RAD229: Final Exam – 2017/2018
You will have 3 hours to complete this Exam

Name: ________________________________
Student ID: ____________________________

General Instructions:
1. There are 12 questions and 10 pages total.
2. You may use notes including lectures, homework, solutions or Matlab script text on the course website, and you may use an electronic device only for reading such material.
3. You may not use Matlab or the Internet, or other calculating devices of any sort. If you have to do calculations, simple estimates will be fine.
4. Please answer questions on the exam, and show your final answer clearly, with appropriate units.
5. You may request more paper if needed. If you use the back of pages, make it clear what question you are answering please!
6. Show your reasoning and work, as this will often earn you partial points.
7. For this exam, please a gyromagnetic ratio value $\gamma/2\pi = 40.0 \text{ MHz/T}$

Please do not proceed to the next page until the exam begins.
**Short Answer Questions (25% of Points)**

1. Identify each of the preparation sequences below. Describe (briefly) how it works and why it is used. Assume 3T, and 2D spin-echo imaging following each preparation, with slice along z, readout on x and phase-encode on y.
   
   i) Fat saturation, works by selectively exciting fat based on the frequency shift from water, then dephasing fat. Used to remove fat from the image.
   
   ii) Inversion. Causes $M_z$ to be negated, so that there is longitudinal recovery prior to the imaging sequence to separate tissue based on $T_1$ differences. Used either to enhance $T_1$ contrast, or to null species as in STIR or FLAIR imaging.
   
   iii) RF played at an off-center frequency
   
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2. Consider $S_1$ and $S_2$ as a single repetition of two different pulse sequences. Assume that magnetization propagation over $S_1$ can be written $M' = A_1M + B_1$ and over $S_2$ can be written $M' = A_2M + B_2$, where $M$ and $M'$ are the magnetization at the start and end. Now consider a sequence $S_3$ that consists of $S_1S_2S_1$, played repeatedly. Write an expression for the steady-state magnetization at the start of sequence $S_3$.

3. Given magnetization described by EPG coefficients as $F_{+1} = 0.5$, $F_{-1} = 0.25$, $Z_1 = 0.5$ (all other states zero), what are the EPG coefficients after a one-cycle gradient with duration $\tau = T_1\ln 2$ and $T_1 = 2T_2$? $\ln(x)$ here is the natural (base e) logarithm.
4. Compare the SNR efficiency of a 20 interleaf spiral to (i) a radial-out sequence and (ii) to a 40 interleaf sequence. In all cases assume that you are scanning a 20cm FOV with 1mm resolution as fast as possible on a standard MRI system, and that the duration of RF pulses is negligible.

5. An RF excitation $B_1(t)$ has a bandwidth of 2kHz, and is used with a gradient to excite a 10mm-thick slice at position 0cm. What do you do to this excitation to simultaneously excite slices centered at 0, 2 and 4cm if the same gradient is used?

6. Consider a 10-interleaf spiral sequence with readout duration of 5ms, and 400Hz off-resonance. (i) Describe, qualitatively, the effect of off-resonance on the image. (ii) Repeat (i) if the same waveform is used to image with spiral-in, spiral out for 10ms per interleaf and 5 interleaves total.
Medium Answer Questions (15% of Points)

7. Consider T2-prepared 3D imaging (256x128x32) using an RF-spoiled gradient echo sequence to form the image, aiming to emulate contrast of a spin-echo sequence with TE=100ms. The T2-prep duration is 100ms, and the sequence samples 200 lines of k-space with TR=5ms before repeating the T2-prep pulse.

![RF-spoiled Imaging Diagram]

a. How many T2-prep pulses must you play (total)?

b. What is a good strategy to sample k-space?

c. Comment on the image contrast and spatial resolution effects

d. Comment on trade-offs of the flip angle
8. EPI. Assume an object \( m(x,y) \) within the FOV. We will image the object using EPI with 128 ky lines total, but there is a \( \pm \beta \) (radian) phase difference between leftward and rightward going lines that is uncorrected.

a. For single-shot EPI, what is the resulting image?

\[
\text{Let the image be } m(x,y). \text{ The odd lines form an image } 0.5\exp(i\beta)[m(x,y)+m(x,y+\text{FOV}/2)] \\
\text{The even lines form an image } 0.5\exp(-i\beta)[m(x,y)-m(x,y+\text{FOV}/2)] \\
\text{The result is } m(x,y)\cos(\beta) + isin(\beta)m(x,y+\text{FOV}/2)
\]

b. If the image \( m(x,y) \) is as shown below (left), draw the expected image for \( \beta=\pi/10 \) from (a).

