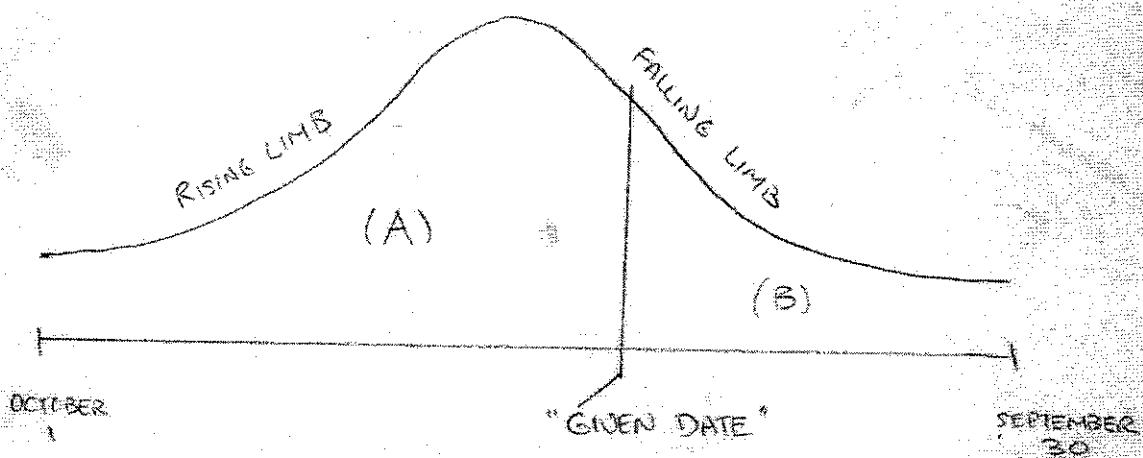


SUMMARY

Historically, two problems inherent to the interpretation of Article V undermine attempts to formulate an administrative process for the Compact:

(1) Timing Problem

Annual hydrograph:



Rising Limb Characteristics

- excess flow at gage
- reservoirs fill to maximum volume
- minimum demand for water

Falling Limb Characteristics

- minimal water at gage
- reservoirs release storage
- maximum demand for water

2

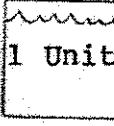
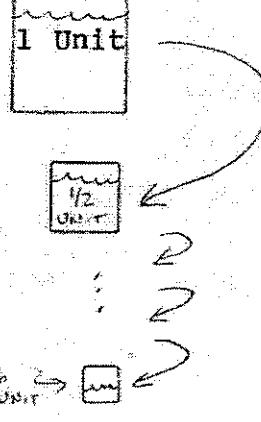
The hydrology and distinct water supply/demand characteristics of the rising and falling limbs are responsible for a timing problem which dictate that each limb of the hydrograph be apportioned separately according to Article V.

If each limb is not apportioned separately, the article V calculation would have the effect of dividing flow at the downstream gage for use upstream at a later date. This is a physical impossibility. Secondly, (A) and (B) should be apportioned separately so that each state may quantify that portion of (A) which can be put in storage and used to supplement irrigation after the given date. Flow apportioned during (A), and that water which has been stored will not be re-apportioned after the given date. Likewise, Post-1950 diversions satisfied from water apportioned before the given date, are not included in the apportionment after this date.

(2) The Water Budget Problem

Conceptually, Article V undermines attempts to prepare an adequate water budget because, it confuses the apportionment by mixing together two "kinds" of water - a depletable flow and a diverted flow. Depletable flow is water which is physically available "in the bucket". The diverted flow (or divertable flow equivalent), is a measure of how many times water in the bucket

could be used and reused. Mixing these two kinds of water together confuses units, which makes balancing a water budget nearly impossible. In order for the apportionment formula in Article V to work properly, the water budget requires us to use similar units. The only way to balance the Article V water budget is to convert all depletion values to their divertable flow equivalents. Conversely, divertable flow equivalents could be converted back to depletable flows when it is necessary to calculate the quantity of water to be released from storage.

DEPLETABLE FLOW	DIVERTABLE FLOW EQUIVALENT
<u>Examples from Art.V</u>	<u>Examples from Art. V:</u>
a) post-1950 Δ's	a) post-1950 diversions in MT
b) flow at the downstream gage	b) post-1950 diversions in WY
"What water physically exists."	"A measure of how this water is used and revised." revised
	 assume 50% return flow
SUBTOTAL 1 Unit	SUBTOTAL 1.99999 Units
1.20? ← 40%	ARTICLE V 2.9999 "UNITS" → 60% 1.79?

So: How do we convert a depletion or depletable flow to its divertable flow equivalent?

This kind of conversion is already being done on a theoretical and practical level.

THEORY

Hydrologic computer models developed by both Wyoming and Montana incorporate monthly return flow factors. The factors vary from month to month and from basin to basin, but still have the effect of changing the inflow to a specific node to a "divertable flow equivalent" as the use and re-use of water is accounted for through that particular node.

PRACTICE

Water commissioners in Wyoming are responsible for releasing water from reservoirs on regulated streams to meet downstream demands. The quantity of water released from storage is not equal to the sum of the individual diversions for any specific time period. The release is based on converting from the "divertable flow equivalent" back to the depletable flow. (Actual volume in storage.) The decision on what quantity of water will be released from storage may be based on experience, or trial and error. Furthermore, computers could be programmed to aid in the decision making process, especially on larger streams.

6

Calculation of divertable flow equivalent conversion factors:

Problem: Assuming a 50% return flow, determine the divertable flow equivalent of 1 unit of stored water.

$$1 \text{ Unit} = 1 + 0.5 + 0.25 + 0.125 + 0.0625 \dots$$

which mathematically can be expressed as

$$1 [1 + \text{Limit}_{\rightarrow \infty} \sum_{x=1}^{x=1} (.50)^x] = 2.0 \text{ units}$$

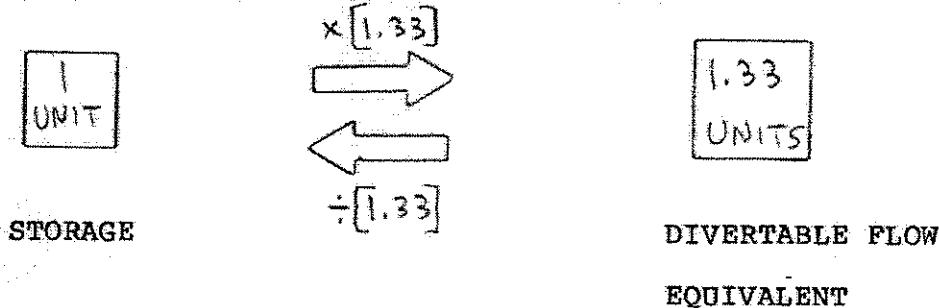
(divertable flow
equivalent)

