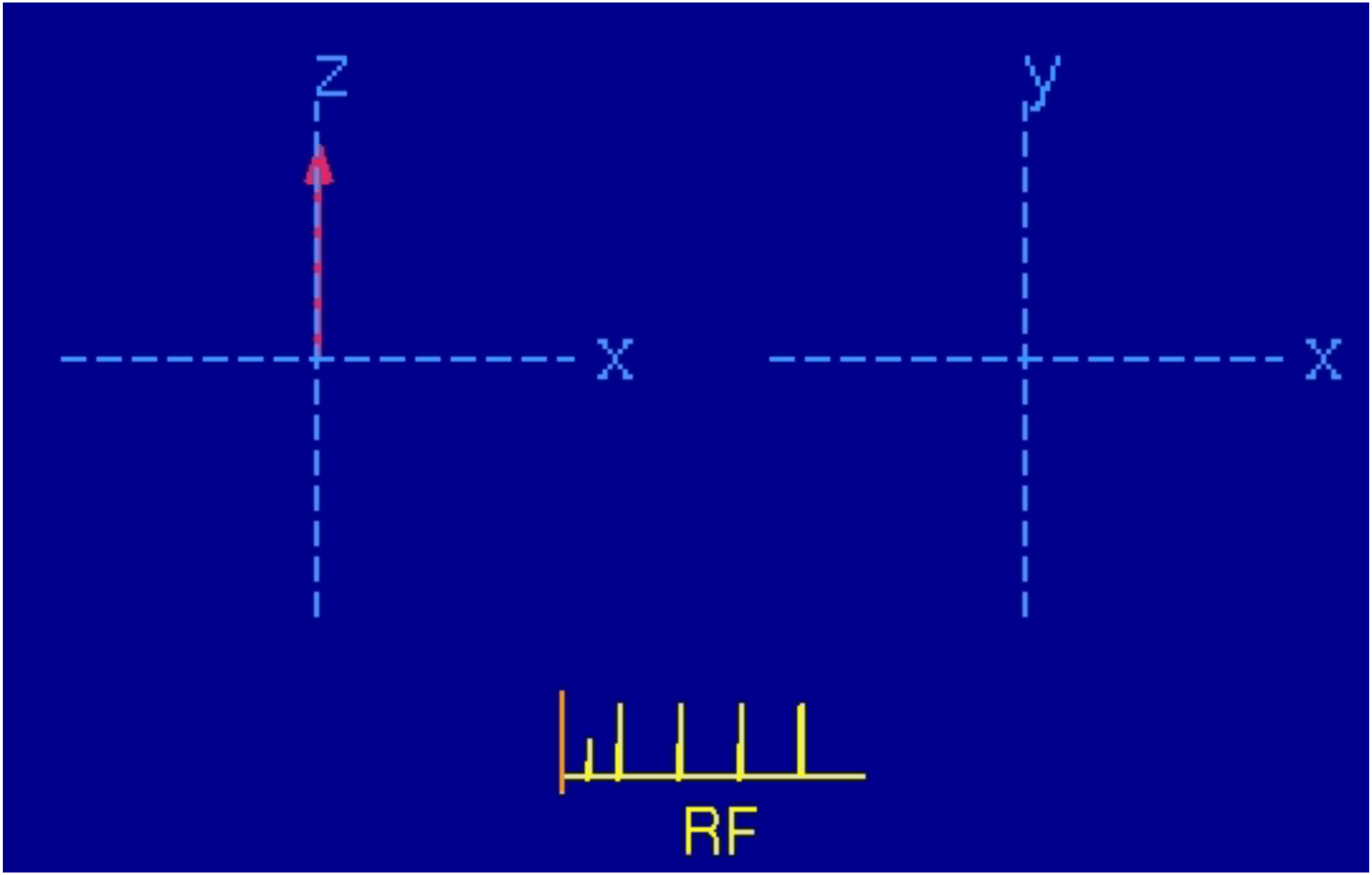


Courtesy of Kim Butts Pauly



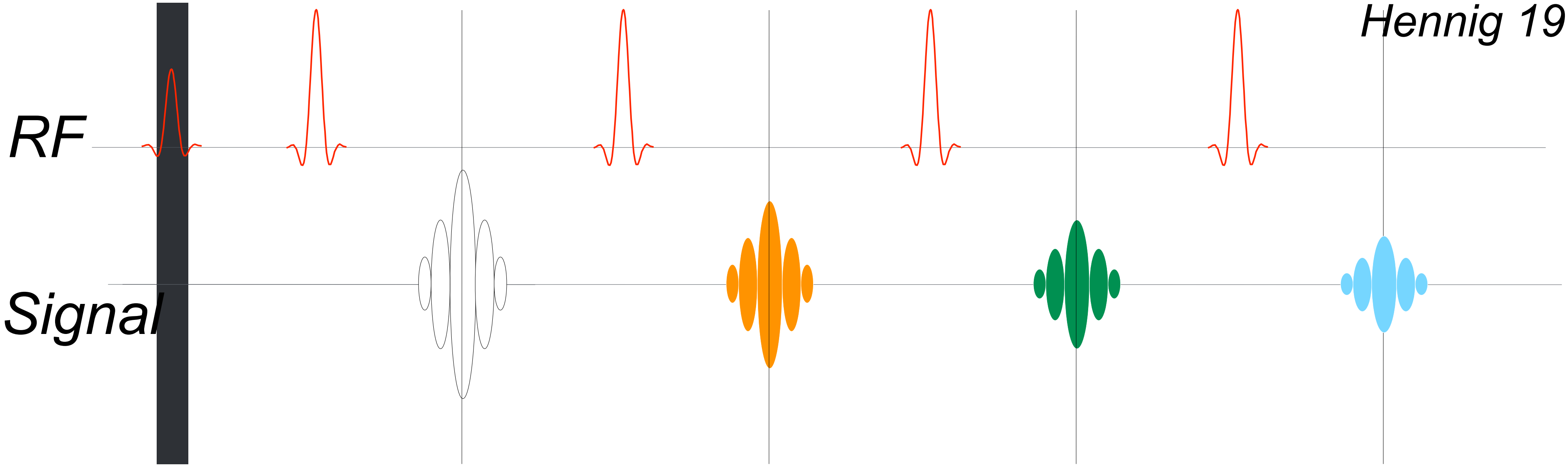
Multi-Echo Trains: RARE, TSE, FSE

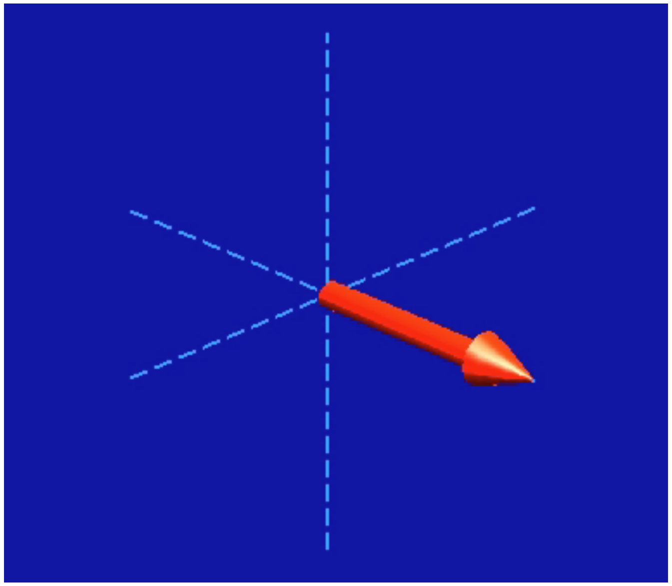
Hennig 1986

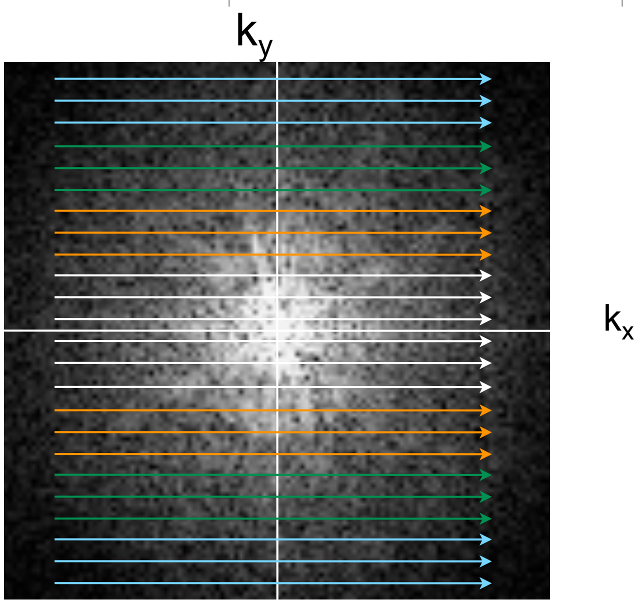


Spin-Echo-Train Imaging

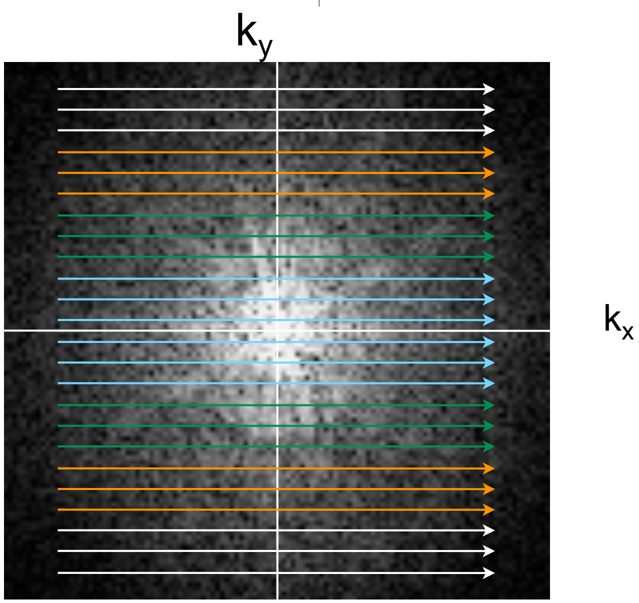
Hennig 1986





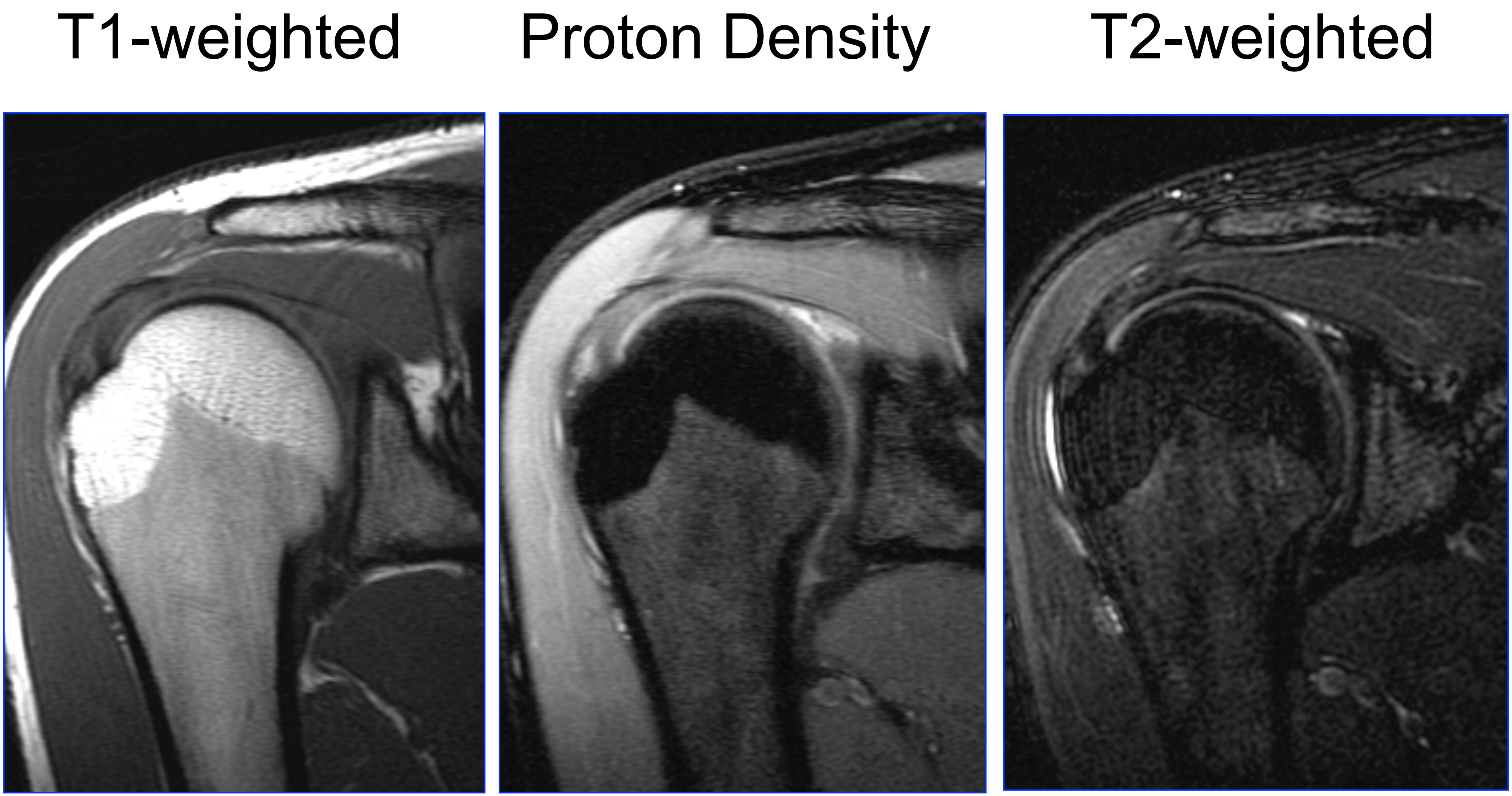


PD-weighted k-space



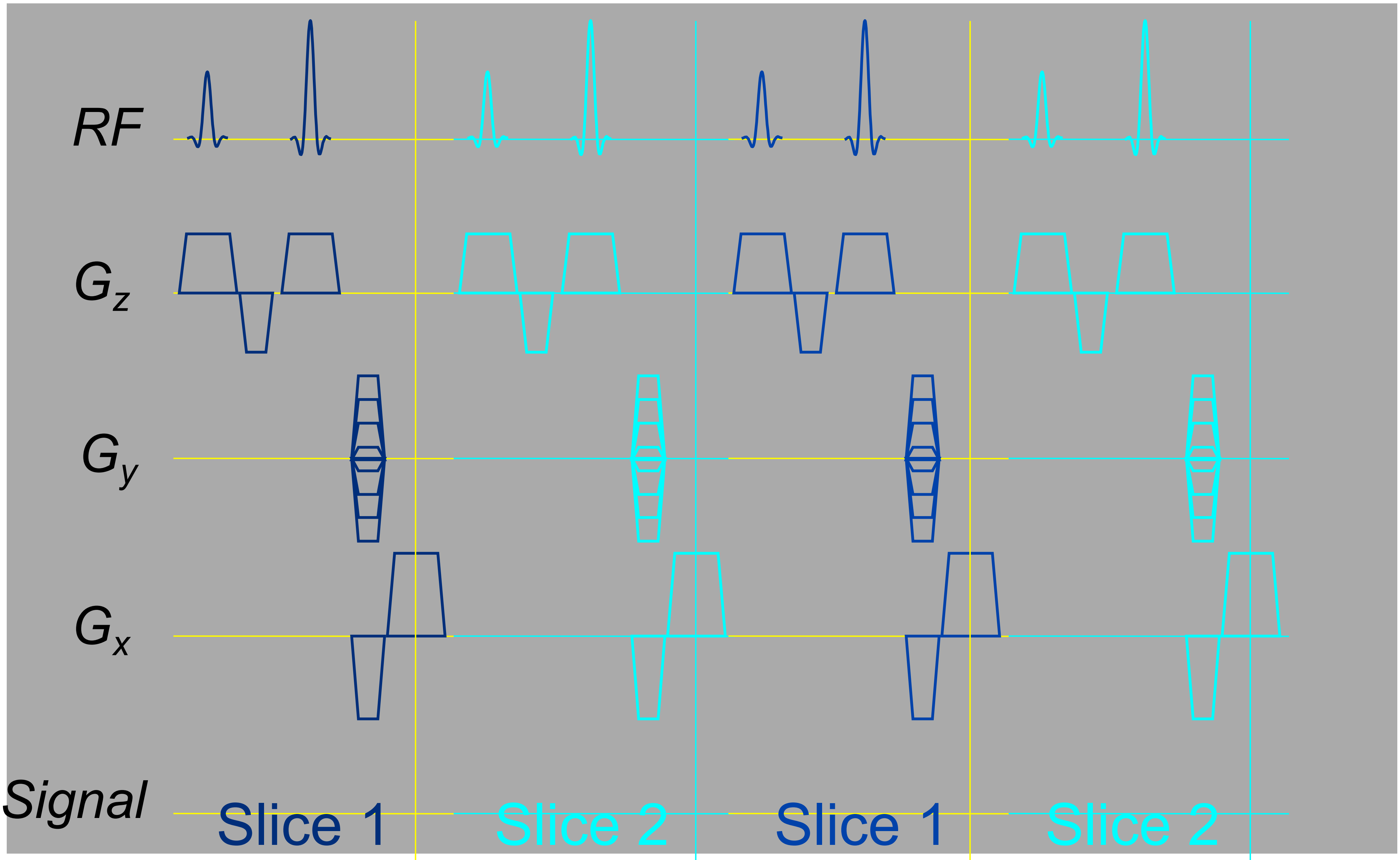
T_2-weighted k-space

Spin-Echo Contrast Variations



(Coronal shoulder images showing rotator cuff tear)

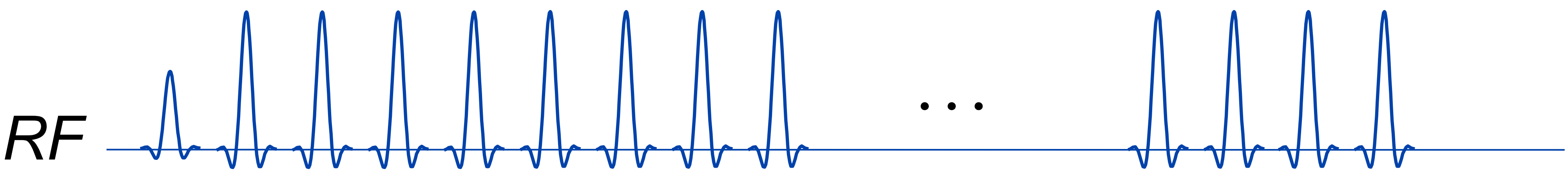
Interleaved T_1-weighted Imaging

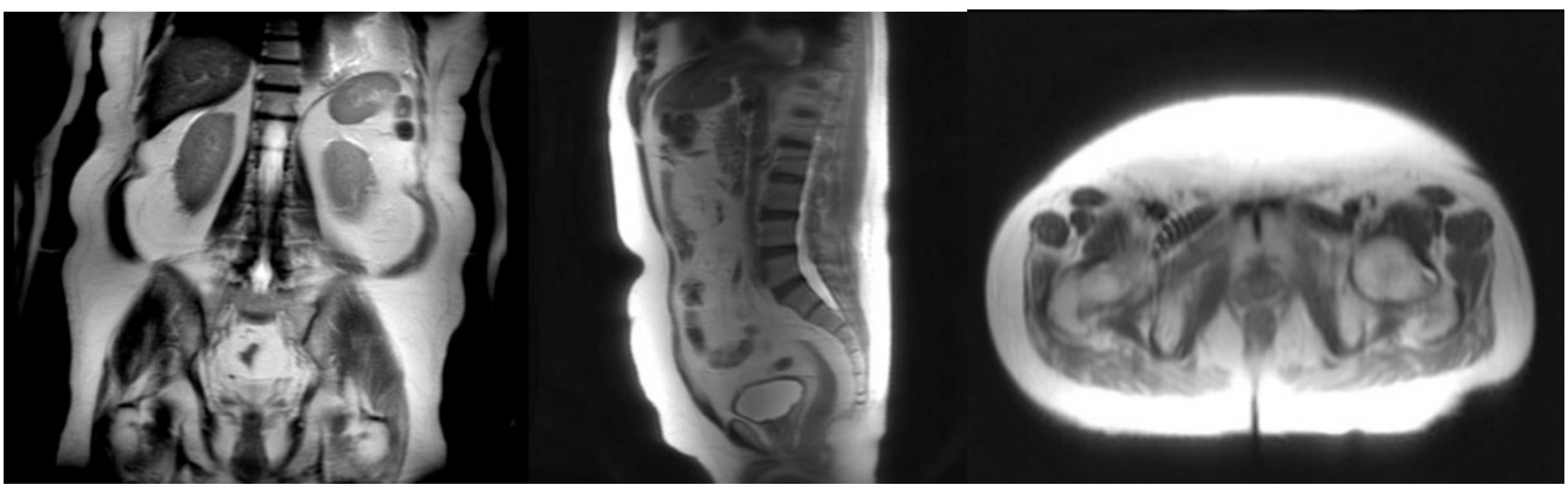


Single-Shot FSE (SSFSE, HASTE)

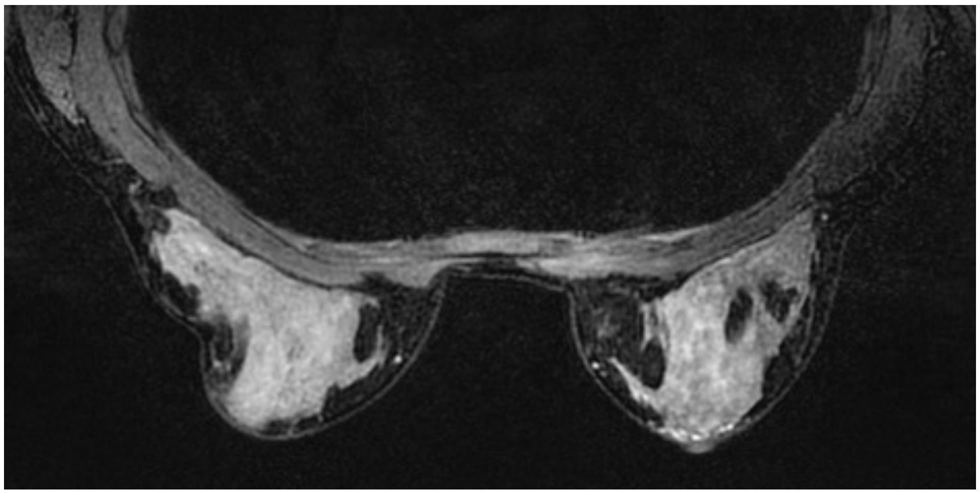
Entire image acquired in single echo train

- Lower resolution
- Significant echo-train blurring
- Robust to motion

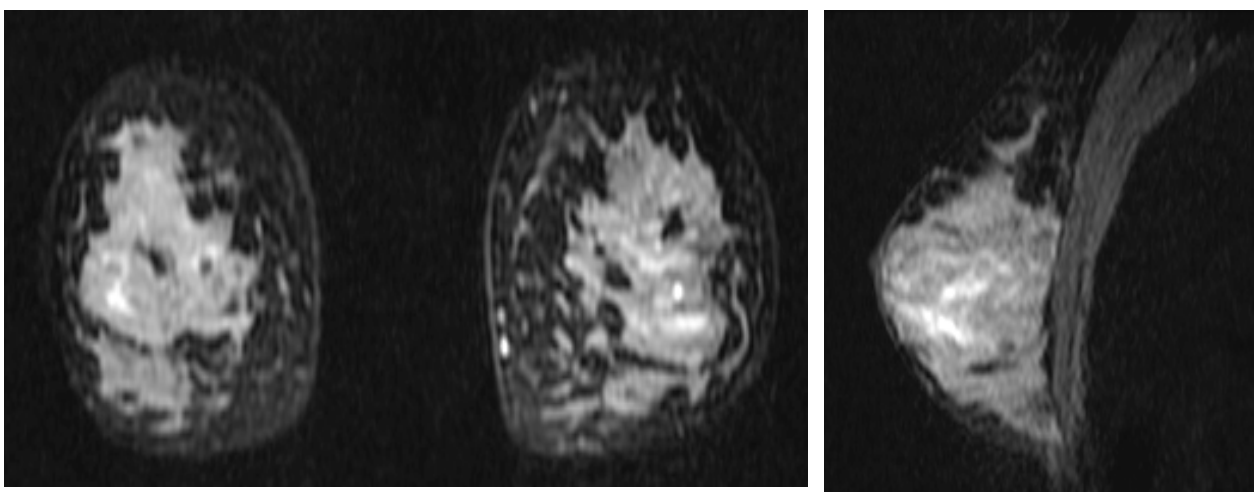




3D Spin Echo Train Methods



- Originally quite long
- Extended echo-trains help
- Phase-encode orders vary k_y and k_z modulation
- No interleaving(!)



Courtesy Ananth Madhuranthakam

Spin Echo Variations - Summary

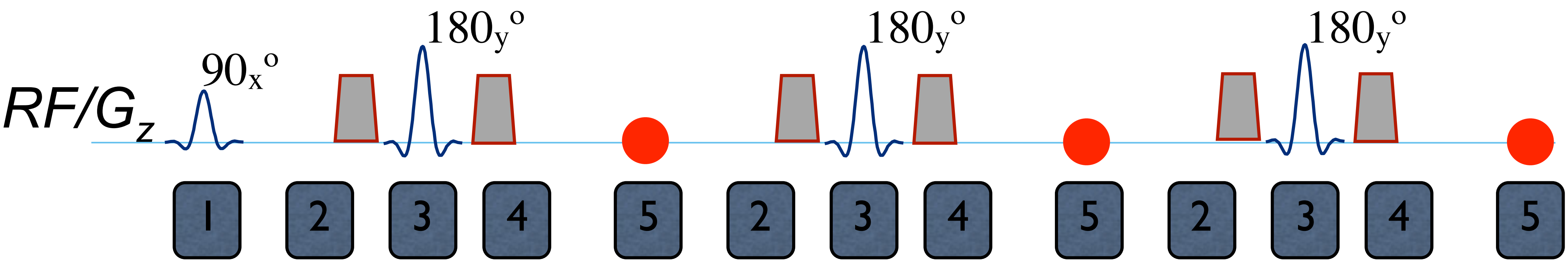
- 2D Interleaved:
  - Single-echo
  - Echo-train PD or T2 (FSE, RARE, TSE)
  - STIR, FLAIR, Fast-recovery options
- Single-shot (SSFSE, HASTE)
- 3D: (Cube, SPACE, VISTA)

Spin-Echo Signals

- Basics: T2 decay models, EPG
- Reduced refocusing angles
    - CPMG
    - Pseudo-steady-states
- Modulated refocusing angles

Spin-Echo Signals - Warmup!

Spin Echo Train Example



- Simulate

    1. 90^o excitation

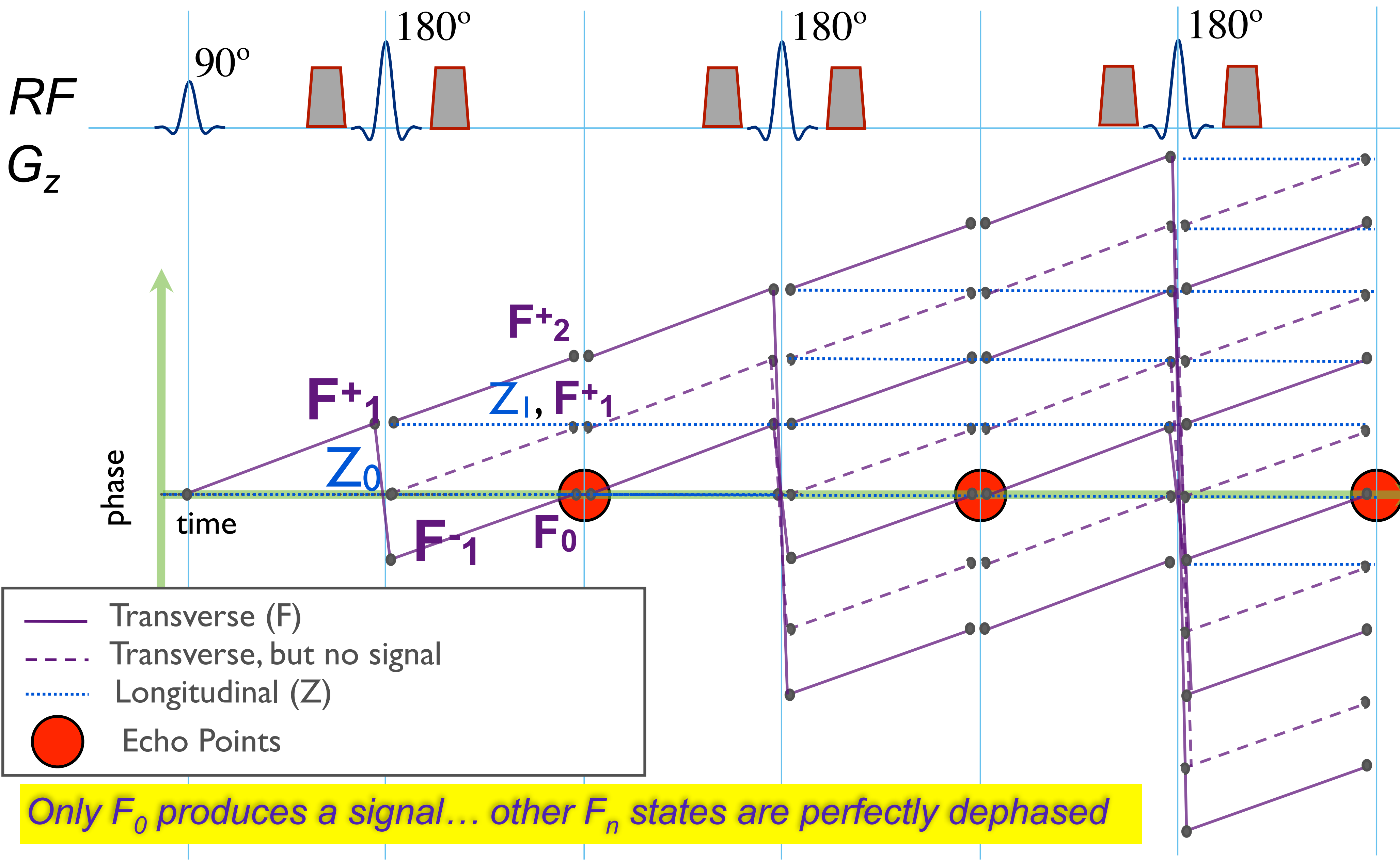
  Repeat:

    2. Relaxation and crusher gradient
    3. Refocusing pulse
    4. Relaxation and crusher gradient
    5. Signal at spin echo

- *Vary refocusing angle and/or phase…*

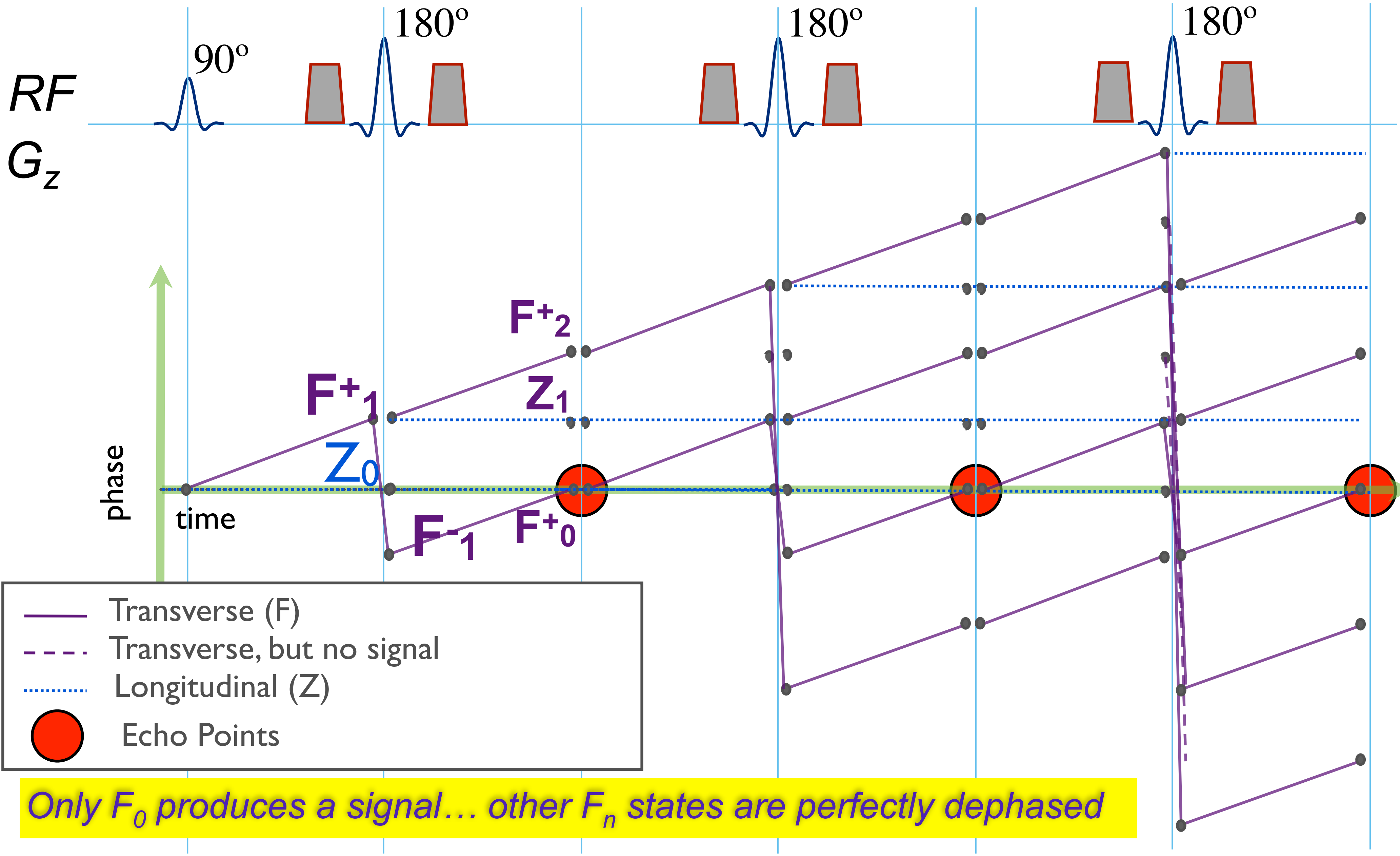
epg_cpmg.m

Review: Spin-Echo Coherence Pathways



Only F_0 produces a signal… other F_n states are perfectly dephased

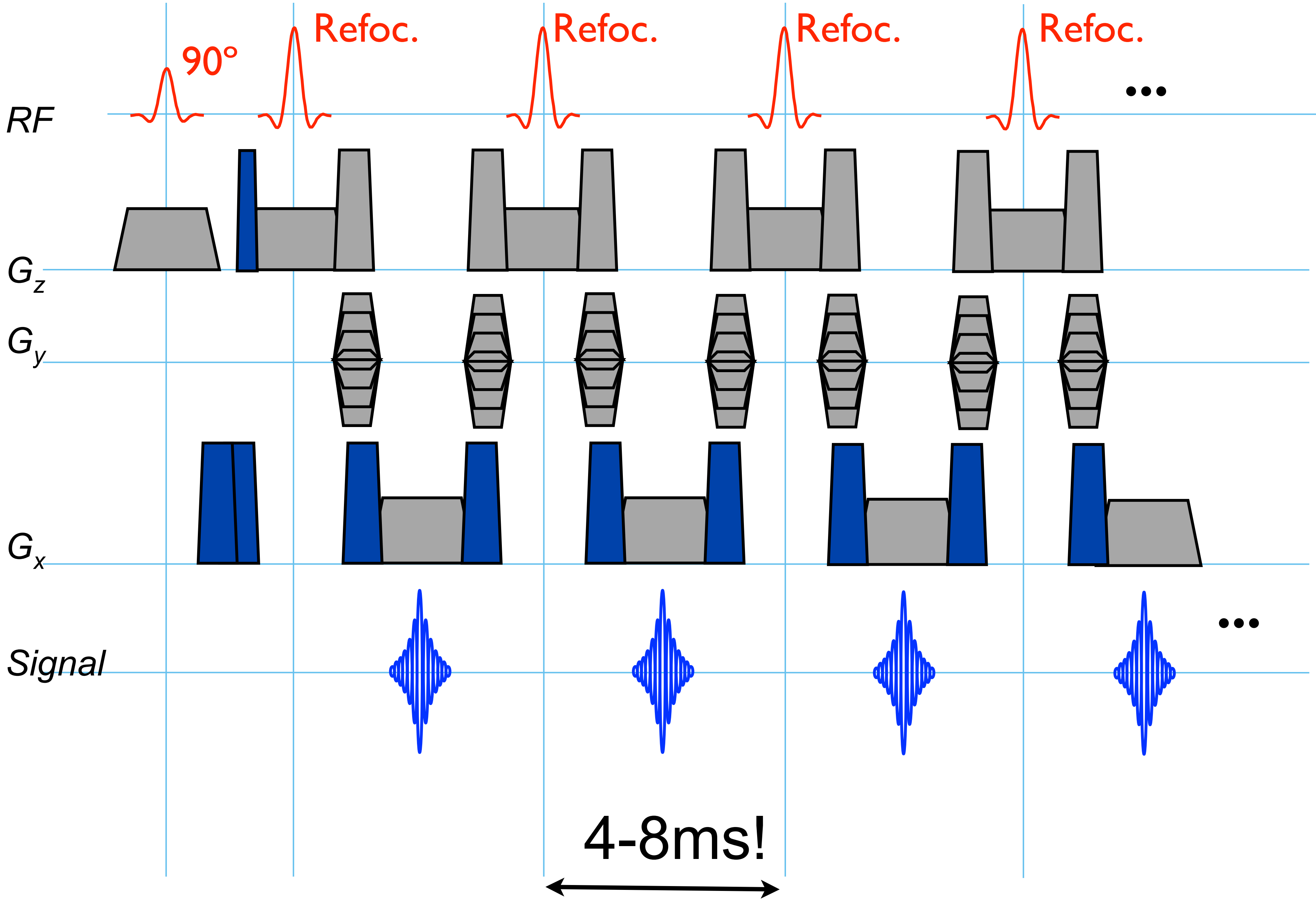
Effect of Crusher Pulses - Eliminate Pathways



Only F_0 produces a signal… other F_n states are perfectly dephased

Crushers Review Question

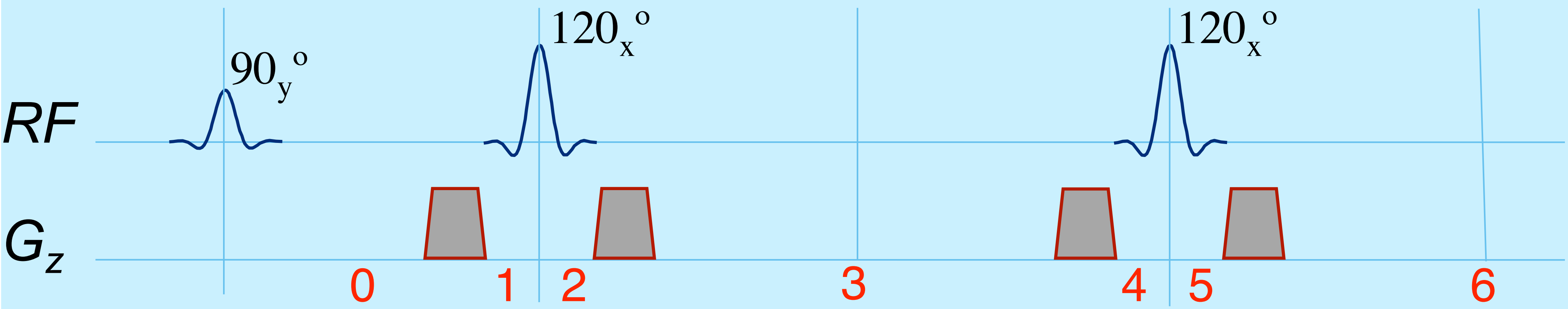
Standard CPMG Sequence... FAST!



CPMG Sequences

- Most spin-echo train sequences use CPMG
- CPMG = Carr Purcell Meiboom Gill
  - 90_x , 180_y , 180_y , 180_y , ...
  - 90_x , -180_x , 180_x , -180_x , ... (alternate ref. frame)
  - \-   +90º, +270º, +90º
- Consider the "dephased" disc:
  - If the 180 angle is lower, CPMG "corrects"

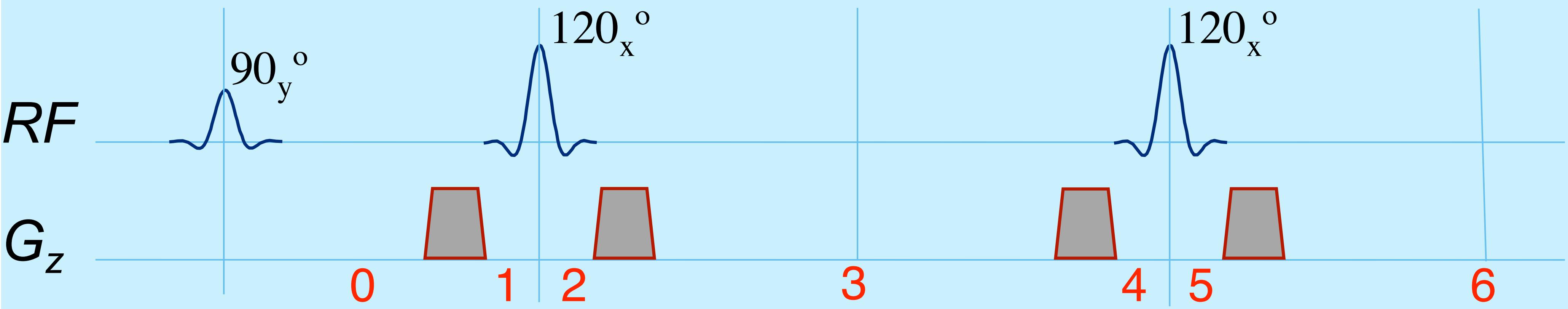
Example: CPMG




```
>> Q0 = [1;1;0];
>> Q1 = epg_grad(Q0)

     0     1
     0     0
     0     0

>> Q2 = RR*Q1

   0.00 + 0.00i   0.25 + 0.00i
   0.00 + 0.00i   0.75 + 0.00i
   0.00 + 0.00i   0.00 - 0.43i

>> Q3 = epg_grad(Q2)

   0.75 + 0.00i   0.00 + 0.00i   0.25 + 0.00i
   0.75 + 0.00i   0.00 + 0.00i   0.00 + 0.00i
   0.00 + 0.00i   0.00 - 0.43i   0.00 + 0.00i
```

```
>> Q4 = epg_grad(Q3)

   0.00 + 0.00i   0.75 + 0.00i   0.00 + 0.00i   0.25 + 0.00i
   0.00 + 0.00i   0.00 + 0.00i   0.00 + 0.00i   0.00 + 0.00i
   0.00 + 0.00i   0.00 - 0.43i   0.00 + 0.00i   0.00 + 0.00i

>> Q5 = RR*Q4

   0.00 + 0.00i  -0.19 + 0.00i   0.00 + 0.00i   0.063 + 0.00i
   0.00 + 0.00i   0.94 + 0.00i   0.00 + 0.00i   0.19 + 0.00i
   0.00 + 0.00i   0.00 - 0.11i   0.00 + 0.00i   0.00 - 0.11i

>> Q6 = epg_grad(Q5)

   0.94 + 0.00i   0.00 + 0.00i  -0.19 + 0.00i   0.00 + 0.00i   0.06 + 0.00i
   0.94 + 0.00i   0.00 + 0.00i   0.19 + 0.00i   0.00 + 0.00i   0.00 + 0.00i
   0.00 + 0.00i   0.00 - 0.11i   0.00 + 0.00i   0.00 - 0.11i   0.00 + 0.00i
```

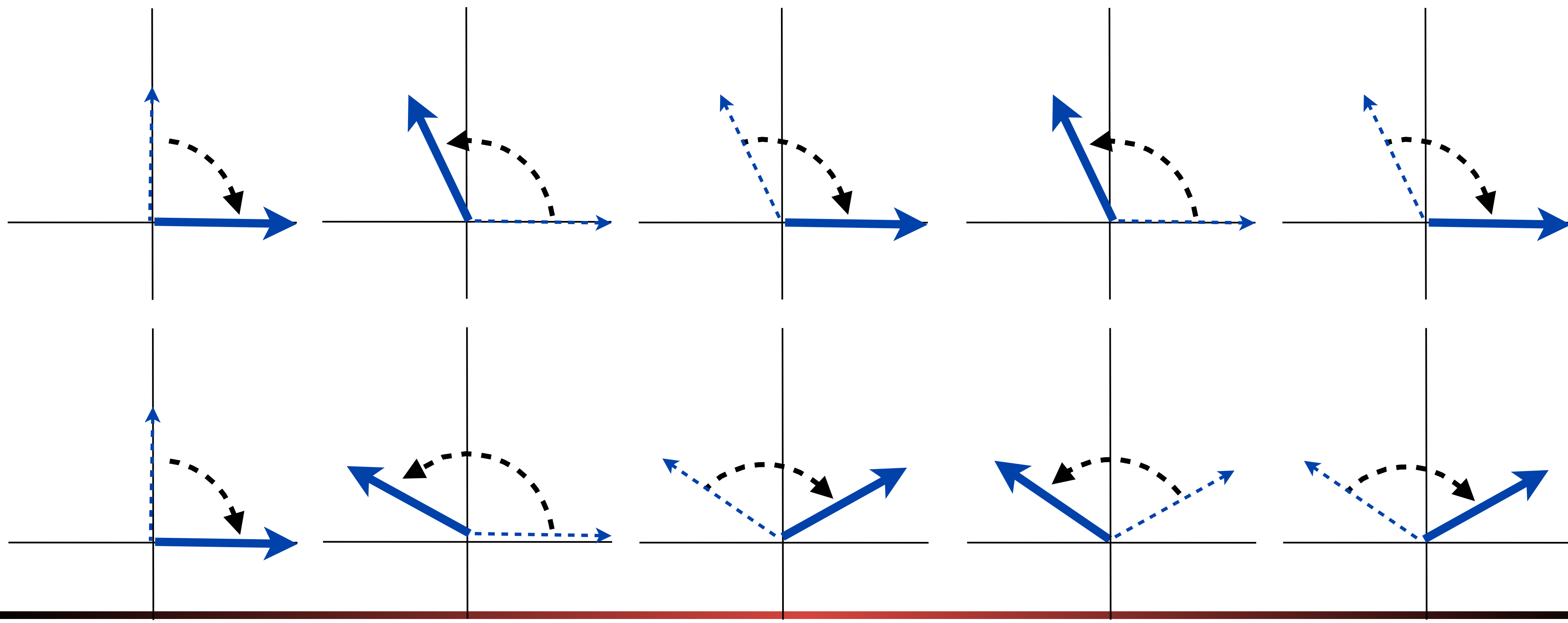
Example: Non-CPMG



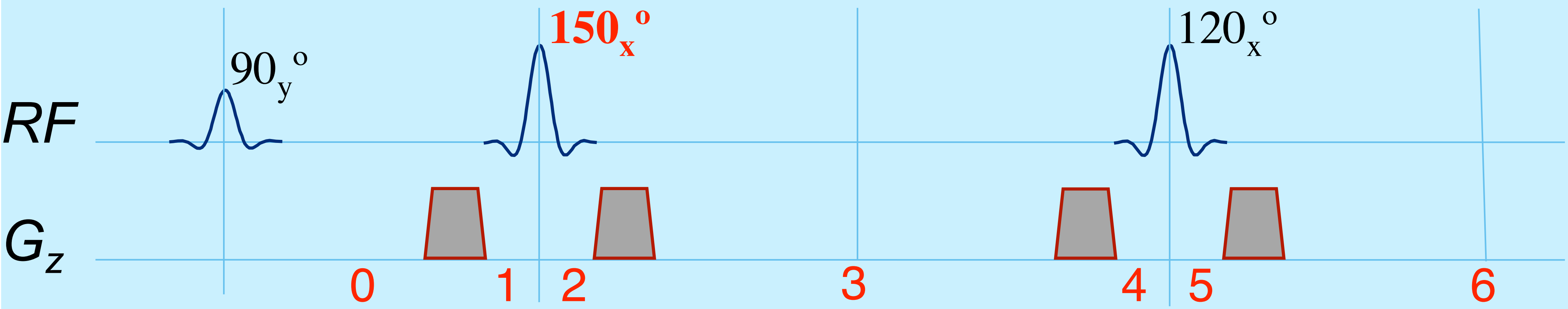

```
>> Q0 = [i;-i;0];
>> Q1 = epg_grad(Q0)

  0.00 + 0.00i   0.00 + 1.00i
  0.00 + 0.00i   0.00 + 0.00i
  0.00 + 0.00i   0.00 + 0.00i

>> Q2 = RR*Q1

  0.00 + 0.00i   0.00 + 0.25i
  0.00 + 0.00i   0.00 + 0.75i
  0.00 + 0.00i   0.43 + 0.00i

>> Q3 = epg_grad(Q2)

  0.00 - 0.75i   0.00 + 0.00i   0.00 + 0.25i
  0.00 + 0.75i   0.00 + 0.00i   0.00 + 0.00i
  0.00 + 0.00i   0.43 + 0.00i   0.00 + 0.00i

>> Q4 = epg_grad(Q3)

  0.00 + 0.00i   0.00 - 0.75i   0.00 + 0.00i   0.00 + 0.25i
  0.00 + 0.00i   0.00 + 0.00i   0.00 + 0.00i   0.00 + 0.00i
  0.00 + 0.00i   0.43 + 0.00i   0.00 + 0.00i   0.00 + 0.00i

>> Q5 = RR*Q4

  0.00 + 0.00i   0.00 - 0.56i   0.00 + 0.00i   0.00 + 0.06i
  0.00 + 0.00i   0.00 - 0.19i   0.00 + 0.00i   0.00 + 0.19i
  0.00 + 0.00i  -0.54 + 0.00i   0.00 + 0.00i   0.11 + 0.00i

>> Q6 = epg_grad(Q5)

  0.00 + 0.19i   0.00 + 0.00i   0.00 - 0.56i   0.00 + 0.00i   0.00 + 0.06i
  0.00 - 0.19i   0.00 + 0.00i   0.00 + 0.19i   0.00 + 0.00i   0.00 + 0.00i
  0.00 + 0.00i  -0.54 + 0.00i   0.00 + 0.00i   0.11 + 0.00i   0.00 + 0.00i
```

Intuition: Stabilization Pulse

- Often use reduced refocusing angles (Hennig 2000)
  - 90_x , -120_x , 120_x , -120_x , ...
- Consider the "on-resonant" spins
  - 90x , -150_x , 120_x , -120_x , ...



Example: CPMG (prep)




```
>> Q0 = [1;1;0];
>> Q1 = epg_grad(Q0)
```

0	1
0	0
0	0

```
>> Q2 = RR1*Q1
```

000. + 0.00i	0.07 + 0.00i
0.00 + 0.00i	0.93 + 0.00i
0.00 + 0.00i	0.00 - 0.25i

```
>> Q3 = epg_grad(Q2)
```

0.93 + 0.00i	0.00 + 0.00i	0.07 + 0.00i
0.93 + 0.00i	0.00 + 0.00i	0.00 + 0.00i
0.00 + 0.00i	0.00 - 0.25i	0.00 + 0.00i

```
>> Q4 = epg_grad(Q3)
```

0.00 + 0.00i	0.93 + 0.00i	0.00 + 0.00i	0.07 + 0.00i
0.00 + 0.00i	0.00 + 0.00i	0.00 + 0.00i	0.00 + 0.00i
0.00 + 0.00i	0.00 - 0.25i	0.00 + 0.00i	0.00 + 0.00i

```
>> Q5 = RR*Q4
```

0.00 + 0.00i	0.02 + 0.00i	0.00 + 0.00i	0.02 + 0.00i
0.00 + 0.00i	0.92 + 0.00i	0.00 + 0.00i	0.05 + 0.00i
0.00 + 0.00i	0.00 - 0.28i	0.00 + 0.00i	0.00 - 0.03i

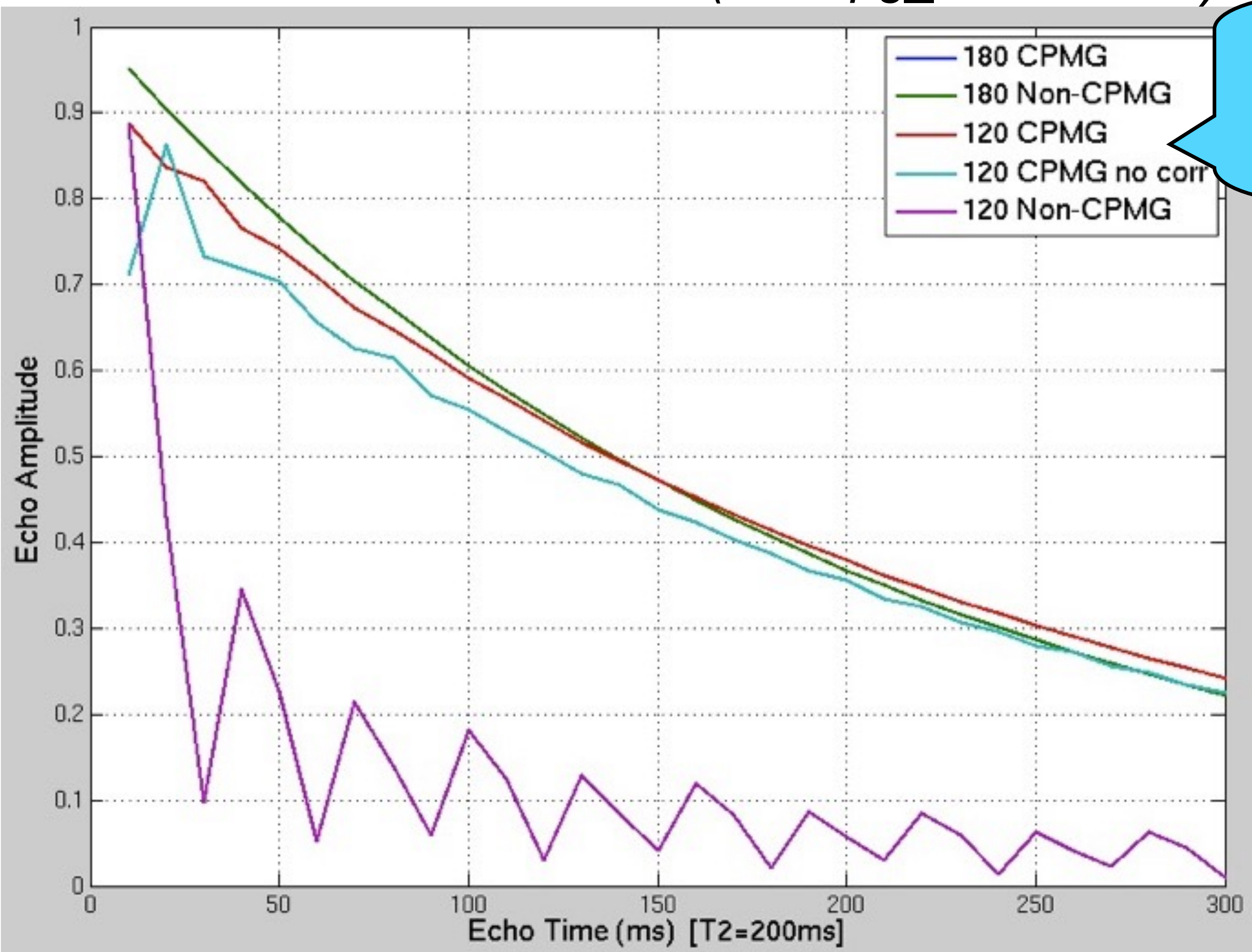
```
>> Q6 = epg_grad(Q5)
```

0.92 + 0.00i	0.00 + 0.00i	0.02 + 0.00i	0.00 + 0.00i	0.02 + 0.00i
0.92 + 0.00i	0.00 + 0.00i	0.05 + 0.00i	0.00 + 0.00i	0.00 + 0.00i
0.00 + 0.00i	0.00 - 0.28i	0.00 + 0.00i	0.00 - 0.03i	0.00 + 0.00i

Spin Echo Train Results

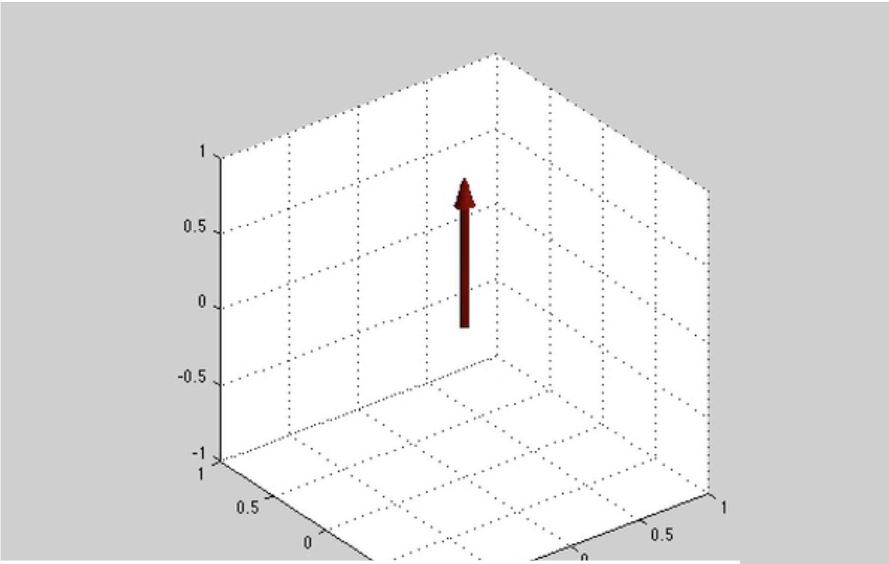
- *Repeated with α_φ refocusing pulses, 10ms echo spacing*

(see epg_echotrain.m)

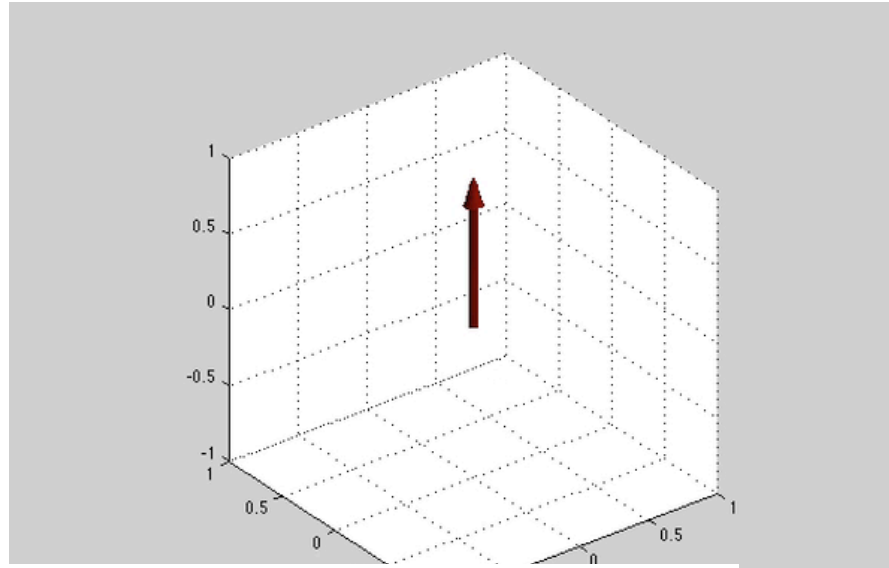


3rd line uses 90-150-120-120
4th line uses 90-120-120-120
(Hennig J et al. 2000; 44: 938)

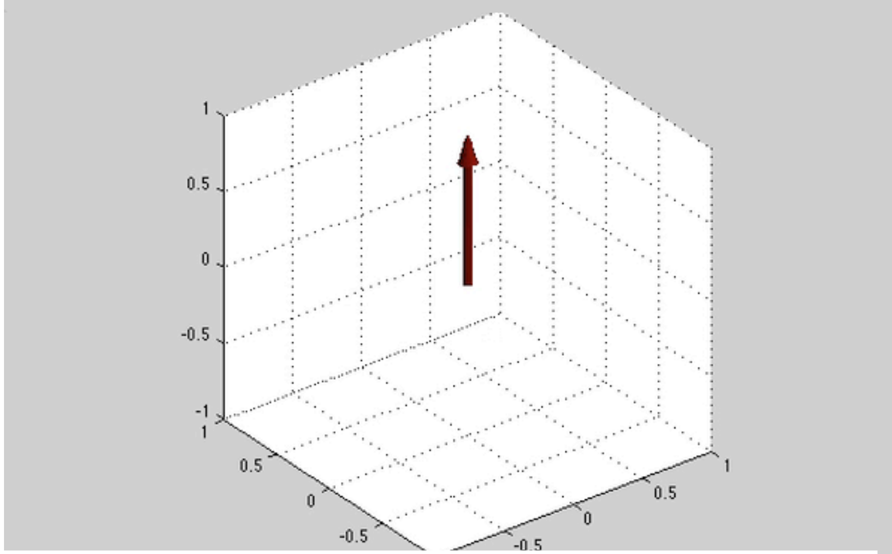
CPMG Cases: Examples



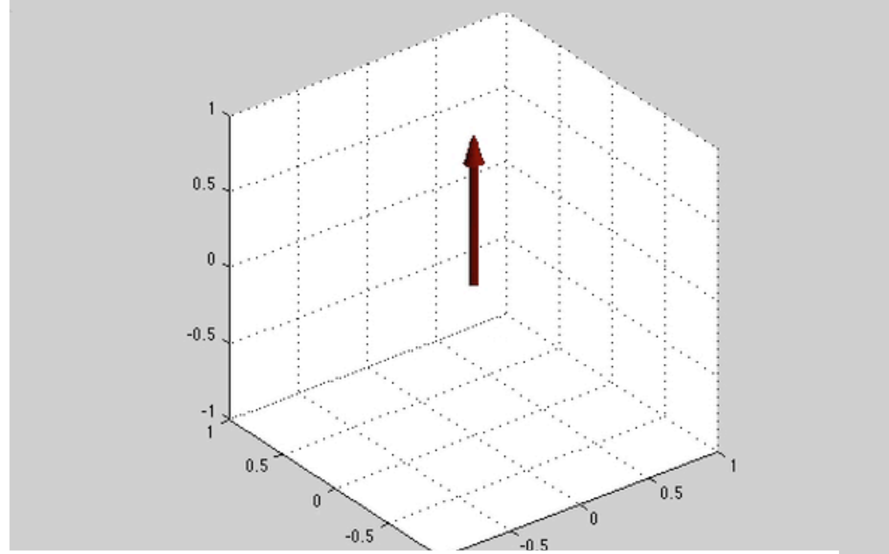
CPMG (90_{-y},120_x, 120_x, 120_x, …)



CPMG (90_{-y},150_x, 120_x, 120_x, …)



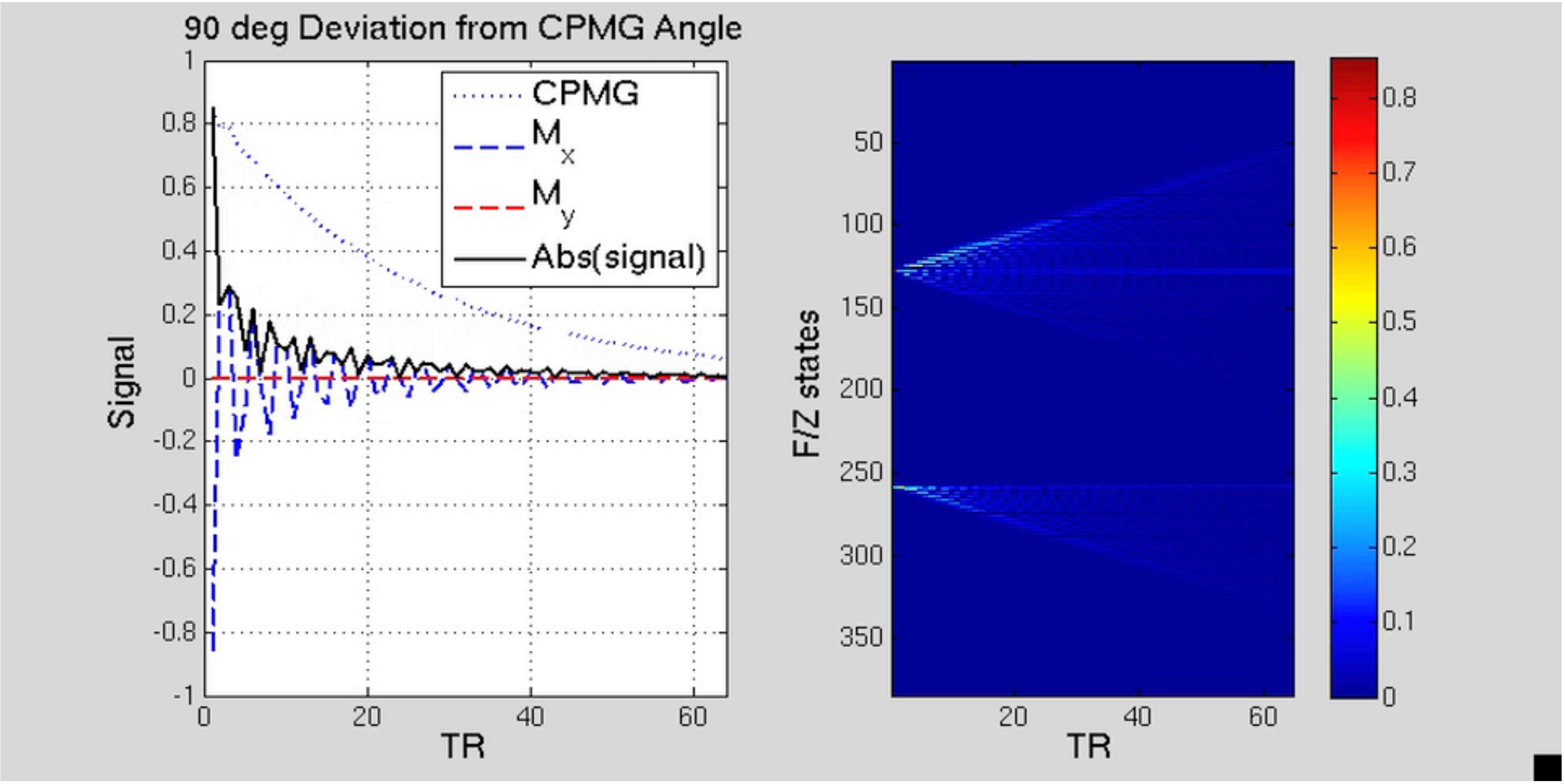
Non-CPMG (90_{-y},150_{-y}, 120_{-y}, 120_{-y}, …)



CPMG (90_{-y},-150_{-y}, $+120_{-y}$, -120_{-y}, …)

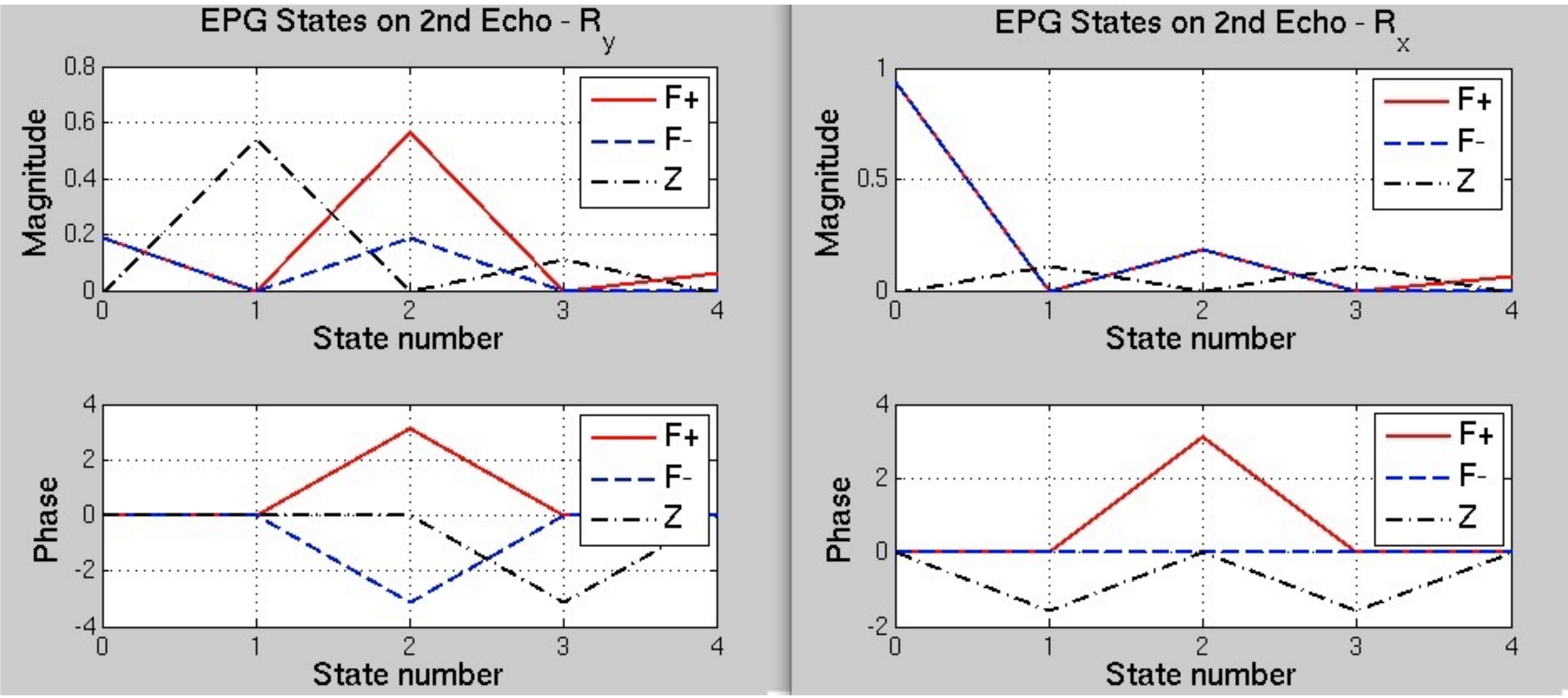
CPMG: Effect of Phase

- Compares $90^o_{-\pi/2}$ - α_φ for $\varphi=[0,\pi]$ and $\alpha=105^o$
- CPMG ($\varphi=0$) shown for reference



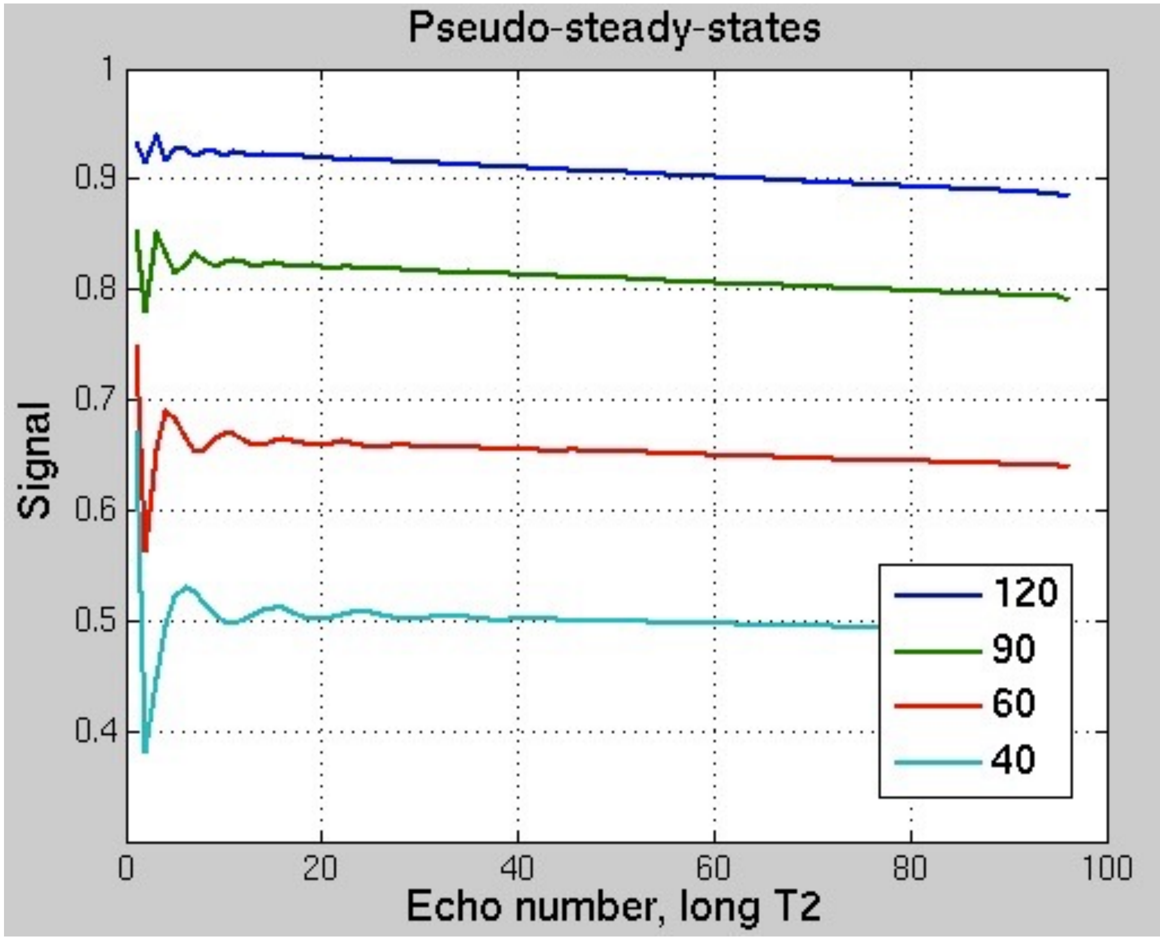
CPMG: EPG States

- Compare 90^o_y - α_y (non-CPMG) to 90^o_y - α_x (CPMG)
- F/Z states on 2nd spin-echo after perfect 90º pulse



Pseudo-Steady States

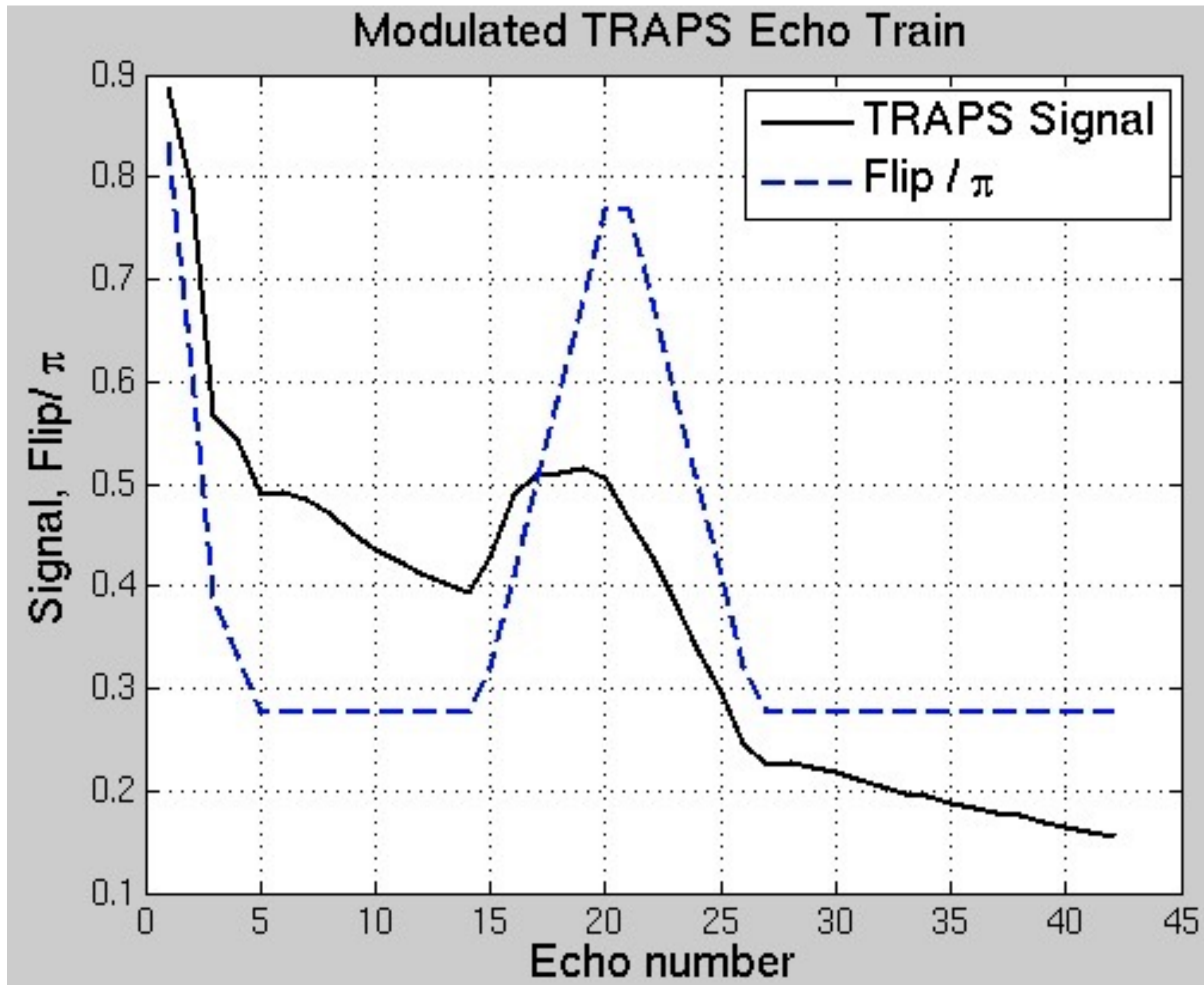
- Reduced flip angles
- “Stabilization” pulse



TRAPS

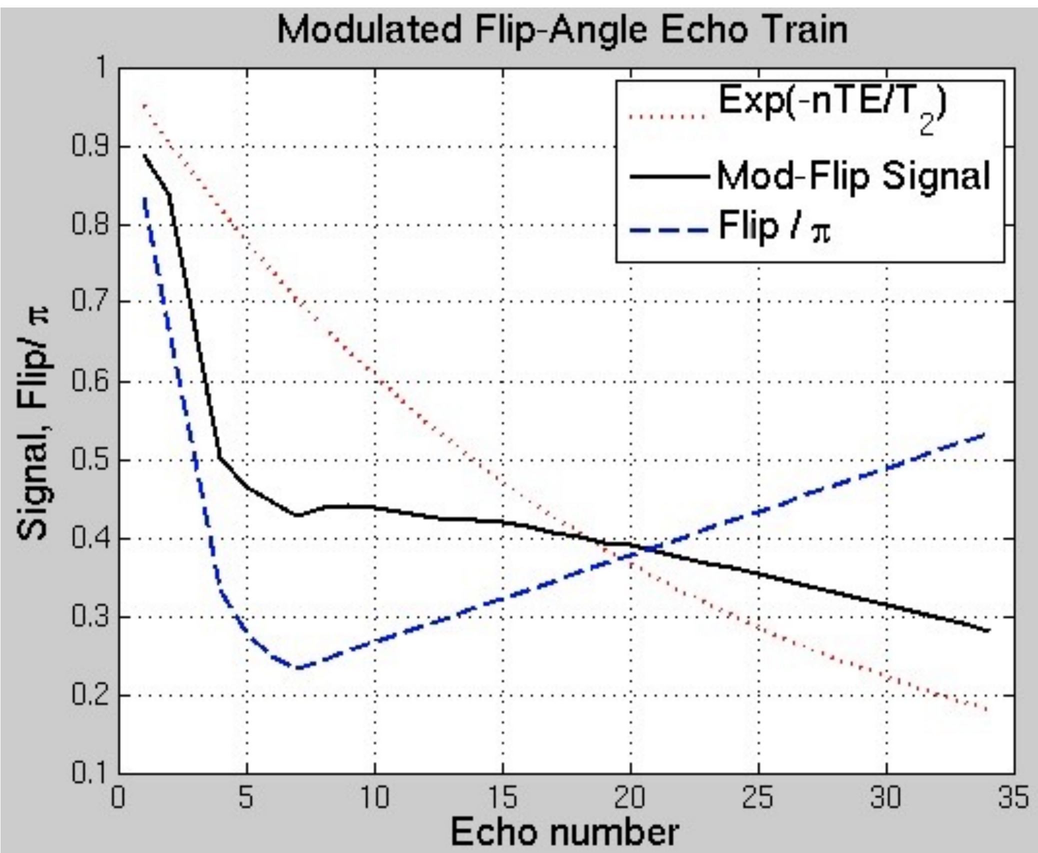
Hennig 2003

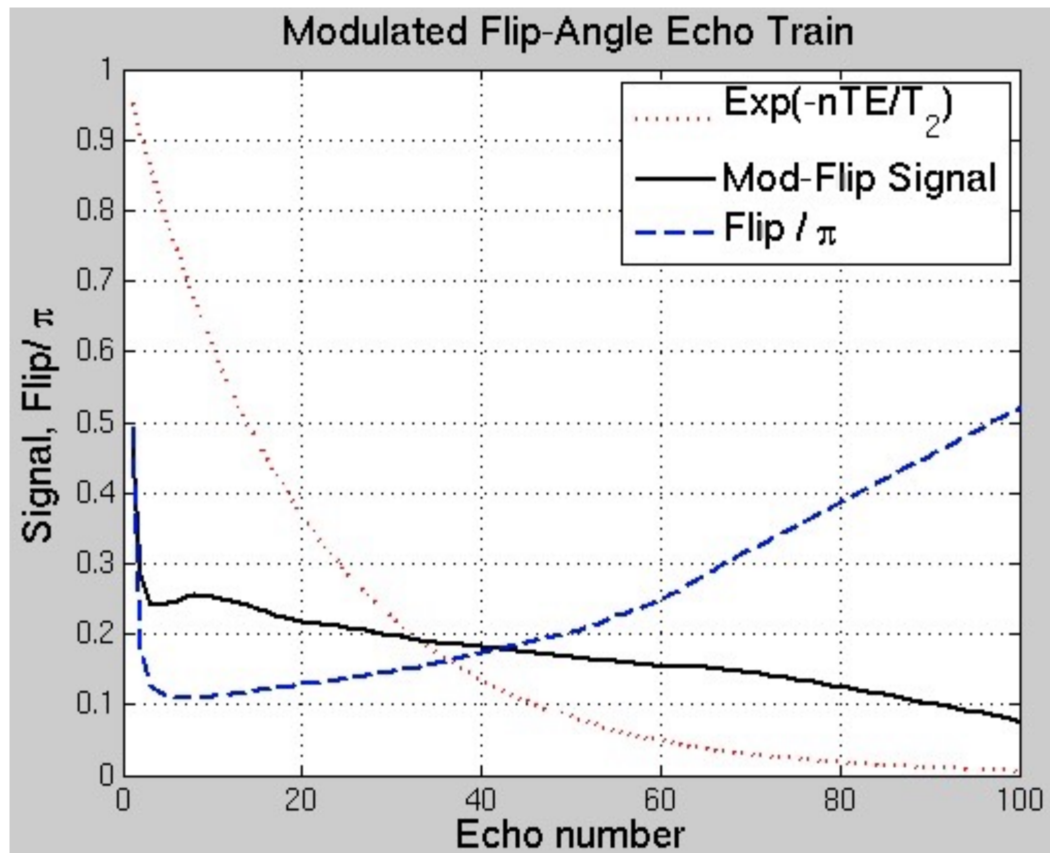
- Transition to Pseudo-steady-states
- Enhance signal at k-space center (sequential k_y)



Modulated Refocusing Angles

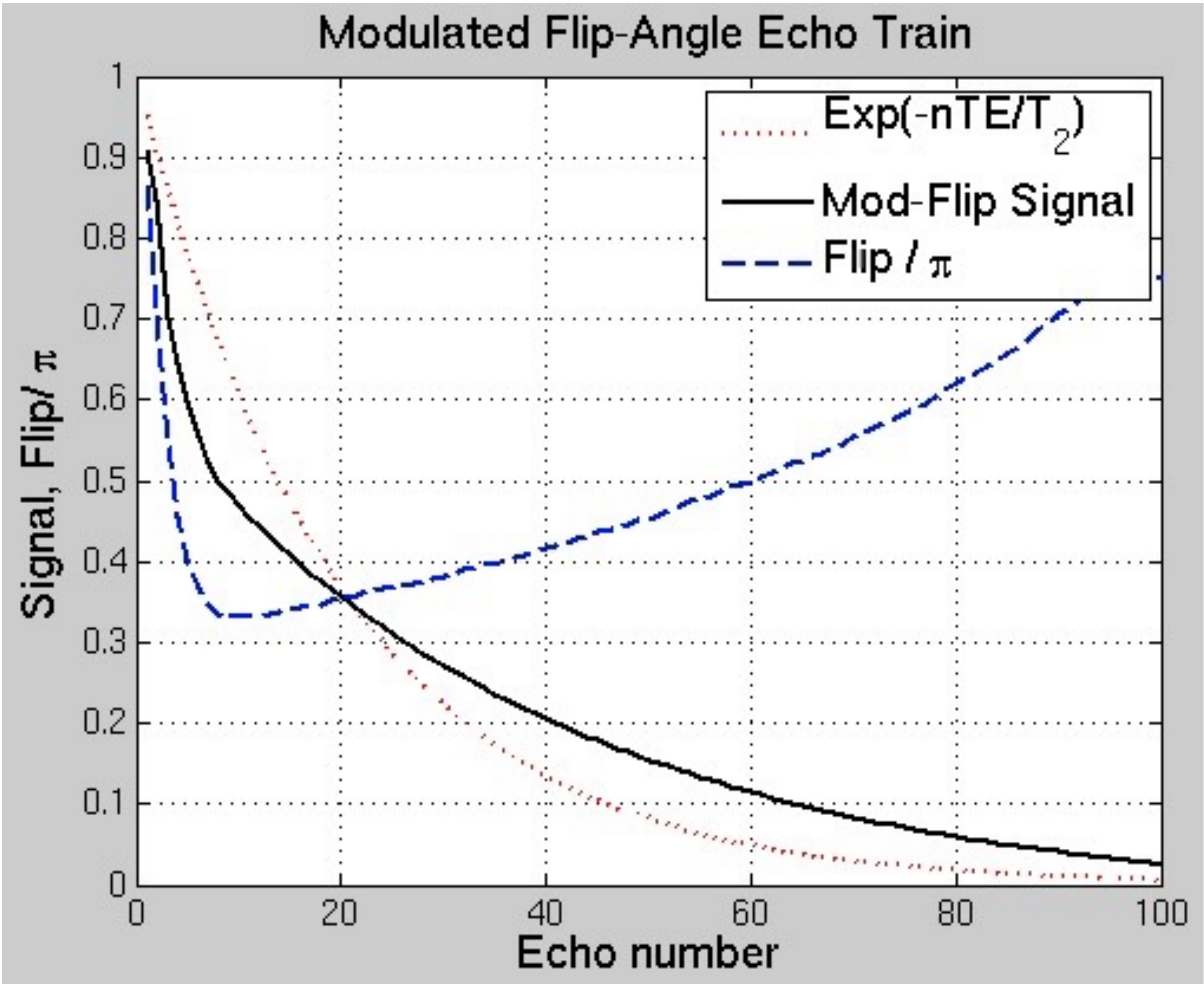
- Variable flip-angles with CPMG
- Different schemes to “optimize” signal over echo train
- “optimize” varies(!)
- AUC vs SAR vs flatness?





