RAD229: Final Exam – 2015/2016
You will have 3 hours to complete this Exam

Name: ________________________________
Student ID: ____________________________

General Instructions:

1. There are 11 questions and 9 pages total.
2. You may use notes including lectures, homework, solutions or Matlab script text on the course website, and you may use an electronic device only for reading such material.
3. You may not use Matlab or the Internet, or other calculating devices of any sort.
4. Please answer questions on the exam, and show your final answer clearly, with appropriate units.
5. You may request more paper if needed. If you use the back of pages, make it clear what question you are answering please!
6. Show your reasoning and work, as this will often earn you partial points.
7. For this exam, please a gyromagnetic ratio value $\gamma/2\pi = 40.0 \text{ MHz/T}$
8. The following plot, which we have used repeatedly, is given for the above $\gamma$ value and gradients with maximum amplitude 50mT/m and maximum slew rate 200mT/m/ms:

![Minimum Gradient Duration vs Area (γ/2π=40MHz/T)](image)

Please do not proceed to the next page until the exam begins.
Short Answer Questions (25% of Points)

1. For a 50mT/m constant gradient, used as a readout gradient, with 1mm resolution and 20cm FOV, what is (i) the readout bandwidth, (ii) the bandwidth per pixel and (iii) the readout duration.

\[ \gamma / 2\pi G = 2000 \text{ kHz/m} = 20 \text{ kHz/cm}. \]

(i) \( \text{BW} = 400 \text{ kHz or } +/\text{- }200 \text{ kHz}. \)

(ii) \( \text{BW/pixel} = 400 \text{ kHz} \times (1 \text{ mm}/200 \text{ mm}) = 2 \text{ kHz/pixel}. \)

(iii) \( 1/400 \text{ kHz} = 2.5\mu s \text{ sample spacing} \times 200 = 500 \text{ ms}. \)

2. What are two methods to efficiently acquire an image with \( T_2 \) weighting, using a 3D acquisition? Some \( T_1 \) weighting is okay, but the contrast should be predominantly \( T_2 \)-weighted. Briefly explain the advantages and disadvantages of your two methods with respect to each other.

- \( 3D \) \( T_2 \)-weighted FSE, with a modulated flip angle train. The flip angle train will introduce some \( T_1 \) weighting, but the sequence is very immune to off-resonance.
- \( T_2 \)-prepared bSSFP, sampling with a centric weighting. Some contrast will be lost at higher \( k \)-space and there is more sensitivity to off-resonance but this approach is probably faster than \( 3D \) FSE.

3. EPG is formulated using the assumption that the magnetization across a voxel behaves uniformly, and that the voxel has a rectangular profile \( \text{rect}(x/\delta x) \) where \( \delta x \) is the resolution. Under these assumptions, the signal using EPG is assumed to be that in the \( F^0 \) state. Explain how, given all EPG states (that is \( F^+_n, F^-_n, \) and \( Z_n \)), you can calculate the signal if the voxel shape is \( p(x) \) instead of \( \text{rect}(x/\delta x) \).

The transformation from EPG to \( M_{xy} \) can provide this solution – after generating \( M_{xy}(x) \), you can simply calculate the convolution of \( M_{xy} \times p(x) \) evaluated at \( x=0 \).
4. Consider the signal $s_1$ from a CPMG sequence $(90_{\pi/2} - \alpha_0 - \alpha_0 - \alpha_0)$ and $s_2$ from an ‘anti-CPMG’ sequence $(90_0 - \alpha_0 - \alpha_0 - \alpha_0)$ where $\alpha = 120^\circ$ and the subscripts represent the phase of the RF pulses. Both sequences have appropriate crusher pulses and the time from 90 to first $\alpha$ is half that of the time between $\alpha$ pulses. A researcher argues that for the sequence $(90_\phi - \alpha_0 - \alpha_0 - \alpha_0)$, that the signal $s_\phi$ can be written as a simple linear combination of $s_1$ and $s_2$. Is he or she correct? Explain clearly.

5. At the Ernst angle ($\alpha_e$), consider a balanced SSFP and perfectly spoiled sequence. Immediately after the excitation, what are the relative values of (i) $|M_{xy}|$ and (ii) $M_z$ between these sequences.
Medium Answer Questions (30% of Points)

6. **Diffusion and CPMG:** A researcher decides to use a CPMG spin-echo train for diffusion-weighted imaging, but simply stretches out all the crusher gradients so that they have width 10ms, and separation 13.33ms. This brings the echo spacing up to 30ms. The gradients have amplitude 50mT/m, and infinitely fast slew rates.

   a. What is the $b$ value for the signal on the first spin echo, assuming 180º refocusing pulses?

   $$b = (2\pi \times 40 \text{ kHz/mT} \times 50 \text{mT/m}) \times (10 \text{ms}) \times (13.33 - 3.33 \text{ms})^2 \approx 144 \text{ s/mm}^2$$

   b. What is the $b$ value for the signal on the second spin echo, assuming 180º refocusing pulses?

   This is just double (a) since the signal should be attenuated twice

   $$b \approx 288 \text{ s/mm}^2$$

   c. Draw an EPG phase coherence pathway diagram showing the 90º excitation, refocusing pulses with arbitrary refocusing flip-angles $\alpha^\circ$, and the first 2 echoes. Label the EPG states shown in the diagram.

