The difference $\hat{p}_1 - \hat{p}_2$ just as it does to $\hat{p}$, and $\hat{p}$. So we can use a $z$-test.

The central limit theorem applies to the

$\hat{p}$'s estimate by $\hat{p}_1 = 55\%$, $\hat{p}_2 = 58\%$.

$H_0$ is stated as $\hat{p}_1 - \hat{p}_2 = 0$.

$H_1$ is $\hat{p}_1 - \hat{p}_2 \neq 0$.

$p$ is unusual if $p$ is equal to $\hat{p}_1 - \hat{p}_2$.

It is common to look at the difference

$\hat{p}_1 - \hat{p}_2$ proportion of all likely voters approving this month

$\hat{p}_1 - \hat{p}_2$ proportion of all likely voters approving last month

We want to assess whether

Evidence to conclude that the trend has changed?

This month, a poll of 1,200 likely voters resulted in a rate of 58%. Is this significant?

Last month, the President's approval rating in a sample of 1,000 likely voters was 55%.

The two-sample $z$-test
\[ z = \frac{0.2020}{0.03} = \frac{\frac{0.991}{\sqrt{2}}}{0 - (1d - 2\bar{d})} \]

\[ SE_{\text{observed difference}} + SE_{\text{expected difference}} = \]

An important fact is that \( P \) and \( P' \) are independent; then

\[ SE_{\text{difference}} = \frac{SE_{\text{observed difference}}}{(1d - 2\bar{d})} = \frac{SE_{\text{expected difference}}}{(1d - 2\bar{d})} = z \]

We can use a z-test for the difference \( P - P' \):

The two-sample z-test
Since the standard deviations are large for husbands and also for the wives, even if they were independent, the small differences in ages would not be significant. The two-sample t-test is not applicable since the two samples are not independent.

<table>
<thead>
<tr>
<th>Husbands' age</th>
<th>Wives' age</th>
<th>Age difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>27</td>
<td>0</td>
</tr>
<tr>
<td>66</td>
<td>66</td>
<td>0</td>
</tr>
<tr>
<td>31</td>
<td>32</td>
<td>1</td>
</tr>
<tr>
<td>70</td>
<td>71</td>
<td>1</td>
</tr>
<tr>
<td>41</td>
<td>43</td>
<td>2</td>
</tr>
</tbody>
</table>

The ages of the couples:

Do husbands tend to be older than their wives?

The paired-difference test
The independence assumption is in the sampling of the couples.

\[ SE(\bar{d}) = \frac{s_d}{\sqrt{n}} \]

where \( s_d \) is the standard deviation of the differences of the \( n \) couples.

**H0**: Population difference has mean zero

**Paired-t-Test**

Since we have paired data, we can simplify and analyze the differences obtained from each pair.

The paired-difference test
The sign test lacks the virtue of easy interpretation due to the analogy to the binomial.

The p-value of the sign-test is less significant than that of the paired t-test. This is because the latter uses more information, namely the size of the differences. On the other hand, the sign test has the virtue of easy interpretation due to the analogy to the binomial.

The test statistic for the sign test is given by

\[ z = \frac{\text{sum of } (+)}{\text{sum of } (-)} \]