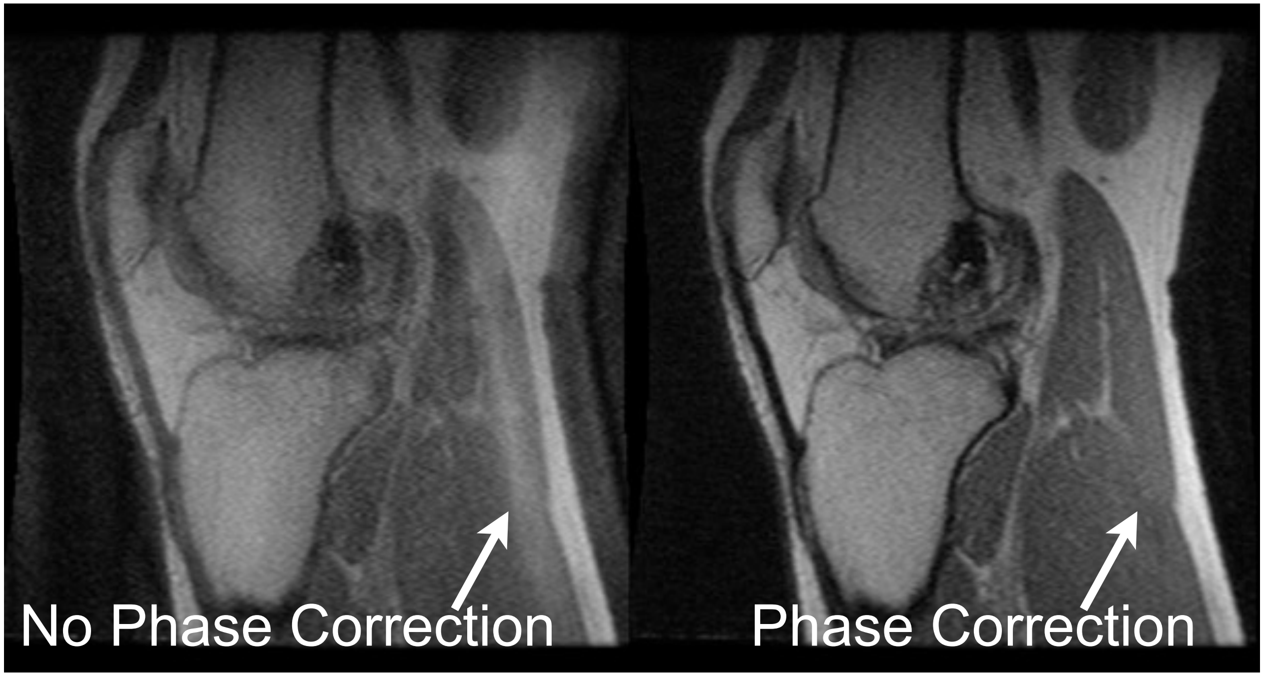
Modulated Refocusing Angles: XETA

- “Extended” exponential -- Busse 2006
- T_2 contrast with extended echo train



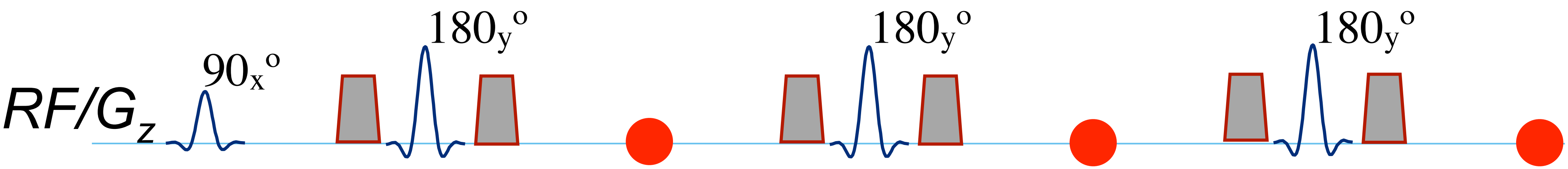
Phase Correction

- Eddy-current variations are a problem
  - Between refocusing pulses eddy currents are the same - so less problematic
  - 90-180 eddy currents differ, causing loss of the 90º phase difference for CPMG
- Linear corrections by modifying G_X and G_Z areas



Hinks, 1993

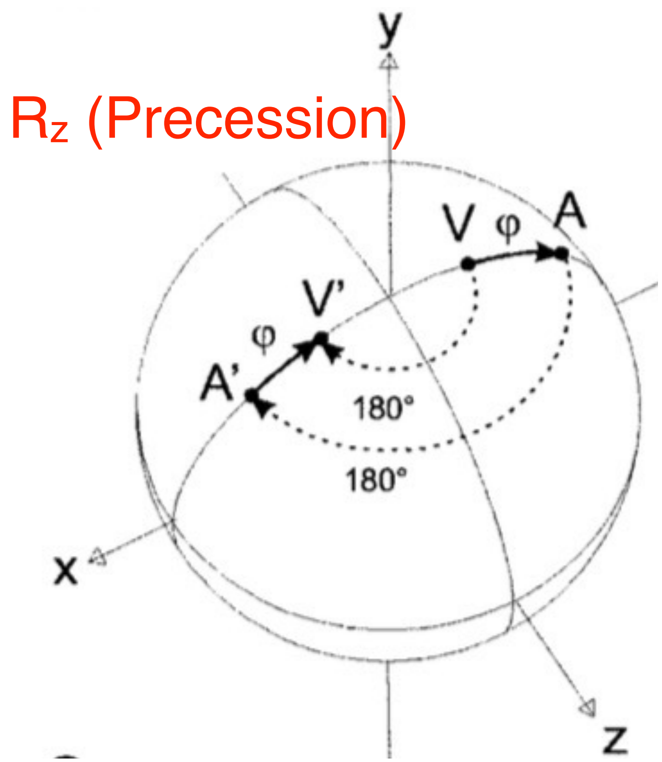
CPMG Summary

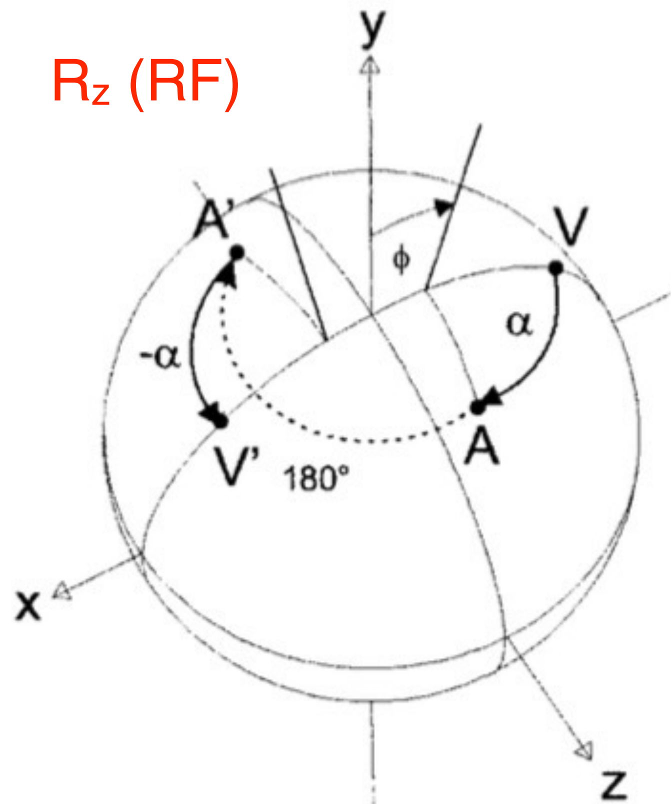


- CPMG: Refocusing pulses "self-correct"
  - 90_x, α_y, α_y, α_y, … or 90_x, $-\alpha_x$, α_x, $-\alpha_x$, …
- Prep Pulse: First refocusing pulse balances echoes
- Non-CPMG: Signal oscillates and decays quickly
- CPMG allows reduced, variable refocusing angles
- Eddy-current-induced phase can prevent CPMG

Hyperechoes

Hennig 2001



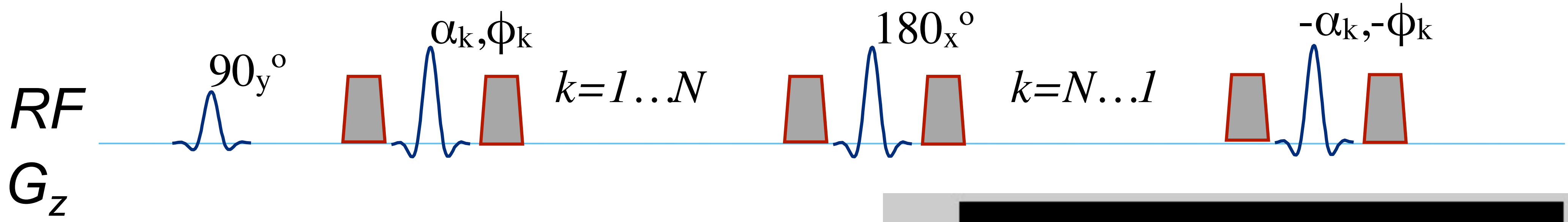


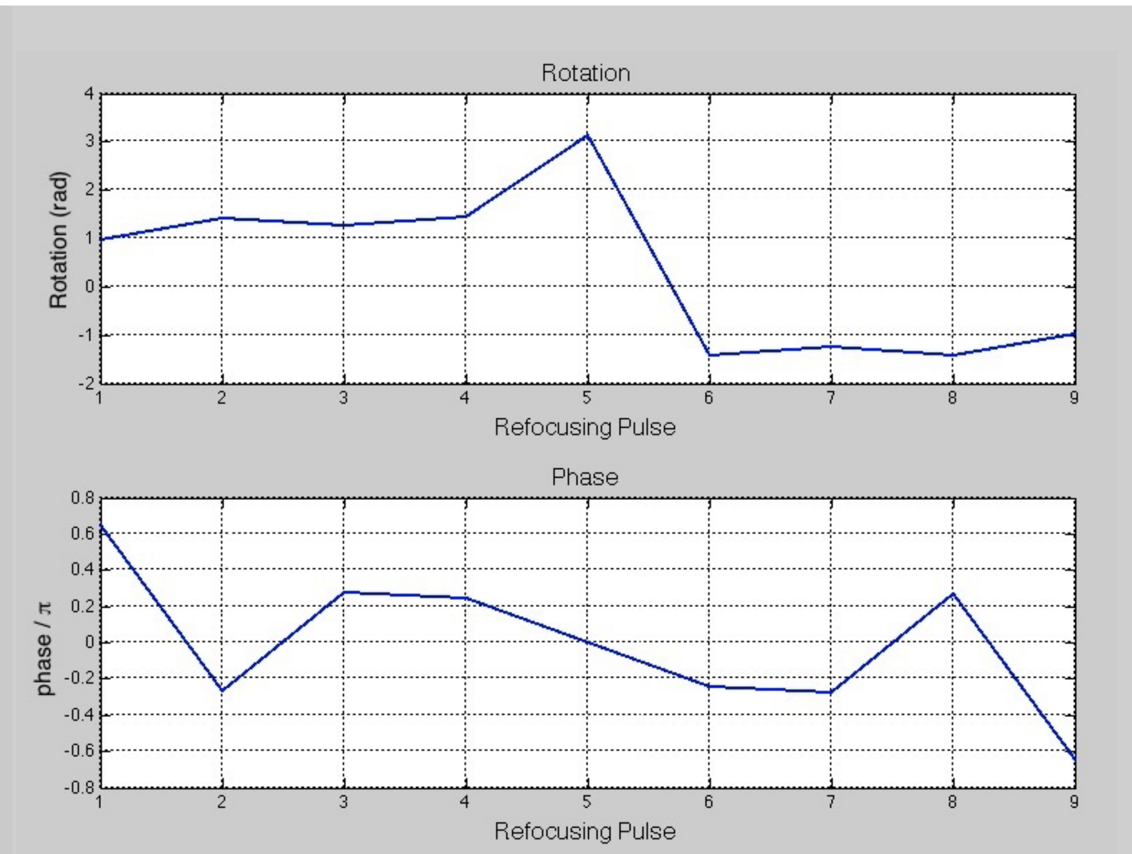
- Symmetry around 180_y°:
  - $R_z(\beta)\ R_y(180°)\ R_z(\beta) = R_y(180°)$
  - $R_\varphi(\alpha)\ R_y(180°)\ R_{-\varphi}(-\alpha) = R_y(180°)$
- The following reduces to $R_x(180°)$, *with φ defined w.r.t x*

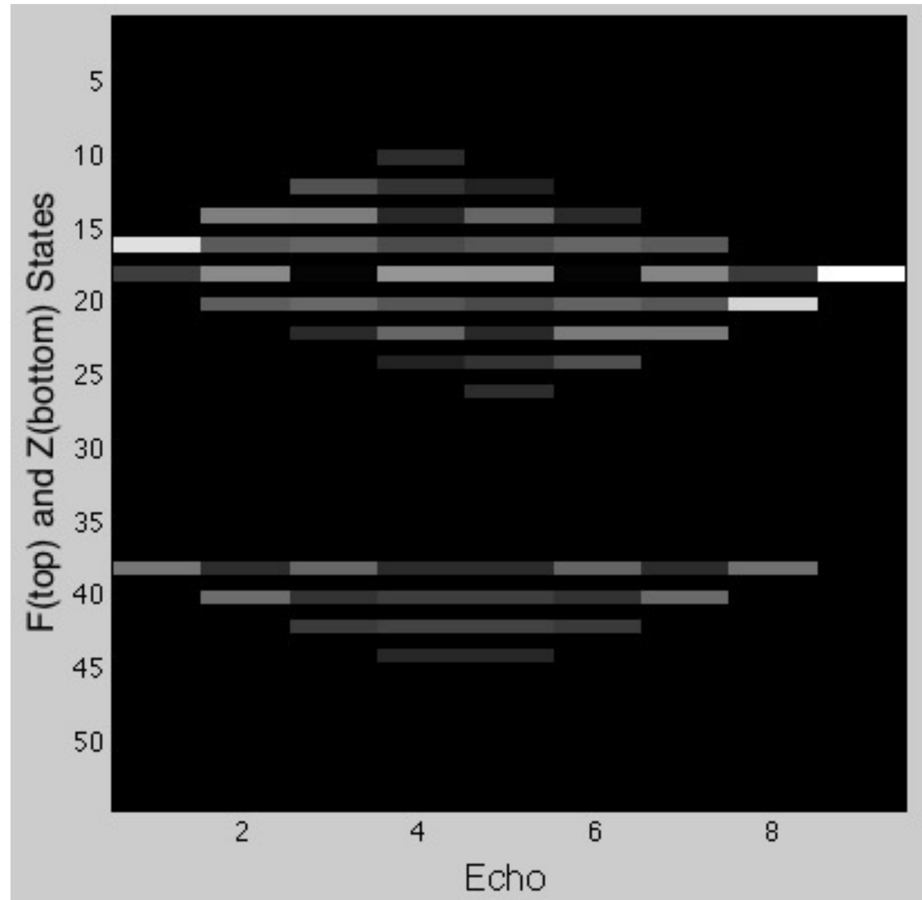
  $(\alpha_1,\varphi_1), (\alpha_2,\varphi_2), \ldots (\alpha_N,\varphi_N), (180°,0), (-\alpha_N,-\varphi_N), \ldots, (-\alpha_2,-\varphi_2),(-\alpha_1,-\varphi_1)$

Hyperecho Example

Hennig 2001





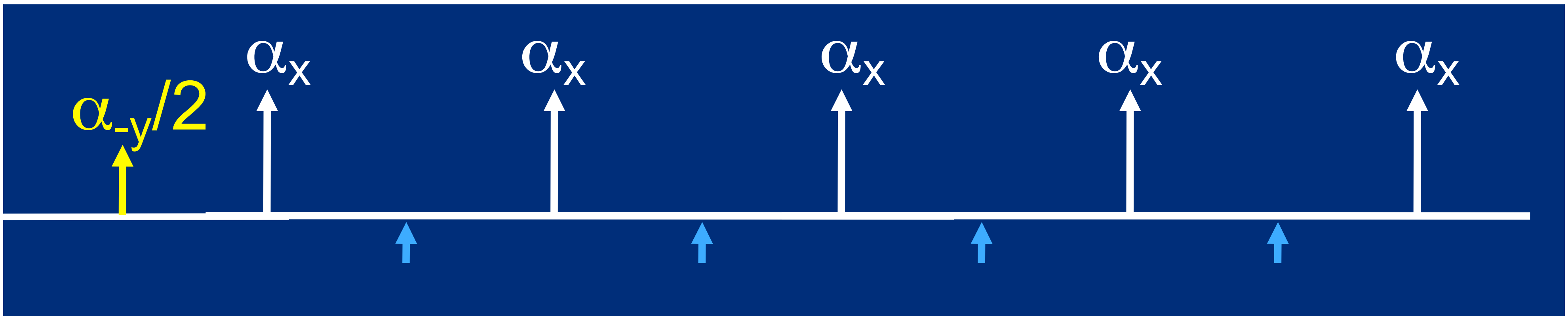


Random α, φ. N=4

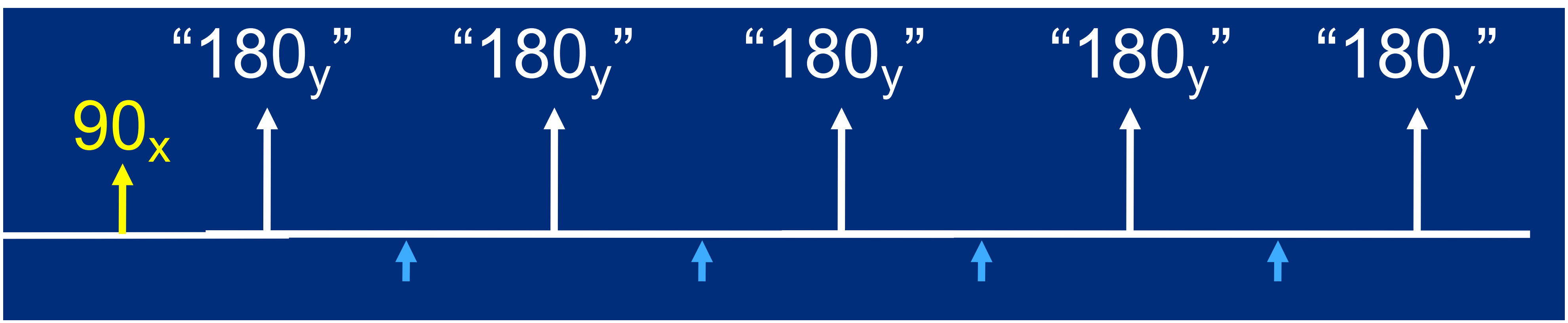
Phase Diagram (epg_cpmg)

SSFP vs Fast Spin Echo

bSSFP



FSE



Spin-Echo vs Balanced SSFP

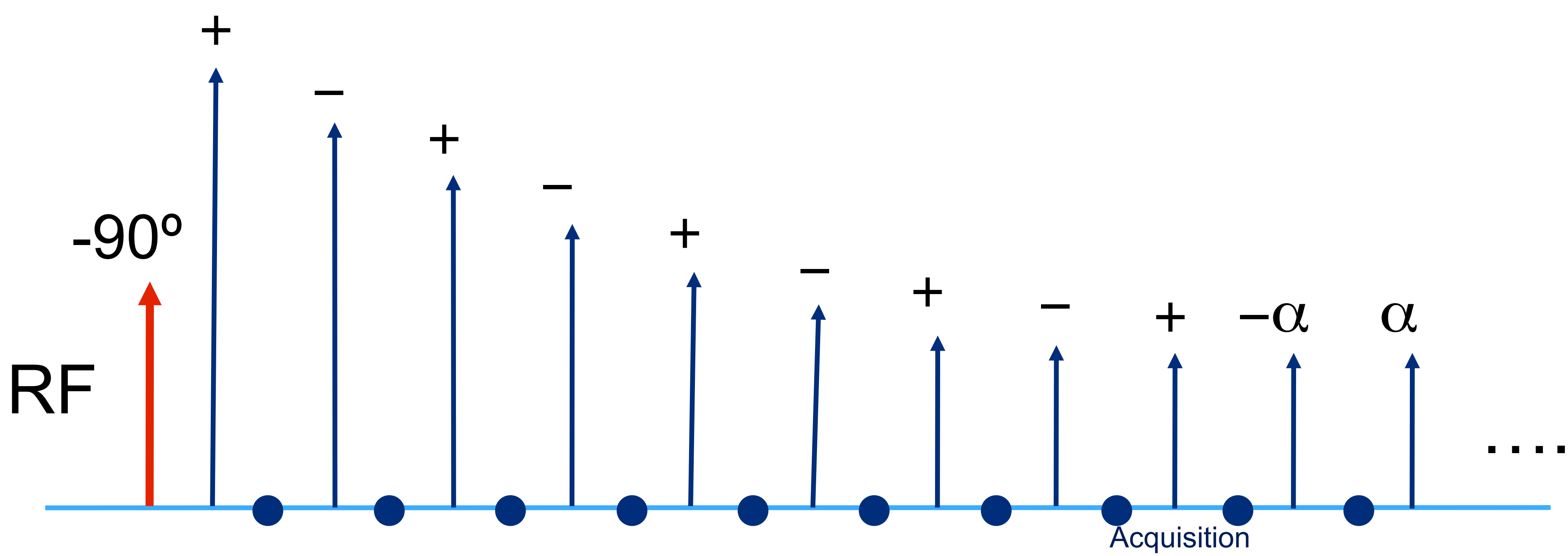
- RARE is bSSFP with high-flip angles and crushers
- 90-TE/2 pulse is like the α/2-TR/2 pulse
  - But steady-state is eliminated by crushers
  - Transient state is imaged

cTIDE:

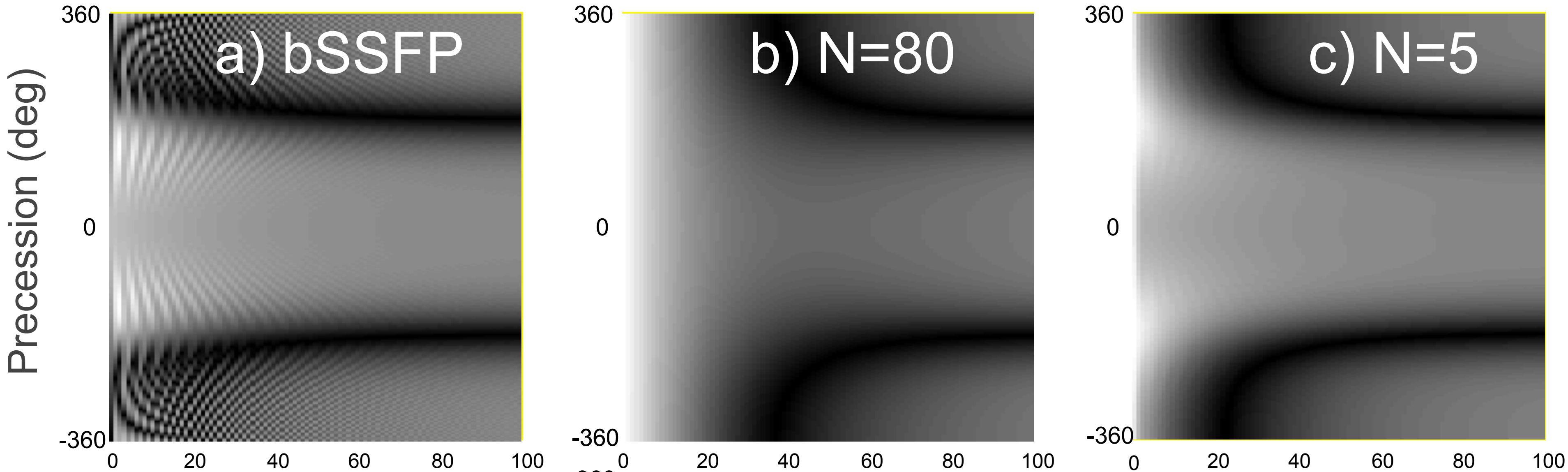
Hennig, et al., 2002

"Continuous Transition into Driven Equilibrium"

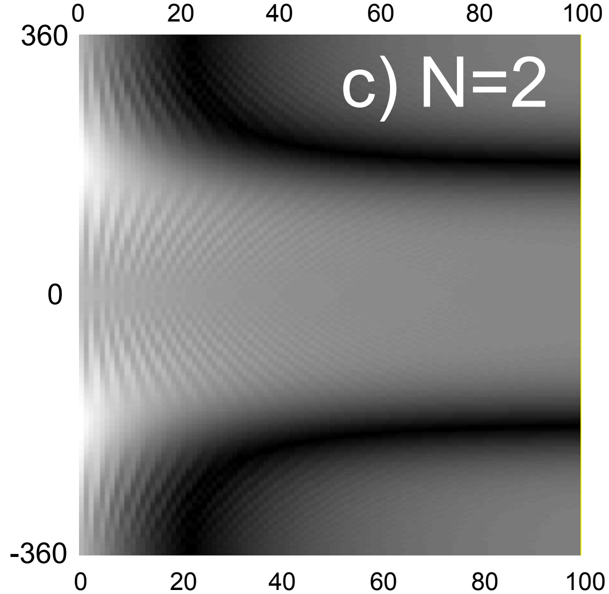
- Start with 90º pulse.
- Ramp down from 180º to α
- Looks a lot like modulated spin-echo train
- Usually acquire data during pseudo steady state

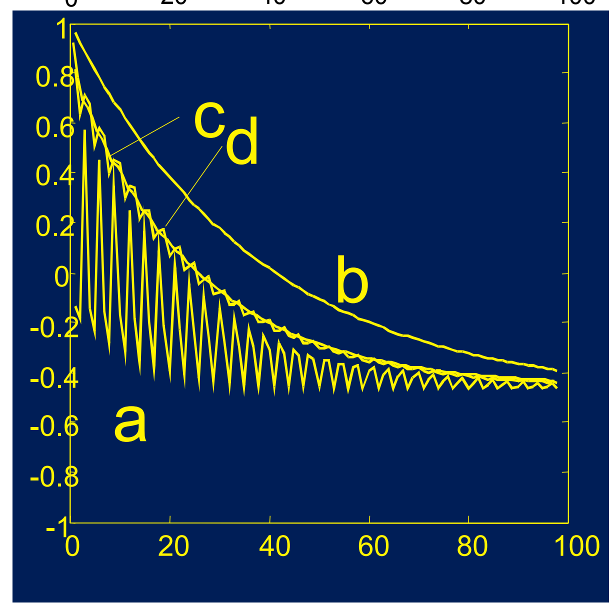


Off-resonance Behavior of TIDE



TR #





a: trueFISP

b: N=80

c: N=5

d: N=2
(90° - 90°+α/2 - α...)

(Courtesy of Jurgen Hennig, Univ. of Freiburg)

Spin-Echo Trains: Additional Points

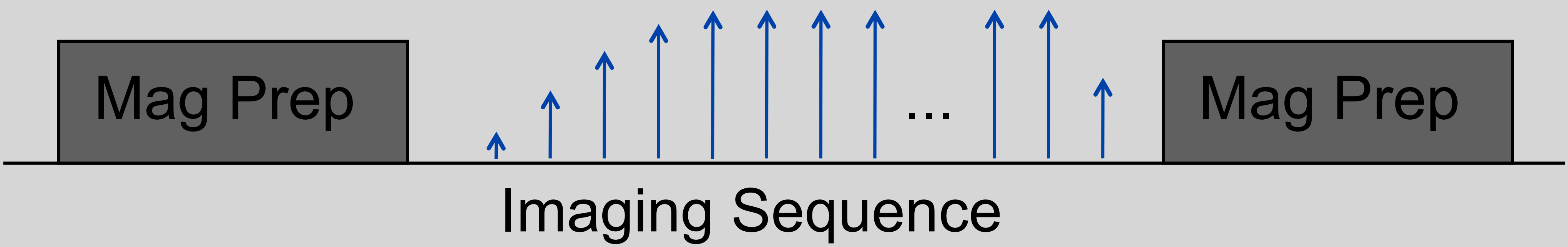
- J-coupling: Relaxation mechanism in fat
  - Rapid refocusing decreases relaxation rate, so fat is brighter on FSE/RARE than SE
- MT: Interleaved multislice
  - Slice-selective pulses are off-resonance to others
  - MT saturation effect - suppresses some signals

Summary: Spin-Echo Sequences

- Basic spin echo
- Echo-trains: RARE, FSE, TSE
  - Efficient T2 and PD contrast
  - Extreme cases: SSFSE/HASTE
  - 3D Echo trains
- Signal considerations
  - CPMG / Reduced refocusing angles
  - Modulated echo trains

Magnetization Preparation Sequences

- Acquisition method may not give desired contrast
- "Prep" block adds contrast (and/or encoding)
  - MP-RAGE = Magnetization prepared rapid acquisition with gradient echo (Mugler, ~1990)
  - Inversion-recovery (IR) prep
  - Fat saturation
  - T_2-prep
  - Diffusion-weighted imaging



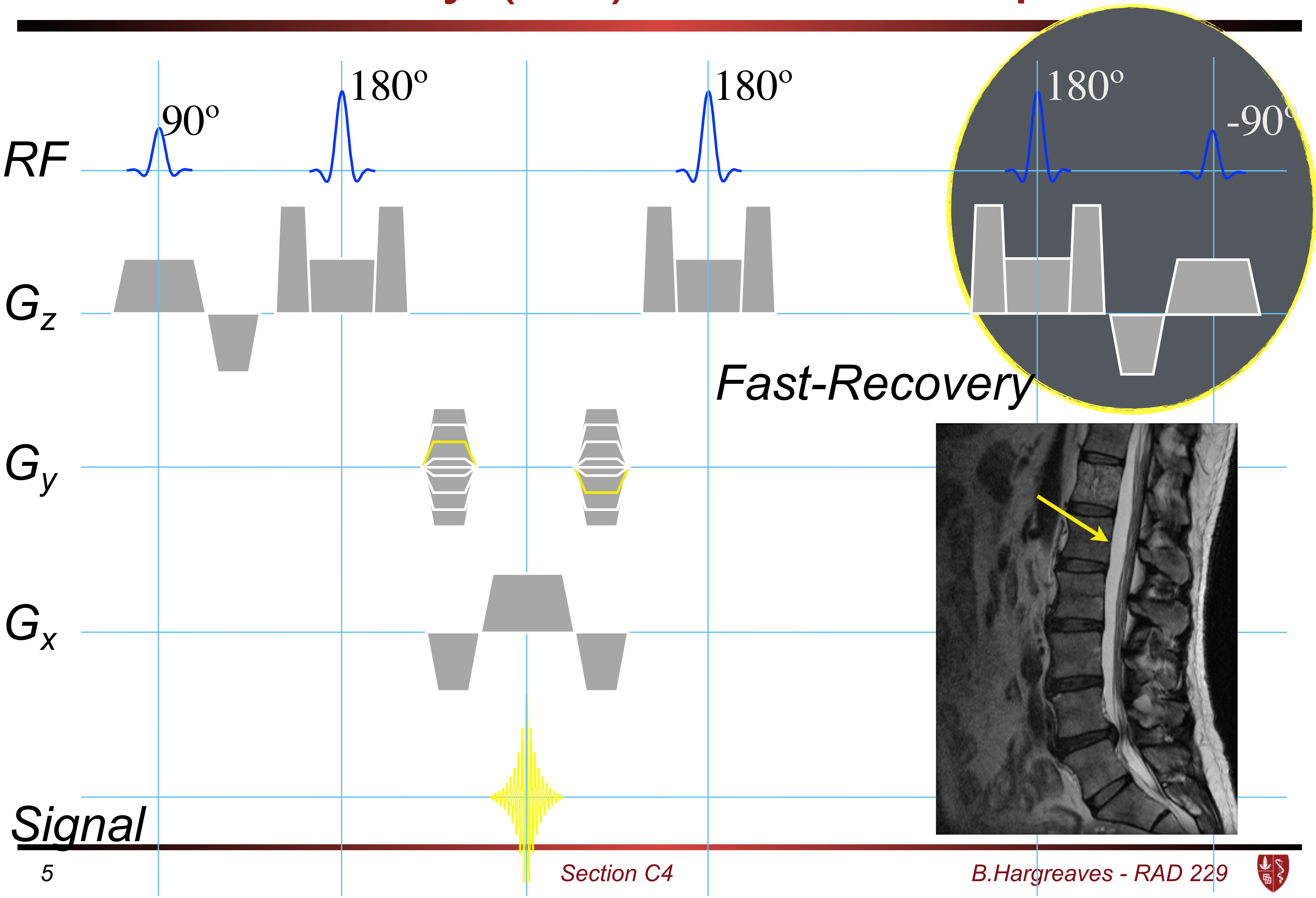
(From Previous) Challenge: Diffusion

Challenge: Diffusion (Solution)

Contrast Review

- Spin Echo
  - PD, T1, T2
  - Echo-train effects
- Gradient Echo
  - bSSFP, Gradient Spoiled (T2/T1)
  - RF spoiled (T1)
  - PD (how?) is inefficient, T2 is not possible

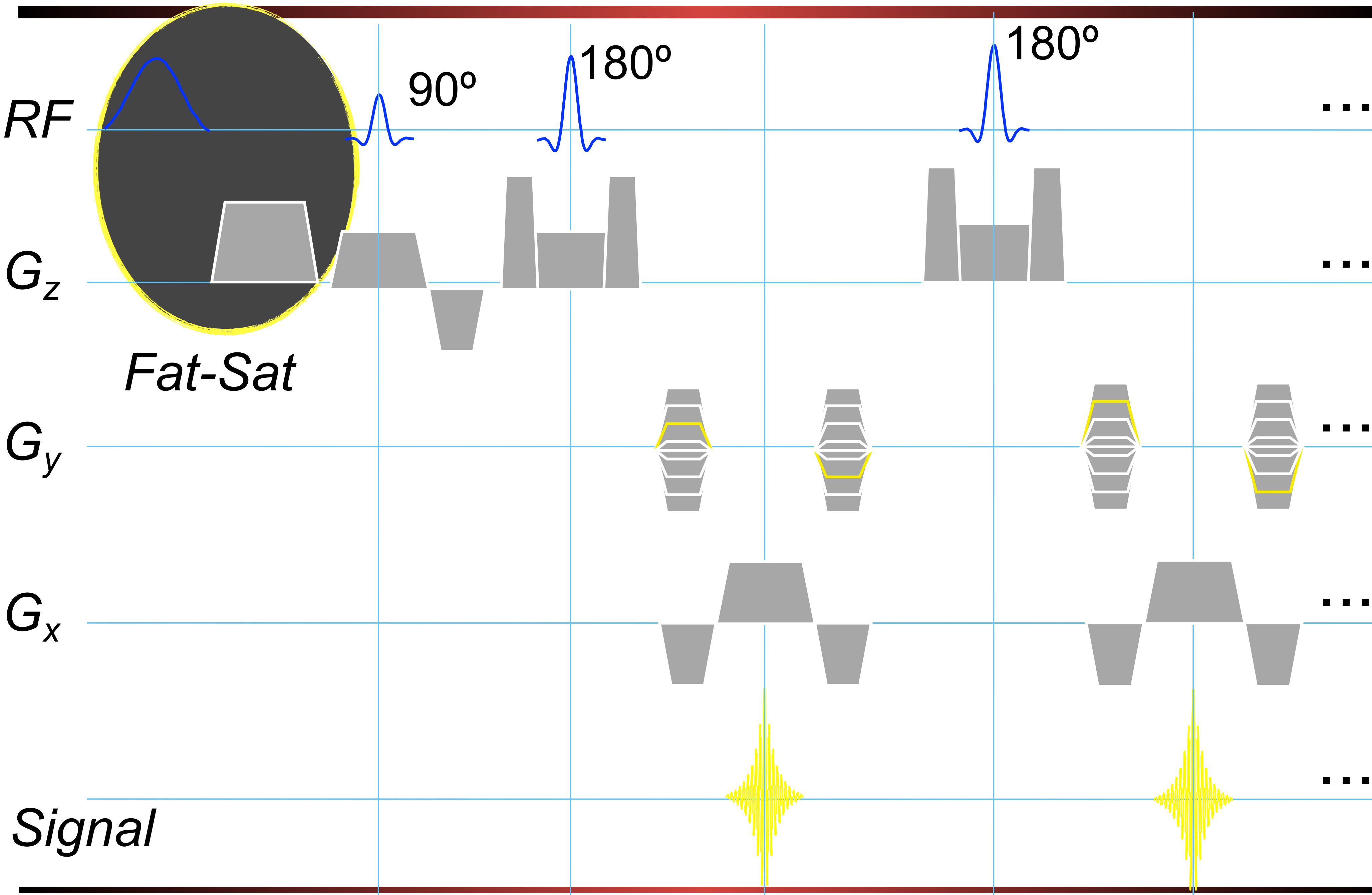
Fast Recovery (FR) or Driven Equilibrium



Saturation or Nulling

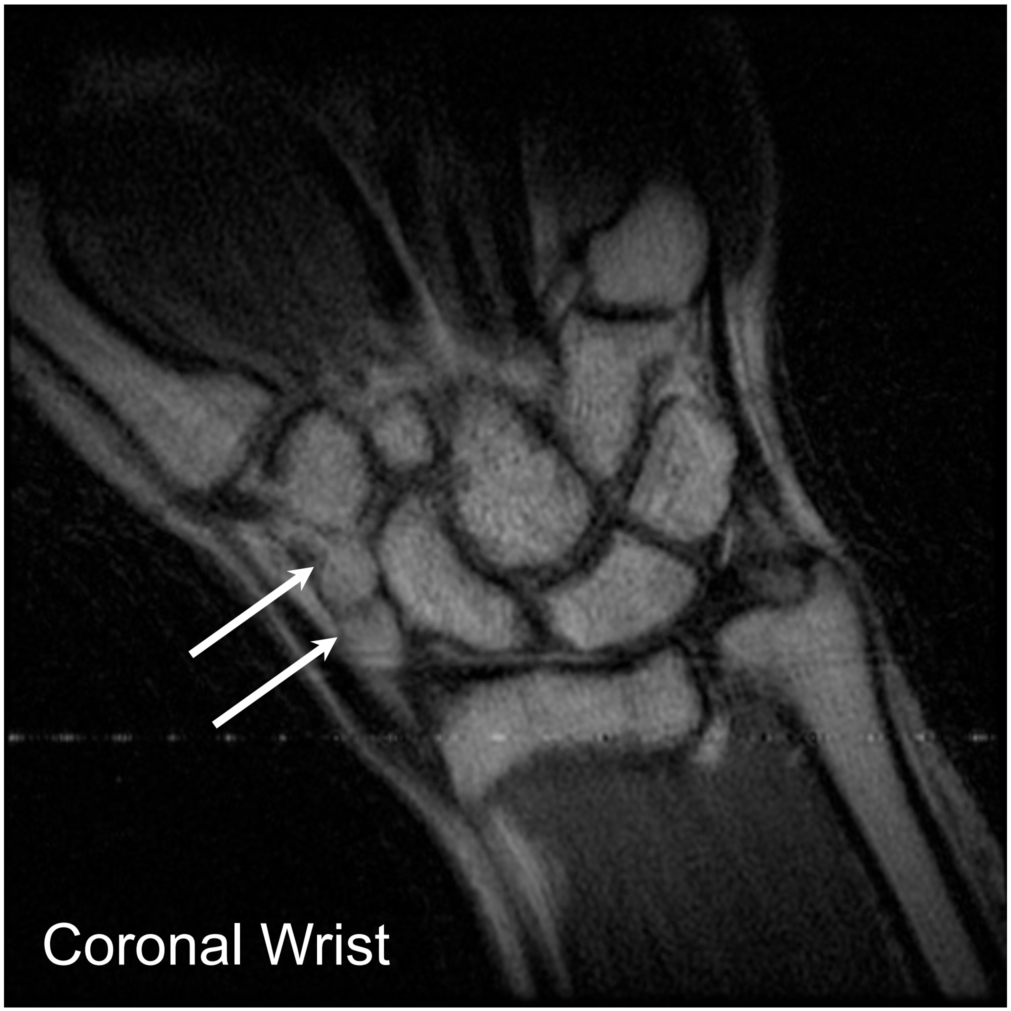
- Eliminate the signal from something
  - Chemical species
  - Regions of image
- Advantages
  - Minimal cost (example, can do short TE)
  - Increase dynamic range for desired signal
- Disadvantages
  - Exciting unwanted signal - it can come back!

Fat-Saturated FSE



Fat Suppression for Contrast

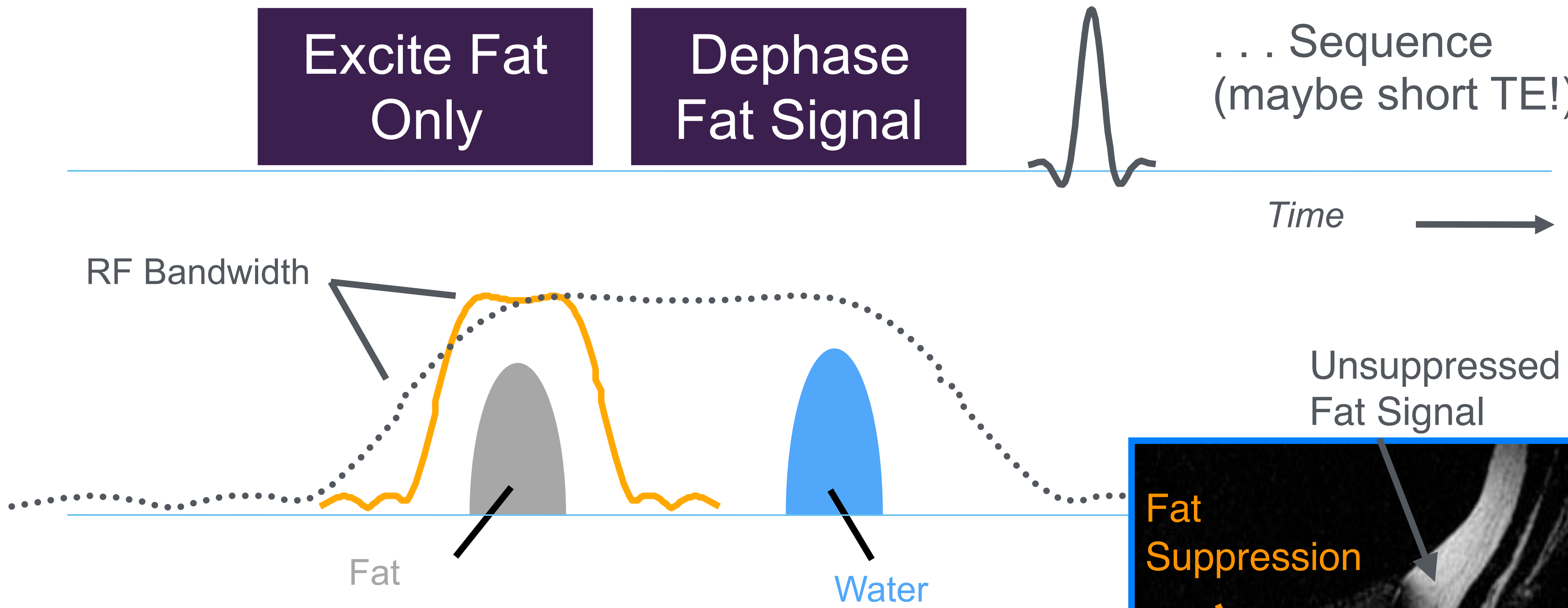
PD FSE

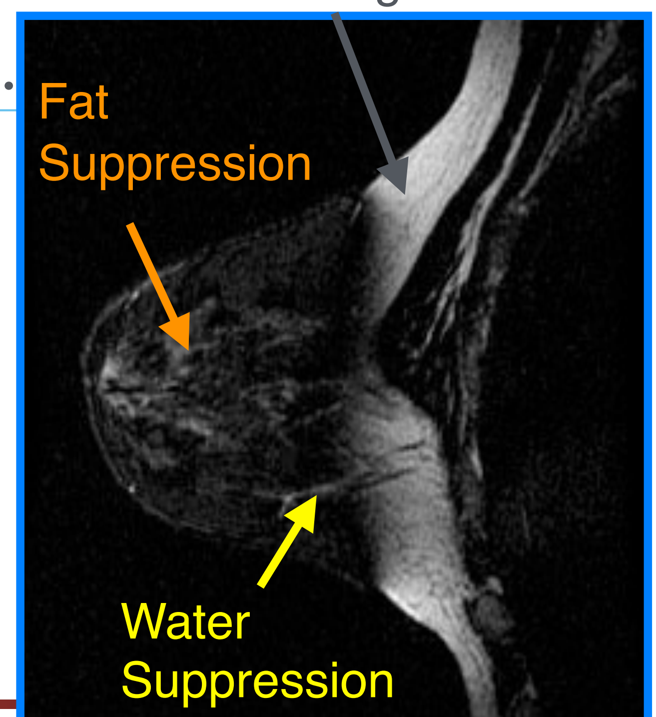


Fat-Sat PD FSE

Radial cyst was otherwise iso-intense with fat

Fat Saturation





- Chemically-selective excitation
- Dephaser gradient
- Normal imaging sequence

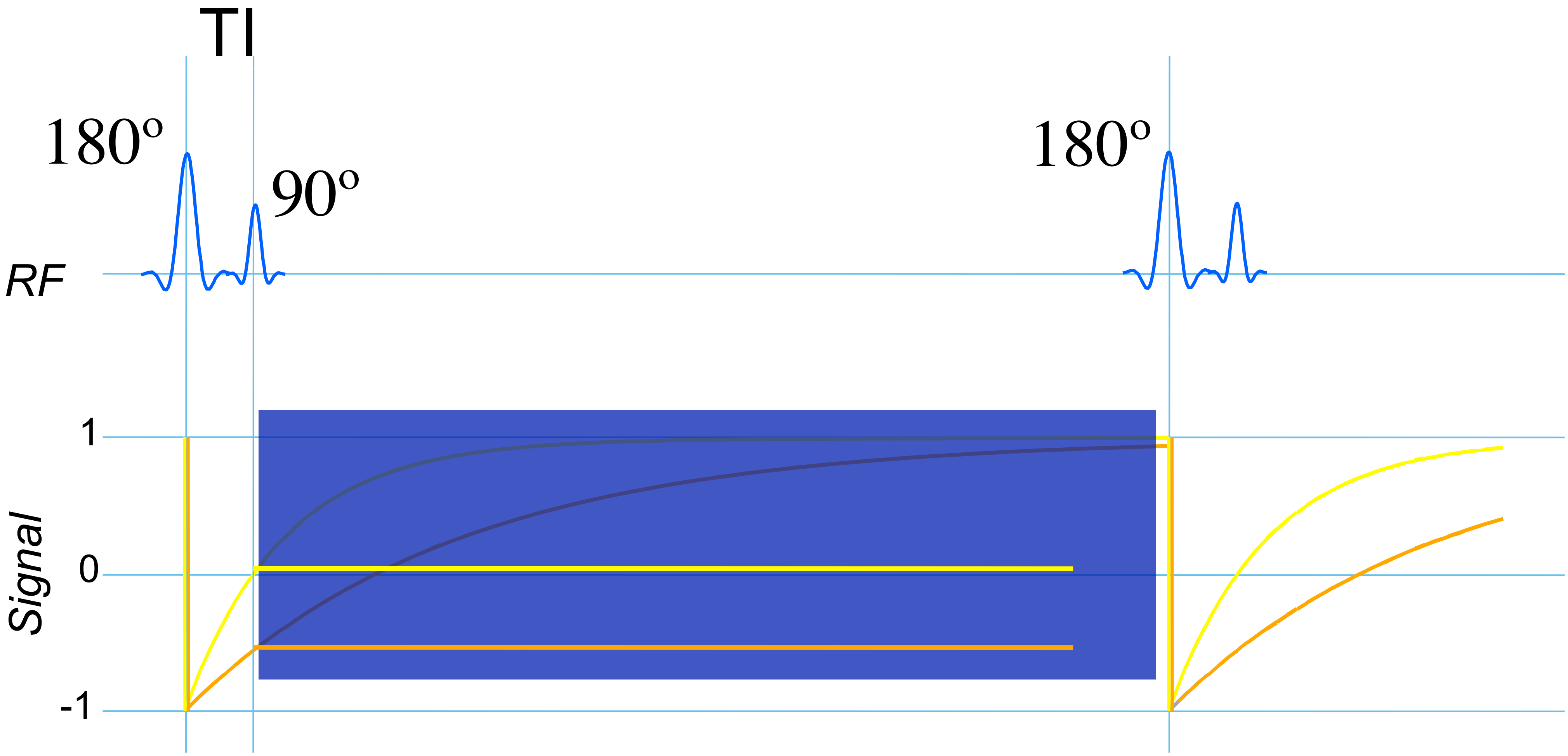
Effect of Fat Saturation

Fat-Saturated (PD)

Not Fat-Saturated (T1w)

Questions: Fat Saturation

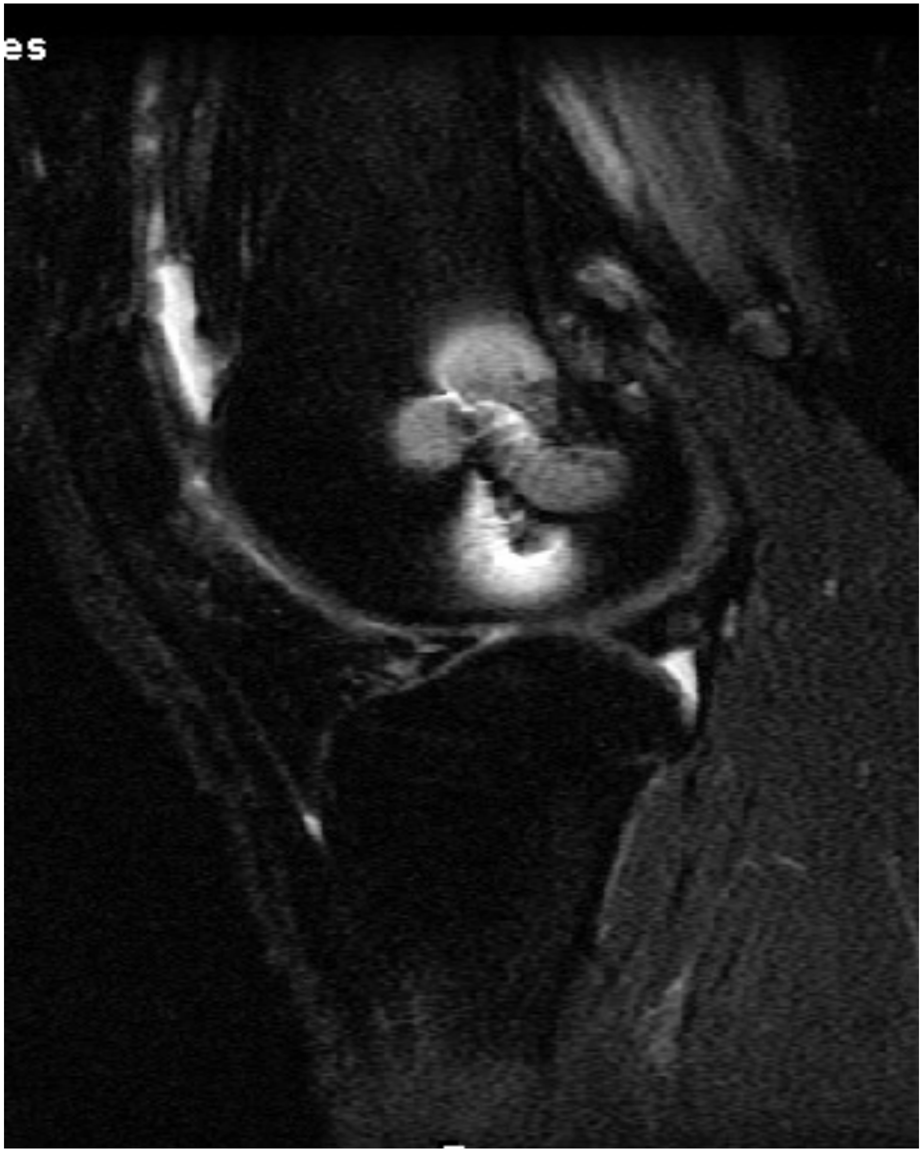
Inversion-Recovery



Fat suppression based on T_1

Short TI Inversion Recovery (STIR)

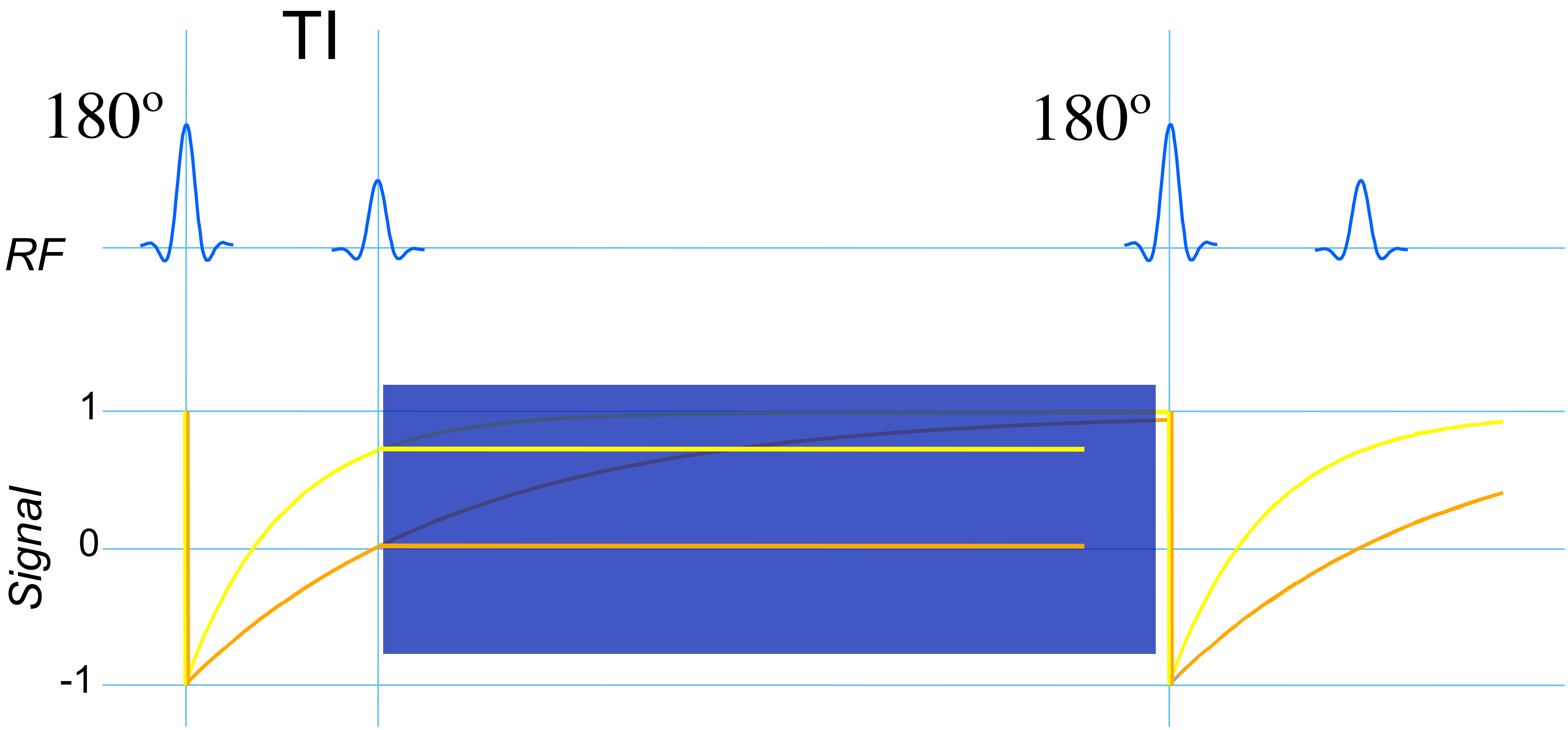
Fat Suppression near B_0 Inhomogeneity



Fat Sat

STIR

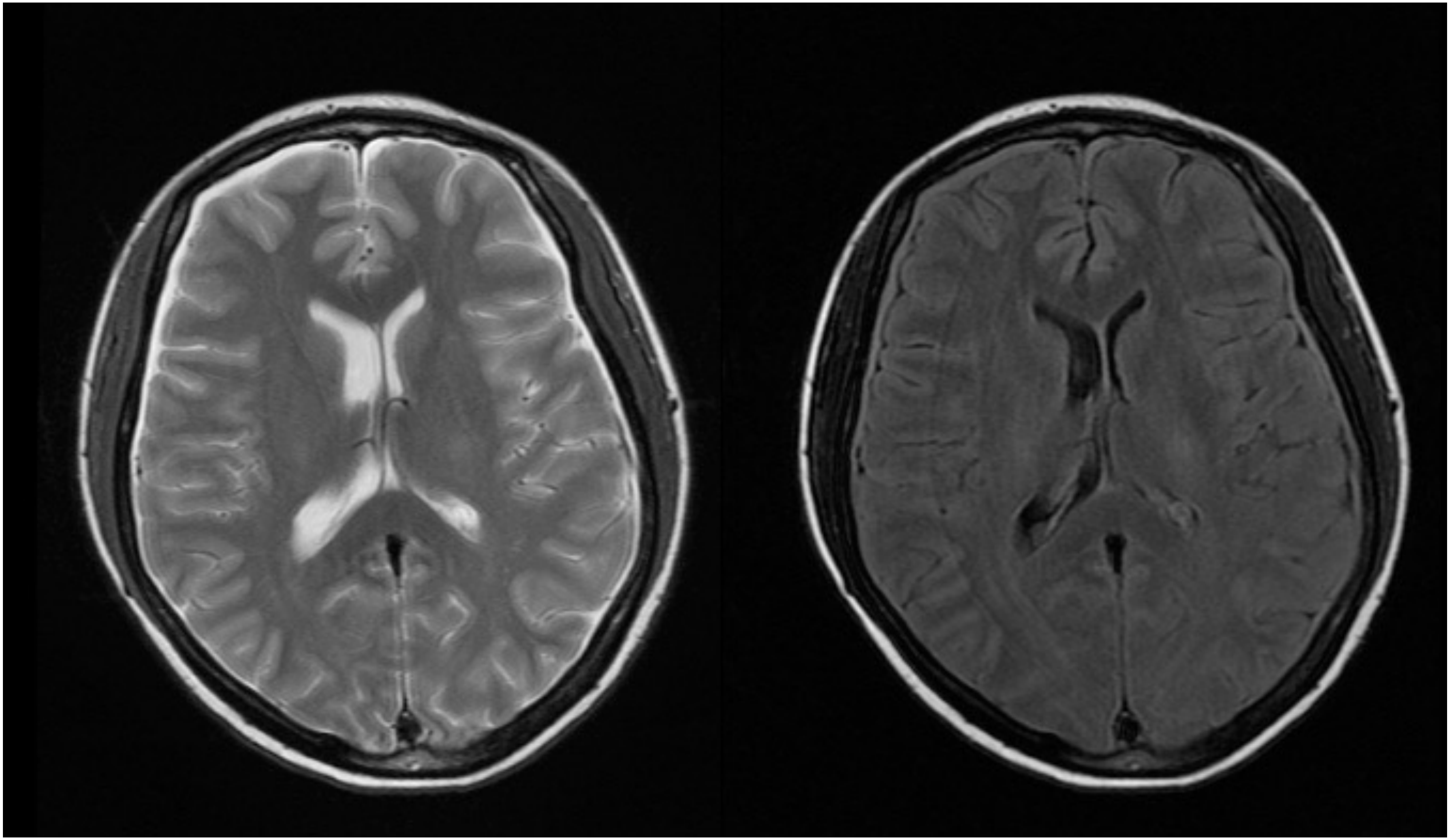
Fluid Attenuated Inversion-Recovery



Fluid suppression based on T_1

FLAIR

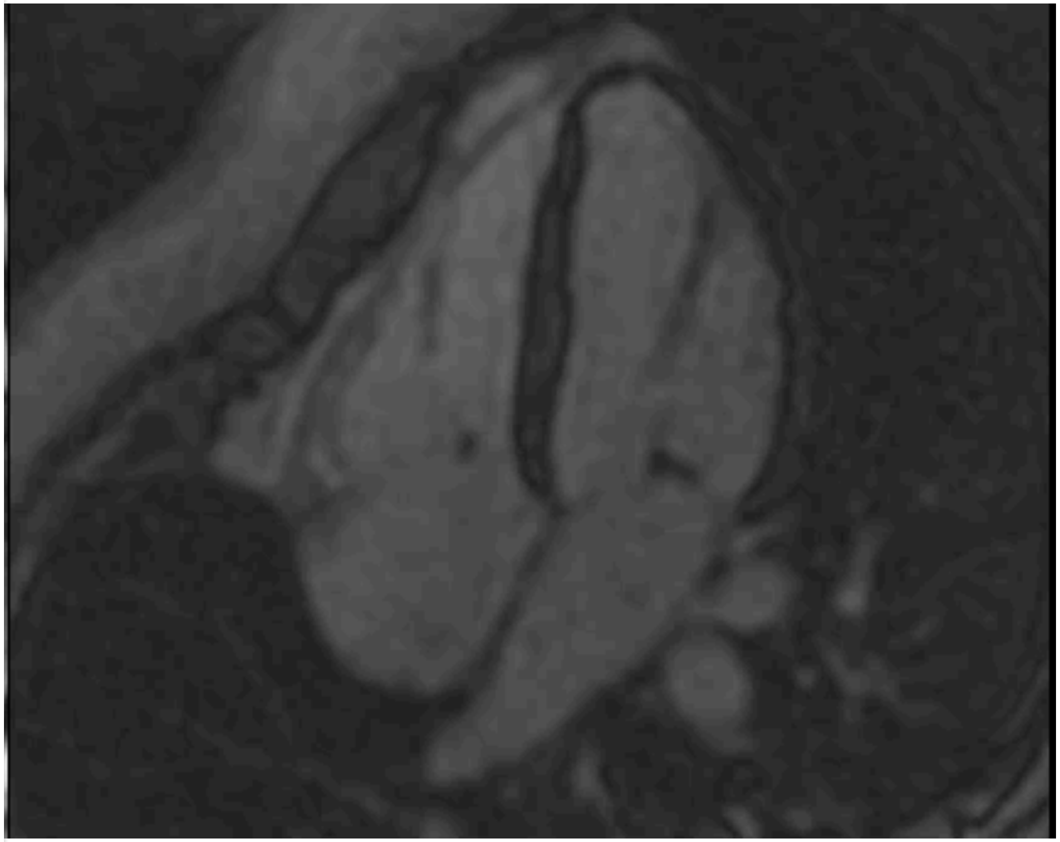
Long Inversion Time (TI) - FLAIR



Long TI suppresses fluid signal

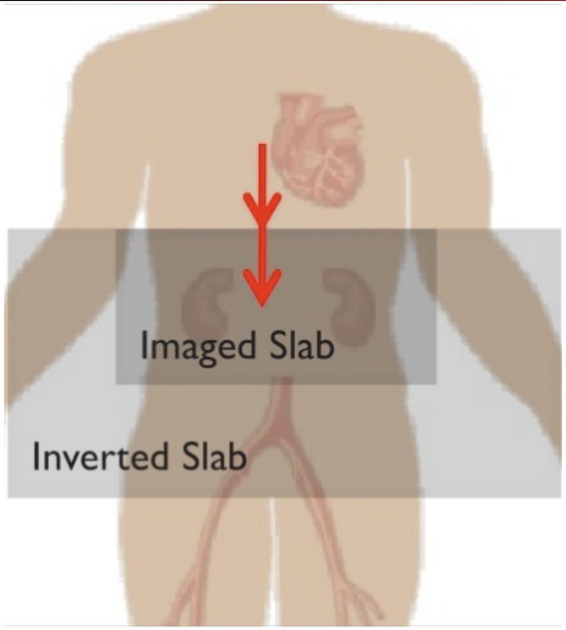
IR Prep to enhance T1 contrast

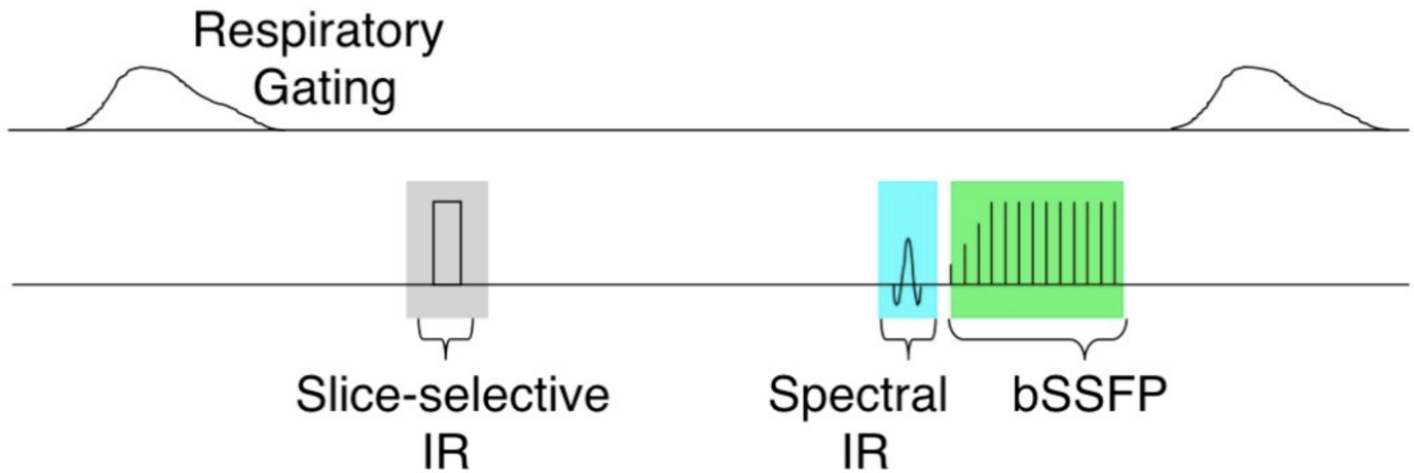
- Often used with GRE (MP-RAGE)
- Example: Cardiac CINE, IR at start (note septum)



IR-Prep RF-Spoiled

Mag-Prep: Inflow-enhanced MRA



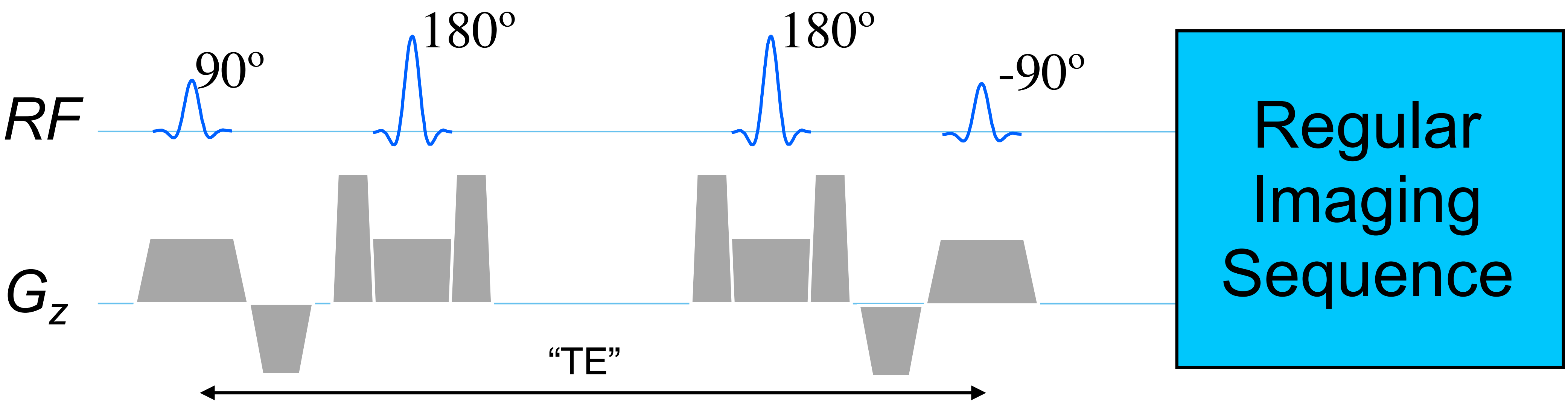


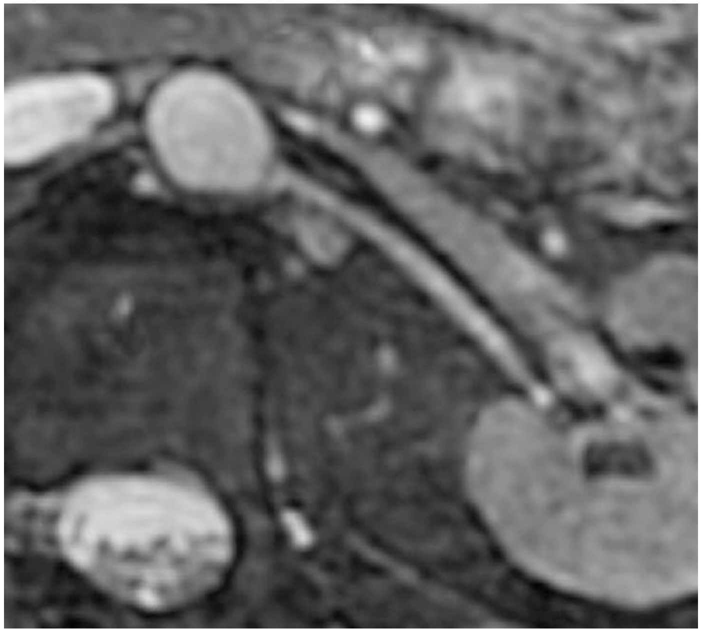
Preparation:
- Background Suppression
- Fat Suppression

Courtesy Pauline Worters

Questions: Inversion-Recovery

T2-Prep (Enhance T2 contrast)

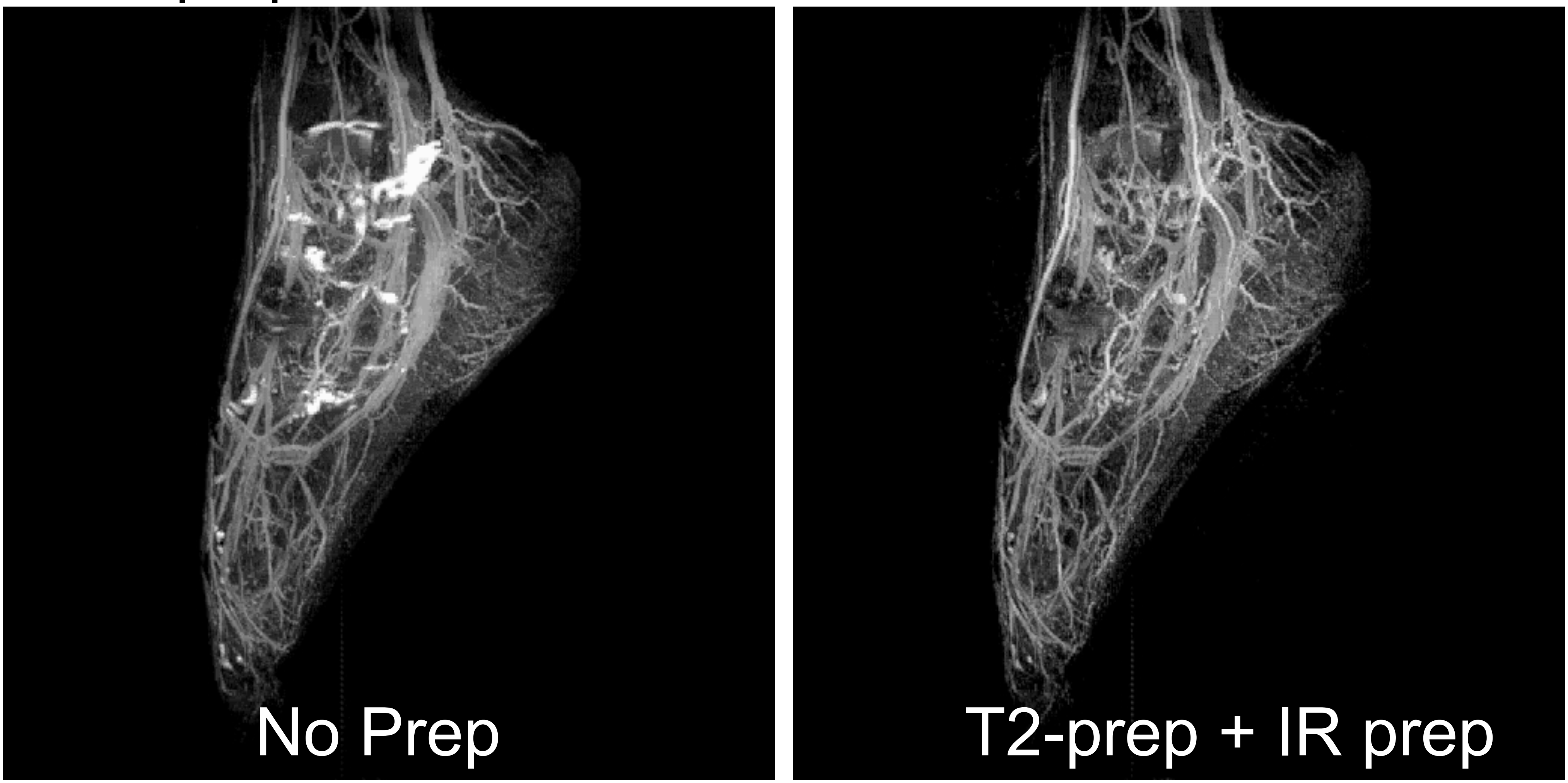




T2-prep + Fat-Sat Renal Artery

T2-Prep: Flow-Independent Angiography

- Inversion: Suppress synovial fluid
- T2-prep: Arterial-venous contrast

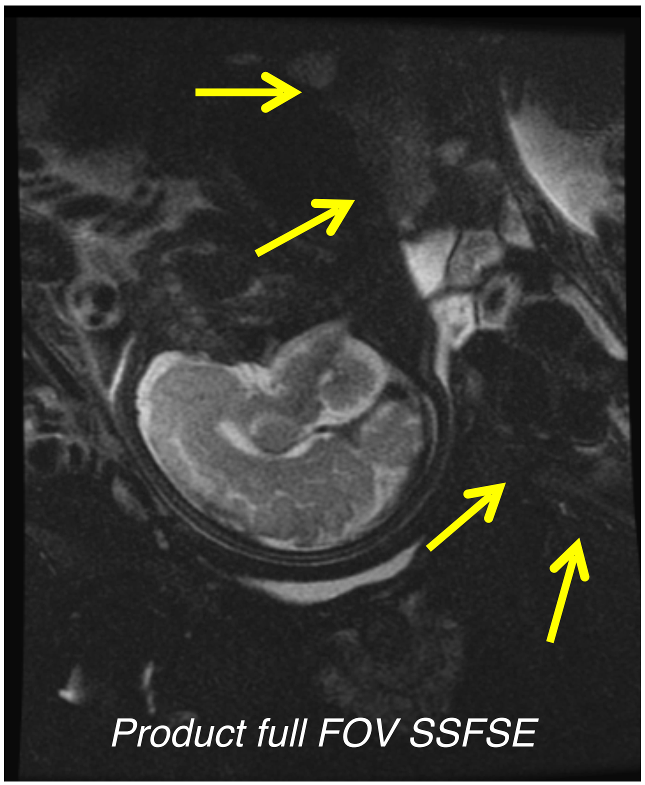


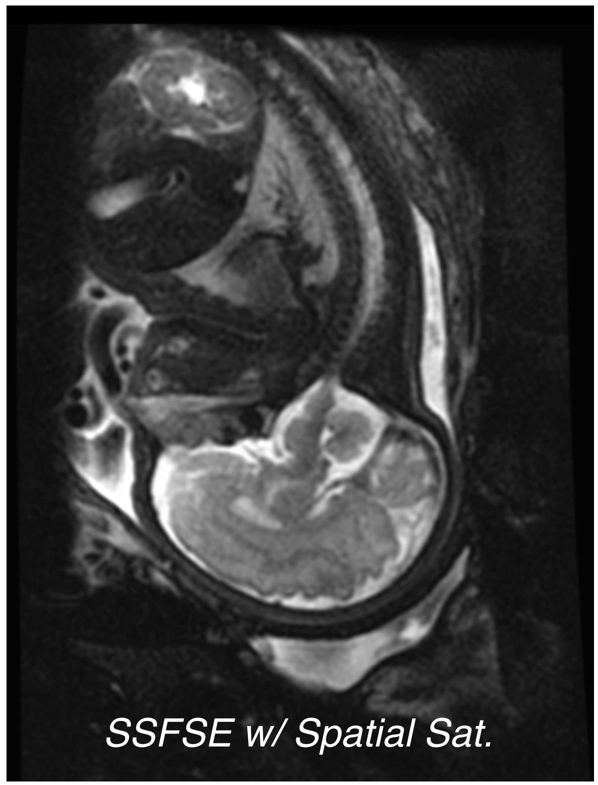
Courtesy Neal Bangerter

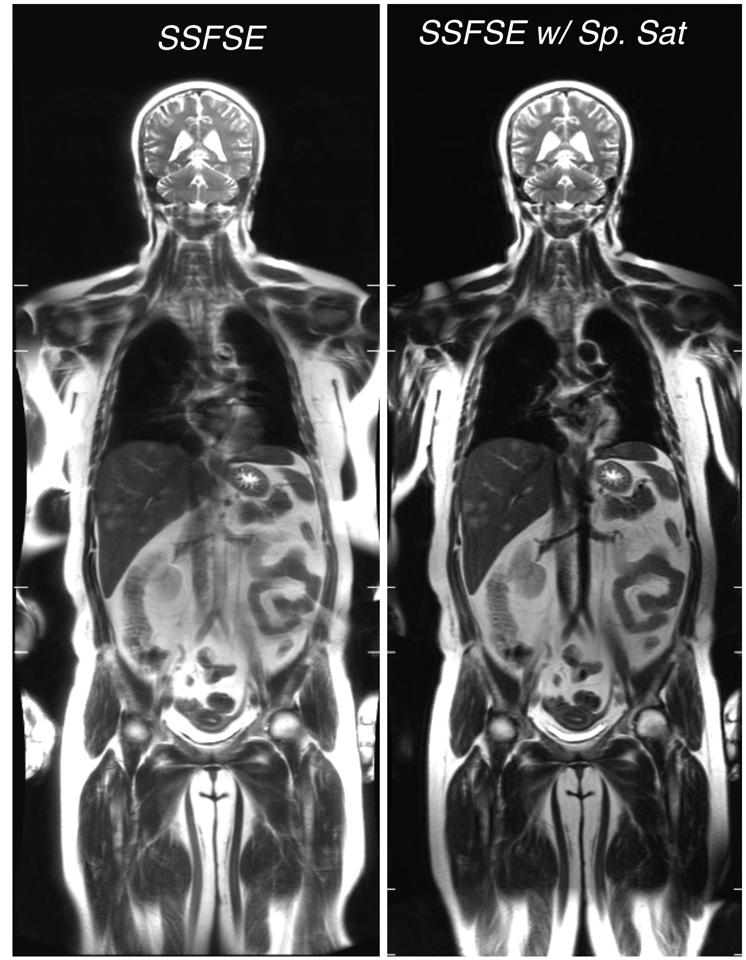
Questions: T2-prep

Spatial Saturation

- Reduced FOV imaging
- Saturate “bands” outside FOV to prevent aliasing

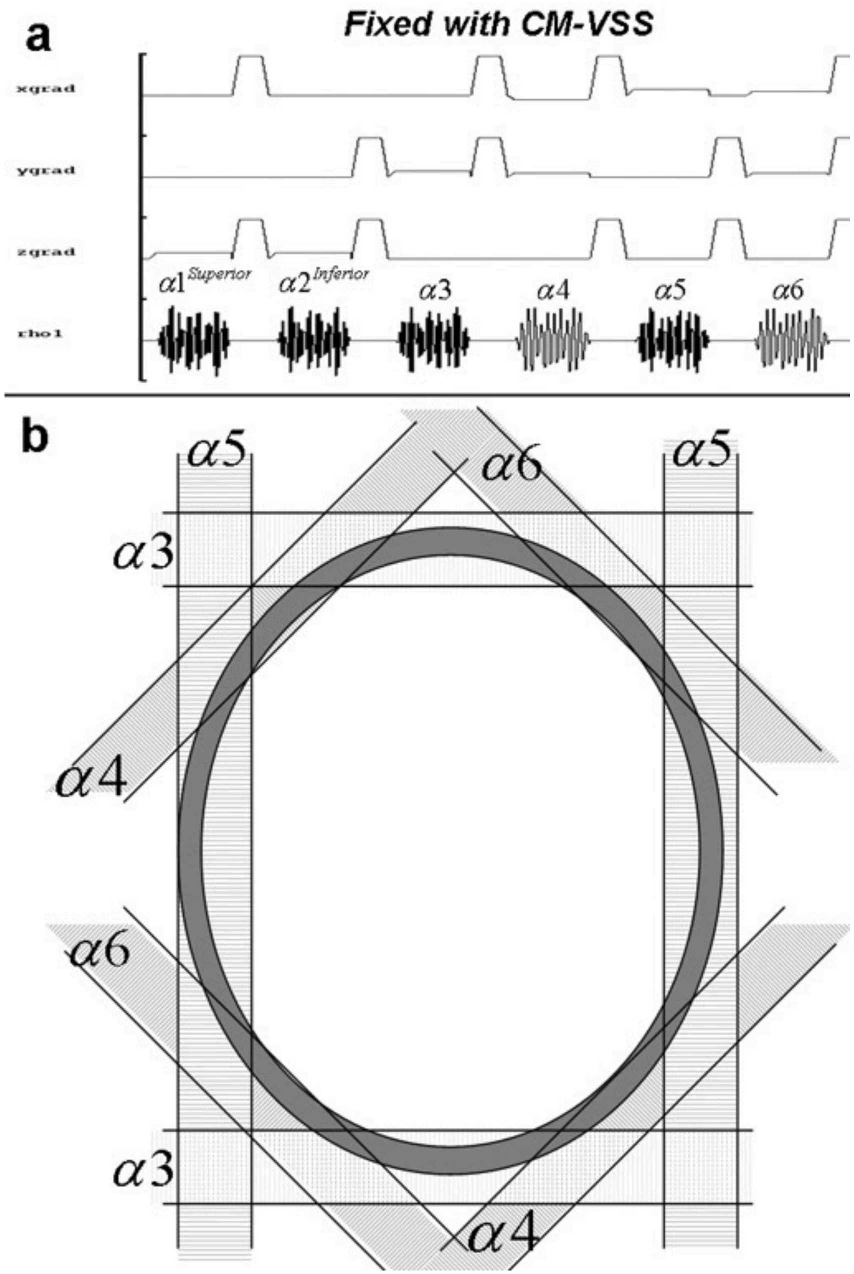






Spatial Saturation

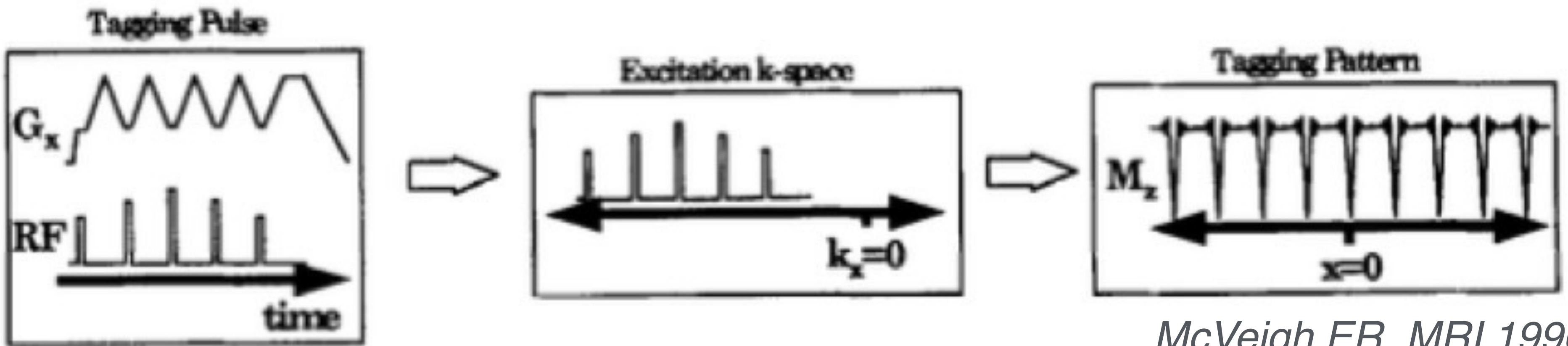
- Use with arbitrary sequences
  - Save time with reduced FOV
- Very selective w/o time penalty
- Cosine modulate (dual-band)
  - *Osorio JA, et al. MRM 2009*



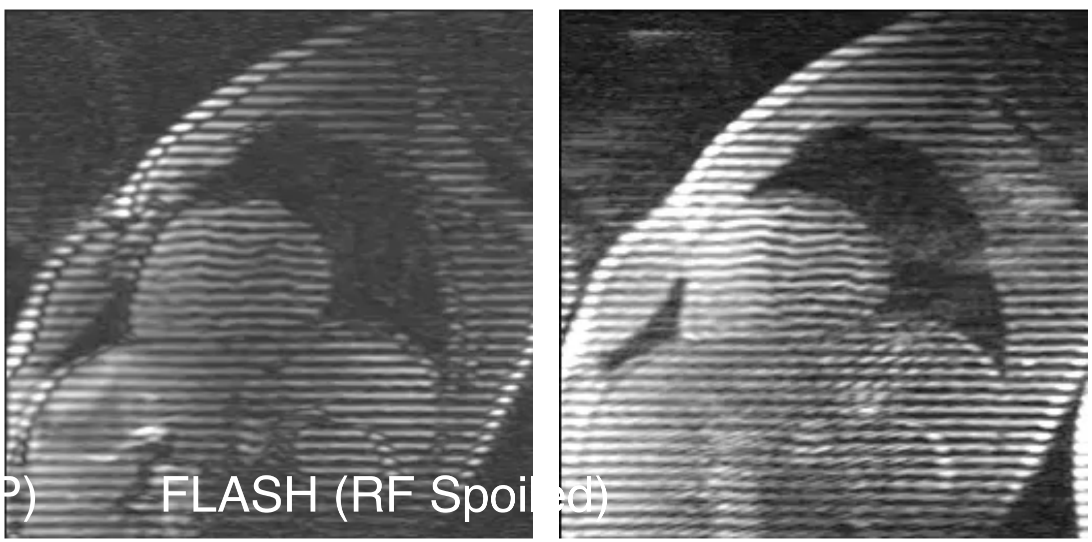
Myocardial Tagging

Zerhouni E, 1988

- Spatially selective saturation pattern (lines, grid)
- Often ‘cine’ acquisition

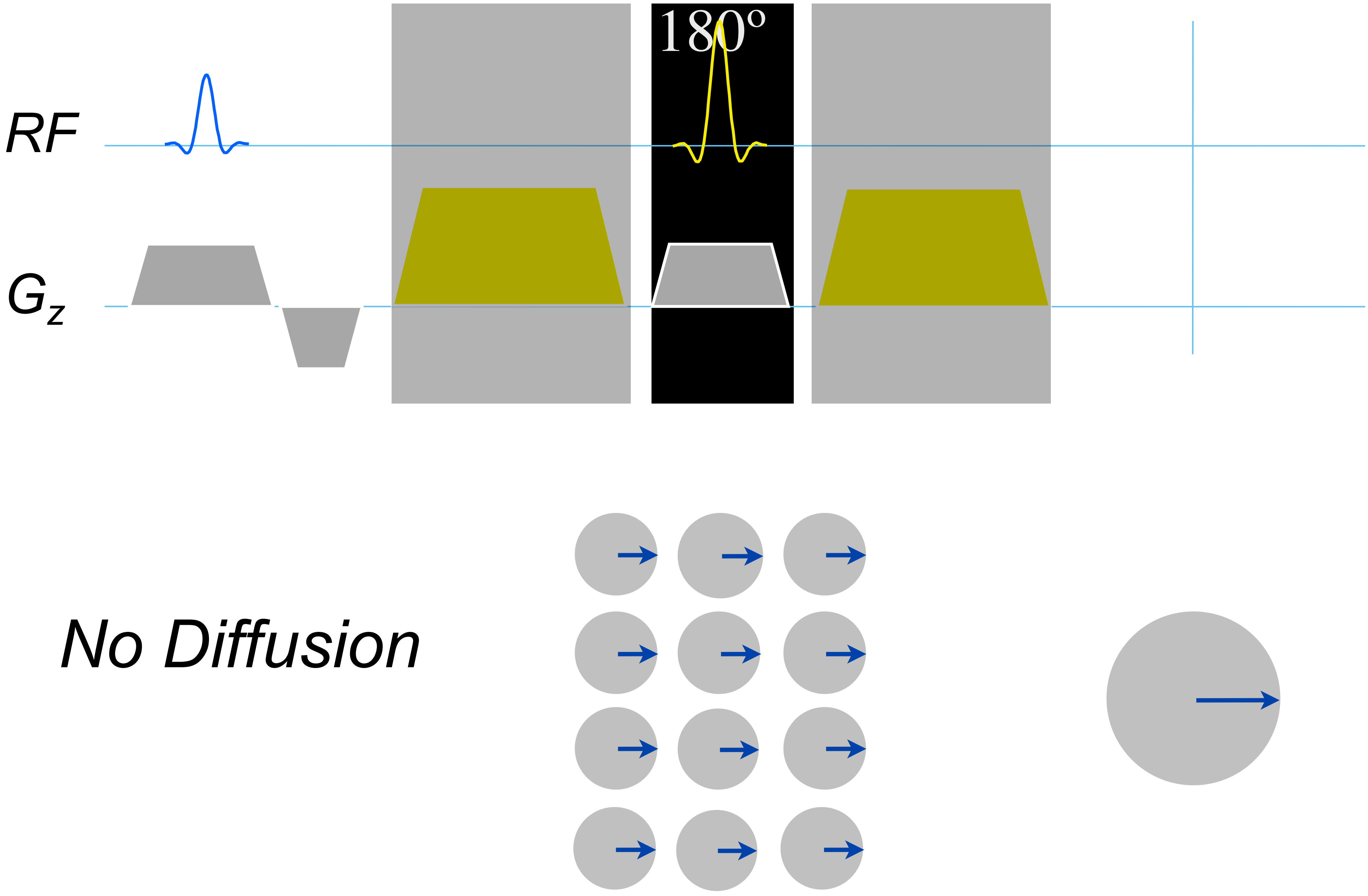


McVeigh ER, MRI 1996

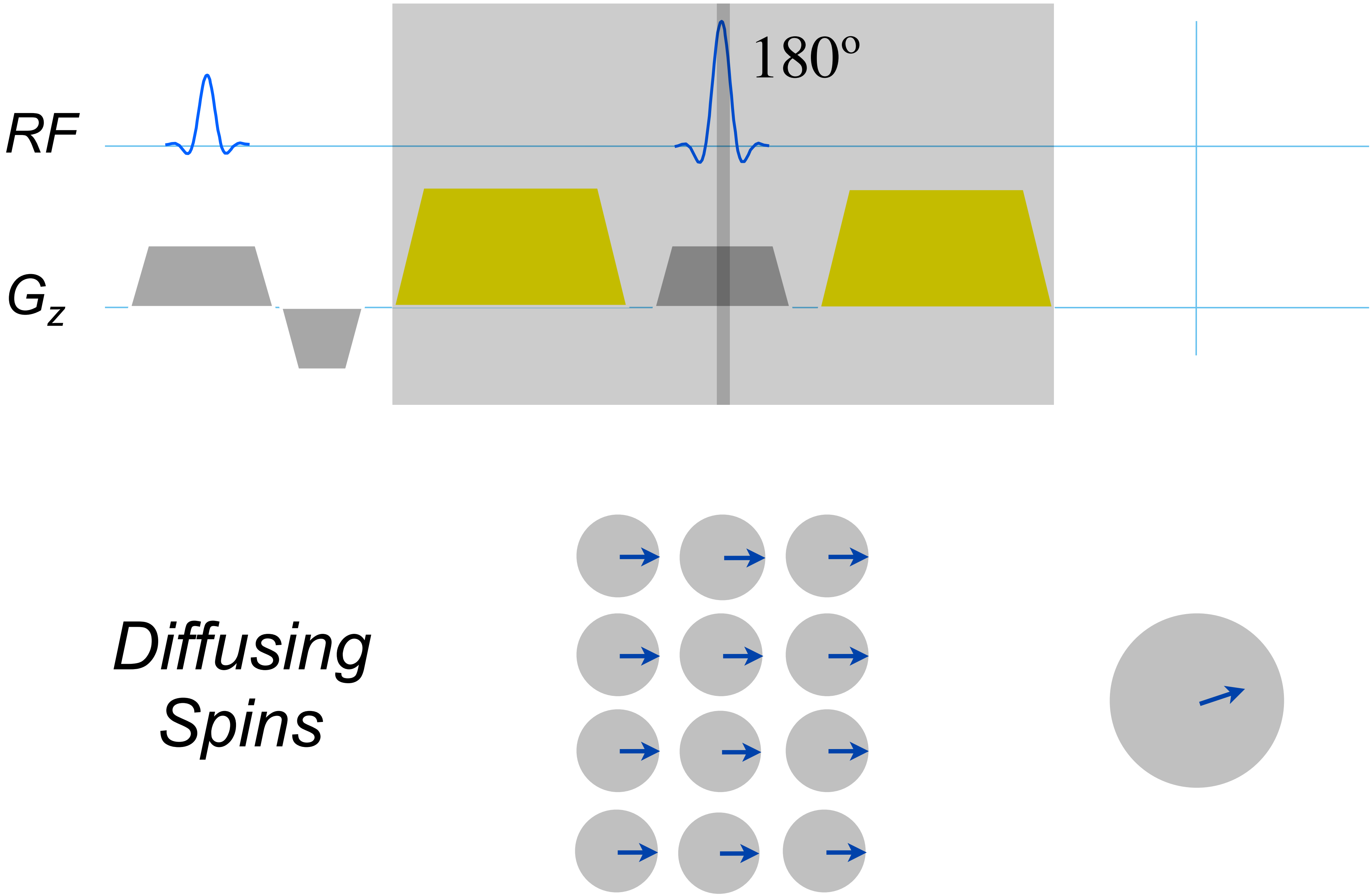


Courtesy J. Zwanenburg (MRM 2003)

Diffusion-Weighted Imaging (DWI)

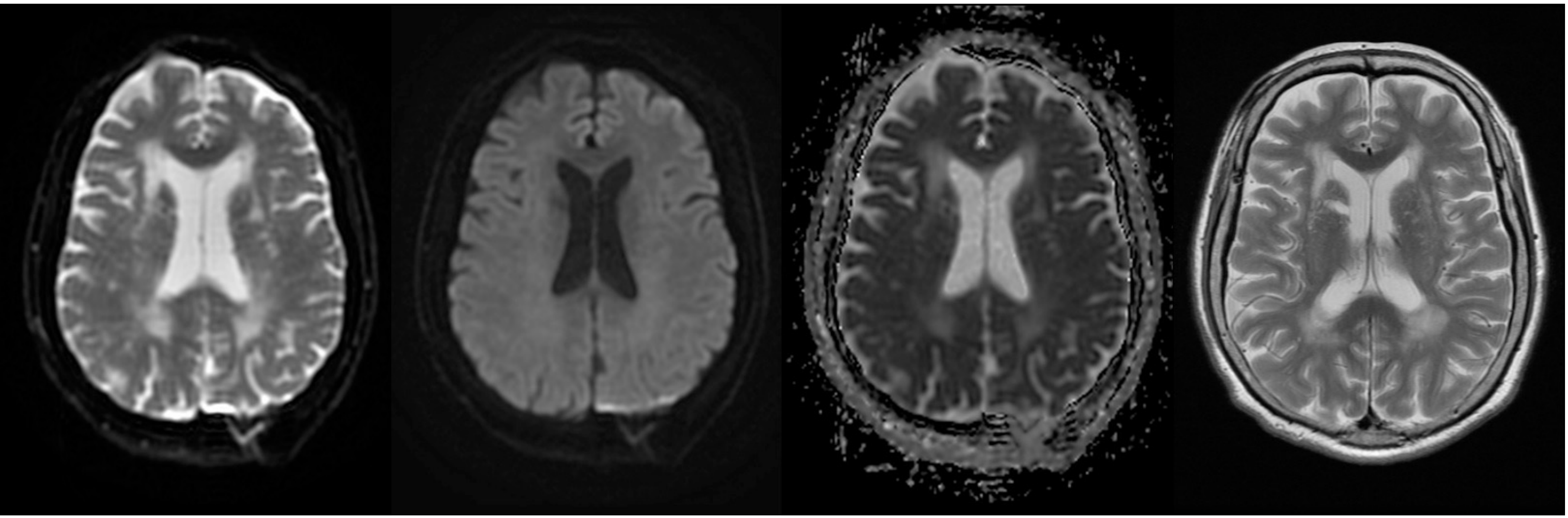


Diffusion-Weighted Imaging (DWI)

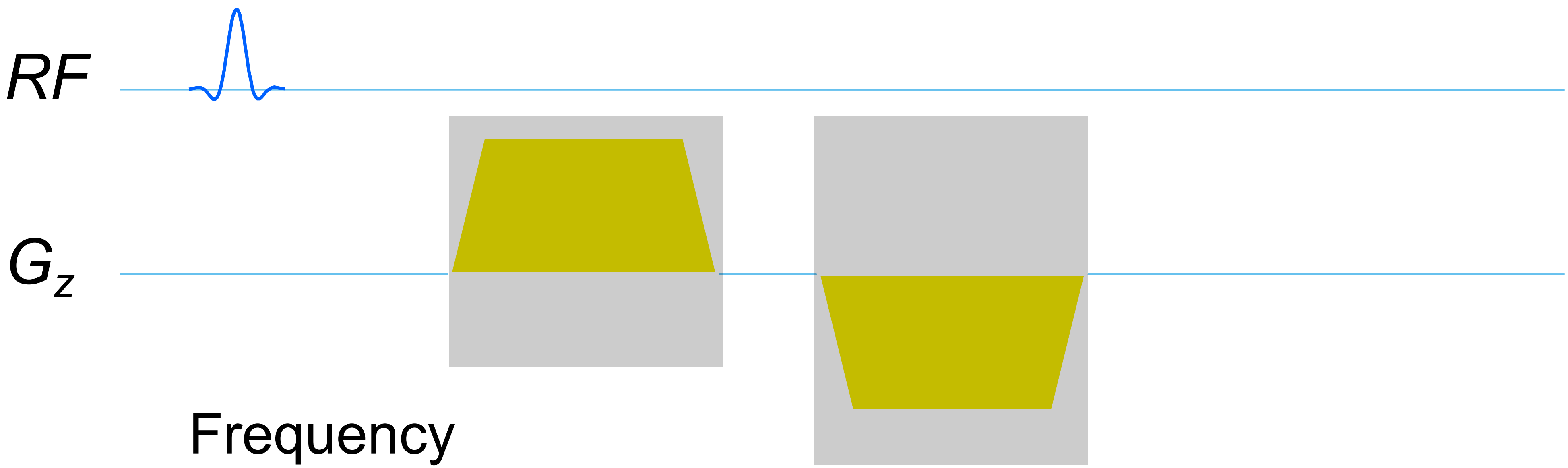


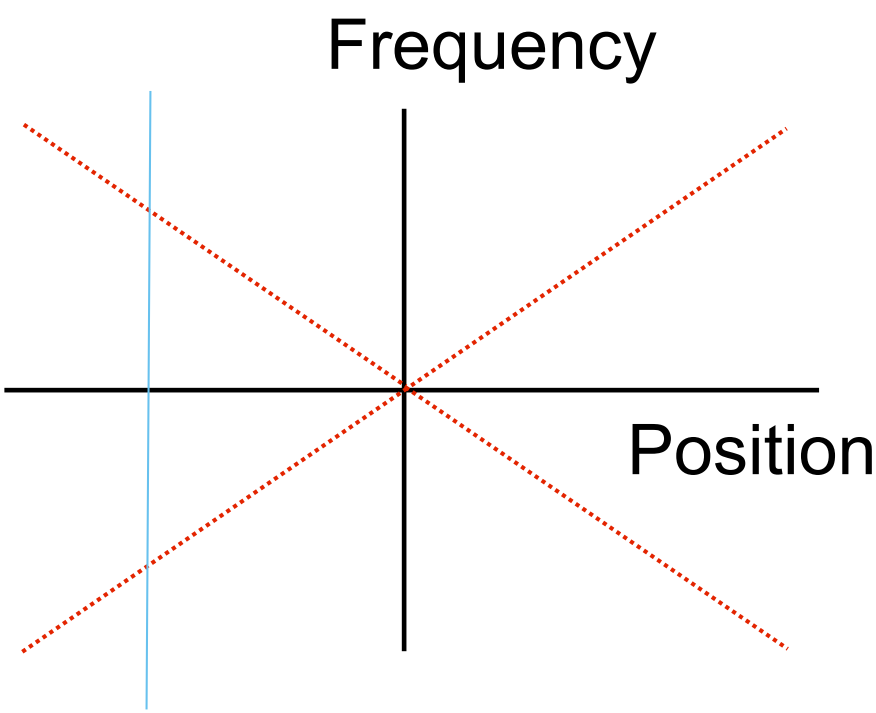
Diffusion-Weighted Imaging (DWI)

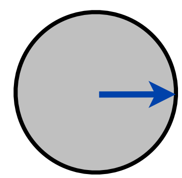
Low b-value High b-value ADC T2 FSE



Phase Contrast







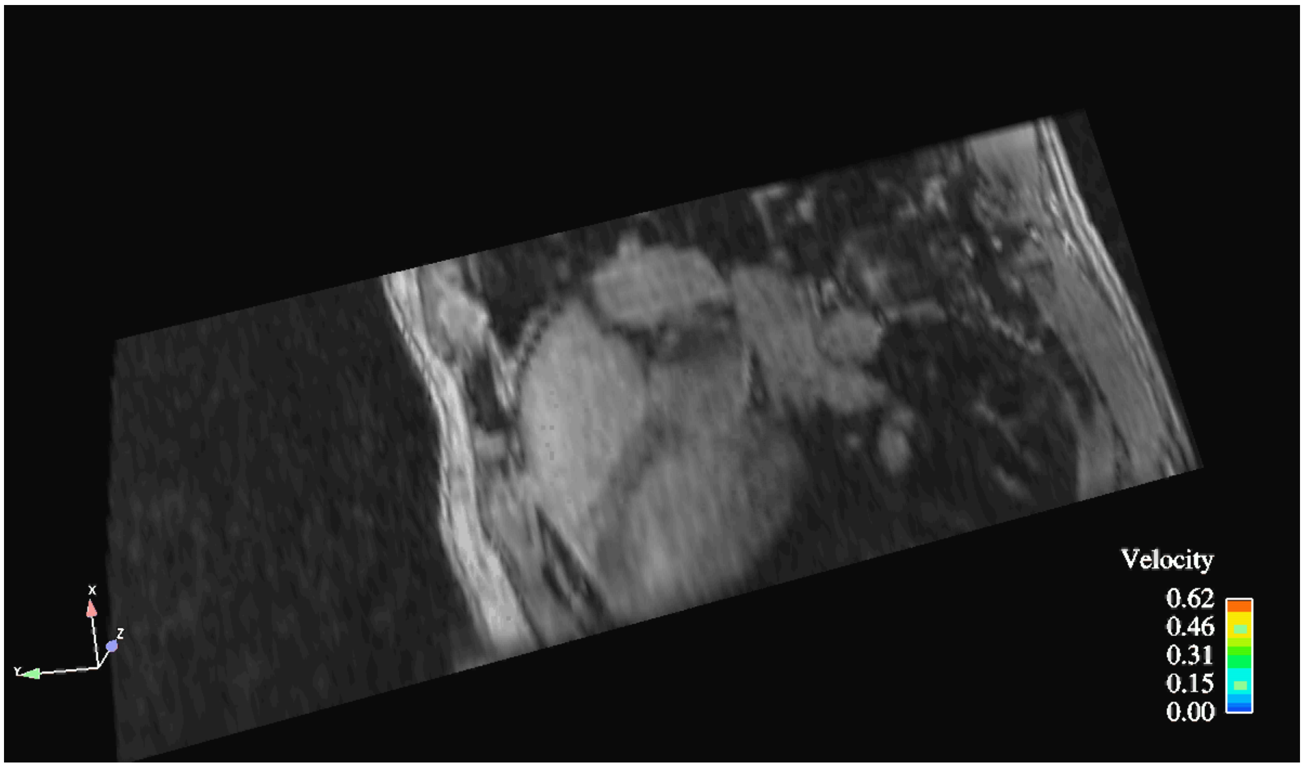
Phase is not zero!
(any position)

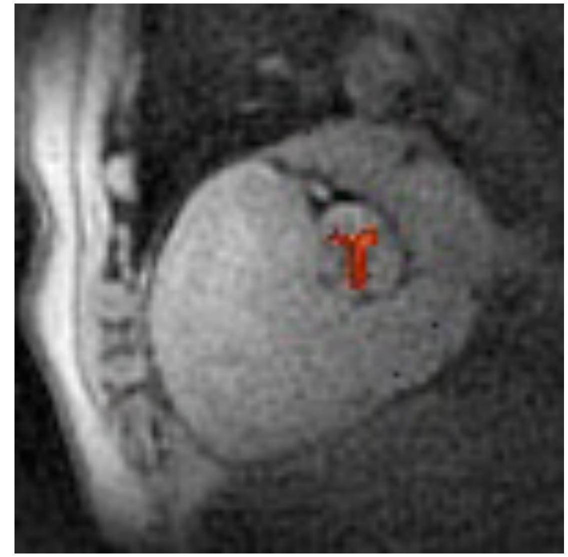
x

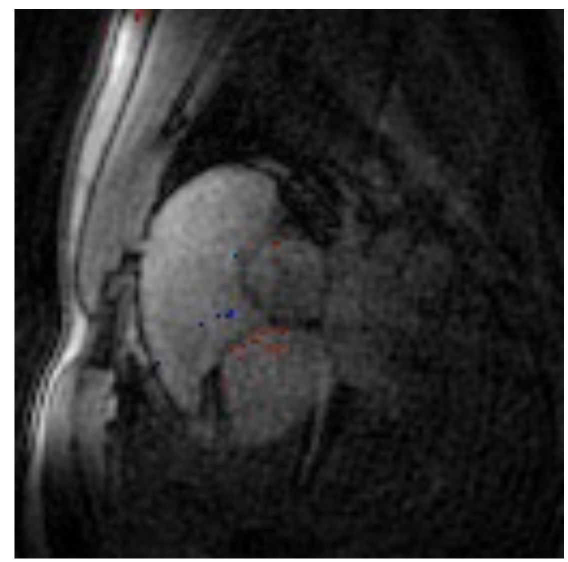
$$\phi = \gamma(x \int G_x dt + x' \int G_x t dt)$$

"Zero Moment" "First Moment"

Flow Encoded Imaging





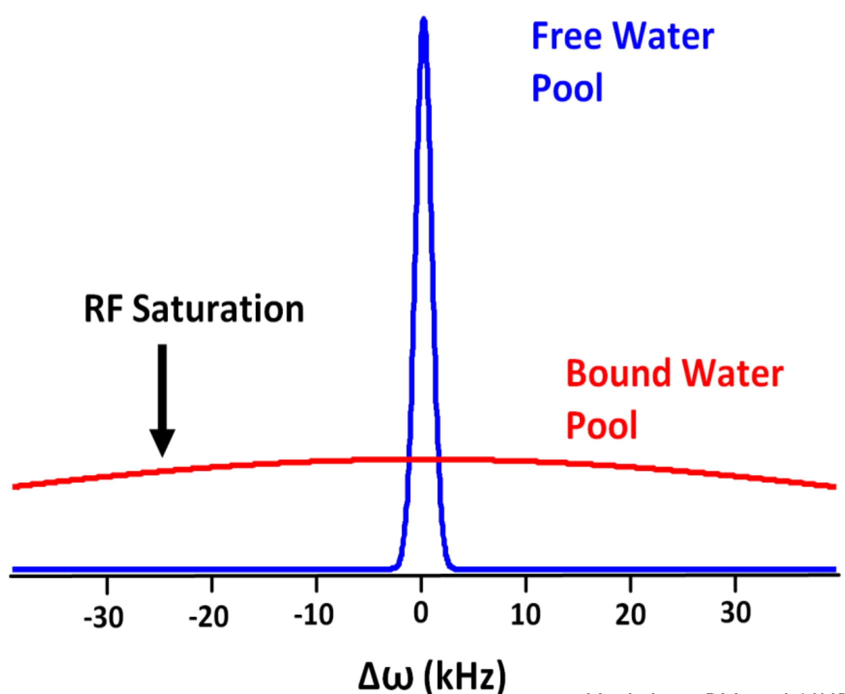


Marcus Alley

Krishna Nayak

Magnetization Transfer (MT)

- Saturate very-short-T_2 water bound to macromolecules
- MT effect causes saturation of free water (signal loss)
- More RF generally causes more MT saturation (adverse)
- CEST: Saturation at specific frequency



Henkelman RM et al. *NMR in Biomedicine* 2001; 14(2):57-64.

$$MTR = \frac{M_0 - M_{sat}}{M_0}$$

Courtesy of Feliks Kogan

MT and EPG!

- *"EPG-X: An Extended Phase Graph formalism for systems with Magnetization Transfer or Exchange." Shaihan J Malik, Rui PAG Teixeira, Joseph V Hajnal. ISMRM Workshop on MR Fingerprinting*

- Add state for bound Mz

$$[\tilde{F}_n^a \; \tilde{F}_{-n}^{*a} \; \tilde{Z}_n^a \;\; \tilde{Z}_n^b]^T$$

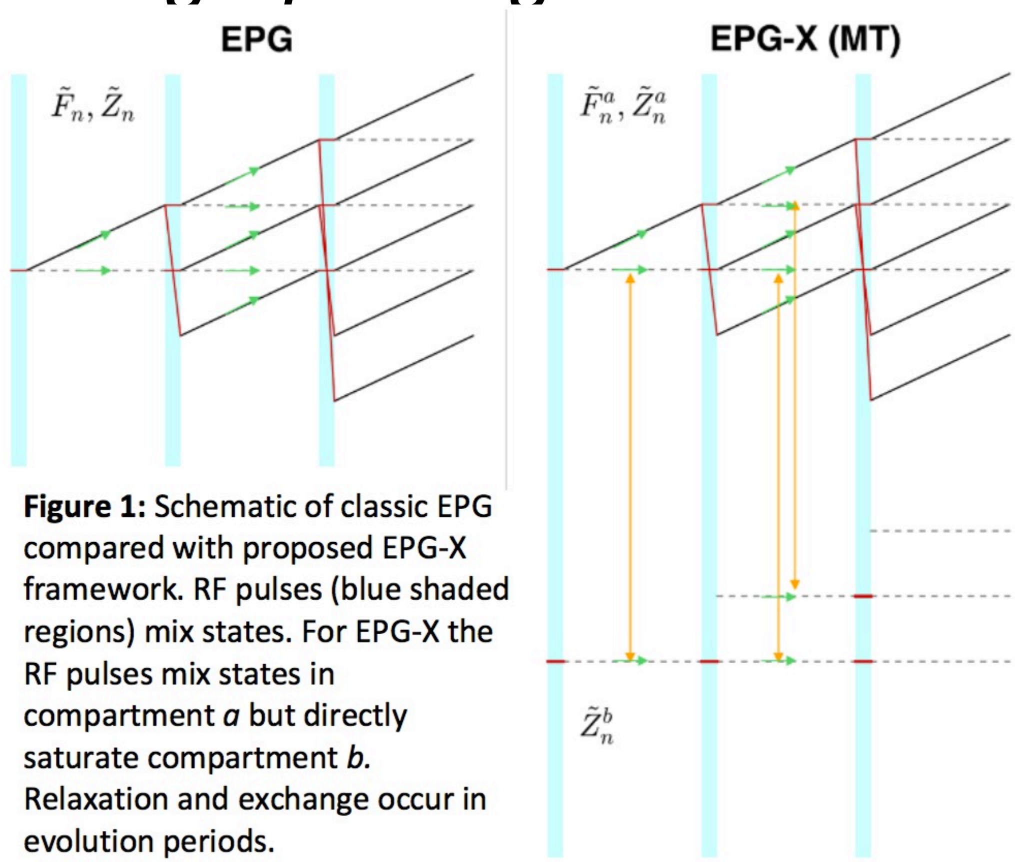


Figure 1: Schematic of classic EPG compared with proposed EPG-X framework. RF pulses (blue shaded regions) mix states. For EPG-X the RF pulses mix states in compartment *a* but directly saturate compartment *b*. Relaxation and exchange occur in evolution periods.