Likewise:

Return Flow Factor	Divertable Flow Equivalent
50%	$[1 + \text{Limit}_{\rightarrow \infty} \sum_{x=1}^{x=1} (.50)^x]$ ≈ 2.00
33 1/3%	$[1 + \text{Limit}_{\rightarrow \infty} \sum_{x=1}^{x=1} (.333)^x]$ ≈ 1.50
25%	$[1 + \text{Limit}_{\rightarrow \infty} \sum_{x=1}^{x=1} (.25)^x]$ ≈ 1.33
10%	$[1 + \text{Limit}_{\rightarrow \infty} \sum_{x=1}^{x=1} (.10)^x]$ ≈ 1.11

For Example

Assuming an average return flow of 25%

**STRATEGY FOR ARTICLE V CALCULATIONS:**

- (1) Be consistent with units. Do all calculations with actual diversions, or divertable flow equivalents.
- (2) Break the hydrograph into two parts. Calculate a rising limb and falling limb apportionment. Storage from the rising limb will supplement irrigation during the falling limb.
- (3) For the rising limb, quantify the storage limit based on the total flow at the gage, total post-1950 changes in storage, and total post-1950 water use from Oct 1 to the end of the rising limb.

(4) Post-1950 uses on the falling limb side of the hydrograph satisfied from storage, do not enter the Article V allocation for the falling limb. That is:

$$\boxed{\text{Falling limb post-1950}} \quad \boxed{\text{Total post-1950}} \quad - \quad \boxed{\text{Divertable flow equivalent released from storage to satisfy those post-1950 diversions.}}$$
$$\text{diversions for Article} \quad = \quad \text{diversions}$$

$$SL_M = [(A+B+C+D+E) \times \%_M] - E$$

$$W_M = [(A+B+C+D+E)] \times \%_M = SL_W$$

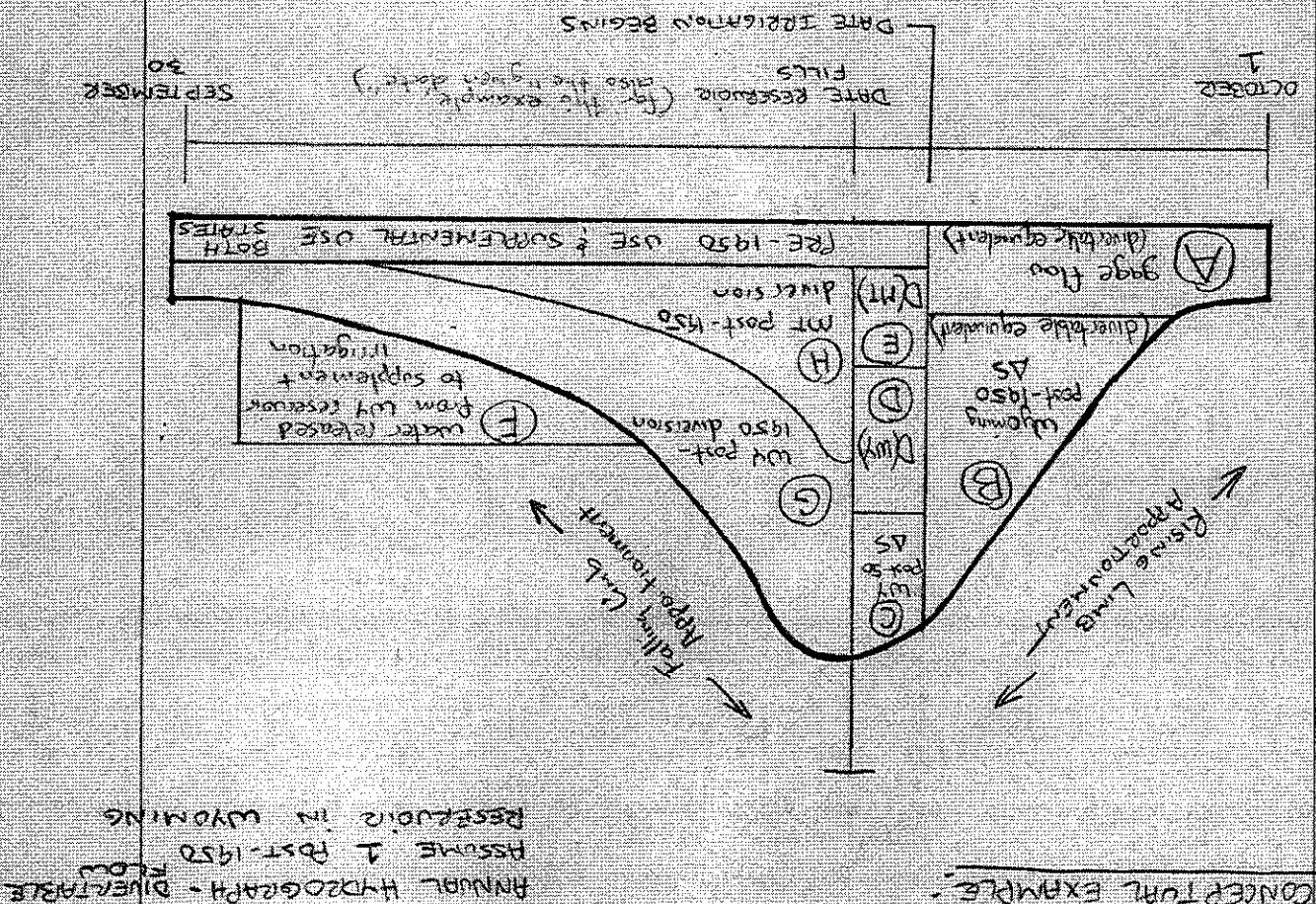
② How much of the head loss appropriaed flow can be physically stored. (This defines the storage limit)

$$MT = ((A+B+C+D+E) \times \%_M)$$

$$W_H = ((A+B+C+D+E) \times \%_M)$$

① How much discharge can each share legally appropriate? and which side of the hydraulics can each share legally appropriate?