   ![EPG Phase Coherence Pathway Diagram](image)

   d. If the refocusing pulse amplitudes are reduced, explain what happens to the signal sensitivity to diffusion in cases (a) and (b).
7. **Sequence Limits & Efficiency:** A 90°-180°-180°-180° spin-echo-train sequence has 16 echoes with 6ms inter-echo spacing.

   a. What is the duration to play the pulses, rounded to the nearest 10ms, if the RF pulses last 1.5ms?

   b. At 3T, an additional 100ms of “dead time” is required because the RF power (heating / SAR) is at the limit. If we can reduce the main (B₀) magnetic field (and RF modulation frequency) and keep everything else constant, what is the maximum B₀ that will allow us to play the pulses without the need for any dead time?

   c. Assuming polarization is proportional to magnetic field strength to the 0.9 power i.e. (B₀)⁰.⁹, write an expression for the SNR efficiency vs. B₀.

   d. What B₀ maximizes the SNR efficiency? (Explain)
8. **Balanced SSFP**: The signal vs. frequency profiles below are from a balanced SSFP sequence with constant parameters, but at different flip angles.

![Signal vs. Frequency Profiles](image)

- a. What is the repetition time (TR) in ms?
  
  
  b. What is the echo time (TE) in ms?
  
  c. What is the approximate ratio $T_2/T_1$?
  
  d. What is the flip angle for profile (i)?
  
  e. What is the flip angle for profile (ii)?
9. **Circular sampling**: Assume that you have infinite maximum gradient amplitude and slew rate, but can sample with a maximum rate of 250,000 samples / second (4µs sample period). You would like to fully sample k-space on circular trajectories as shown (arbitrary scale) to achieve a resolution of 0.5mm, and FOV of 20cm. Ignore prewinder/rewinder gradients in this problem.

   a. How many circular trajectories are required?

   b. What is the approximate readout duration for the outermost trajectory?

   c. Write expressions for, and draw gradient waveforms to sample this outermost circular trajectory?

   \[
   G_x = A \cos\left(\frac{2\pi}{4.8\text{ms}} t\right)
   \]

   \[
   G_y = A \sin\left(\frac{2\pi}{4.8\text{ms}} t\right)
   \]

   \[A = \frac{1}{\gamma \left(\frac{1}{4.8\text{ms}} \right) \left(\frac{1}{1\text{mm}} - 1\right)}\]

   d. For the trajectory with half the diameter of that in (c), repeat part (c) assuming you minimize the trajectory duration.

   \[
   G_x = A \cos\left(\frac{4\pi}{4.8\text{ms}} t\right)
   \]

   \[
   G_y = A \sin\left(\frac{4\pi}{4.8\text{ms}} t\right)
   \]

   e. What is the approximate efficiency (\(\eta\)) of the overall sampling trajectory compared to that of a Cartesian trajectory? *Explain your answer.*
10. **Interleaved EPI:** An interleaved EPI sequence with 4 interleaves (shown) is used to image an object with signal \(s(x,y)\), with perfect correction for delays and odd/even line phase effects, and FOV of \(F\) in both \(x\) and \(y\). *Ignore initial phase-encode and readout dephaser gradients throughout this question – just follow the trajectory shown.*

a. Assuming ramp sampling, 40x40cm FOV and 2x2 mm resolution, what is the duration of each EPI interleaf?

b. Draw the readout \((G_x)\) gradient for the first 5 k-space lines, labeling widths and heights.

c. What is the displacement of fat (in mm) with respect to water at 3.0T?

d. In terms of \(s(x,y)\), what are the images reconstructed from each interleaf (1,2,3, and 4) separately (to within a linear phase in \(y\))?  

e. If a contrast injection happens exactly halfway through the scan and *instantly* doubles the signal across the whole object, quantitatively describe the overall image if the interleaves are acquired in order 1, 3, 2, 4 in the diagram above.

f. Repeat (e) if the interleaves are acquired in order 1, 2, 3, 4.
11. **Spiral Imaging.** Consider the Archimedean spiral-in / spiral-out trajectory shown below, with \( \mathbf{k} = B \theta(t) e^{i|\theta(t)|+i\varphi} \), where \( \theta(t) = At \), \( \varphi \) is the angle of each interleaf, \( A \) and \( B \) are proportionality constants, \( k_x = \text{real}\{\mathbf{k}\} \) and \( k_y = \text{imag}\{\mathbf{k}\} \). An image is acquired with \( N \) interleaves using this trajectory, using \( \varphi = [0,1,2,\ldots,N-1] \times (\pi/N) \).

   a. What are constants \( A \) and \( B \)?

   b. What are the resolution and field-of-view (FOV), for \( N=5 \)?

   c. Draw the k-space trajectory for one interleaf, labeling distances/locations appropriately.

   d. Assuming adequate reconstruction, sketch the point-spread function resulting from this trajectory, labeling relevant distances/locations (including FWHM) appropriately.

   e. Is the PSF real-valued or complex valued? *Explain.*

   f. Explain how the PSF changes for 250Hz off-resonance, and does that change the answer to (e)?