Other Preparations

- Double IR: Non-selective, then selective
  - "Black Blood"
- Multiple IR: Null multiple species simultaneously
- Arterial spin labeling (Invert blood, subtract reference)
- Diffusion preparation (tip-up)
  - Motion-sensitized driven equilibrium (MSDE)
  - Null vessel signal

Summary of Magnetization Prep

- Suppression: Spatial, Fat, Blood, Fluid
- Contrast: Inversion, T2-prep, Diffusion
- Encoding: Flow/motion, Diffusion, Tagging

Midterm Review - 2017

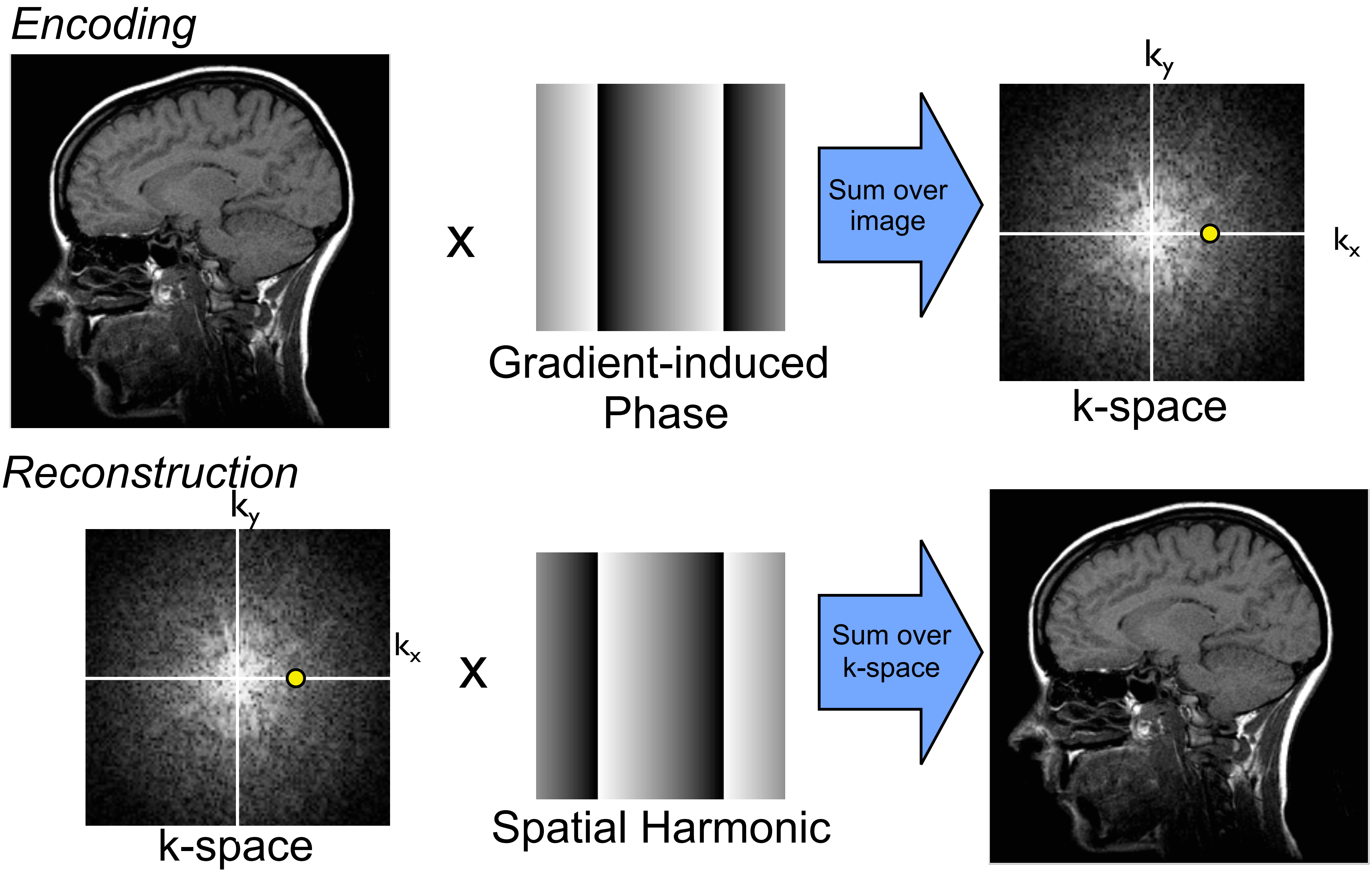
- EE369B Concepts
- Noise
- Simulations with Bloch Matrices, EPG
- Gradient Echo Imaging

About the Midterm…

- Monday Oct 30, 2017.
- CCSR 4107
- Up to end of C2

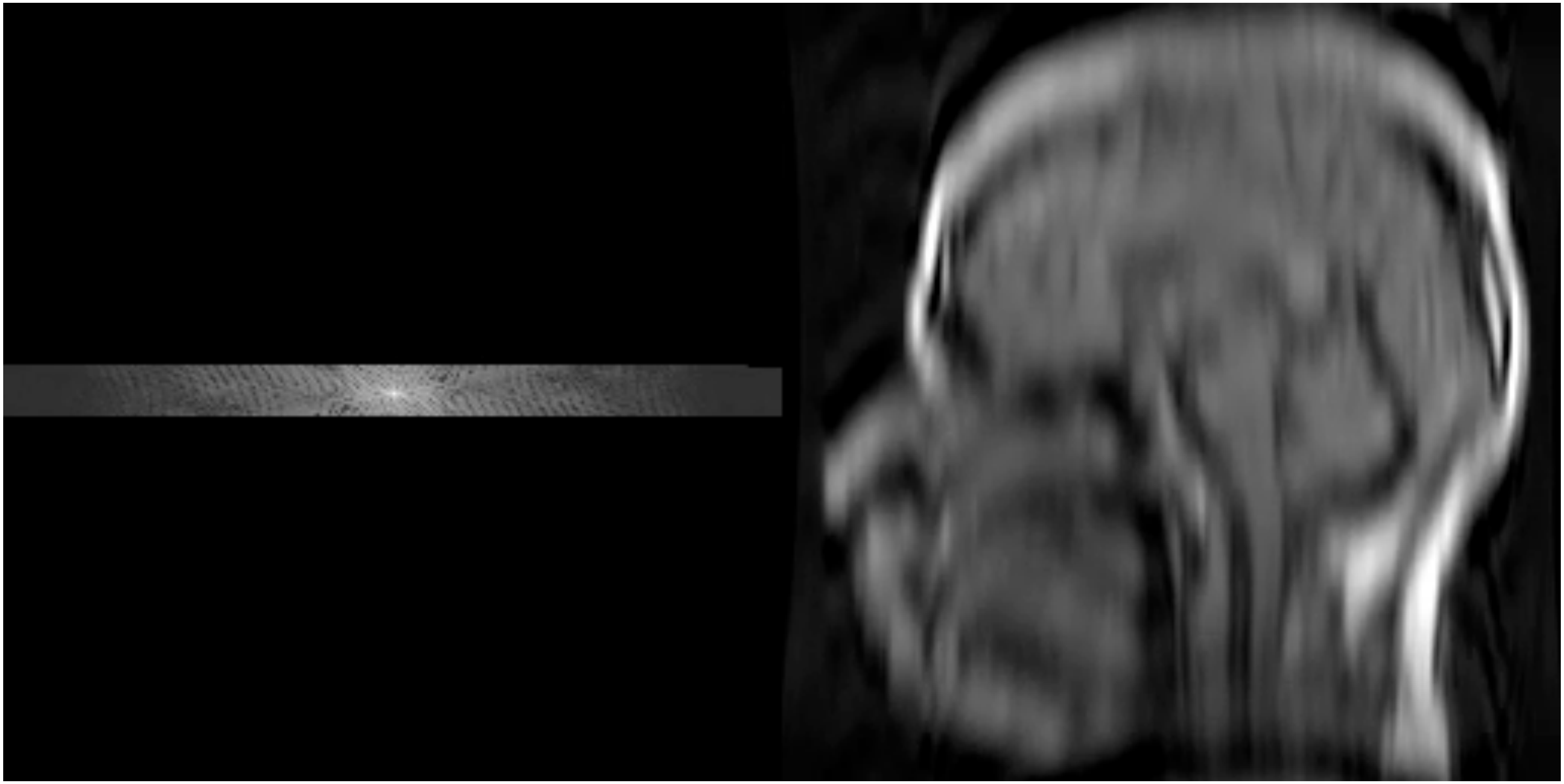
1. Write your name legibly on this page.
2. You may use notes including lectures, homework, solutions or Matlab script text on the course website.
3. You may **not use Matlab or the Internet.**
4. Please answer questions on the exam, showing your final answer clearly.
5. Show your reasoning and work, as this will often earn you partial points.
6. You may request more paper if needed.

Fourier Encoding and Reconstruction



k space Extent and Image Resolution

Data Acquisition "k" space Image Space



Fourier Transform

$\Delta x = 1/(2k_{max})$

Question 1

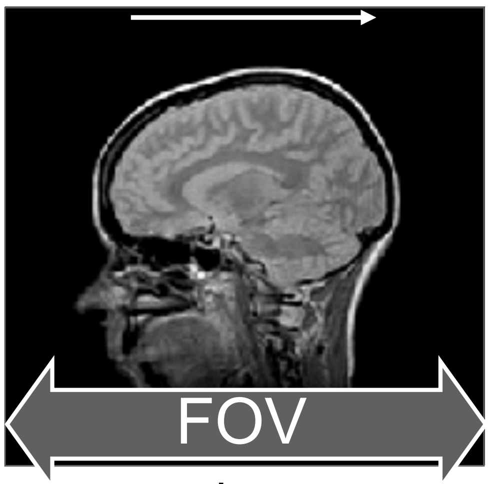
Sampling and Field of View

- Sampling density determines FOV
- Sparse sampling results in *aliasing*

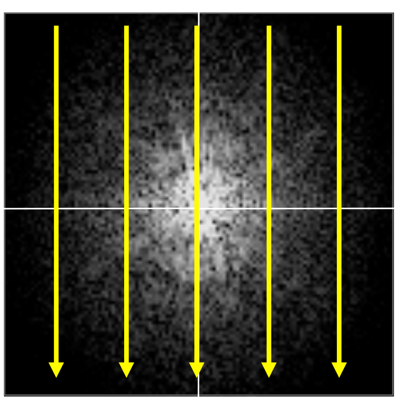
$$FOV = 1/\Delta k_y$$

Phase-Encode

Readout

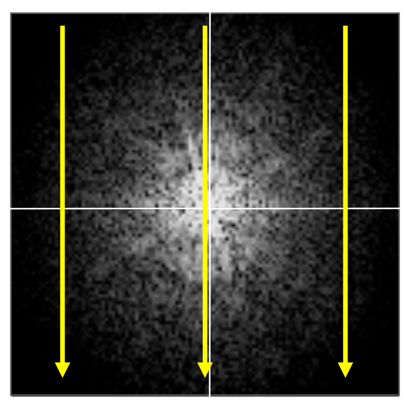


k_{read}

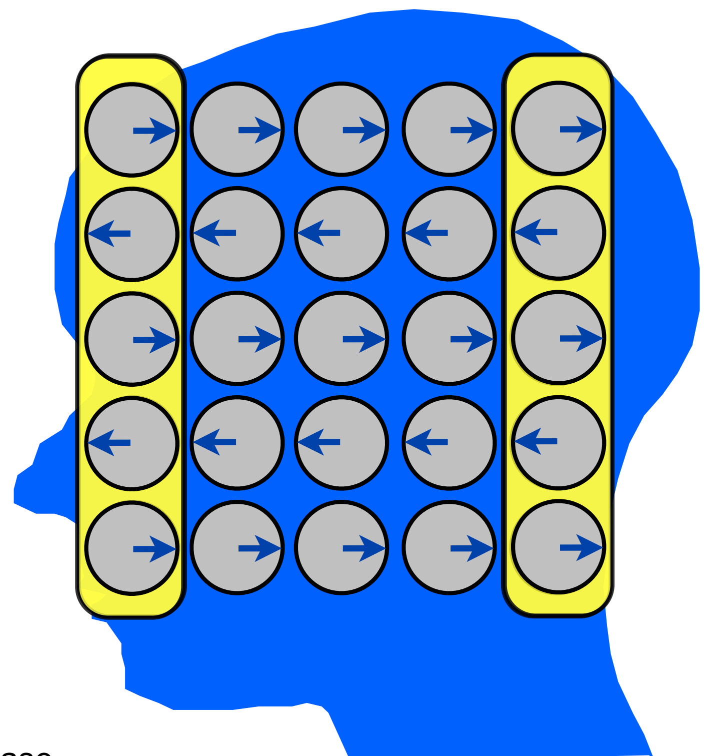


k_{phase}

k_{read}

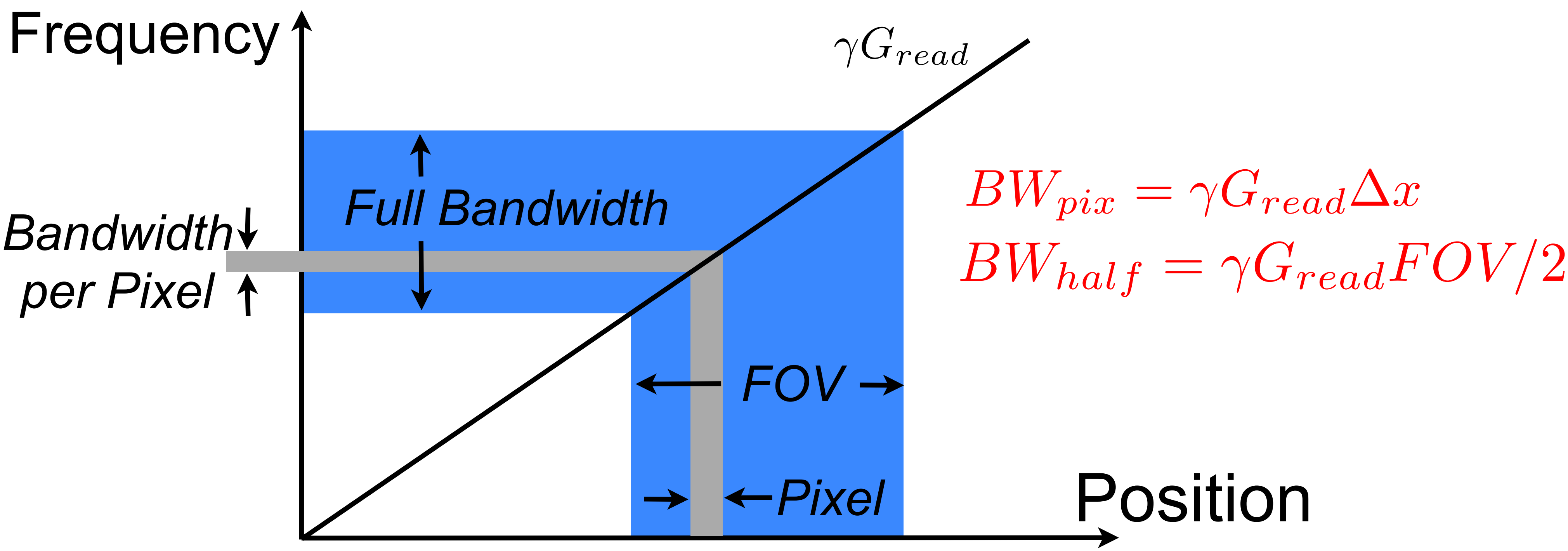


k_{phase}



Readout Parameters

- Bandwidth linked to readout
  - "half-bandwidth" (GE) = 0.5 x sample rate
  - Same as Filter bandwidth (baseband)
- Pixel-bandwidth often useful



Question 2

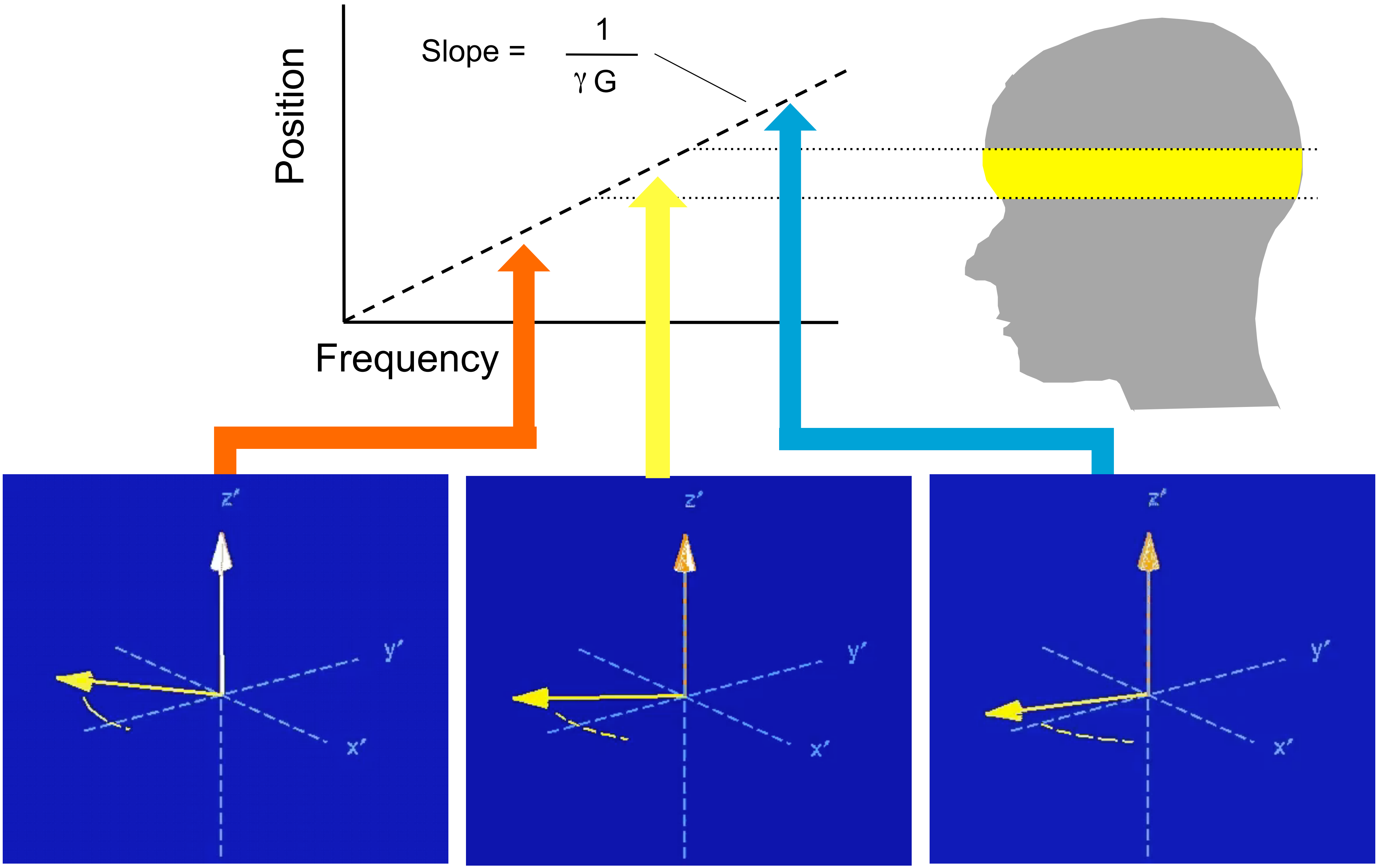
Imaging Example

- Desired Image Parameters:
  - 512 x 512, over 20cm FOV
  - 62.5 kHz bandwidth

- What are the...
  - Sampling *period?*
  - Readout duration?
  - Gradient strength?
  - Bandwidth per pixel?
  - k-space extent?

Period= 1/(2*62.5kHz)=8us,
Duration=8us * 512 = 4ms,
Gradient = 125kHz/ 0.20m / 42.58 kHz/mT ~ 15 mT/m (1.5 G/cm)
pix-BW ~250Hz/pixel
K-extent = 0.5/.4mm = -1.25 mm^(-1) to +1.25 mm^-1 = 25cm^(-1)

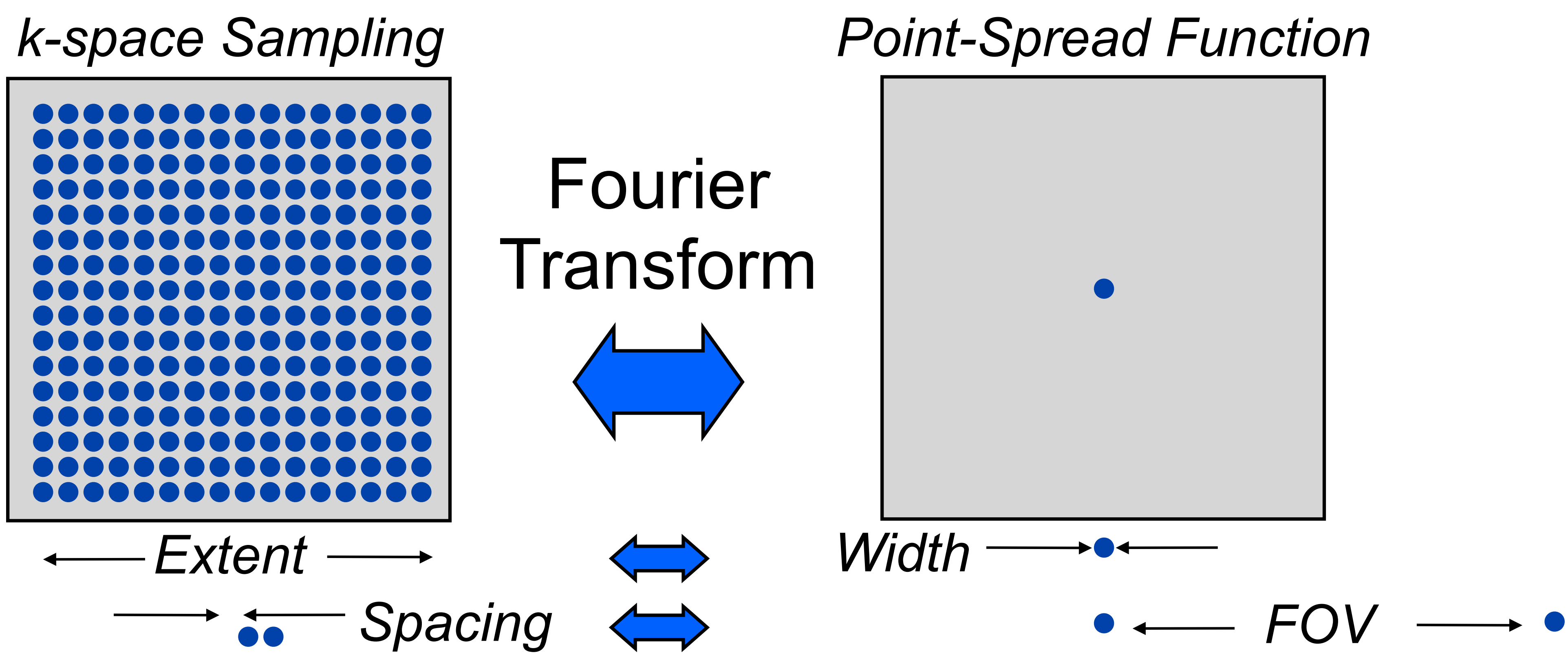
Selective Excitation



Question 3

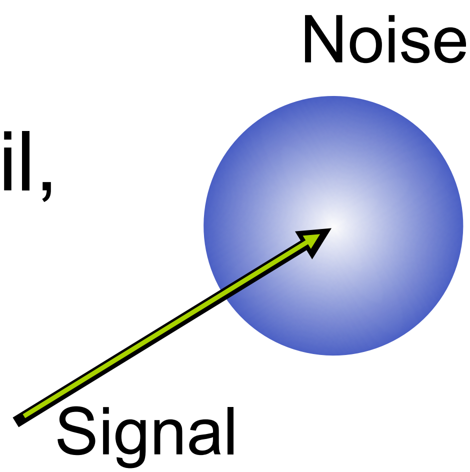
Sampling & Point-Spread Functions

- PSF = Fourier transform of sampling pattern
  - Extent/Windowing of sampling = PSF width/ripple
- Regular undersampling = replication/aliasing



SNR: Signal-to-Noise Ratio

- Signal: Desired voltage in coil
- Noise: Thermal, electronic
  - Thermal dominates, depends on coil, patient size
- SNR = average signal / σ
  - Gaussian noise (FT is gaussian)
  - N averages = sqrt(N) increase
- Magnitude noise is Rician; can obtain σ



SNR Efficiency

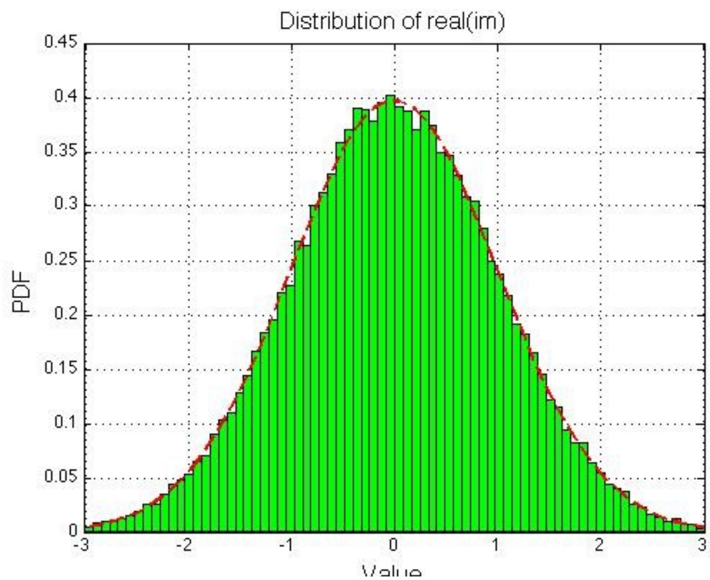
- Often want to compare SNR of different sequences
- If times differ, comparison can be made fair by use of SNR efficiency:

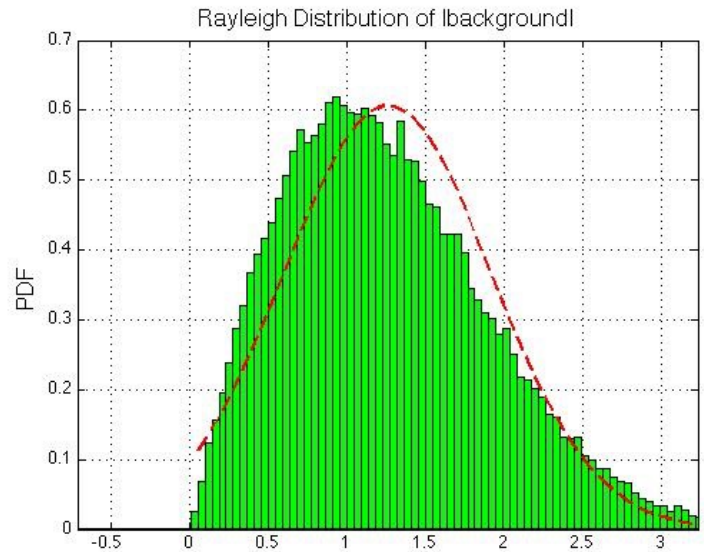
$$\eta_{SNR} = \frac{SNR}{\sqrt{T_{scan}}}$$

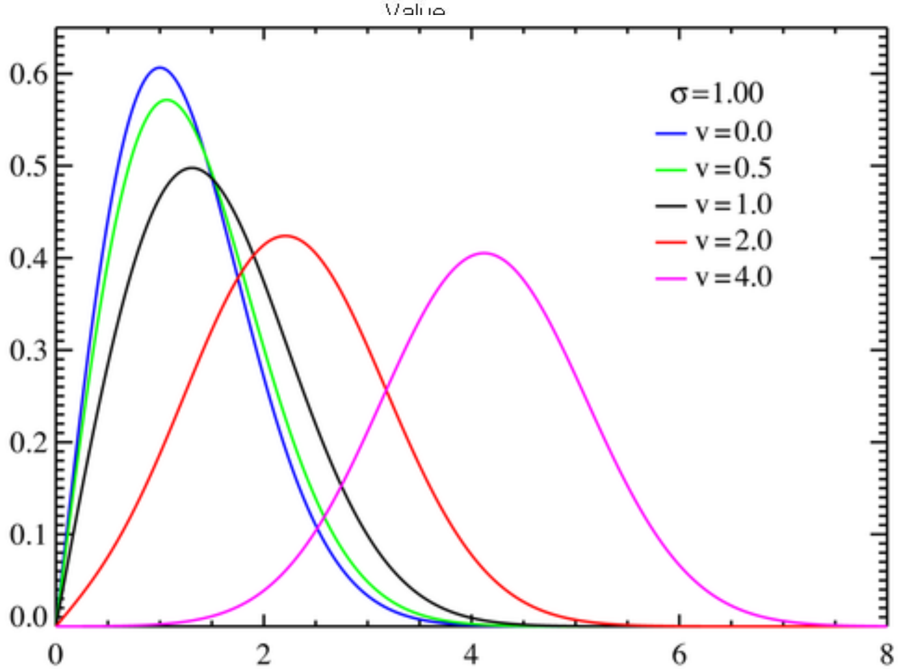
- In many cases:

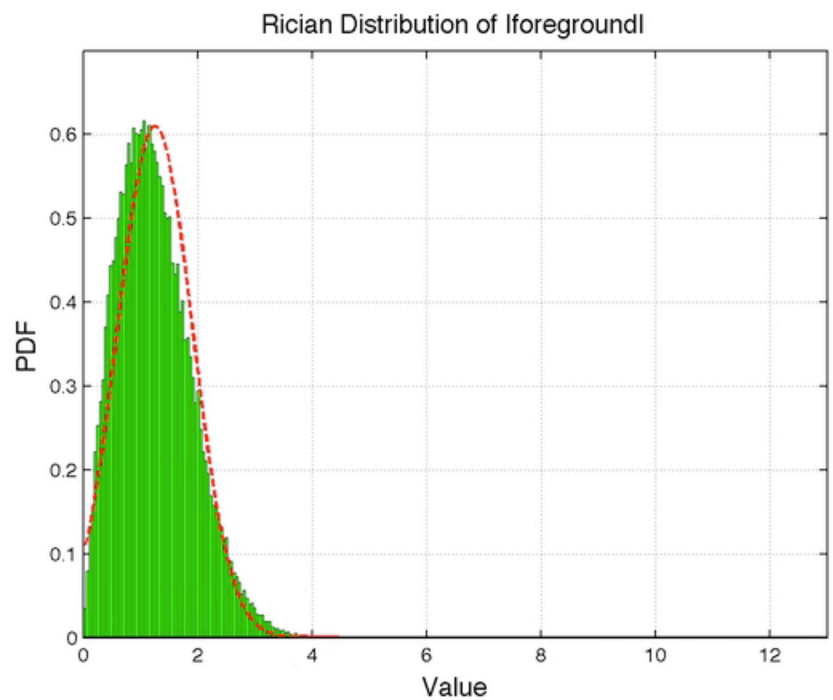
$$\eta_{SNR} = \frac{SNR}{\sqrt{TR}}$$

Basic Noise Statistics







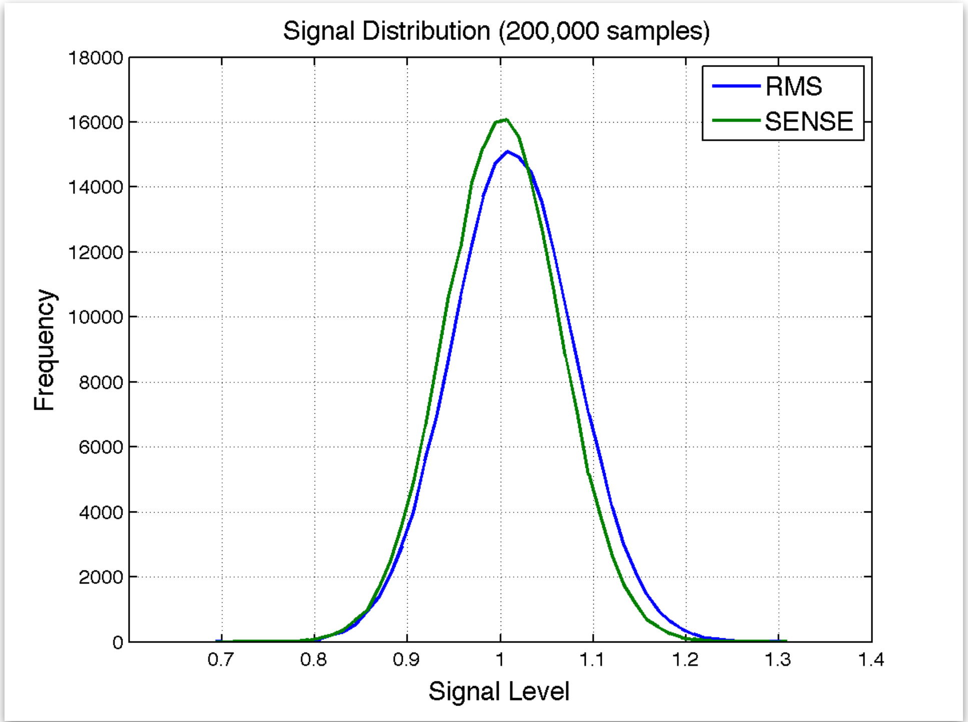


Multiple Coils - Noise Statistics

- RMS combination: S_i=coil image, C_i = sensitivity
  - Image = sqrt($S_1^2 + S_2^2$…)
    - = sqrt[$(mC_1+n_1)^2 + (mC_2+n_2)^2$ + …]
  - Signal bias, noise (slightly) non-gaussian

exampleB4_9.m

- R=1 SENSE combination
  - (linear) $$S_{SENSE} = \sum_{i=1}^{N_{coils}} \alpha_i S_i$$
  - No bias, Gaussian noise
  - Phase preserved



SENSE Reconstruction

- $S_i(k)$ = signal from coil i,
- R = reduction factor
- C_{ij} = sensitivity of coil i, to aliased pixels $j=1\ldots R$
- m = aliased image for coils (N_c x 1)
- $m = \mathcal{F}\{ S(k) + n \}$ (includes all channels)
- Ψ is noise covariance matrix

$$\hat{M} = \left(C^H \Psi^{-1} C\right)^{-1} C^H \Psi^{-1} \mathrm{m}$$

$$g = \sqrt{\left[\left(C^H \Psi^{-1} C\right)^{-1}\right]_{x,x} \left[C^H \Psi^{-1} C\right]_{x,x}}$$

$$SNR = \frac{SNR_0}{g\sqrt{(R)}}$$

Bloch/Matrix Simulations

- $M = [M_x\ M_y\ M_z]^T$
- RF and precession ~ 3x3 rotation matrices
- Relaxation ~ 3x3 diagonal multiplication + M_z recovery
- Propagation of A/B: Pre-multiply both by A, add new B
- Steady states: $M_{ss} = AM_{ss} + B \quad = (I-A)^{-1}B$

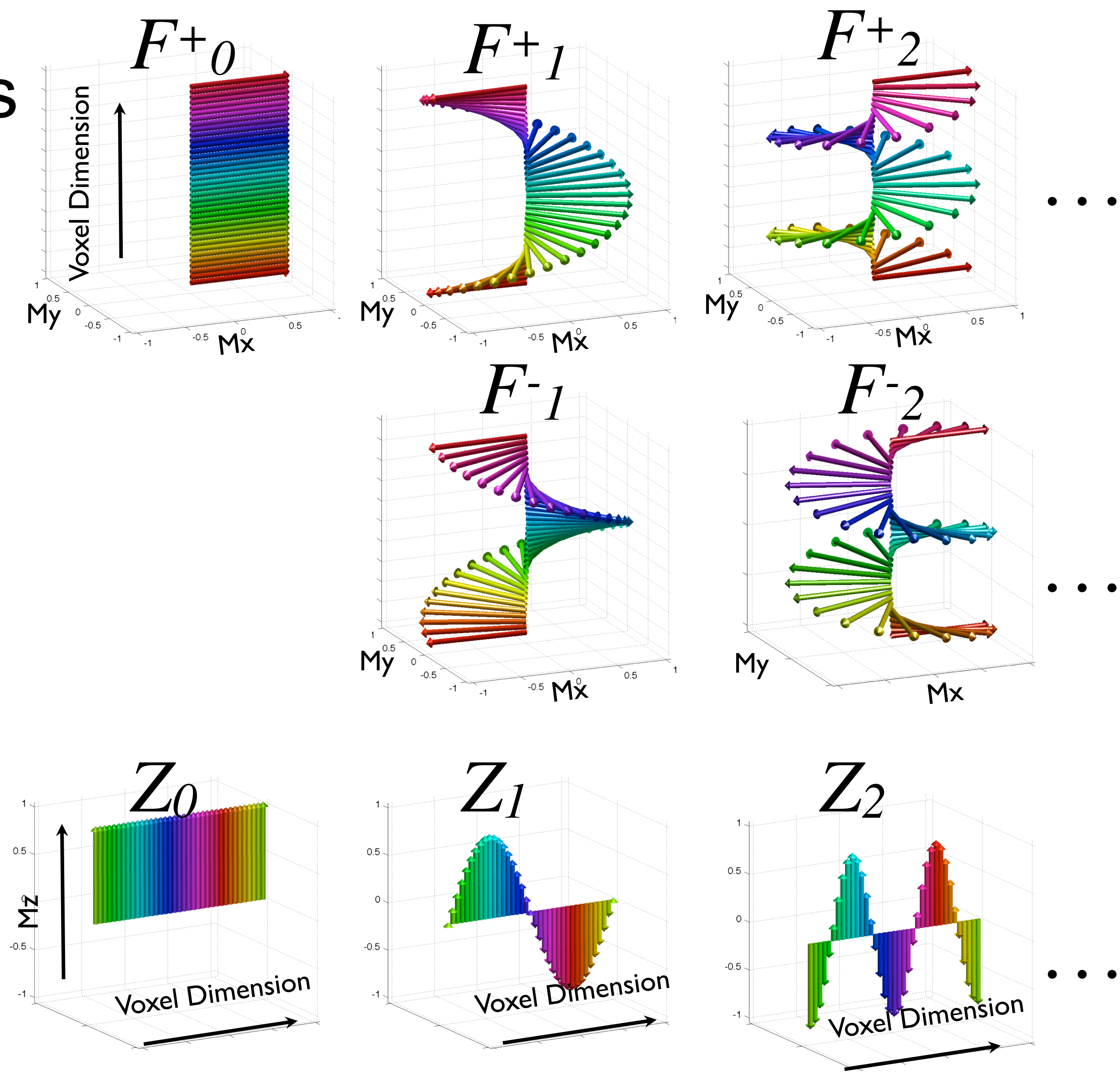
Question 4

EPG

- Forward / Reverse Transforms
- Propagation of states (matrix)
- Coherence pathway diagrams

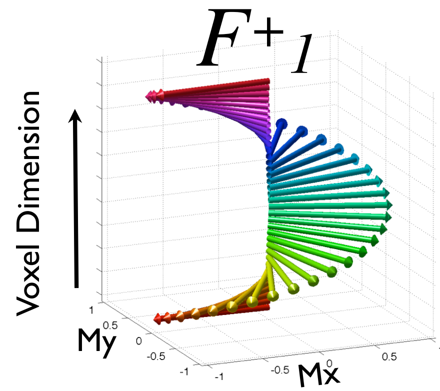
EPG Basis: Graphical

- F_n and Z_n are basis *coefficients*
- Transverse basis are simple phase twists *(sign of superscript or subscript indicates direction)*
- Longitudinal basis are sinusoids



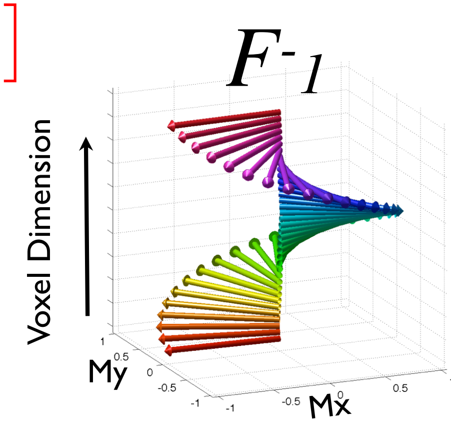
EPG Basis: Mathematically

- Transverse basis functions (F_n) are just phase twists:



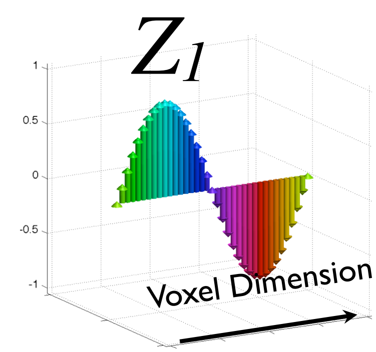
$$M_{xy}(z) = \sum_{n=-\infty}^{\infty} F_n^+ e^{2\pi i n z}$$

$$M_{xy}(z) = F_0^+ + \sum_{n=1}^{\infty} \left[F_n^+ e^{2\pi i n z} + (F_n^-)^* e^{-2\pi i n z} \right]$$



- Longitudinal basis functions (Z_n) are sinusoids:

$$M_z(z) = Real\left\{ Z_0 + 2\sum_{n=1}^{N} Z_n e^{2\pi i n z} \right\}$$



F_n and Z_n are the coefficients, but we sometimes use them to refer to the basis functions ("twists") they multiply

Although there are other basis definitions, this is consistent with that of Weigel et al. J Magn Reson 2010; 205:276-285

Magnetization to EPG Basis

- F^+ states:

$$F^{+}{}_{n} = \int_0^1 M_{xy}(z) e^{-2\pi i n z} dz$$

- F^- states:

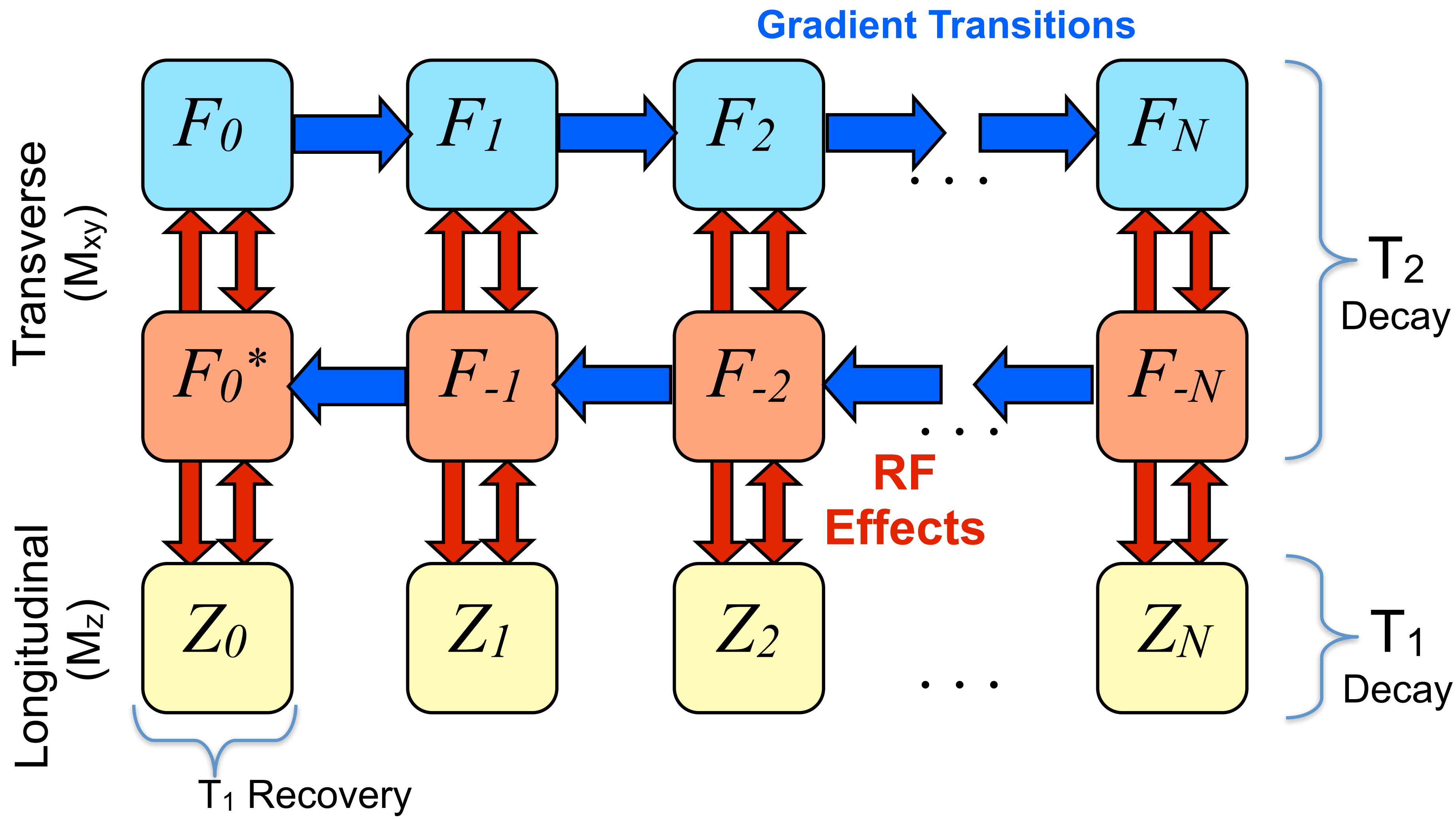
$$F^{-}{}_{n} = F^{*}{}_{-n} = \int_0^1 M_{xy}^{*}(z) e^{-2\pi i n z} dz$$

- Z states:

$$Z_n = \int_0^1 M_z(z) e^{-2\pi i n z} dz$$

Note redundancy $F^{-}{}_{n} = (F^{+}{}_{-n})^{*}$
(Can use $F^{+}{}_{n}$ and $F^{-}{}_{n}$ for n>0, or just $F^{+}{}_{-n}$ (all n))

Phase Graph "States" (Flow Chart)



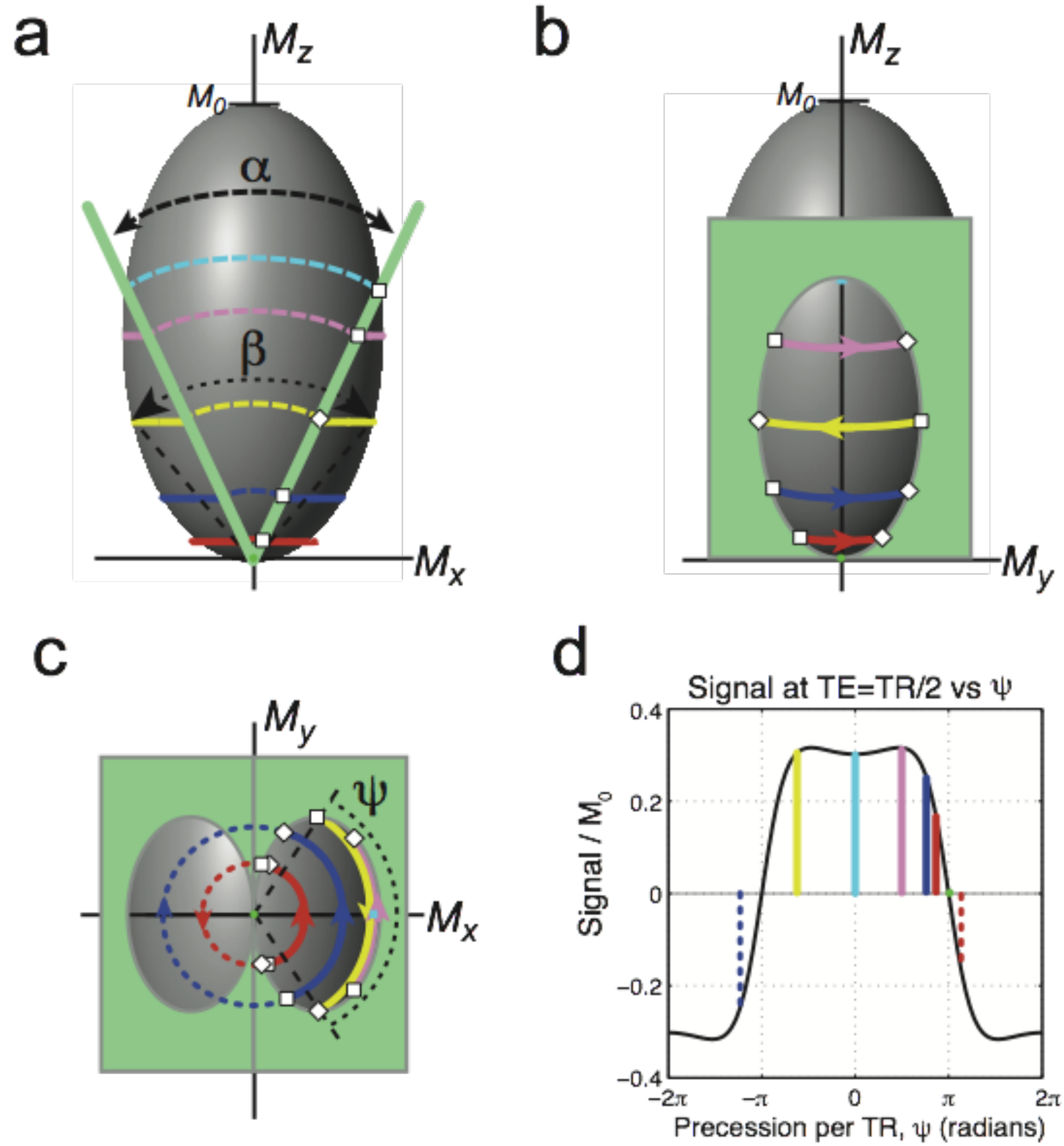
Question 5

bSSFP - Geometric Interpretation

- Ellipsoid
- Effective flip angle
- Precession
- Freq response

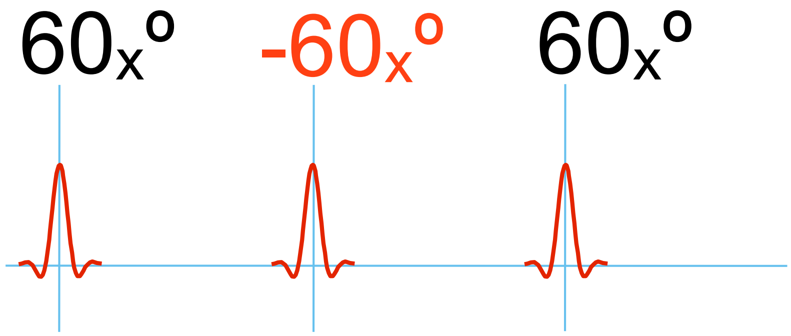
Geometric Interpretation - bSSFP

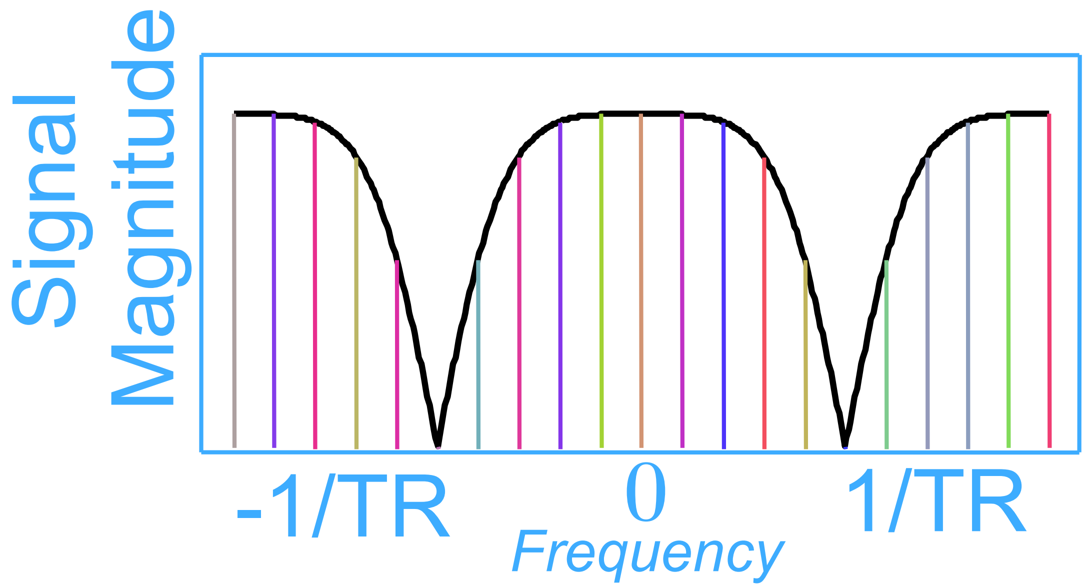
- Ellipsoid height M_0, radius $(M_0/2)\mathrm{sqrt}(T_2/T_1)$
- Flip angle is α
- Effective flip angle β

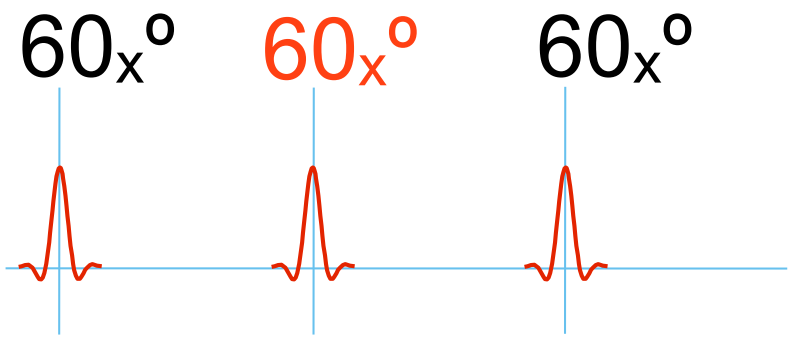


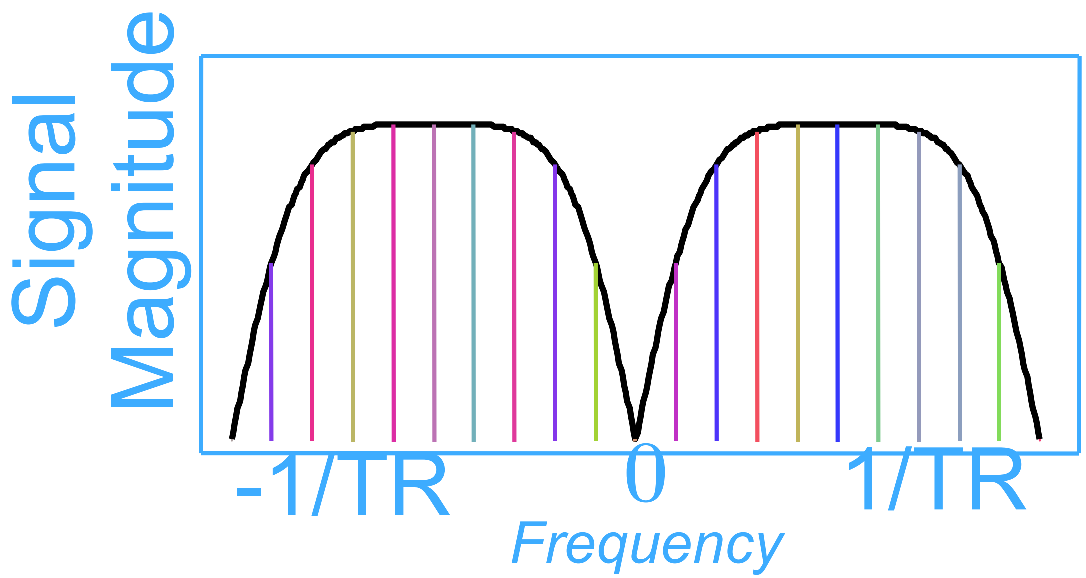
Phase Cycling

Hinshaw 1976









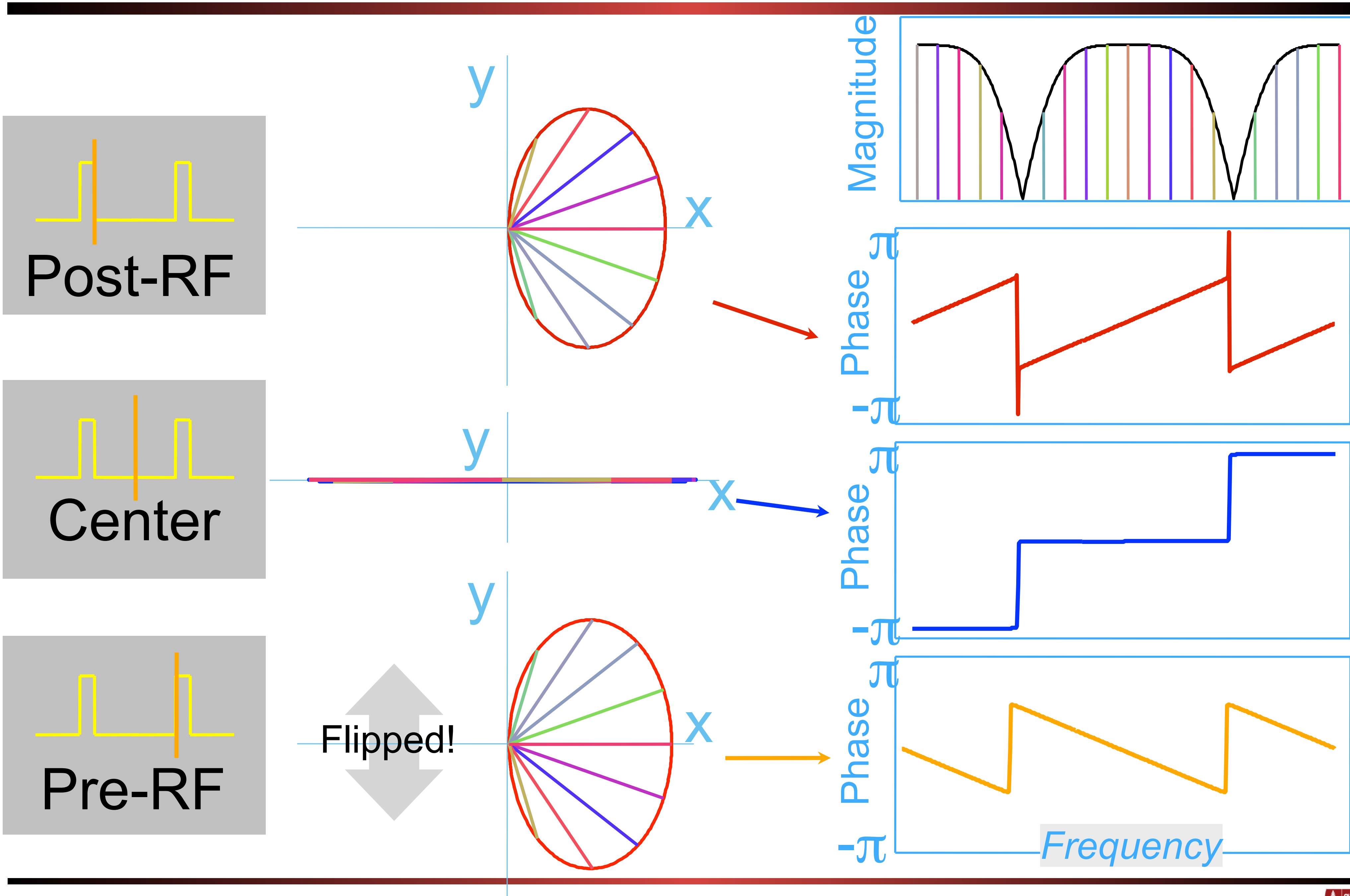
Question 6

Question 7

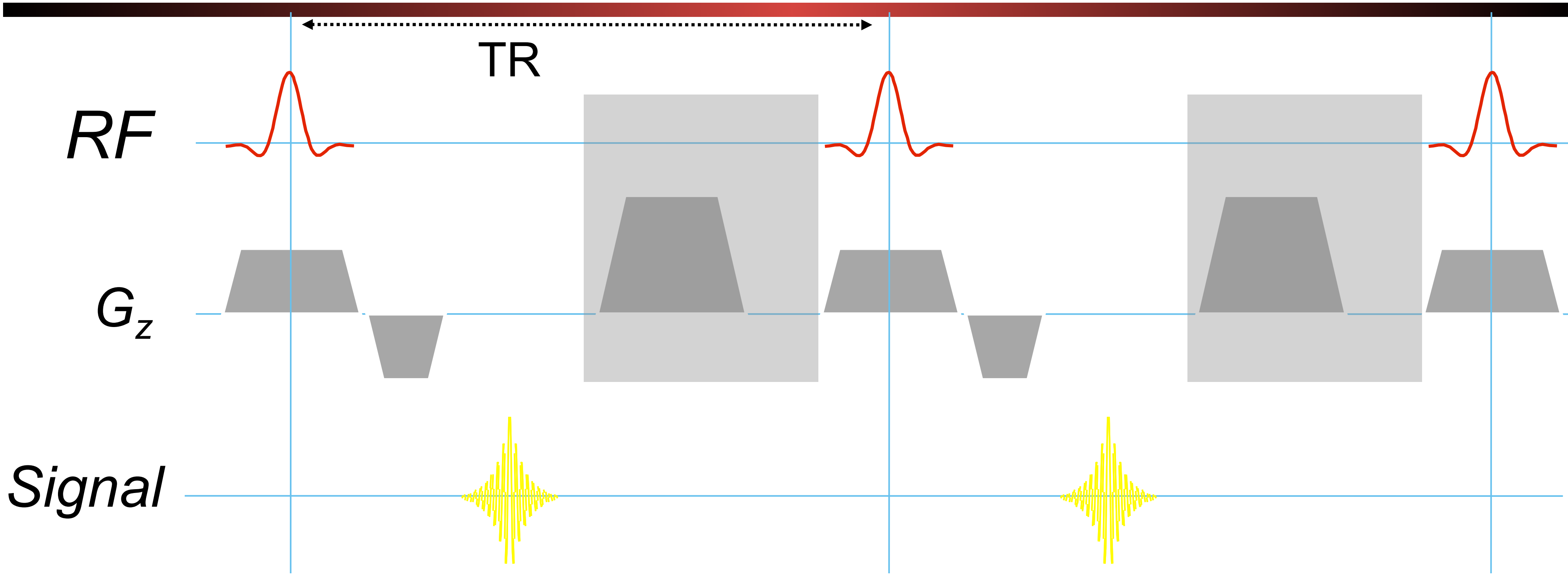
Gradient Spoiling

- Averaging, at appropriate time
- Forward / Reverse

Signal vs Frequency: Phase

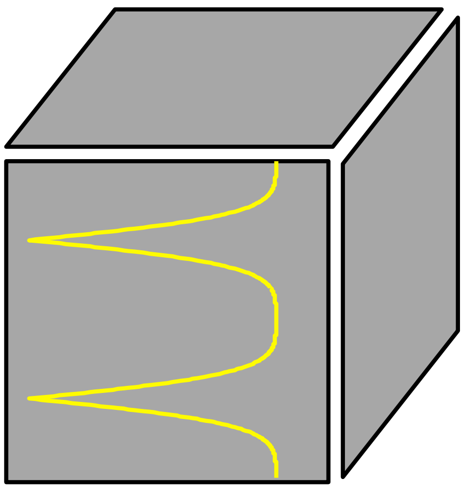


Gradient Spoiling



Precession across a voxel dominated by spoiler:

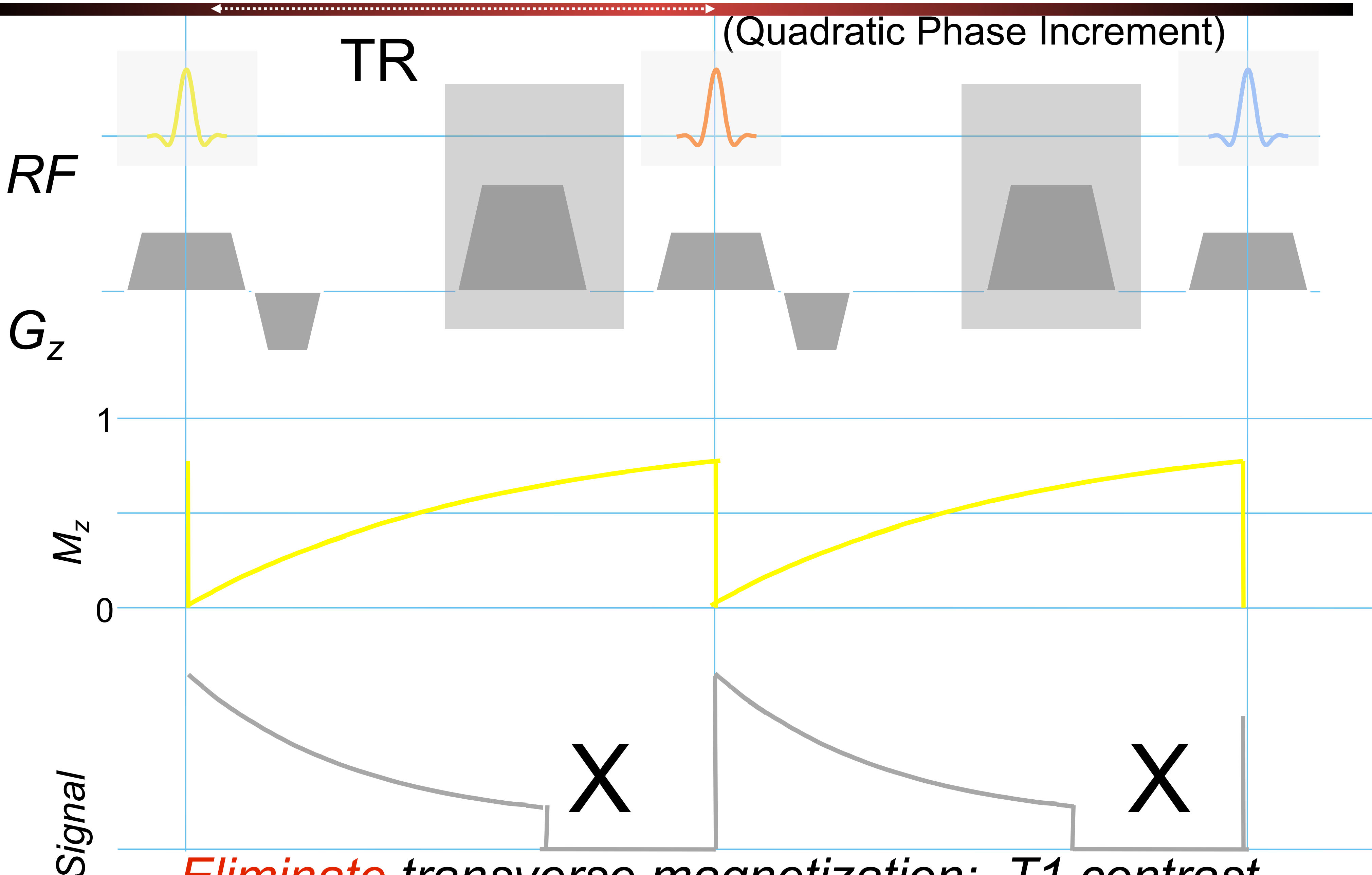
- Each spin has a different precession
- Average of balanced SSFP



Question 8

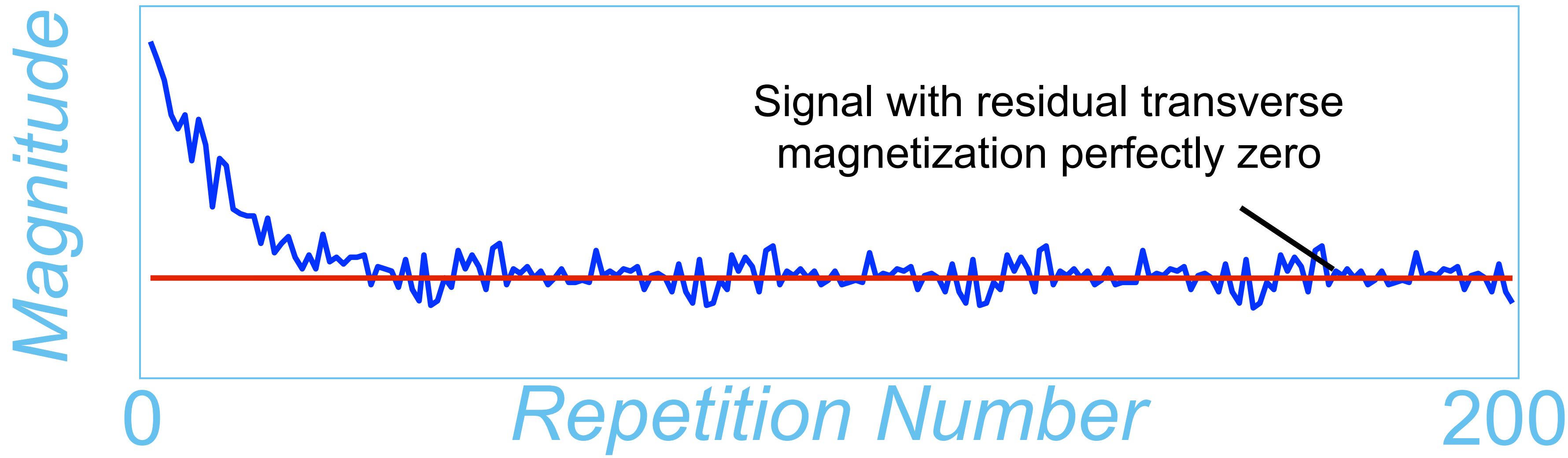
Question 9

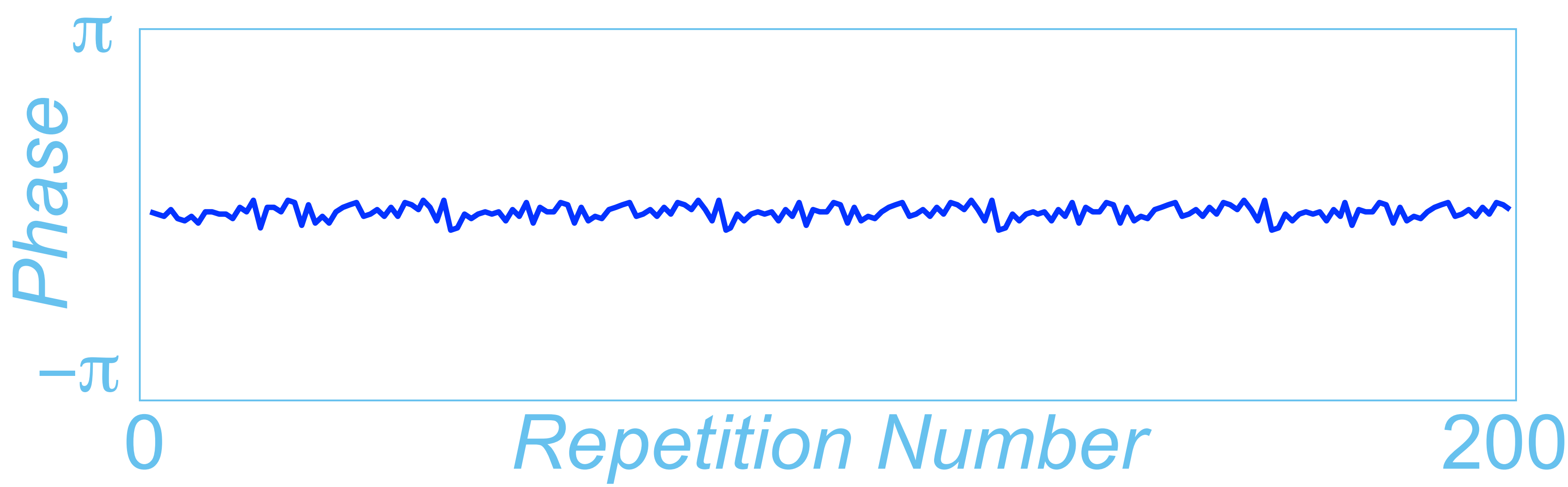
RF-Spoiled Gradient Echo



Eliminate transverse magnetization: T1 contrast

RF Spoiled Signal



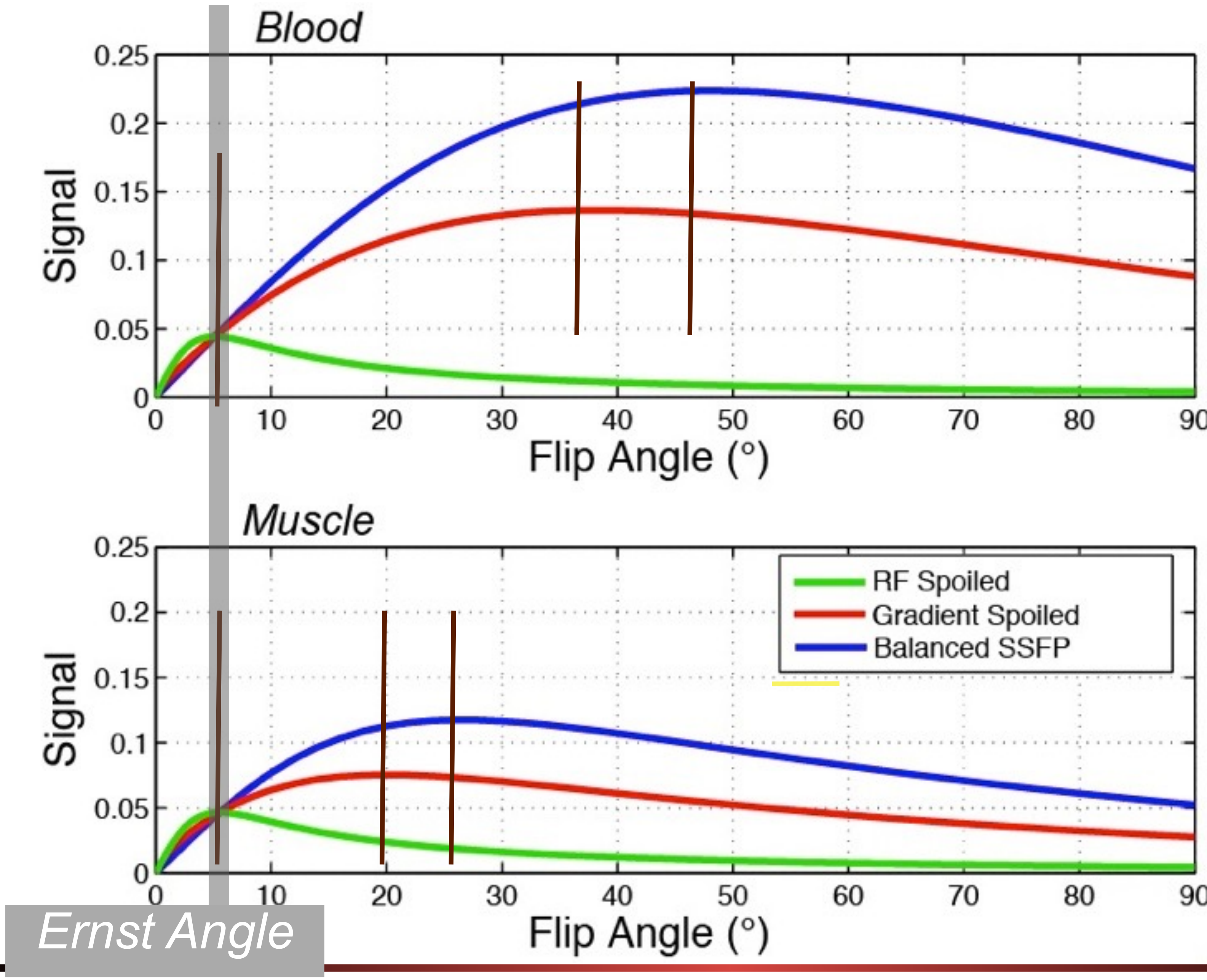


Question 10

Gradient Echo Sequence Comparison

Sequence	Balanced SSFP	Gradient Echo	RF-Spoiled
Spoiling	None	Gradient	RF + Gradient
Transverse Magnetization	Retained	Averaged	Cancelled
Contrast	T_2/T_1	T_2/T_1	T_1
SNR	High (but Banding)	Moderate	Lower

Flip Angle Selection



Buxton 1990

Summary

- EE 369B Review:
  - Imaging principles / review
  - SNR considerations
- Bloch/Matrix Simulations
- Extended Phase Graphs
- Gradient Echo Sequences

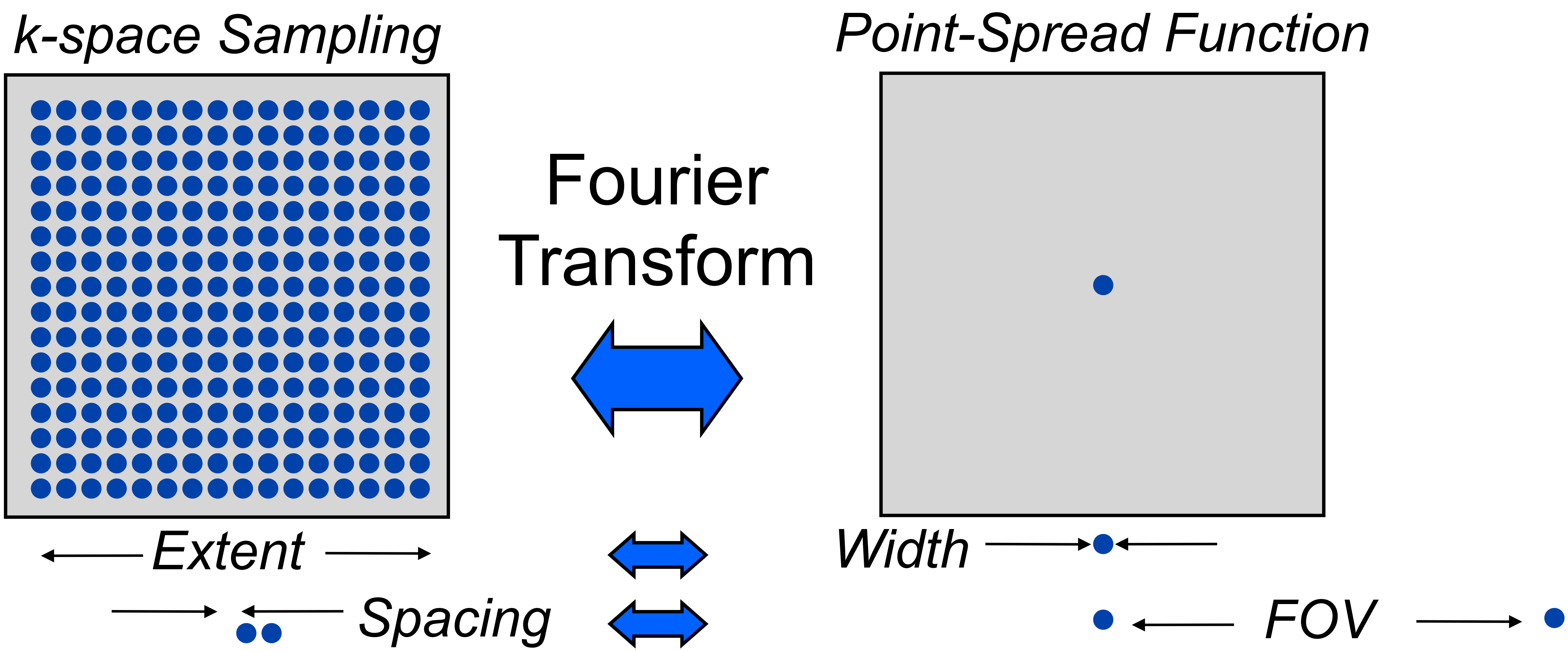
Try to use intuition as much as math!

Sampling, Ordering, Interleaving

- Sampling patterns and PSFs
- View ordering
  - Modulation due to transients
  - Temporal modulations
- Timing: cine, gating, triggering
- Slice interleaving
  - Sequential, Odd/even, bit-reversed
  - Arbitrary
- Simultaneous Multislice / “Multiband” MR

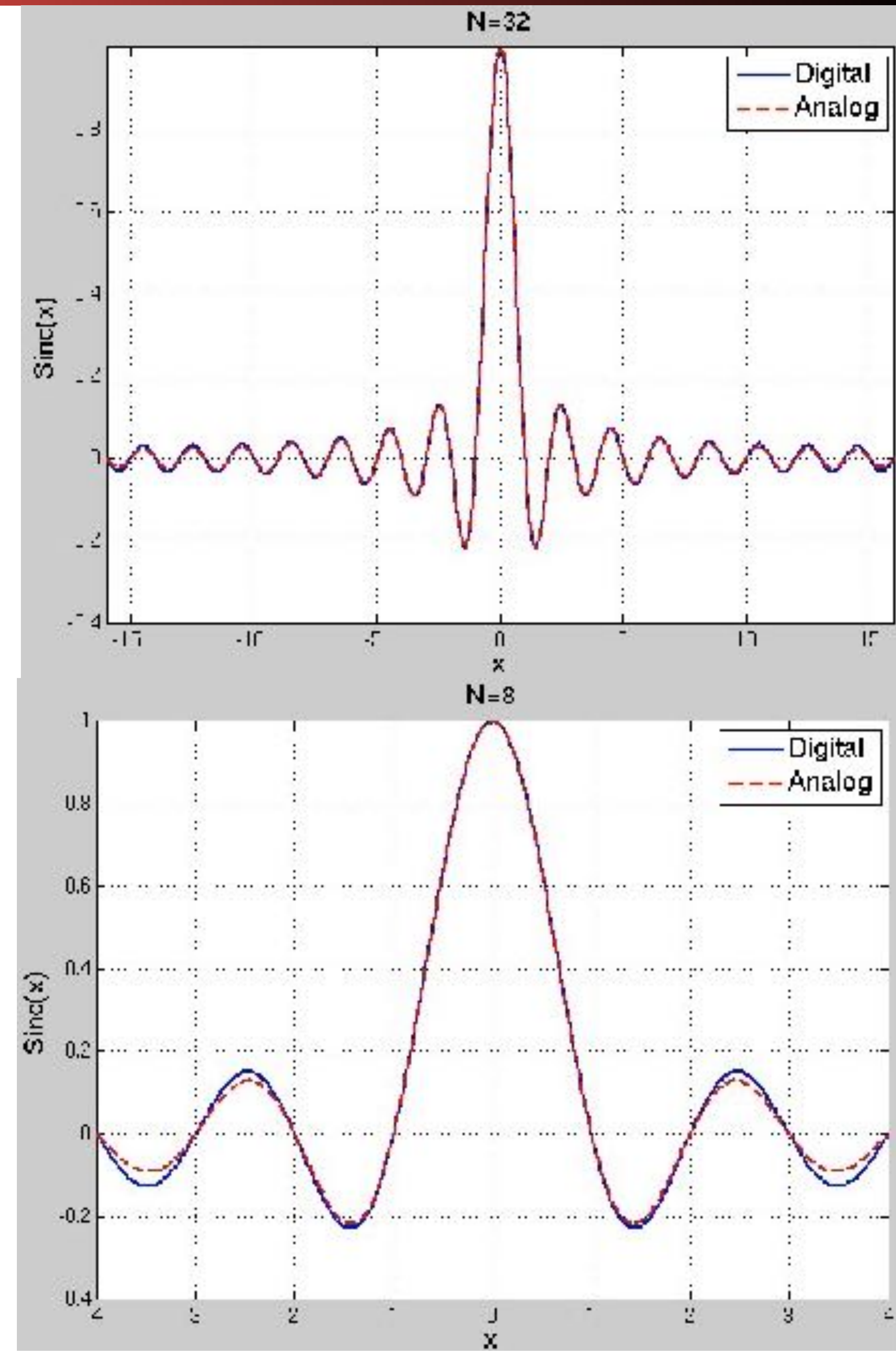
Sampling & Point-Spread Functions

- PSF = Fourier transform of sampling pattern
  - k-space: Extent, Density, Windowing
  - PSF: Width, Replication, Ripple (side-lobes)



The “Discrete” sinc function

$$h(x) = \Delta k \frac{\sin(\pi N \Delta k x)}{\sin(\pi \Delta k x)}$$

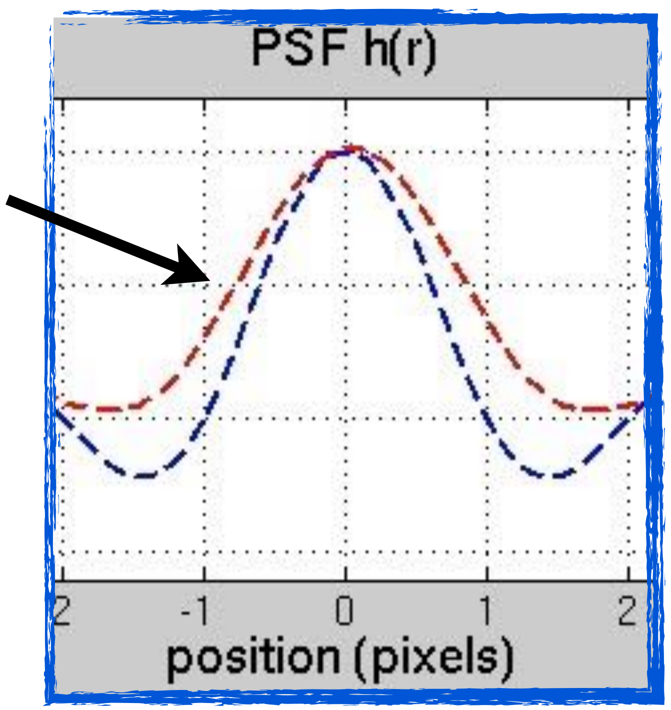


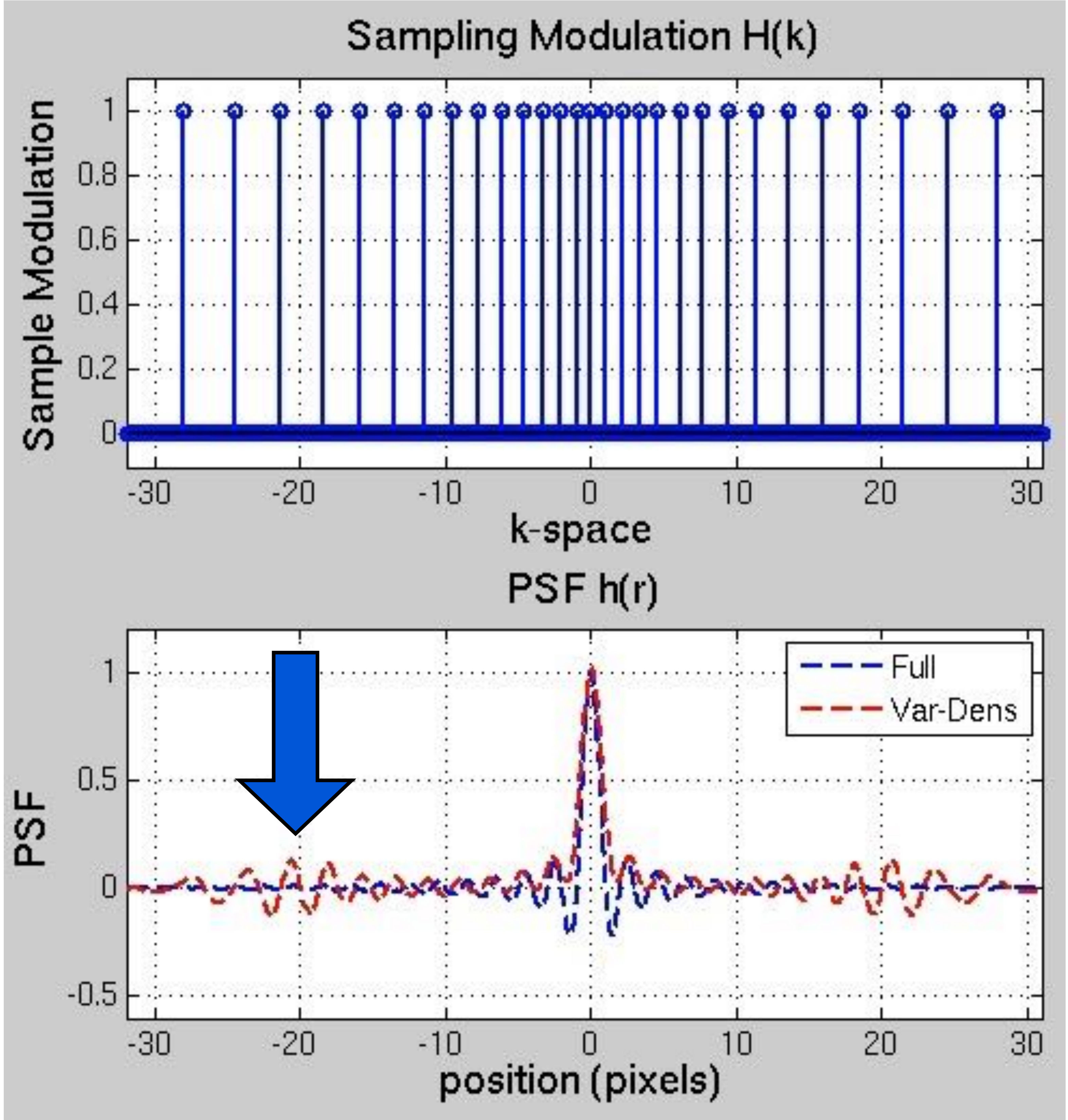
- Function of extent
- Shows challenge of low N

Variable Density Sampling

- 2x undersamling
- Δk linear with k
- Minor Aliasing
- PSF broadens

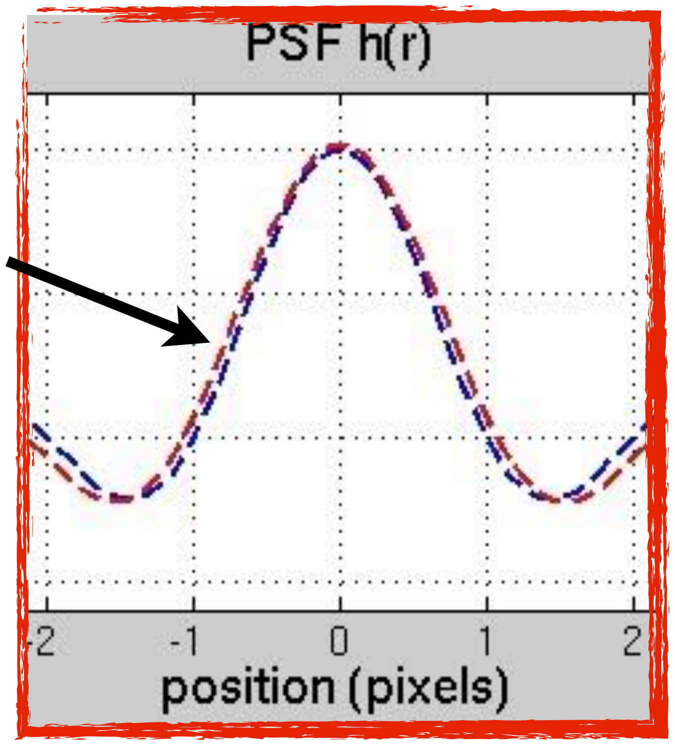
exampleE1_vds.m

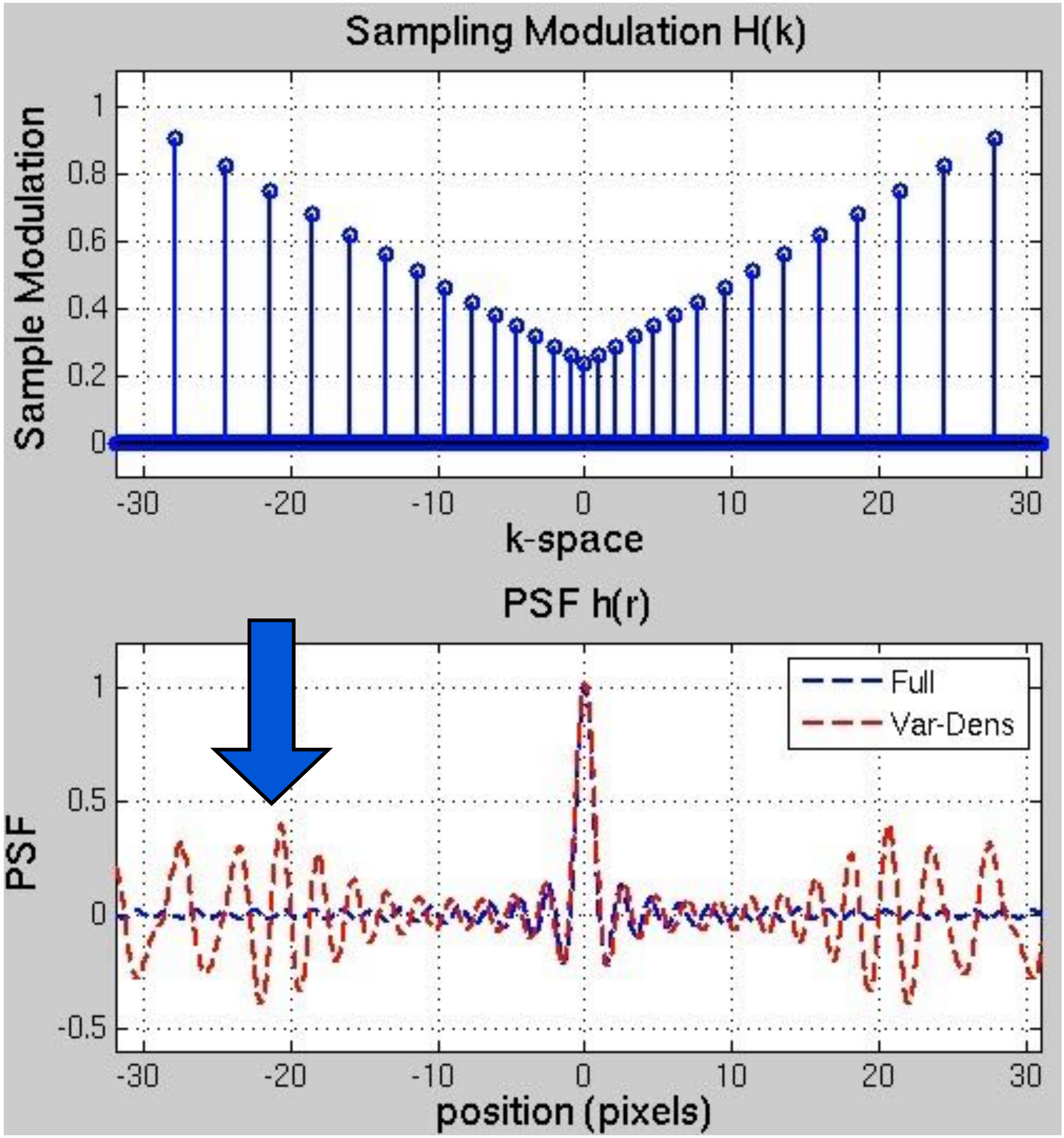




Variable Density Sampling: Density Compensated

- Multiply by Δk
- No PSF Broadening
- Higher ringing
  (center less dominant)
- need to apodize





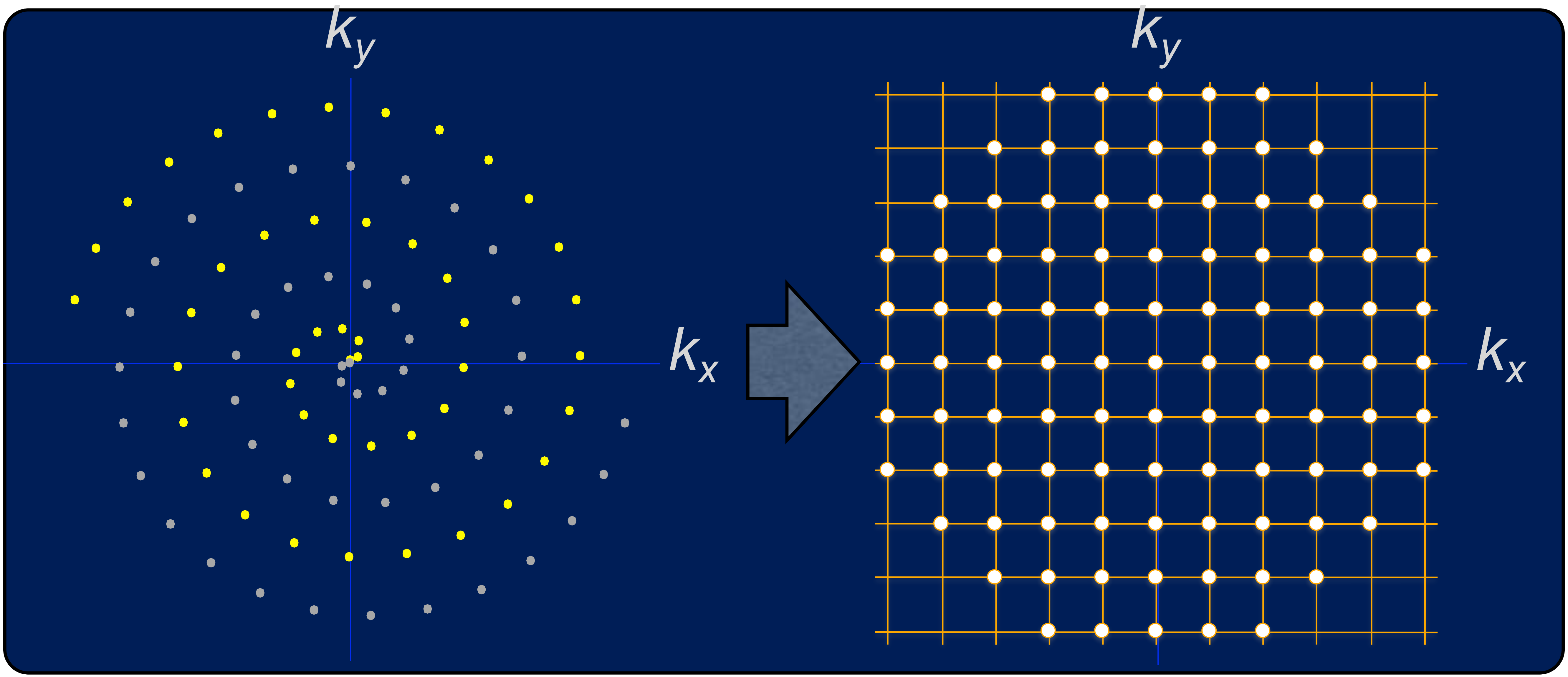
exampleE1_vds.m

Question - SNR?