RISING LINE APPROPRIEMENT



FALLING LIMB APPORTIONMENT

- ① what quantity of post-1950 diversions can legally be appropriated from natural stream flow by each state?

$$\text{Wyoming: } \frac{G}{H+G} = .60$$

$$\text{Montana: } \frac{H}{H+G} = .40$$

- ② what quantity of post-1950 diversions can Wyoming legally appropriate from natural stream flow, and also from storage -

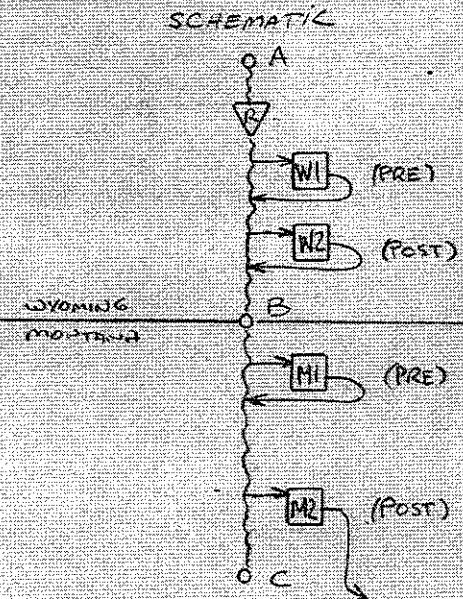
$$\text{Total} = G + F$$

Yellowstone Compact Allocation Example

1. All values in acre-feet

2. Inflows to system at point A:

Oct	430
Nov	320
Dec	150
Jan	150
Feb	100
March	200
April	400
May	5990
June	3990
July	3190
Aug	1500
Sept	500
	2000



3. Reservoir is located in Wyoming and it has a post-1950 priority date. There are no reservoirs in Montana.

4. There are four water users on the river. Two are located in Wyoming (W1, W2) and two are in Montana (M1, M2). W1 and M1 have pre-1950 priorities. W2 and M2 have post-1950 priorities. Diversion requirements in acre-feet/month are as follows:

	Montana	Wyoming
pre-1950	M1 = up to 2000	W1 = up to 2000
post-1950	M2 = up to 2000	W2 = up to 2750

5. "Gwen date" for this example is day reservoir fills.
(May 31)

6. 50% of what is diverted returns to the stream each month

7. Because M2 is located very close to point C (compact gage), all return flow enters stream below point C

8. Apportionment - Wyoming - 60%

Montana - 40%

9. Each month's allocation is forecasted at the beginning of the month based on forecasts of inflow and demand. At the end of the month, the apportionment is quantified using actual water-use data.

