RAD229: Final Exam – 2014/2015
You will have 3 hours to complete this Exam

Name: ________________________________
Student ID: ____________________________

General Instructions:
1. There are 10 questions and 12 pages total.
2. You may use notes including lectures, homework, solutions or Matlab script text on the course website, and you may use an electronic device for reading such material.
3. You may not use Matlab or the Internet, or other calculating devices of any sort.
4. Please answer questions on the exam, and show your final answer clearly.
5. Show your reasoning and work, as this will often earn you partial points.
6. You may request more paper if needed.
7. For this exam, please a gyromagnetic ratio value $\gamma/2\pi = 40.0$ MHz/T
8. The following plot, which we have used repeatedly, is given for the above $\gamma$ value and gradients with maximum amplitude 50mT/m and maximum slew rate 200mT/m/ms:

The EPG RF rotation matrix $R(\alpha, \phi)$ is

$$
R(\alpha, \phi) = \begin{bmatrix}
\cos^2(\alpha/2) & e^{2i\phi} \sin^2(\alpha/2) & -ie^{i\phi} \sin \alpha \\
e^{-2i\phi} \sin^2(\alpha/2) & \cos^2(\alpha/2) & ie^{-i\phi} \sin \alpha \\
-i/2e^{-i\phi} \sin \alpha & i/2e^{i\phi} \sin \alpha & \cos \alpha
\end{bmatrix}
$$

Please do not proceed to the next page until the exam begins.
Short Answer Questions (15% of Points)

1. Describe, intuitively and briefly, the effect on an image of the following k-space modulation effects:
   a. Linear phase modulation by N cycles over the full k-space in the $k_x$ direction
   b. A unit step discontinuity modulation of k-space magnitude or phase
   c. Smooth decrease of magnitude from k-space center outward
   d. Smooth change of phase from k-space center outward
2. Given the signal contours below (vs $T_1$ and $T_2$), what is the type of contrast, and describe (briefly) spin-echo sequences and parameters (TR, TE) and/or variations studied that could produce this contrast. *You do not need the parameter numbers to be exact!*

![Diagram A](image1.png)

![Diagram B](image2.png)

![Diagram C](image3.png)
3. **Fat/Water Separation:** Consider a 2-point Dixon sequence at 3T where the fat/water chemical shift is 400 Hz (slight simplification from actual 440 Hz). The reconstruction does NOT correct any phase effects due to off-resonance or effects from relaxation.

   a. What are appropriate (minimal) echo times for the acquisition for a gradient-echo sequence?

   b. If a voxel contains only water (amplitude 1), and is actually 200Hz off-resonance, what will the reconstructed water and fat images be? (Ignore relaxation).

   c. If a voxel contains only water and is on-resonance, what are the water and fat signals as a function of $T_2$ (Assume the same $T_2$ for fat and water)

   d. If the individual images at different echo times have SNR of 50 for water-only pixels, what will be the SNR of the water image be?
Medium Answer Questions (25% of Points)

4. **Projection-Reconstruction with Motion.** An object is moving *rigidly* in-plane with periodic “cardiac” motion described \( \Delta x(t) \) and \( \Delta y(t) \), which are both band-limited to 50Hz. The period is at least 50ms, but can vary over time by about 10%.

   a. If the object shift due to motion is \( (\Delta x, \Delta y) \), what is the corresponding k-space modulation pattern?

   b. If no motion correction is performed and a golden-angle ordering is used, explain the impact on the image due to motion.

   c. Using of projections (from \( -k_{\text{max}} \) to \( k_{\text{max}} \)), what is the minimum number needed to determine the position relative to a complete static reference image of the object? Explain how you would measure position.

   d. Describe a projection angle order that ensures that you measure the position at a high frame rate, and describe this rate.

   e. How many projections are needed to acquire a FOV of diameter 20cm with image resolution of 1mm, assuming gradient amplitude and sampling rate are not limitations?
5. **Diffusion:** You are imaging a species that has diffusive motion with a uniform (rather than Gaussian) distribution. The diffusion probability density function is \( p(\Delta x) = \frac{1}{X} \) for \( |\Delta x| < \frac{X}{2} \), where \( X = \sqrt{D\Delta t}/2 \), and \( D \) is the diffusion coefficient (mm\(^2\)/s, for example). Assume magnetization starts at equilibrium.

a. Using the sequence at right, where diffusion gradients have area \( GT \) (mT/m*ms, for example) but are instantaneous, write an expression for the signal at TE as a function of \( G, T, D, \Delta t, TE \) and \( T_2 \).
6. **Sampling and SNR:** A researcher is sampling an image using full-k-space 2D Cartesian sampling, sampling in both \( k_x \) and \( k_y \) over an extent from \(-k_{\text{max}}\) to \( k_{\text{max}}\). The researcher wants to improve the SNR.

   a. What is the impact on (i) SNR, (ii) scan time and (iii) SNR efficiency of increasing the FOV in the *phase encode* direction by a factor of 1.5?