Non-Cartesian Sampling / Gridding

- Irregularly sampled data
- Resample to grid to perform DFT

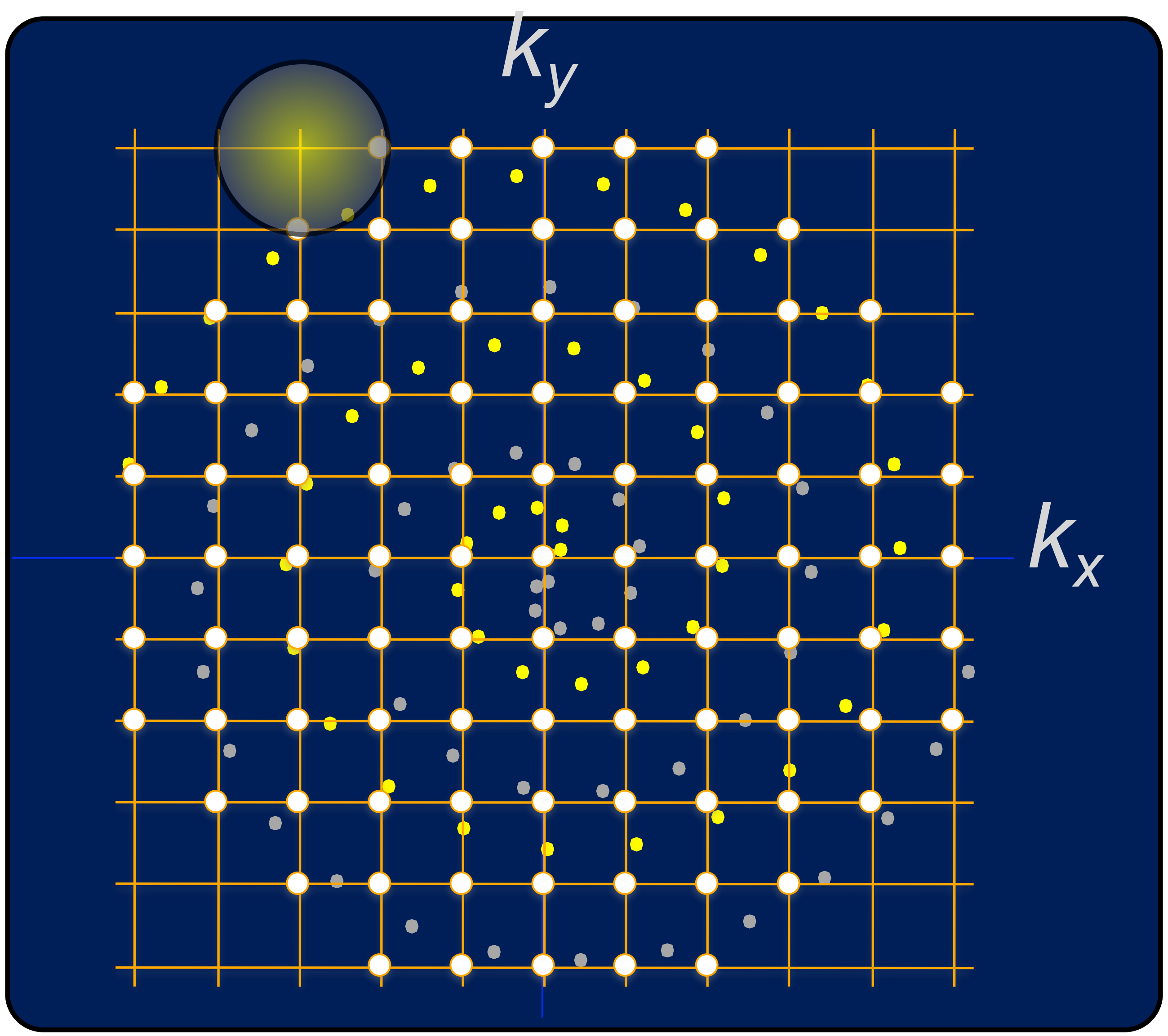
gridmat.m



Gridding Steps

Jackson 1991

- Divide samples by density at location k
  - Want to have uniform signal if we grid 1's
- Convolve sampled k locations with kernel c(k)
- Resample at grid points
- FFT Reconstruction
- De-apodize to undo convolution side effects

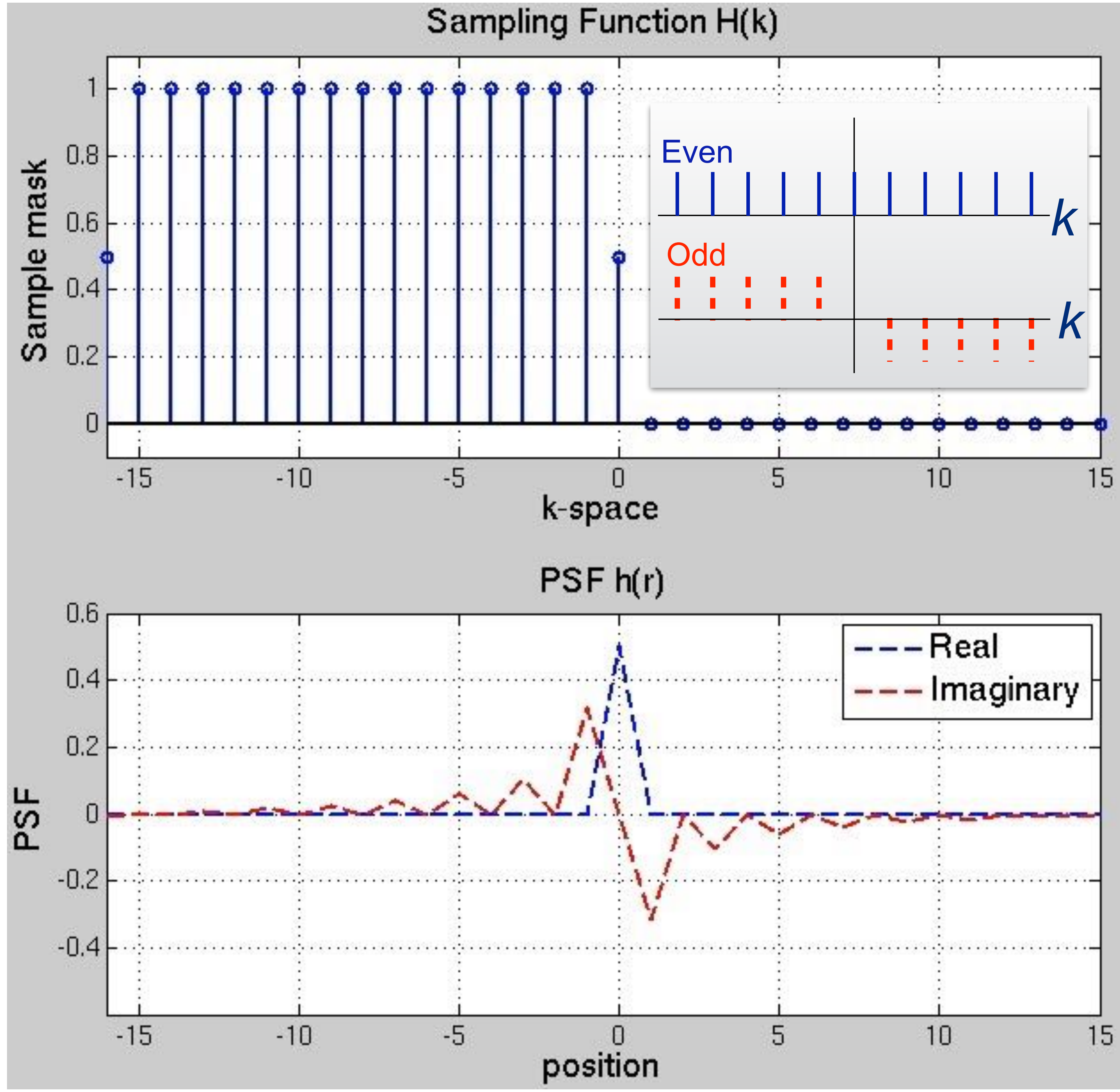


Partial Fourier and PSF

- Full k-space trajectory is $S_f(k)$, psf is $\delta(r)$
- Half-k-space trajectory is $S_h(k)$, PSF is $s_h(r)$
  - $S_h(k)$ is real, with even component $0.5\ S_f(k)$
  - $\mathrm{Real}\{s_h(r)\} = 0.5\ \delta(r)$
- Sampling: $M(k)\ S_h(k) \Leftrightarrow m(r)*[0.5\ \delta(r) + \mathrm{Imag}\{s_h(r)\}]$
- *If m(r) is real, the image is the real-part of* $m(r)*s_h(r)$.
- *How can we remove phase when m(r) is complex?*

Partial k-space PSF - Contiguous

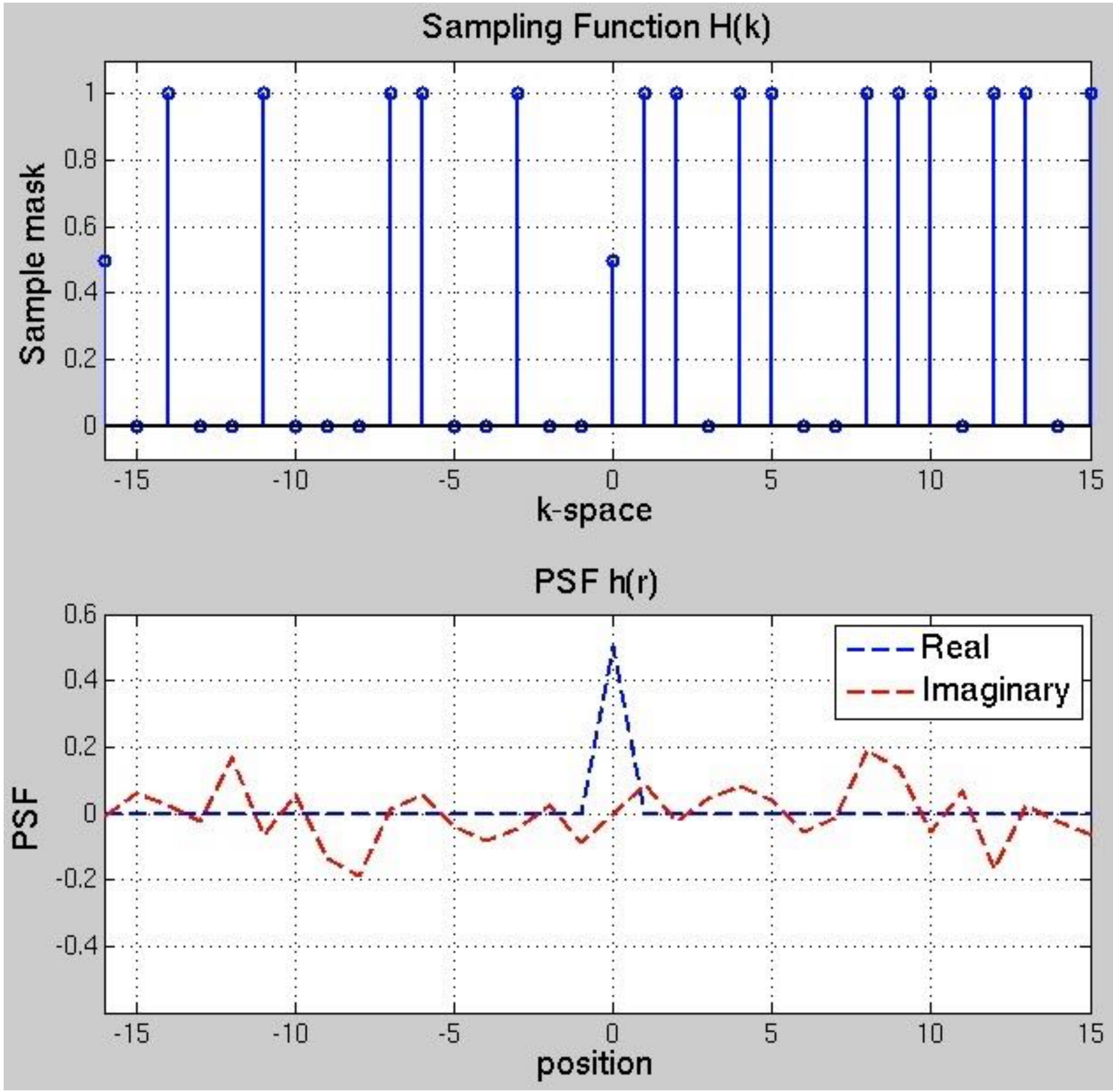
- Odd component is a step function
- Imaginary PSF is "localized"



Example: Partial k-space PSF - Even/Odd

Partial k-space PSF - Random Selection

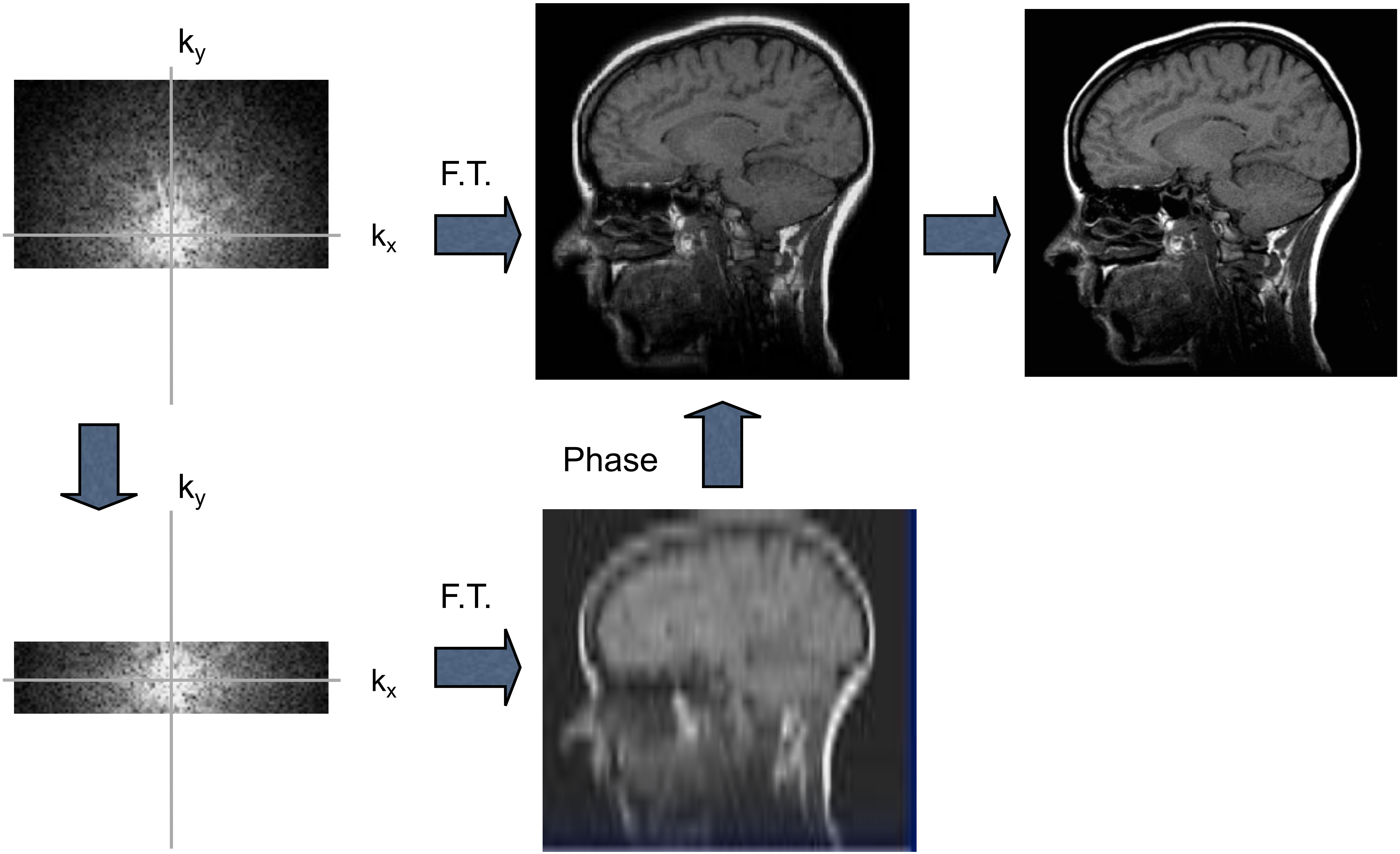
- Odd component is random 0 or 1
- Imaginary PSF is spread out



Homodyne Reconstruction

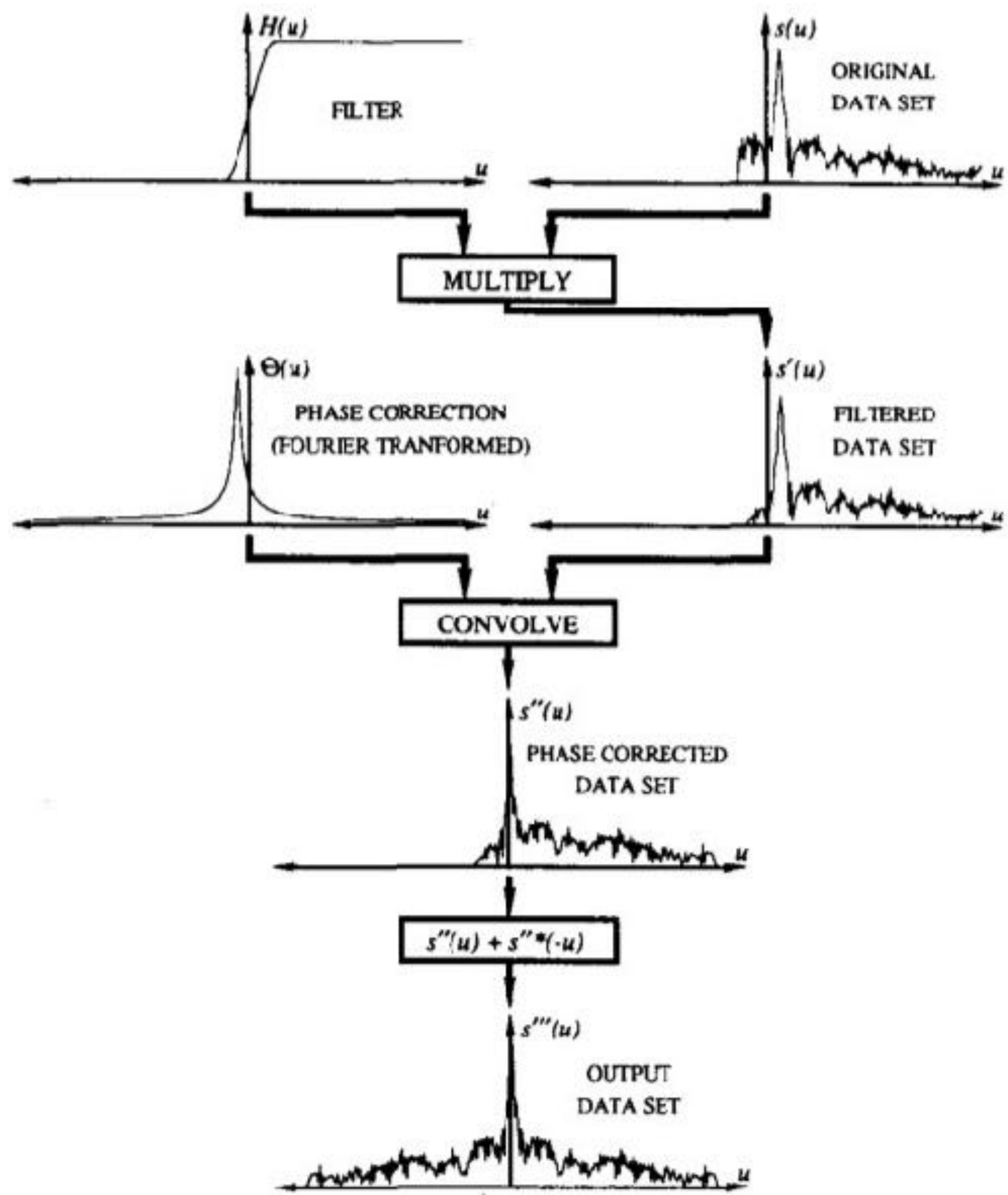
- Sample half k-space plus a little extra
- Symmetric k-space:
  - low-resolution image phase $\phi(r) \approx \angle m(r)$
- Use ramp filter to reconstruct $m(r)*s_h(r)$
- Remove phase: $[m(r)*s_h(r)]\, e^{-i\phi(r)}$
- *If* $s_h(r)$ *is narrow, phase of* $m(r)$ *is canceled, and real-part leaves* $m(r)*\delta(r)$
- *See John Pauly's notes for other recon methods*

Partial Fourier Acquisition/Reconstruction



Homodyne (k-space interpretation)

- From McGibney MRM 1993
- H(u) ~ Density Compensation, reduced ringing
- Assumption Θ(u) is narrow

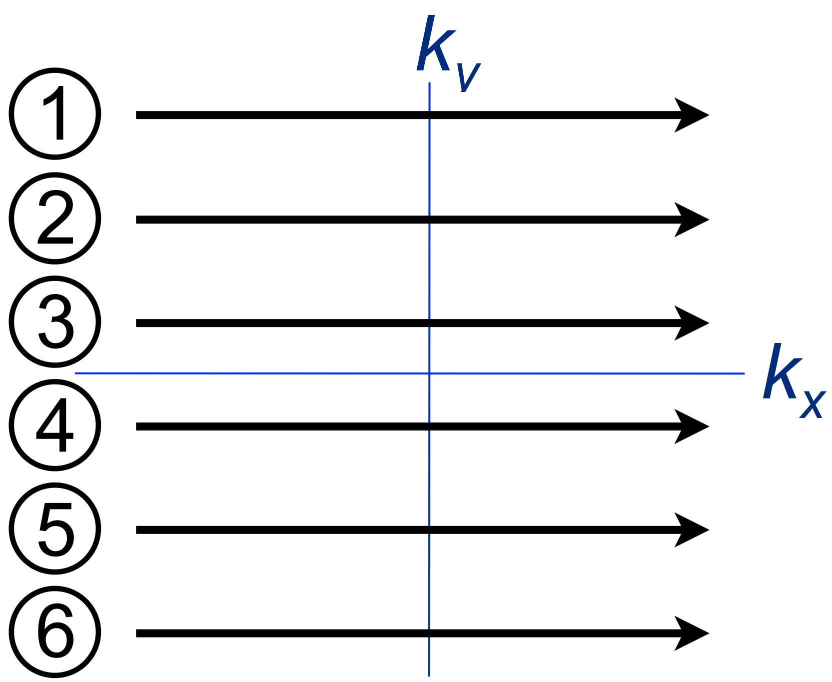


k-space Modulation

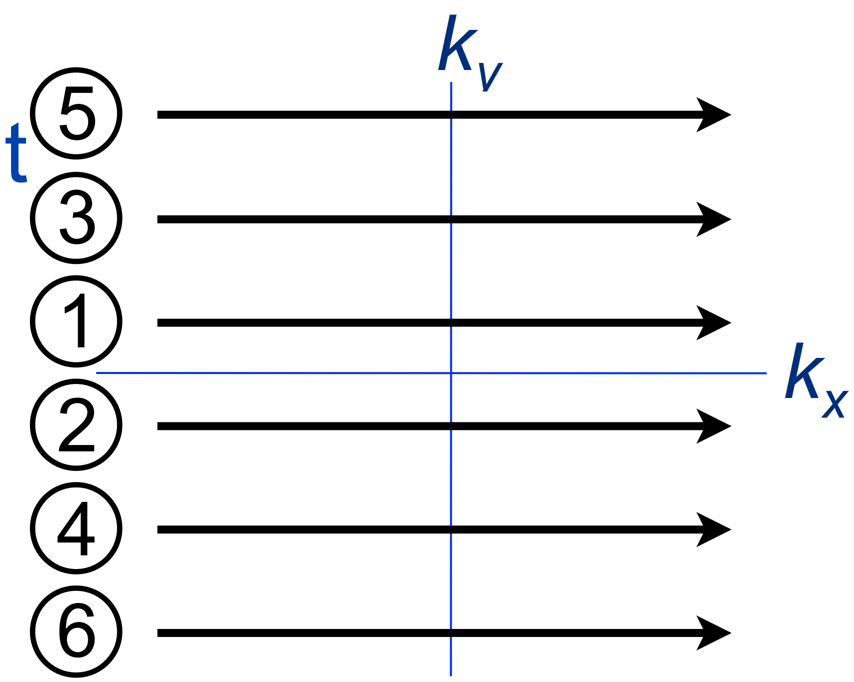
- Many sequences acquire multiple lines with <u>transient</u> magnetization:
  - Echo trains: T_2 and T_2* decay over k-space
  - Magnetization-prepped bSSFP, RF-spoiled transients
  - Off-resonance (EPI, Spiral primarily)
  - Temporal signal effects (non-motion):
    - Contrast uptake, inflow, varying B_0,

View Ordering / Grouping

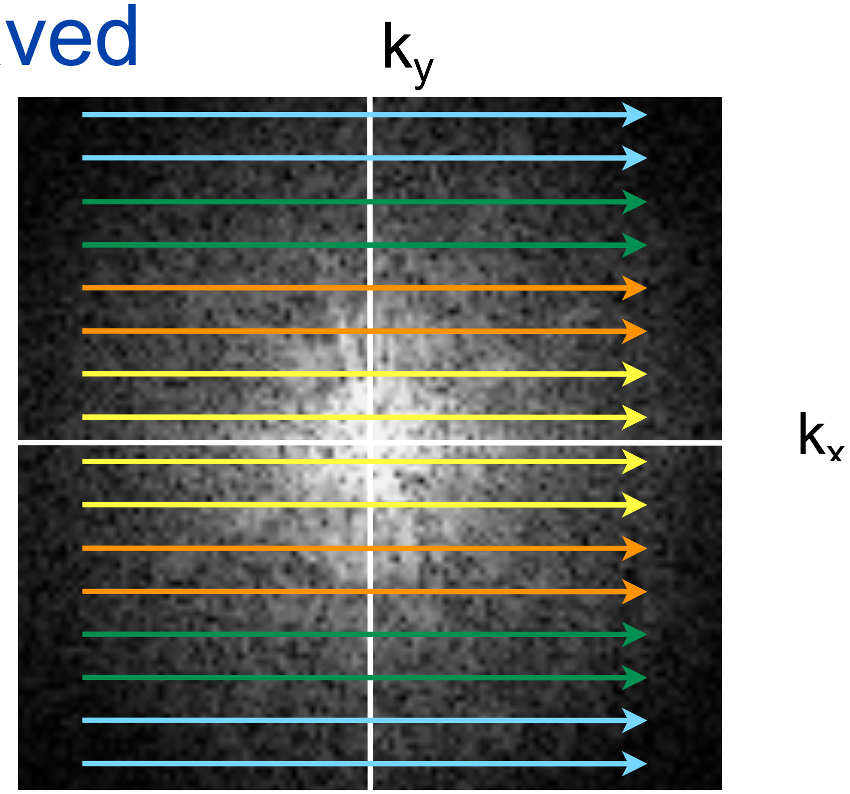
Sequential



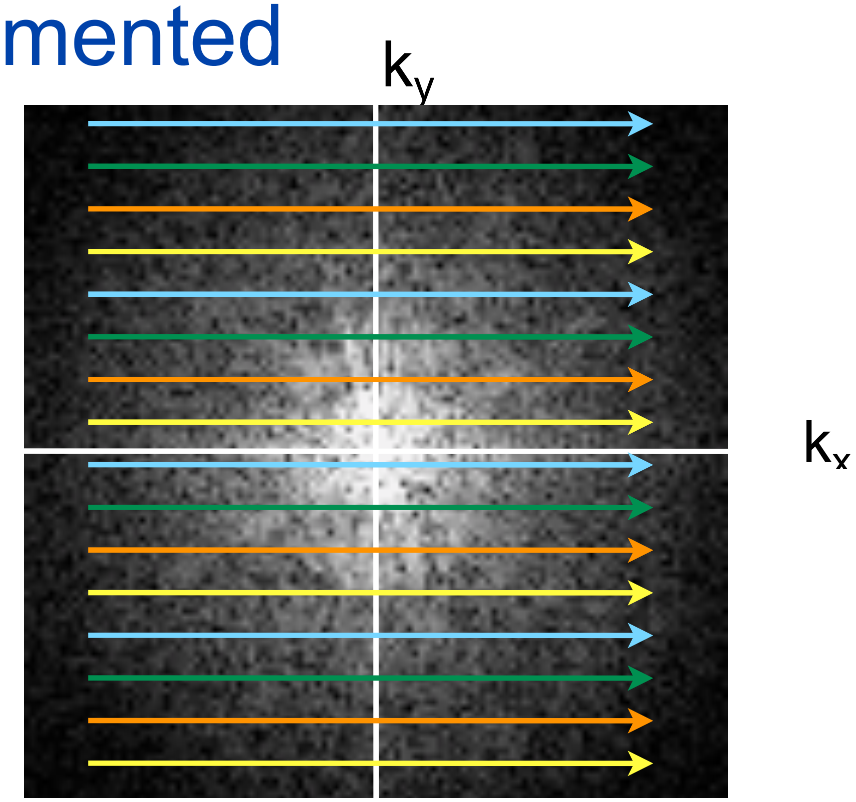
Centric / Center-out



Interleaved

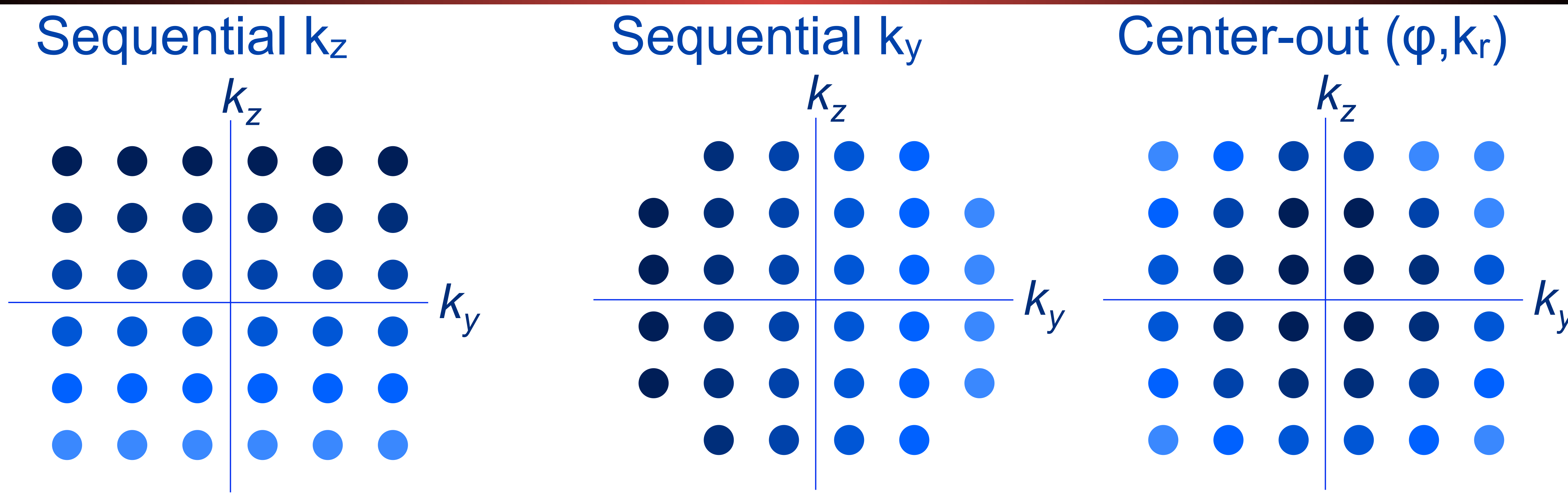


Segmented



Each color is a different “modulation” (echo, time, etc)

3D Image (k_y-k_z) View Ordering/Grouping



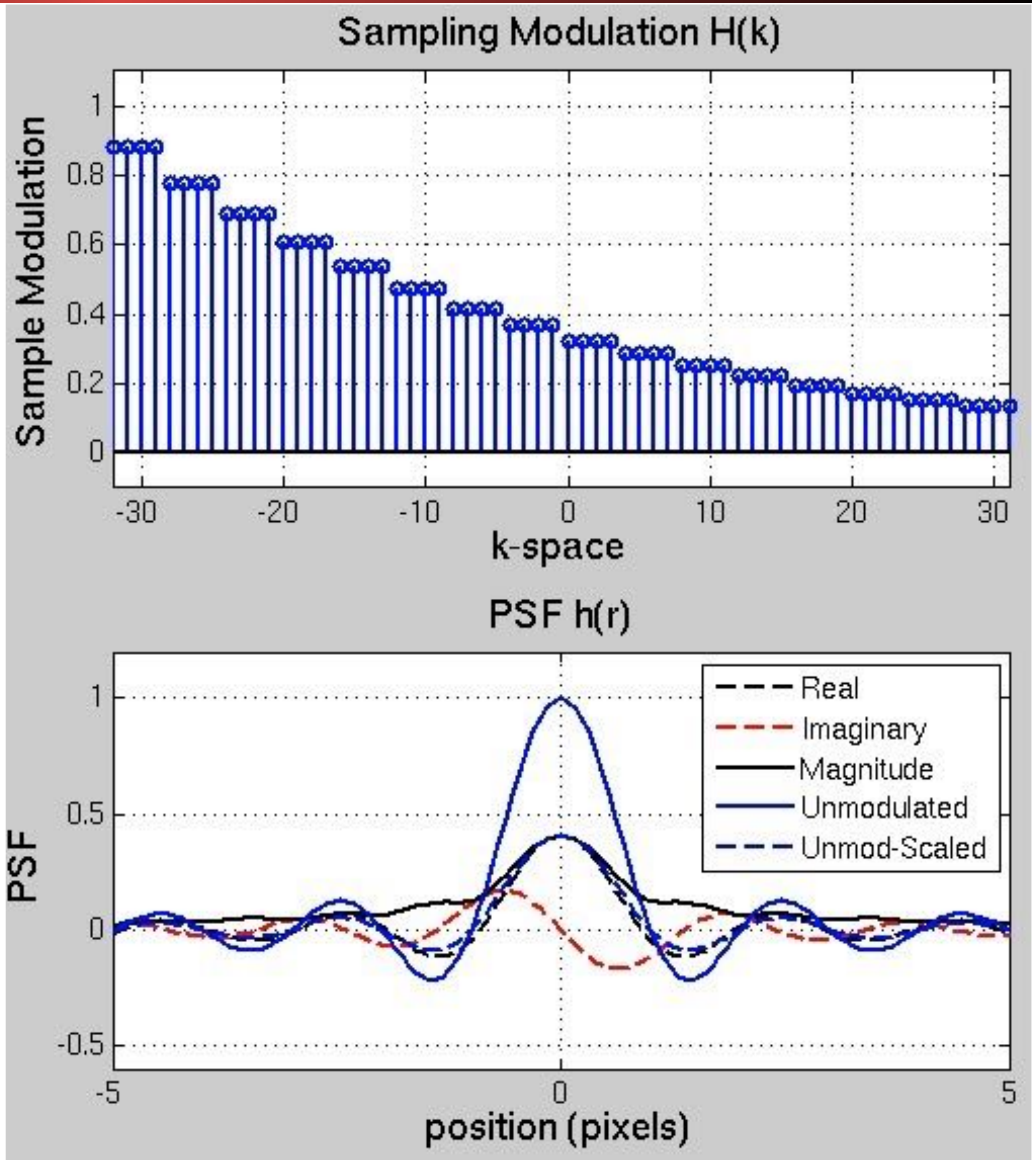
- Centric (ordered by radius first or azimuth (φ) first)
- Segment groups by k_y, k_z, φ, k_r
- Sub-segment groups (k_y, k_z, φ, k_r, randomly)

Modulation and PSFs

- Group k-space samples by intensity
- Reconstruct PSF for each group
- Multiply by modulation and sum

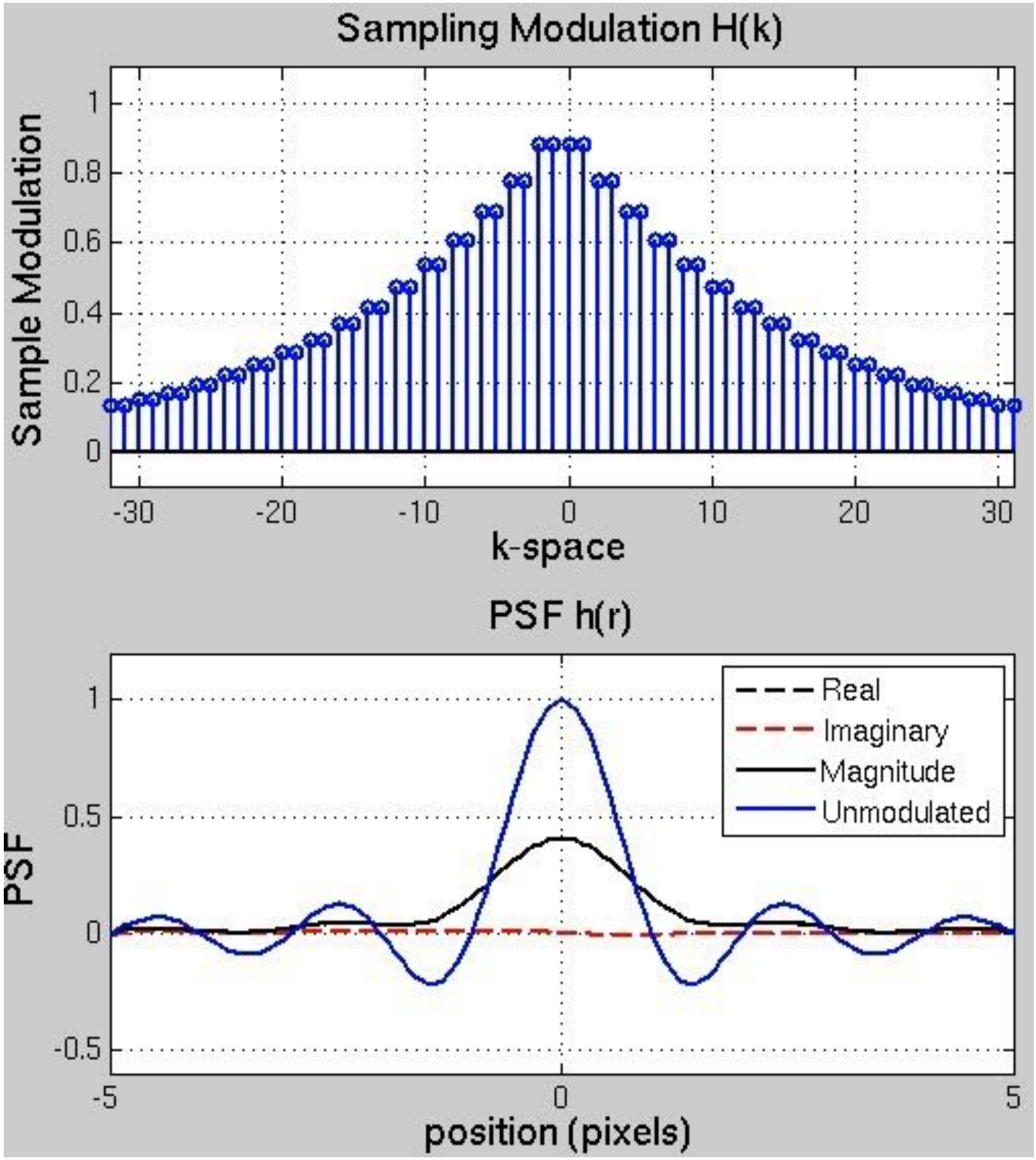
Modulation Example 1: FSE

- Echo Train of $2T_2$
- Peak reduction (area)
- Decompose Modulation into even / odd parts
- real{PSF} good
- |PSF| broadens



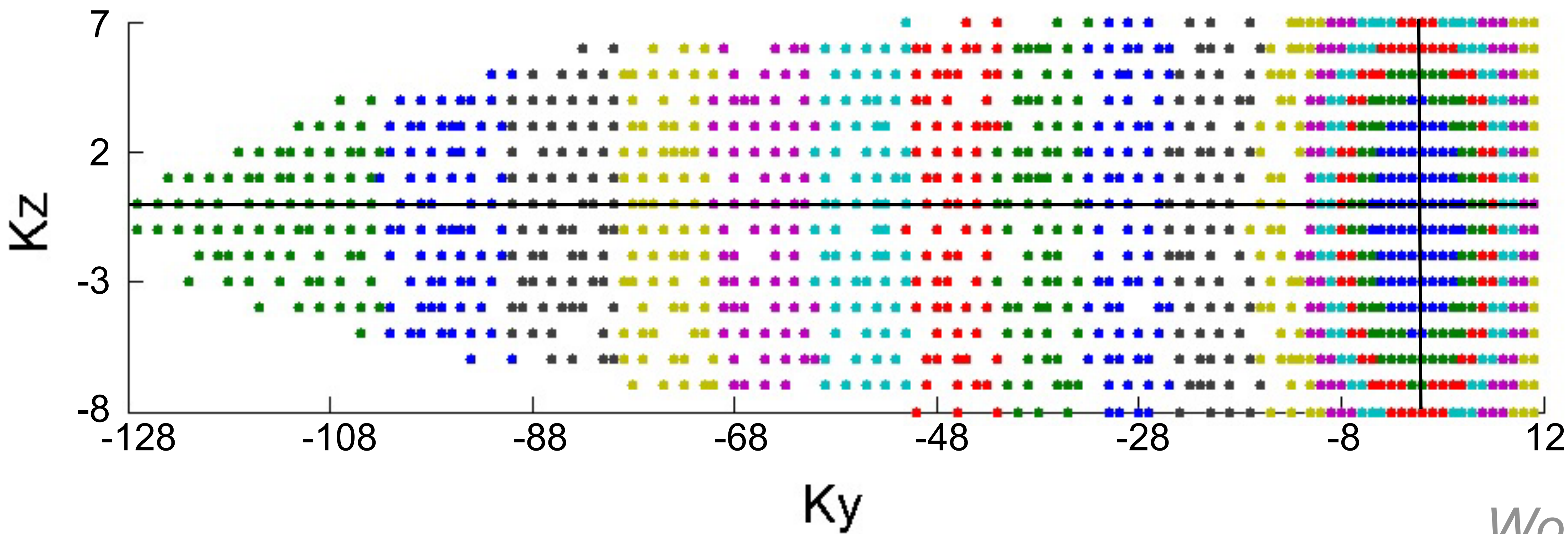
Modulation Example 2: PD FSE

- Echo Train of $2T_2$
- Peak reduction (area)
- Symmetric modulation: Real PSF
- PSF broadens



Example: Echo-Train + CS + Half-Fourier + Elliptic

- 2D k-space sampling variation (k_y-k_z phase encodes)
- "smooth" modulation with echo train
- Random sampling for CS
- Choose trajectories through regions to minimize change (eddy-current)

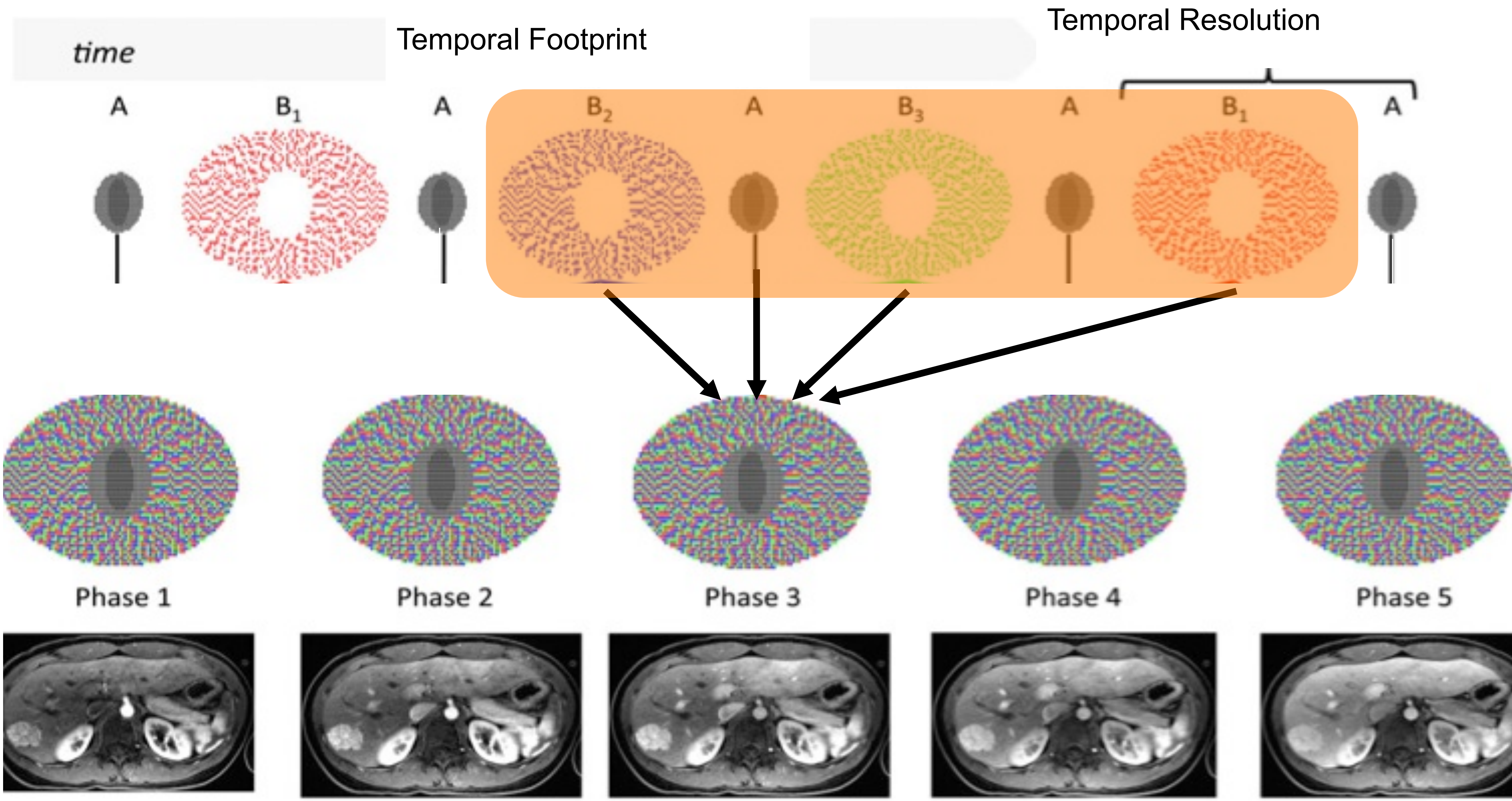


Worters 2011

Question: Temporal Odd/Even Sampling

Temporal Sampling (k_f and k-t)

Temporal Undersampling: DISCO



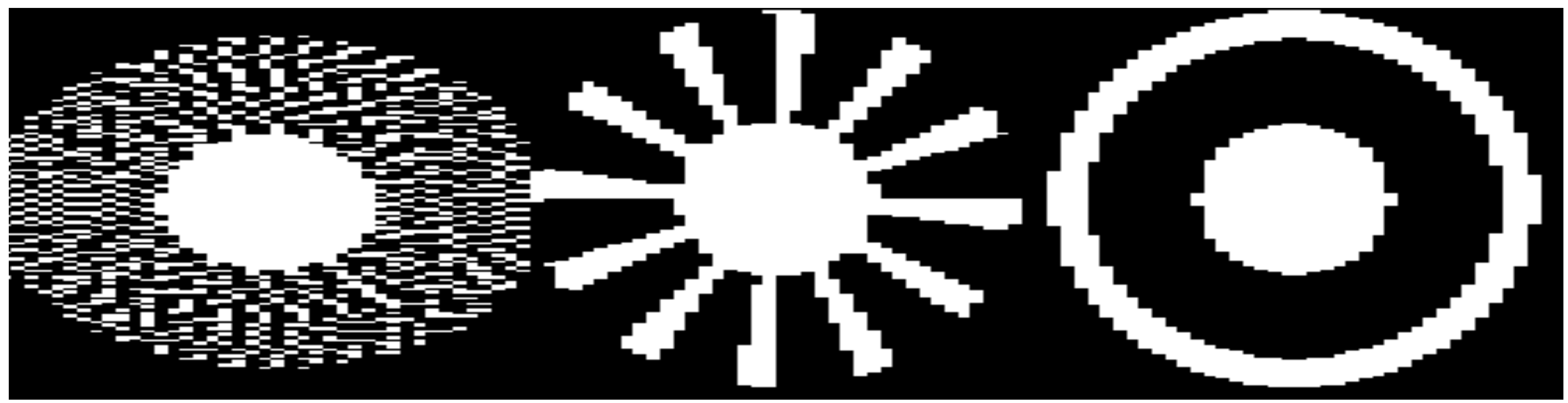
Temporal Undersampling: PSFs

Data Acquisition (k_y-k_z -space)

DISCO, TWIST

Cartesian Acquisition with Projection Reconstruction (CAPR)

Time-Resolved Imaging of Contrast Kinetics (TRICKS)

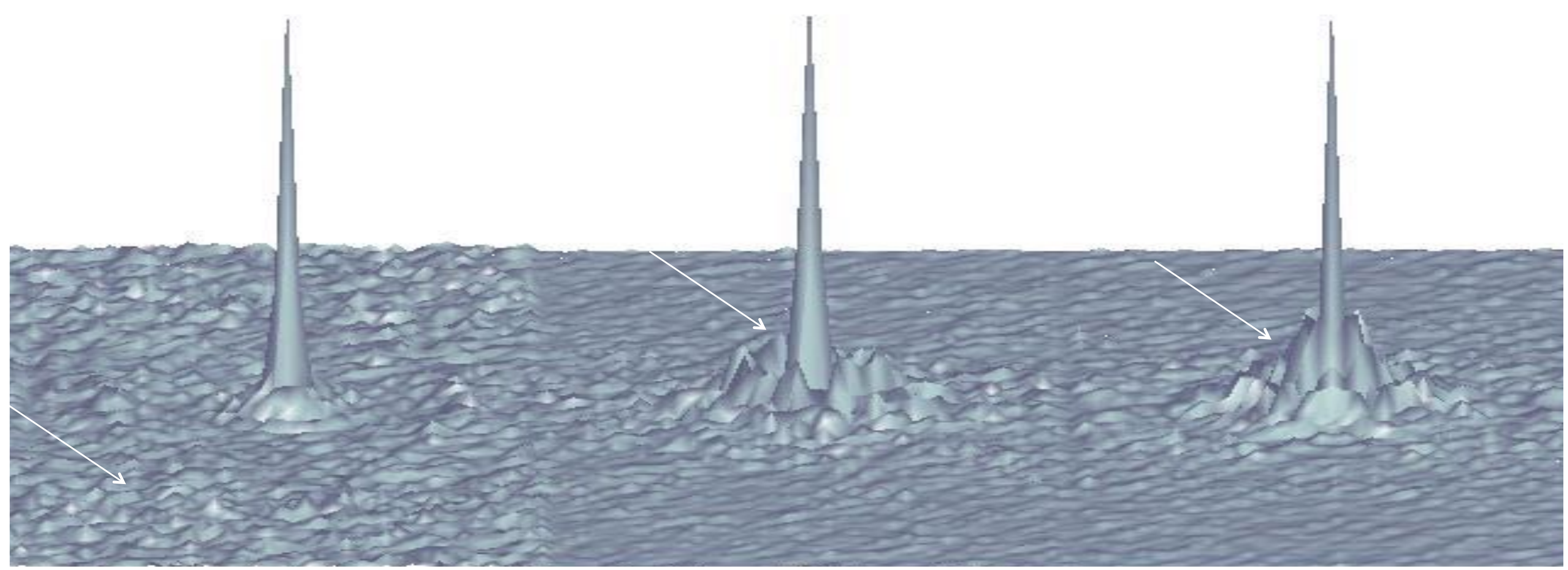


Saranathan 2012

Madhuranthakam 2006

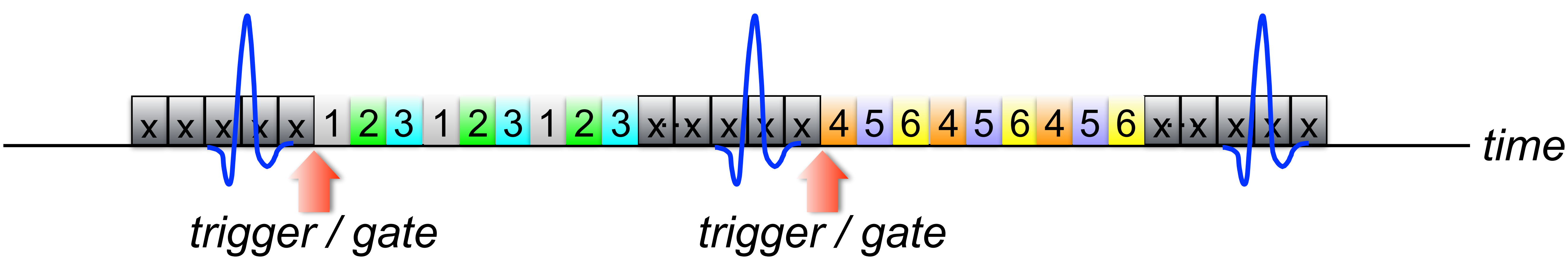
Korosec 1996

Point-Spread Functions



Cardiac/Respiratory Acquisition Timing

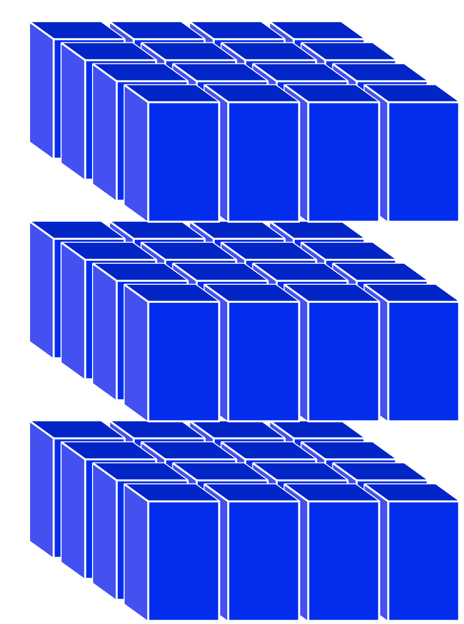
- Cine: Exploit periodicity of cardiac/respiratory cycle
  - Sample N k_y lines repeatedly, next N lines on next heartbeat.
  - High frame rate and spatial resolution
- Triggering: Start acquisition based on external trigger (EKG, plethysmograph, respiratory bellows)
- Gating: Excite continuously, but acquire only after trigger
- *Can combine any/all of these*

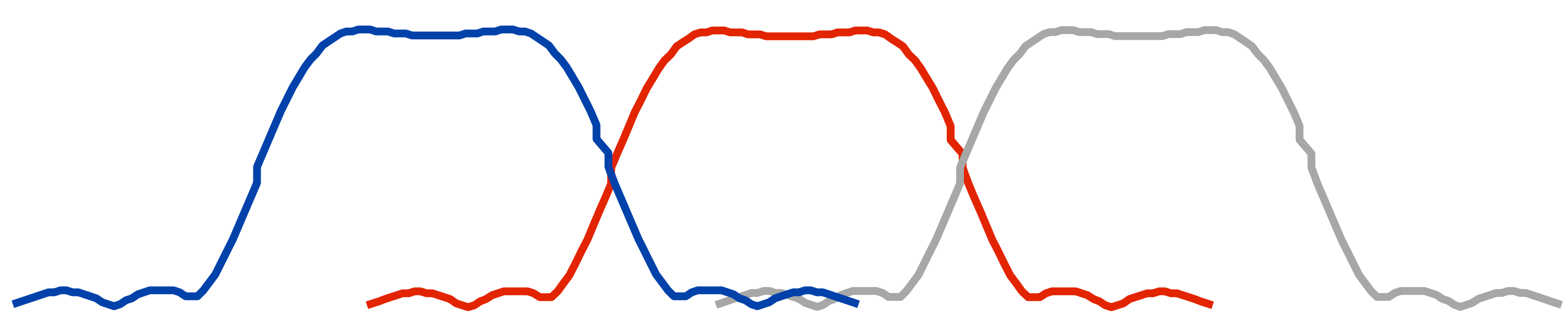


Cine Example

Slice Interleaving

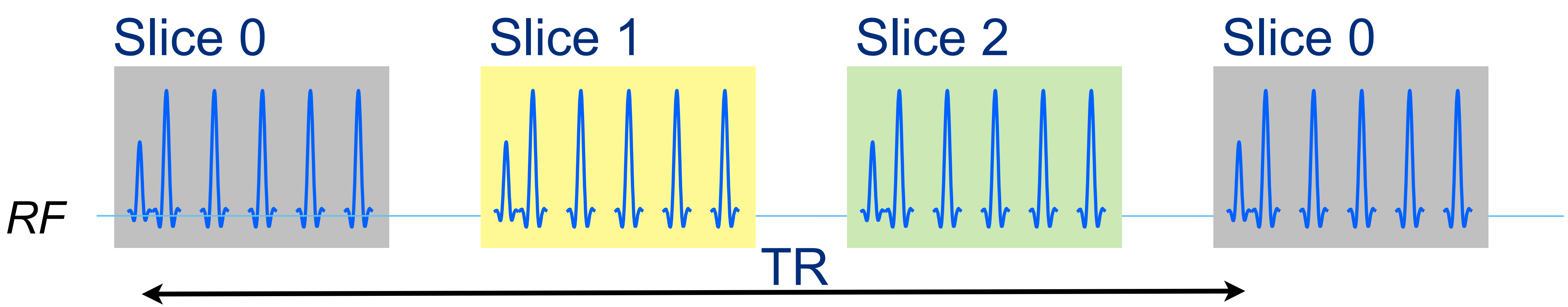
- Multislice acquisitions allow volumetric imaging
- Acquisitions can be *sequential* or *interleaved*
- Interleaving time efficient if there is "dead time"
- Different ways to interleave (reduce adjacent-slice-saturation)
    - Sequential: 0, 1, 2, 3, 4, 5, 6, 7
    - Odd/Even: 0, 2, 4, 6, 1, 3, 5, 7
    - Bit-reversed: 0, 4, 2, 6, 1, 5, 3, 7





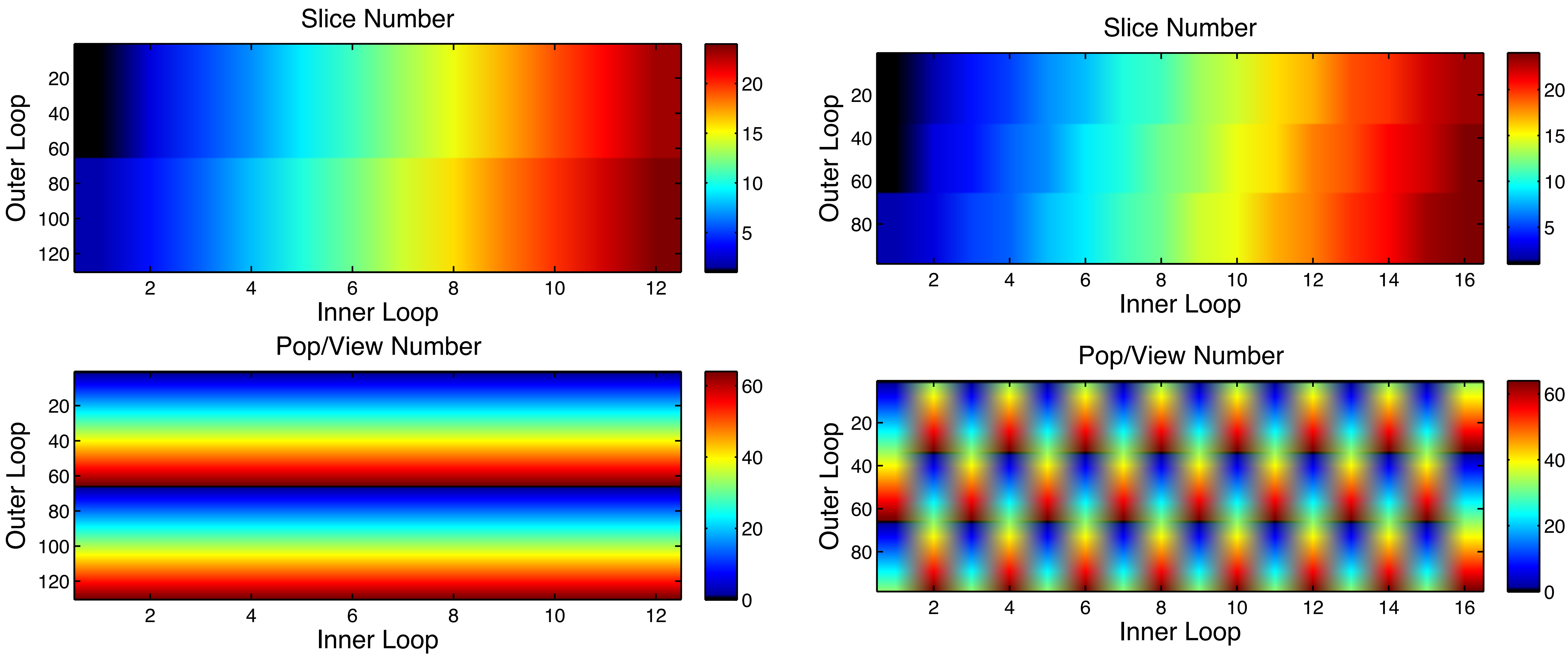
How Many Slices to Interleave?

- Usually specify TR, TI, Echo-train-length (ETL), Resolution, ...
  - Tells “pulse durations” (T_{seq}) and RF power
  - $N_{max} \sim TR / T_{seq}$
  - Can re-order slices in “time slots”
  - Additional slices require another “acquisition”



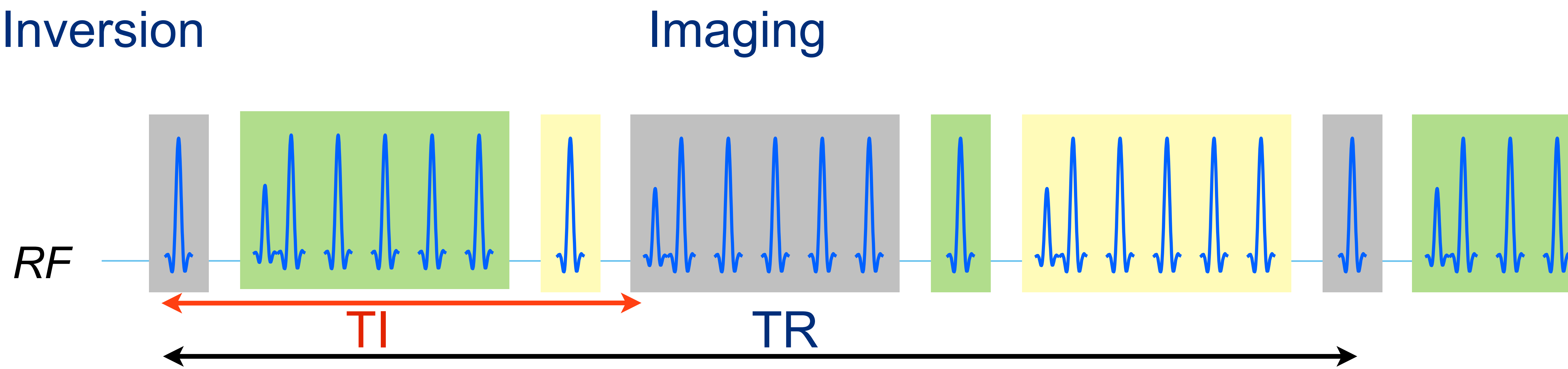
More Flexible Interleaving

- If $N_{slices} > N_{max}$, scan is 2x, 3x, ... longer
- Decoupling phase encode number allows flexible interleaving
- Read-out “matrices” across then down

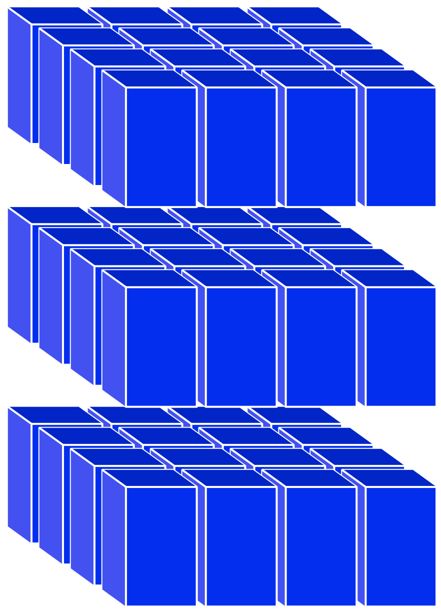


FLAIR / STIR?

- Additional dead-time during TI interval
- Can sometimes interleave other acquisitions
- Additional constraints on TR, TI, T_{seq}



Multiband Imaging

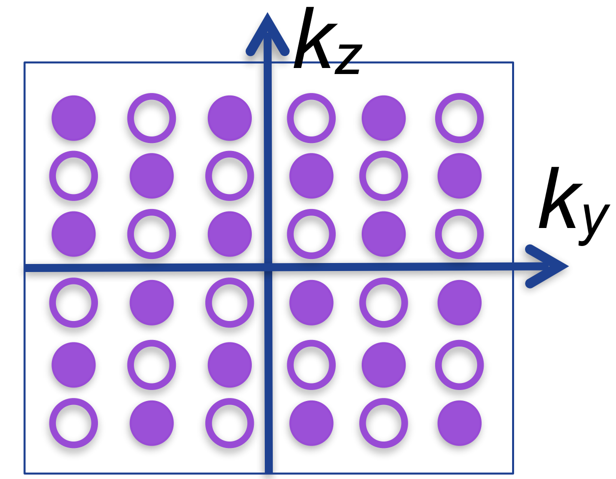


- Imaging multiple slices simultaneously
- Excitation: Multiply RF(t) by cos(t+φ)
  - Increases SAR
- Imaging:
  - Hadamard: Excite "1,1" and "1,-1" pattern, add & subtract
  - POMP: Alternate patterns, increase y FOV
  - Parallel Imaging: Use coils to separate slices
  - Blipped sequences: G_z "blips" induce slice-dependent phase
    - Like 3D k-space with limited excitation
    - Similar to Dixon water/fat: slices are like spectral peaks

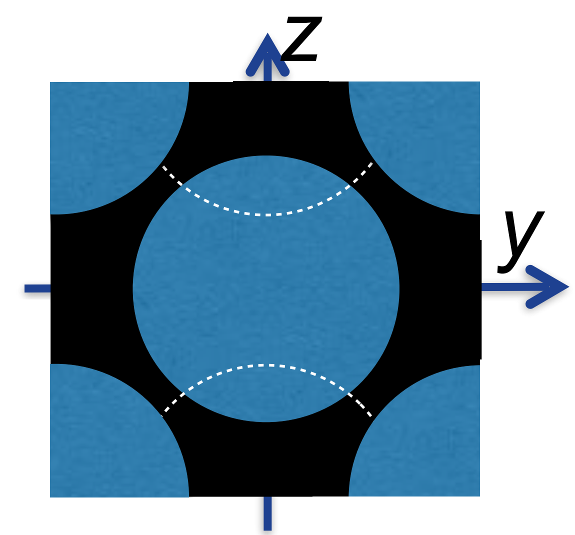
"Controlled Aliasing in Parallel Imaging" (CAIPI)

Breuer F, MRM 2005, 2006

- 3D (ky-kz) sampling:
  - Hexagonal sampling offsets replicas
  - Reduced aliasing (further apart)



- 2D Multislice:
  - Alternating phase during excitation or blips
  - Offset replicas allows in-plane coil sensitivities to help separate slices
  - Can think of as 3D k-space

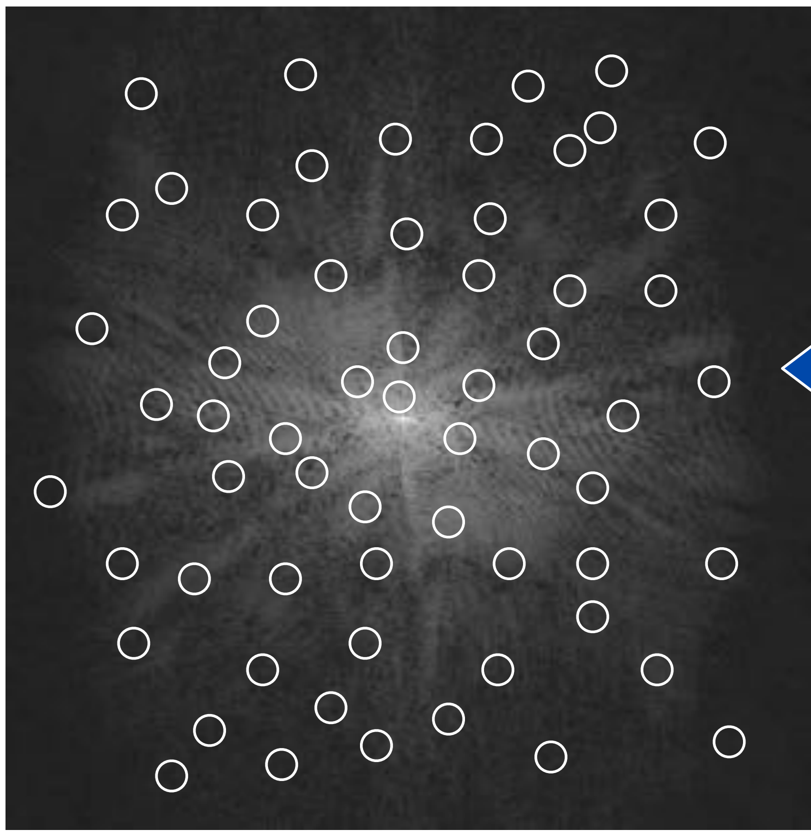


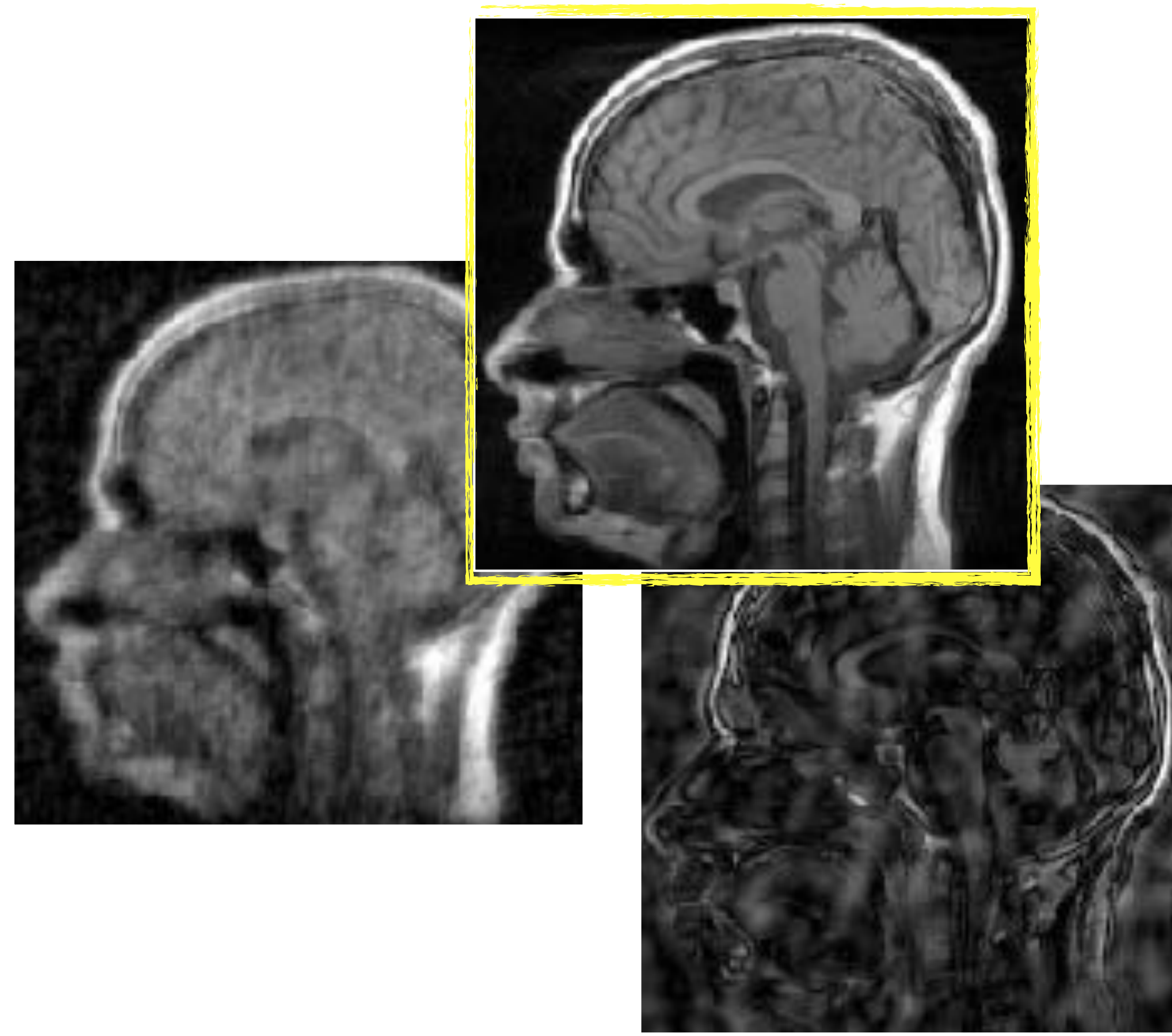
Multiband Example

Compressed Sensing MRI

Lustig, et al. MRM 2007

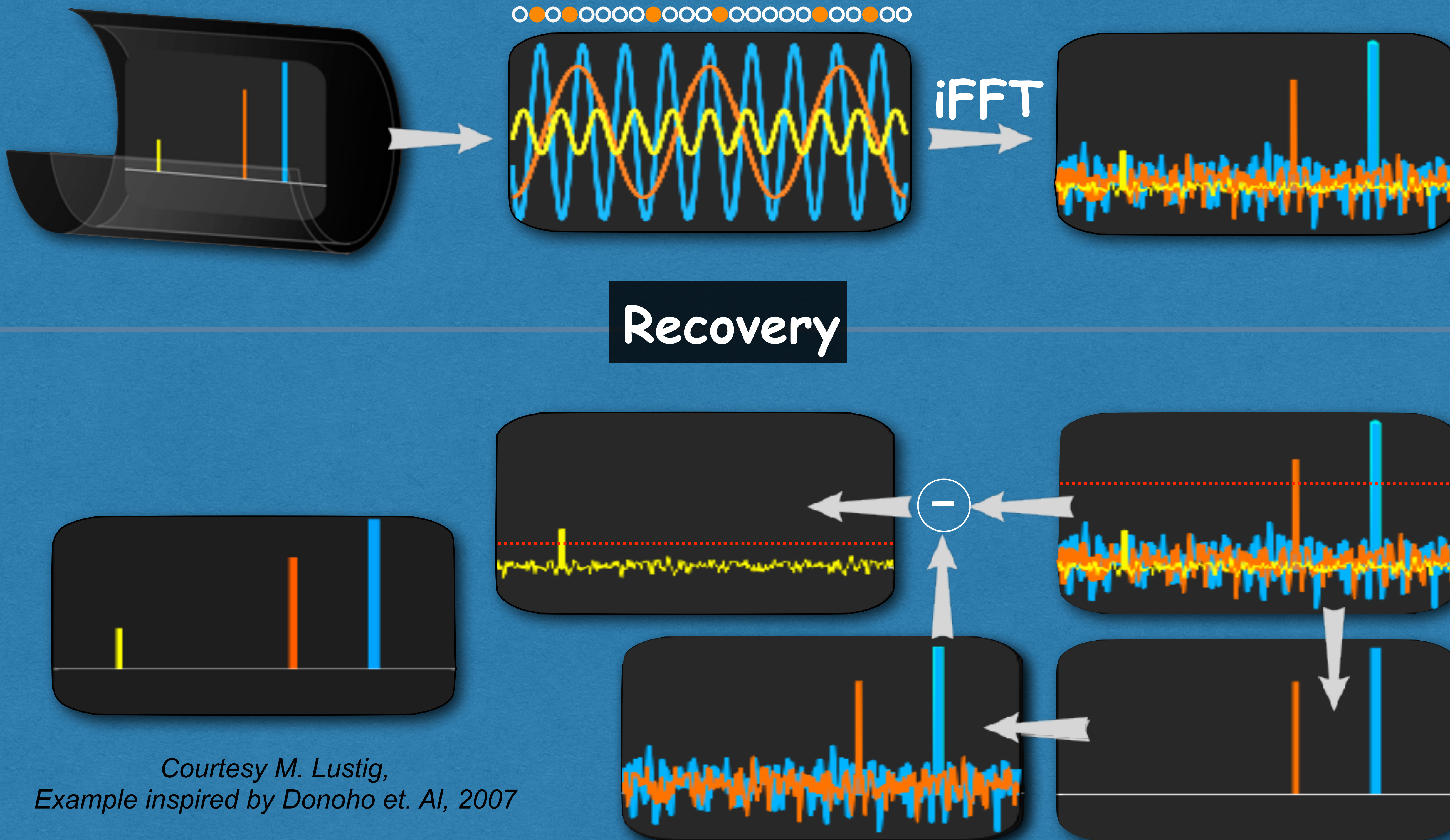
MRI Data





- Sample less data, randomly
- Choose “compressible” image matching data

CS Reconstruction and PSFs

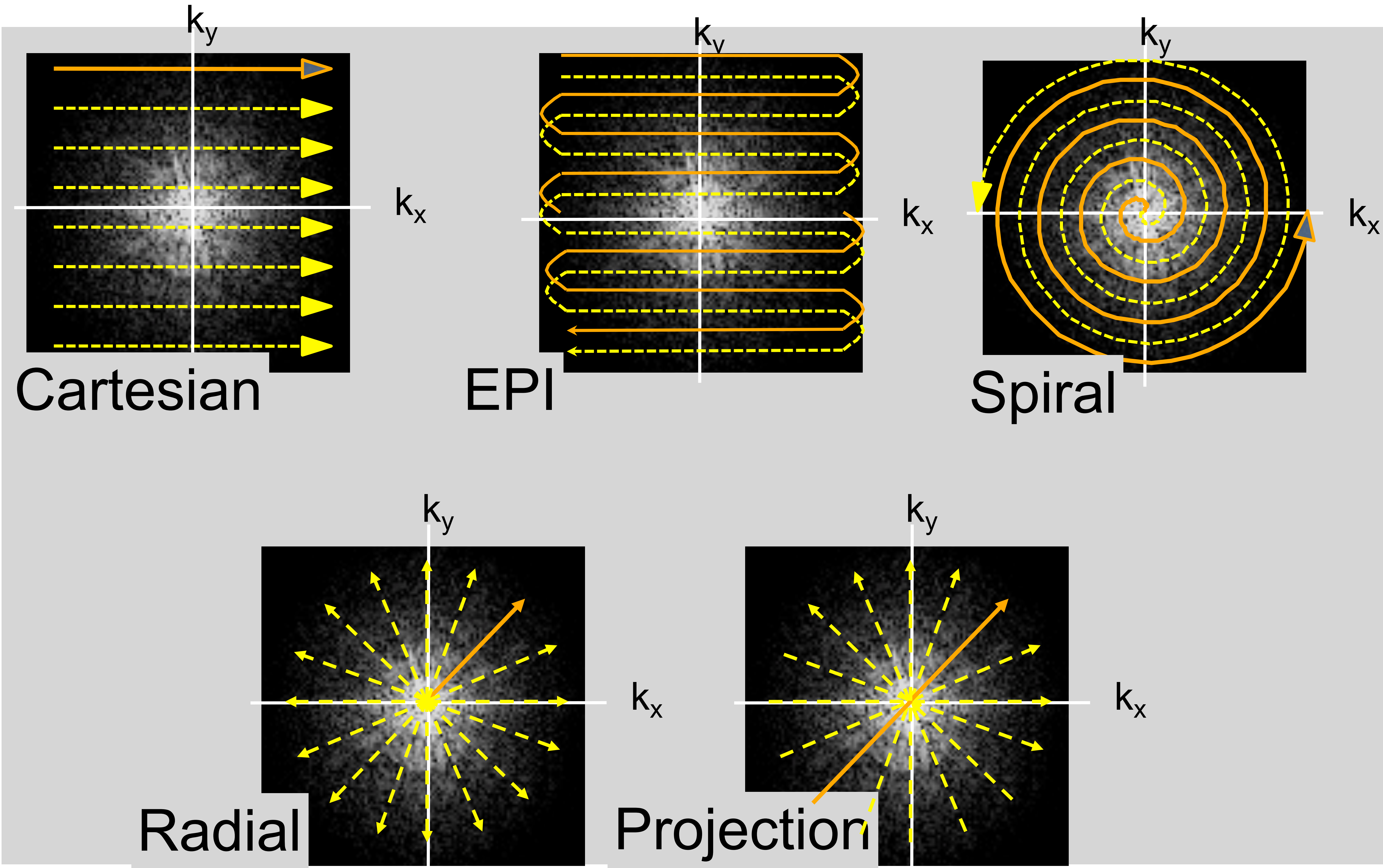


Courtesy M. Lustig,
Example inspired by Donoho et. Al, 2007

Summary

- Sampling and PSFs
  - Resolution, FOV, ringing
  - Variable-density and gridding
  - Partial Fourier
  - View ordering and k-space modulation
  - ky-kz and k-t sampling
- Slice interleaving
- Simultaneous Multislice Imaging

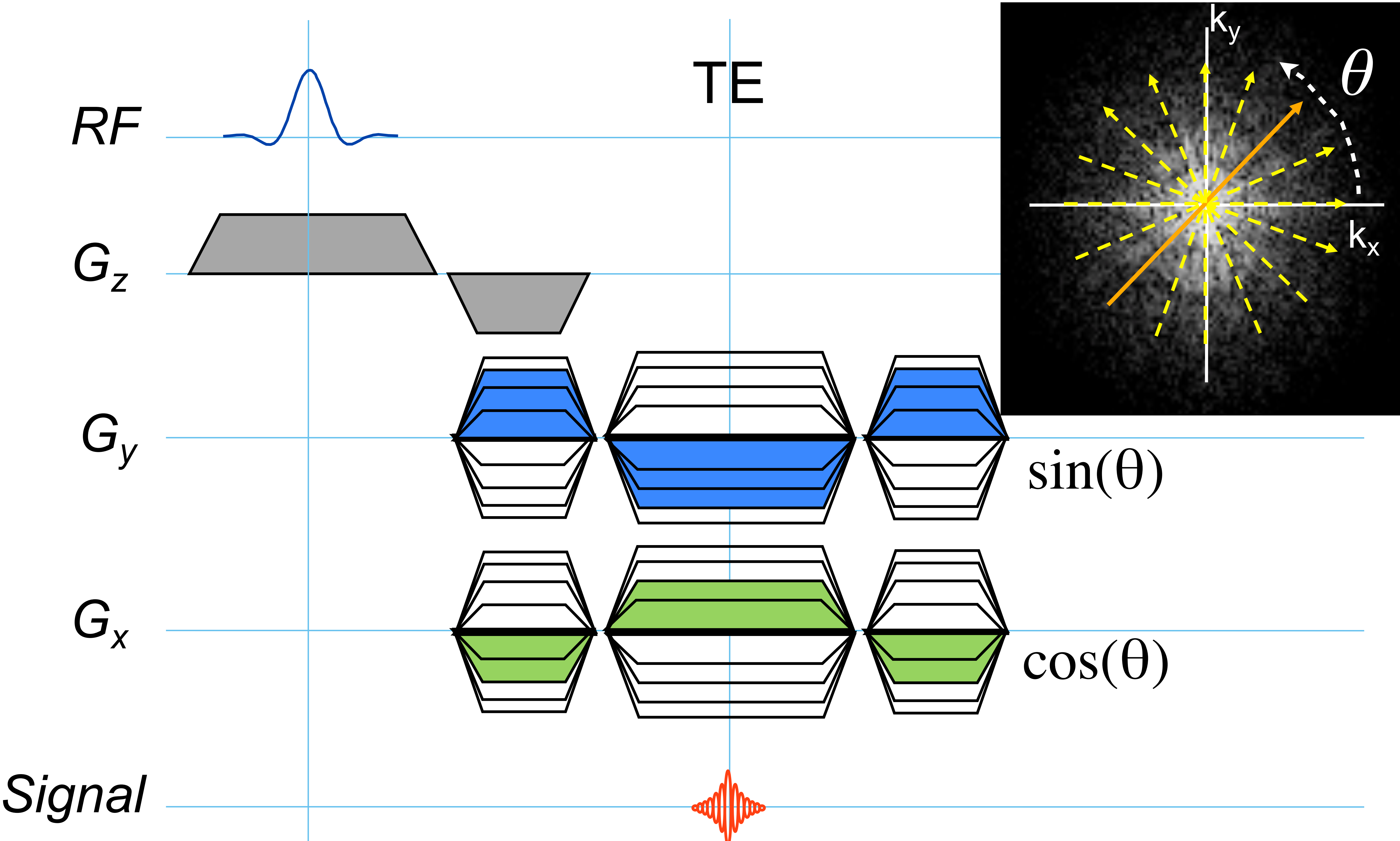
Advanced Imaging Trajectories



Radial and Projection Imaging

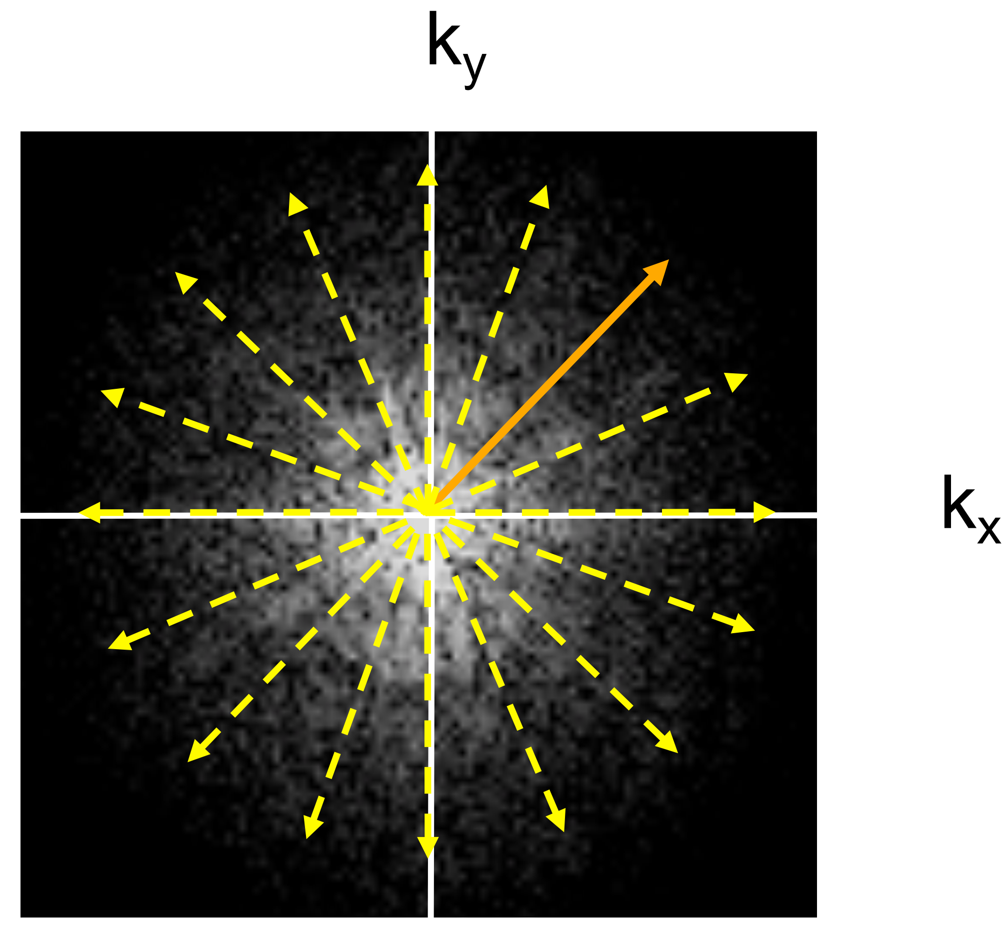
- Sample spokes
  - "Radial out": from $k=0$ to k_{max}
  - Projection: from $-k_{max}$ to k_{max}
- Trajectory design considerations (resolution, #shots)
- Reconstruction, PSF and "streak-like" aliasing
- SNR considerations
- Undersampling
- 3D Projection

Projection-Reconstruction (PR) Sequence

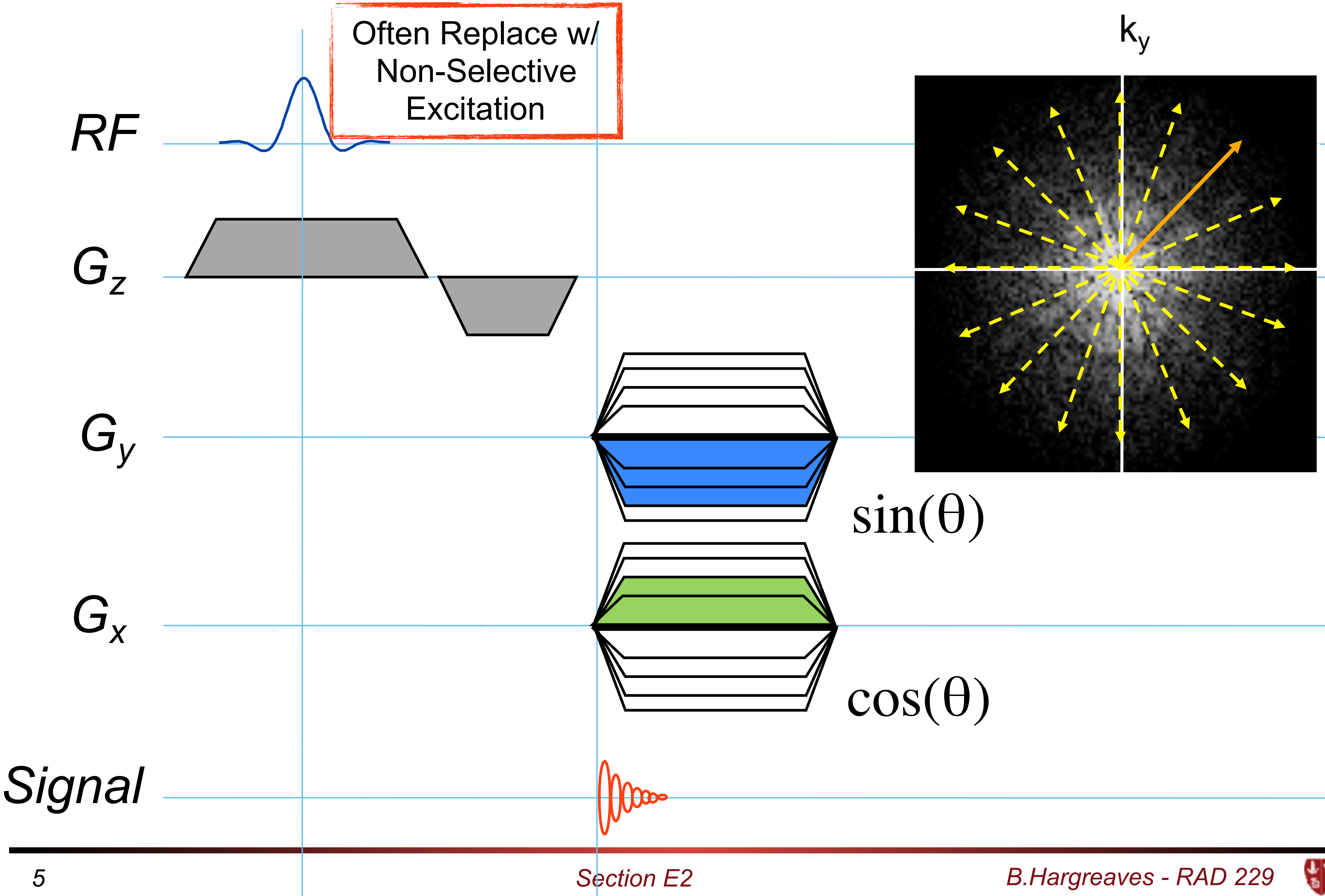


Radial (k=0 outward)

- Similar to Full Projection, but center-out readouts
- Shortest TE (~0) of any sequence
- Low first-moments
- Fastest way to reach high-spatial frequencies
- Impact of delays
- Can do odd/even sampling
- Impact of ramp sampling

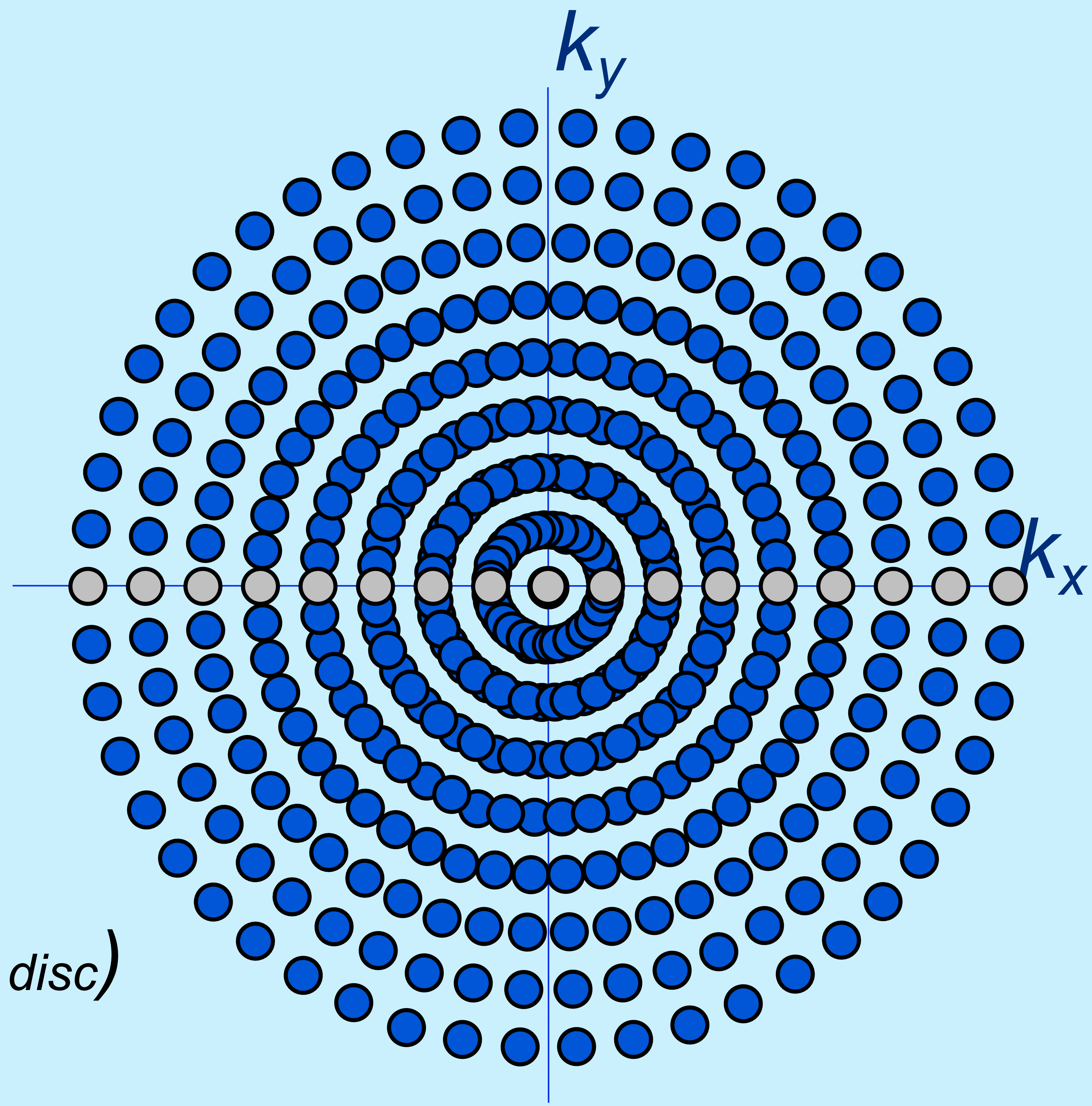


Radial (out) Sequence



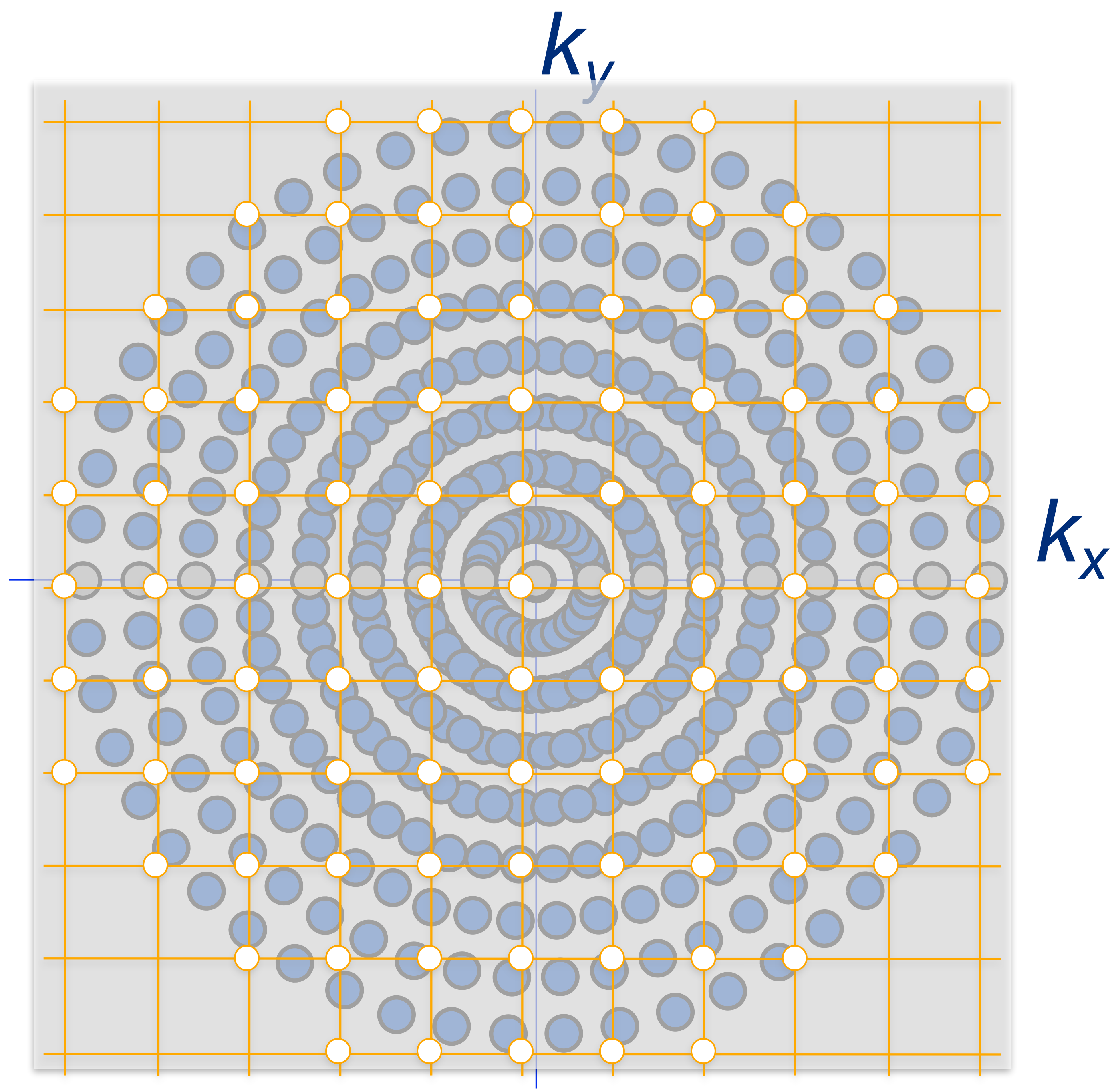
Projection (PR): Design Considerations

- Readout Resolution?
  - Same as Cartesian
- Readout FOV?
  - Same as Cartesian
- Number of angles?
  - $(\pi/2)N_{read}$ *(Full Projection)*
  - πN_{read} *(Radial-Out)*
  - *(Note: N_{read} = # points across disc)*
  - May undersample



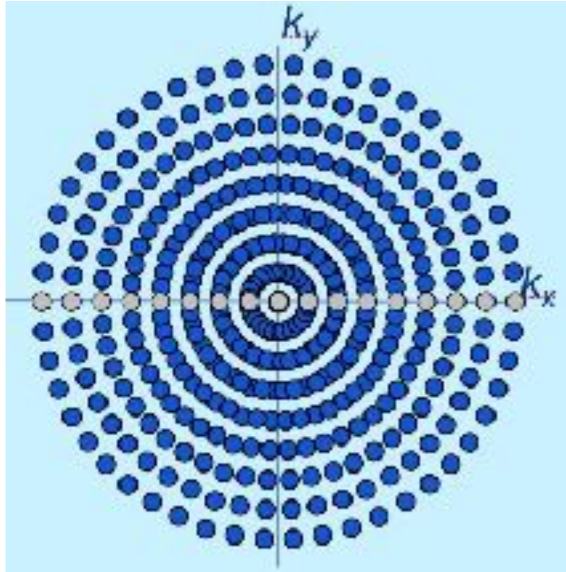
Projection Reconstruction

- Non-Cartesian Sampling
- Filtered back-projection
- Gridding + FFT
  - Both use $|k_r|$ density compensation

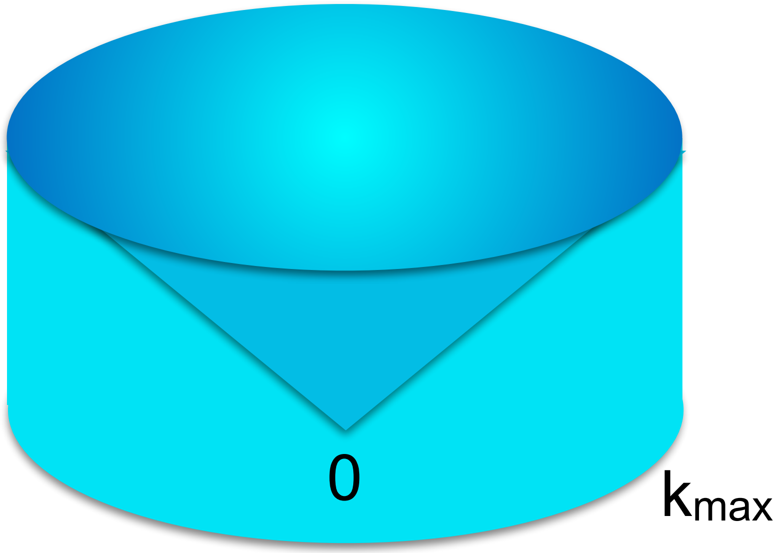


gridmat.m

Radial Sampling Efficiency



- Number of samples per-unit-area of k-space:
  - Cartesian requires NxN samples, Area $4k_{max}^2$
  - Radial requires (π/2)NxN samples, Area πk_{max}^2
  - Radial requires (π/2)/(π/4) = 2x samples / unit area
    - Penalty 1/sqrt(2)
- Noise variance averaged over k-space area
  - average of k_r (Density compensation)
  - "pill-box minus cone" = 2/3
- Overall Efficiency is 1 / sqrt(2 x 2/3) = $\sqrt{3}/2 = 0.87$



Reduced aliasing artifacts using variable-density k-space sampling trajectories. Tsai CM, Nishimura DG. Magn Reson Med. 2000 Mar;43(3):452-8.