9/20/84

ALL VALUES ARE
IN ACRE- FTEXHIBIT
#1 aALLOCATION FOR
OCTOBER 1 TO APRIL 31

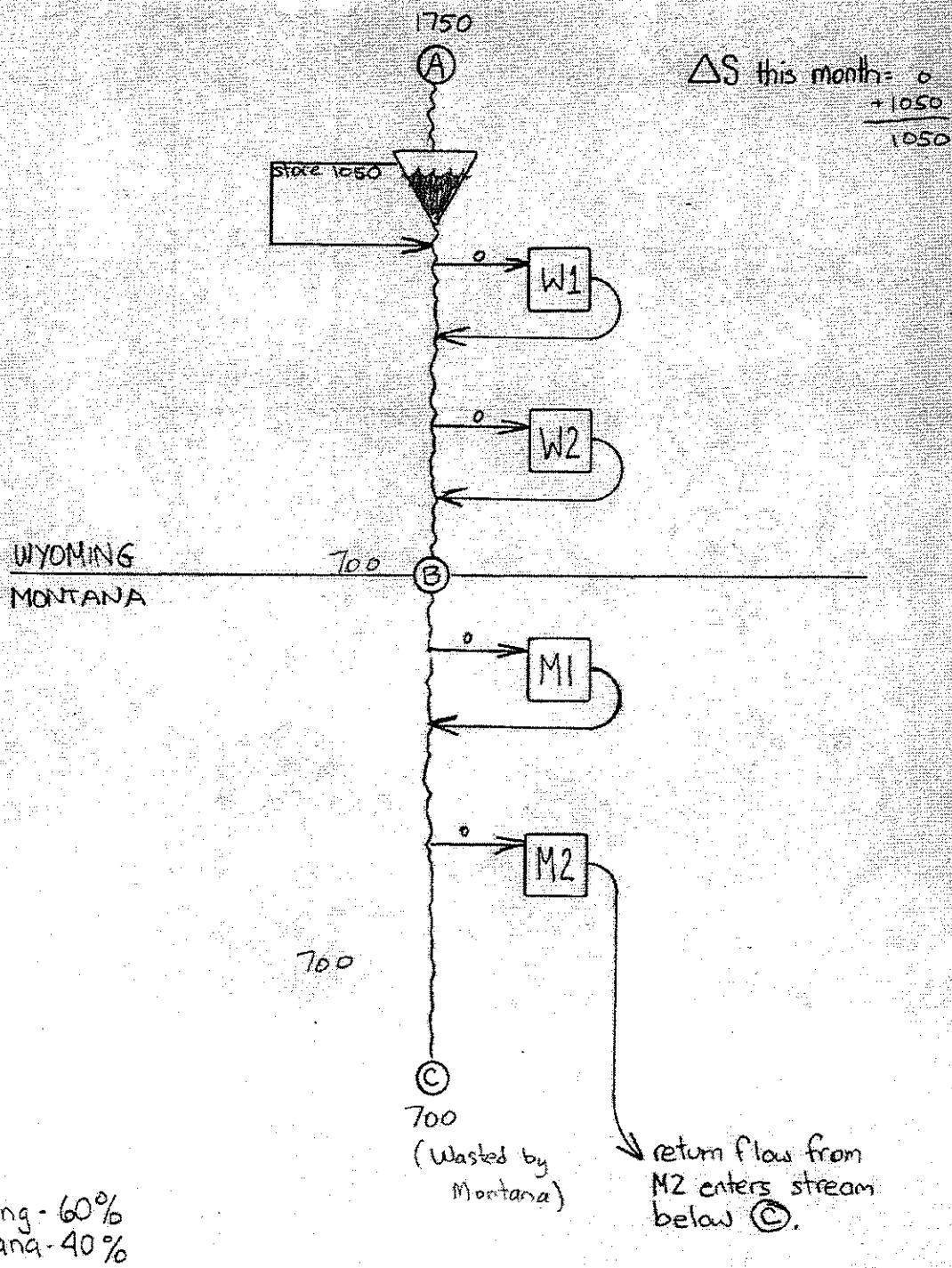
A = inflow to system

B = state line flow

C = flow at gage

→ = agricultural diversion

▽ = post- 1950 reservoir

pre- 1950 W1 = 0
post- 1950 W2 = 0pre- 1950 M1 = 0
post - 1950 M2 = 0

MT-01037

ALL VALUES ARE
IN ACRE - FT

EXHIBIT
#2-a

ALLOCATION FOR
MAY 1 to 31

A = inflow to system

B = state line flow

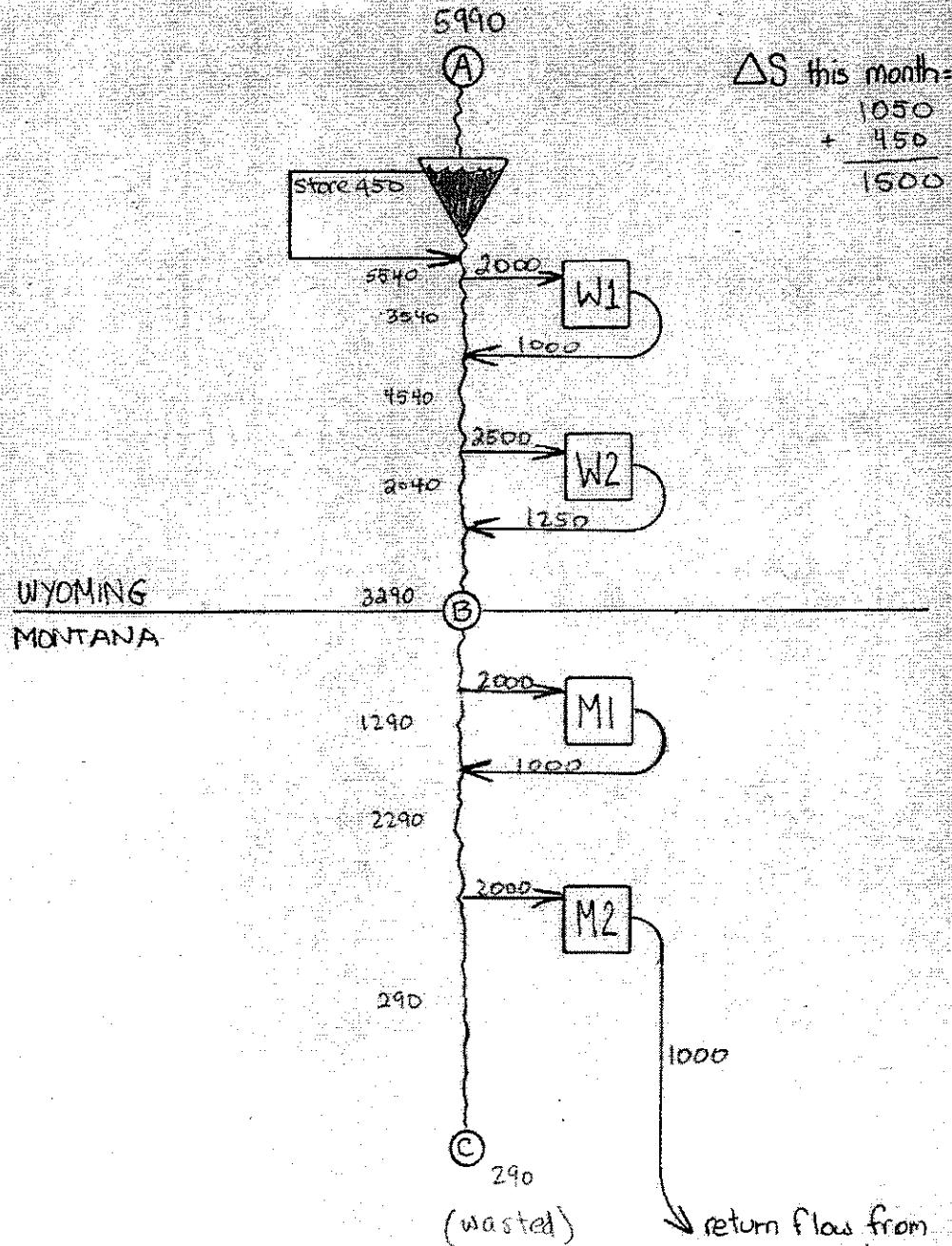
C = flow at gage

agricultural diversion

post-1950 reservoir

pre-1950 W1 = 2000
post-1950 W2 = 2500

pre-1950 M1 = 2000
post-1950 M2 = 2000



Wyoming - 60%
Montana - 40%

MT-01038

RISING LIMB
APPORTIONMENT

EXHIBIT
#2B

- I Convert reservoir changes in storage and flow at the gage to their "divertable flow equivalents".

	<u>TERM</u>	<u>ACTUAL VALUE</u>	<u>X</u>	<u>CONVERSION FACTOR (C)</u>	=	<u>DIVERTABLE FLOW EQUIVALENT</u>
a.	$\Delta S(WY_{post-50})$	1500	x	2.0	=	3000
b.	$\Delta S(MT_{post-50})$	0	x	2.0	=	0
c.	G	(700 + 290) = 990	x	2.0	=	1980

- II Calculate the accumulated diversions $D(WY)$ and $D(MT)$ from October 1 through this date for Wyoming and Montana.

$$a. D(WY) = D(WY)_{previous} + W2_{this\ period}$$

$$2500 = 0 + 2500$$

$$b. D(MT) = D(MT)_{previous} + M2_{this\ period}$$

$$2000 = 0 + 2000$$

- III Calculate the maximum allowable storage limits (SL_W , SL_M) for reservoirs in both states for the period October 1 through today. Use divertable flows or "divertable flow equivalents".

$$a. SL_W = \left[\Delta S(WY_{post-50}) + \Delta S(MT_{post-50}) + D(WY) + D(MT) + G - \right] \times \%_W - D(WY)$$

$$3188 = \left[3000 + 0 + 2500 + 2000 + 1980 \right] \times .60 - 2500$$

$$b. SL_M = \left[\Delta S(WY_{post-50}) + \Delta S(MT_{post-50}) + D(WY) + D(MT) + G - \right] \times \%_M - D(MT)$$

$$1792 = \left[3000 + 0 + 2500 + 2000 + 1980 \right] \times .40 - 2000$$

- IV For the sake of clarity and for future reference, convert each storage limit "divertable flow equivalent" back to an actual volume equivalent.