   b. The researcher instead chooses to sample the central 50\% of the \( k_y \) lines twice, and simply adds the acquired signal at each of these lines before performing the reconstruction. Write an expression for the PSF in the \( y \) direction, i.e. \( p(y) \), and explain the effect of extra sampling on the PSF.

   c. What step must be added to the reconstruction in order to not affect the point-spread function?

   d. Assuming (c) is done correctly, what is the impact on noise spectral distribution (short, qualitative explanation).

   e. Does the SNR efficiency change with the sampling in b and the reconstruction in c, and if so, by what amount?

   f. The researcher has a new idea and instead of (a), increases the FOV in the *readout* direction by a factor of 1.5, while also increasing the sampling rate (and bandwidth) by a factor of 1.5. How does the SNR change?
Long Answer Questions (60% of Points)

7. **CPMG Sequence Design:** Consider a non-selective (3D) CPMG sequence with 180° refocusing pulses where you wish to image with 1mm resolution and a FOV of 40cm in the readout direction in a minimum time. The system gradients are maximum amplitude 50mT/m, maximum slew rate 200mT/m/ms (250µs rise time), and the receiver can sample with a maximum half-bandwidth of 100kHz. The peak B₁ amplitude is 15µT.

   a. What is the minimum RF pulse duration for 180° pulses (rectangular pulses)?

   b. If the z-resolution is 2mm, and crusher pulses (on Gₓ) must dephase 1 cycle over a voxel, what is the duration of each crusher pulse?

   c. What is the readout gradient amplitude (mT/m), and plateau duration (ms)?

   d. What is the minimum echo spacing (in ms)? Echo spacing is the time from one refocusing pulse to the next or (equivalently) from one spin echo to the next.

   e. Draw the pulse sequence (RF, Gz and Gx) from the 90° excitation to the second refocusing pulse that corresponds to the minimum echo spacing. Label all scales and axes.
8. **Spin Echo Trains.** For this question you are to design a CPMG spin-echo-train sequence with constant 100° refocusing pulses over all of the echo train (except the first pulse). Assume the magnetization starts at equilibrium ($M_0$) – you are just looking at dynamics from equilibrium here.

   a. What should the flip angle of the first refocusing pulse (after the 90° excitation) be, and why?

   b. Draw the EPG coherence pathway diagram for the first 3 echoes

   ![EPG coherence pathway](image)

   c. How many distinct pathways lead to the observed signal at the 3rd spin echo? (List the pathways by the EPG states before/after each refocusing pulse.)

   d. Give an expression for the signal on echoes 1 and 2 as a function of TE, $T_1$ and $T_2$. You may include terms such as sin(90°) in this expression.
9. **Echo-Planar Imaging (EPI).** Consider a single-shot, bipolar 2D EPI imaging trajectory used to image the 2D object below, which has amplitude 1 for points within a radius of 75mm of the center and 0 elsewhere. This can be written as \( M(x,y) = \text{circ}(r/A) \) where \( A = 75\text{mm} \) and \( r = \sqrt{x^2 + y^2} \), and \((x,y)\) are image-space coordinates. The object is imaged with 2D EPI with a 25cm FOV and 1mm resolution in both \( x \) and \( y \) directions.

b. Assume that there is a constant phase difference between odd and even phase-encode lines \((k_y)\) of \( \pi/4 \) (i.e. even lines are multiplied by \( \exp(i\pi/8) \) and odd lines by \( \exp(-i\pi/8) \)). Give an expression for the resulting magnitude image, to within a constant scale factor. (writing \(|a+bi|\) is sufficient.)

c. Sketch the resulting magnitude image at right, labeling magnitudes (within a scale factor).

d. Now there is additionally a sampling delay two samples in \( k \)-space along the whole trajectory. (Effectively odd lines are shifted in one direction along readout by 1 sample, even lines shifted in the other direction by one sample.) Give an expression for the magnitude image. (writing as \(|a+bi|\) is sufficient.)

e. Describe differences between the image resulting in (c) and the image in (a,b).
10. **Spiral Design**: You are asked to image with 1mm resolution and 12cm (diameter) FOV using Archimedean spiral imaging using a number of interleaves between 10 and 16. For the Archimedean spiral, $k = A\theta \exp(i\theta)$, and $k_x = \text{real}\{k\}$, $k_y = \text{imag}\{k\}$, where $A$ is a constant.

a. What is the effect of off-resonance (briefly)? What is a good choice of number of interleaves to minimize this effect?

b. Is there any image quality benefit of reducing the number of interleaves slightly here? Explain?

c. Draw the desired k-space trajectory at right for a single interleaf of this trajectory, labeling scales.

d. If we later decide that we need *double* the FOV, how can we use this same waveform to achieve our imaging goal?

e. Write an equation for each of the gradients $G_x(t)$ and $G_y(t)$ if $\theta(t) = Bt$.

(This Question Continues on Next Page)
f. Describe what considerations must be made to address each of the following, or “none” if the system parameter does not affect the waveform design. *Answer briefly.*

   i. Maximum gradient amplitude (\(G_{\text{max}}\))

   ii. Minimum data sampling interval (\(T_s\)) rate

   iii. Maximum gradient slew rate