Radial/Projection: SNR Efficiency (η)

- Radial density $D = k_{max}/k_r$, extent [-1,1]

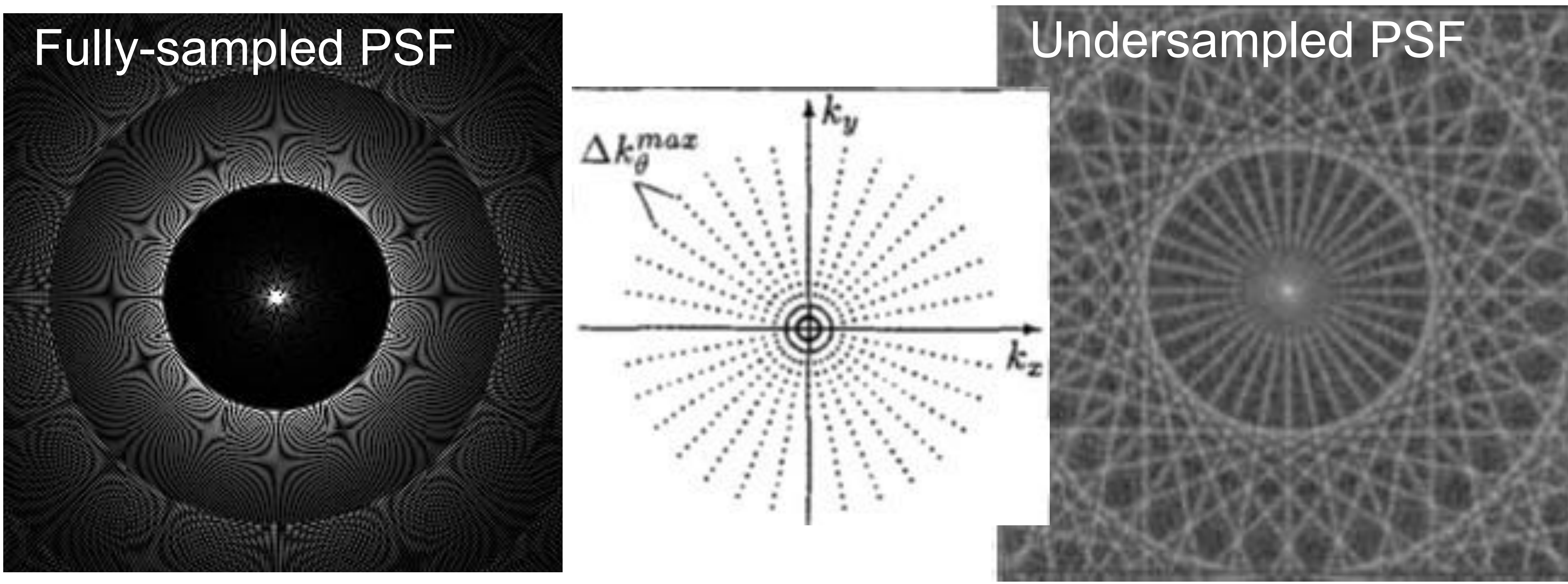
Quantity	Symbol	Cartesian	2D Radial
k-space area	(A)	4	π
#samples needed to cover area	$\int_A D$	4	2π
Integrated density compensation	$\int_A 1/D$	4	2π/3
Efficiency	$\eta = \frac{A}{\sqrt{\int_A D \int_A 1/D}}$	1	$\sqrt{3}/2 = 0.87$

- Note that the density variation affects efficiency

Radial-Outward Design Example

Projection-Reconstruction PSF / Undersampling

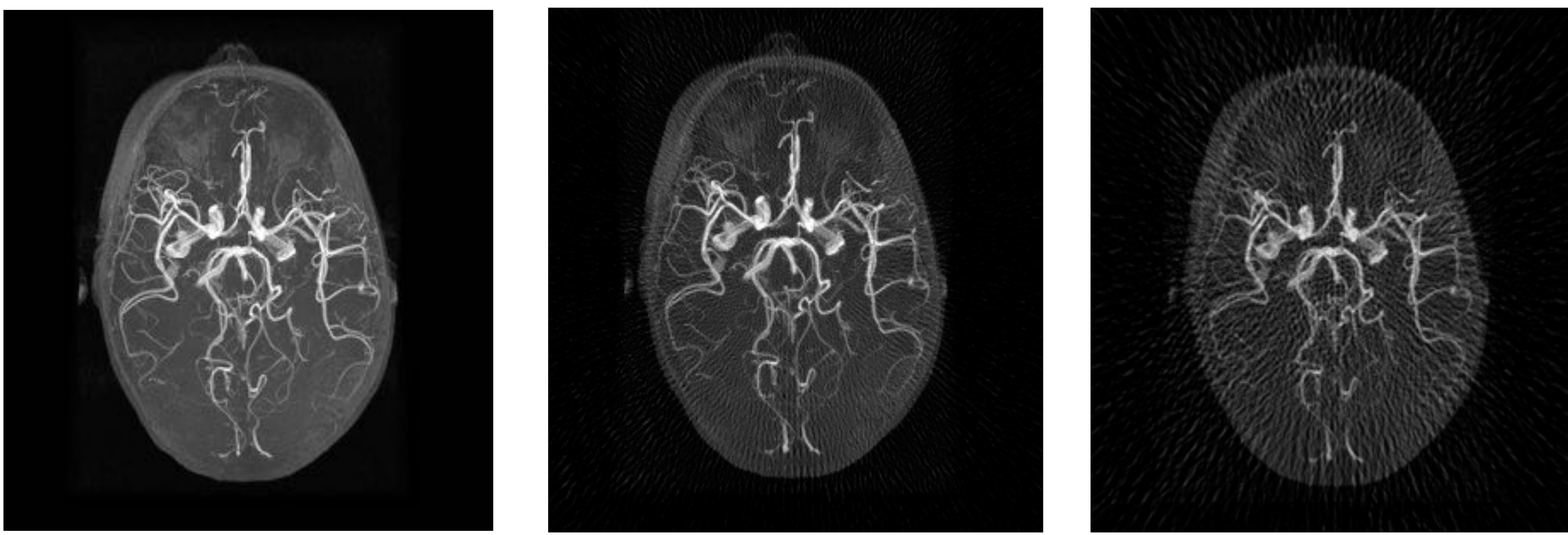
- PSF has a "ring" of aliasing (less coherent)
  - Intuition: No "preferred" direction for coherent peak
- Undersampling tends to result in streak artifacts



From Scheffler & Hennig, MRM 1998

Undersampled PR: Streak Artifacts

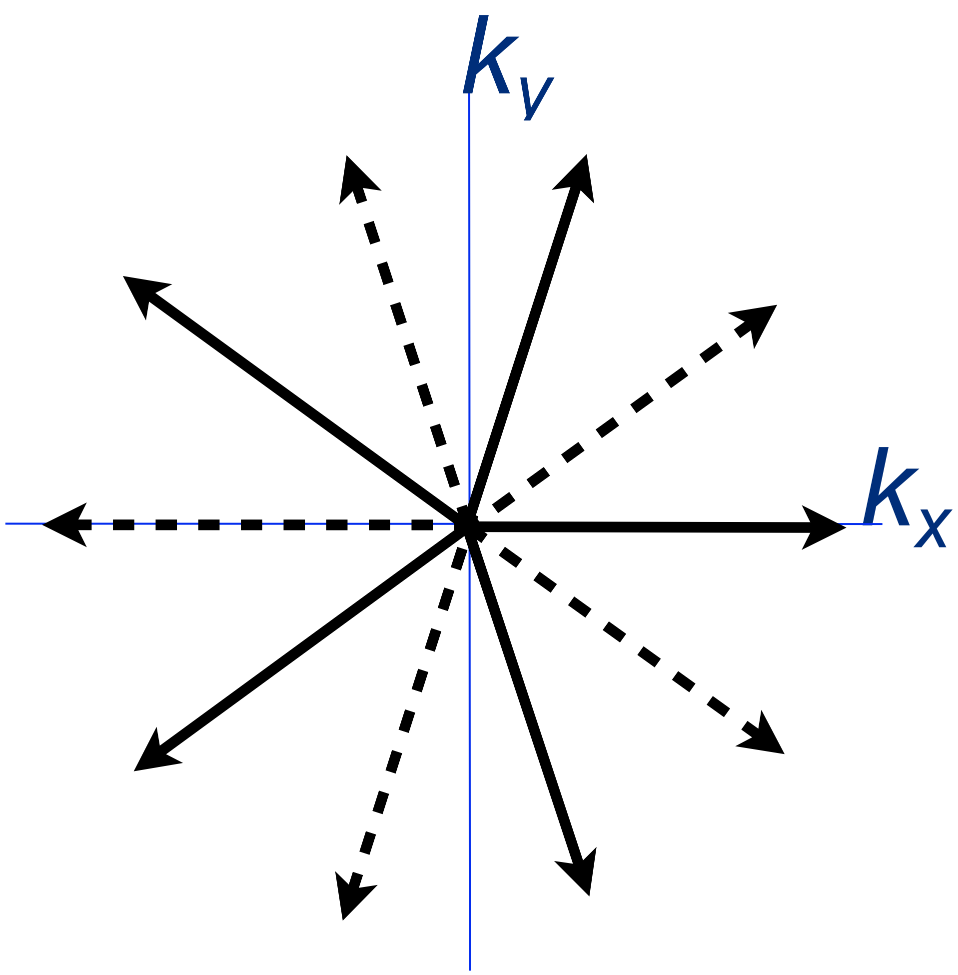
- Reduced sampling leads to streaks
- With reasonable undersampling the artifact is often benign

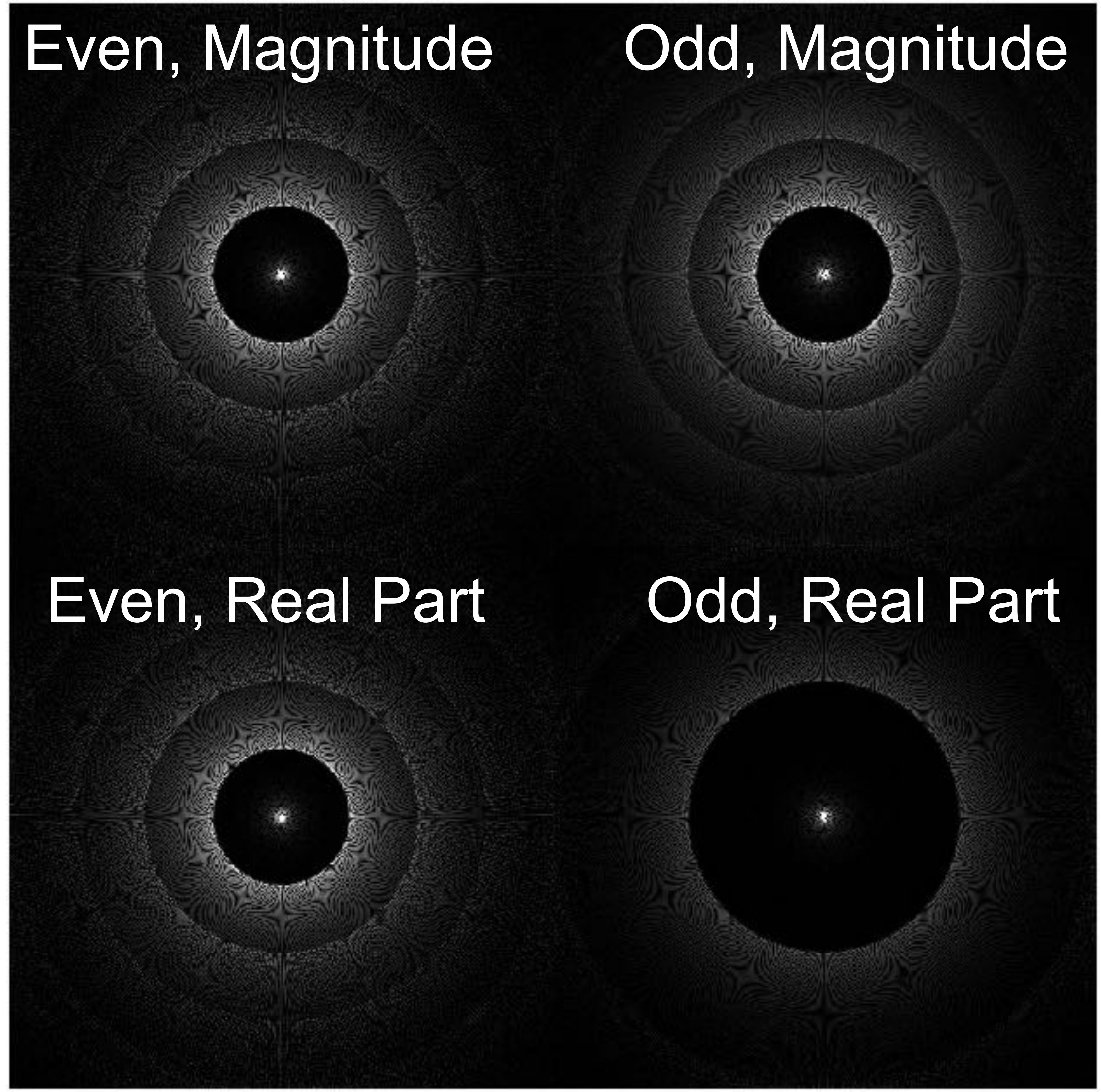


J.Liu, A.Lu, A.Alexander, J.Pipe, E.Brodsky, D.Seeber, T.Grist, W.Block, Univ Wisc.

Radial Out - Odd vs Even #Spokes

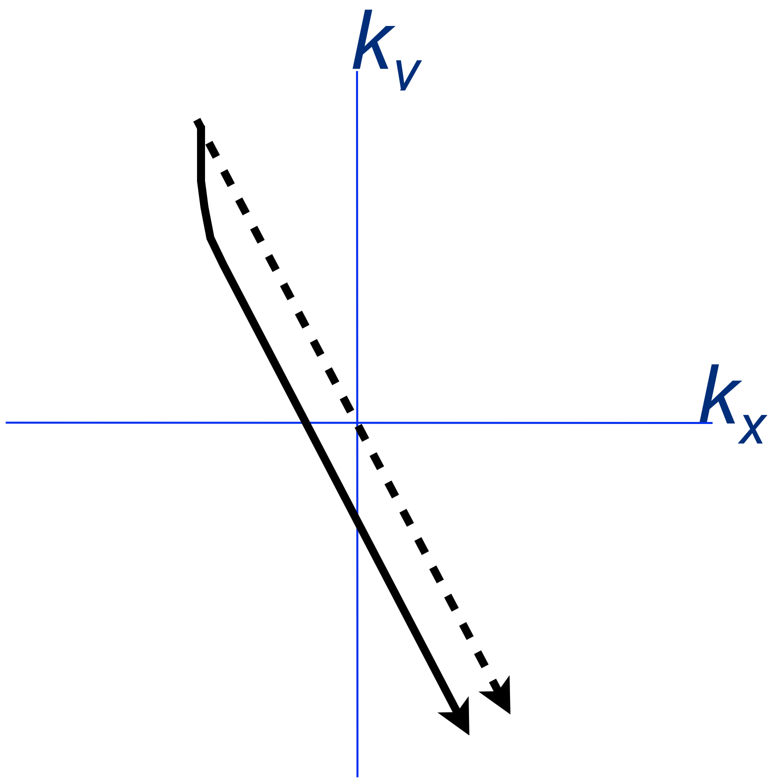
- Odd N is a half-Fourier trajectory
- Difficult to do homodyne, but quadrature aliasing

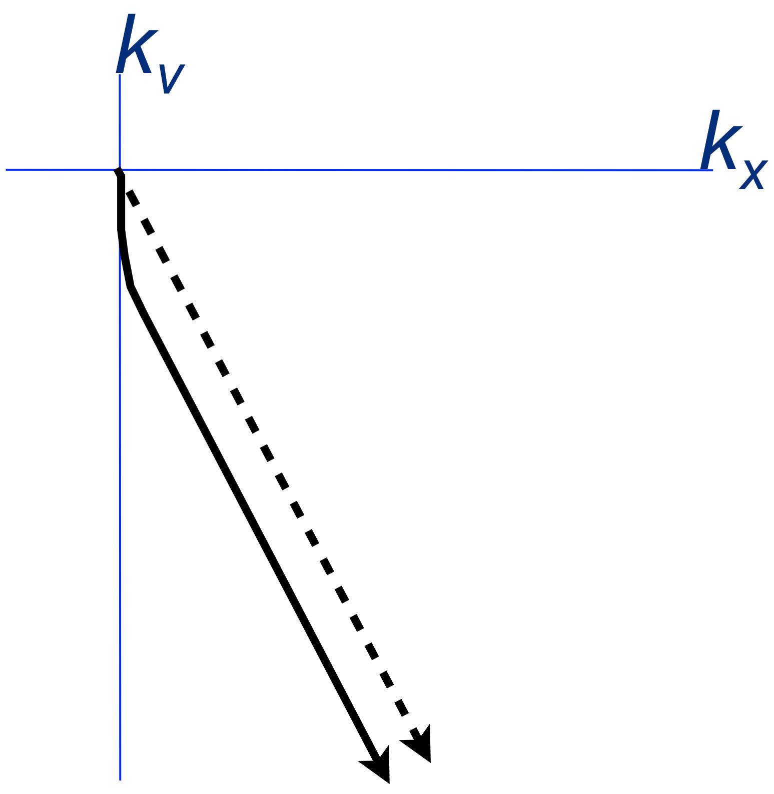


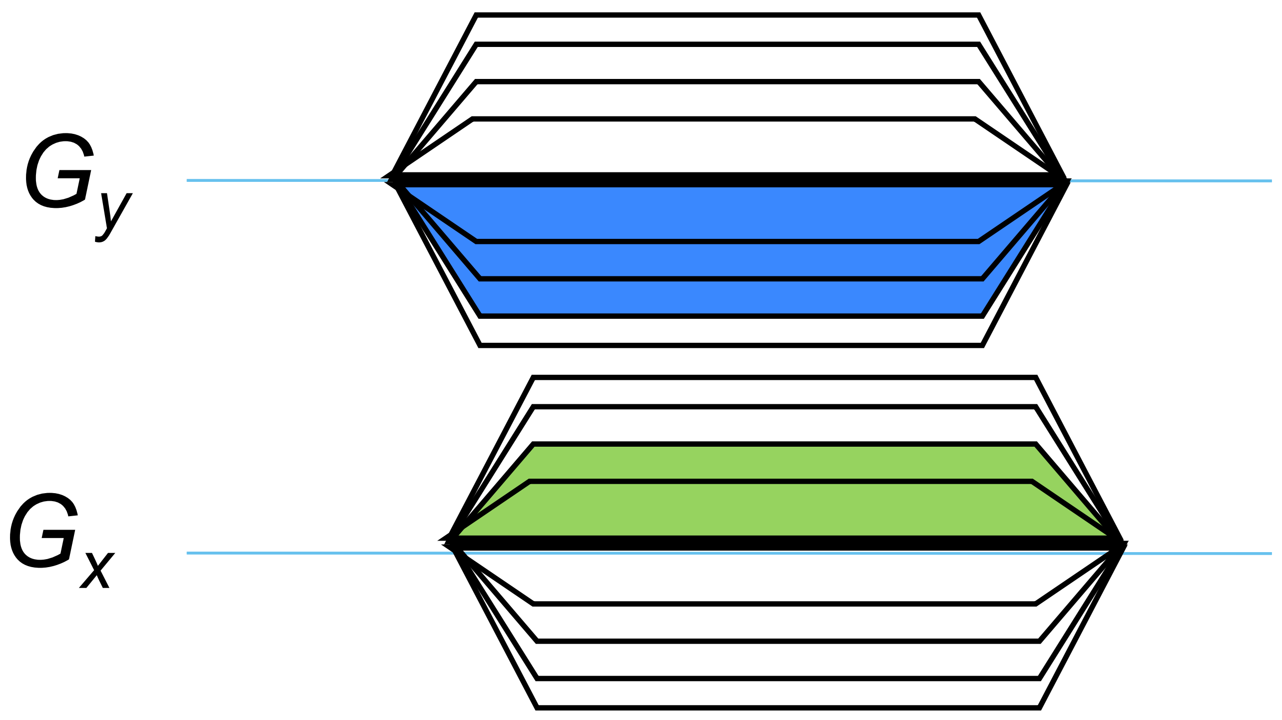


Gradient Delay Considerations

- Full projection
  - Global Delay: Shift center
  - Inter-axis: May miss center
- Radial out:
  - Global Delay: Shifts along traj.
  - Inter-axis: Warping of trajectory

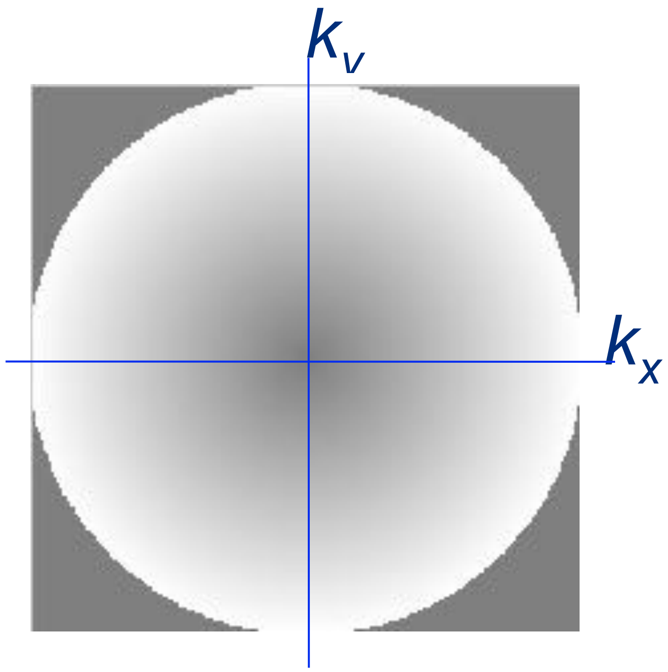




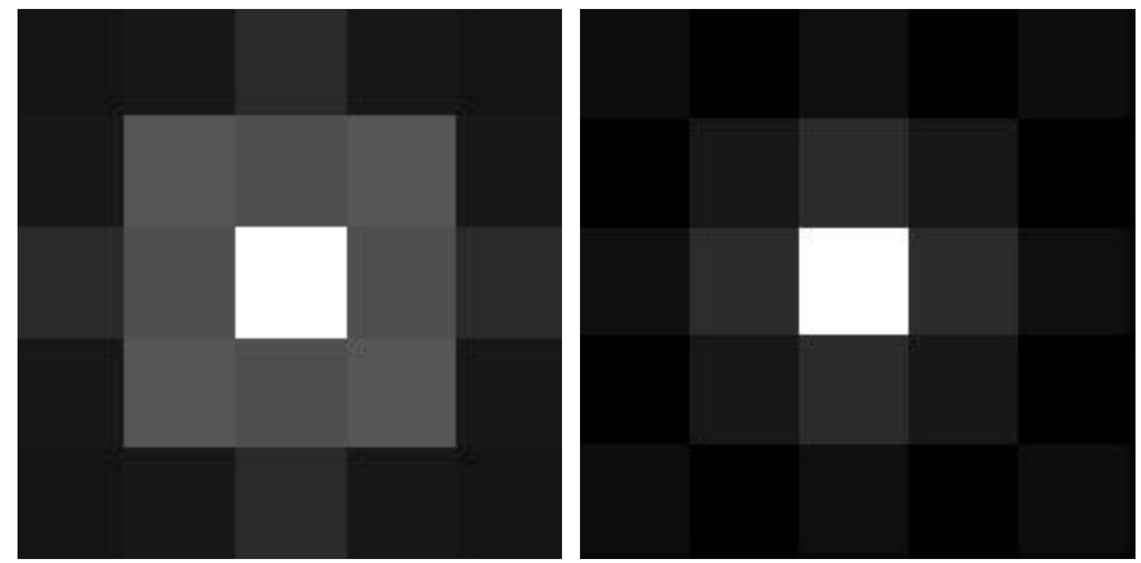


Off-Resonance Effects

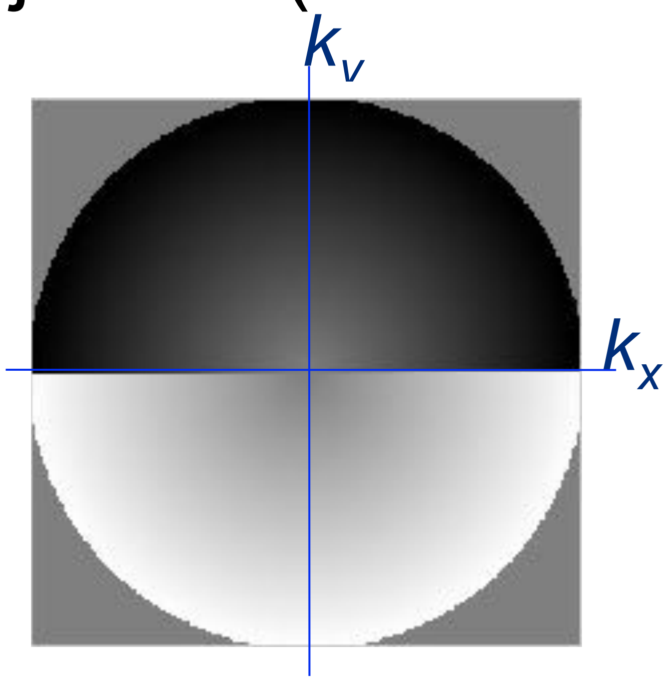
Radial-out (0 to π variation)



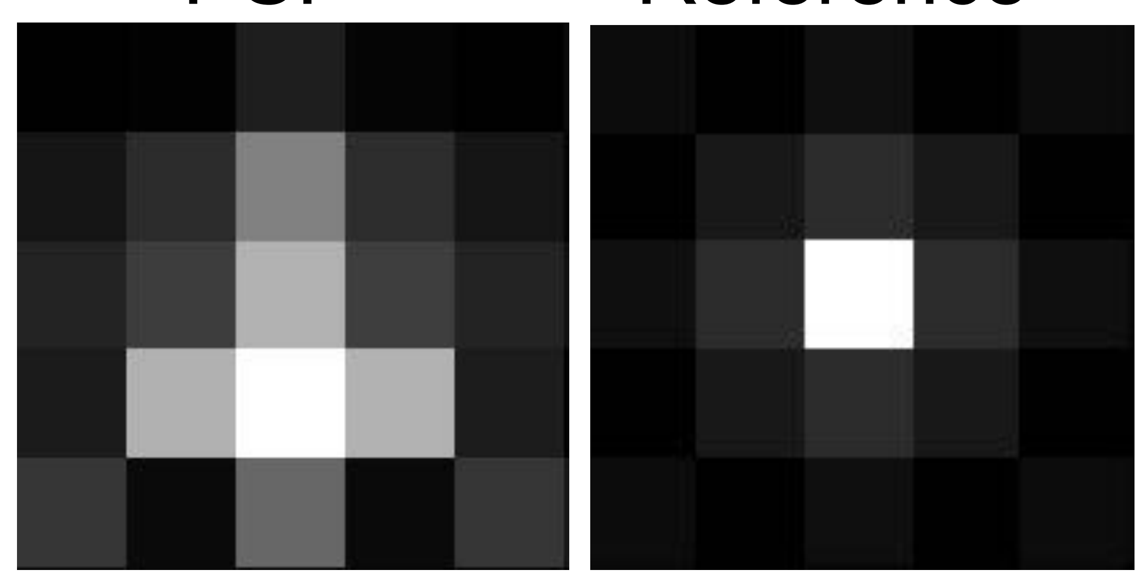
PSF Reference



Full Projection (-π to π variation)

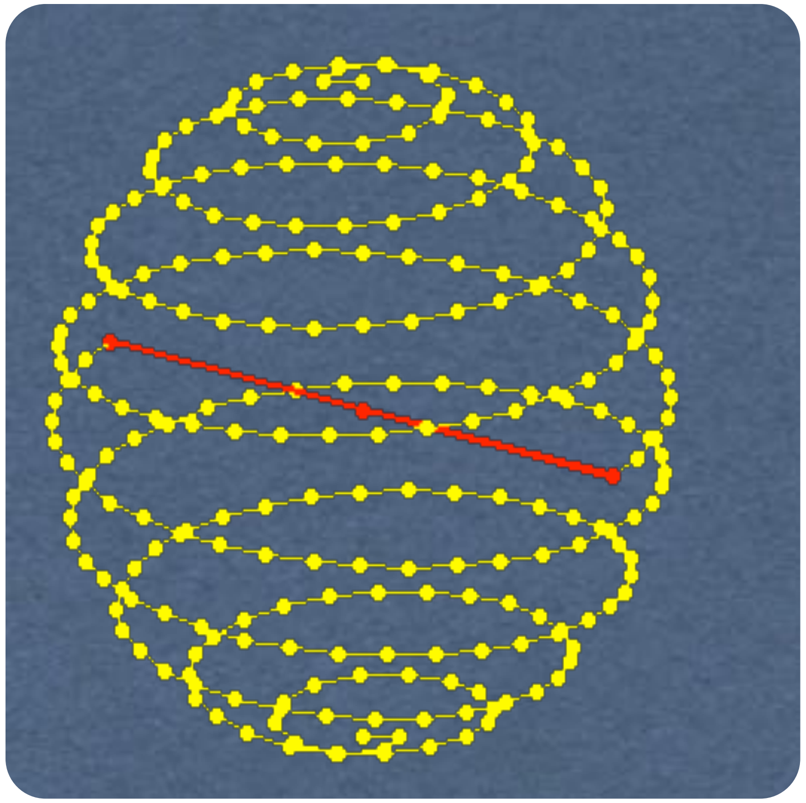


PSF Reference



3D Projection-Reconstruction

- Encode in k_x, k_y, k_z (θ,φ end-point parameterization)
- Density $(1/k_r)^2$, compensate by multiplying by k_r^2
- SNR efficiency now 0.75
- Can undersample more though!



J.Liu, A.Lu, A.Alexander, J.Pipe, E.Brodsky, D.Seeber, T.Grist, W.Block, Univ Wisc.

3D Radial/Projection: SNR Efficiency (η)

- Radial density $D = (k_{max}/k_r)^2$, extent [-1,1]

Quantity	Symbol	Cartesian	3D Radial
k-space volume	(V)	8	4/3π
#samples needed to cover volume	$\int_V D$	8	4π
Integrated density compensation	$\int_V 1/D$	8	4π/5
Efficiency	$\eta = \frac{V}{\sqrt{\int_V D \int_V 1/D}}$	1	$\sqrt{5}/3 = 0.75$

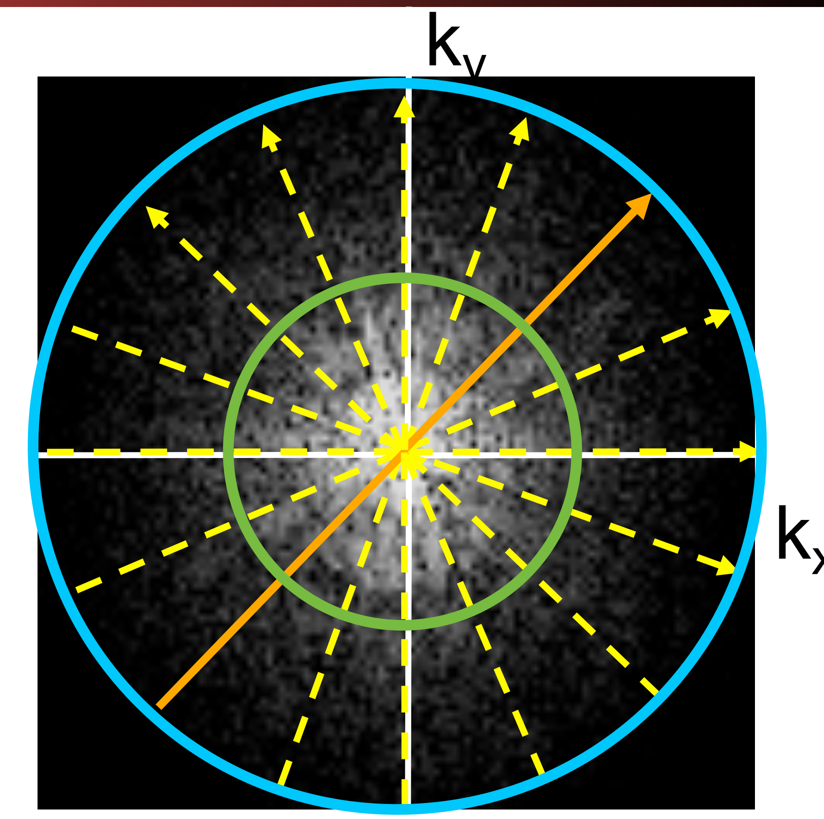
Temporal Radial Patterns

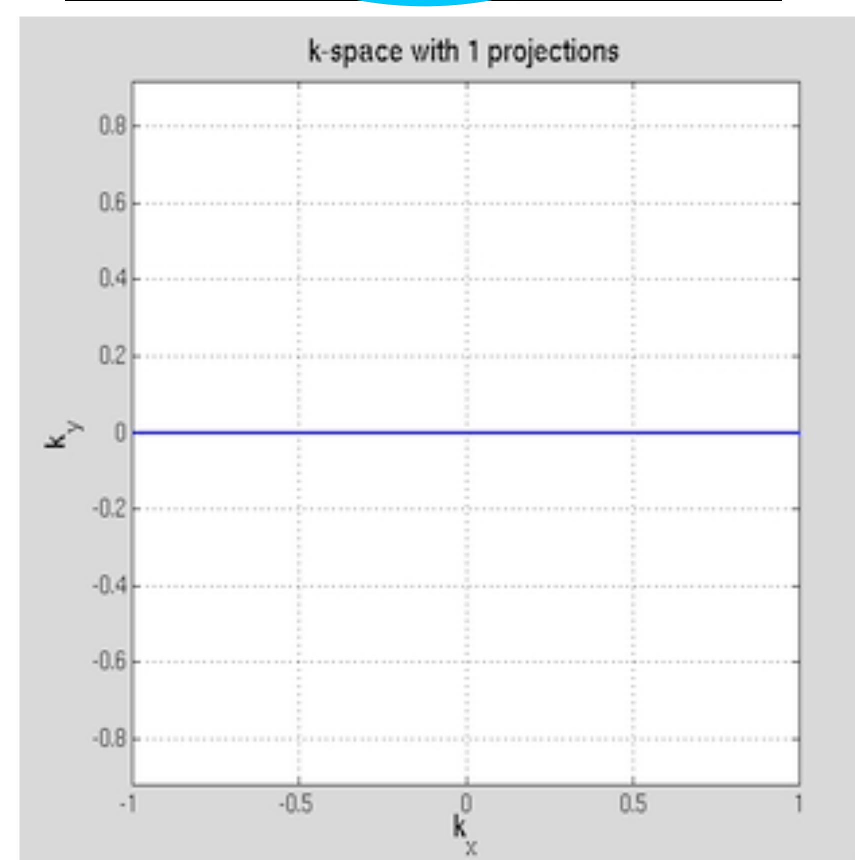
- Reduction R gives a fully-sampled image with R lower resolution
- Vary spatio-temporal resolution
  - "KWIC: k-space weighted image contrast"
  - Field map generation by delaying odd vs even lines
  - Golden-angle increment: 111.246°

    180° / 111.246° = (sqrt(5)+1)/2

    - Arbitrary N_s<N has "uniform" angular spacing

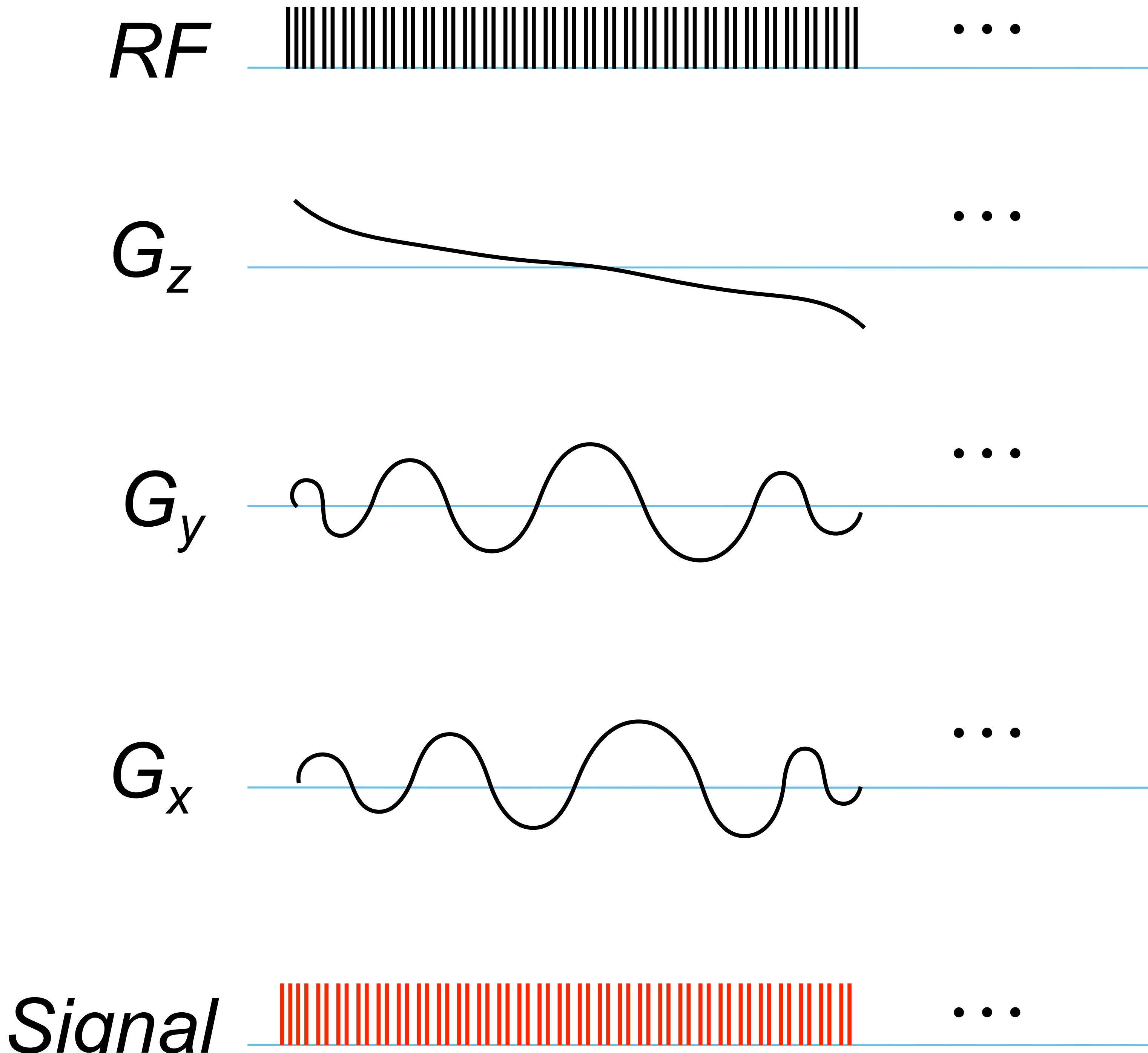
IEEE Trans Med Imaging. 2007 Jan;26(1):68-76.
An optimal radial profile order based on the Golden Ratio for time-resolved MRI.
Winkelmann S, Schaeffter T, Koehler T, Eggers H, Doessel O.





Zero Echo-Time (ZTE)

- Gradients always on!
- Radial sampling
- Very short RF
  - Low flip angle (SAR)
  - Actual RF excites slab with varying direction



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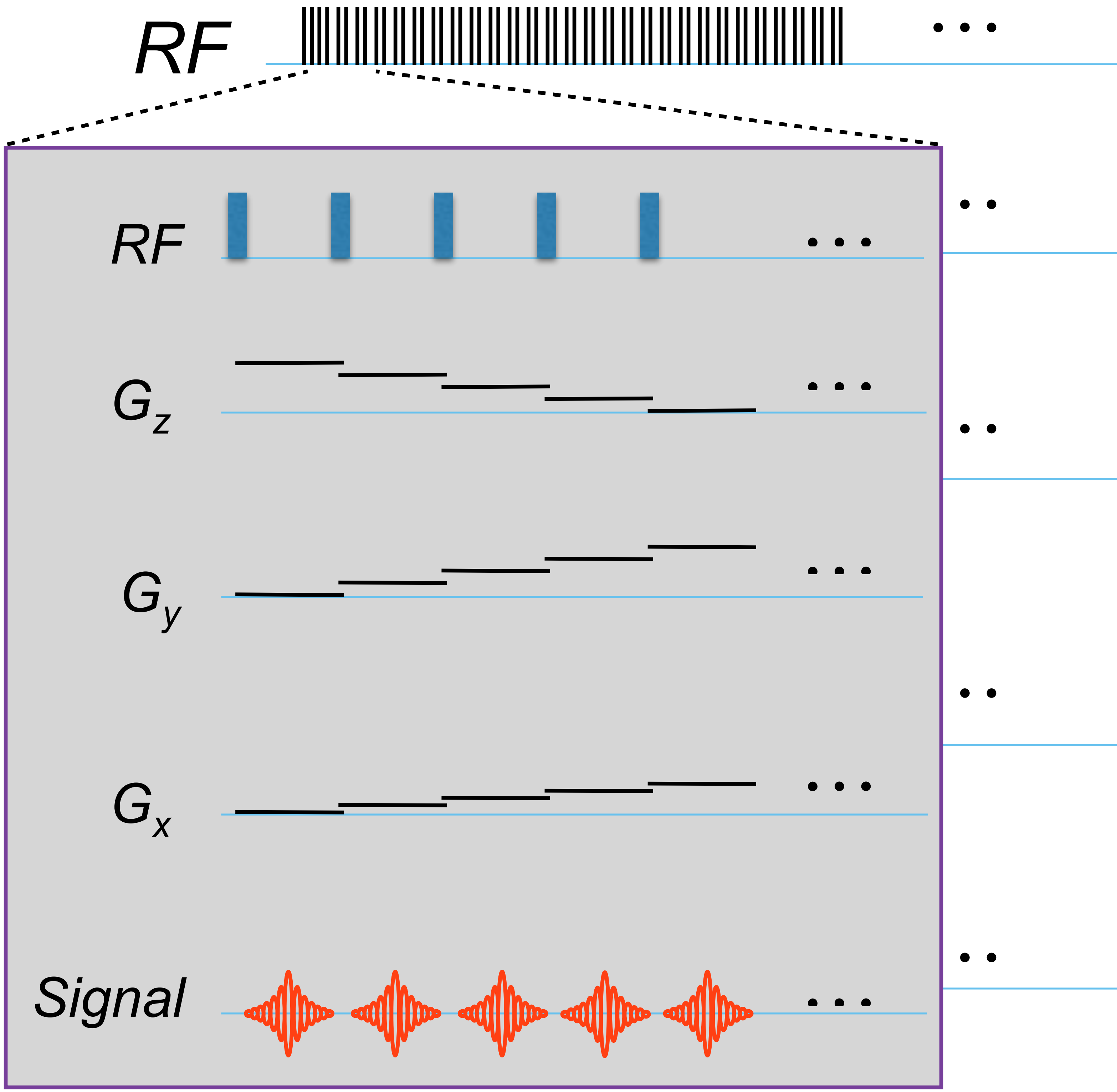
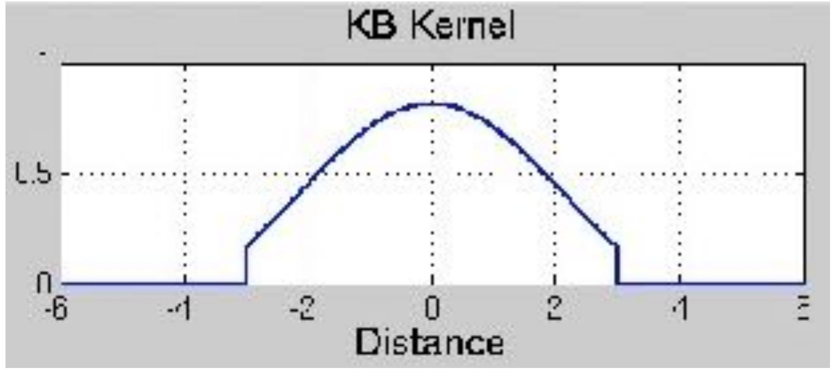


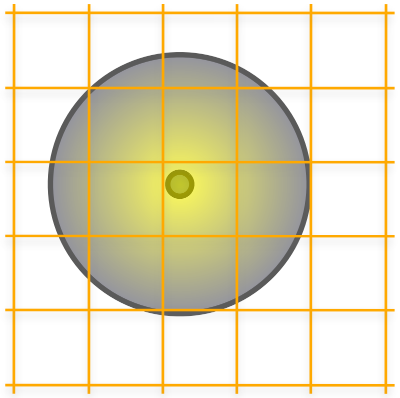
Radial and Projection: Summary

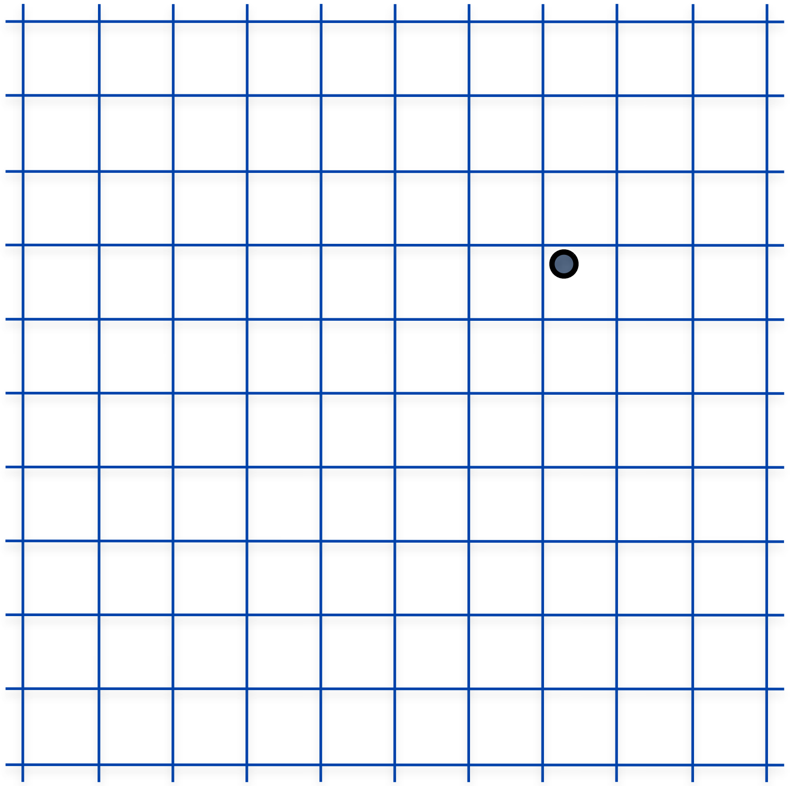
- Non-Cartesian, requires gridding reconstruction
- Incoherent undersampling artifact (similar to CS)
- Short TE (and UTE) imaging
- 2D and 3D options
- No phase-encoding ~ can be efficient
- Off-resonance causes blurring
- SNR efficiency loss due to high-density near center, but resampling the center can be advantageous

Gridding Code: gridmat.m

- Designed to be reasonably fast, but Matlab (readable)
- Uses Kaiser-Bessel interpolation kernel (precalculated)
- For each k-space sample M(k):
  - Build a “neighborhood” of affected grid points k_{grid}
  - Calculate contribution at each grid point:
    - M(k) x kernel($k-k_{grid}$)
  - Add the values to a full-size grid
  - No deapodization





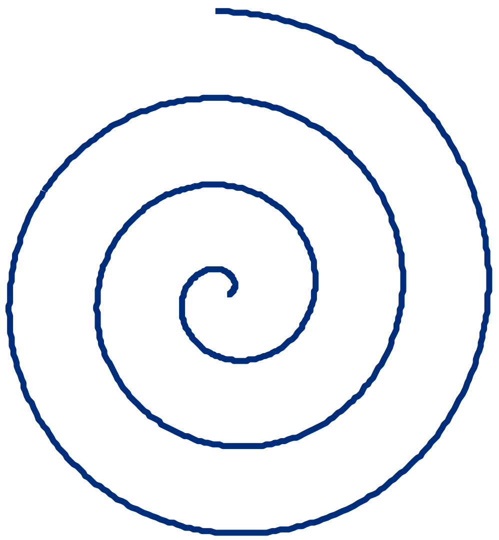


gridmat.m

- Inputs:
  - ksp = list of k-space locations, $k_x + ik_y$
  - kdat = data samples, ie $M(k_x,k_y)$
  - dcf = density compensation factors at each k-space location
  - gridsize = size of grid
- Convention:
  - k is in “inverse reconstructed pixels”
  - $|k| < 0.5$
- Larger gridsize zero-pads image (reduce apodization)
- Scale ksp smaller to “fill” FOV and interpolate pixels

Spiral

- Flexible duration/coverage trade-off
  - Like radial, center-out, TE~0
  - Low first-moments
- Longer readouts maximize acq window
  - Archimedean, TWIRL, WHIRL
  - Variable-density



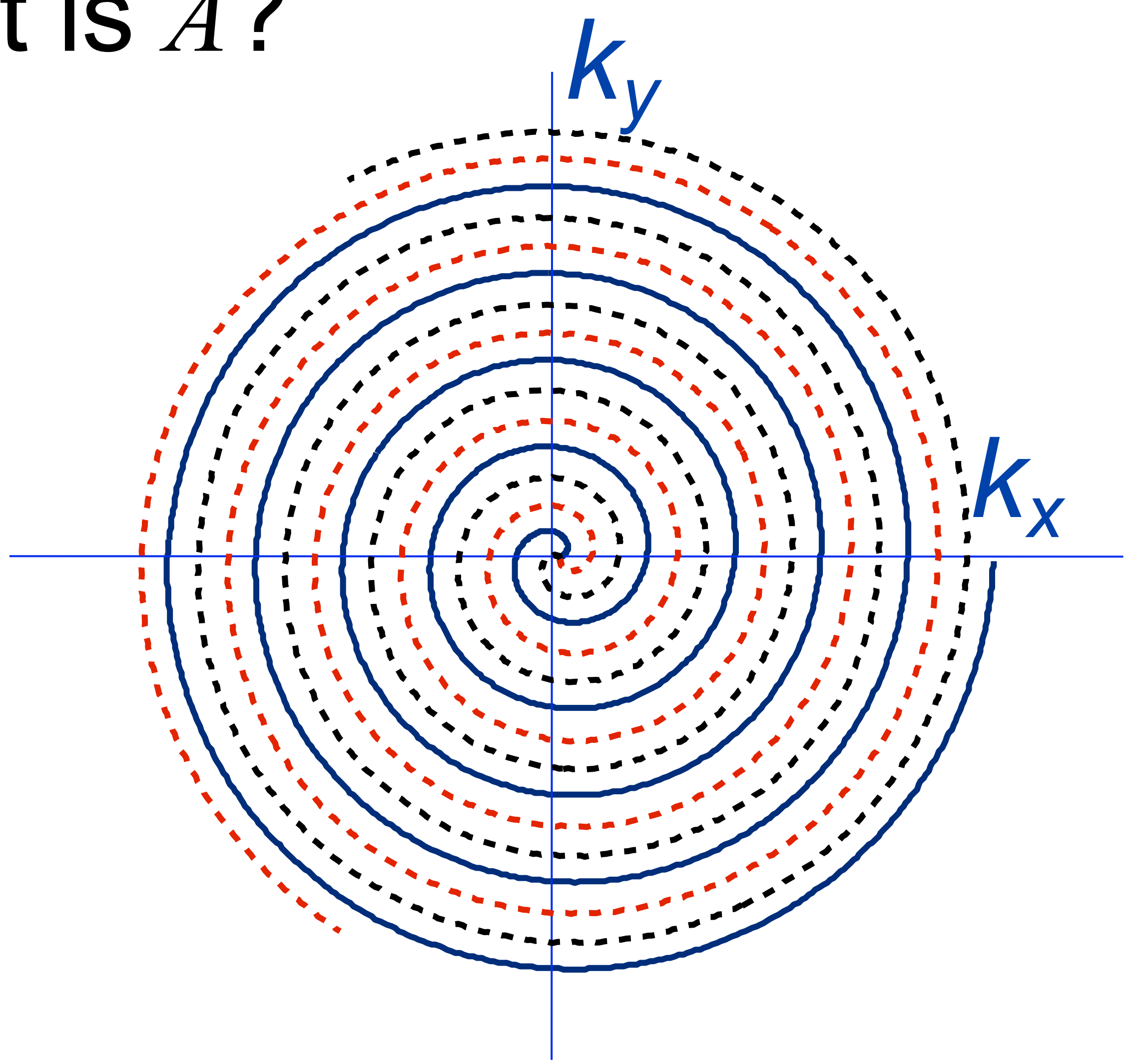
Archimedean Spiral

- Radius proportional to angle: $k(t) = A\,\theta(t)$
- Somewhat uniform density, with N interleaves
- Extreme case: single-shot with N=1
- θ increases 2π per turn… what is A?

$$k(t) = \frac{N\theta}{2\pi\ \mathrm{FOV}}\, e^{i\theta}$$

Stopping point:

$$\theta_{max} = \frac{2\pi}{N} k_{max} \mathrm{FOV}$$



Challenge is to design $\theta(t)$ to meet constraints

Archimeden Spiral Design

- Begin with spiral equation: $k(t) = \frac{N\theta}{2\pi \ \mathrm{FOV}} e^{i\theta}$
- Differentiate to obtain dk/dt and d^2k/dt^2
- Amplitude limit: $dk/dt < \gamma/2\pi \ G_{max}$
- Slew limit: $d^2k/dt^2 < \gamma/2\pi \ S_{max}$

Solution Options

Approximations for $\theta(t)$ *(Glover 1999)*

Consider slew-limited and amplitude-limited regions

Solve numerically at each point

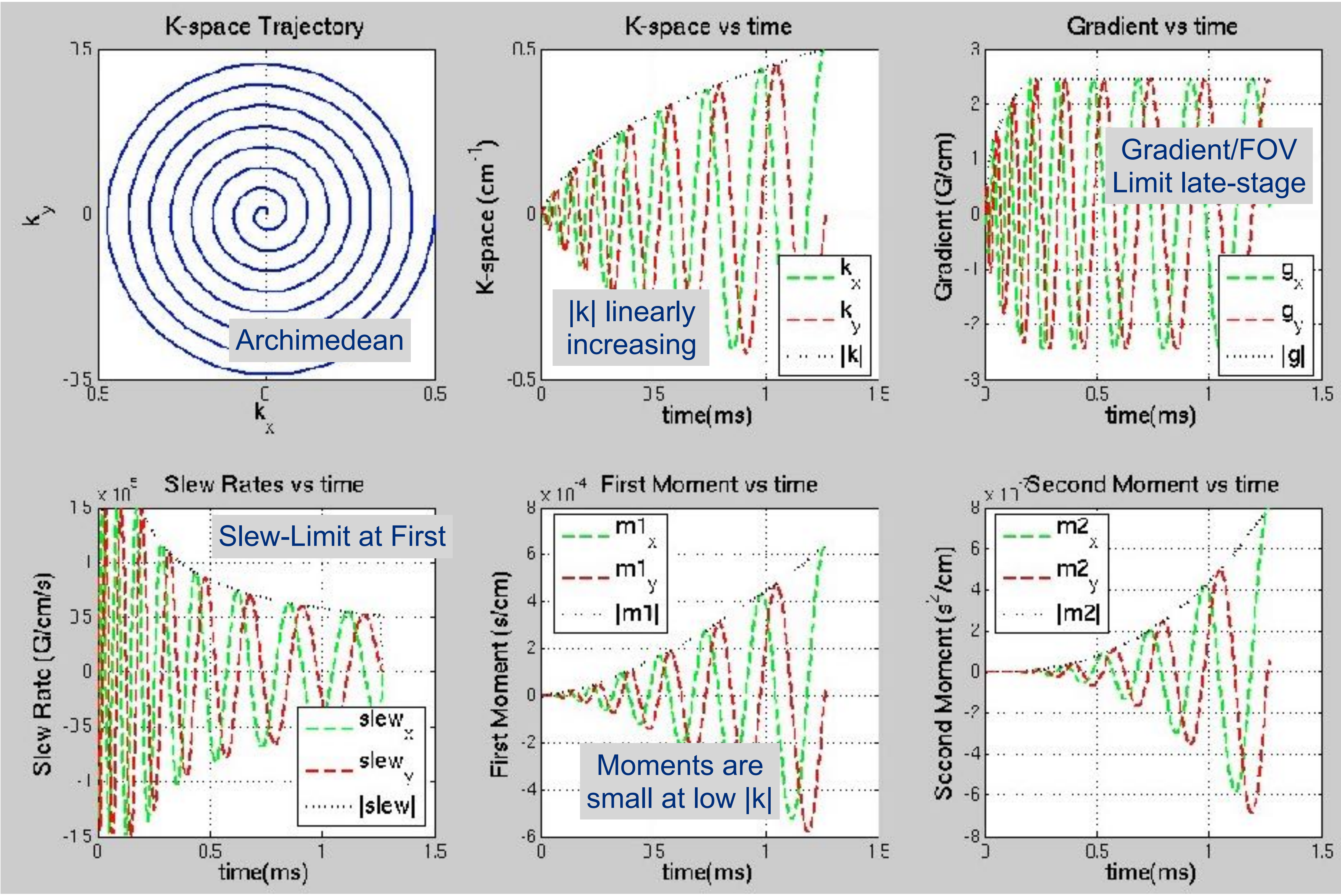
Find all limits, use active limit

Can include circuit model easily

(Meyer, King methods ~ 1995)

Both methods allow variable density design *(Kim 2003)*

Example Spiral Waveforms



Spiral Characteristics

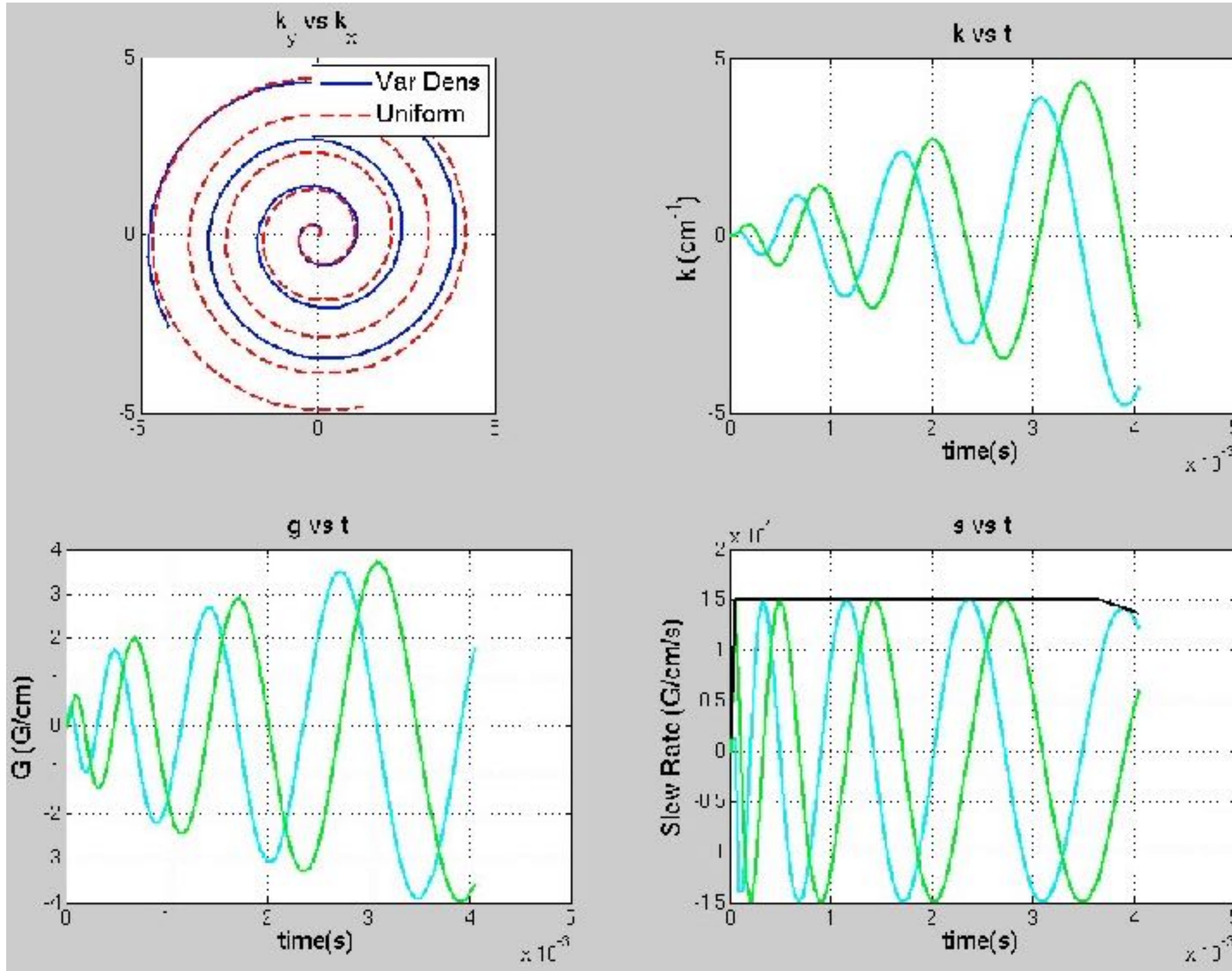
vds.m

Variable-Density Spiral

vds.m

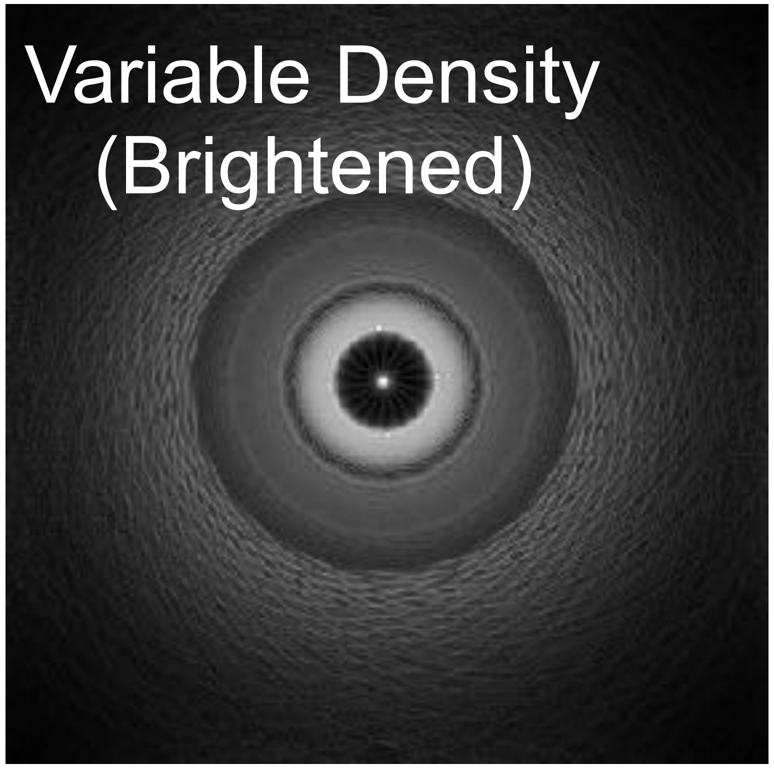
- Undersample outer k-space
- Vary spacing (1/FOV) with $k/k_{max}|$ or θ
- Increase spacing along trajectory

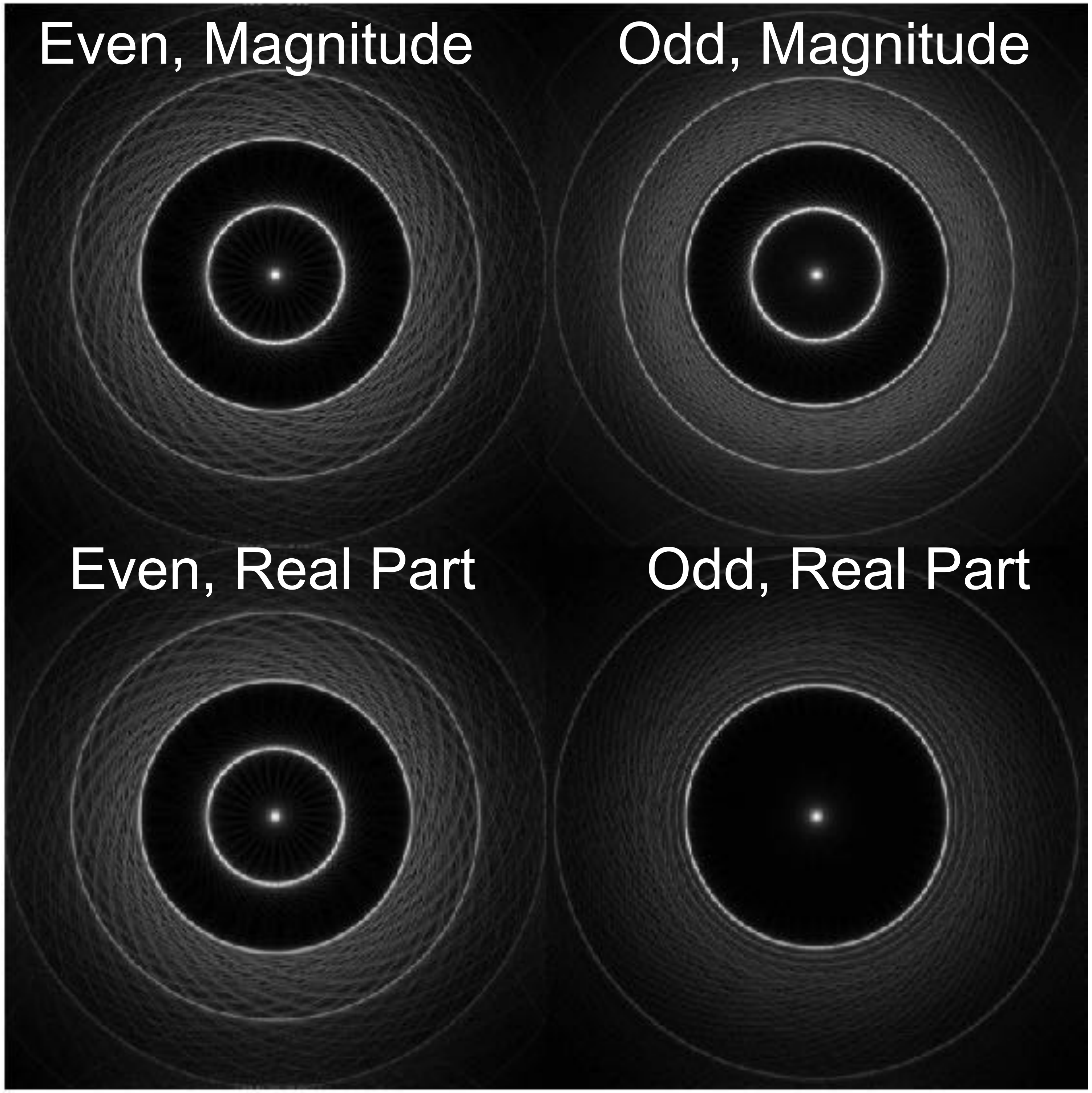
$$k(t) = \frac{N\theta}{2\pi\,\mathrm{FOV}(\theta)}\,e^{i\theta}$$



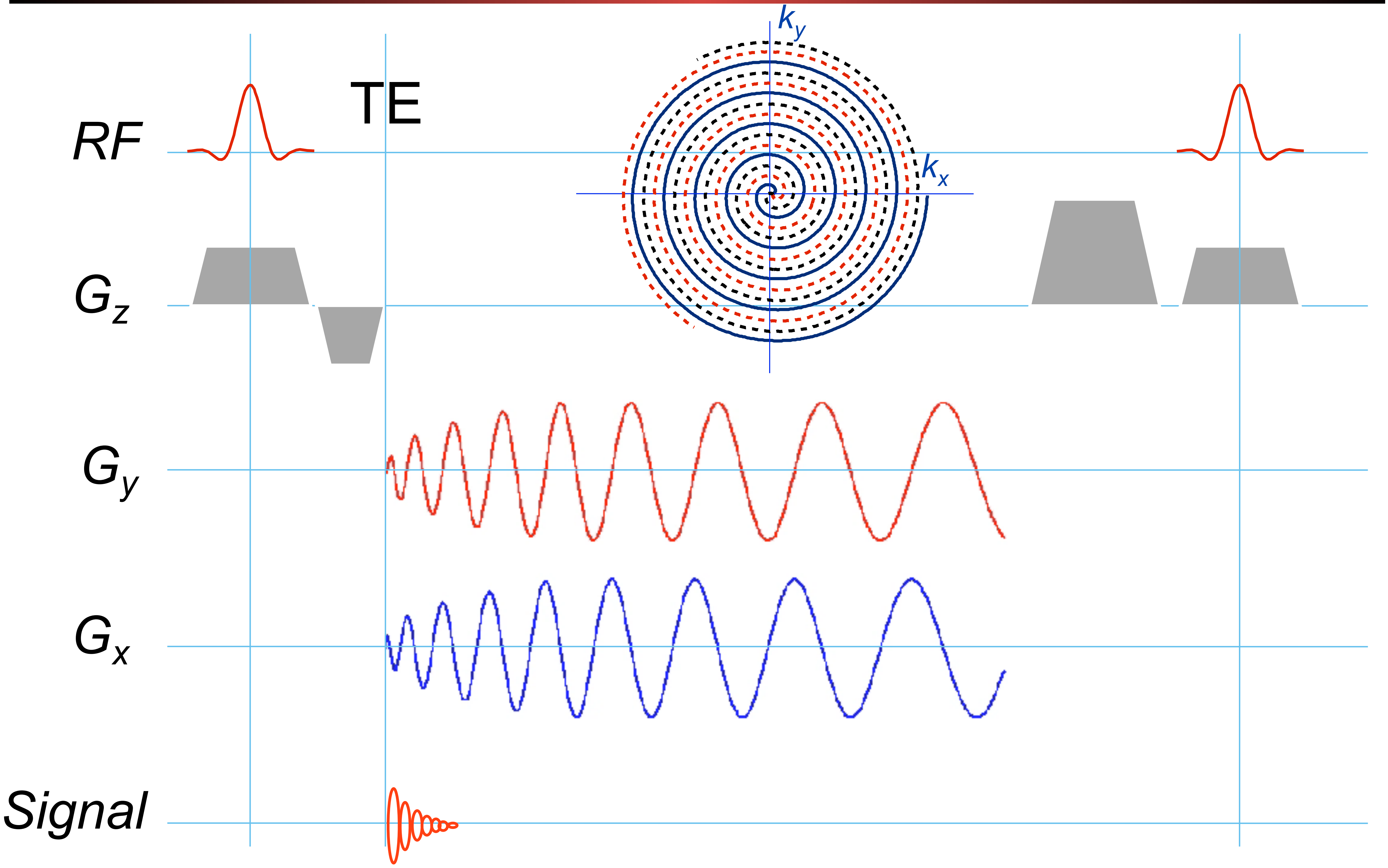
Spiral Point-Spread Functions

- "Swirl" artifacts from undersampling
- Again, odd/even selection applies
- Variable Density: Less coherent



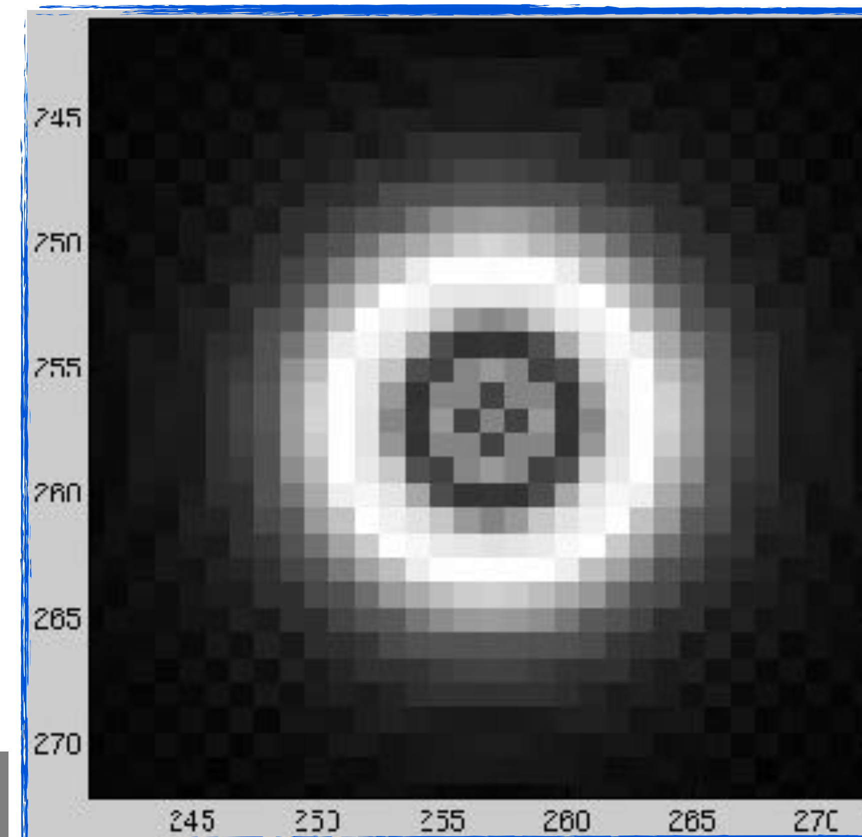


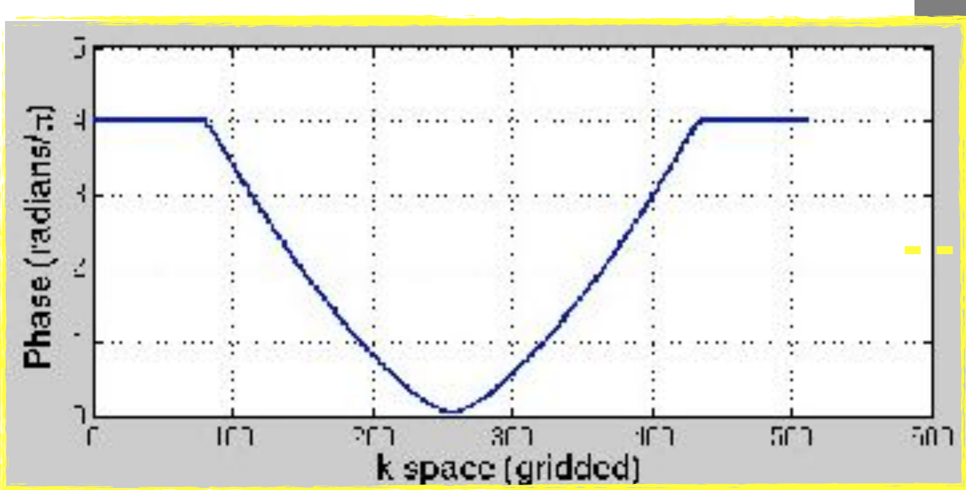
Spiral Imaging Sequence

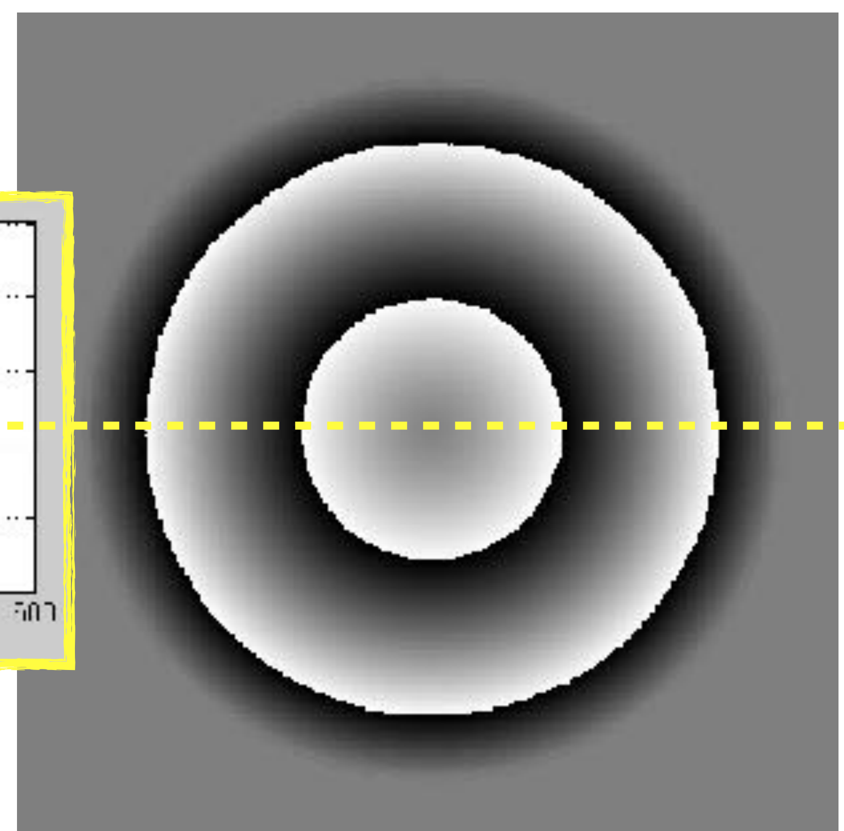


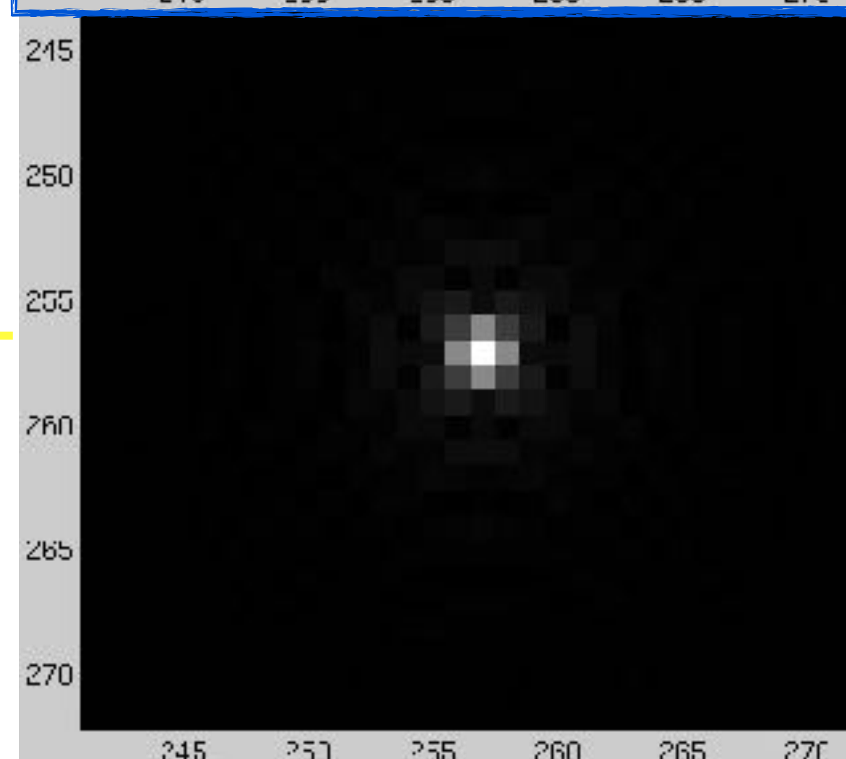
Off-Resonance Sensitivity

- Uniform-density - $\varphi \sim A|k|^2$
- Spiral usually longer than radial
- PSF broadening
- Off-resonance correction in recon

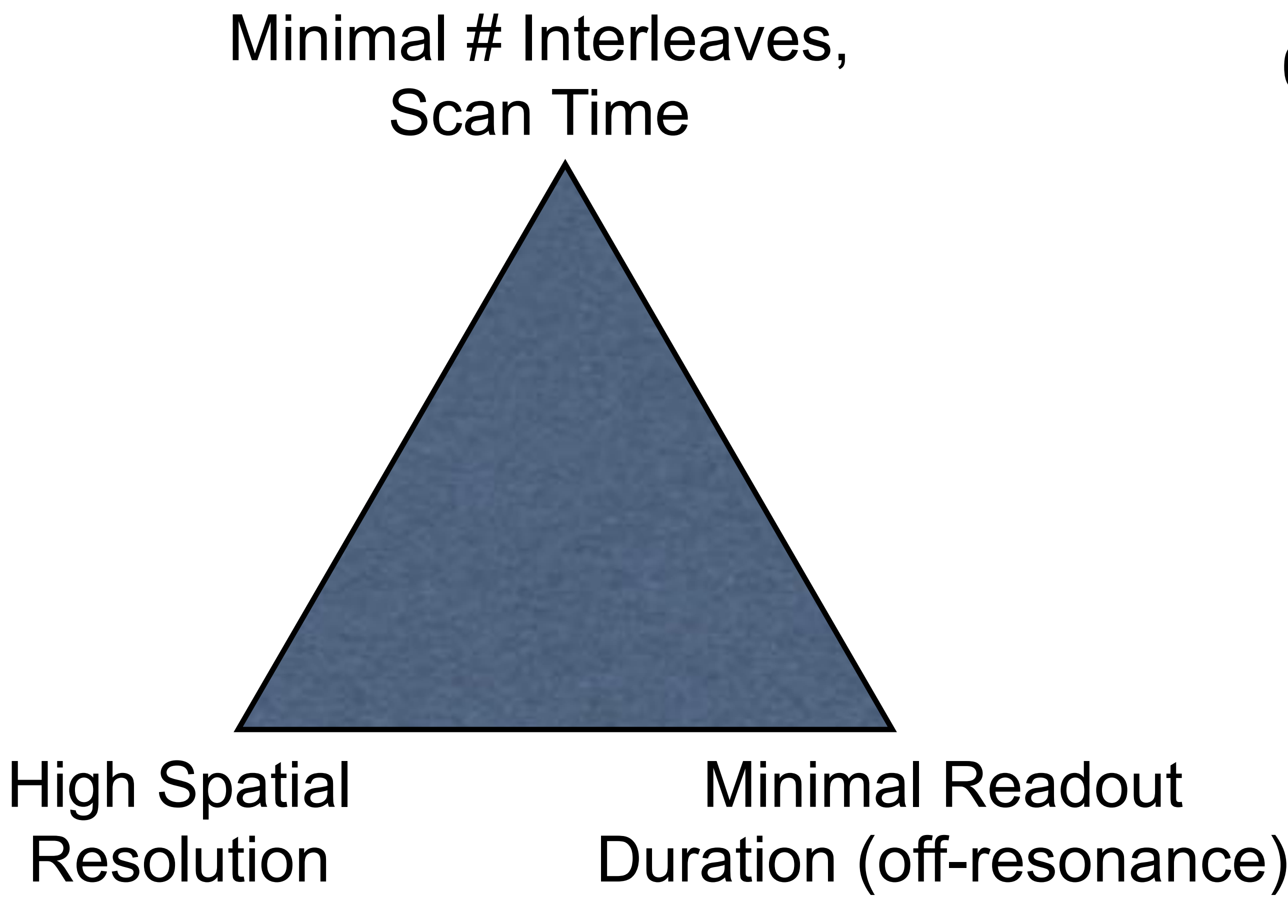


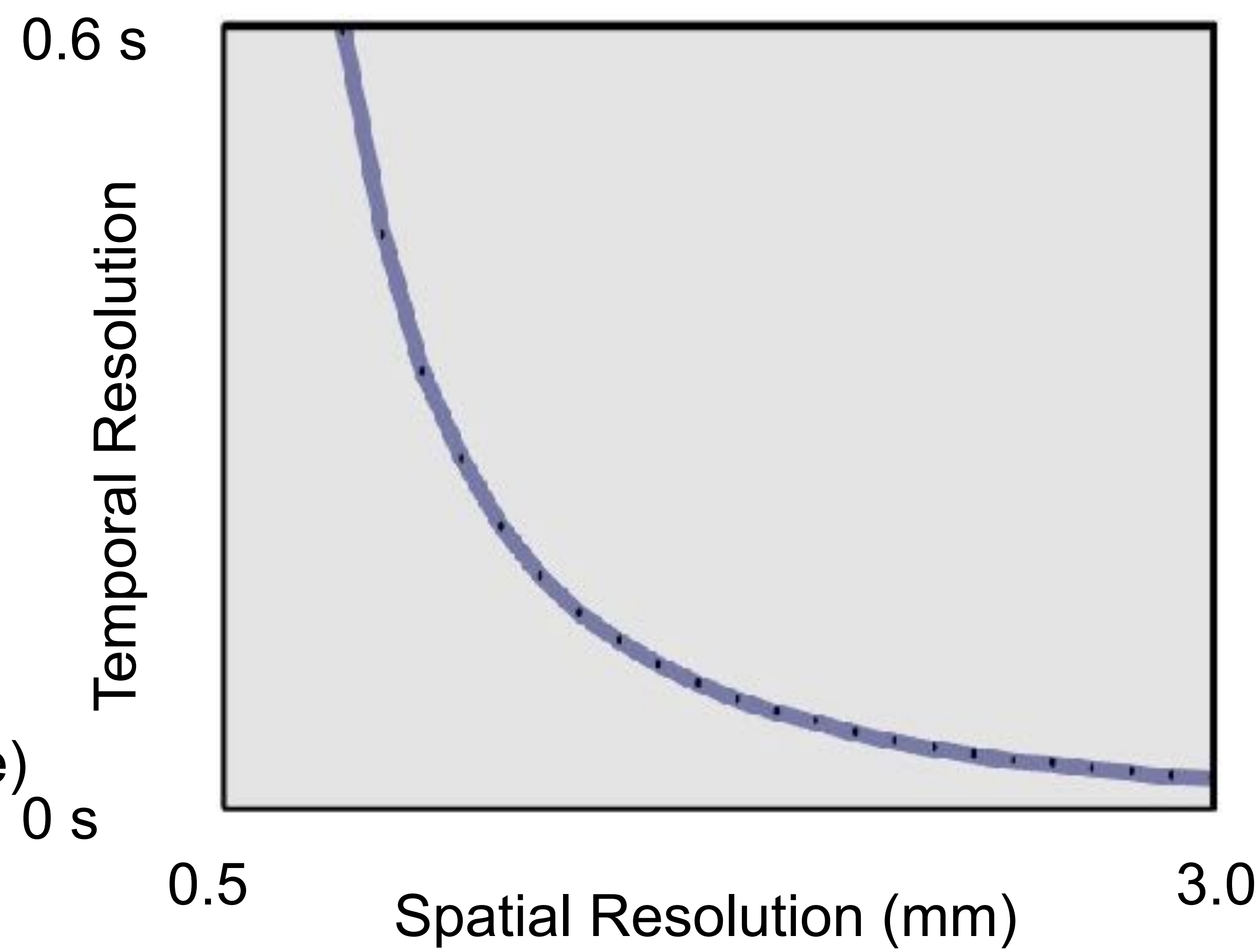






Spiral Design Trade-offs





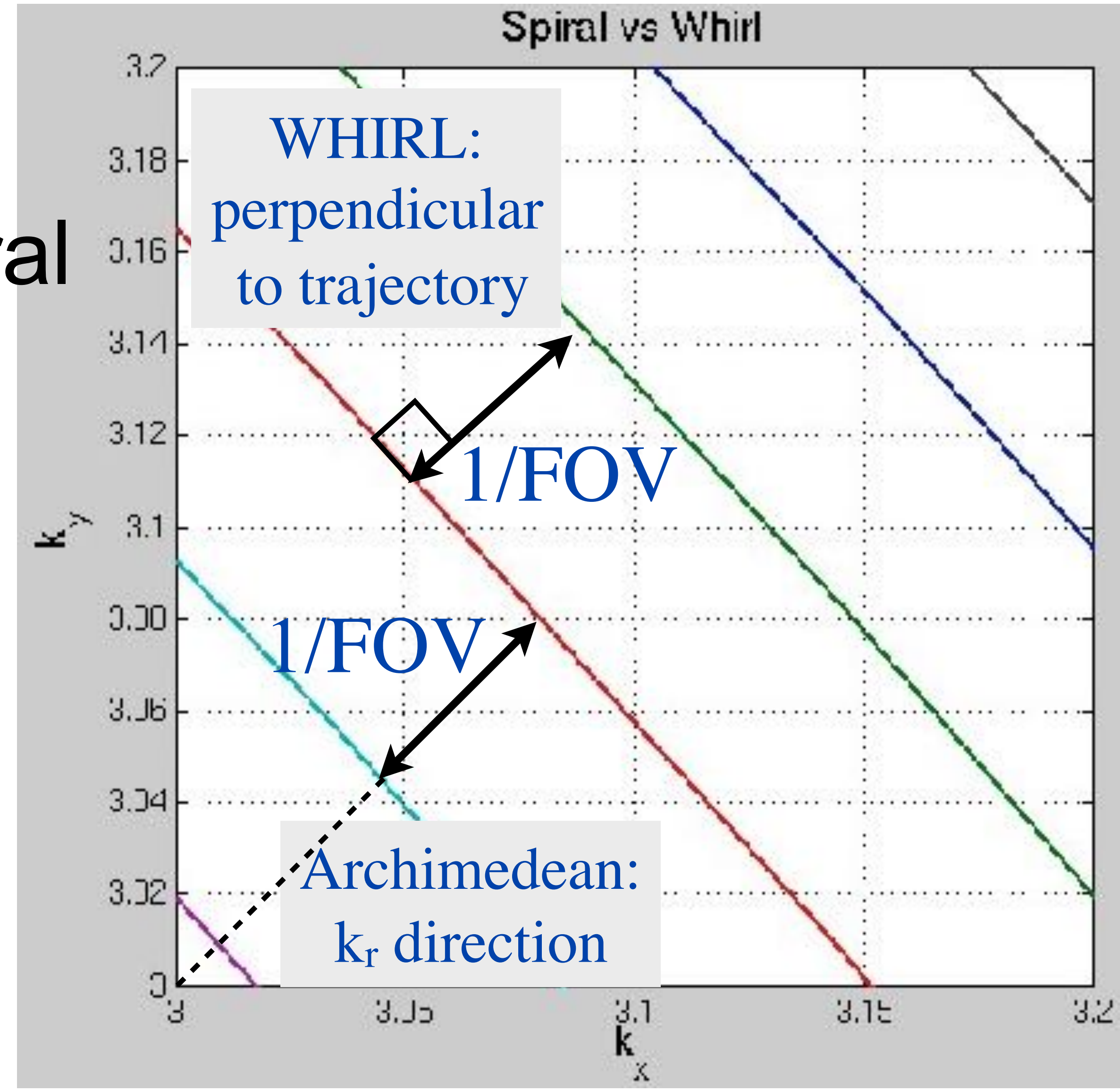
1. Choose tolerable readout duration

2. Trade scan time (#interleaves) for spatial resolution

TWIRL / WHIRL

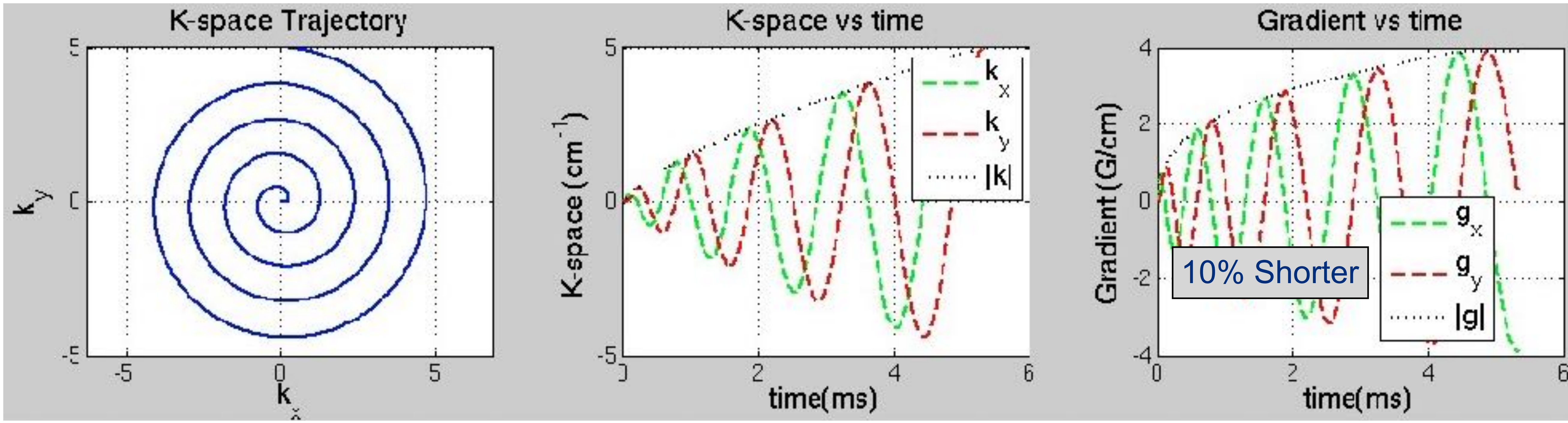
whirl.m

- Faster start, with radial segment
- TWIRL: *Jackson 1990*
  - Radial, then Archimedean spiral
- WHIRL: *Pipe 1999*
  - Non-archimedean spiral
  - constrained by trajectory spacing
  - *Faster spiral,* particularly for many interleaves
  - whirl.m on website

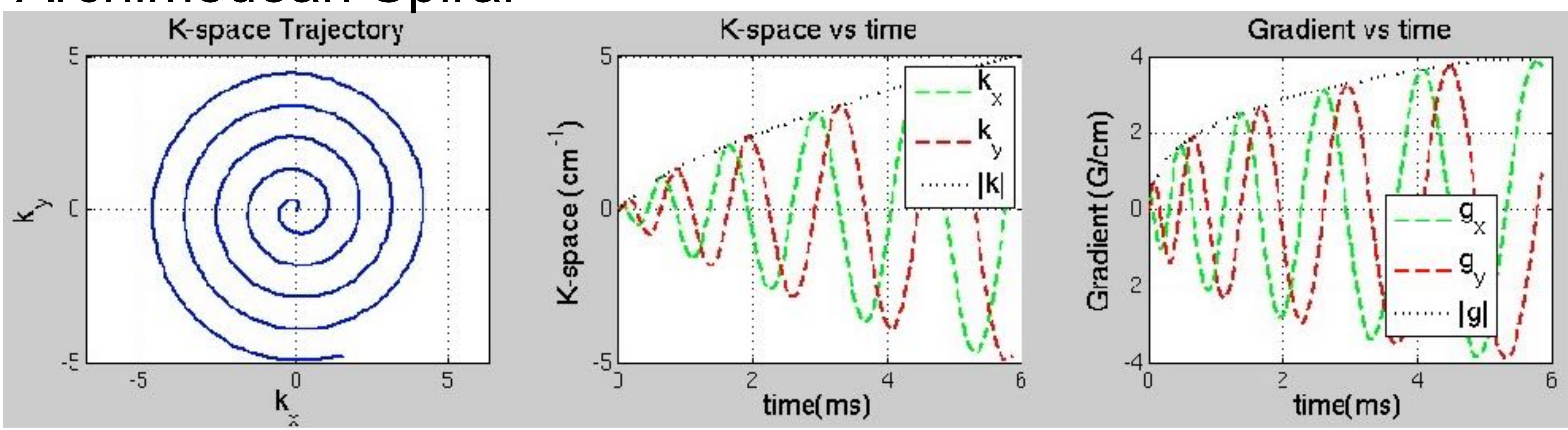


WHIRL vs Archimedean Spiral

WHIRL



Archimedean Spiral

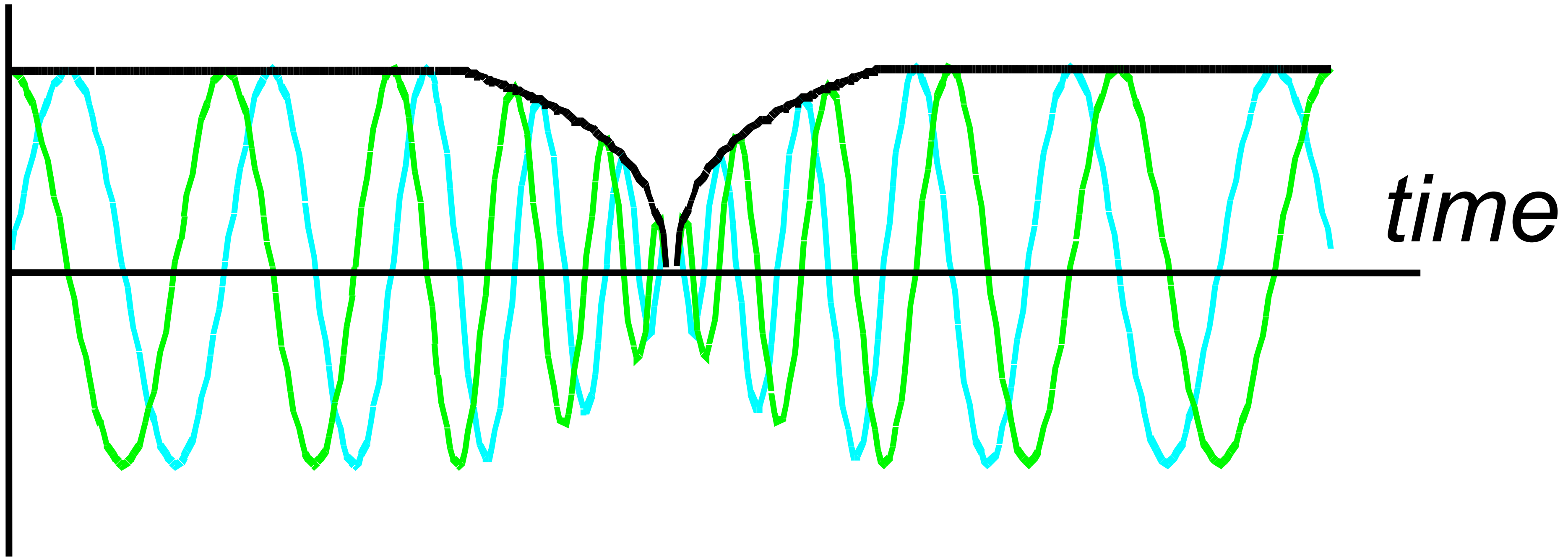


25 interleaves, 24cm FOV, 1mm resolution

Spiral in - Spiral out

Glover 2000

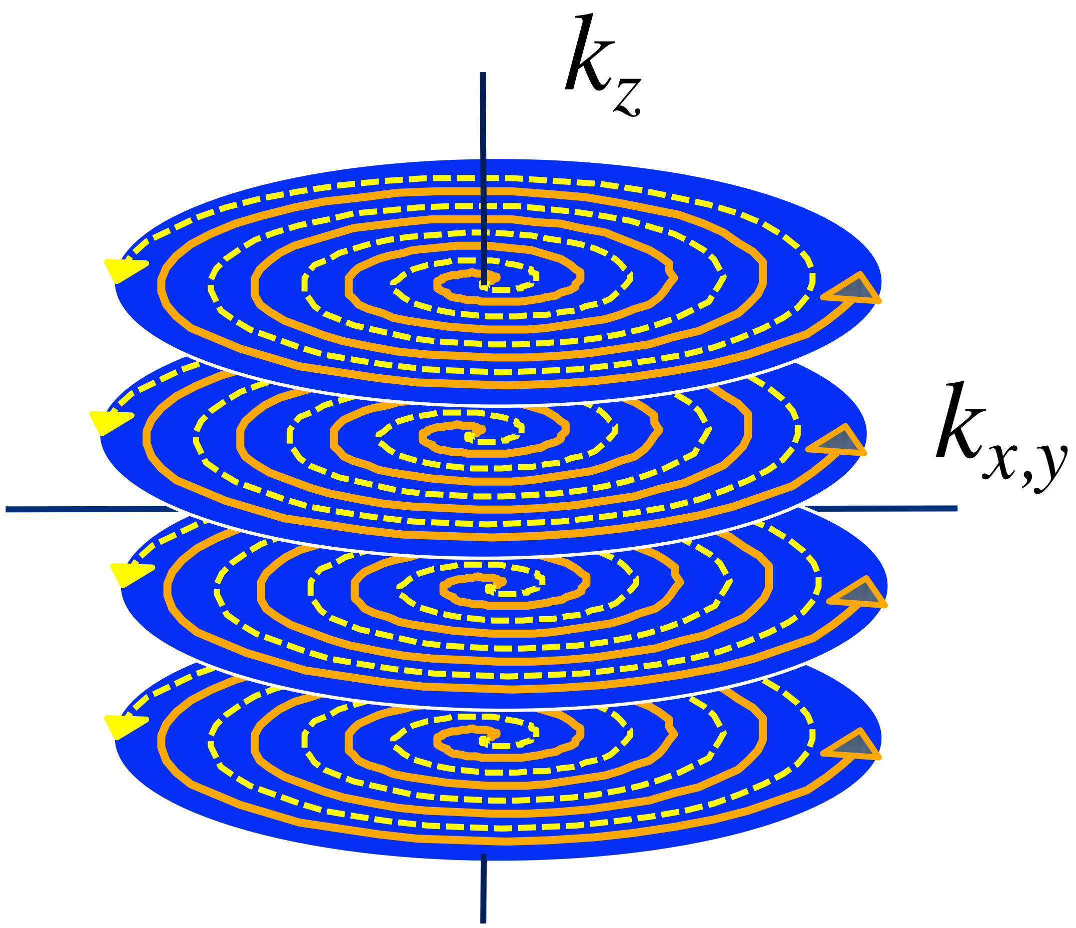
- Useful for delayed TE
- Perhaps also for spin echo
- Simply add time-reversed gradient