a. Actual Volume of storables flow (limit) for Wyoming

$$\text{Actual Volume} = SL_w \div \text{CONVERSION FACTOR}$$

$$1594 = 3188 \div 2.0$$

b. Actual Volume of storables flow (limit) for Montana

$$\text{Actual Volume} = SL_m \div \text{CONVERSION FACTOR}$$

$$896 = 1792 \div 2.0$$

IV Calculate the "divertable flow", if any, which has been stored in excess of each states' storage limit. ($\Delta S_{\text{post-50}^*}$)

a. $\Delta S(WY_{\text{post-50}^*}) = \begin{cases} \Delta S(WY_{\text{post-50}}) - SL_w & \text{if } SL_w < \Delta S(WY_{\text{post-50}}) \\ 0 & \text{if } SL_w \geq \Delta S(WY_{\text{post-50}}) \end{cases}$

CASE

$$\frac{SL_w}{3188} ? \frac{\Delta S(WY_{\text{post-50}})}{3000}$$

$$\Delta S(WY_{\text{post-50}^*}) = 0$$

CASE

b. $\Delta S(MT_{\text{post-50}^*}) = \begin{cases} \Delta S(MT_{\text{post-50}}) - SL_m & \text{if } SL_m < \Delta S(MT_{\text{post-50}}) \\ 0 & \text{if } SL_m \geq \Delta S(MT_{\text{post-50}}) \end{cases}$

CASE

$$\frac{SL_m}{896} ? \frac{\Delta S(MT_{\text{post-50}})}{0}$$

$$\Delta S(MT_{\text{post-50}^*}) = 0$$

VI Calculate the accumulated diversions plus the excess divertable flow which has been stored, from October 1 through this date.

a. $Q(\text{accum WY}) = \Delta S(WY_{\text{post-50}^*}) + D(WY)$

$$2500 = 0 + 2500$$

b. $Q(\text{accum MT}) = \Delta S(MT_{\text{post-50}^*}) + D(MT)$

$$2000 = 0 + 2000$$

VII Calculate the total divertable flow accumulated by both states from October 1 through today.

a. $Q(\text{accum}) = Q(\text{accum WY}) + Q(\text{accum MT})$

$$4500 = 2500 + 2000$$

VII Calculate allocable flow to each state.

a. $Q(\text{alloc WY}) = Q(\text{accum}) \times \%_W$

$$2700 = 4500 \times .60$$

b. $Q(\text{alloc MT}) = Q(\text{accum}) \times \%_M$

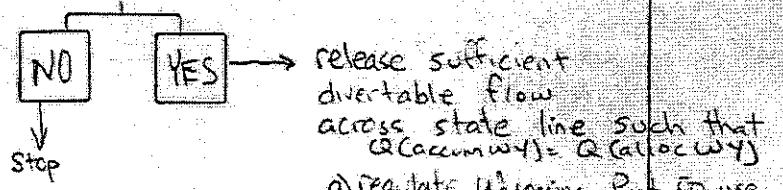
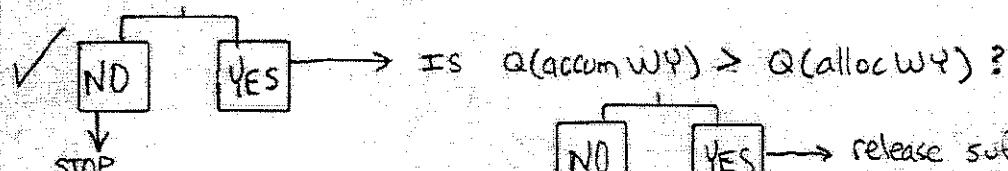
$$1800 = 4500 \times .40$$

IX Compare accumulated divertable flow and allocated divertable flow for each state:

<u>WYOMING (ALLOCATED)</u>	<u>WYOMING (ACCUMULATED)</u>	<u>MONTANA (ALLOCATED)</u>	<u>MONTANA (ACCUMULATED)</u>
2700	> 2500	1800	< 2000

X Determine what action, if any, needs to be taken:

Is more water needed to meet Montana's demand?



a) regulate Wyoming - Post 50 use

b) drawdown storage to meet extra Wyoming demand.