c. Now the same trajectory is used for 2-shot interleaved EPI. Comment on the result, and sketch the resulting image (no need to label amplitudes).

d. If, instead of interleaving, one shot is used to acquire the top half of k-space, and the other is used to cover the bottom both going top-to-bottom, comment on (i) the result from the above phase difference and (ii) any pros/cons of using this approach compared with that of part (c).
Long Answer Questions (60% of Points)

9. **CPMG Sequence**: You are asked to design a CPMG sequence with echo-train length of 20, with 90° refocusing pulses and an initial excitation is 90°, using rectangular (non-selective) waveforms for all RF pulses. Ultimately the goal is a 3D acquisition with 1 x 1 x 1 mm³ resolution, with minimal and smooth decay over the echo train. Assume the sampling bandwidth and slew-rates are both unlimited, and maximum gradient amplitude is 50 mT/m. Crusher pulses (on G_z) should dephase one cycle over the voxel. Crushers on other axes are optional. *State any reasonable assumptions!*

   a. List the pulse train (angles, phases). What are the widths of all RF pulses? Assume maximum RF amplitude of 10µT

   b. What are the amplitude and width of each G_z crusher gradient, and the G_x readout?

   c. What is the minimum echo spacing for the sequence (within 0.1ms)?
d. Draw the RF, Gx, and Gz waveforms for the first 3 spin-echoes of the echo train, labeling amplitudes, phase and widths for RF, and amplitudes and width of Gx and Gz (Gy not needed).

e. For this subject, the maximum RF power, in convenient units, is $20 \mu T^2$. What is the minimum TR?
10. **Flyback EPI:** Consider the sequence shown below, which is used for a 200 x 200 image acquisition over a 20 cm FOV, using 10 shots, each with echo train length of 20?

![Flyback EPI Diagram](image)

a. What is the readout bandwidth in Hz/pixel?

\[ T = \frac{1\text{ms}}{200} = 5\text{µs} = 0.005\text{ms} \]
\[ \text{à 200 kHz full BW, or 1kHz/pixel.} \]

b. What is the readout gradient amplitude?

\[ \gamma/2\pi \text{ Gx maps 1kHz to 1mm so (40kHz/mT) G (0.001m)} = 1kHz, \text{ G = 25mT/m} \]

c. Describe the impact of 100Hz off-resonance on the image as completely as you can.

There is a shift in the readout direction of 100Hz/1kHz or 0.1 pixel or 0.1mm.

The echo period is 1.8ms, so the shift in y is \( (0.1kHz)(20\text{cm})(1.8\text{ms})/10\text{shots} = 3.6\text{mm} \)

d. Explain how the answer to (c) changes if a half-ky acquisition is whereby the echo train length is 12 and 10 shots are still used (sampling 60% of ky).

The ky velocity does not change, so it is completely unchanged.
11. **Spiral:** In this question, use Archimedean spirals with 5 interleaves to make an image that has resolution of 1.25mm and a FOV of 15cm. The system has a maximum gradient amplitude of 50 mT/m and maximum slew rate of 200 mT/m/ms, and arbitrary sampling rates up to a maximum sampling rate of 500,000 samples/second.

a. How many full turns are needed in each spiral?

b. What is the maximum k-space extent, and spacing between samples on adjacent interleaves, both in cm⁻¹?

c. Assuming the spiral is mostly gradient-amplitude limited, what is the approximate duration of each interleaf?

d. Assuming 1ms for each RF pulse, what is the total scan time?

e. With the shortest readout duration possible, and 1.5 ms for RF, phase-encode, pre-winders and ramps (all other stuff!) per TR, what is the total scan time for a Cartesian sequence with the same time between samples?

f. If the maximum sampling rate was 250,000 samples/s (±125kHz bandwidth) how does this affect (d) and or (e)?
12. **bSSFP:** Consider a balanced SSFP sequence, and material with $T_1 = 300\text{ms}$, $T_2 = 100\text{ms}$.

   a. For a flip angle that alternates between $90^\circ$ and $-90^\circ$, what is the steady-state signal for TR=$4\text{ms}$, TE=$2\text{ms}$, on-resonance?

   b. What flip angle would give the peak signal for this material, on resonance, and what is this signal?

   c. Draw the distribution of signals in the $m_x$-$m_z$ plane for the flip angle in (b) and all off-resonances

   d. For the flip angle in (b), give an *expression* for the off-resonance precession $\varphi$ over TR that will lead to the same signal at TE as in (a)?

   e. Sketch the signal vs frequency (general shape is fine), for the given parameters in (a), but flip angle in (b). Labeling critical frequencies and amplitudes as possible from previous parts.