3D Methods: Spiral Stack, TPI, Cones

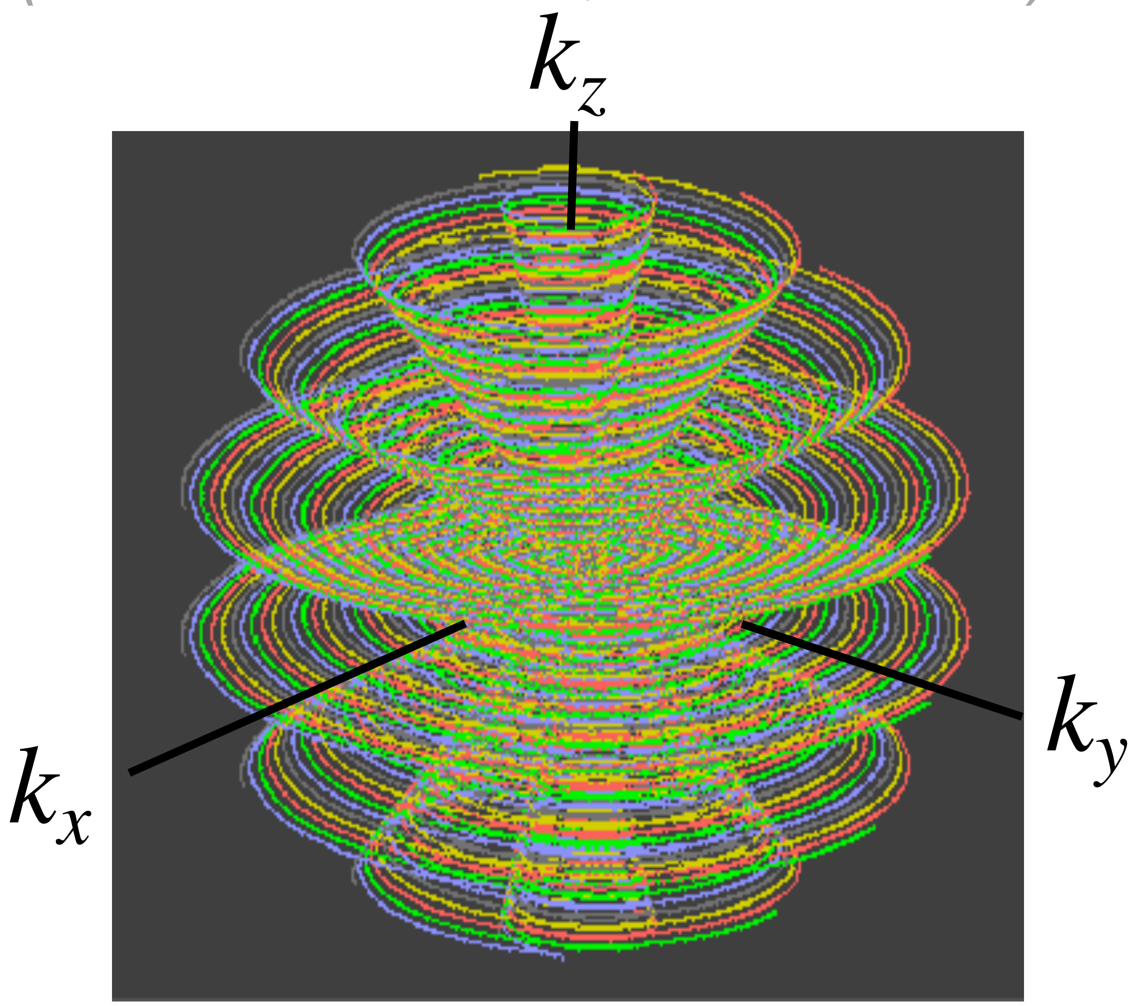
Stack of Spirals

(Irarrazabal, 1995)



Cones, Twisted-Projections

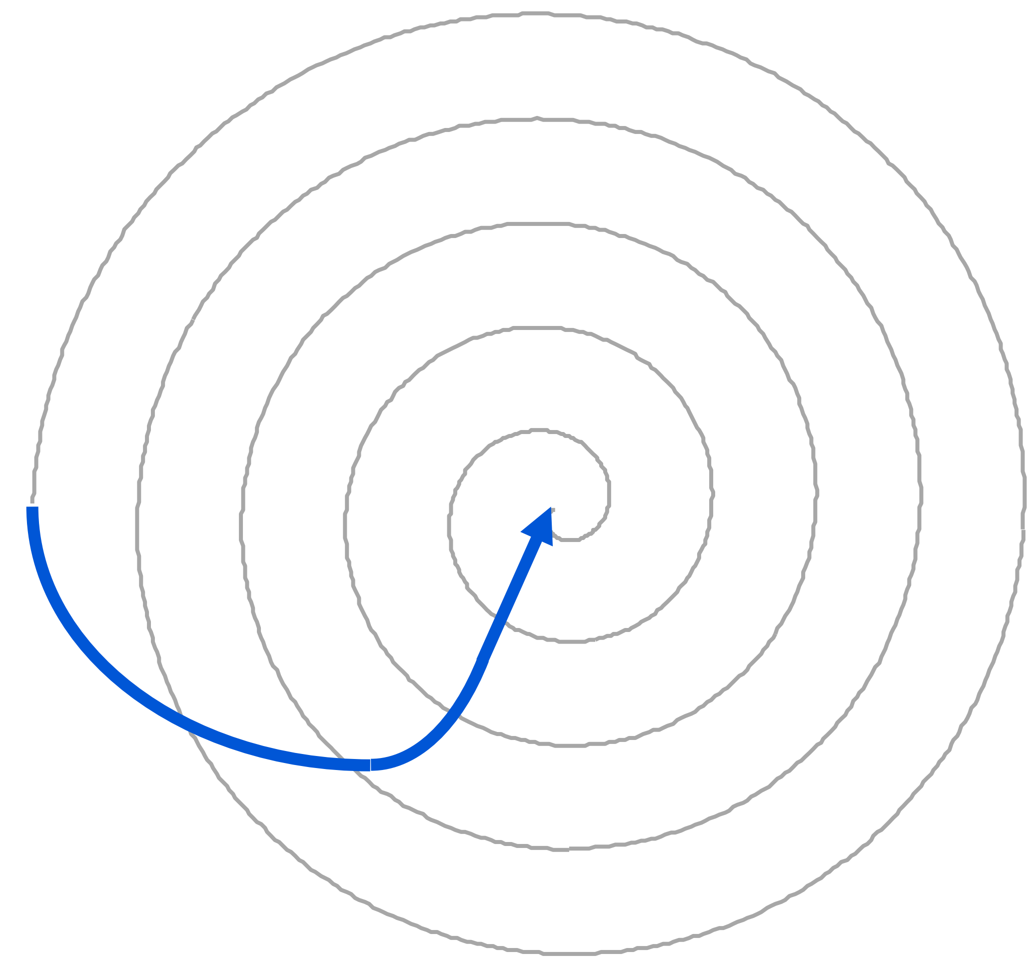
(Irarrazabal 1995, Boada 1997)



- *Many variations (spherical stack of spirals)*
- *Density-compensated cones, TPI*
- *3D design algorithms get very complicated*

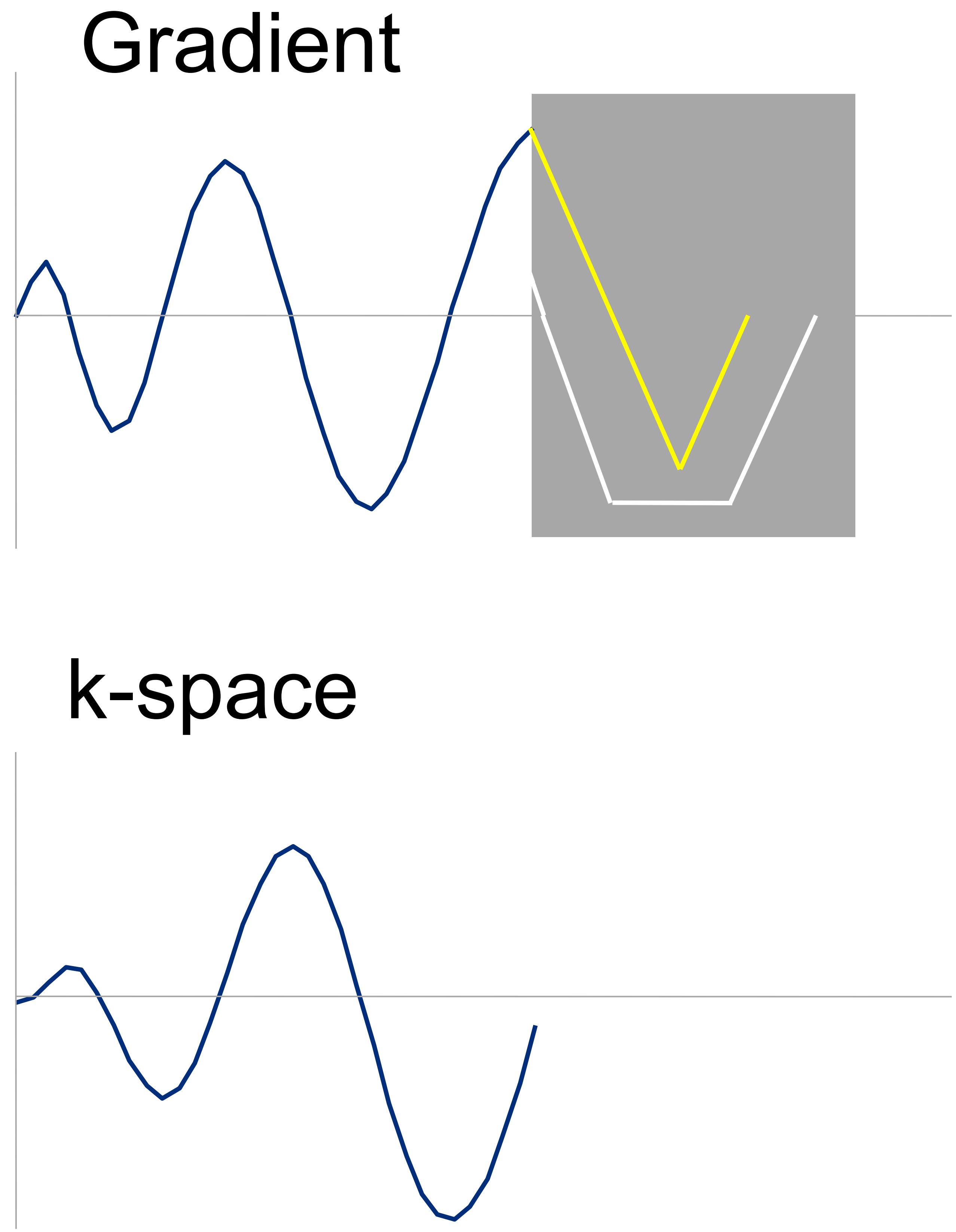
Rewinders and Prewinders

- "Preparatory gradients"
  - Spiral rewinders
  - Phase-encode/rewind
  - spoilers
- Consider 3D rotation again
- Arbitrary path
- Speed always helps efficiency



Approaches to “Rewinder” Design

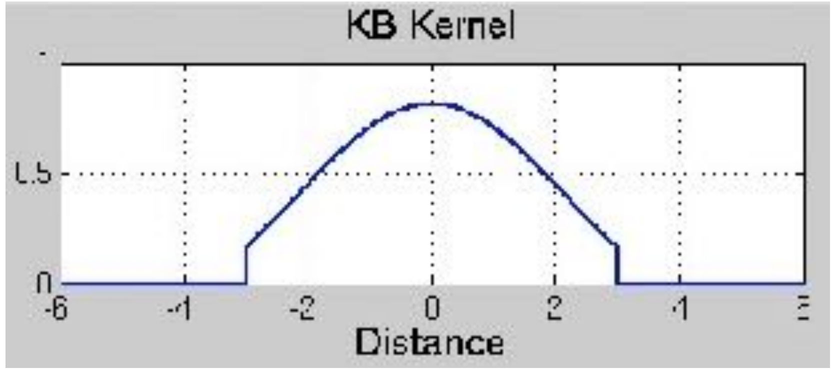
- Goal: Bring G and k to zero quickly
- Just use trapezoids
- Problem:
  - How much “power” to use on each axis
  - Finite segment method solutions (Meyer 2001)
  - Convex Optimization

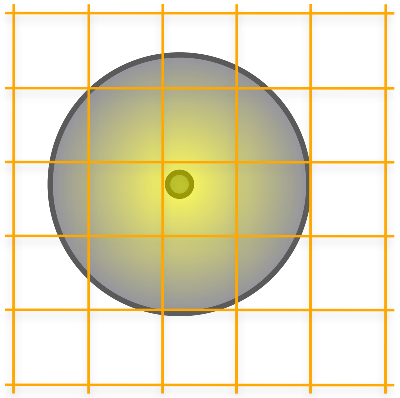


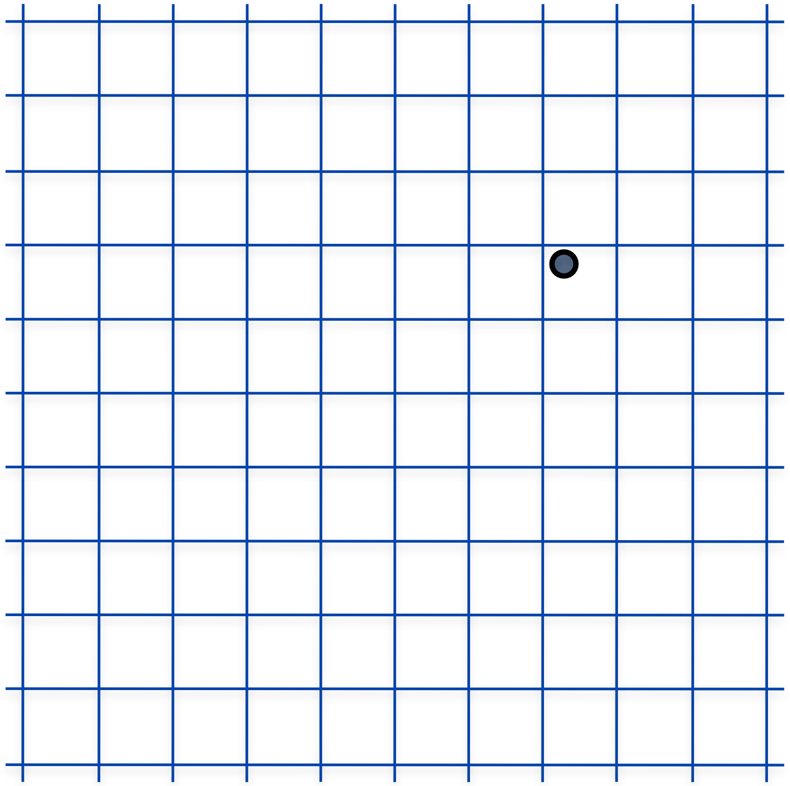
Spirals and Gradient/RF Delays

- Delays can have a few effects:
  - Mis-mapping of center of k-space, causes a low-frequency “cloud”
  - In outer k-space, delays cause rotation of the image
  - Other effects can be similar to radial
- Often measure actual gradient waveforms (later I hope)
- Study in homework!

Gridding Code: gridmat.m







- Designed to be reasonably fast, but Matlab (readable)
- Uses Kaiser-Bessel interpolation kernel (precalculated)
- For each k-space sample M(k):
  - Build a “neighborhood” of affected grid points k_{grid}
  - Calculate contribution at each grid point:
    - M(k) x kernel($k-k_{grid}$)
  - Add the values to a full-size grid
  - No deapodization

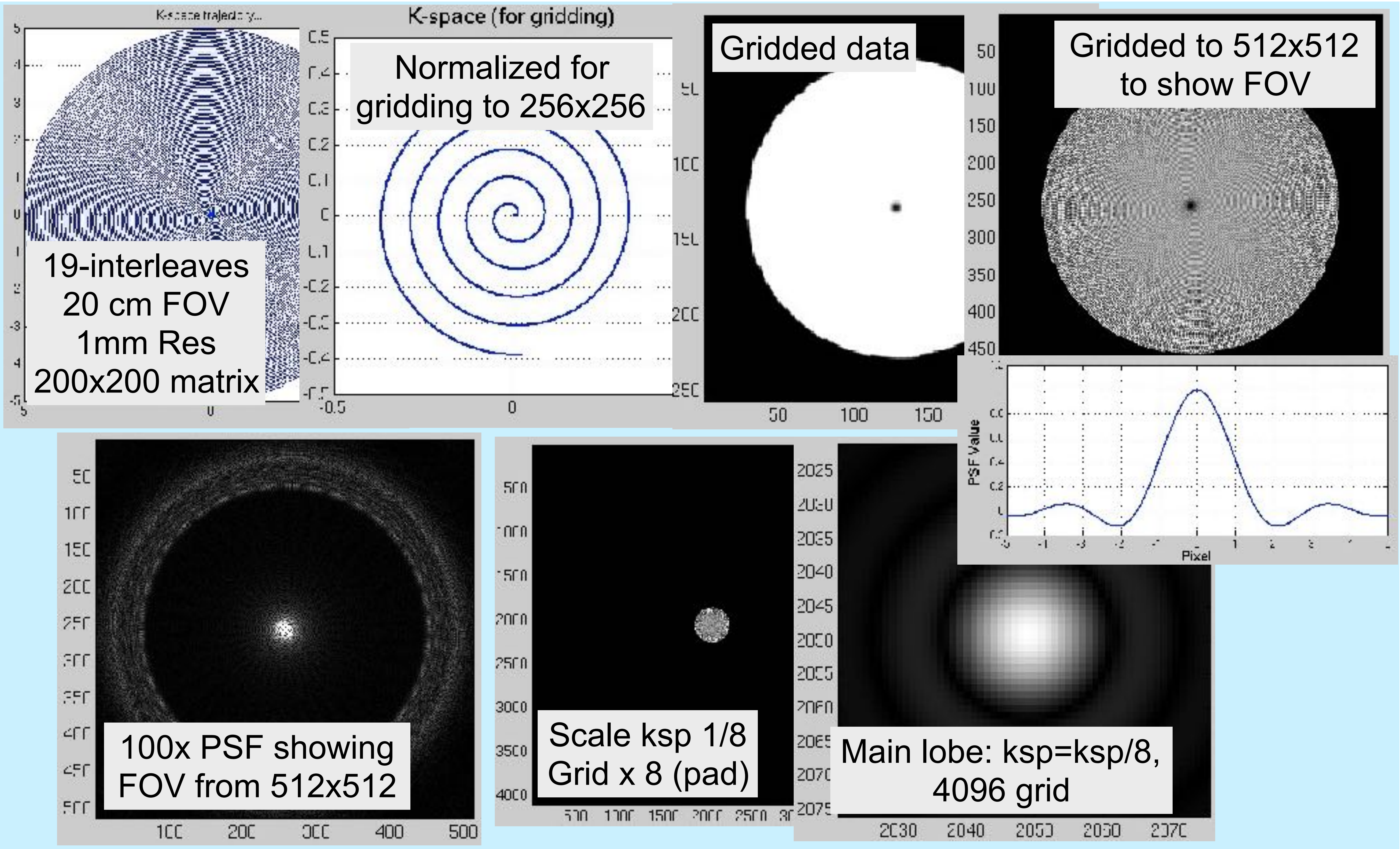
gridmat.m

- Inputs:
  - ksp = list of k-space locations, $k_x + ik_y$
  - kdat = data samples, ie $M(k_x,k_y)$
  - dcf = density compensation factors at each k-space location
  - gridsize = size of grid
- Convention:
  - k is in “inverse reconstructed pixels”
  - $|k| < 0.5$
- Larger gridsize zero-pads image (reduce apodization)
- Scale ksp smaller to “fill” FOV and interpolate pixels

Density Calculation Options

- Radial Density: $1/k_r$ (2D) or $1/k_r^2$ (3D)
- Spiral: Approximate $DCF = \vec{k} \times \vec{G}$
- Gridding: Can grid 1s *to k-space points,* then sum.
- Iterative (Pipe): $W_{k+1} = W_k / (W_k ** C)$
    - W = weights (DCFs), C = convolution kernel
- Voronoi regions (areas/volumes around each point, and calculate area/volume)

Gridding Example (FOV / Res)



Spiral Design Example

Spiral Design Example

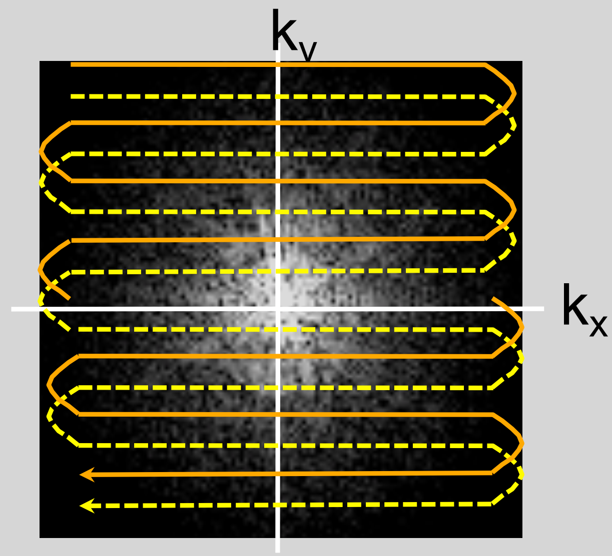
- For resolution of 1mm, 24cm FOV and 12 interleaves
- How many turns in each interleaf?
- How far does each interleaf "travel?"
- Estimate the duration of each interleaf

Spiral Summary

- Flexible duration/coverage trade-off
  - Center-out: TE~0, Low first-moments
- Archimedean, TWIRL, WHIRL, variable-density
- PSF with circular aliasing, swirl-artifact outside
- Off-resonance sensitivity, correct in reconstruction
- Variations: Spiral in/out, 3D TPI, 3D Cones
- Rewinder design

EPI

- Faster "Cartesian" approach
- Single-shot, Interleaved, segmented, half-k-space
- Delays, etc -> Phase corrections
- Flyback EPI
- GRASE

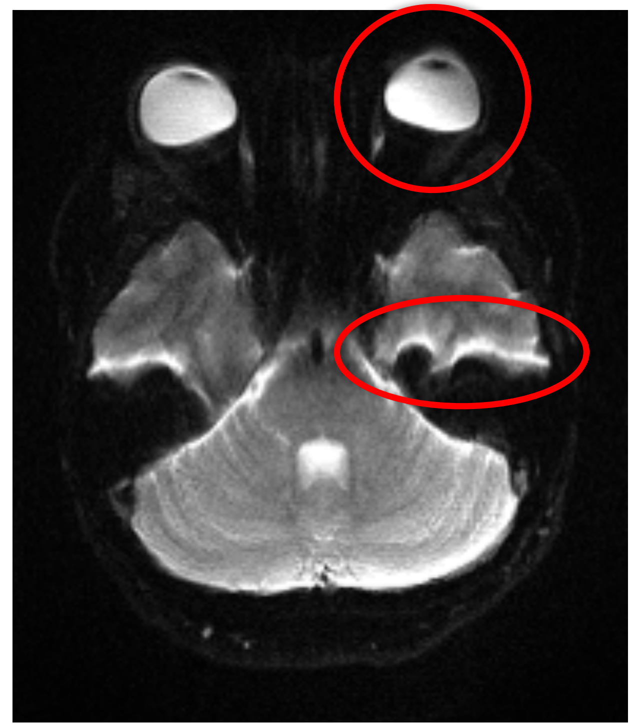


Thanks to Samantha Holdsworth!

EPI: Speed vs Distortion

Fast Spin Echo (FSE)

Slow ~ 3mins

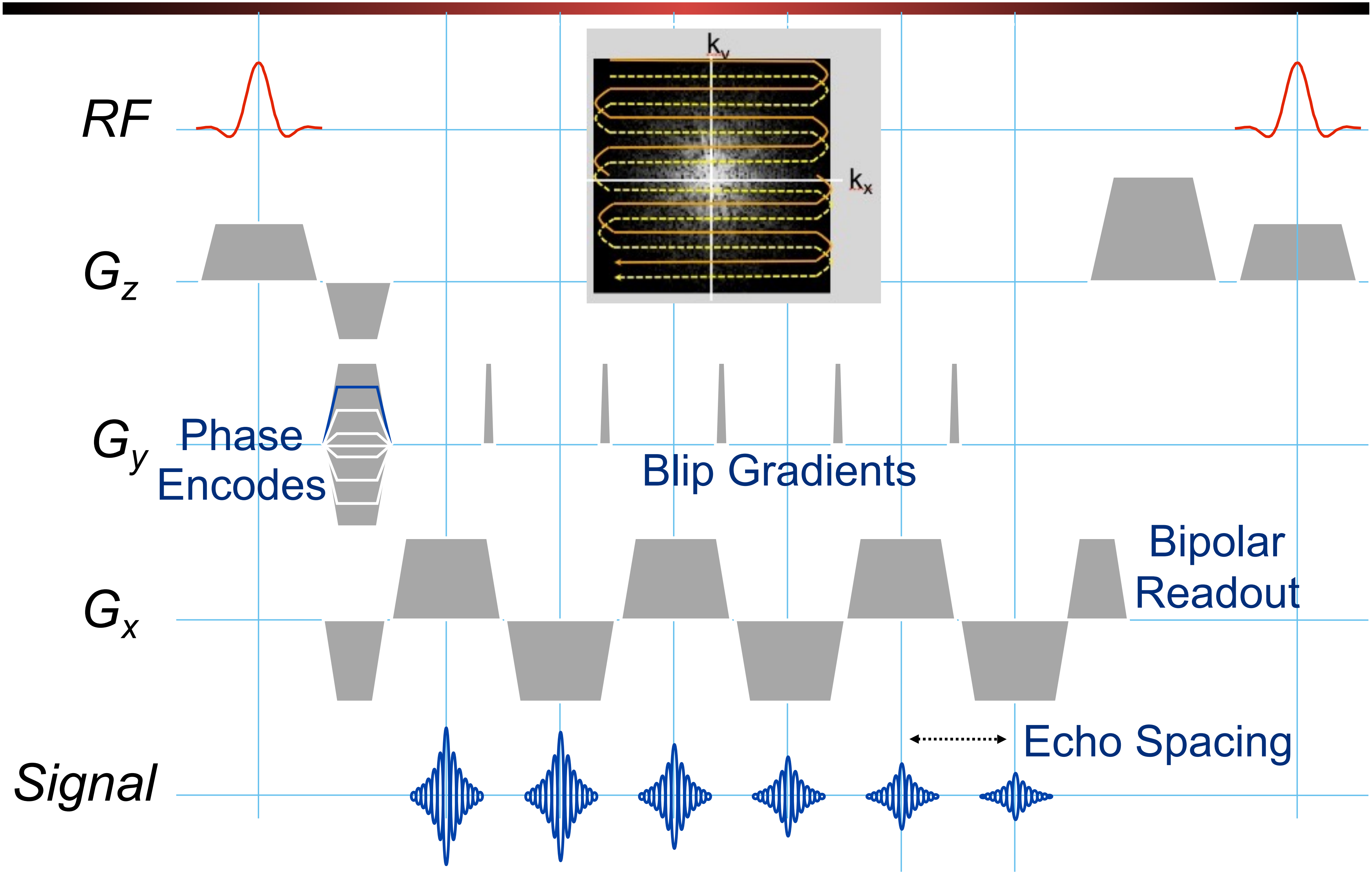


Echo Planar Imaging (EPI)

Faster ~ 10 seconds

(T2-weighted image. Full brain coverage. Same target resolution.)

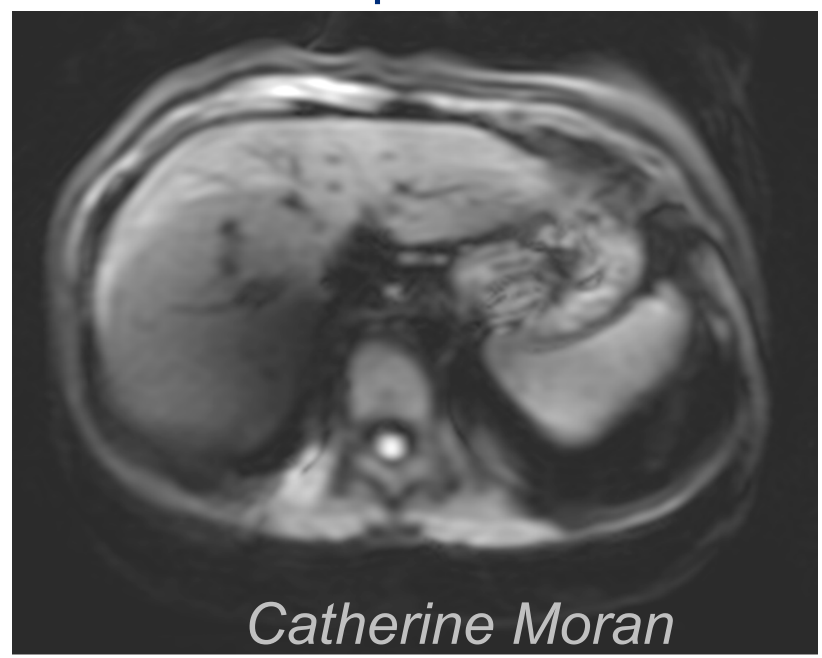
Echo-Planar Imaging (EPI)



EPI Calculations

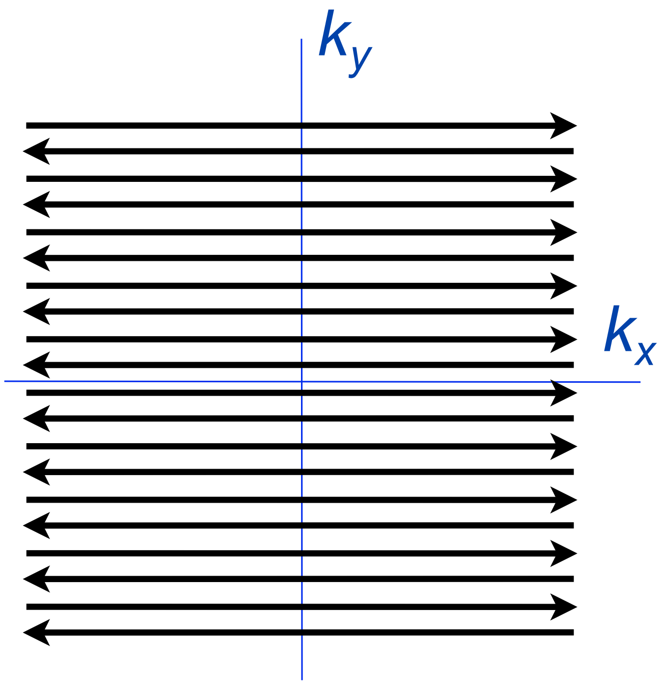
- T = ESP = Echo spacing. 1/T = effective bandwidth
  - Limited by gradients, readout resolution/duration
- Δk_y = 1/FOV
- Δk_y / T = k_y velocity (Hz/cm)
- Displacement = Δf (FOV) (T)
- T_2* decay over “echo train”
  - exp(-ETL x T /T_2*)

Fat/Water Displacement in EPI

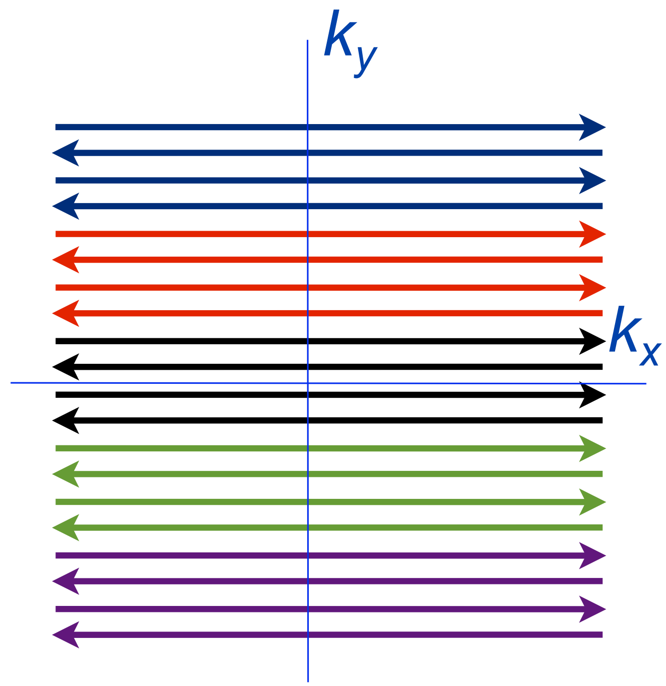


EPI Variations

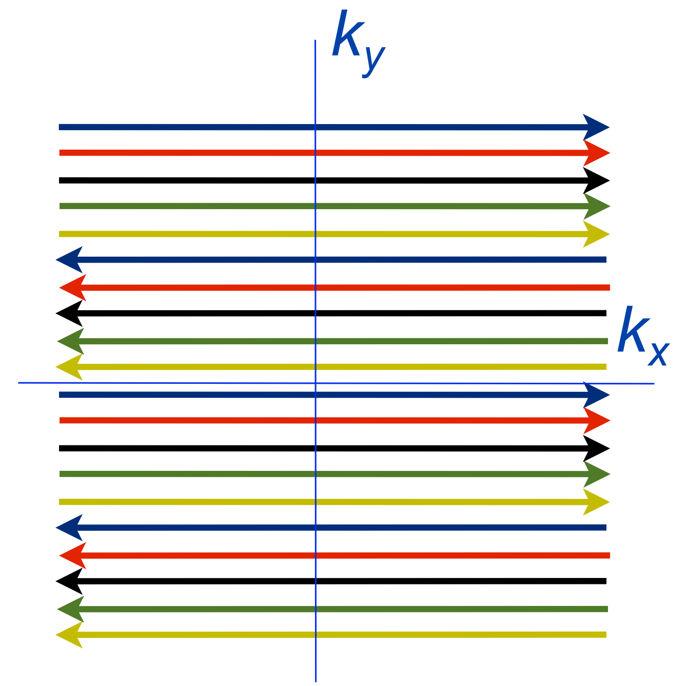
Single-shot



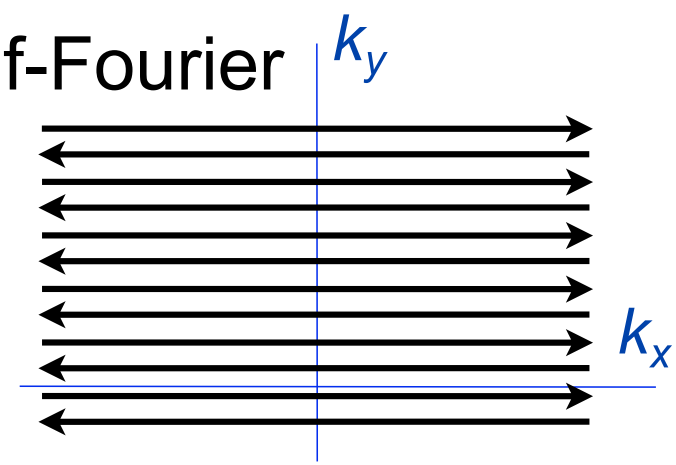
Segmented



Interleaved



Half-Fourier

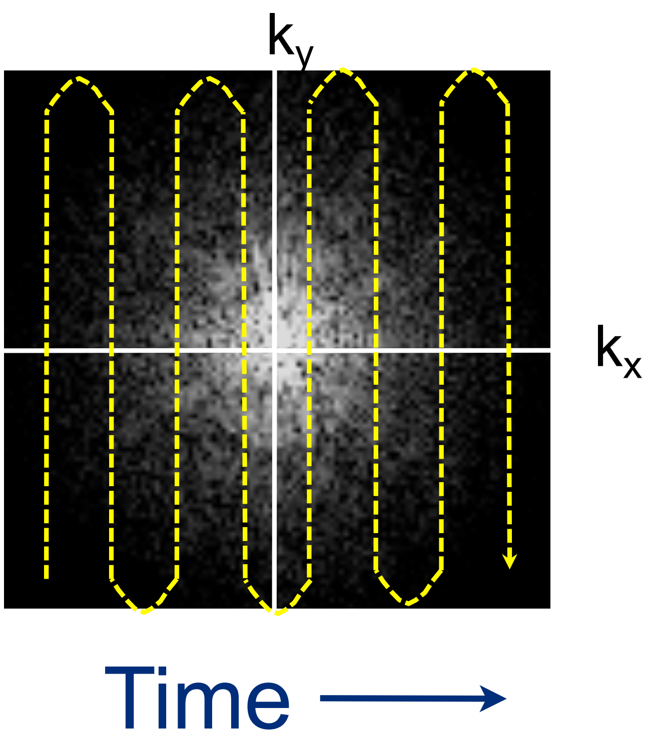


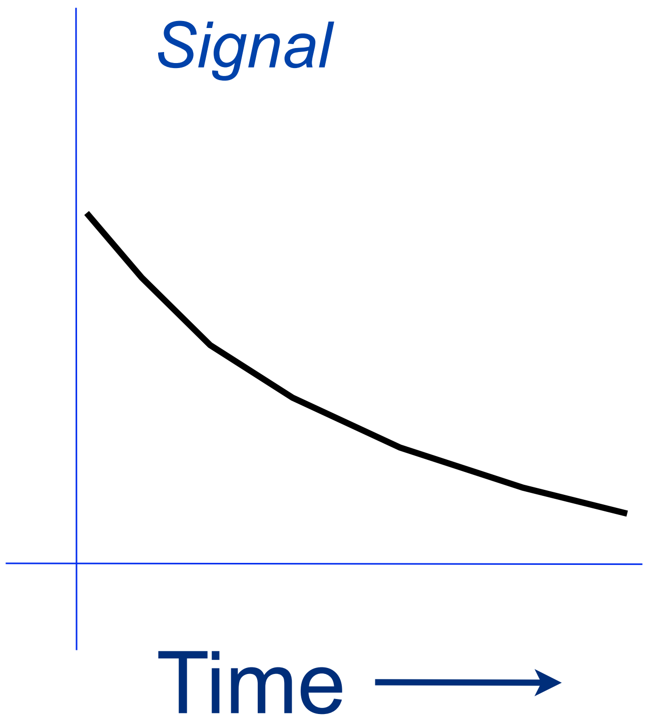
Interleaved and Single-Shot EPI

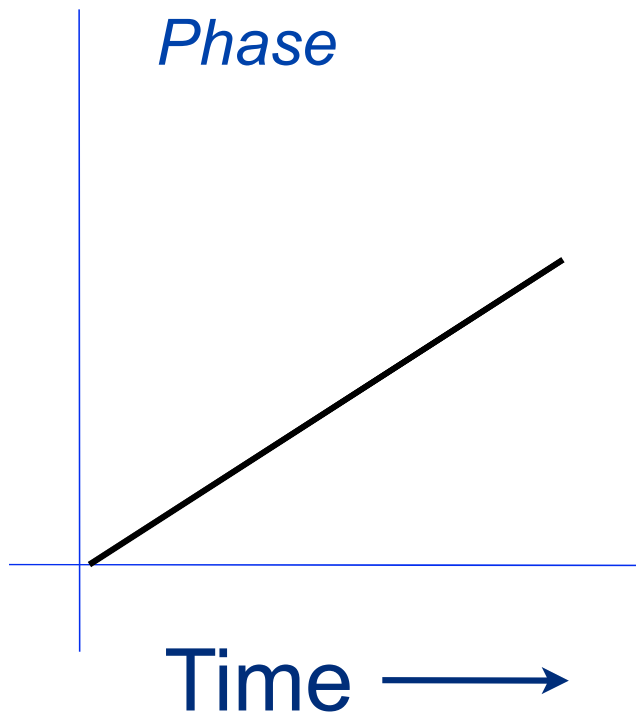
- Single-shot EPI:
  - All lines on one shot - reduces impact of motion
- Segmented EPI:
  - Acquire ETL *consecutive* lines - not used much
- Interleaved EPI (N_y = ETL x $N_{interleaves}$:
  - Acquire ETL lines per shot
  - Reduces T2* and distortion by $N_y/N_{interleaves}$
- Half-Fourier (k_y) often used (all methods)

Signal Modulation in EPI

- "Blip" direction traversal is slow
- T_2^* similar to echo-train T_2 modulation in FSE
- Low "effective bandwidth"
- Usually ignore readout direction effects





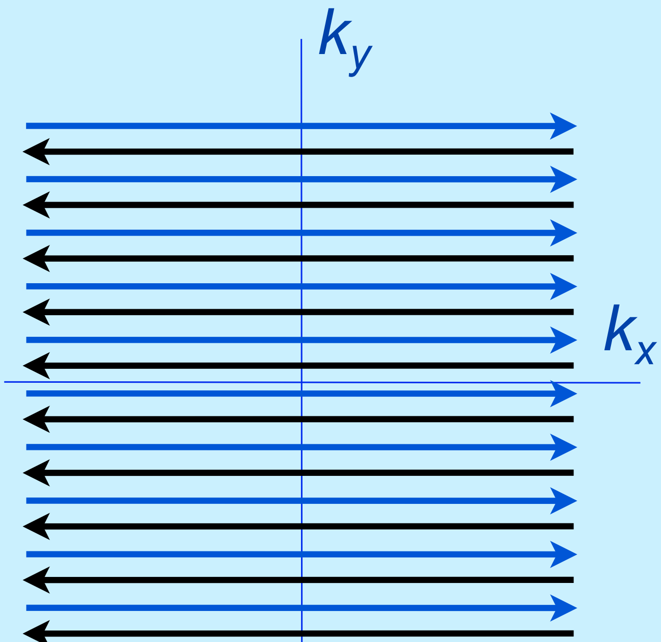


Signal/Phase Modulation

- *T2 = 100ms, Echo-spacing 1ms, 128 lines (full k_y)*
  - What is the signal loss?
    - k_y=0 at 64ms, so $e^{-0.64}$.
  - What is the fat/water displacement (3T) per FOV?
    - (0.44kHz)(1ms) = 0.44 cycles/ky line... 0.44 FOV!
    - *Use fat suppression!*
  - How do these change with 3x parallel imaging?
    - $e^{-0.21}$ and 0.13 FOV
  - With 2x reduced FOV?
    - (Like 2x PI) $e^{-0.32}$ and 0.4 $FOV_{orig}/2$

Other Effects - Single-Shot (SS) EPI

- What are some effects of bidirectional readouts?



SS EPI - Odd/Even Decomposition

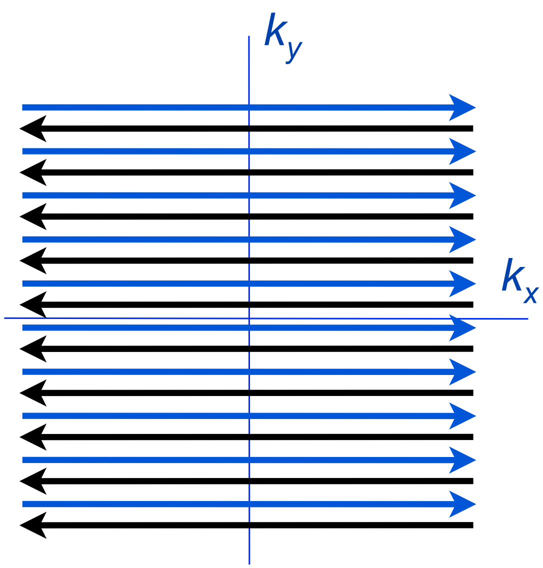
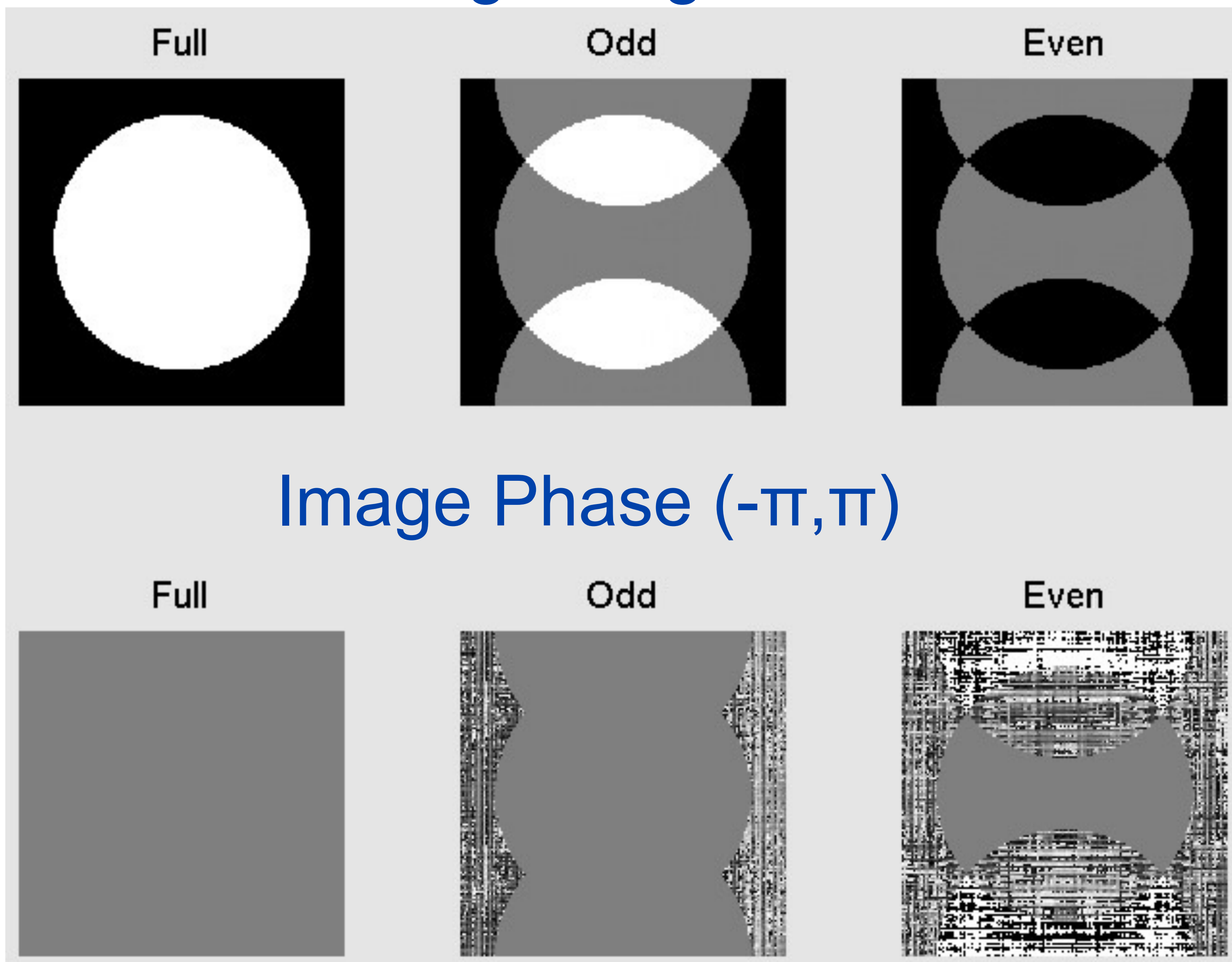
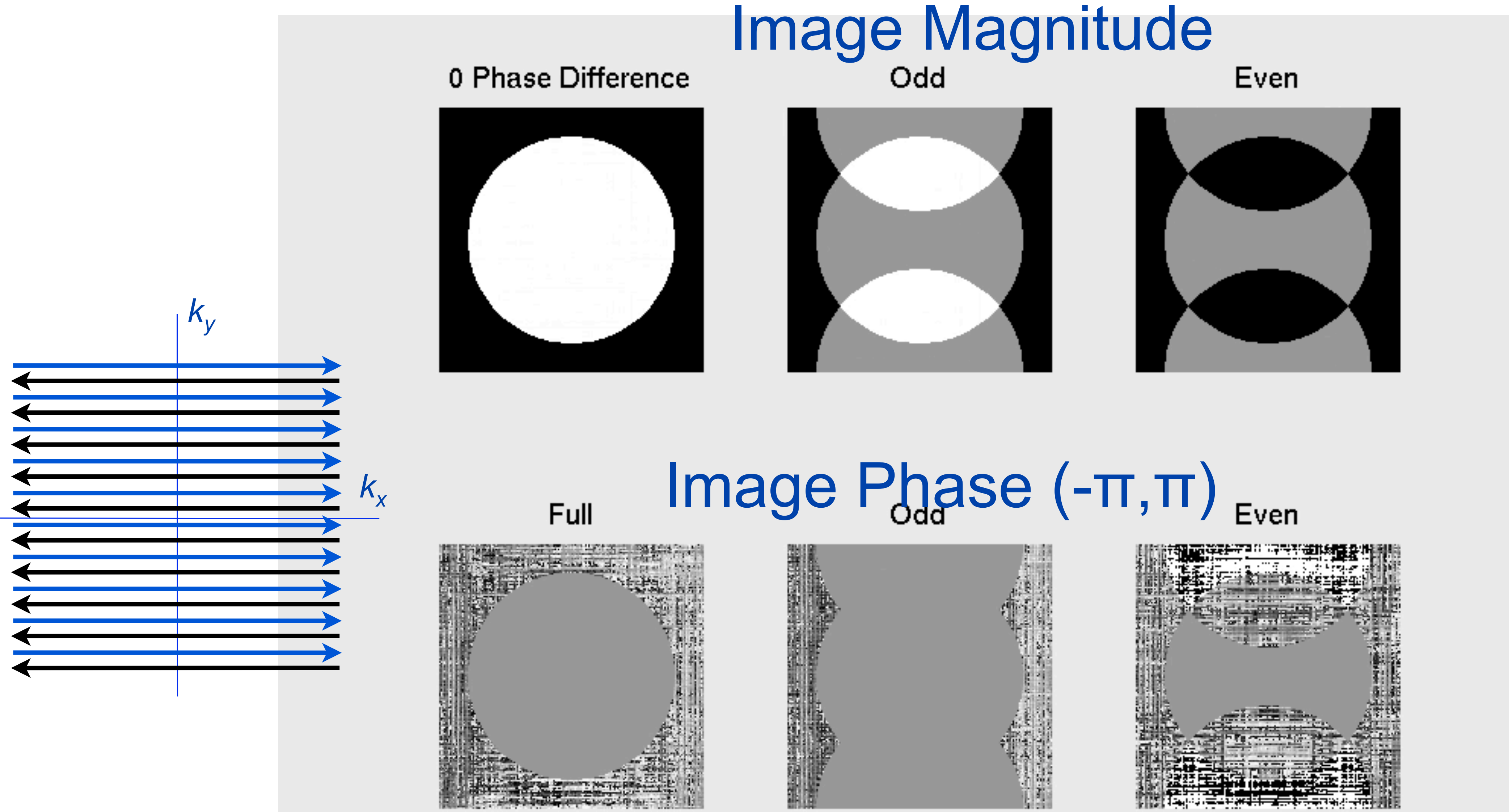


Image Magnitude

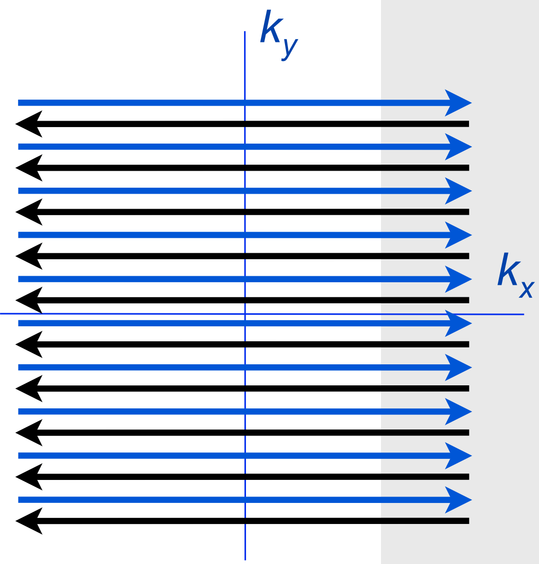


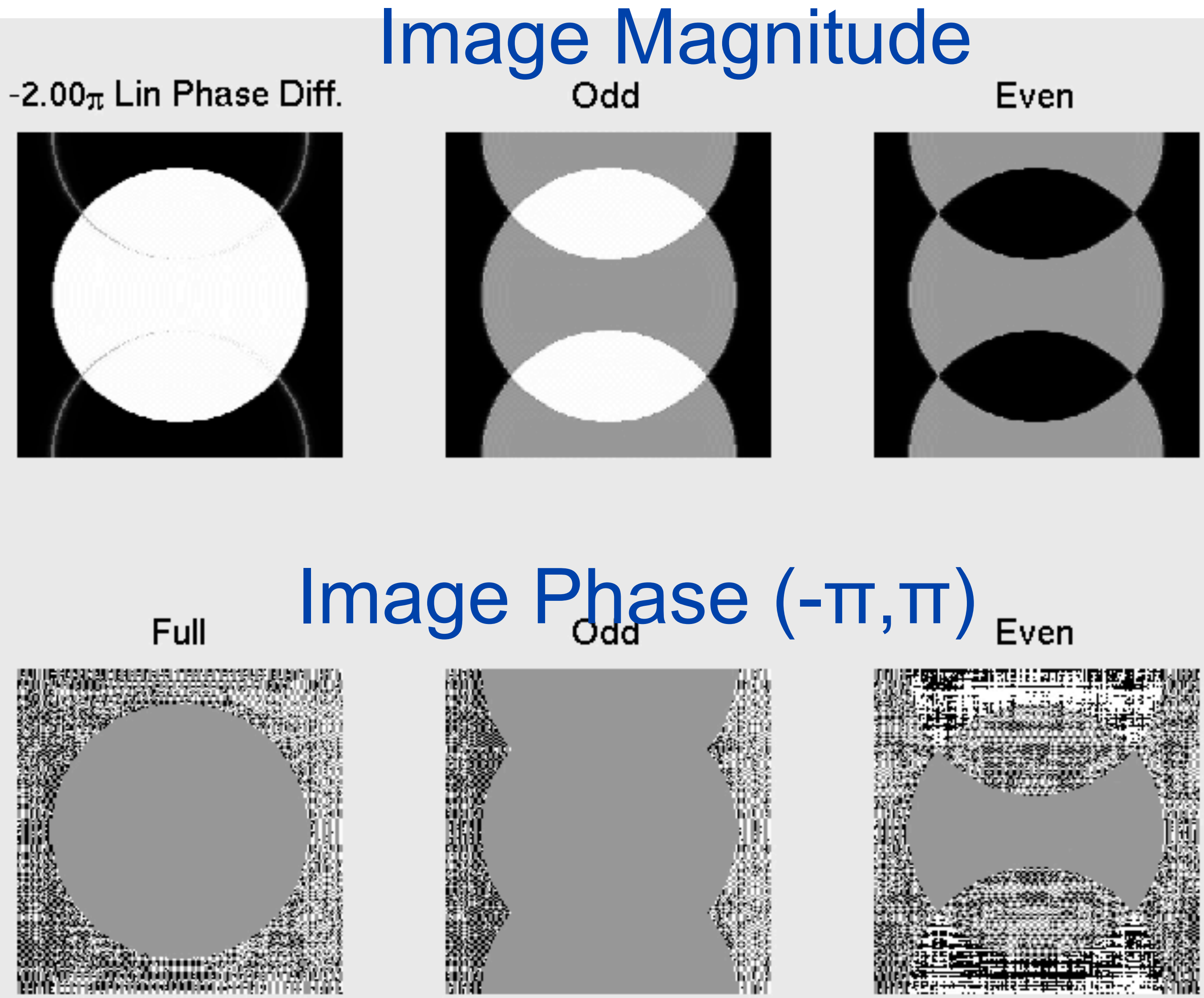
SS EPI - Alternating Constant Phase



(Phase largely due to B0 eddy Currents)

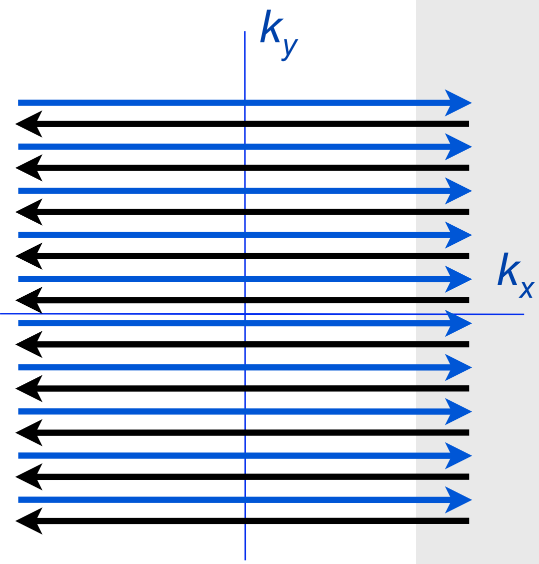
SS EPI - Linear k-space Phase

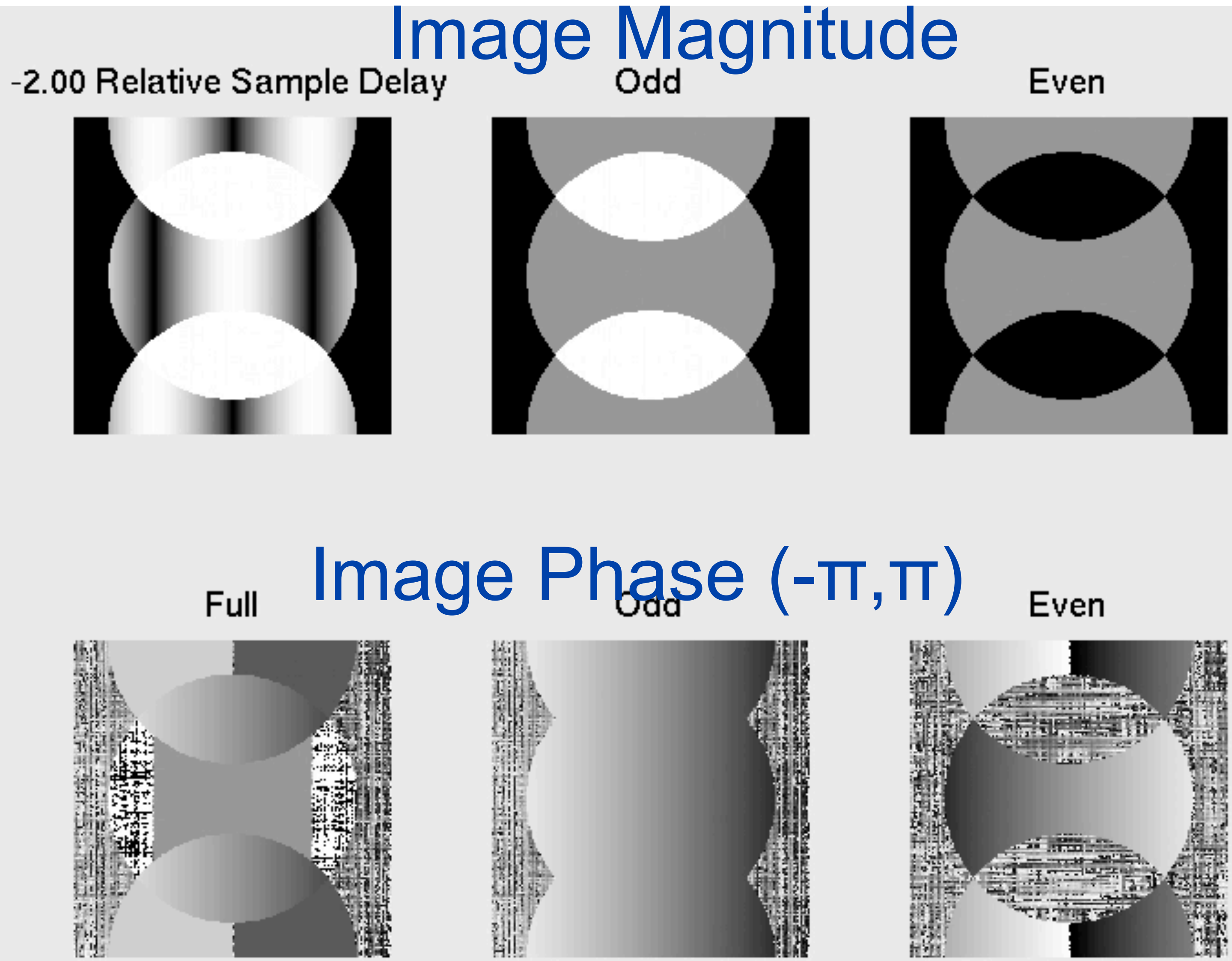




(Phase largely due to off-resonance)

SS EPI - k-space Delays





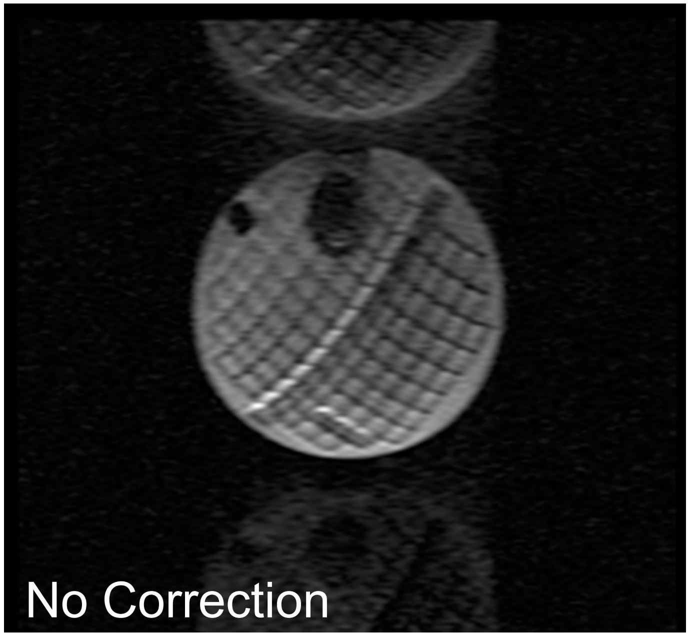
(Gradient delays and Eddy Currents)

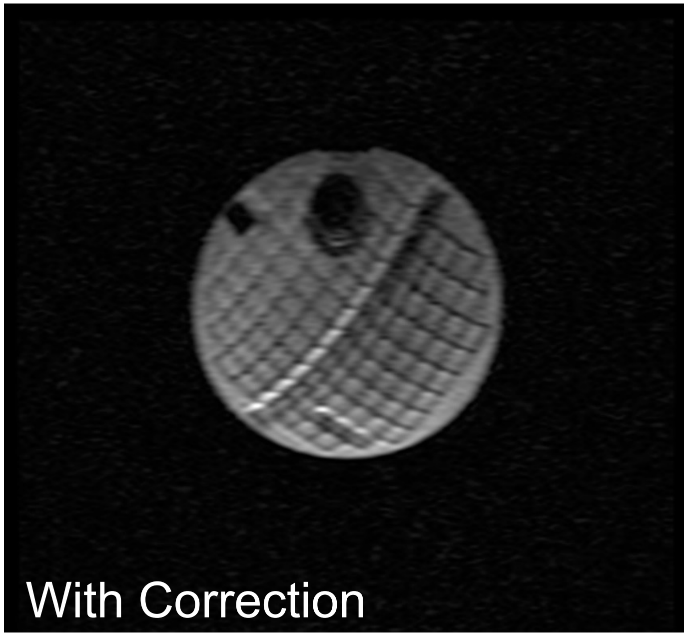
SS EPI: Odd/Even Effects Summary

- Constant phase (image or k-space)
  - coherent ghosts
  - due to eddy currents or sequence imperfections
- Linear phase in k-space
  - component images displaced (high x-freq ghosts)
  - due to off-resonance
- Delays in k-space
  - x-varying ghosts in y
  - due to eddy currents or gradient delays

EPI Phase Correction

- Turn off k_y blips and phase-encodes
- Acquire projections along k_x and FT in x
- Estimate constant and linear phase of each x line
  - Typically both alternate, but early lines may differ as eddy-currents not in steady state.

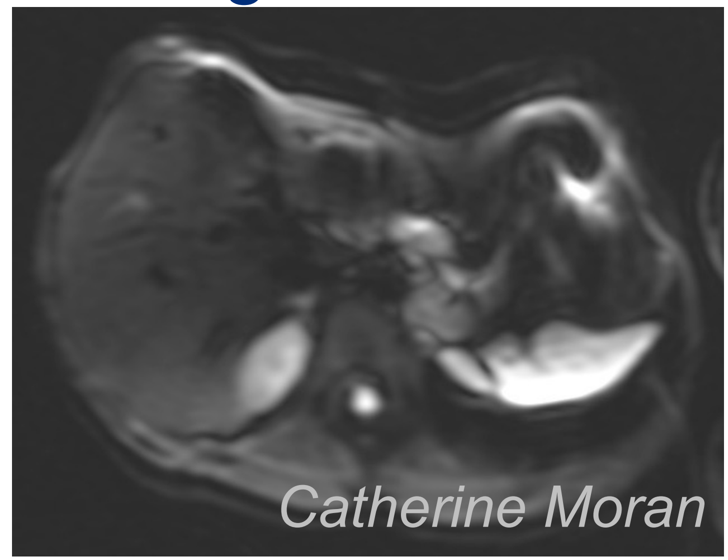




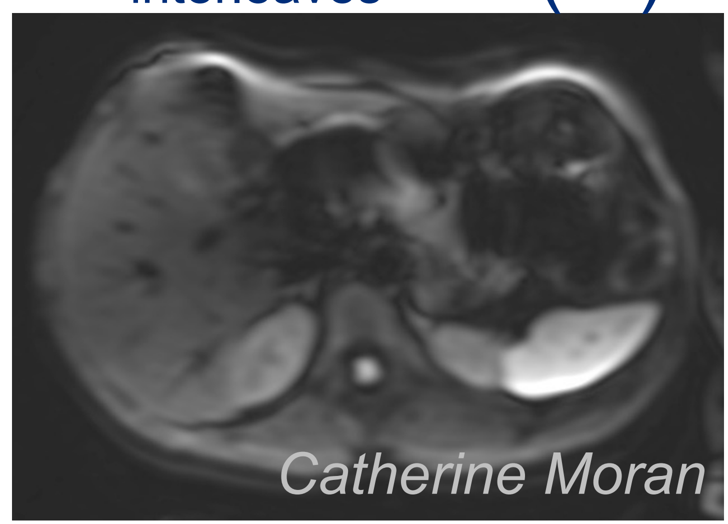
Single-Shot vs Interleaved EPI

- N/2 ghosts vs $N/(2N_{interleaves})$ ghost effects
- Phase correction is very similar
- Interleaved EPI:
  - Reduces sensitivity to T2*, off-resonance
- Single-shot EPI:
  - Faster, reduces sensitivity to motion (especially for DWI)

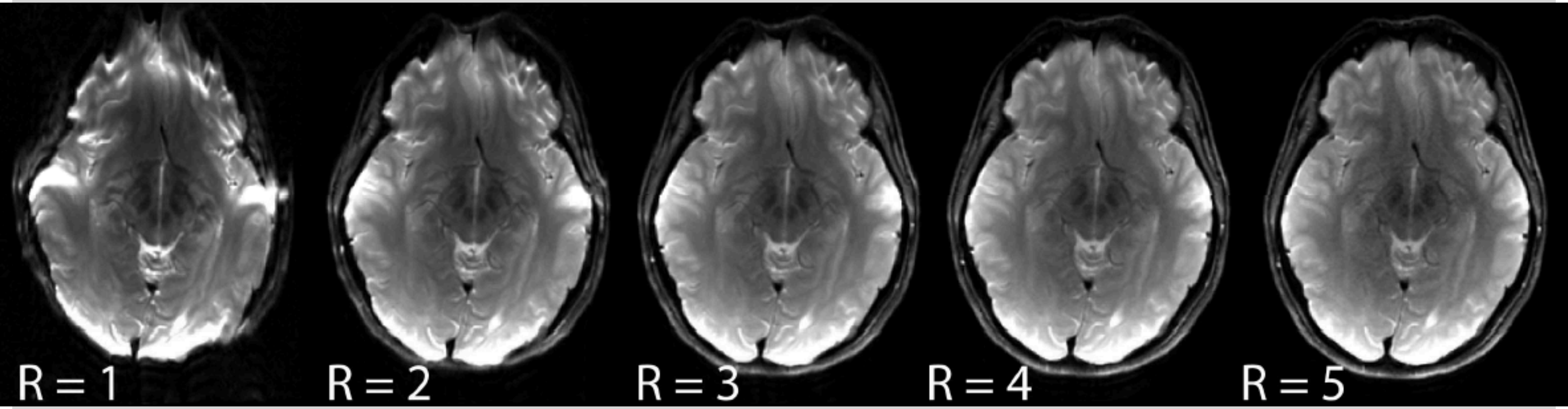
Single-Shot EPI



$N_{interleaves}$ = 2 (PI)



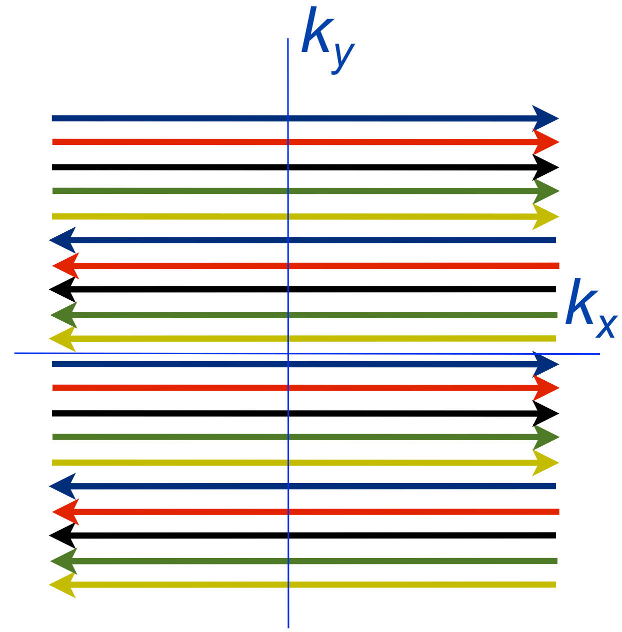
Example of EPI with parallel imaging

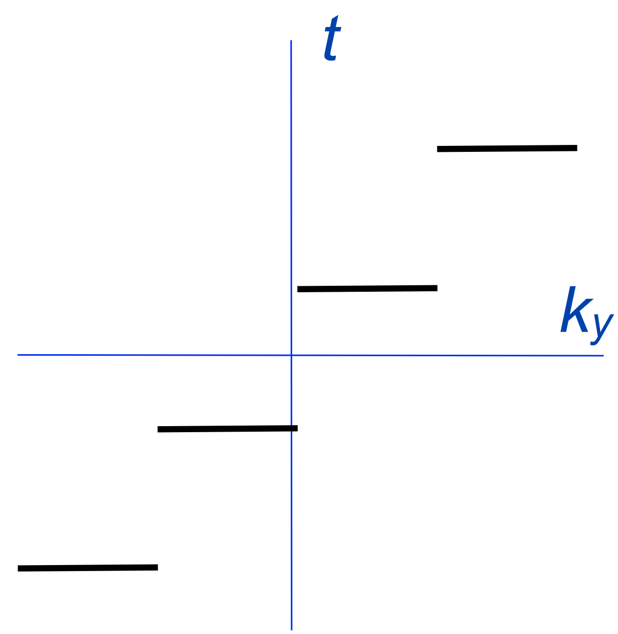


(Different parallel imaging acceleration factors. T2-weighted image. Same target resolution. Scan time matched)

Stair-step Modulation in Interleaved EPI

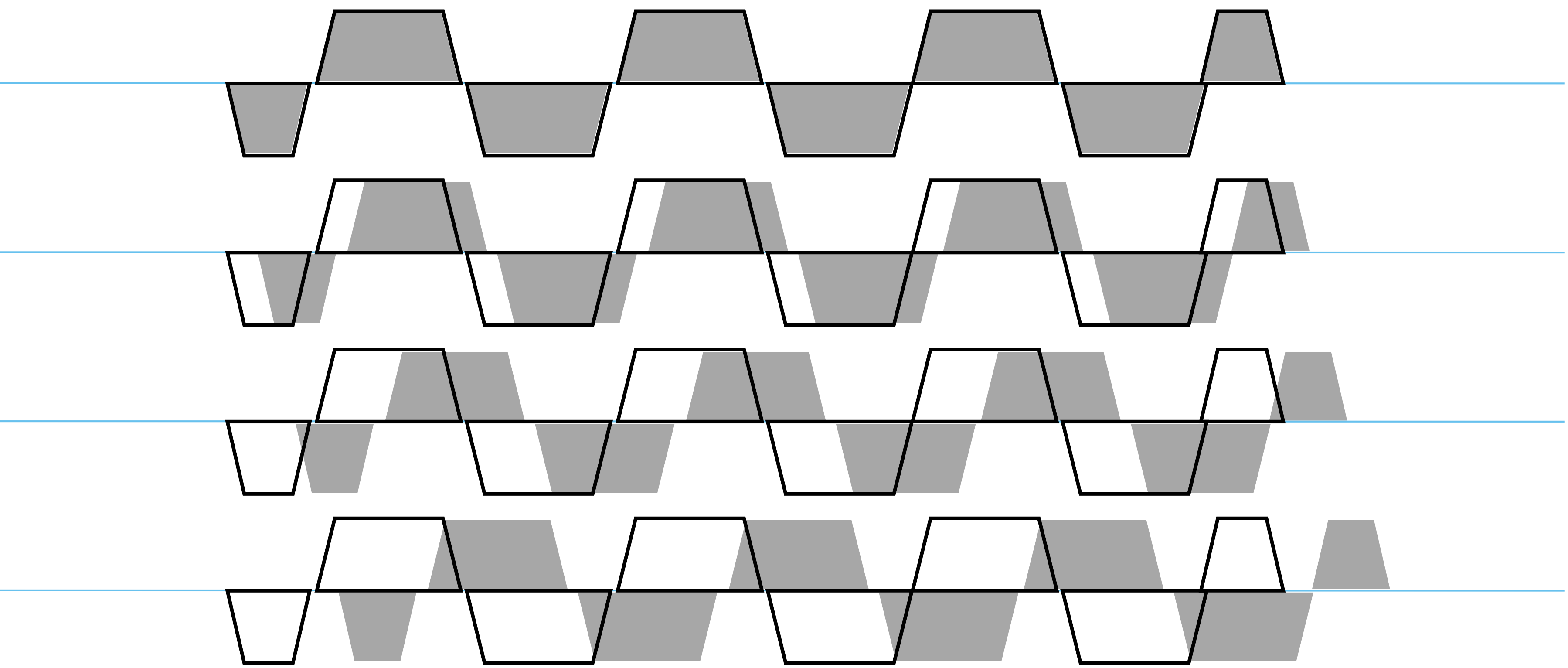
- Lines in a Segment of k-space all acquired at similar time
- Boundaries have a discontinuity in time, thus amplitude and phase
- What might this cause in the image?





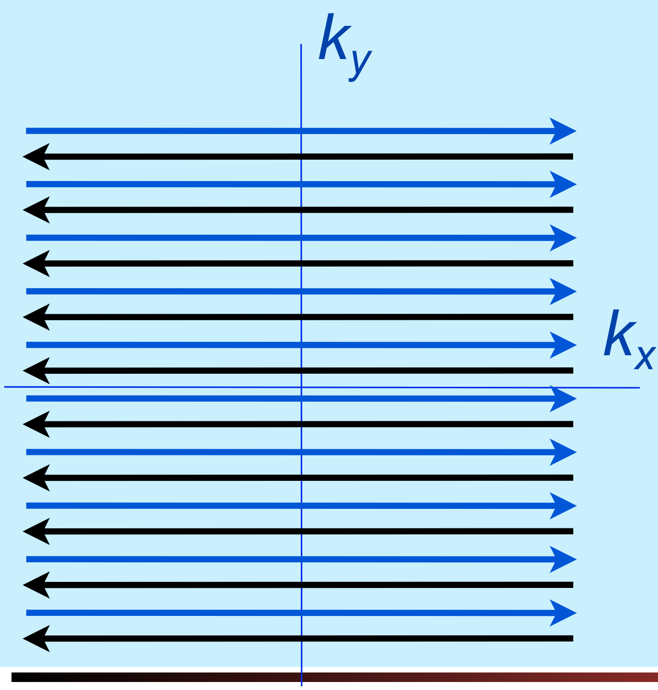
Interleaved EPI: Smoothing Phase

- Time T between echo n and n+1
- Desire smooth $k_y(t)$ overall
- Delay m^{th} interleaf by $(m/N)T$ (N=4 here)



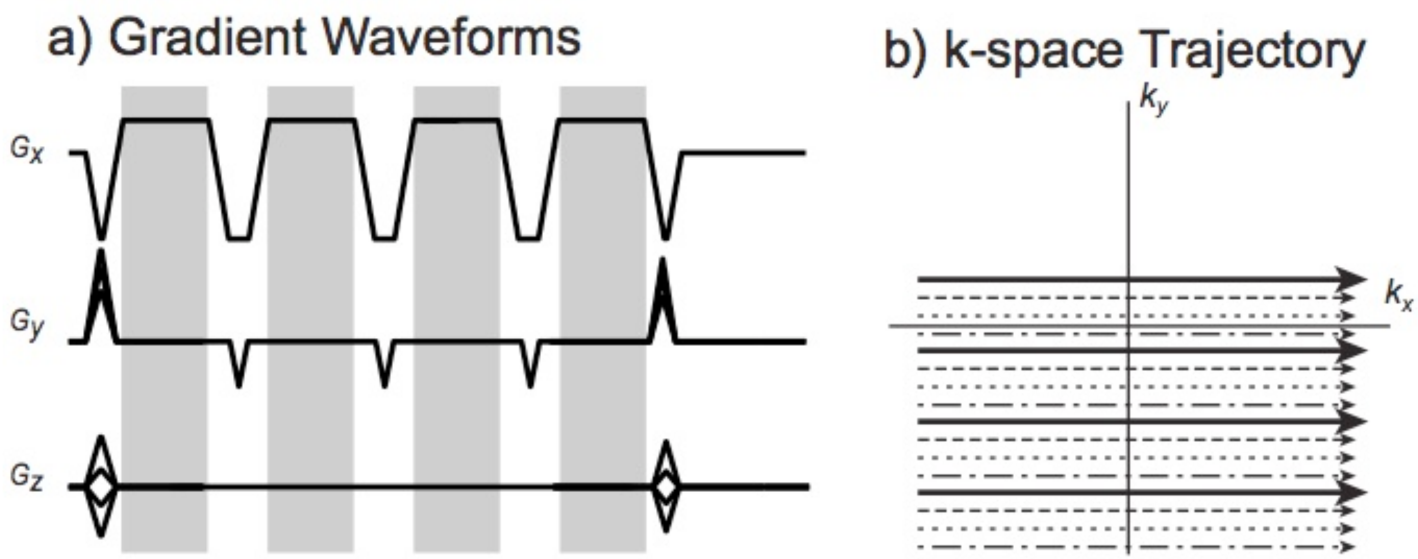
EPI Design Example

- We want to sample a 30cm FOV at 1mm resolution as fast as possible using EPI with less than 1cm displacement between fat and water at 3T



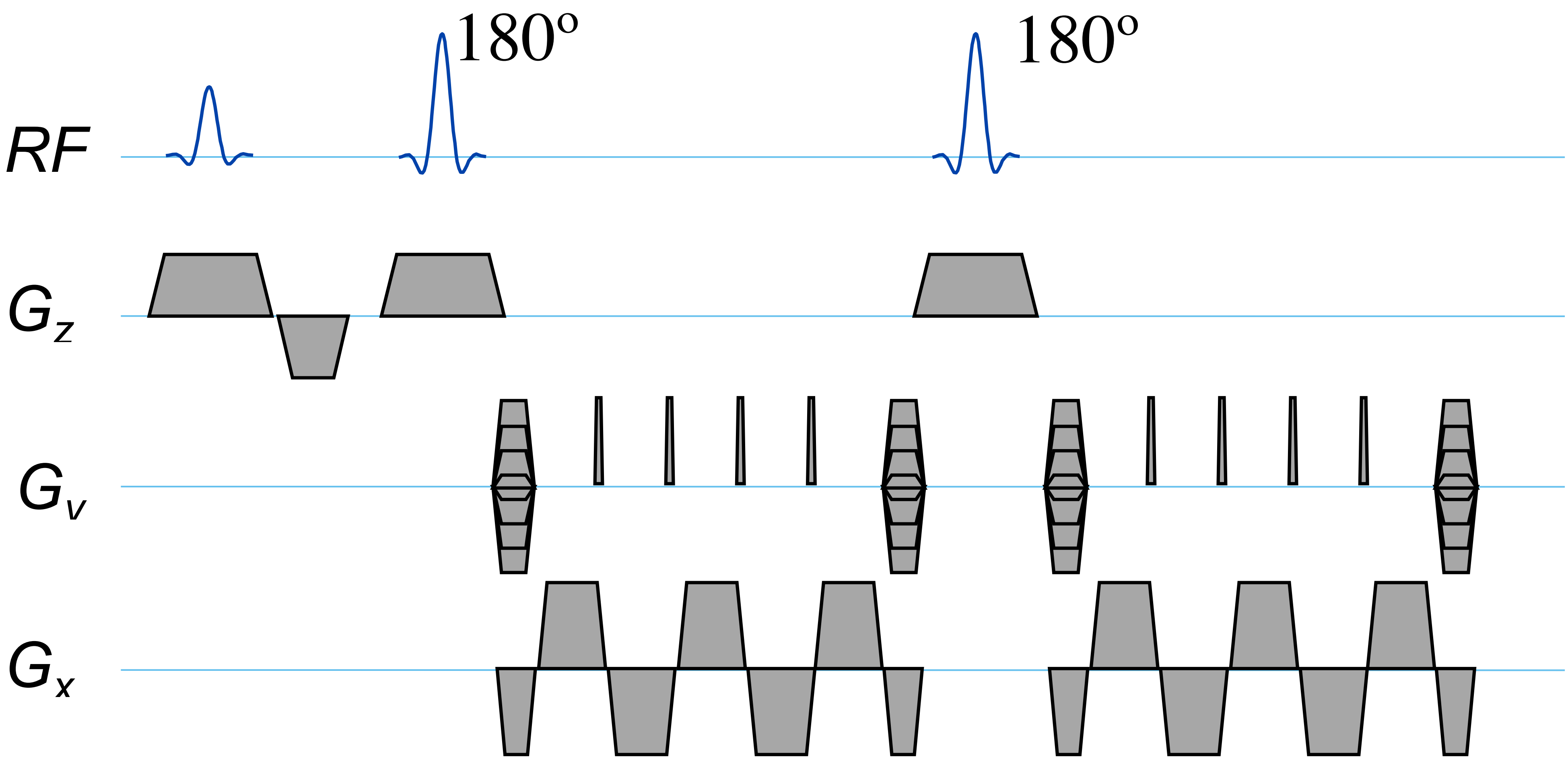
Flyback EPI

- Readout in only one direction
- Completely avoids odd/even line sensitivity
- Slower, but useful when flyback is fast
- Still sensitive to off-resonance



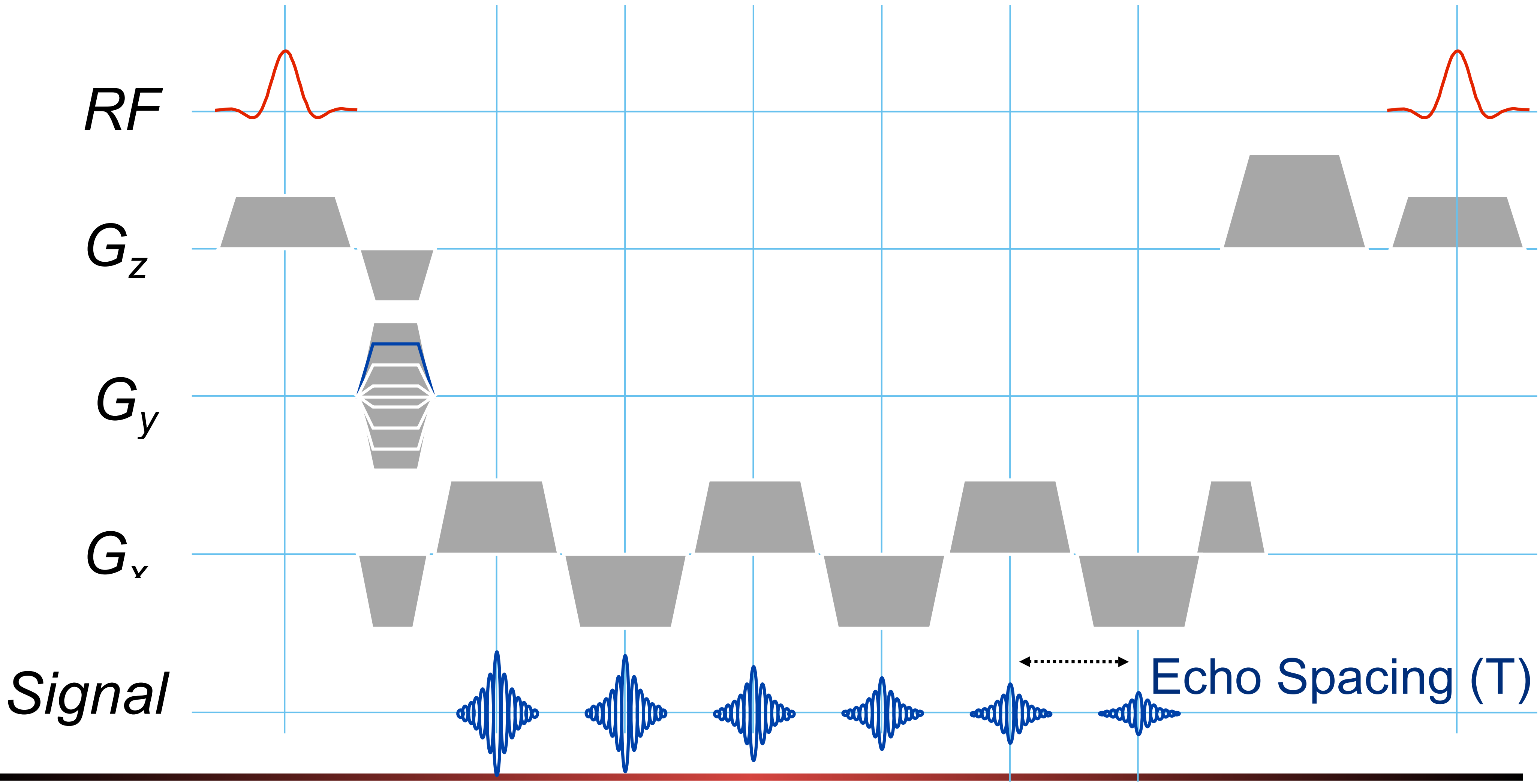
GRASE (Gradient and Spin Echo)

- Helps improve efficiency of spin echo
- Both T2 and T2* modulation! (3D can spread over y and z)



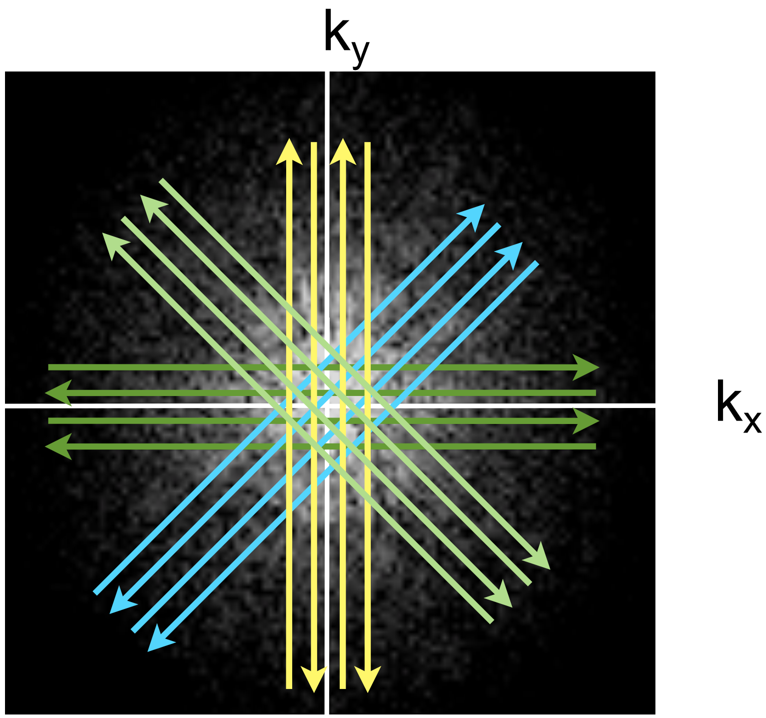
EPSI (Echo-planar Spectroscopic Imaging)

- No k_y blips, or repeat k_y pattern every N echoes
- Spectral FOV of 1/T or 1/(NT)

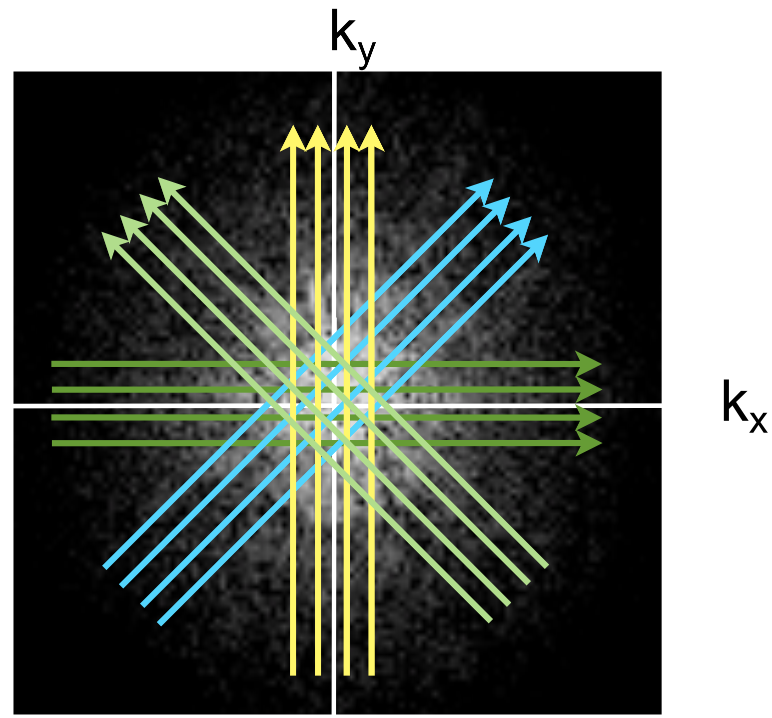


Propellor (EPI or FSE)

- Rotated low-ky-res acquisitions (“blades”)
- Self-navigating (low-res image every blade)
- Individual blades corrected for phase, delays and gridded
- Robust to motion

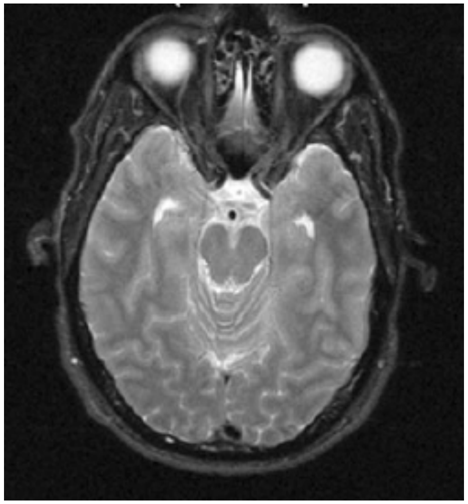


EPI Propellor

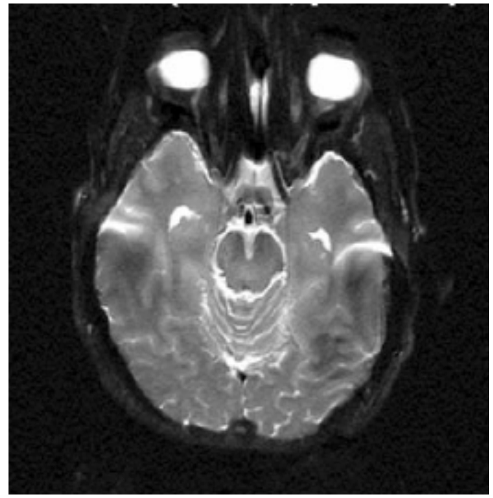


FSE Propellor

Propeller EPI

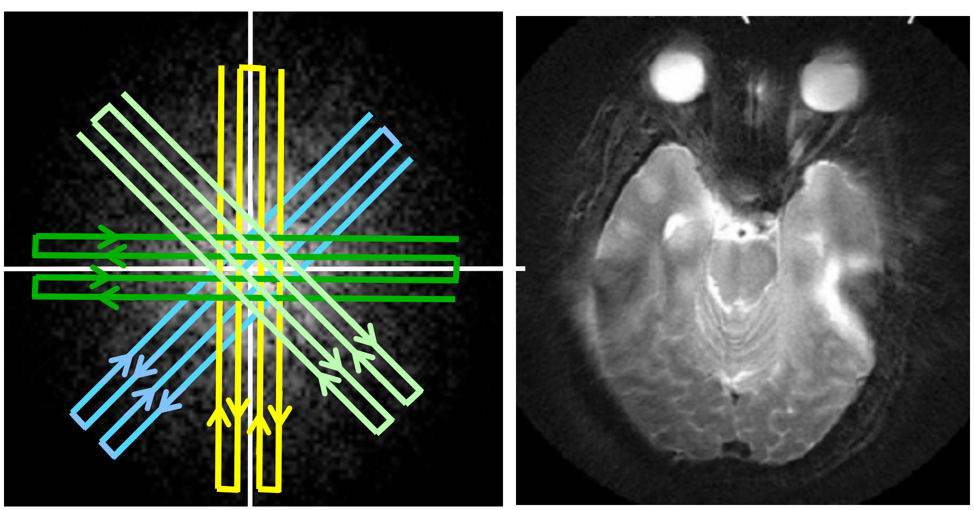


FSE

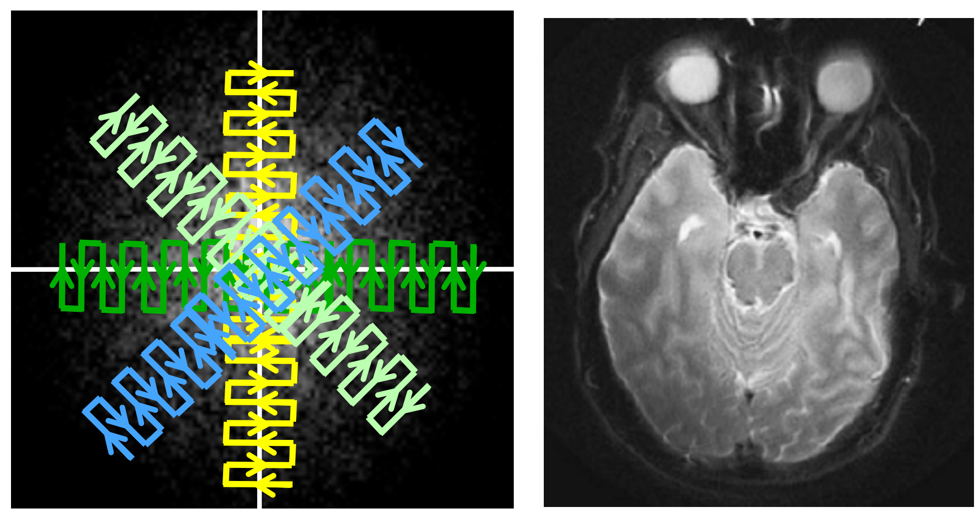


EPI – positive k_y blips

EPI – negative k_y blips



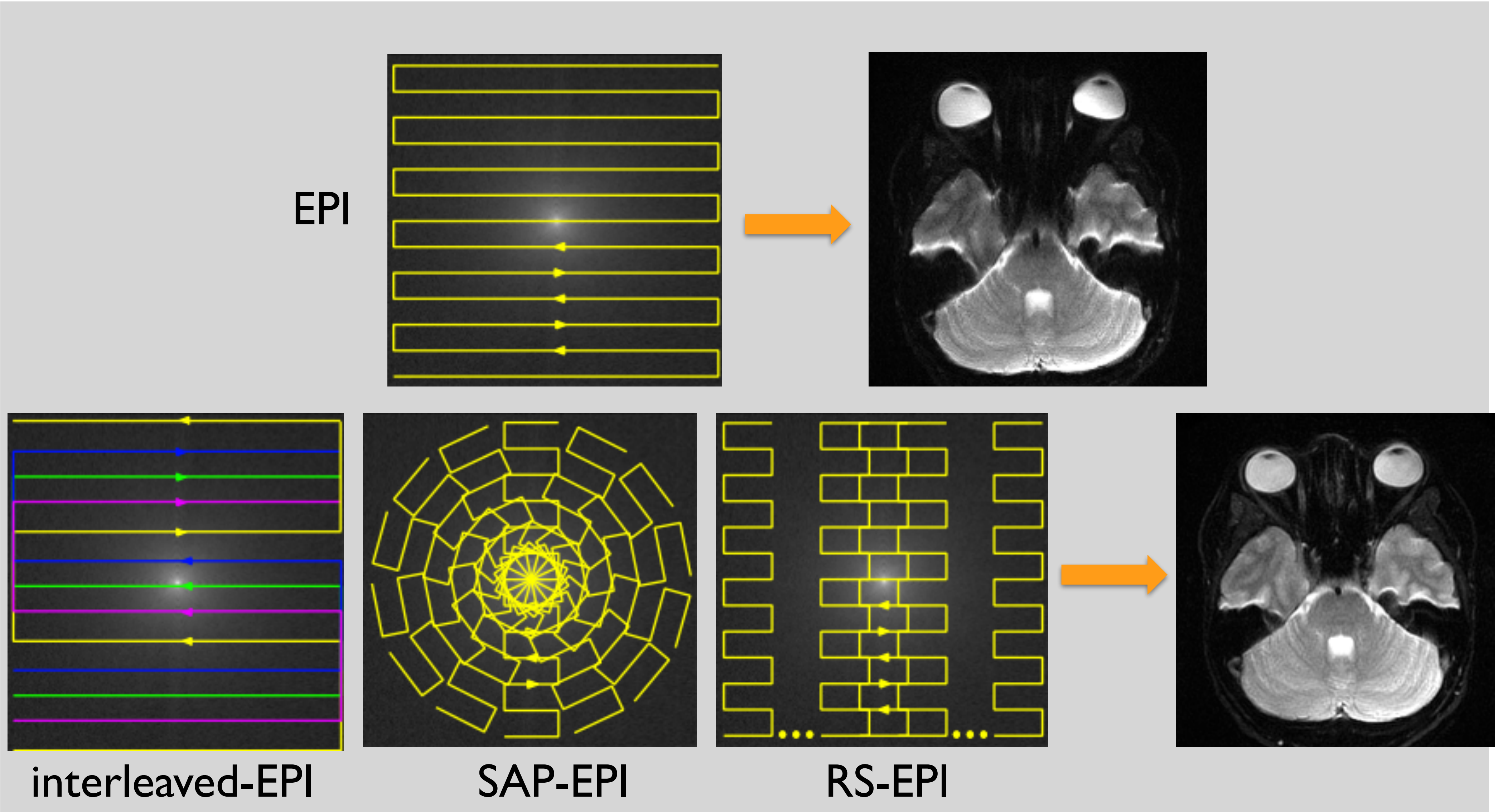
EPI Propeller



Short-Axis EPI Propeller

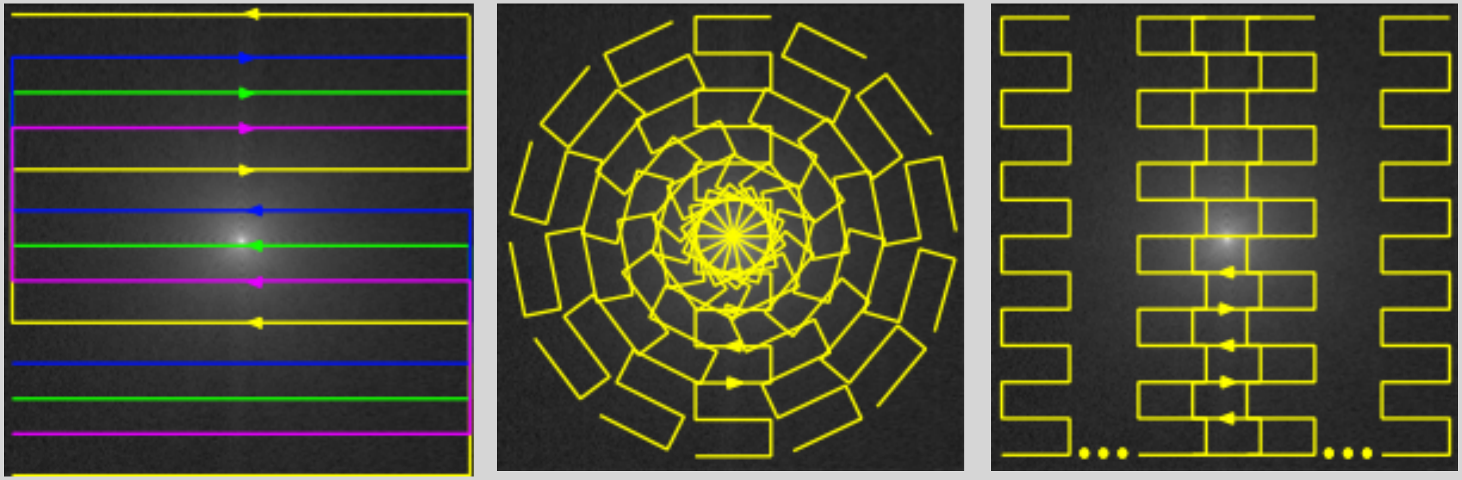
Skare et al. MRM (2006)

Interleaved EPI and other pseudo-EPI approaches



$$Distortion_y \propto FOV_y T_{esp}$$

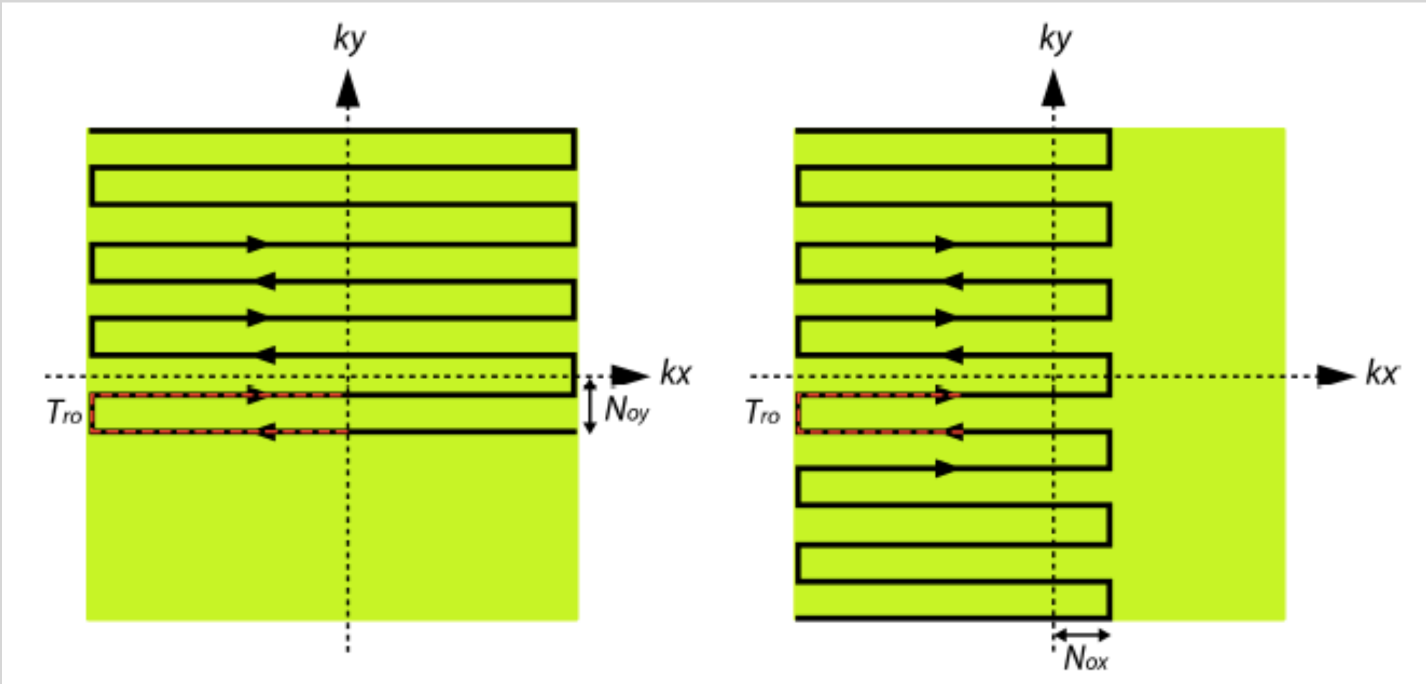
Important differences between interleaved EPI and other pseudo-EPI approaches



interleaved-EPI | SAP-EPI "short-axis propeller EPI" | RS-EPI "readout-segmented EPI"

	Advantages	Disadvantages
Interleaved EPI	*Easier to implement/ reconstruct, not slewing all the time (more efficient)*	*Motion between interleaves causes ghosting – harder to correct*
SAP-EPI and RS-EPI	*Each 'segment' acquired at full FOV -> can correct for motion between segments*	*Slewing a lot. Residual distortion for each "SAP-EPI segment" combines to give overall image blurring.*

Half-Fourier EPI approaches



Half-Fourier in k_y

Compared with full-Fourier:

- Reduced T2* effects
- Reduced minimum TE

(most common)

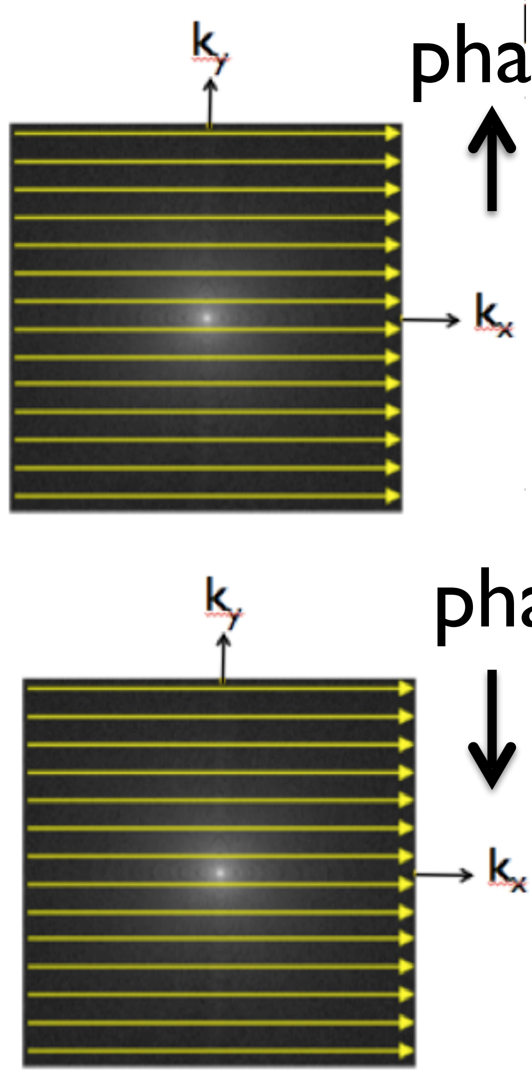
Half-Fourier in k_x

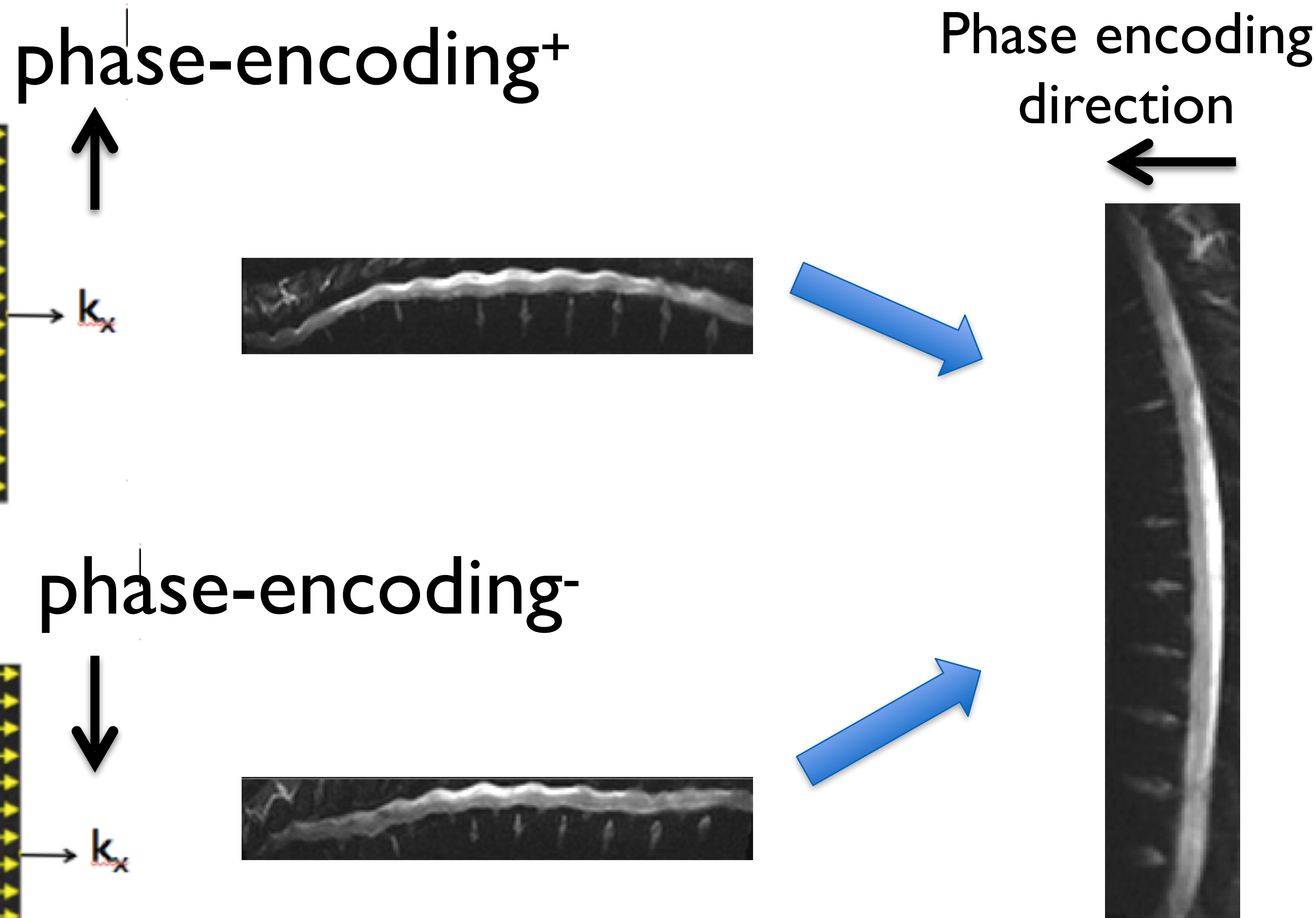
Compared with full-Fourier:

- Reduced distortion
- Slightly reduced T2* effects
- Slightly reduced minimum TE

Other distortion reduction strategies

Reversed Gradient Polarity Method (RGPM[1])





[1] Chang H, Fitzpatrick J. **A technique for accurate magnetic resonance imaging in the presence of field inhomogeneities**. IEEE Trans Med Imaging. 1992;11:319–329. .