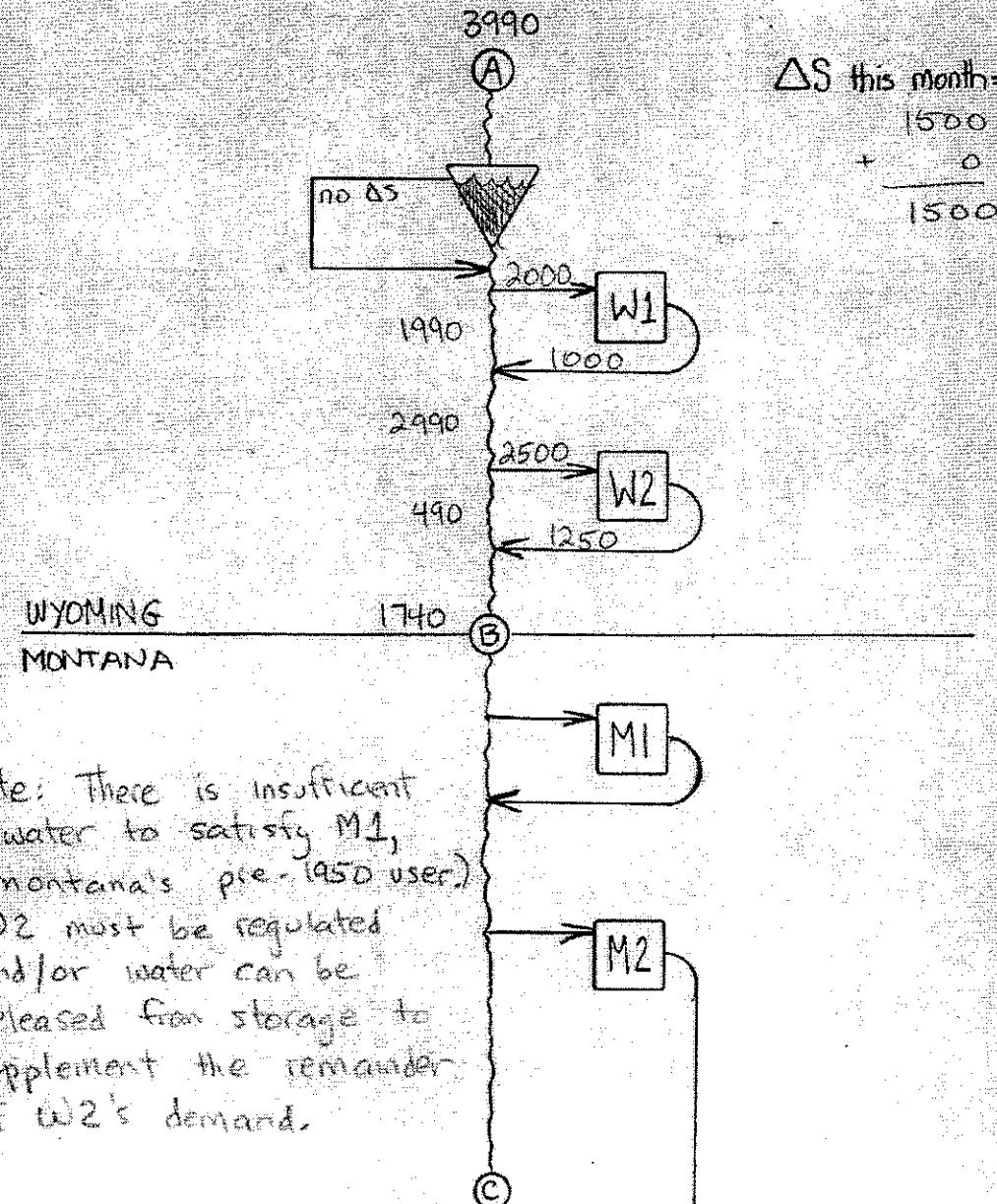
ALL VALUES ARE
IN ACRE - FT

EXHIBIT
HF 3 a

ALLOCATION FOR
June 1 - 30th

- A = inflow to system
- B = state line flow
- C = flow at gage
- \square = agricultural diversion
- ∇ = post-1950 reservoir

pre- 1950	W ₁ = 2000
post- 1950	W ₂ = 2500
pre- 1950	M ₁ = N/A
post - 1950	M ₂ =



return flow from
M₂ enters stream
below C.

ALL VALUES ARE
IN ACRE-FT

EXHIBIT
4 a

ALLOCATION FOR
June 1 to 30th

2nd
Iteration

A = inflow to system

B = state line flow

C = flow at gage

\rightarrow B = agricultural diversion

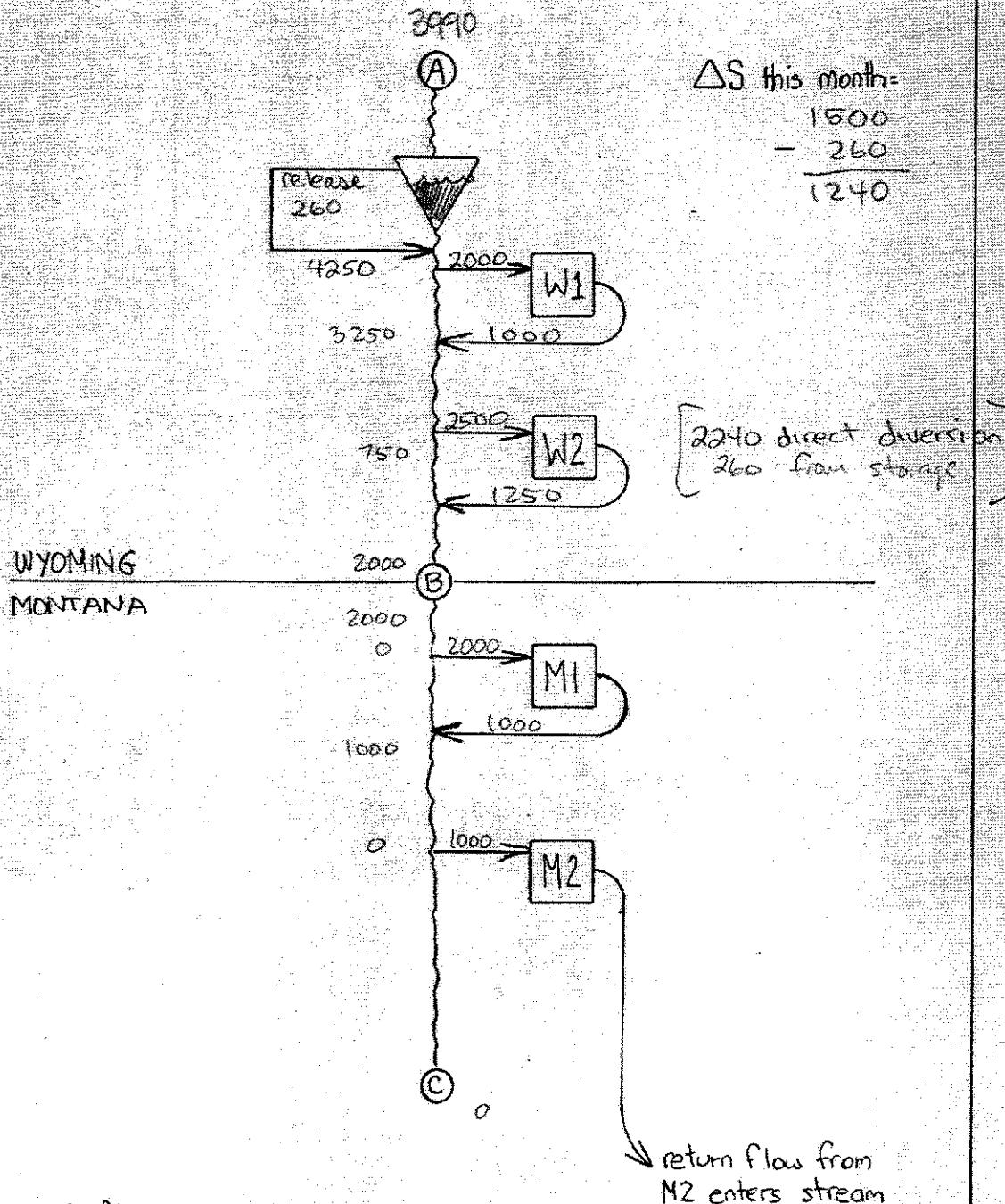
∇ = post-1950 reservoir

pre-1950 W1 = 2000

post-1950 W2 = 2500

pre-1950 M1 = 2000

post-1950 M2 = 1000



Wyoming - 60%
Montana - 40%

return flow from
M2 enters stream
below C.