EPI Other Considerations

- Readouts: Trapezoid gradients
- Phase encode/Blips: Consider quantization to avoid boundary artifacts
- May sample on ramps
  - Regrid data, slight sensitivity to off-resonance
- Parallel imaging: How to calibrate?
- Partial k_x to reduce echo spacing
- Partial k_y to reduce T2* effects (not off-resonance)
- Off-resonance correction in reconstruction may help

EPI Summary

- Very fast imaging trajectory
- Single-shot, Interleaved or Segmented
- Bidirectional EPI requires phase correction
- Sensitive to T2* and Off-resonance (blur and distortion)
- Much more widely used than spiral (currently)
- Variations: Flyback, GRASE, Propellor

Quantitative Sequences

- Basic Quantitative Sequences
  - Gradient Measurement
  - Fat/Water Separation
  - B_0 and B_1 mapping
  - T_1, T_2 and T_2* mapping

Gradient Measurement

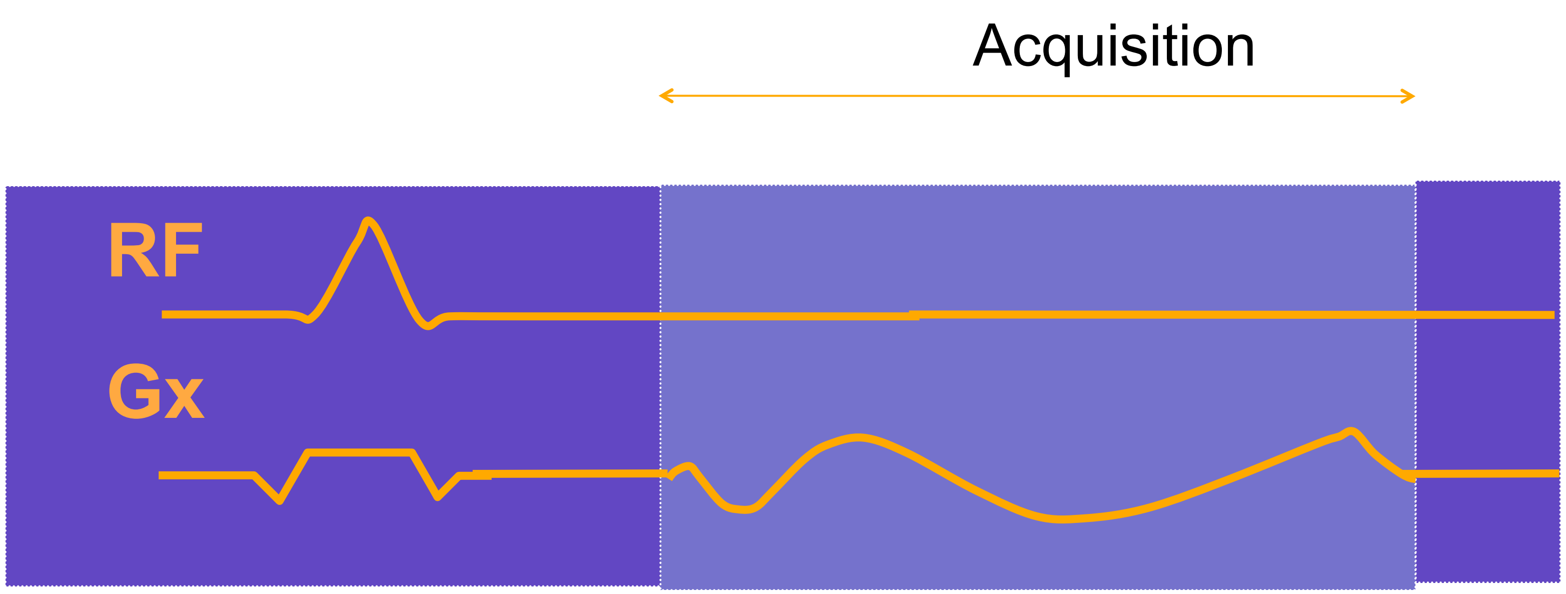
- Duyn method
- Modifications

Duyn Method - Pulse Sequence

- Excite a thin slice (position x) along the *same* axis you are measuring
- $d\phi/dt = \gamma\, G(t)\, x$

$$\phi = \gamma \int_0^t G(t) x dt = 2\pi k_x(t) x$$

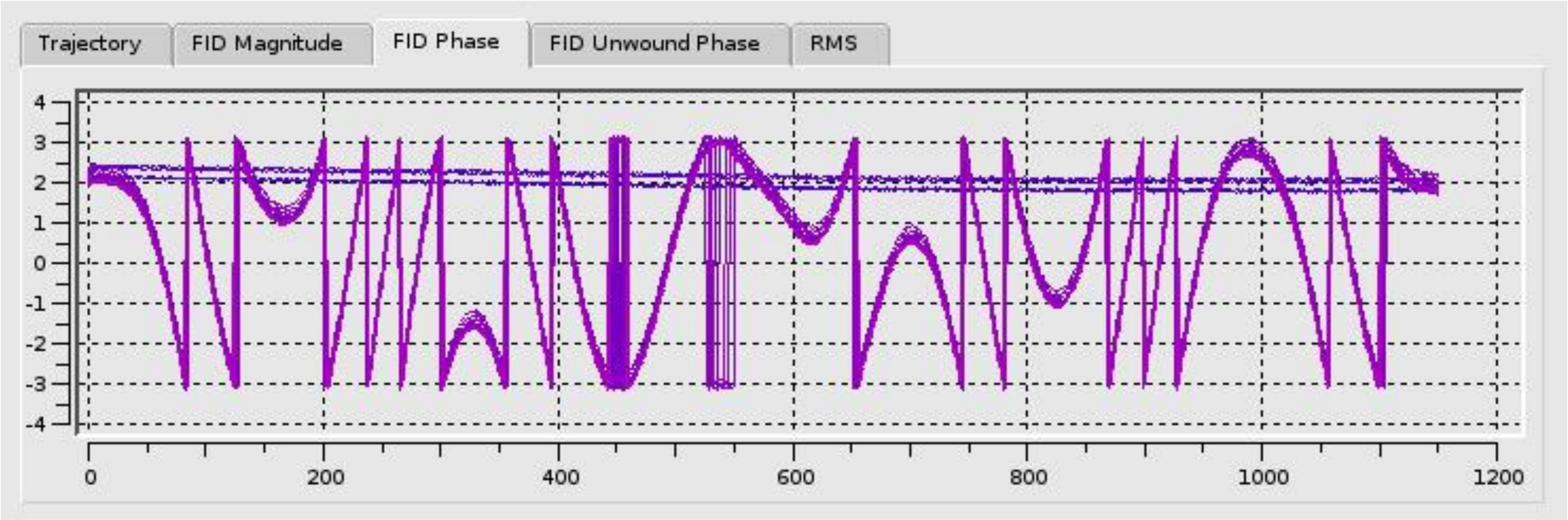
- Can measure baseline without $G(t)$ or with $-G(t)$ to help correct off-resonance



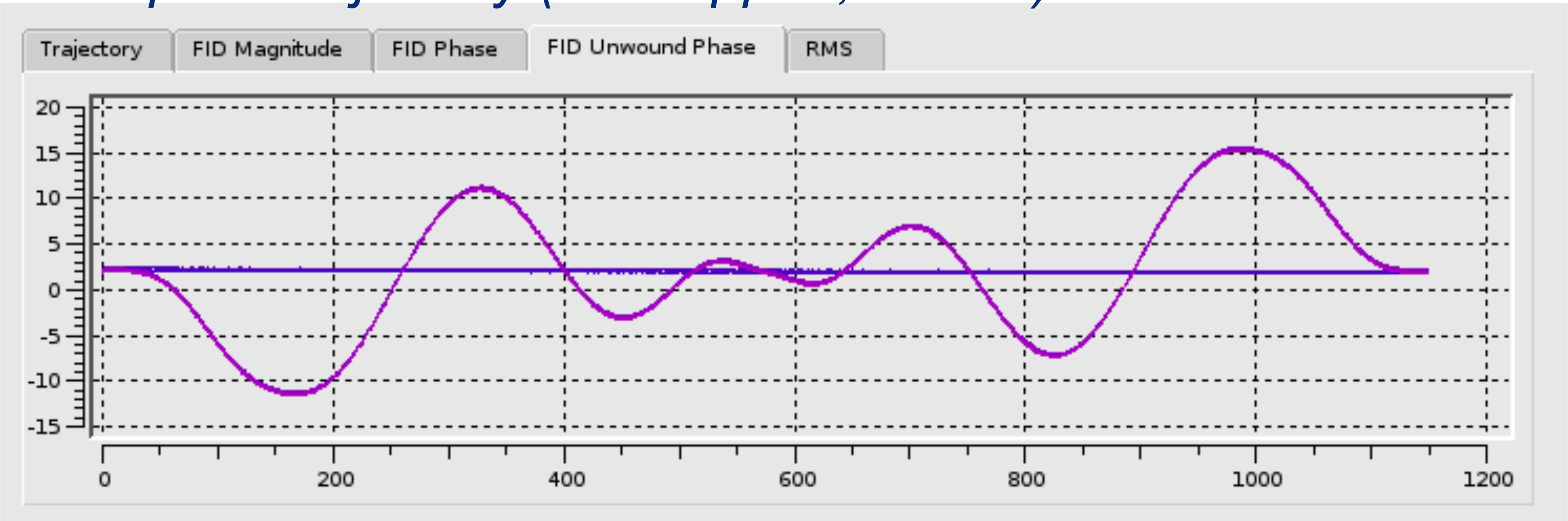
(Courtesy Paul Gurney)

Gradient Measurement Example

Signal Phase



k-space trajectory (unwrapped, scaled)



(Courtesy Paul Gurney)

Gradient Measurement

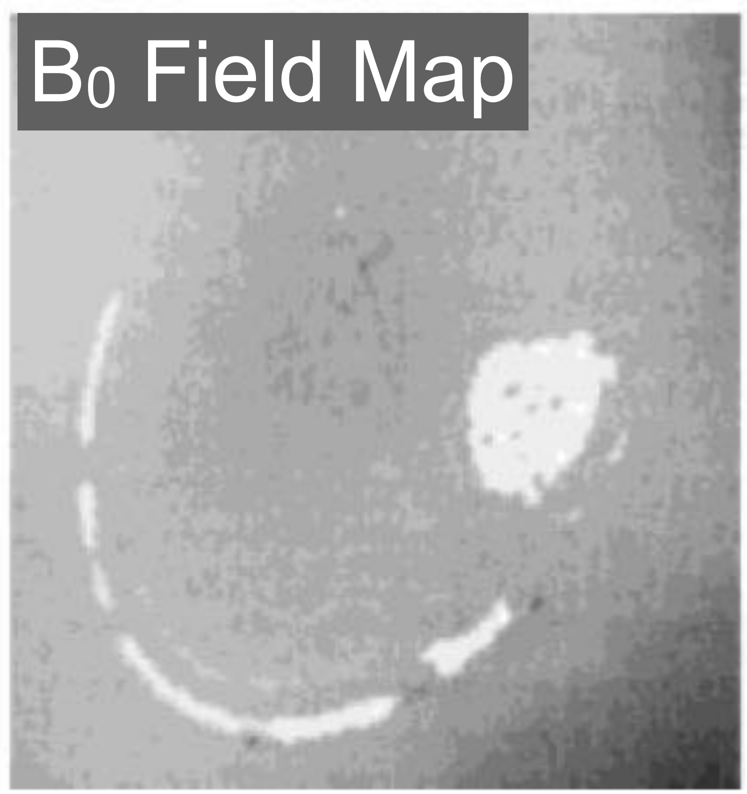
- Can play opposite gradients, excite opposite slices
- Separate different effects:
  - off-resonance (independent of gradient)
  - eddy currents (linear with gradient G)
  - concomitant gradient terms (G^2)

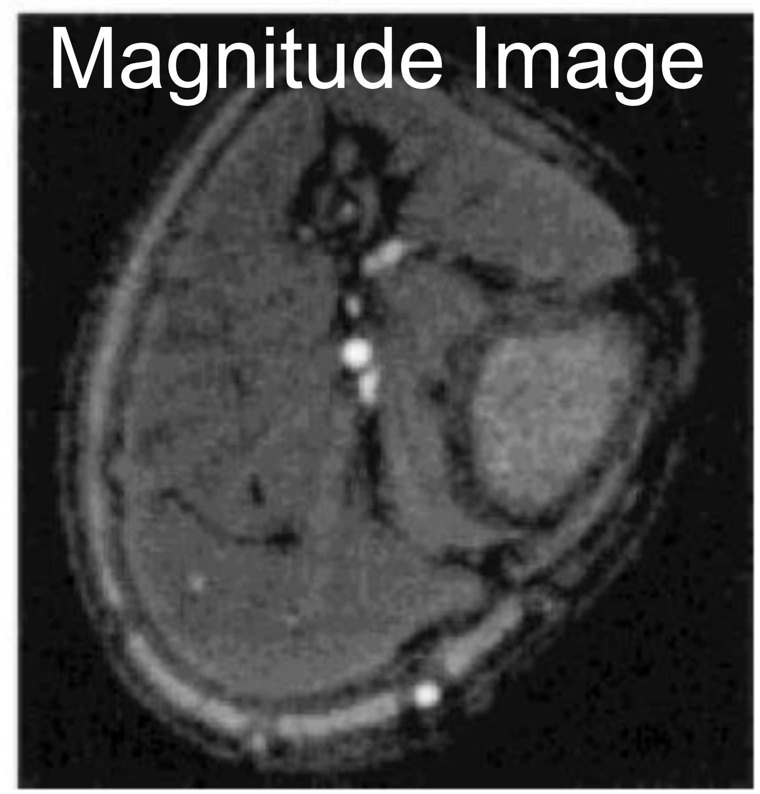
B0 Mapping

- Simple multi-echo sequences
- Fat/water in-phase
- IDEAL/Dixon built in

B_0 Mapping

- Signal phase: $\phi(TE) = \phi_0 + 2\pi\, TE\, \Delta f)$
- ϕ_0 includes terms from excitation, coil, other
- Simple dual-echo method: $$\Delta f = \frac{\angle\left[s(TE_2)s^*(TE_1)\right]}{2\pi(TE_2 - TE_1)}$$
- Assumes Δf only due to B_0 variation

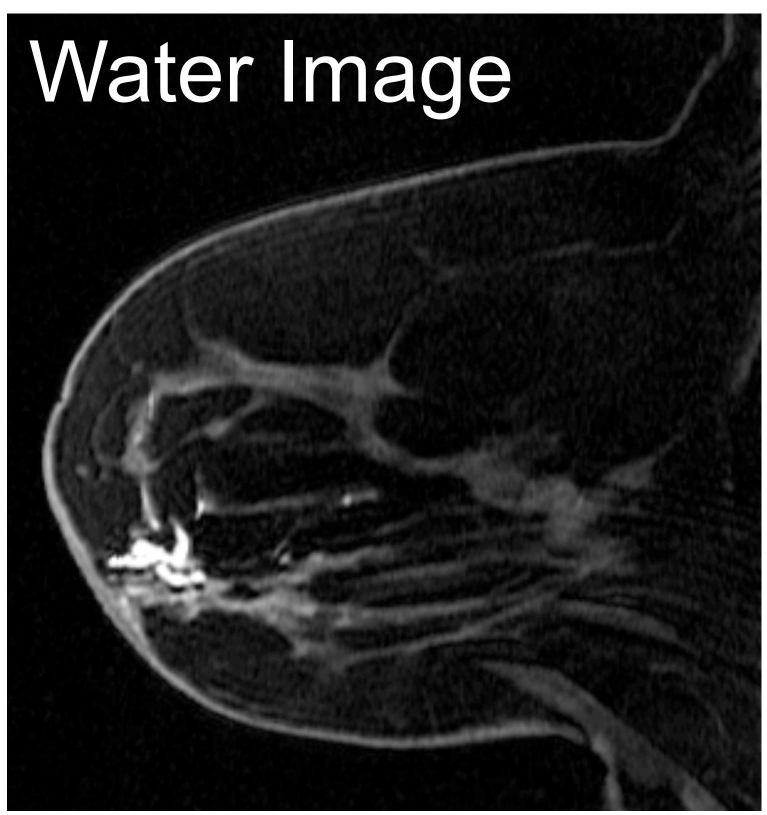


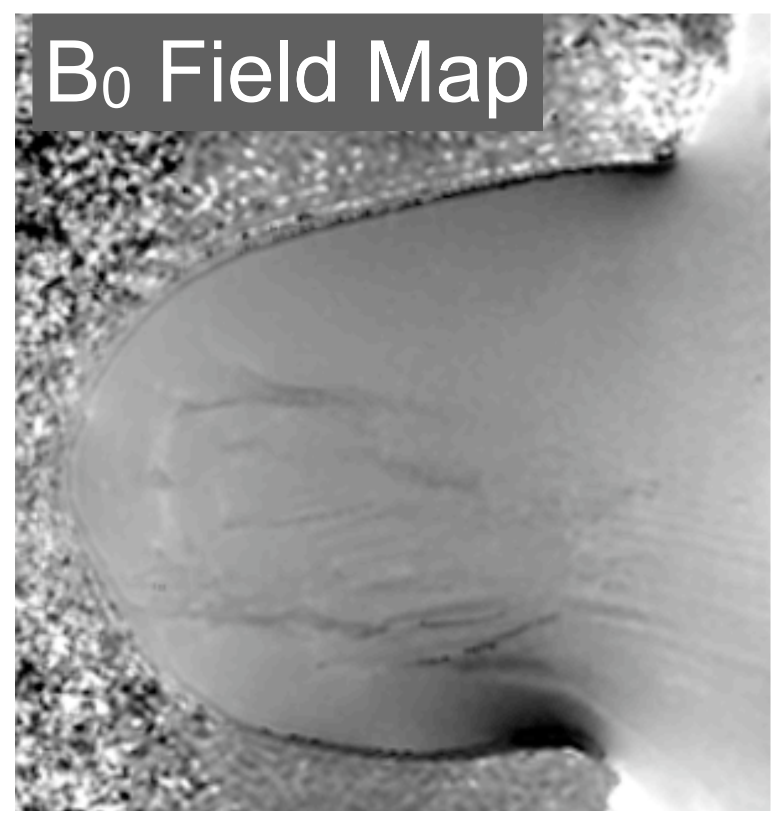


(From Nayak & Nishimura, MRM 2000)

B_0 Mapping (with Fat)

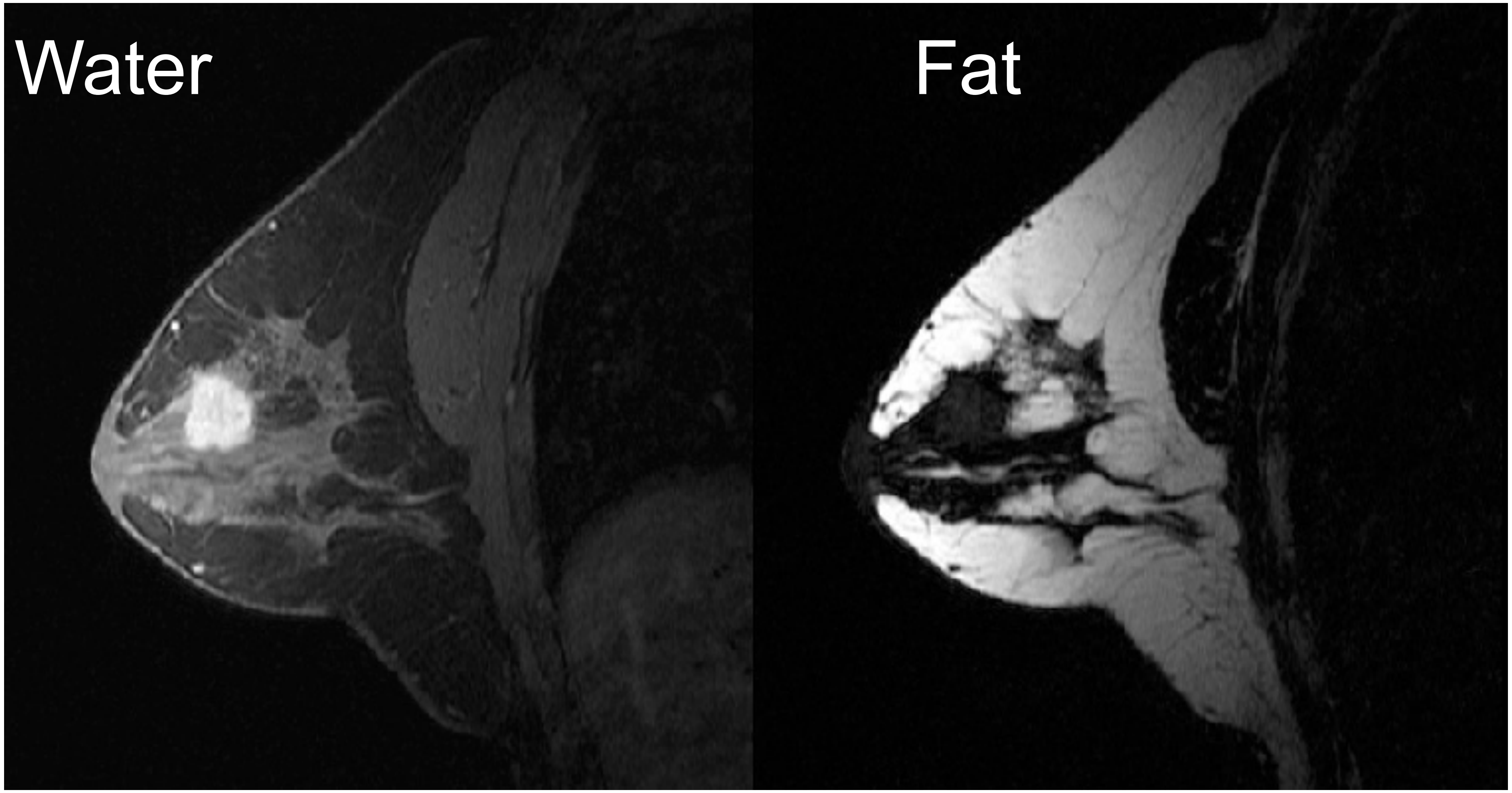
- Signal: $\hat{S}_i = [W + Fe^{2\pi i \Delta\theta_{cs} TE_i}]e^{2\pi i \Delta\psi TE_i}$
- Use “in-phase” TEs
- lgorithms (eg. IDEAL) can fit $W, F\ \Delta\psi$
- B_0 map less sensitive to presence of fat



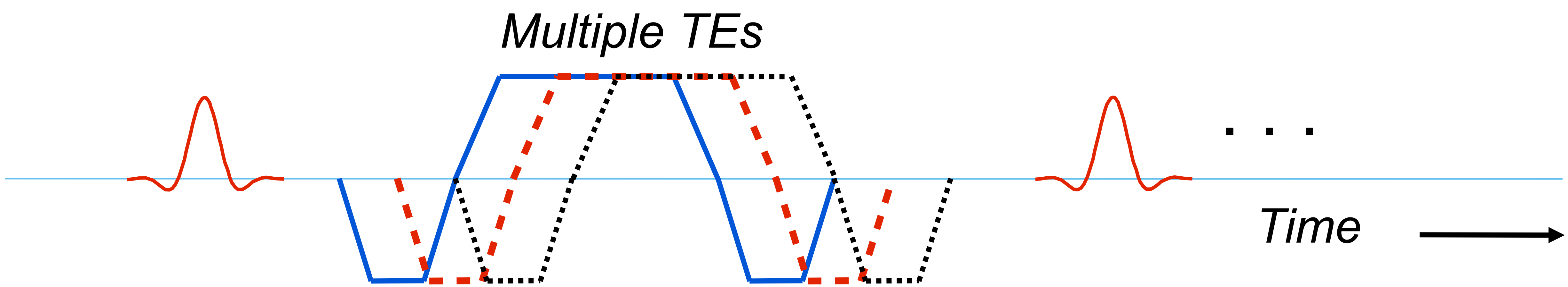


Fat/Water Separation

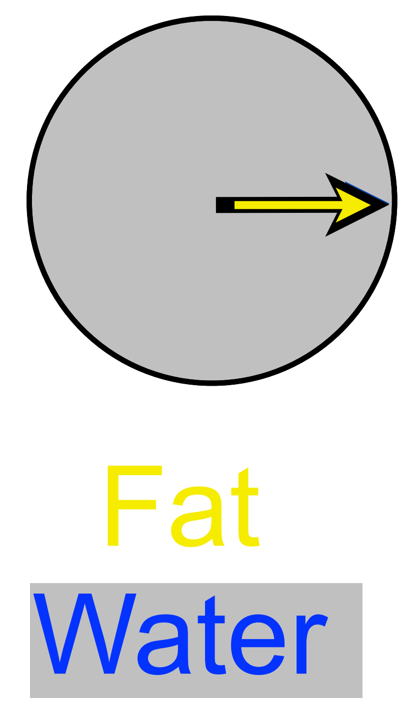
- Concepts: Phase-based separation
- 2-point models
- 3+ point models



Phase-Based Fat/Water Separation

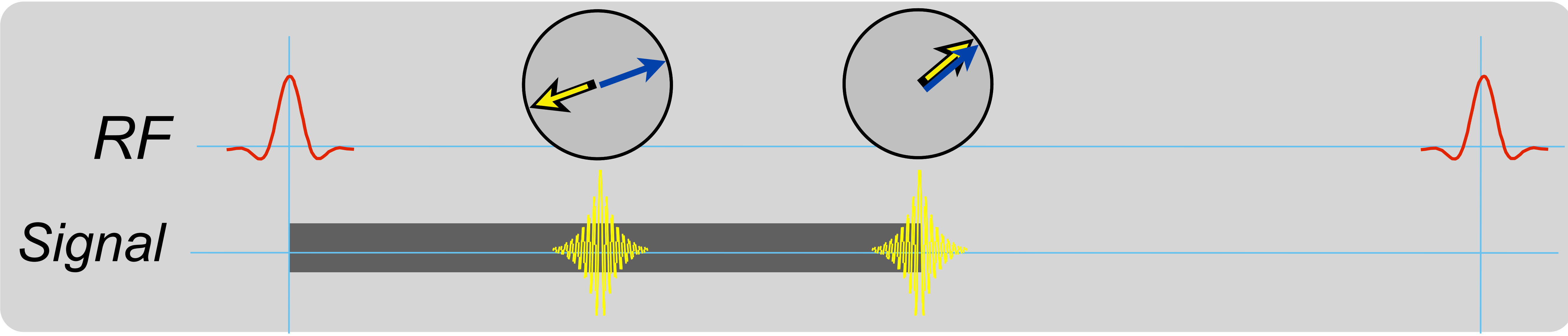


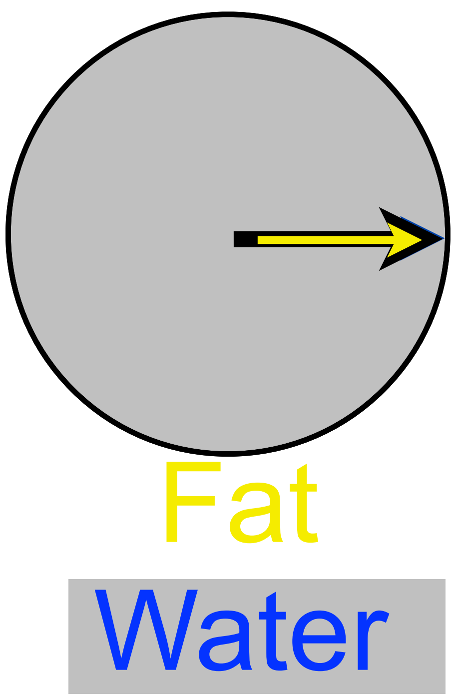
- Estimate phase or field map
- Decompose into water and fat images
- 2-point and 3-point Dixon imaging
- Least-squares separation “IDEAL”

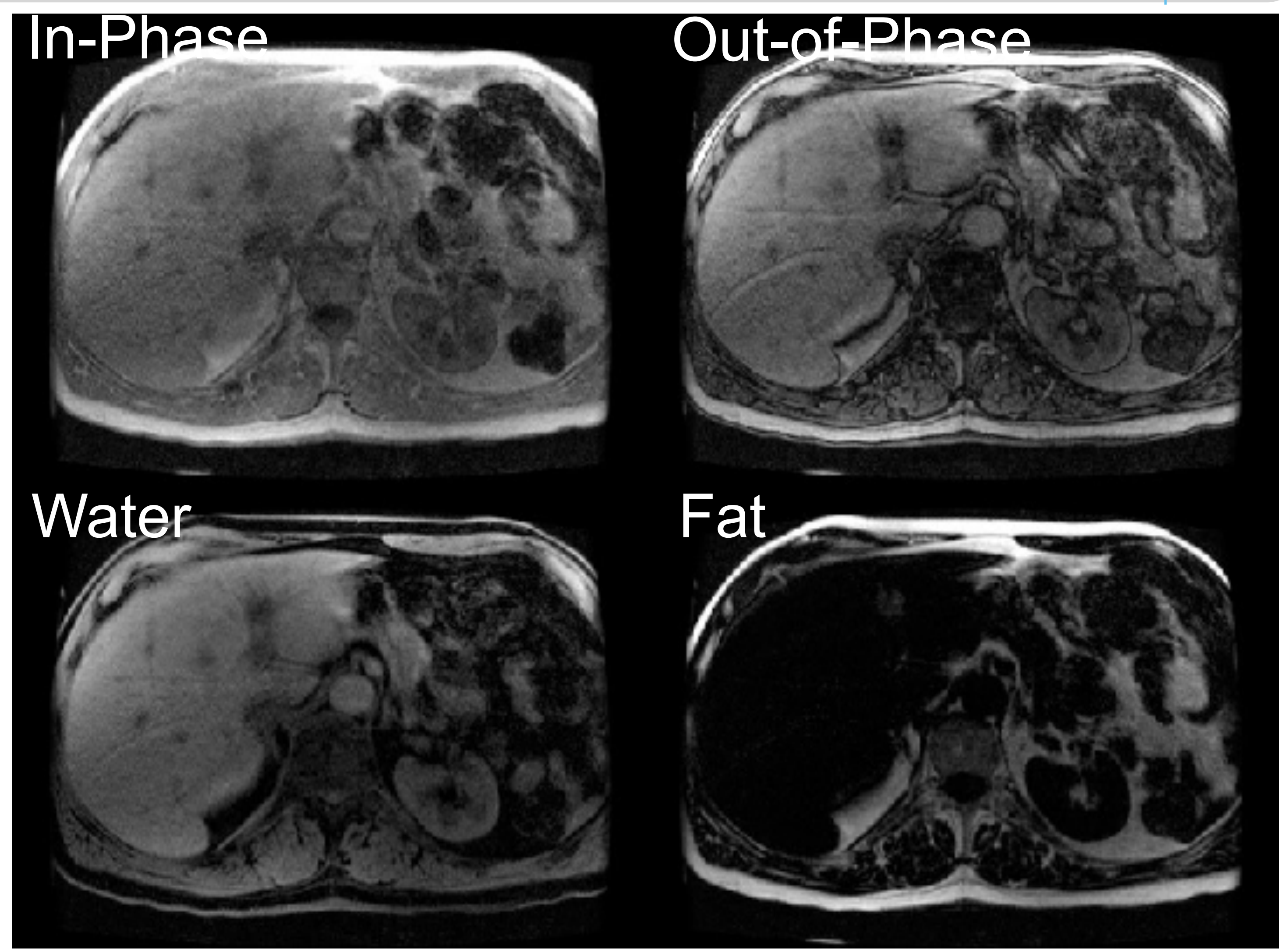


(Dixon 1984, Glover 1991, Xiang 1997, Reeder 2003, 2004)

2pt Dixon-Based Imaging

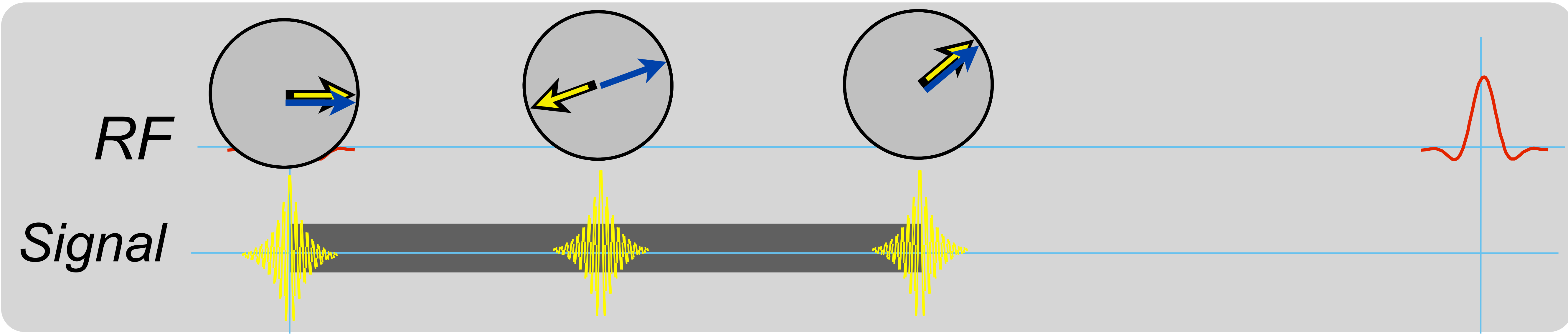


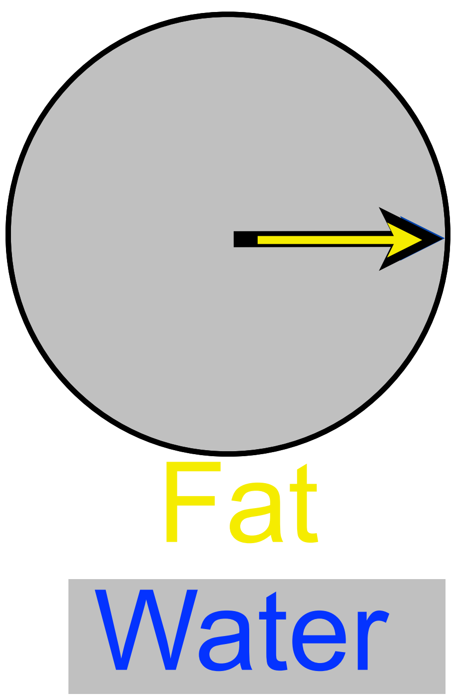




(Dixon 1986, Ma 2002, 2004)

3pt Dixon-Based Imaging





- TE = [0, 0.5, 1] / $\Delta\theta cs$ (in-, out-, in-phase)
  - $S_0 = W+F$
  - $S_1 = (W-F)\, e^{-\pi\, \psi/\Delta\theta cs}$
  - $S_2 = (W+F)\, e^{-2\pi\, \psi/\Delta\theta cs}$
- Estimate ψ from S_2/S_0
- Calculate W and F

(Glover 1990)

Least-Squares Fat/Water Separation

- Fat/Water model: $\Delta\theta cs = 440Hz$ (3T), $\Delta\psi$ = B_0 freq

$$\widehat{S}_i = [W + Fe^{2\pi i \Delta\theta_{cs} TE_i}]e^{2\pi i \Delta\psi TE_i}$$

- Acquire $S_i = S(TE_i)$ *for* $i=1,2,3$ *(or more)*

- Residual:

$$R = \sum_{i=1}^{N} \left| S_i - \widehat{S}_i \right|^2$$

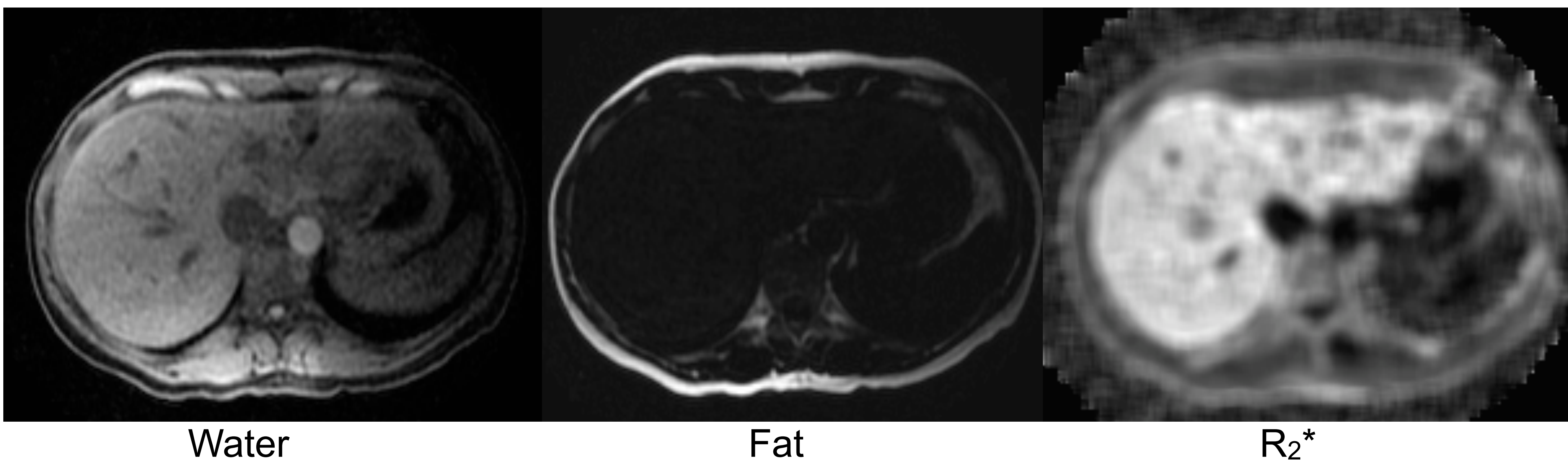
- Find $W, F, \Delta\psi$ to minimize residual

Reeder 2004

IDEAL R_2^* (Further Steps)

- Can fit water, fat, R_2^* ($1/T_2^*$)
- Can multiply W,F by *multipeak* model - better fits and fewer spurious solutions

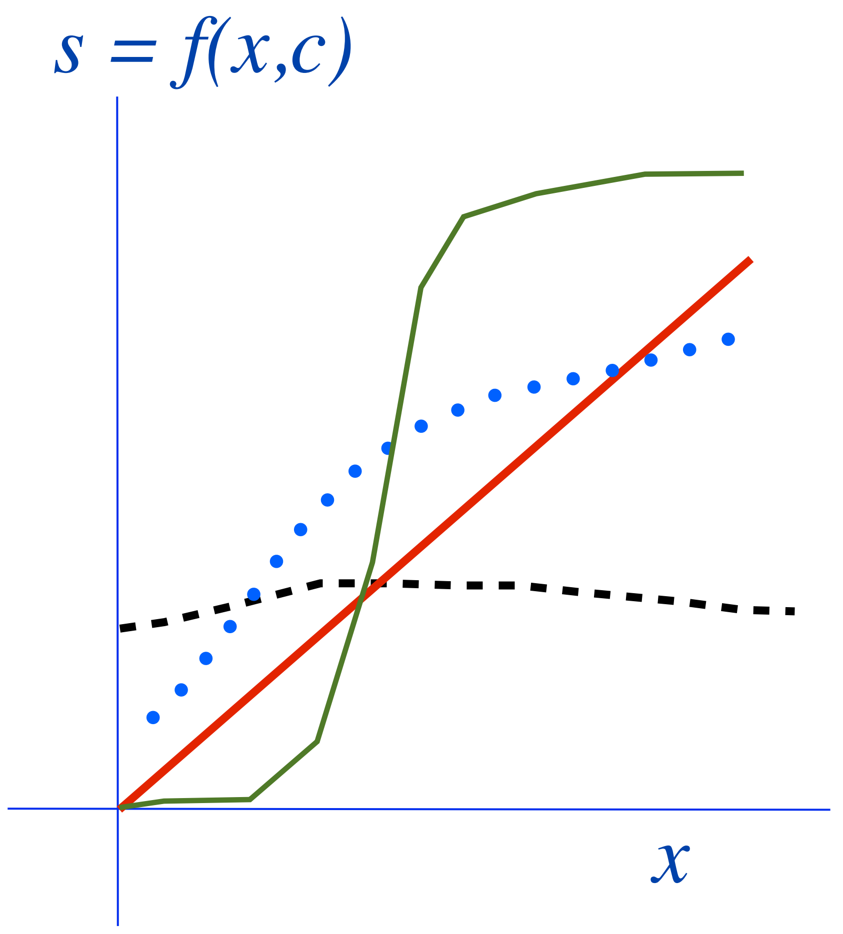
$$S_f(TE) = F \sum_{j=1}^{N} \alpha_j e^{2\pi i \Delta\theta_j TE}$$



Yu 2006, Courtesy Scott Reeder

Mapping Principles

- Generally want to measure some parameter x
- Measurement $s = f(x, c)$
- Invert function *analytically* or by *curve-fit*
- Desire:
  - High sensitivity (ds/dx)
  - Low sensitivity to confounders (c): *ignore, remove or correct*
  - Dynamic range of x



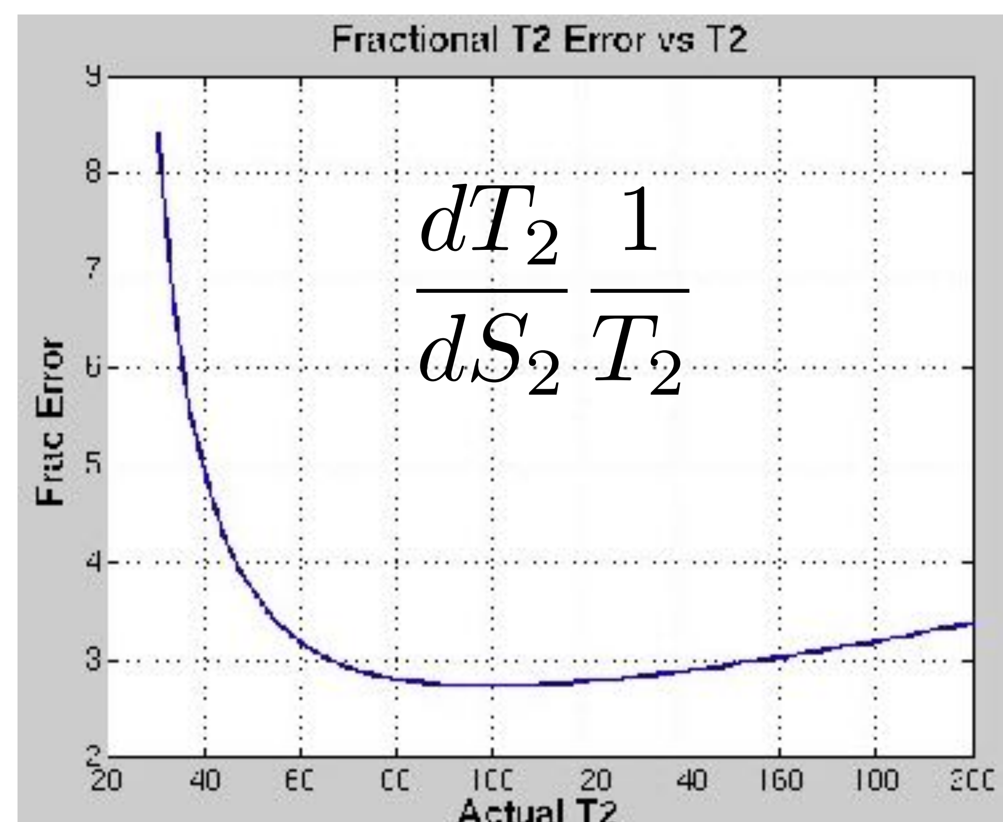
Example: T_2 Fitting (Sensitivity)

- $s_i = s_0 \exp(-TE_i/T_2)$
  - 2 unknowns (s_0 and T_2)
  - Ex: 2 measurements ($i=1,2$)
    - Analytic solution
    - Removed effect of s_0
- "Best" case is TE_1,TE_2=0,T_2

$$T_2 = \frac{TE_1 - TE_2}{\ln S_2 - \ln S_1}$$

$$\frac{dT_2}{dS_2} = \frac{TE_2 - TE_1}{S_2(\ln S_2 - \ln S_1)^2}$$

$$\frac{dT_2}{dS_2}\frac{S_2}{T_2} = \frac{T_2}{(TE_2 - TE_1)}$$



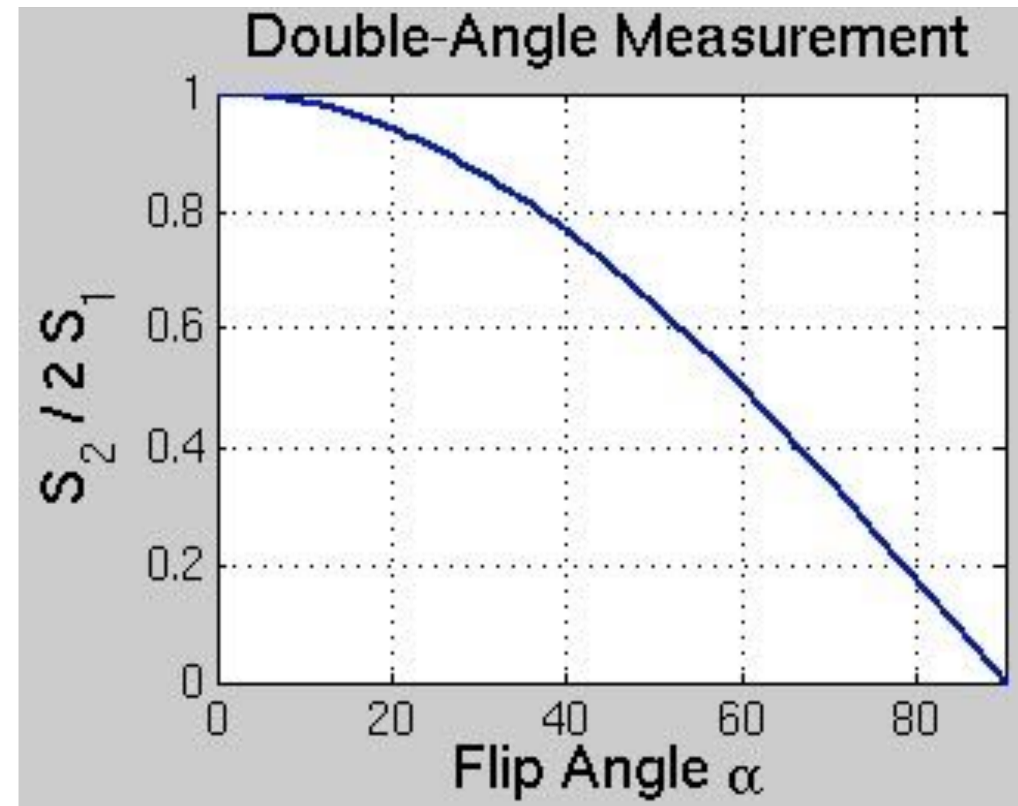
$$\frac{dT_2}{dS_2}\frac{1}{T_2}$$

B1 Mapping

- Double-angle method, SDAM (Insko 1993)
- Stimulated Echo
- AFI (Yarnik 2007)
- Phase-sensitive (Morrell 2008)
- Bloch-Siegert (Sacolick 2010)

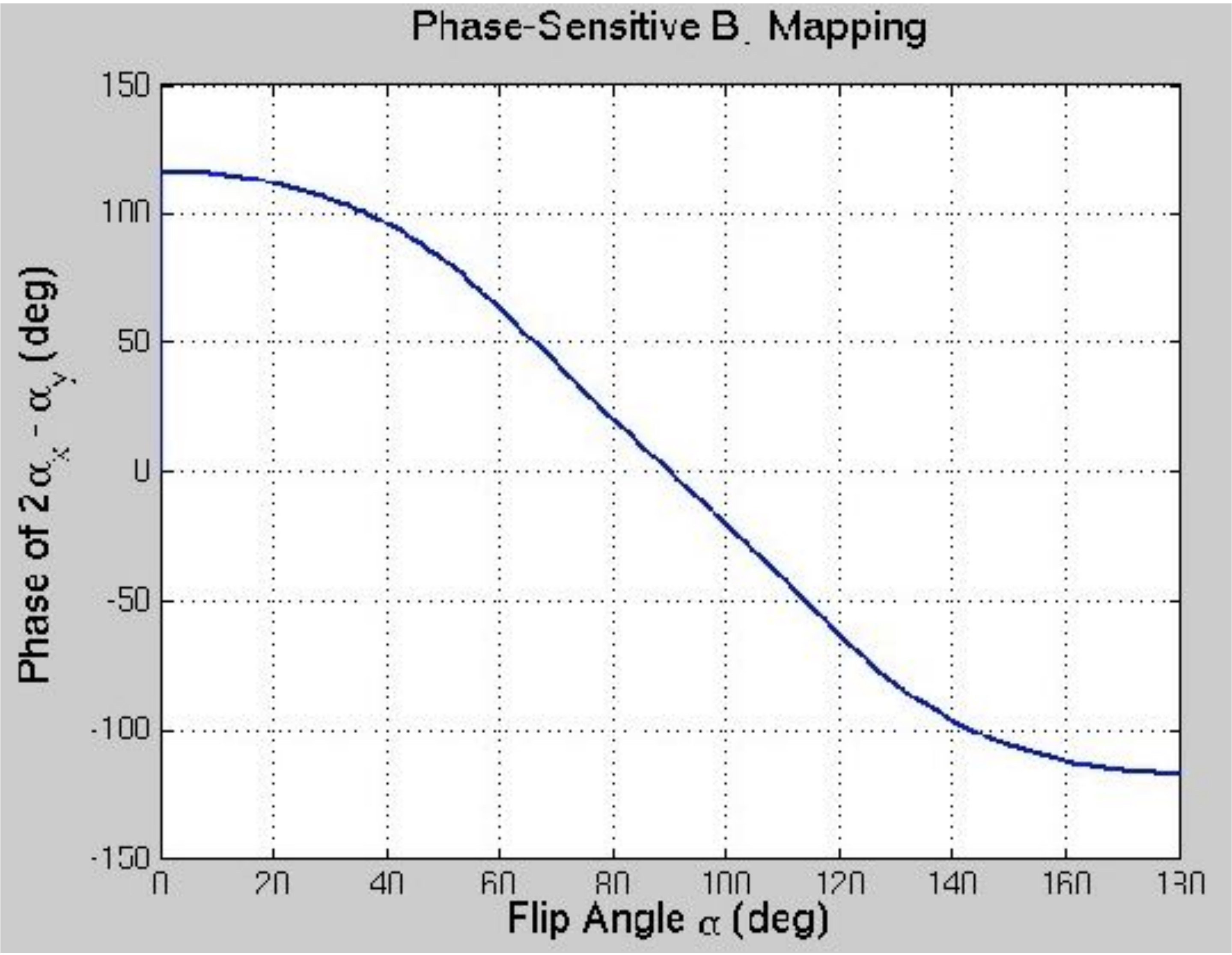
Double-Angle Mapping (DAM)

- Assume complete decay/recovery over TR
- Signal proportional to sin(α)
- Acquire S_1 with flip angle α
- Acquire S_2 with flip angle 2α:
  - $S_1 = A\sin(\alpha)$
  - $S_2 = A\sin(2\alpha) = 2A\cos(\alpha)\sin(\alpha)$
  - $\alpha = \cos^{-1}(S_2/2S_1)$
- Very slow, works better at higher flip angles



Phase-Sensitive B_1 Mapping

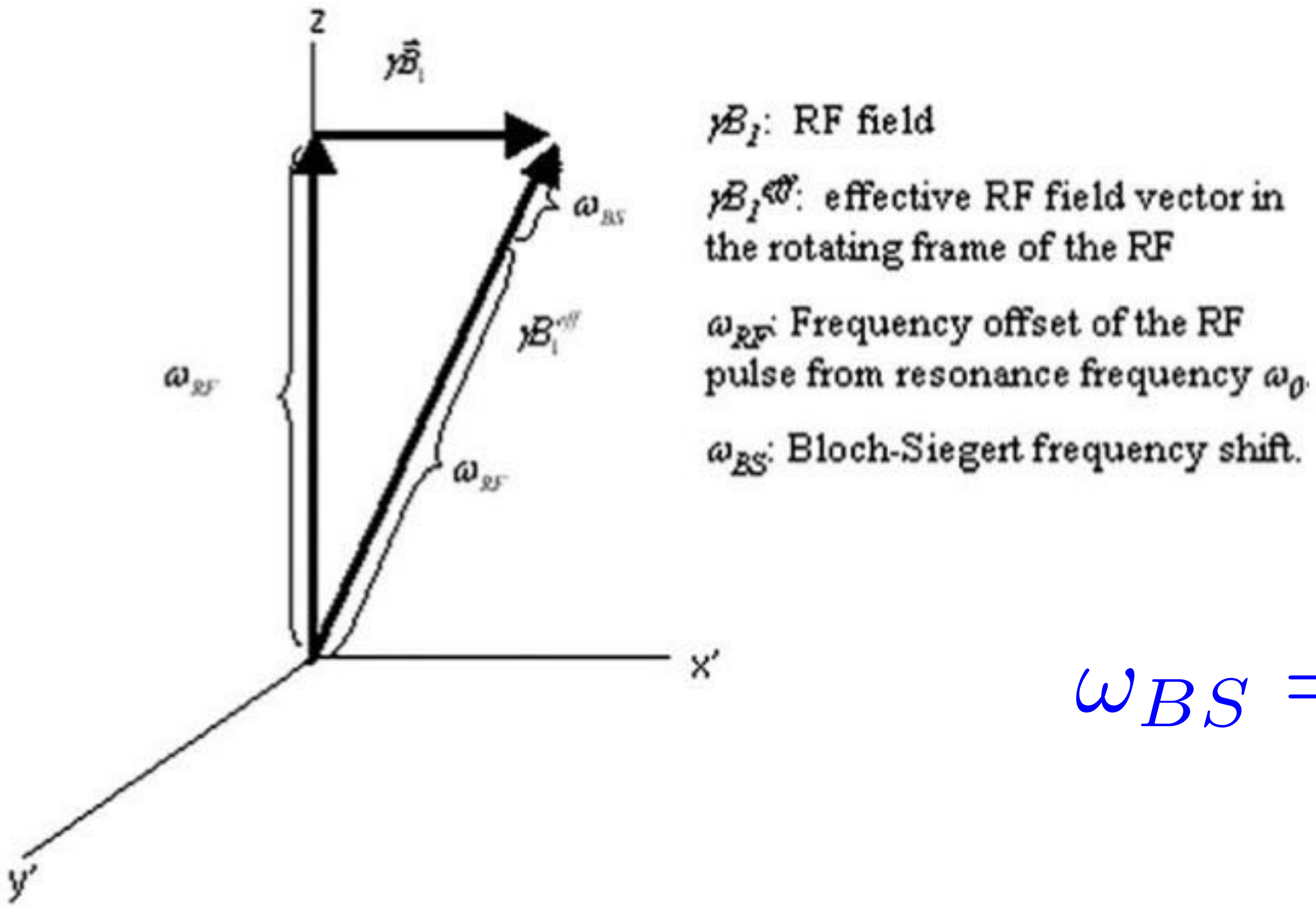
- Play $2\alpha_x$-α_y RF (90º phase change)
- Observe signal phase vs α
- Better range, but still slow



Morrell 2008

Bloch-Siegert Shift Mapping

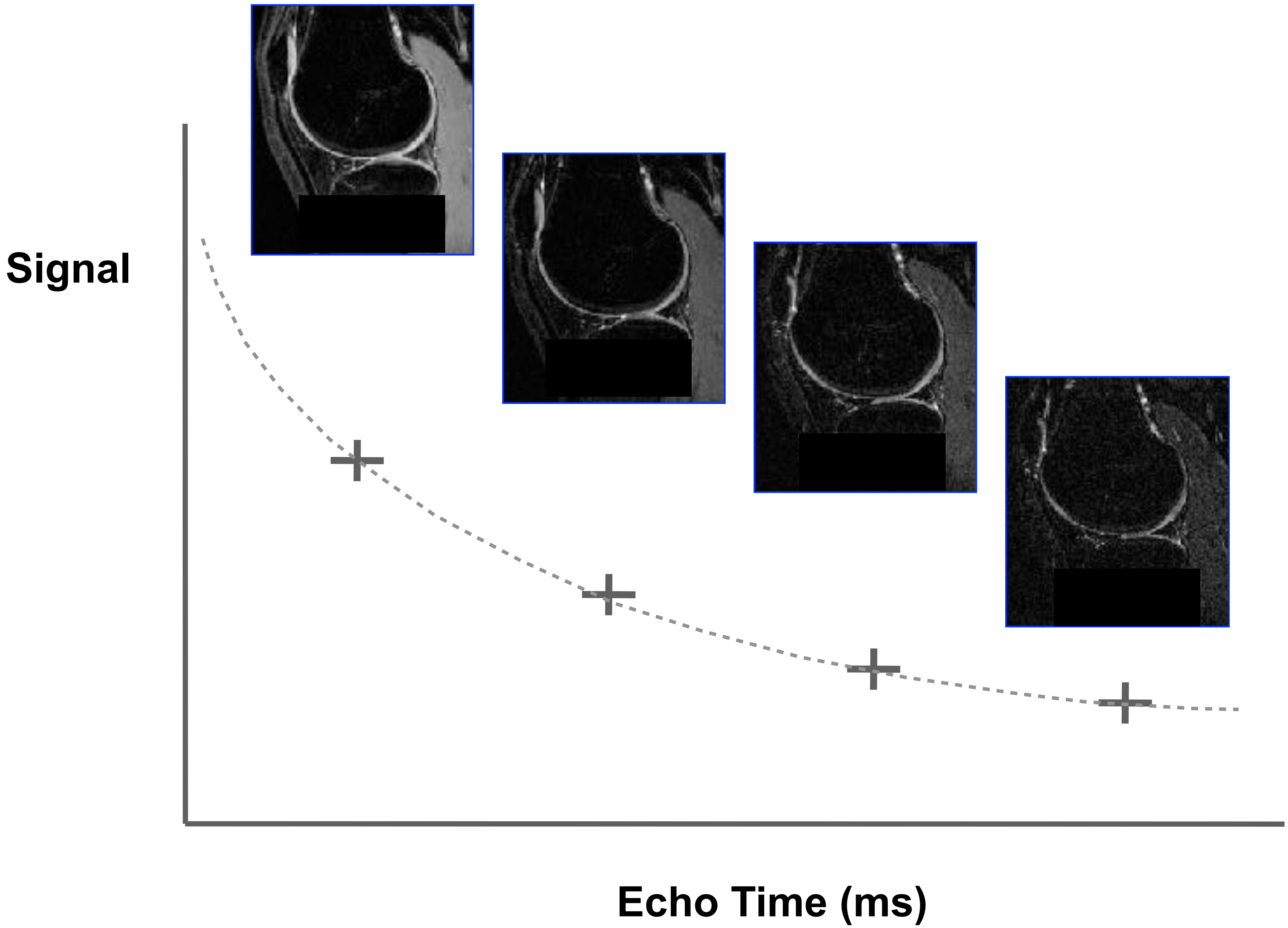
- Play off-resonant pulse after excitation
- Phase is encoded by pulse



$$\omega_{BS} = \frac{(\gamma B_1(t))^2}{2\omega_{RF}(t)}$$

Sacolick 2010

T_2 Mapping

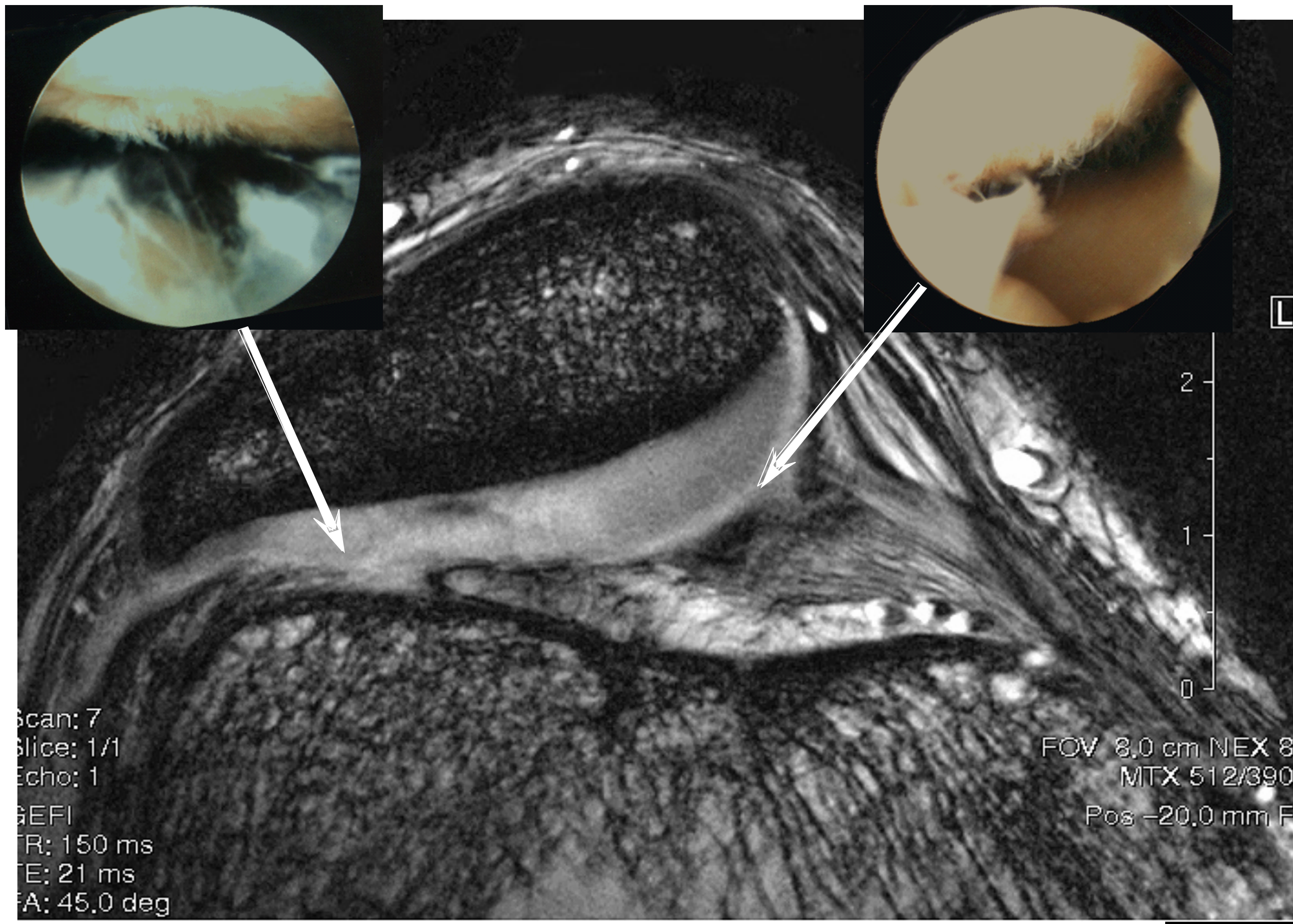


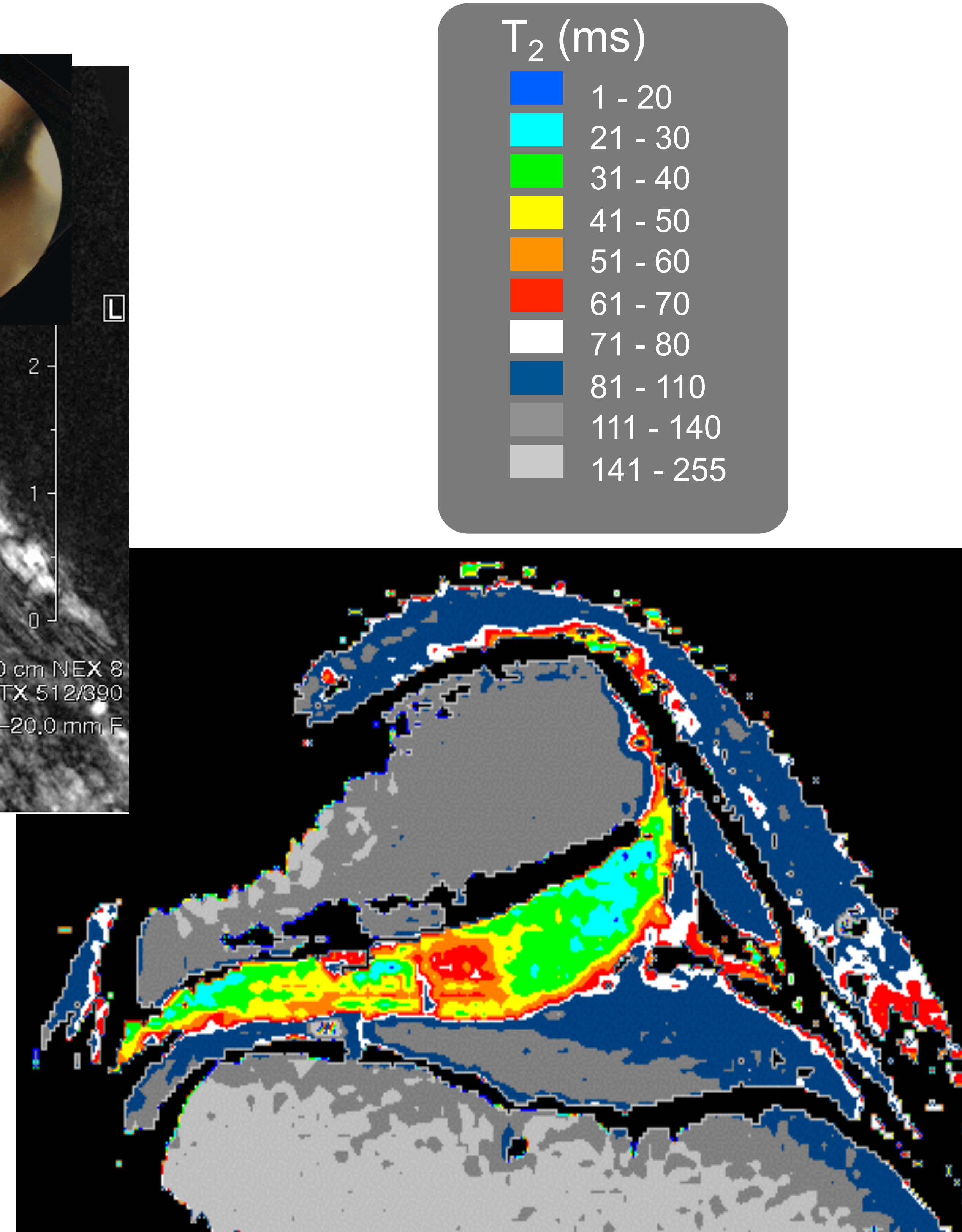
Dardzinski BJ, et al. Radiology, 205: 546-550, 1997.

T2-Mapping Options

- Basic Exponential Decay Model:
  - Single-Echo Spin Echo, repeated at many TEs
  - CPMG with image at each TE
  - T2-preparation with arbitrary readout
  - *Fit by linear regression to* $ln(S_i)$
- Double-echo Steady-State:
  - Simple fit using “effective” echo times *TE* and *2TR-TE*

Example of T_2 Mapping

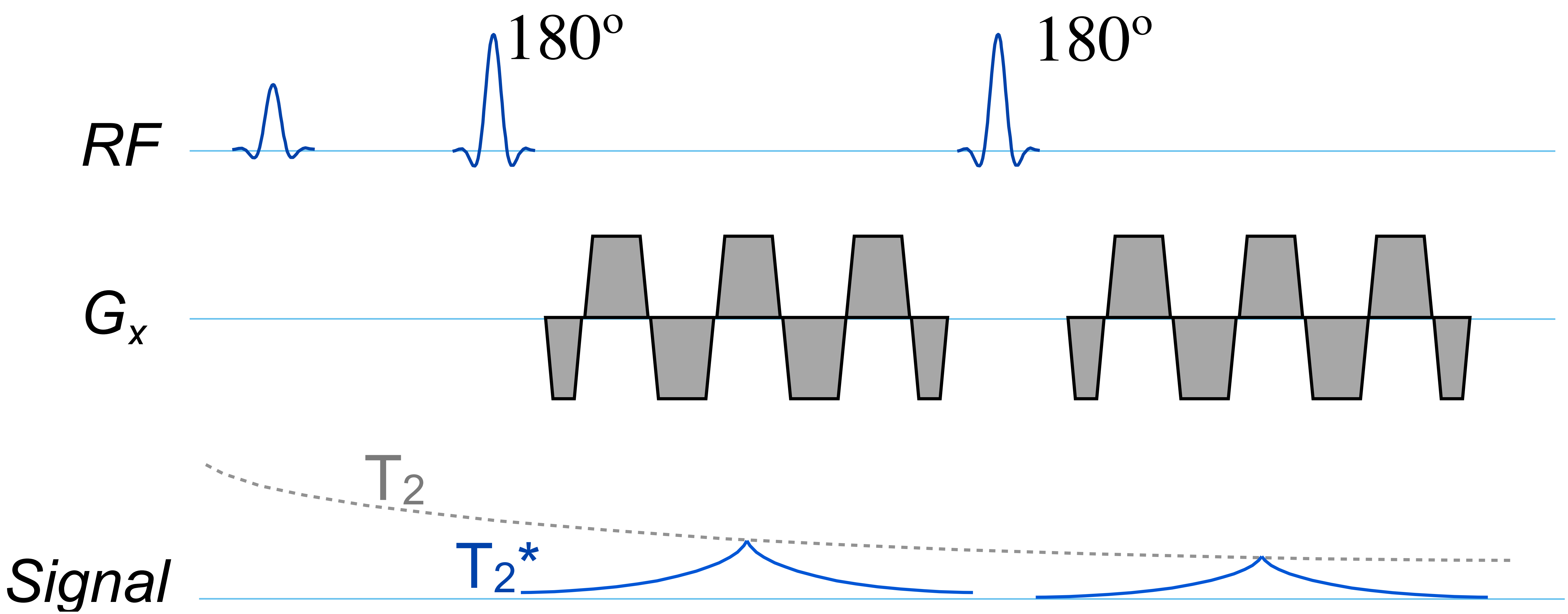




Dardzinski, et al.

T_2 and T_2* Mapping

- Multi-echo spin-echo (T_2) and gradient-echo (T_2*)
- Can combine using GRASE / GESFIDE methods

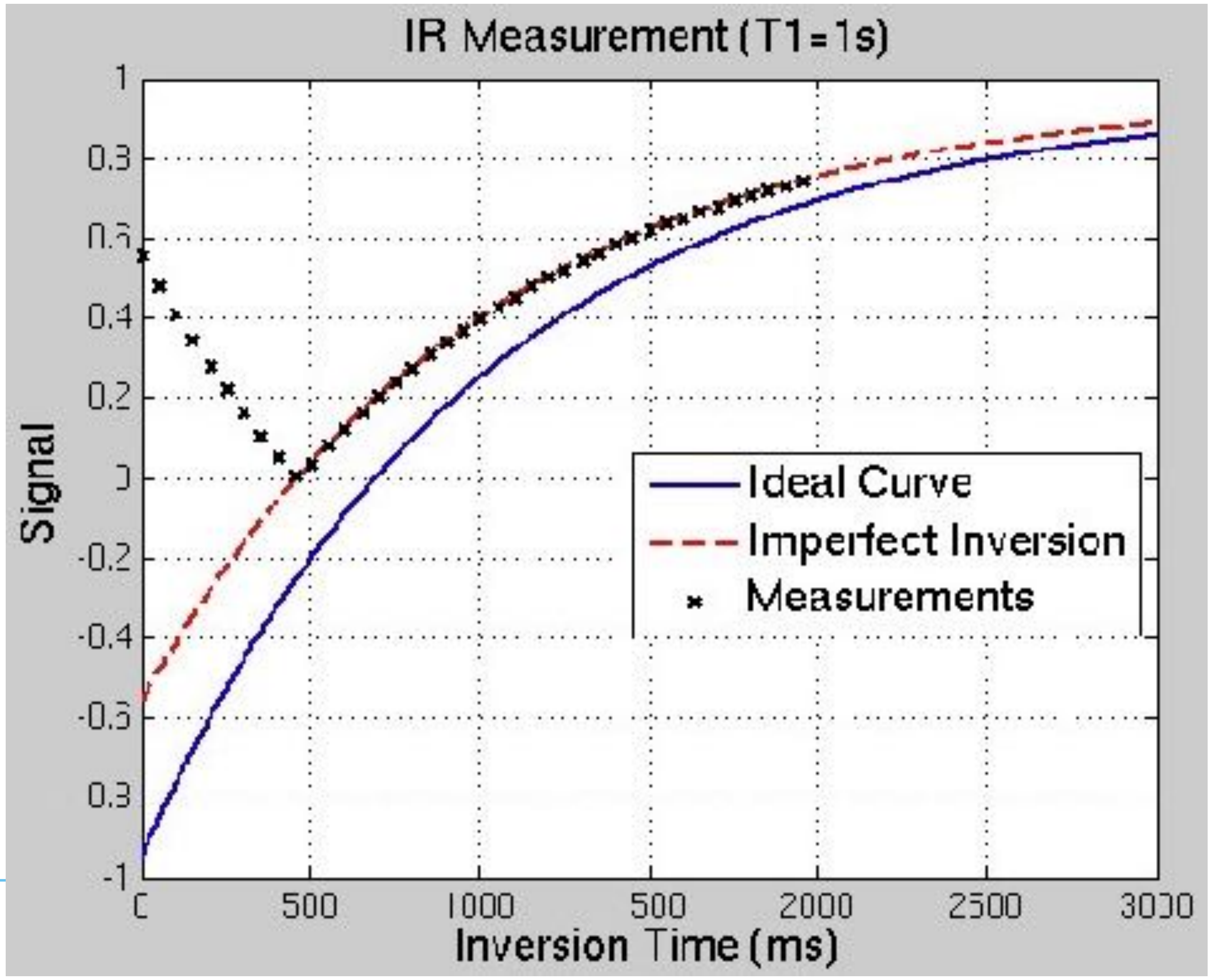


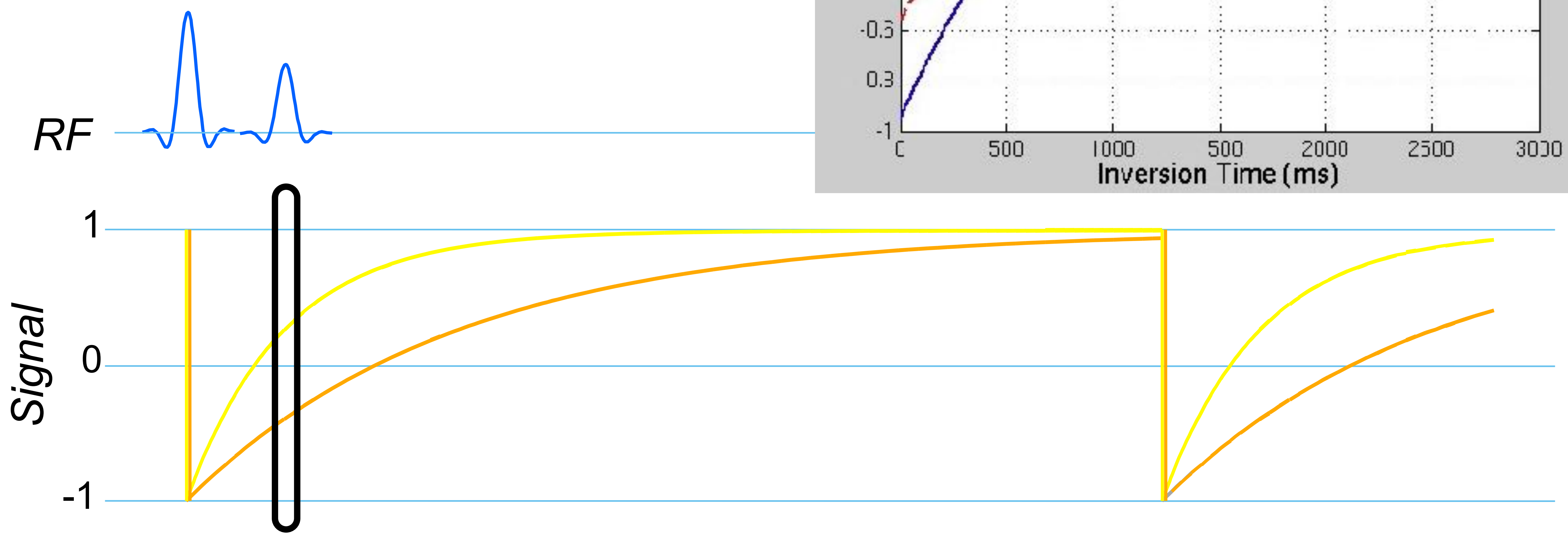
T1 mapping

- IR spin echo
- Saturation-recovery
- Look-locker
- VFA/DESPOT1
- MPnRAGE

IR Spin Echo

- Vary Inversion Time (TI)
- Fit M_0, T_1, Inversion "efficiency"
- Can fit TR in case of incomplete recovery
- Slow!



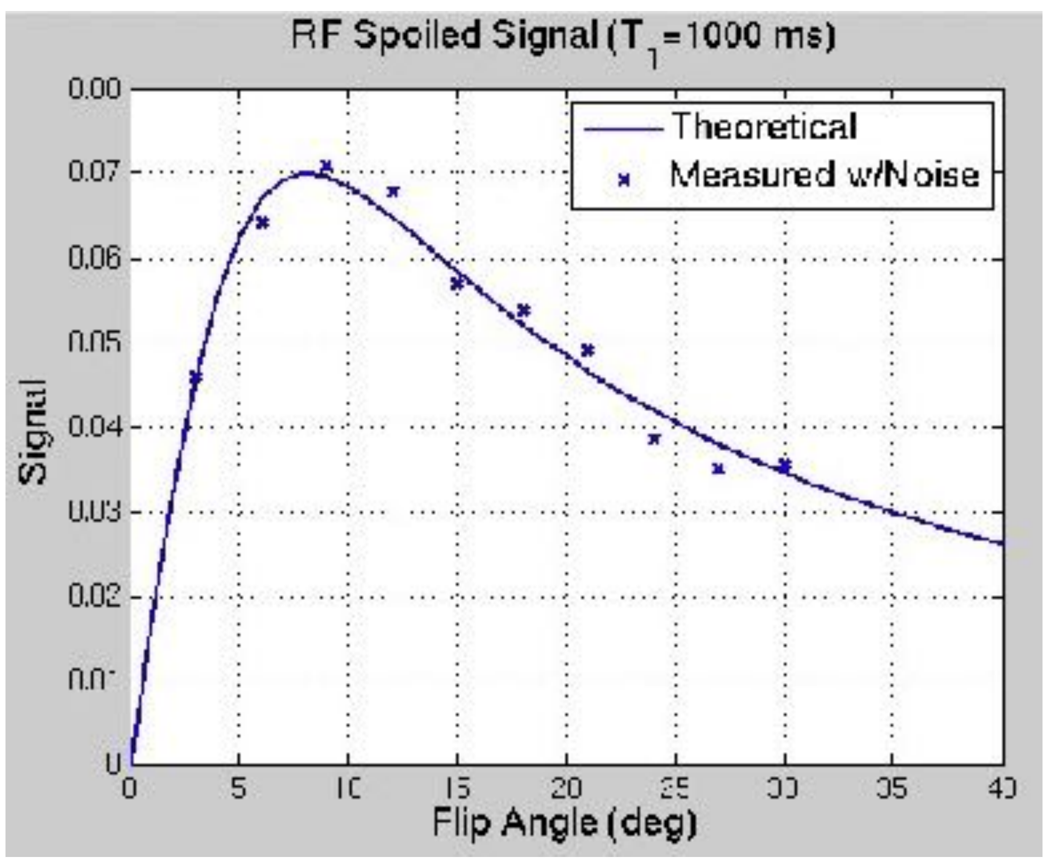


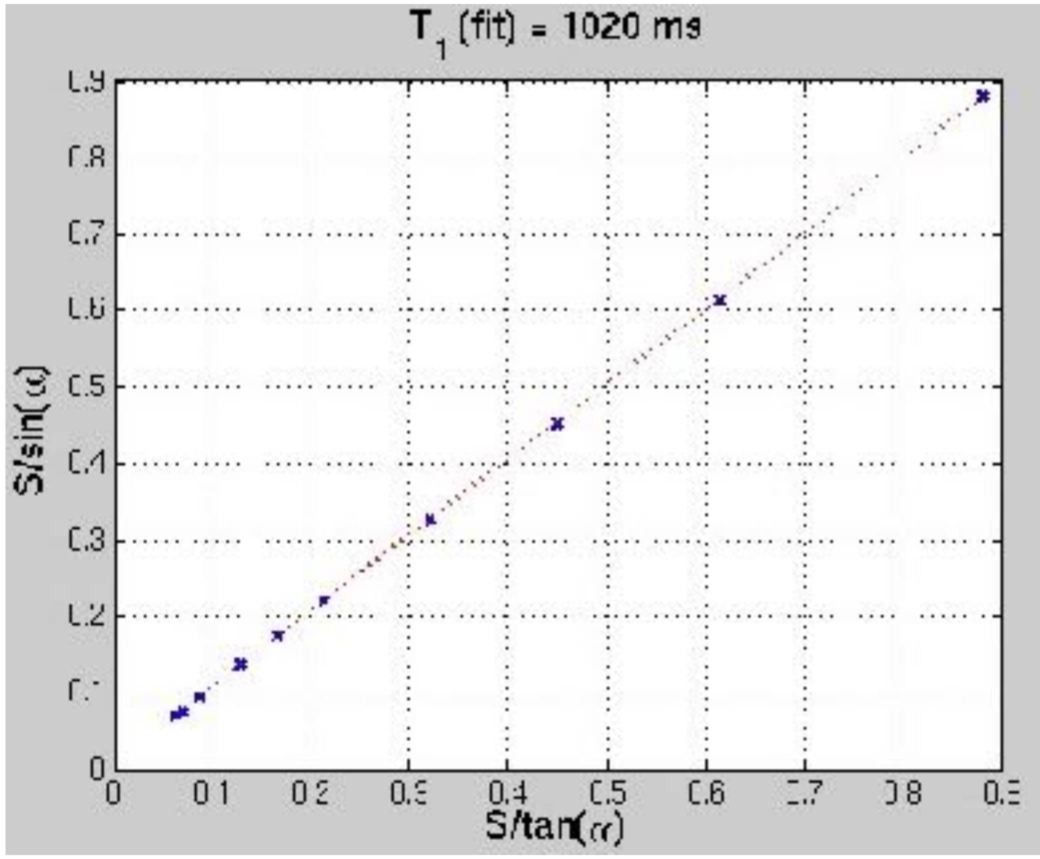
VFA / DESPOT1

- Variable Flip-Angle RF-spoiled
- Measure both sides of Ernst peak
- Linearize, regress
- Slope = E_1
- Sensitive to B_1, and flip angles depend on T_1

$$S = M_0 \frac{(1 - E_1)\sin\alpha}{1 - E_1 \cos\alpha}$$

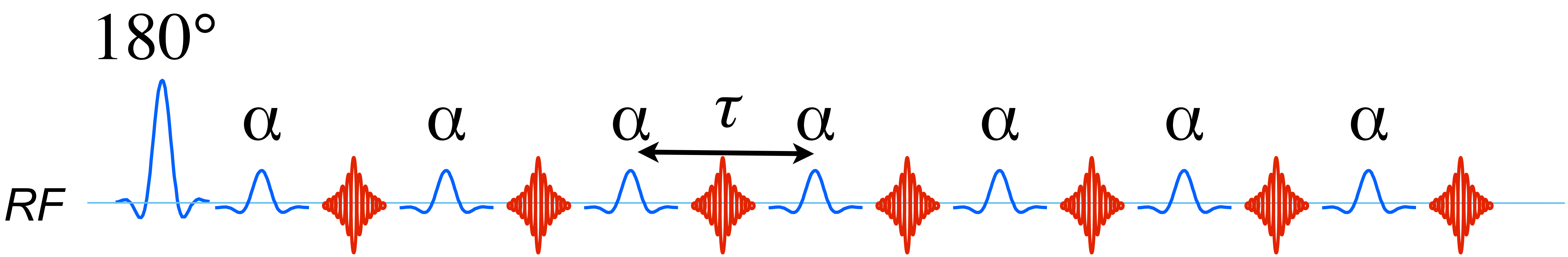
$$\frac{S}{\sin\alpha} = E_1 \frac{S}{\tan\alpha} + M_0(1 - E_1)$$





Look-Locker

- IR sequence, with repeated, small tips instead of 90º
- Much faster than true IR T_1 mapping
- Must correct for signal change ~ $T_{1,eff}$



$$\frac{1}{T_{1,eff}} = \frac{1}{T_1} - \frac{ln(\cos\alpha)}{\tau}$$

Karlsson MRI 2000

Summary: Quantitative/Mapping Methods

- Gradient Measurement
- Fat/Water Separation
- B_0 and B_1 mapping
- T_1, T_2 and T_2* mapping

Course Review

- Midterm Review:
  - EE369B Concepts
  - Simulations with Bloch Matrices, EPG
  - SNR

Bloch/Matrix Simulations

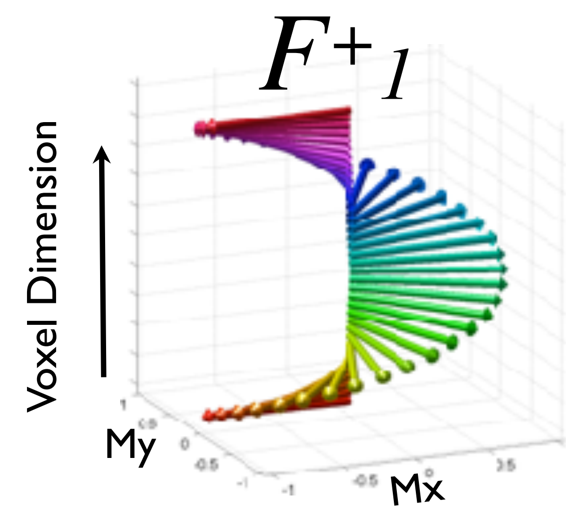
- $M = [M_x \; M_y \; M_z]^T$
- RF and precession ~ 3x3 rotation matrices
- Relaxation ~ 3x3 diagonal multiplication + M_z recovery
- Propagation of A/B: Pre-multiply both by A, add new B
- Steady states: $M_{ss} = AM_{ss} + B \quad = (I-A)^{-1}B$

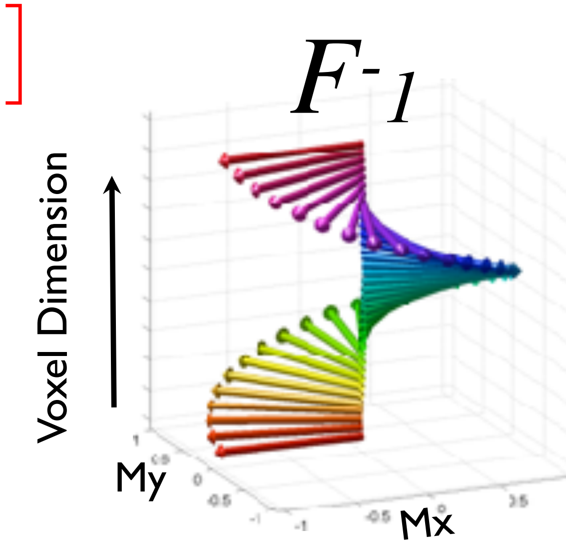
EPG Basis: Mathematically

- Transverse basis functions (F_n) are just phase twists:

$$M_{xy}(z) = \sum_{n=-\infty}^{\infty} F_n^+ e^{2\pi i n z}$$

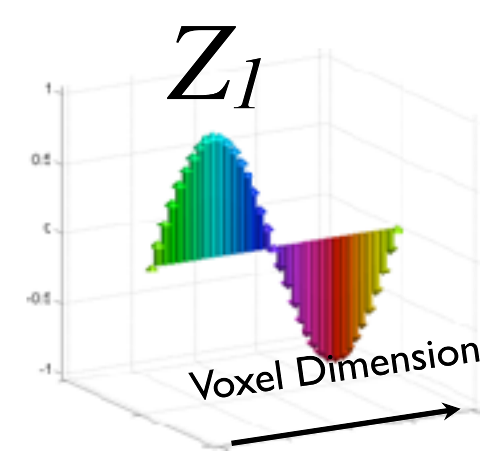
$$M_{xy}(z) = F_0^+ + \sum_{n=1}^{\infty} \left[F_n^+ e^{2\pi i n z} + (F_n^-)^* e^{-2\pi i n z} \right]$$





- Longitudinal basis functions (Z_n) are sinusoids:

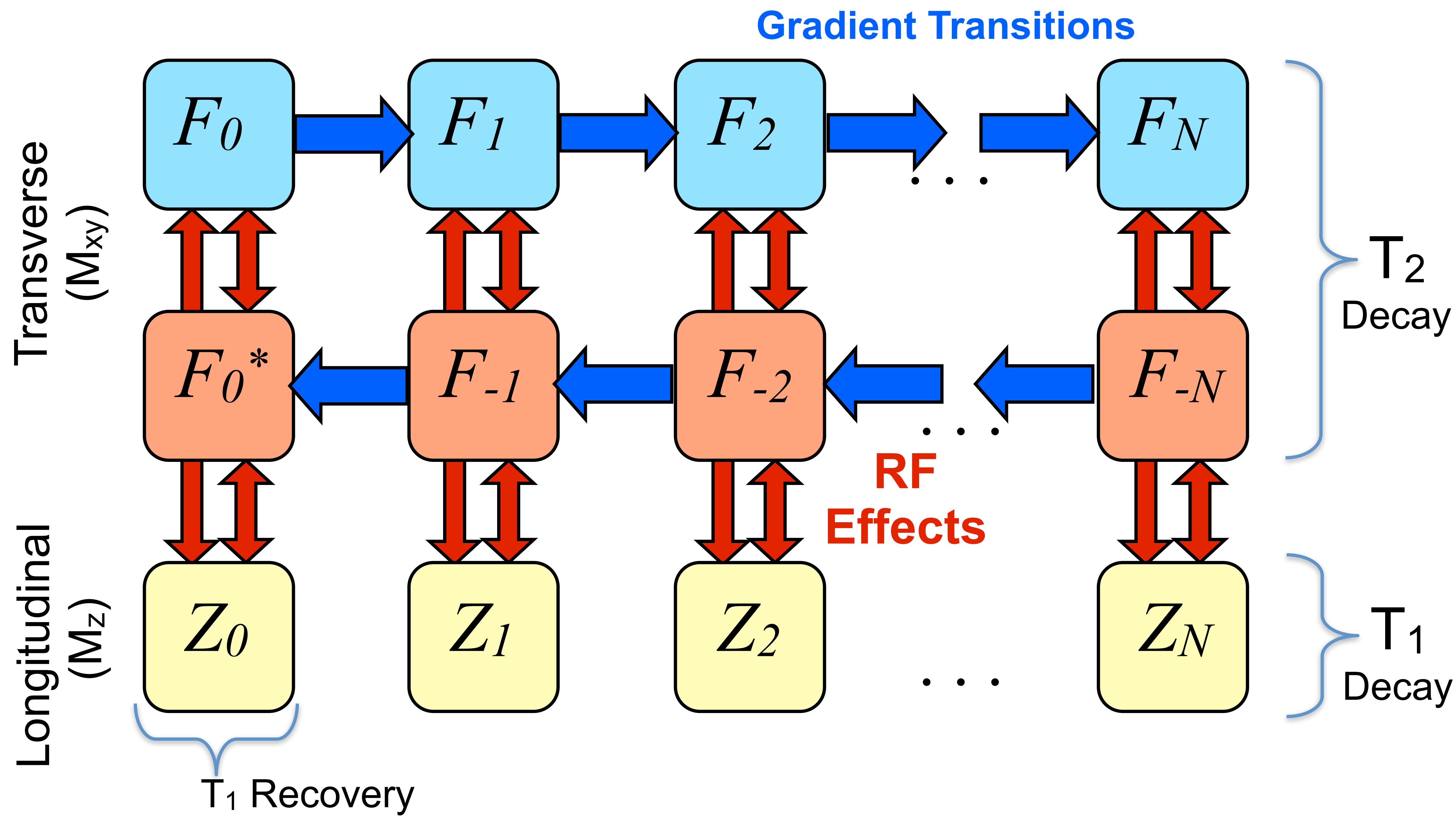
$$M_z(z) = Real \left\{ Z_0 + 2 \sum_{n=1}^{N} Z_n e^{2\pi i n z} \right\}$$



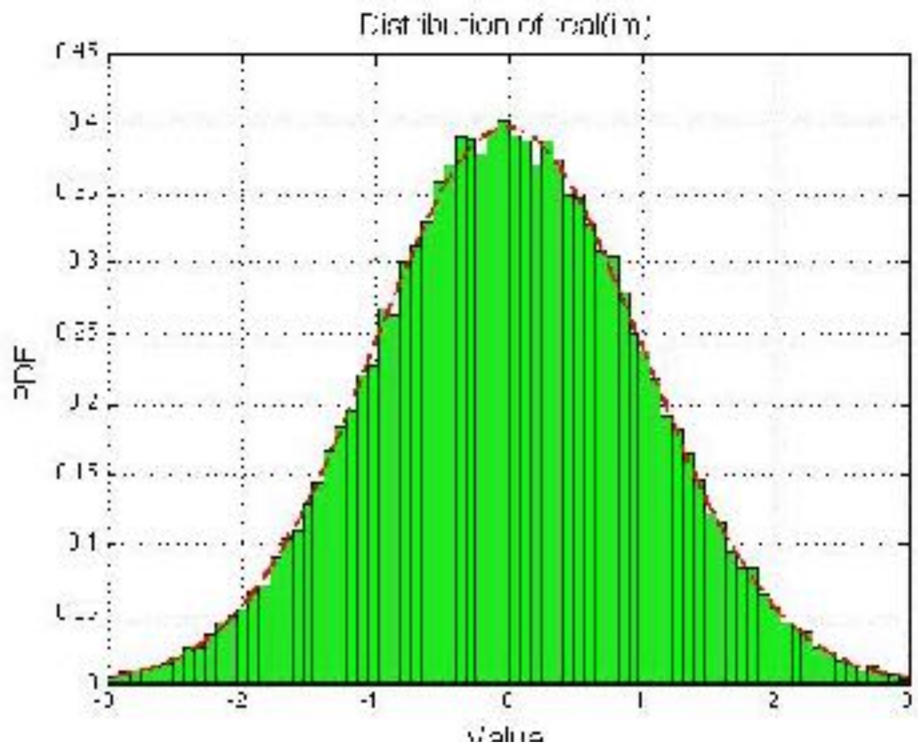
F_n and Z_n are the coefficients, but we sometimes use them to refer to the basis functions ("twists") they multiply

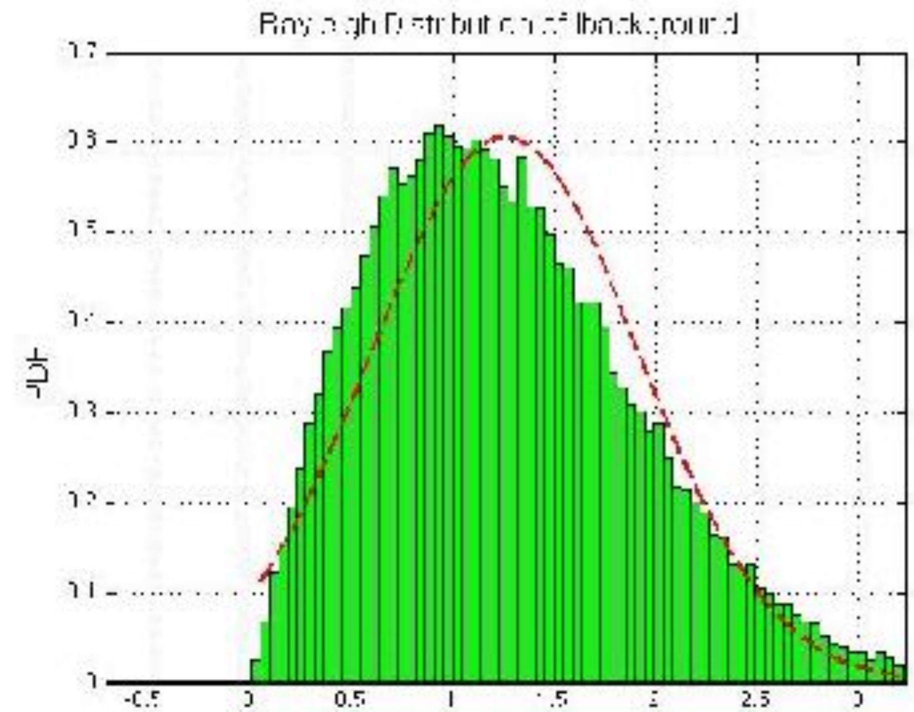
Although there are other basis definitions, this is consistent with that of Weigel et al. J Magn Reson 2010; 205:276-285

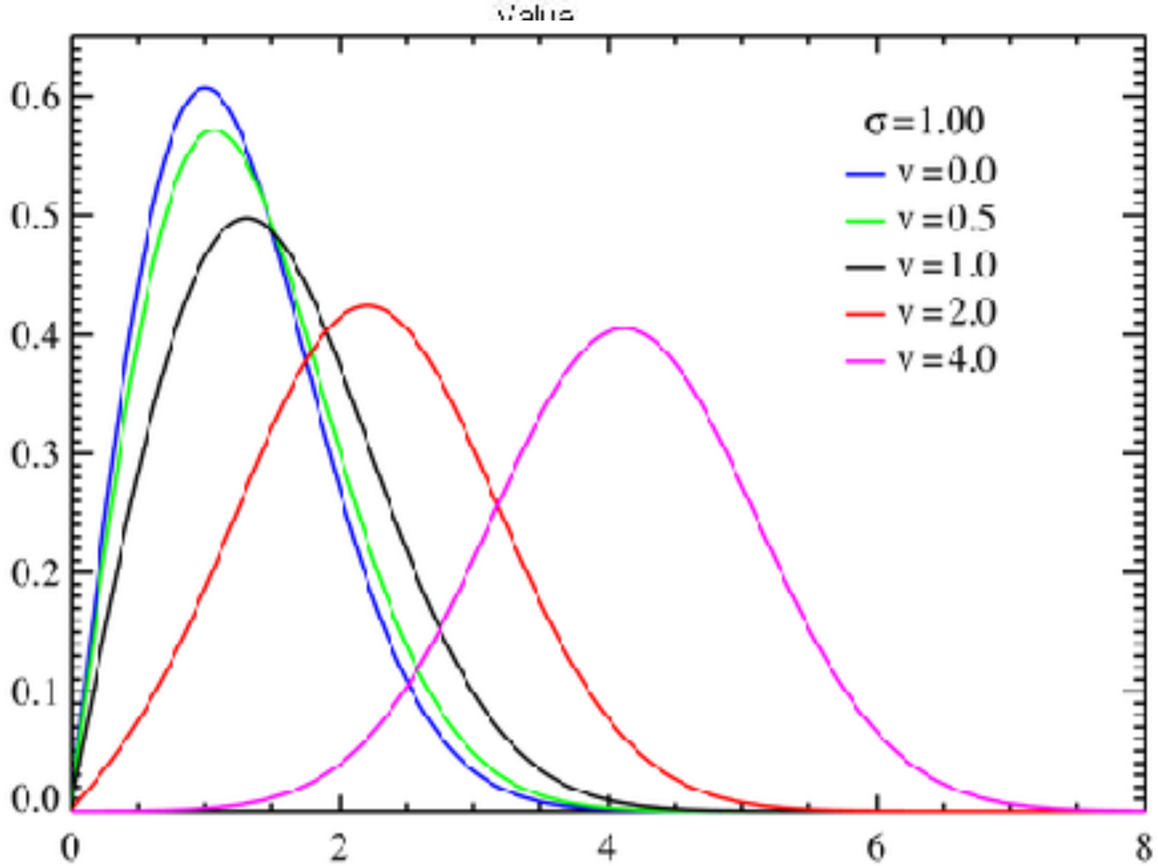
Phase Graph "States" (Flow

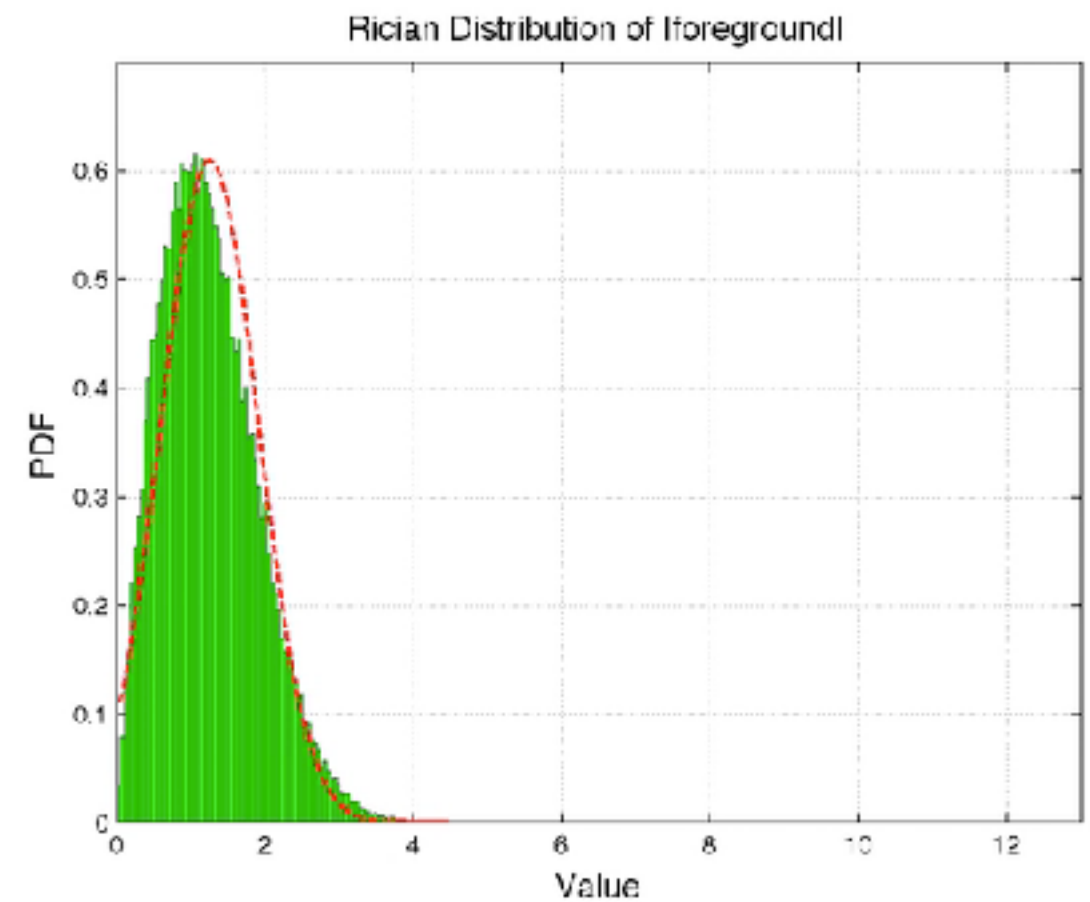


Basic Noise Statistics









Question 1

Post-Midterm Course Review

- EE 396B, Bloch & EPG, SNR
- After Midterm:
  - Spin-Echo Methods
  - Sampling
  - Radial, Spiral, EPI

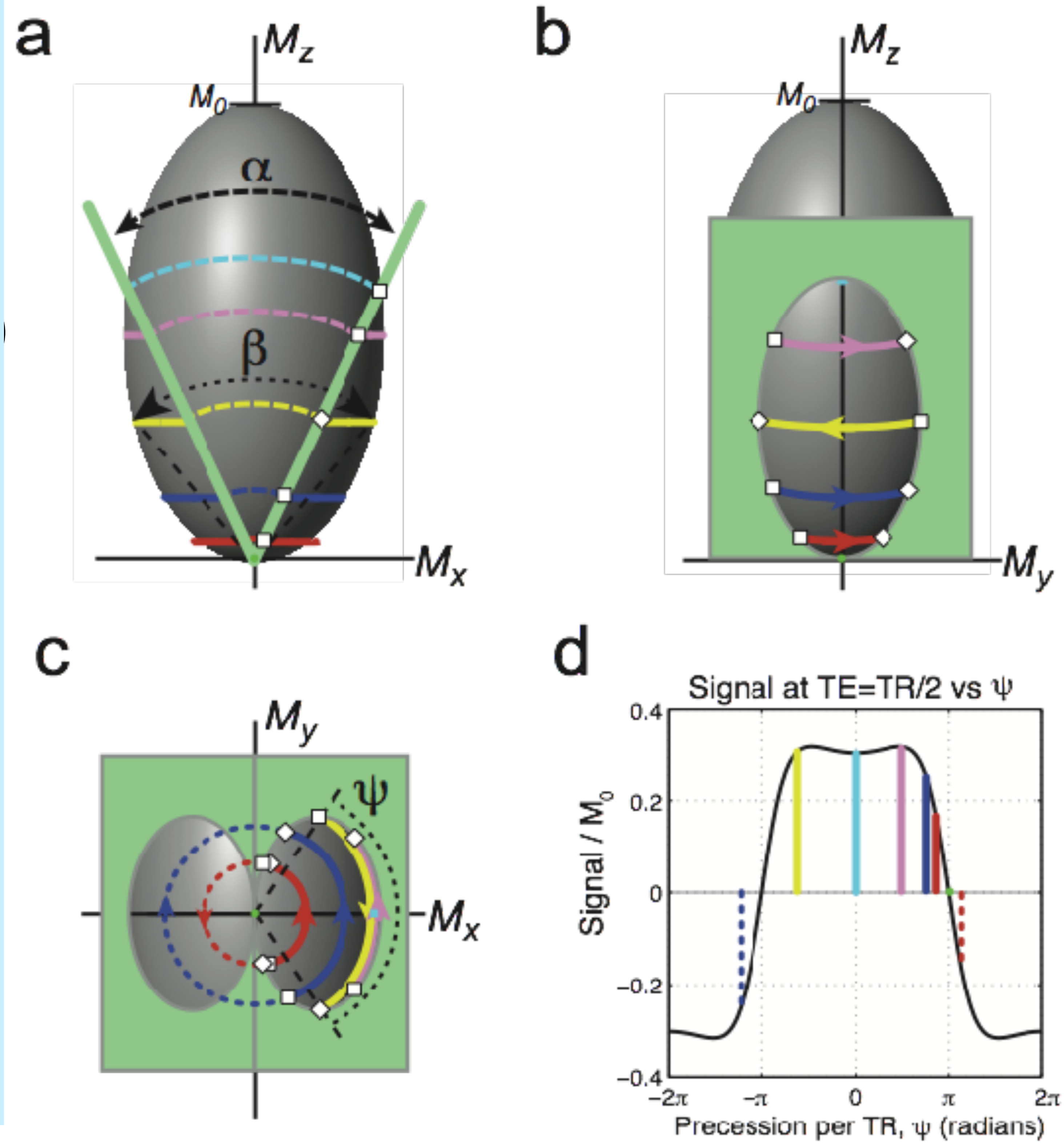
bSSFP - Geometric Interpretation

- Ellipsoid
- Effective flip angle
- Precession
- Freq response

Geometric bSSFP - Question 1

What is α?
What is β?

What is the max radius of the ellipsoid?

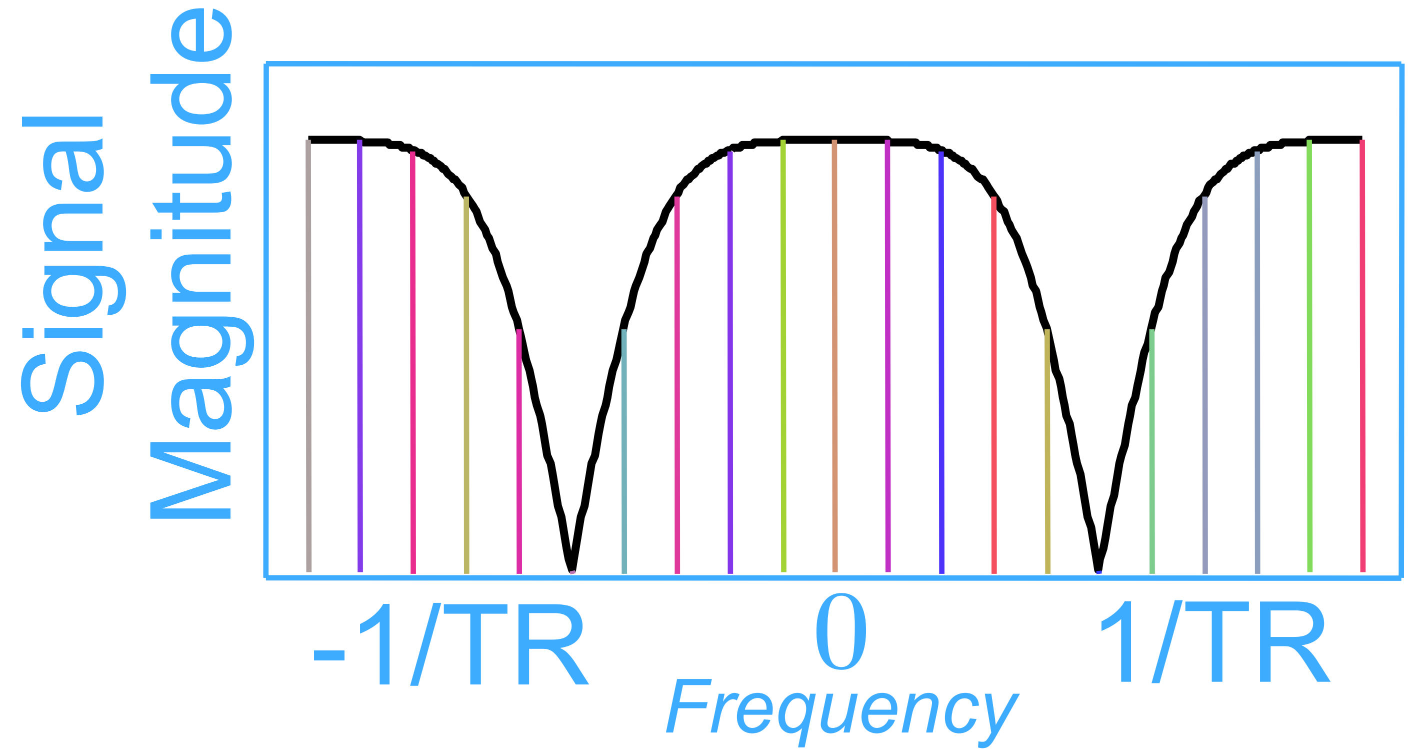


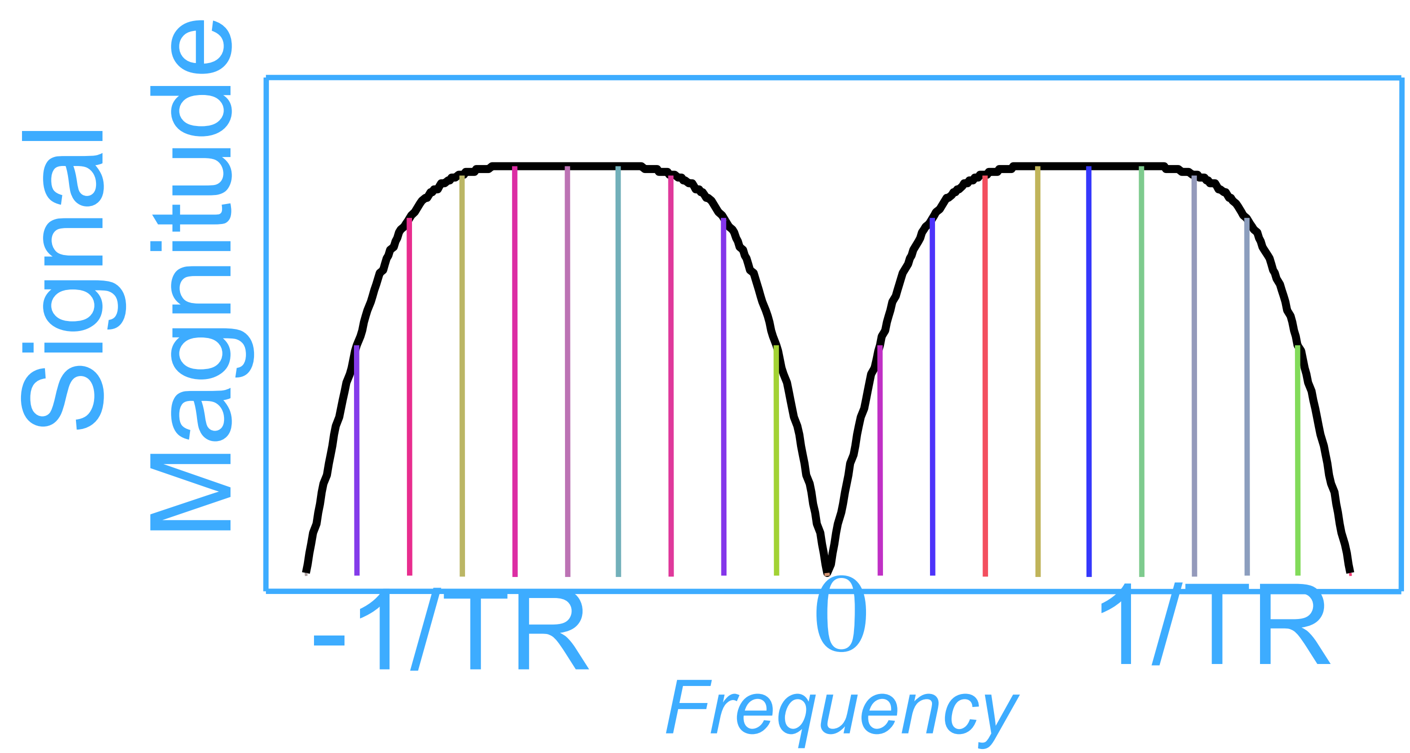
Phase Cycling - Question 2

Hinshaw 1976

Which signal corresponds to alternating RF sign?

What can you say about the distribution of spins on the ellipsoid?



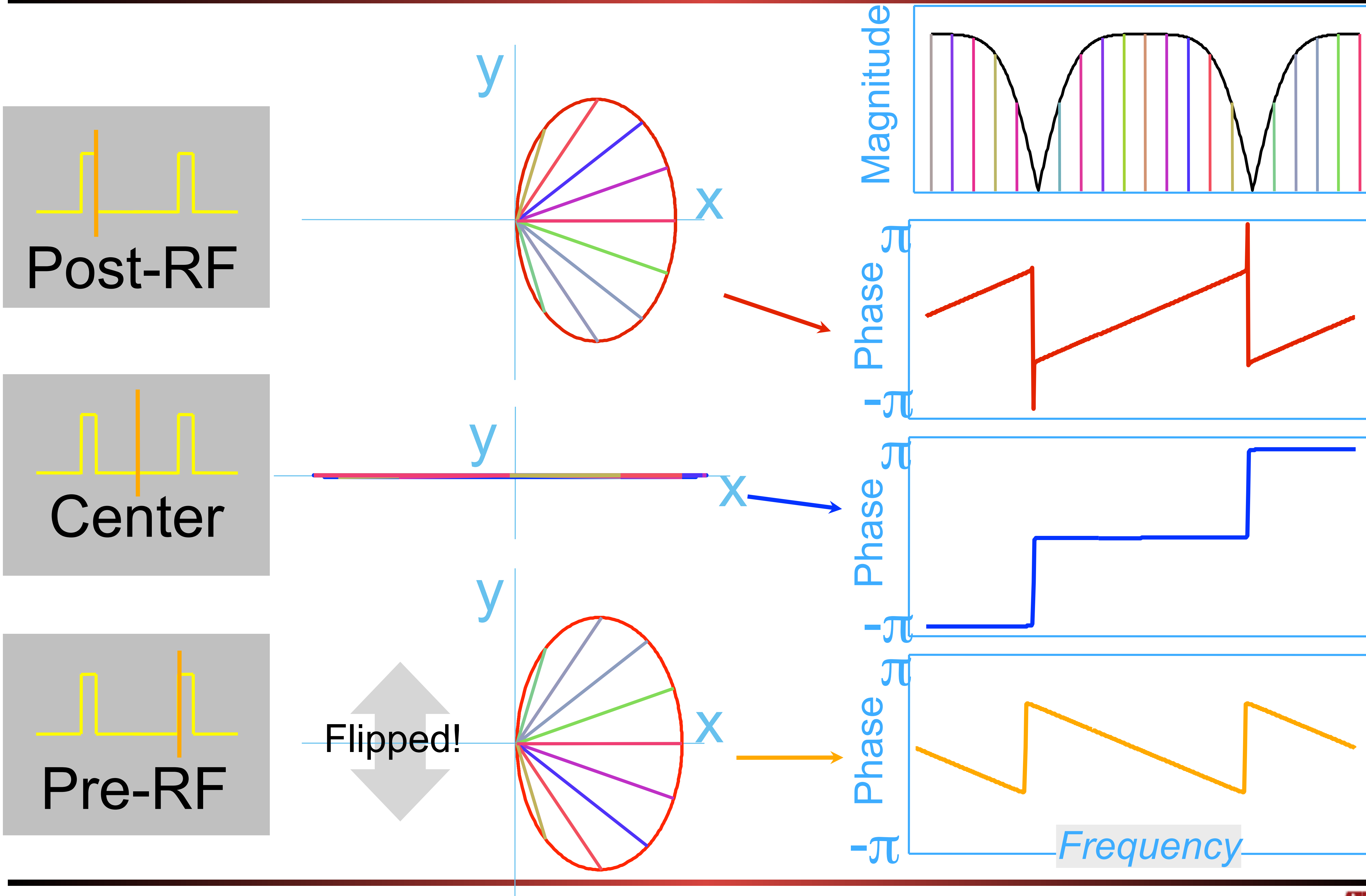


Question 3

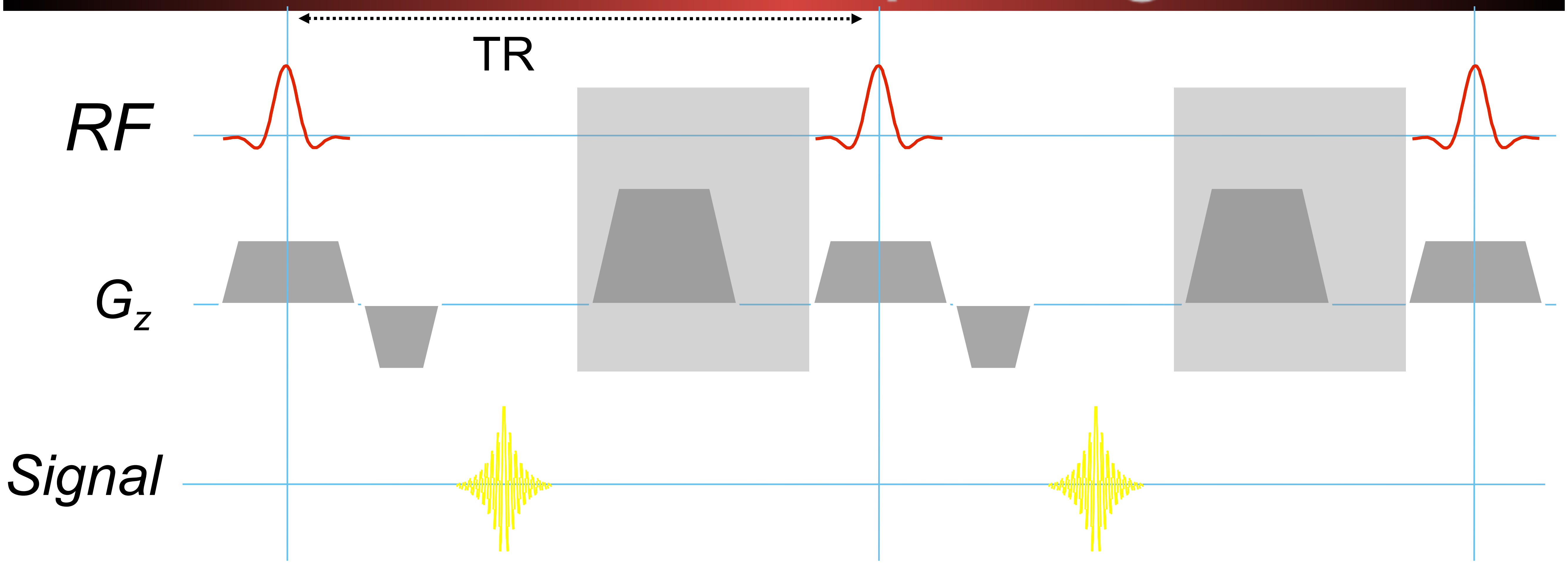
Gradient Spoiling

- Averaging, at appropriate time
- Forward / Reverse

Signal vs Frequency: Phase

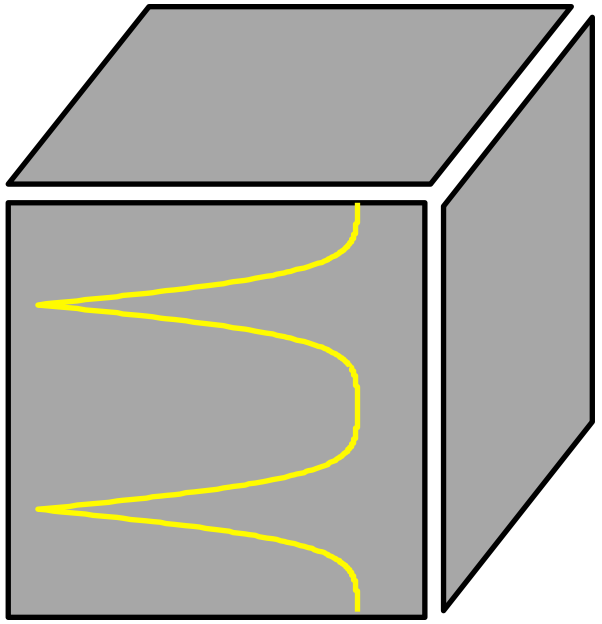


Gradient Spoiling



Precession across a voxel dominated by spoiler:

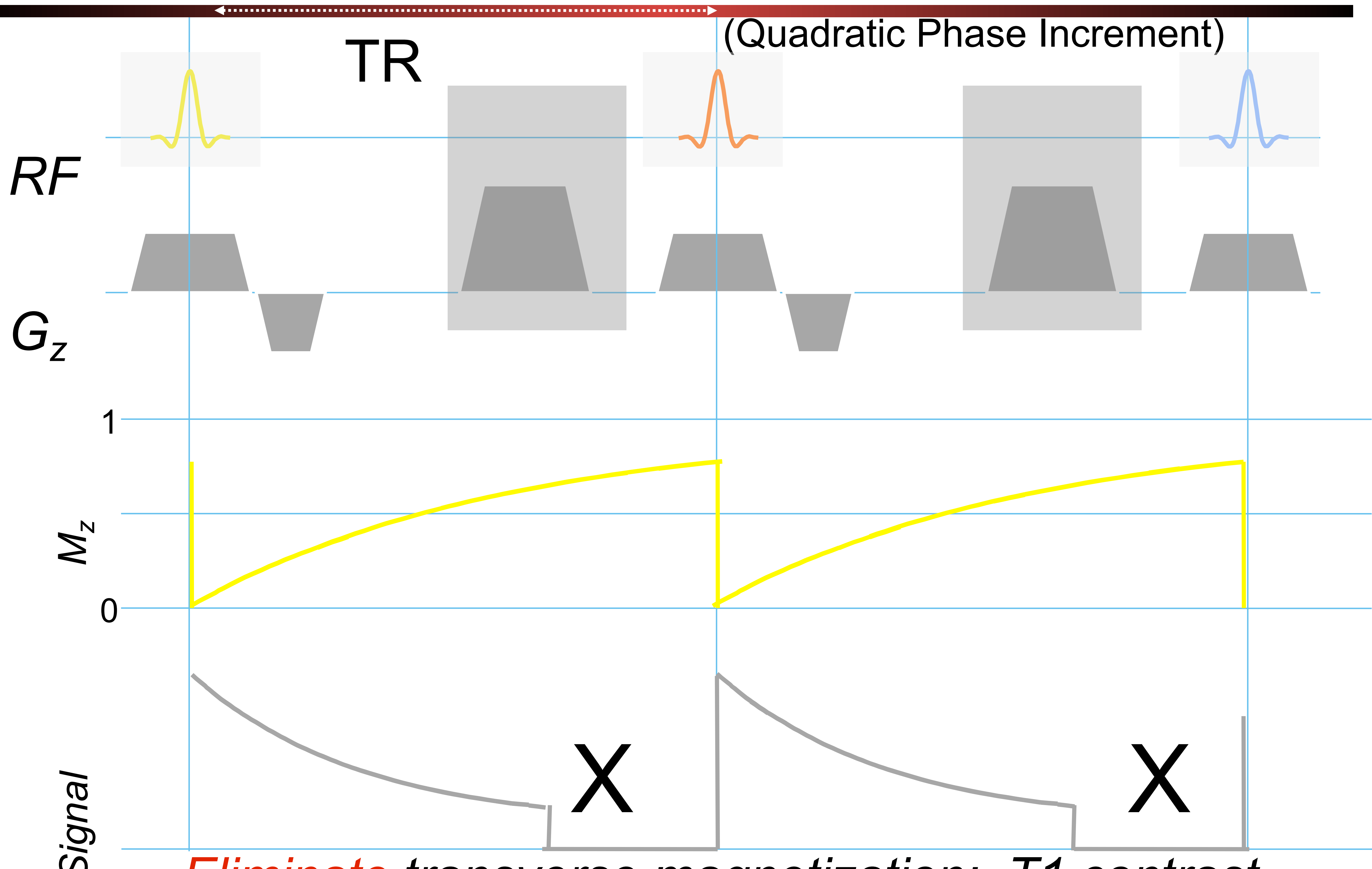
- Each spin has a different precession
- Average of balanced SSFP



Question 4

Question 5

RF-Spoiled Gradient Echo



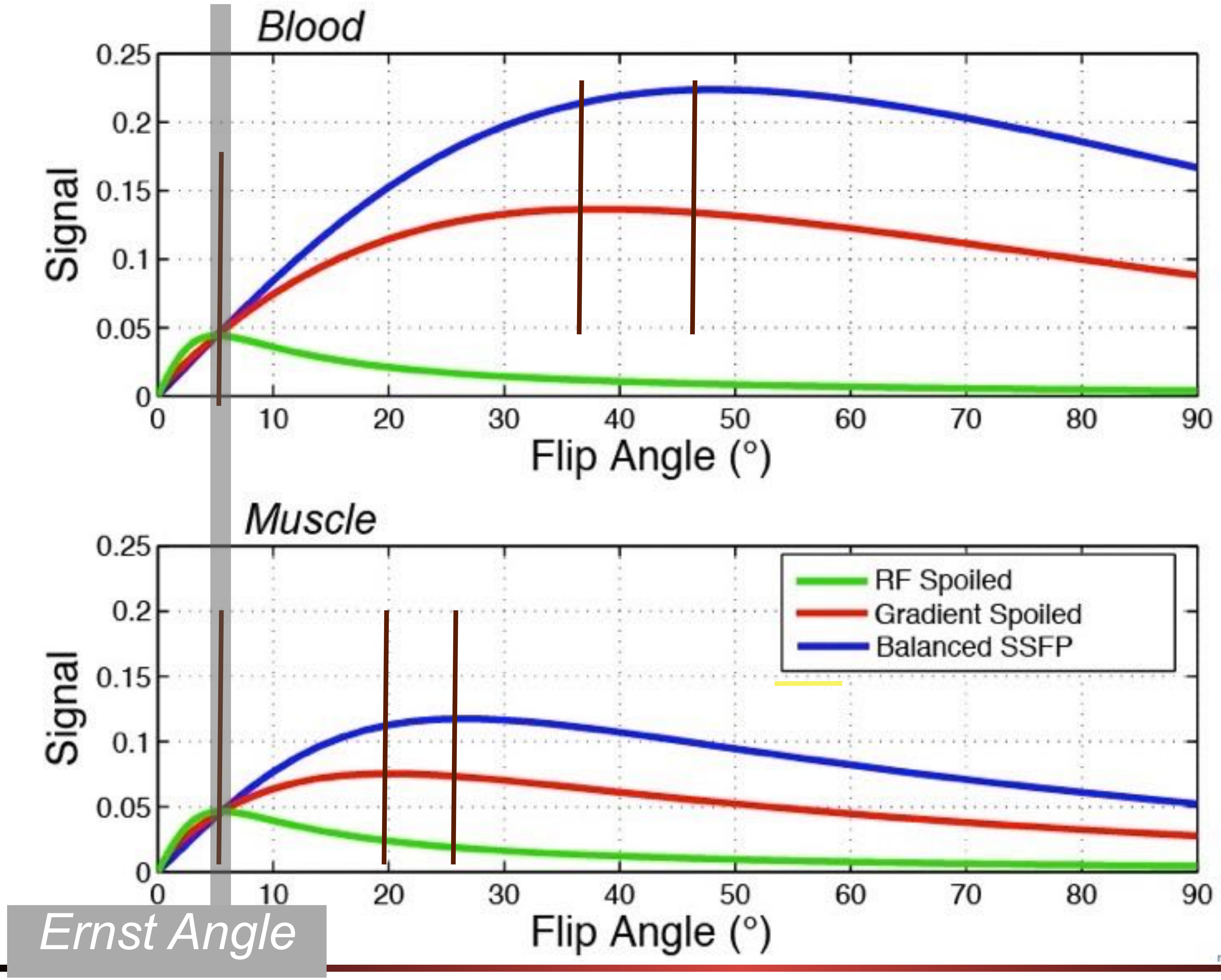
Eliminate transverse magnetization: T1 contrast

Question 6

Gradient Echo Sequence Comparison

Sequence	Balanced SSFP	Gradient Echo	RF-Spoiled
Spoiling	None	Gradient	RF + Gradient
Transverse Magnetization	Retained	Averaged	Cancelled
Contrast	T_2/T_1	T_2/T_1	T_1
SNR	High (but Banding)	Moderate	Lower

Flip Angle Selection



Buxton 1990

Spin Echo Sequences

- 2D Interleaved:
  - Single-echo
  - Echo-train PD or T2 (FSE, RARE, TSE)
  - STIR, FLAIR, Fast-recovery options
- Single-shot (SSFSE, HASTE)
- 3D: (Cube, SPACE, VISTA)

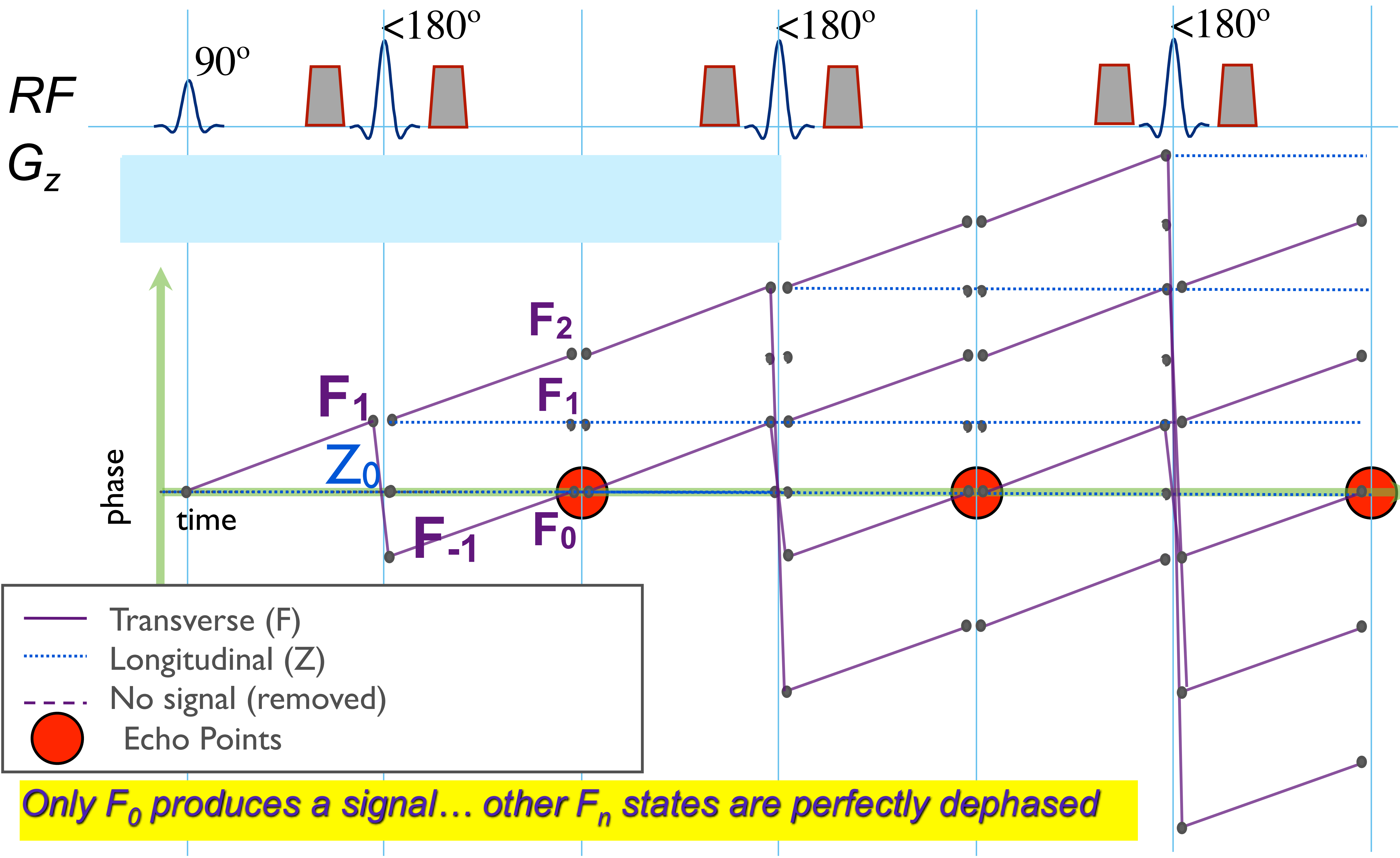
Question 7: Spin-Echoes - Warmup!

- Why do we not play perfect 180º pulses?

Spin Echo Trains

- CPMG: 90°_x - 180°_y - 180°_y - 180°_y - ...
- T_2 decay over echo train ~ modulation
- Reduced flip angles reduce SAR
- "Prep" ($90^\circ + \alpha/2$) pulse reduces oscillation
- Crusher pulses prevent parasitic signal

Effect of Crusher Pulses - Eliminate Pathways



Only F_0 produces a signal… other F_n states are perfectly dephased

Reduced / Modulated Flip Angles

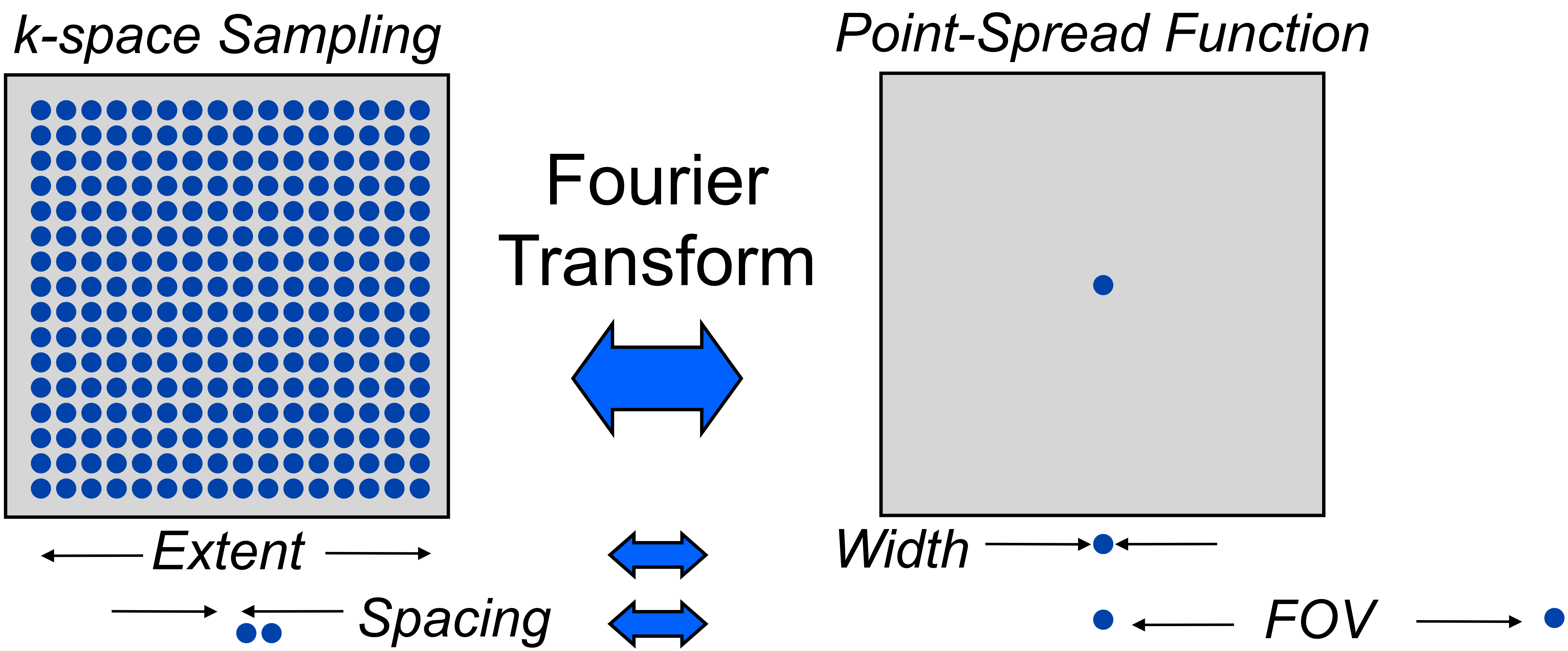
- B_1 variations and SAR prevent use of 180º pulses
- Signal only enters on 90º pulse
- Reduced flip angles - less signal, more T1 contrast
- Modulated flip angles “shape” signal
  - Flatter
  - Slower decay
  - Specific Echo time

Sampling Considerations

- Sampling and PSFs
  - Resolution, FOV, ringing
  - Variable-density and gridding
  - Partial Fourier
  - View ordering and k-space modulation
  - ky-kz and k-t sampling
- Slice interleaving

Sampling & Point-Spread Functions

- PSF = Fourier transform of sampling pattern
  - k-space: Extent, Density, Windowing
  - PSF: Width, Replication, Ripple (side-lobes)

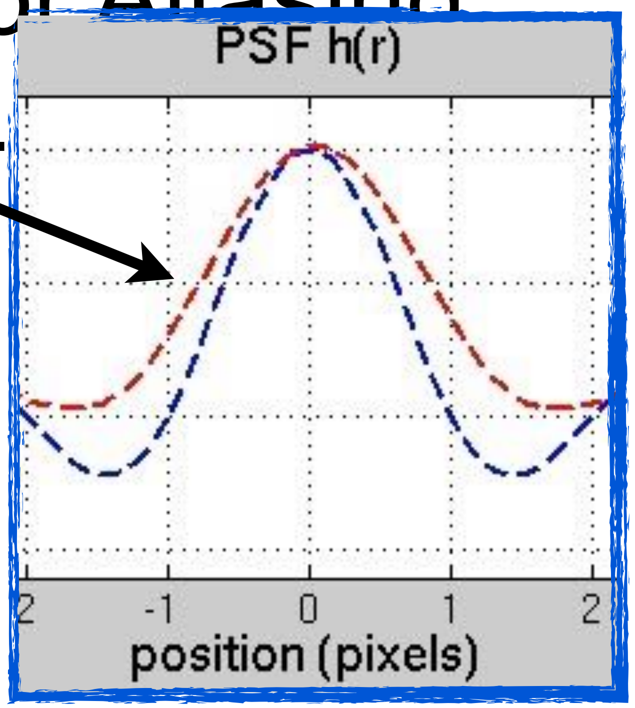


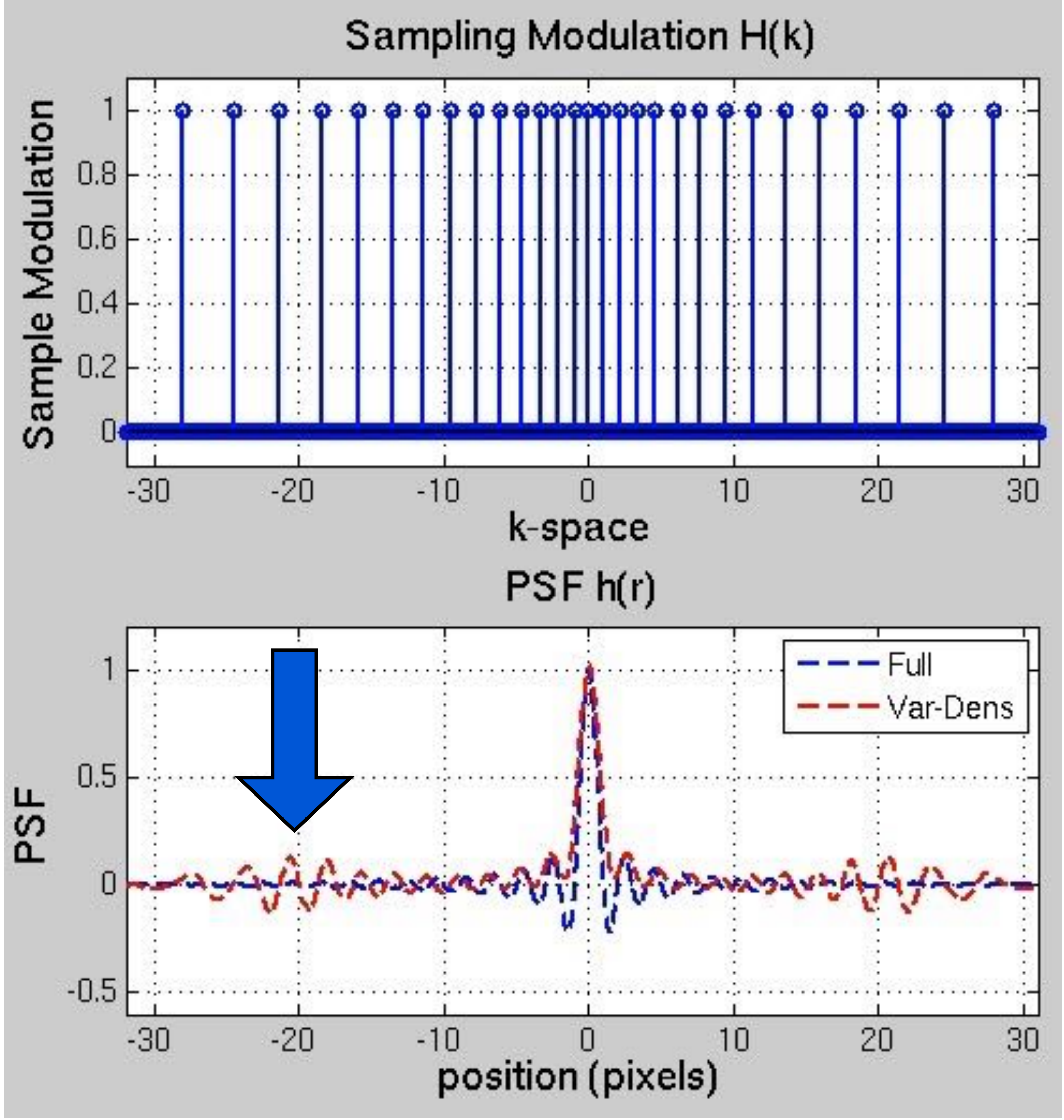
Question 8: PSFs

- If you sample "continuously" from $-k_{max}$ to k_{max} what is the PSF in 1D?
  - *$p(x) = sinc(2\, k_{max}\, x)$*
- If the sample spacing is Δk, how does p(x) change ("intuitively")? (ignore "discrete" sinc)
  - *$q(x) = \Sigma\, p(x + n/\Delta k)$, where n includes all integers*
- If we apply a triangle window to k-space, how does that affect the sampling p(x)?
  - *$r(x) = p(x/2)^2$ or $q(x/2)^2$*

Variable Density Sampling

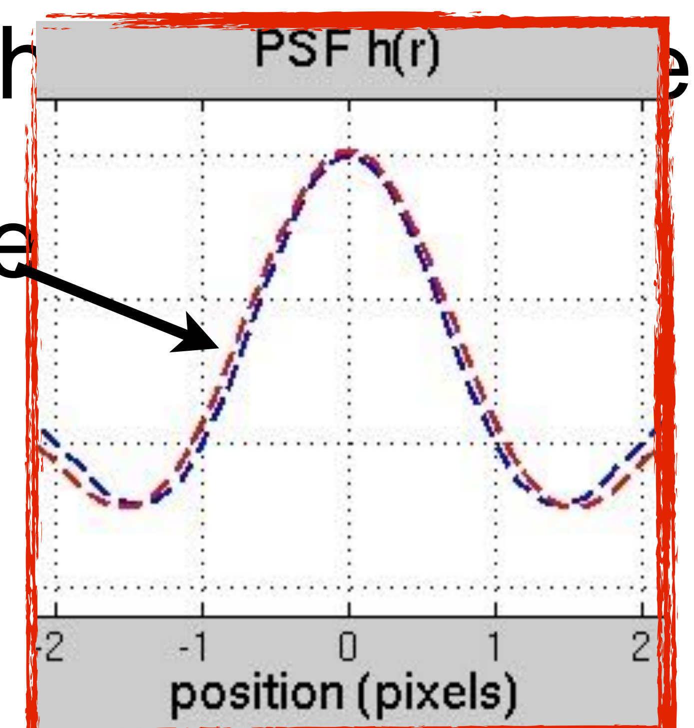
- 2x undersamling
- Δk linear with k
- Minor Aliasing
- PSF

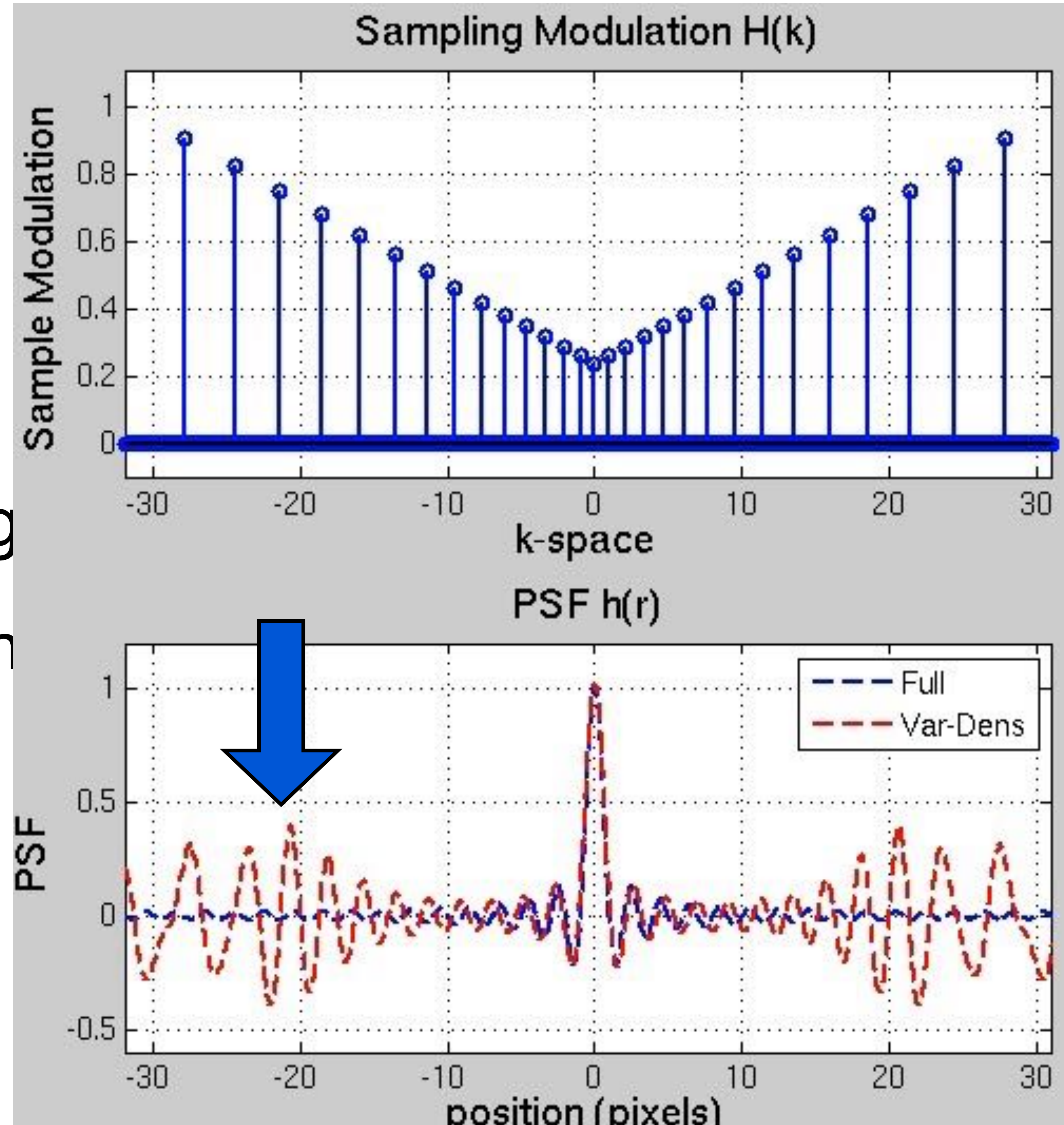




Variable Density Sampling:

- Multiply by 1/Δk
- No PSF Broadening
- High
- nee



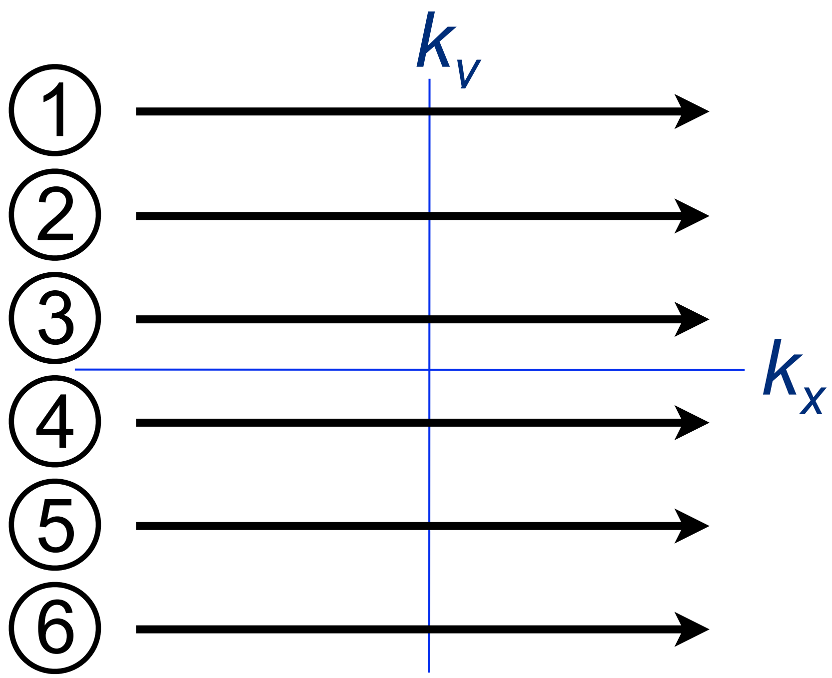


Question 9: SNR Efficiency

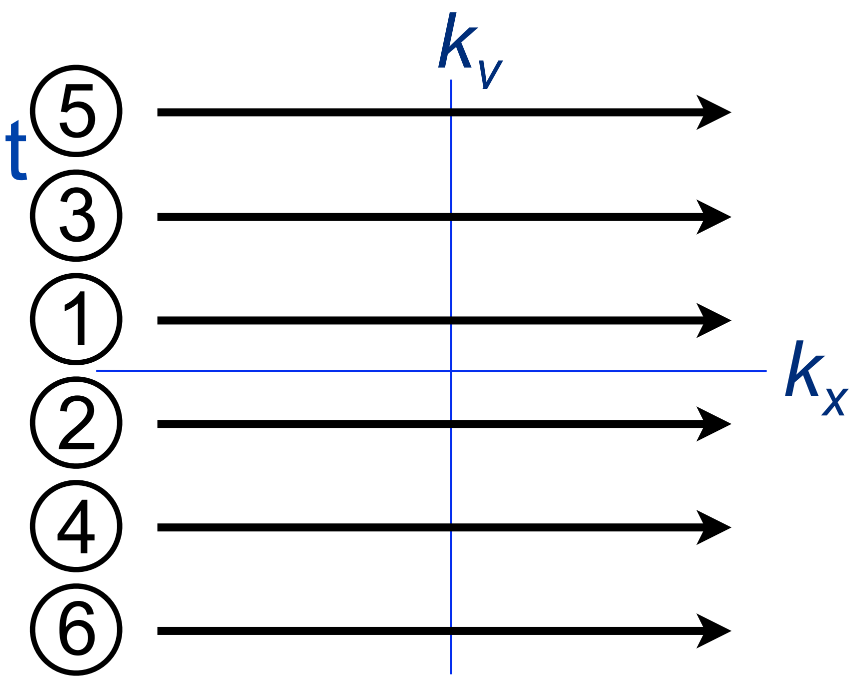
- If we resample **one** point in a 1D set, what do we have to do to ensure the PSF is still a sinc()?

View Ordering / Grouping

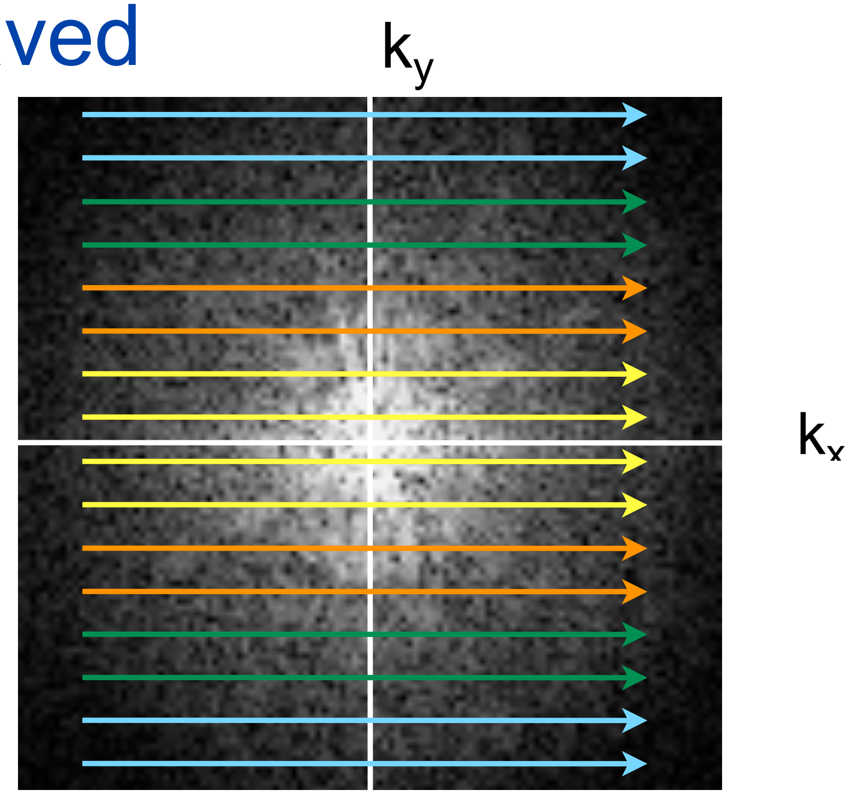
Sequential



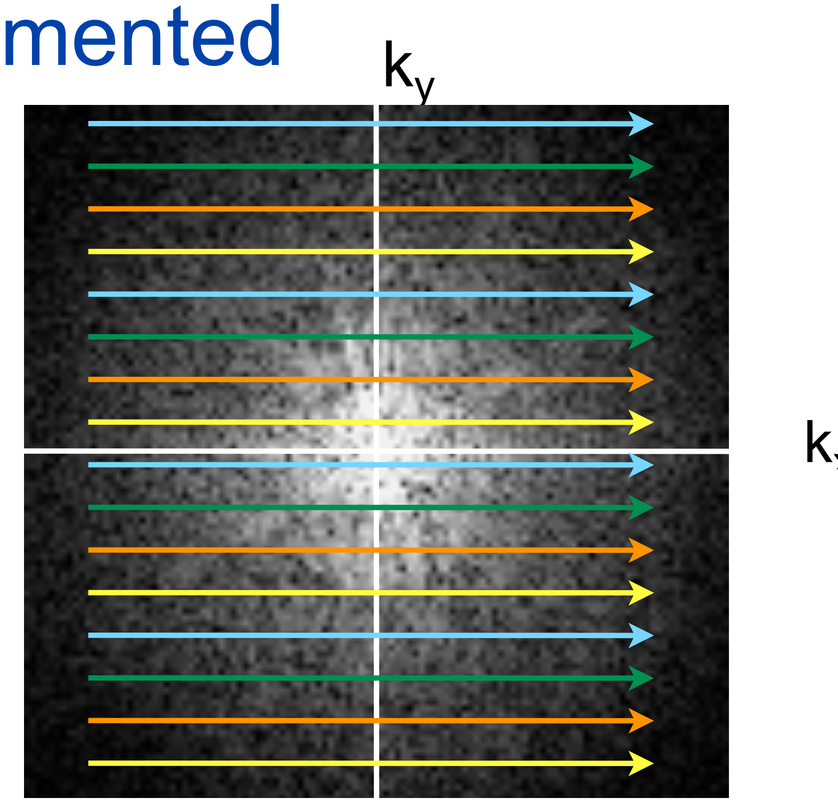
Centric / Center-out



Interleaved



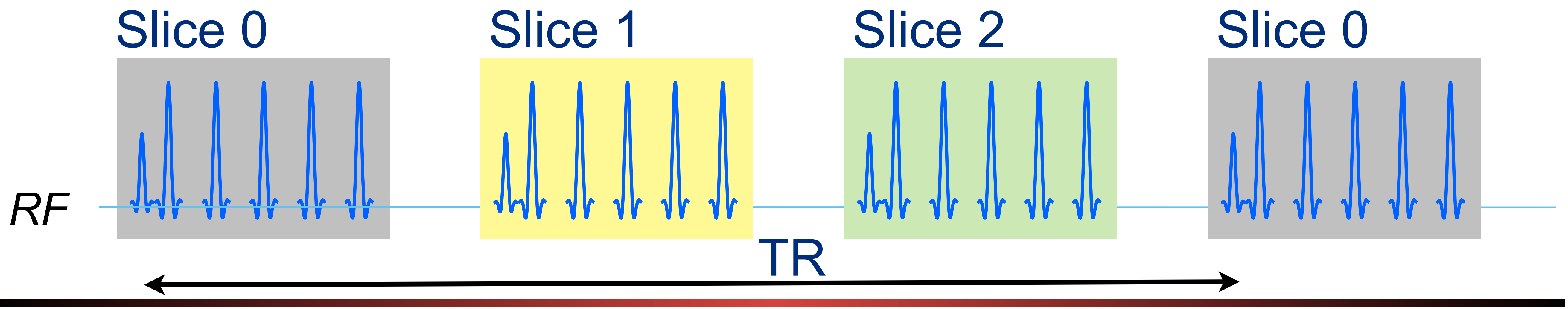
Segmented



Each color is a different “modulation” (echo, time, etc)

How Many Slices to Interleave?

- Usually specify TR, TI, Echo-train-length (ETL), Resolution, ...
  - Tells "pulse durations" (T_{seq}) and RF power
  - $N_{max} \sim TR / T_{seq}$
  - Can re-order slices in "time slots"
  - Additional slices require another "acquisition"



Question 10: Interleaving

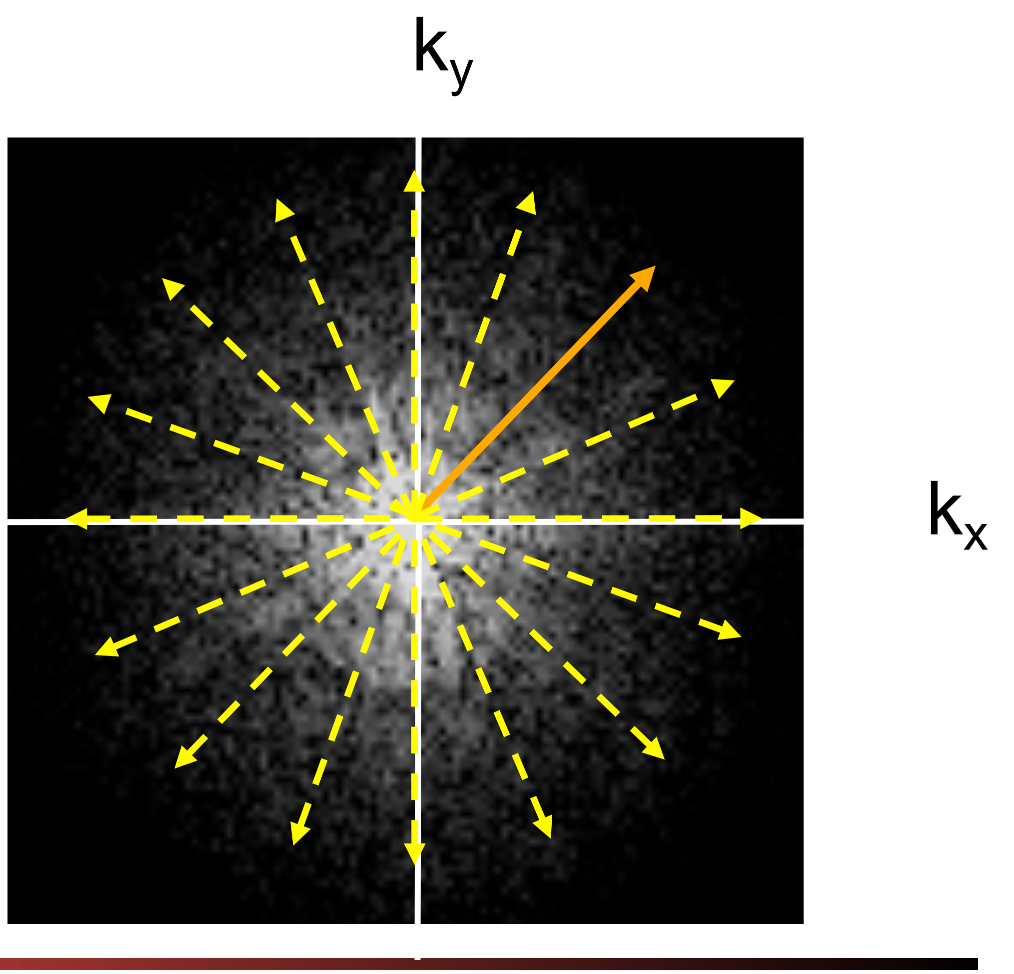
- Consider a spin-echo train to image 15 slices, 256x256 with an echo spacing of 10ms, and echo-train length 16.

Radial and Projection: Summary

- Non-Cartesian, requires gridding reconstruction
- Incoherent undersampling artifact (similar to CS)
- Short TE (and UTE) imaging
- 2D and 3D options
- No phase-encoding ~ can be efficient
- Off-resonance causes blurring
- SNR efficiency loss due to high-density near center, but resampling the center can be advantageous

Radial (k=0 outward)

- Similar to Full Projection, but center-out readouts
- Shortest TE (~0) of any sequence
- Low first-moments
- Fastest way to reach high-spatial frequencies
- Impact of delays
- Can do odd/even sampling
- Impact of ramp sampling



Question 11: PR Design

Radial/Projection: Recon and SNR

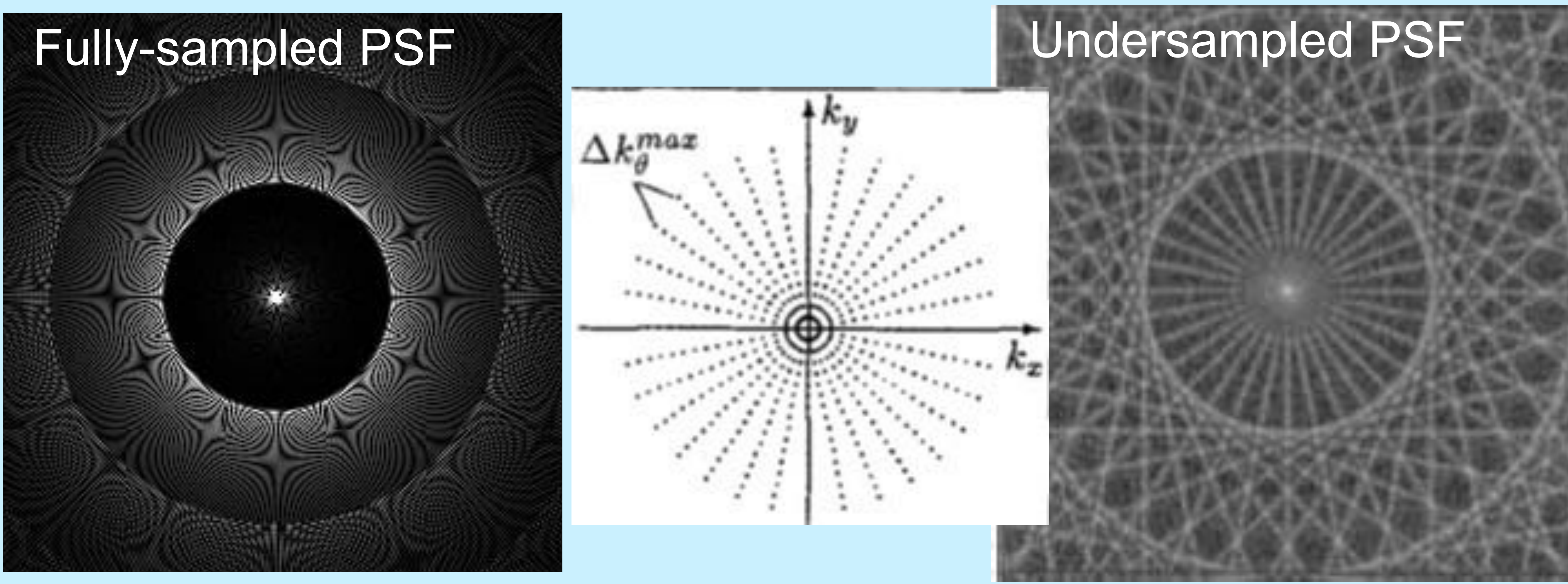
- Usually use gridding
  - Density ($D = k_{max}/k_r$)
  - compensate by multiplying by $1/D = k_r$
- How does this affect SNR?
  - More samples required to cover a given area
  - Noise variance is altered by gridding reconstruction
  - Noise is colored ("Speckle" or "salt and pepper")
  - Efficiency:

$$\eta = \frac{A}{\sqrt{\int_A D \int_A 1/D}}$$

~ 0.87 for Uniform-density projections

Projection-Reconstruction PSF /

- PSF has a “ring” of aliasing (less coherent)
  - Intuitition: No “preferred” direction for coherent peak
- Undersampling tends to result in streak artifacts



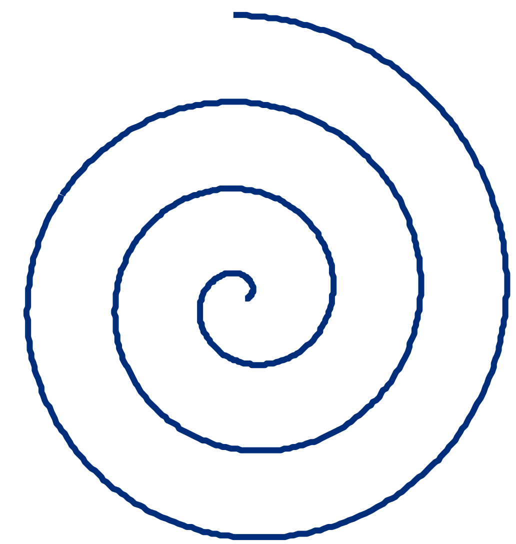
From Scheffler & Hennig, MRM 1998

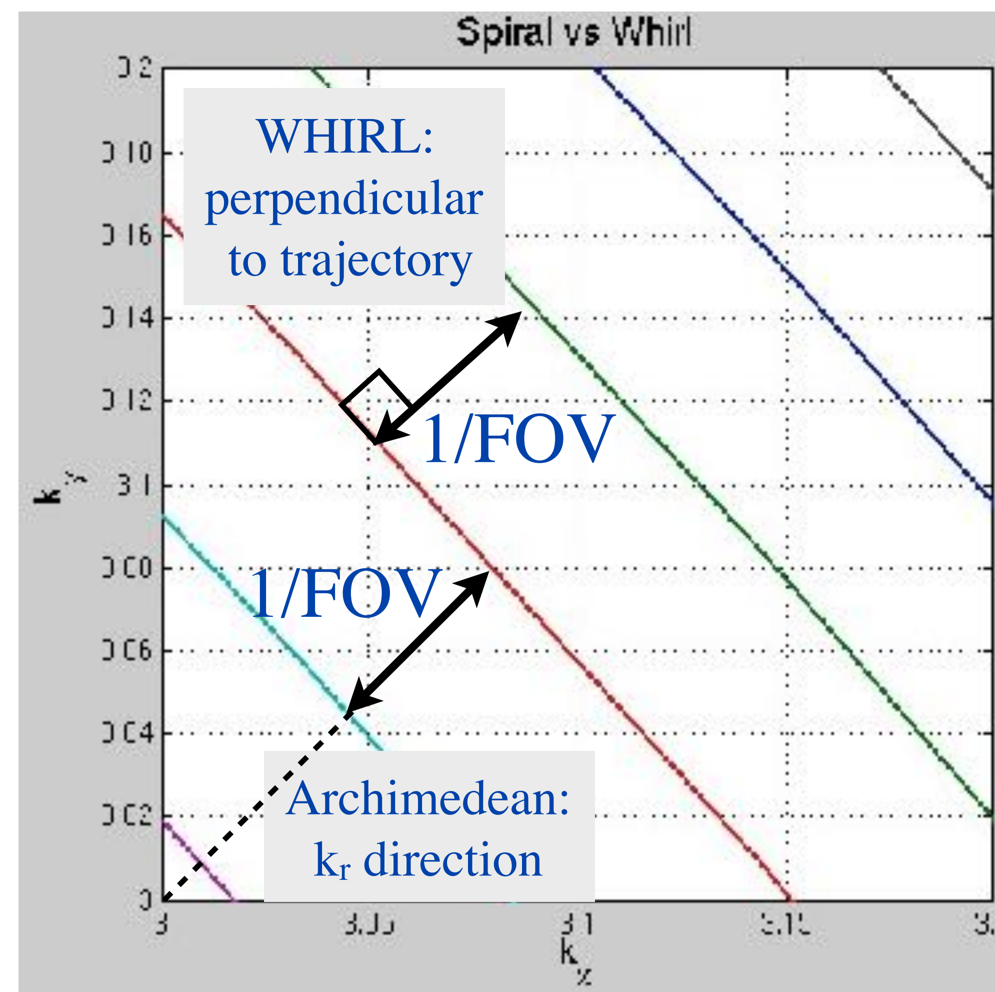
Spiral Summary

- Flexible duration/coverage trade-off
  - Center-out: TE~0, Low first-moments
- Archimedean, TWIRL, WHIRL, variable-density
- PSF with circular aliasing, swirl-artifact outside
- Off-resonance sensitivity, correct in reconstruction
- Variations: Spiral in/out, 3D TPI, 3D Cones
- Rewinder design

Spiral

- Design
  - Resolution = extent
  - Spacing = FOV
  - #Interleaves <> duration
- Longer readouts maximize acquisition window
- Variable-density





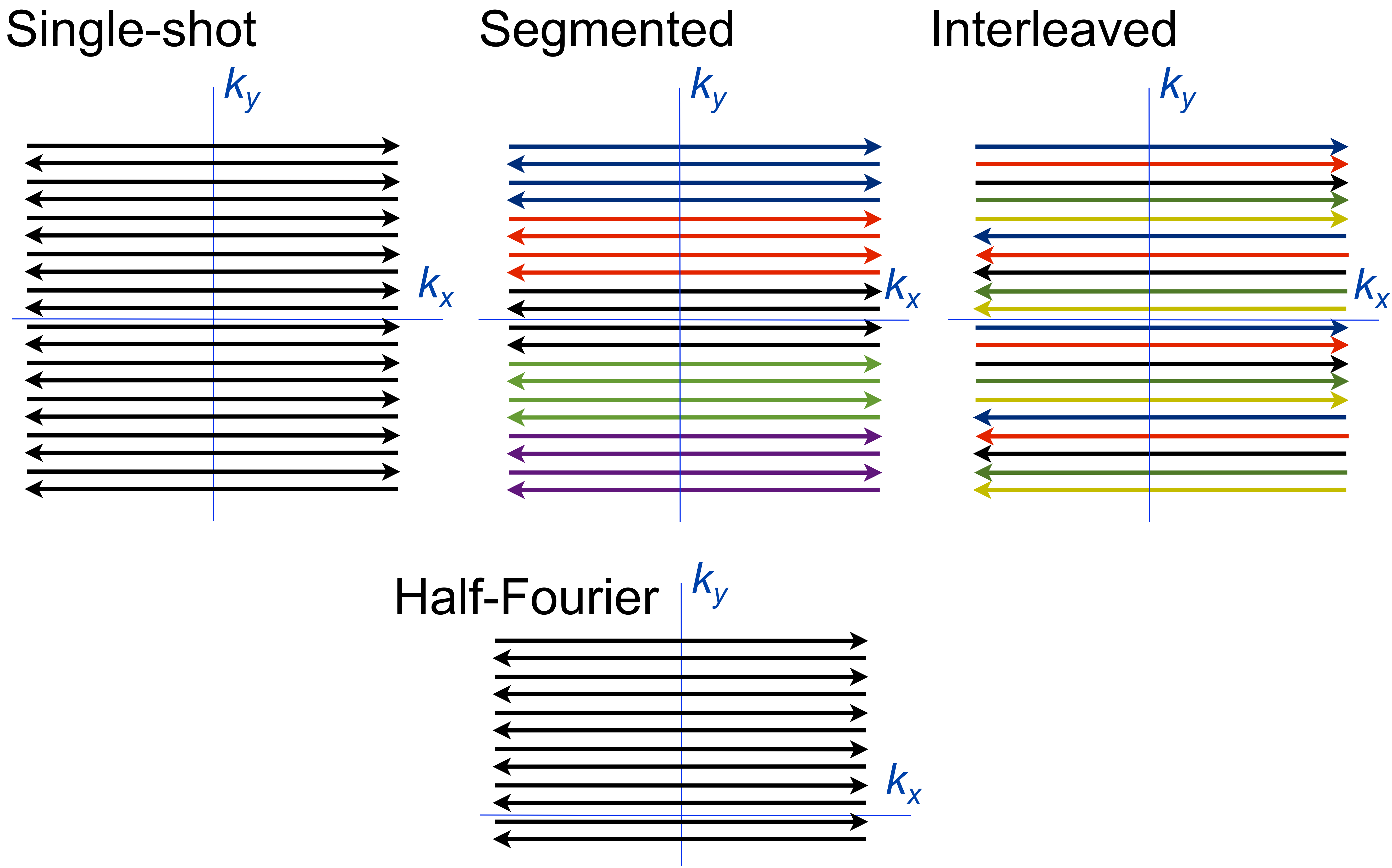
Question 12: SPIRAL

- An Archimedean spiral has 20 complete turns and reaches an k-space radius of $2mm^{-1}$. If we use 10 interleaves, what are the FOV and resolution?

EPI Summary

- Very fast imaging trajectory
- Single-shot, Interleaved or Segmented
- Bidirectional EPI requires phase correction
- Sensitive to T2* and Off-resonance (blur and distortion)
- Much more widely used than spiral (currently)
- Variations: Flyback, GRASE, Propellor

EPI Variations



Question 13: EPI Odd/Even Effects

- What is does each effect cause, and why might it occur?

Summary: Quantitative/Mapping

- Gradient Measurement
- Fat/Water Separation
- B_0 and B_1 mapping
- T_1, T_2 and T_2* mapping

Mapping Concepts

- Sensitivity to a parameter
- Dynamic range of parameter
- Confounding variables

Mapping: Question 14

- Complete the chart…

Method	Main Parameter of Interest	Confounder(s)

Motion Artifact Suppression

- “Patient Independent”
  - Rapid scanning (resolve motion)
  - Breath-holding
- “Patient Dependent”
  - Clever ordering of k-space
  - Triggering and gating of signals
  - Measure motion and correct
    - cylindrical, orbital navigators
    - butterfly navigators

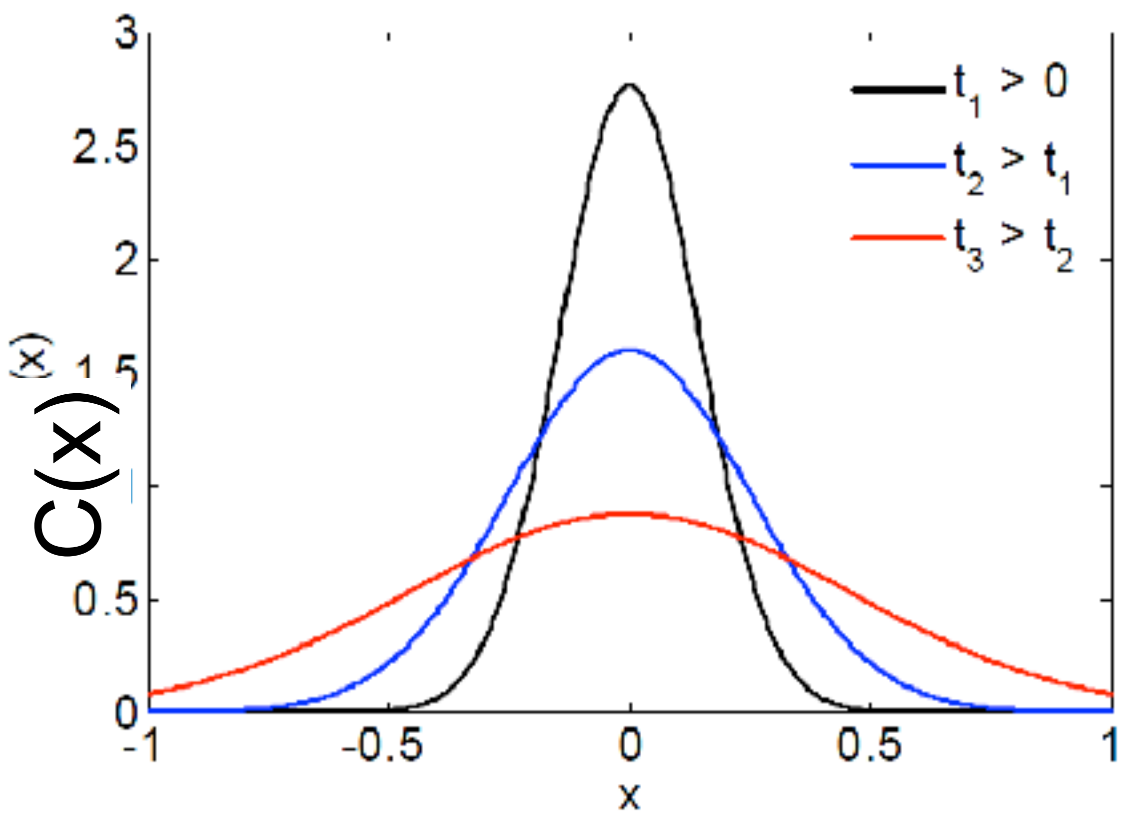
Diffusion MRI: **(Not Tested!)**

- Diffusion MRI: a marker of tissue microstructure.
- What is diffusion and how do we model it?
- Sensitizing the MRI signal to diffusion.
- Diffusion MRI signal equations.
- Mapping diffusion coefficients.
- Effects of motion.
- Eddy currents.

Gaussian Spread of Particles

- Patterns of water diffusion in tissue reflect the tissue microstructure .

Mean squared displacement given by the Einstein relation can also be interpreted as the variance of the spread of positions after a period of time.



$$\sigma^2 = 2Dt$$

For n dimensions :

$$\sigma^2 = 2nD\Delta$$

- Sensitizing the MRI signal to water diffusion is a way to indirectly get information about tissue microstructure.

The Bloch-Torrey Equation

Torrey H.C. *Physical Review* 1956.

- Patterns of water diffusion in tissue reflect the tissue microstructure .

$$\frac{d\mathbf{M}}{dt} = \gamma(\mathbf{M} \times \mathbf{B_0}) + \begin{pmatrix} -\frac{M_x}{T_2} \\ -\frac{M_y}{T_2} \\ \frac{M_0 - M_z}{T_1} \end{pmatrix} + D\nabla^2\mathbf{M}$$

$$\mathbf{M}_+ = M_0 e^{\frac{-t}{T_2}} e^{-bD}$$

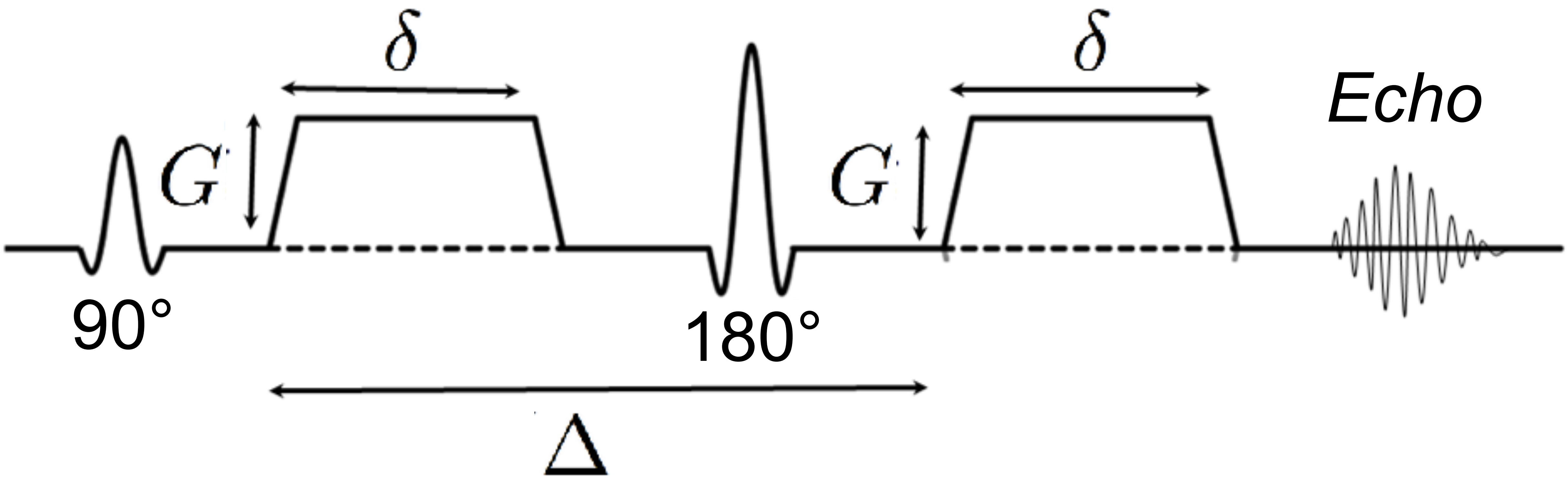
with

$$b = \gamma^2 \int_0^{TE} \left(\int_0^t G(t')dt' \right)^2 dt$$

- Sensitizing the MRI signal to water diffusion is a way to indirectly get information about tissue microstructure.

b-value

- Patterns of water diffusion in tissue reflect the tissue microstructure .

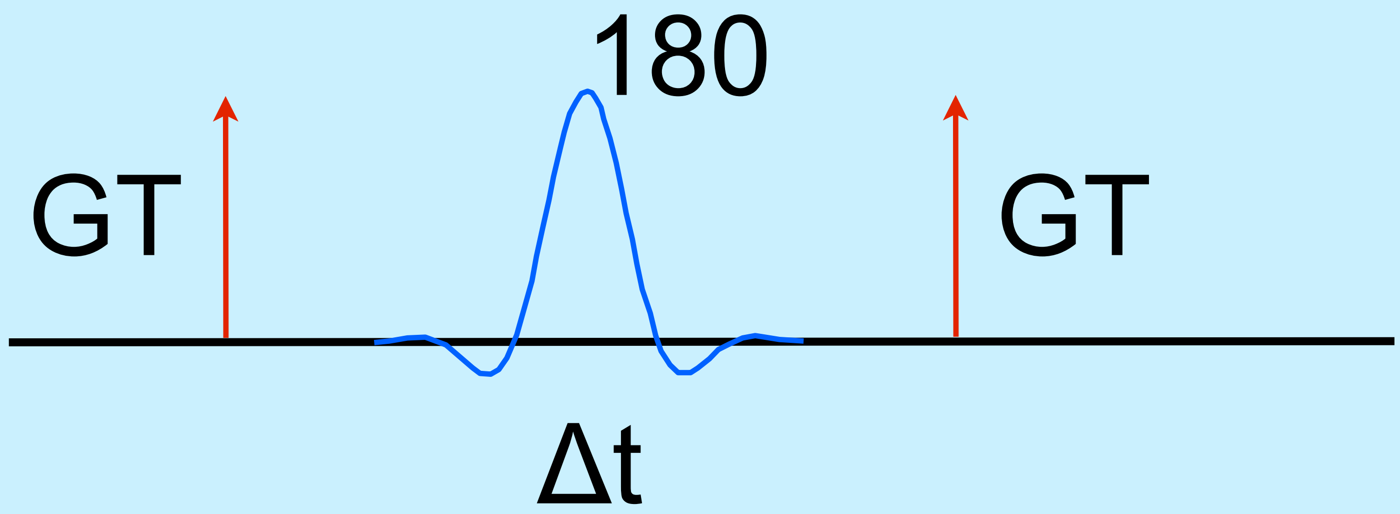


$$b = \gamma^2 \int_0^{TE} \left(\int_0^t G(t')dt' \right)^2 dt$$

- Sensitizing the MRI signal to water diffusion is a way to indirectly g ue microstructur

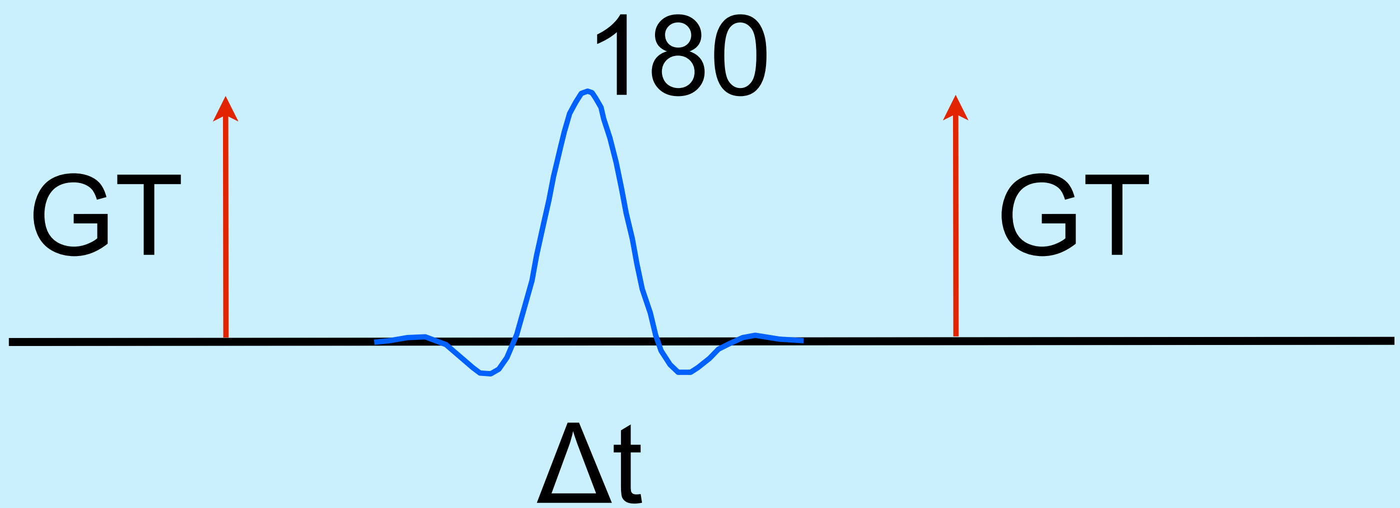
$$b = (\gamma G \delta)^2 (\Delta - \frac{\delta}{3})$$

Diffusion Example



- 1D Gaussian Diffusion: $\Delta l = \sqrt{2D\Delta t}$
- Imagine a sequence with 2 gradients of area GT, with a 180 refocusing pulse between.
- What is the expected value of the spin echo signal as a function of D, Δt, GT, ignoring T_2?
  - b = $(\gamma GT)^2 T$, signal = exp(-bD)

Diffusion Example



$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

$$\sigma = \Delta l = \sqrt{2D\Delta t}$$

- Phase vs x *is* $\phi = \gamma GT\, x$, x *is displacement*
- Expected value is expected value of cos(ϕ)

$$\int \cos(\gamma GTx) \frac{1}{\sqrt{4\pi D\Delta t}} e^{-\frac{x^2}{4D\Delta t}} dx$$

$$\int_{-\infty}^{\infty} e^{-ax^2} \cos(kx)\, dx = \sqrt{\frac{\pi}{a}}\, e^{-k^2/4a}.$$

$$= \frac{\sqrt{4\pi D\Delta t}}{\sqrt{4\pi D\Delta t}} e^{-(\gamma GT)^2 D\Delta t}$$

$$= e^{-(\gamma GT)^2 D\Delta t} \quad = e^{-bD}$$

$$b = (\gamma GT)^2 \Delta t$$

Question 15: Diffusion

- We now replace the “delta-function” gradients with gradients of duration T=10ms, still with area GT.

Post-Midterm Course Review

- EE 396B, Bloch & EPG, Gradient Echo Methods
- After Midterm:
  - Spin-Echo Methods
  - Sampling
  - Radial, Spiral, EPI
  - Measurement and Mapping