MT-01043

FALLING LINE 3
APPORTIONMENTEXHIBIT
#4B

I

Calculate the accumulated diversions $D(WY)$ and $D(MT)$ from Oct 1 through this date for Wyoming and Montana:

a. $Q(\text{accumWY}) = D(\text{WY}) \text{ previous} + W2 \text{ this period}$

$$4740 = 2500 + 2240$$

b. $Q(\text{accumMT}) = D(\text{MT}) \text{ previous} + M2 \text{ this period}$

$$3000 = 2000 + 1000$$

II

Calculate the total divertable flow accumulated by both states from October 1 through today:

a. $Q(\text{accum}) = Q(\text{accumWY}) + Q(\text{accumMT})$

$$7740 = 4740 + 3000$$

III

Calculate the allocable flow to each state:

a. $Q(\text{allocWY}) = Q(\text{accum}) \times \%_w$

$$4644 = 7740 \times .60$$

b. $Q(\text{allocMT}) = Q(\text{accum}) \times \%_m$

$$3096 = 7740 \times .40$$

IV

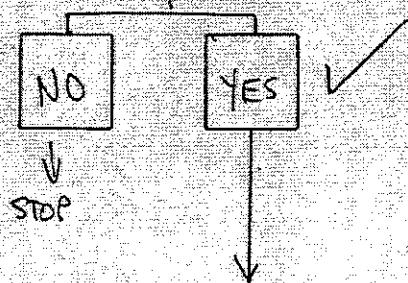
Compare accumulated divertable flow and allocable divertable flow for each state:

a. $\frac{\text{Wyoming (allocated)}}{4644} < \frac{\text{Wyoming (accumulated)}}{4740}$

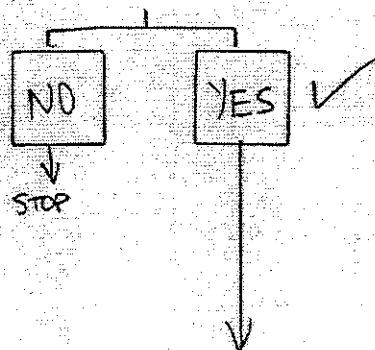
b. $\frac{\text{Montana (allocated)}}{3096} > \frac{\text{Montana (accumulated)}}{3000}$

V Determine what action, if any, needs to be taken.

a. Is more water needed to meet Montana's demand?



Is $Q(\text{accumWY}) > Q(\text{allocWY})$?



release sufficient
divertable flow across
state line such that
 $Q(\text{accumWY}) = Q(\text{allocWY})$

a. regulate Wyoming Post 1950
use.

✓ b. drawdown storage to meet
extra Wyoming demand.

ALL VALUES ARE
IN ACRE- FT

EXHIBIT
44-5-a

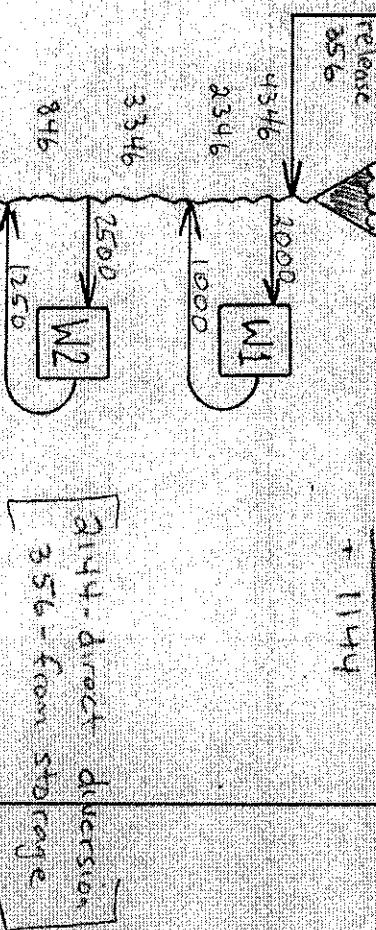
Allocation For
June 1 to 30th

3rd
Iteration

A = inflow to system
 B = state line flow
 C = flow at cage
 = agricultural diversion
 = post-1950 reservoir

pre-1950	$W_1 = 2000$
post-1950	$W_2 = 2500$
pre-1950	$M_1 = 2000$
post-1950	$M_2 = 1096$

$$\Delta S \text{ this month:} \\ 1500 - 356 + 1144$$



WYOMING
MONTANA

2096

WYOMING

MONTANA

1096

2000

1000

96

2000

1000

3346 - direct diversion
356 - from storage

Wyoming - 60%

Montana - 40%

return flow from
M2 enters stream
below C.

FALLING LIMS
APPORTIONMENT

EXHIBIT
- 5B -

I

Calculate the accumulated diversions $D(WY)$ and $D(MT)$ from Oct 1 through this date for Wyoming and Montana:

a. $Q(\text{accumWY}) = D(\text{WY}) \text{ previous} + W2 \text{ this period}$

$$4644 = 2500 + 2144$$

b. $Q(\text{accumMT}) = D(\text{MT}) \text{ previous} + M2 \text{ this period}$

$$3096 = 2000 + 1096$$

II

Calculate the total divertable flow accumulated by both states from October 1 through today:

a. $Q(\text{accum}) = Q(\text{accumWY}) + Q(\text{accumMT})$

$$7740 = 4644 + 3096$$

III

Calculate the allocable flow to each state:

a. $Q(\text{allocWY}) = Q(\text{accum}) \times \%_w$

$$4644 = 7740 \times .60$$

b. $Q(\text{alloc MT}) = Q(\text{accum}) \times \%_m$

$$3096 = 7740 \times .40$$

IV

Compare accumulated divertable flow and allocable divertable flow for each state:

a. Wyoming (allocated) Wyoming (accumulated)

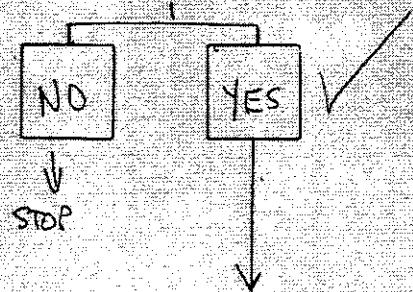
$$4644 = 4644$$

b. Montana (allocated) Montana (accumulated)

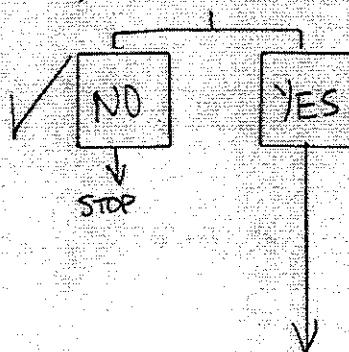
$$3096 = 3096$$

V Determine what action, if any, needs to be taken.

a. Is more water needed to meet Montana's demand?



Is $Q(\text{accumWY}) > Q(\text{allocWY})$?



release sufficient
divertable flow across
state line such that
 $Q(\text{accumWY}) = Q(\text{allocWY})$

a. regulate Wyoming Post-1950
use.

b. drawdown storage to meet
extra Wyoming demand.

ALL VALUES ARE
IN ACRE - FT

EXHIBIT
6 a

ALLOCATION FOR
JULY 1 to 31st

- A: inflow to system
- B: state line flow
- C: flow at gage
- \rightarrow : agricultural diversion
- ∇ : post-1950 reservoir

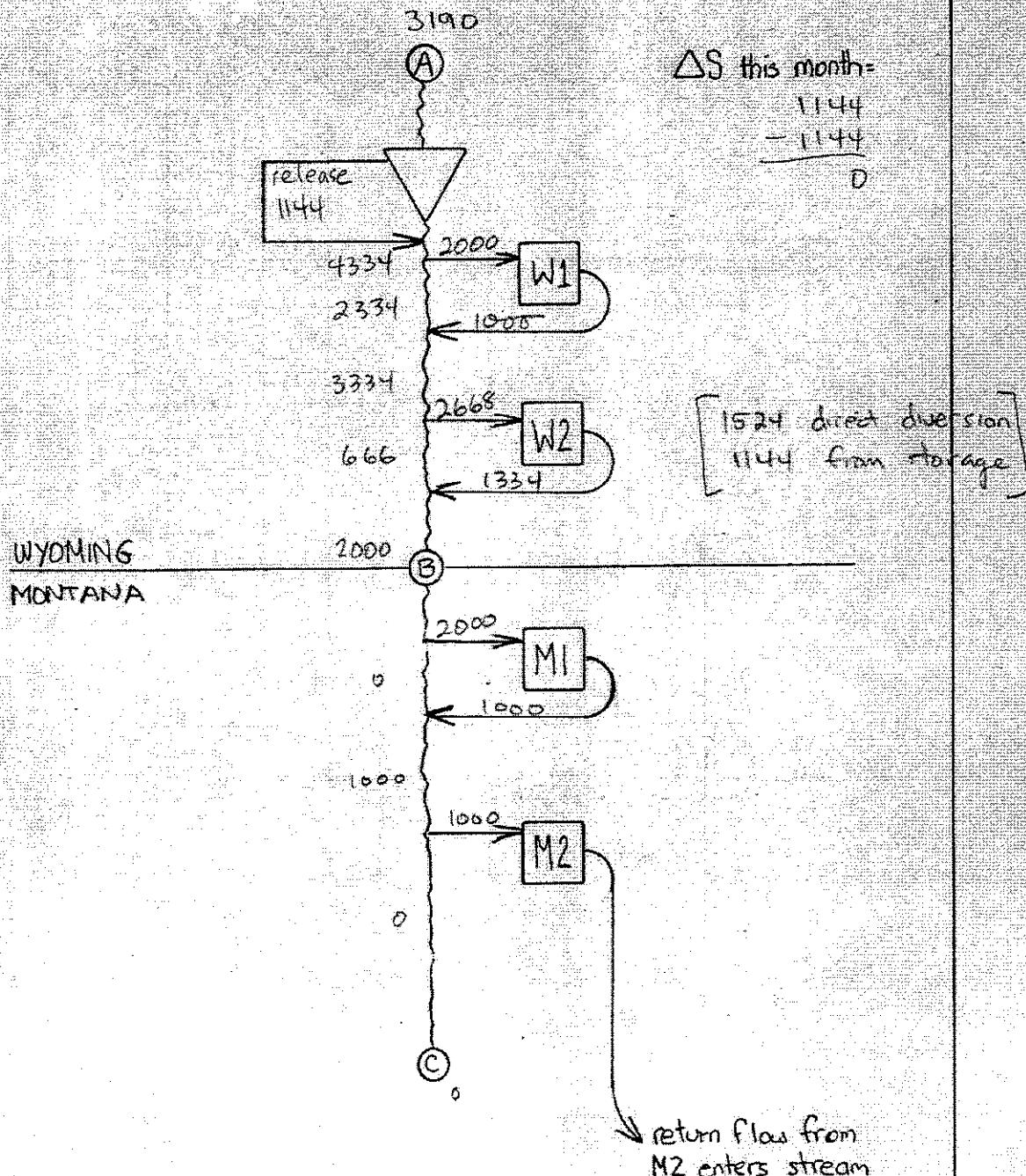
pre-1950 W1 = 2000
post-1950 W2 = 2668

pre-1950 M1 = 2000
post-1950 M2 = 1000

3190

ΔS this month =

$$\begin{array}{r} 1144 \\ - 1144 \\ \hline 0 \end{array}$$



Wyoming - 60%
Montana - 40%

return flow from
M2 enters stream
below C.

FALLING LIMIT
APPORTIONMENT

EXHIBIT
#6B

I

Calculate the accumulated diversions $D(WY)$ and $D(MT)$ from Oct 1 through this date for Wyoming and Montana:

a. $Q(\text{accumWY}) = D(\text{WY}) \text{ previous} + W2 \text{ this period}$

$$6168 = 4644 + 1524$$

b. $Q(\text{accumMT}) = D(\text{MT}) \text{ previous} + M2 \text{ this period}$

$$4096 = 3096 + 1000$$

II

Calculate the total divertable flow accumulated by both states from October 1 through today:

a. $Q(\text{accum}) = Q(\text{accumWY}) + Q(\text{accumMT})$

$$10,264 = 6168 + 4096$$

III

Calculate the allocable flow to each state:

a. $Q(\text{allocWY}) = Q(\text{accum}) \times \%w$

$$6158 = 10264 \times .60$$

b. $Q(\text{allocMT}) = Q(\text{accum}) \times \%m$

$$4106 = 10264 \times .40$$

IV

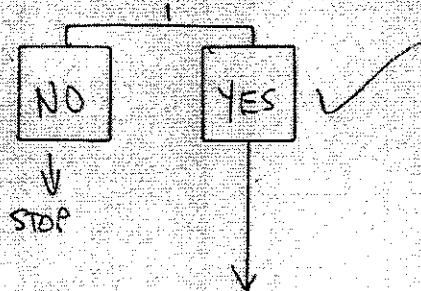
Compare accumulated divertable flow and allocable divertable flow for each state:

a. Wyoming (allocated) Wyoming (accumulated)
 $6158 < 6168$

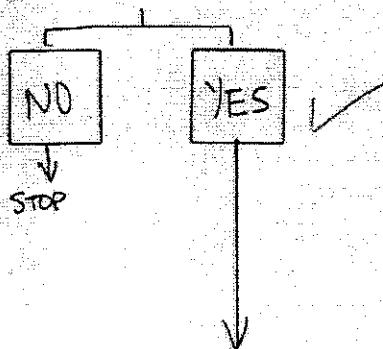
b. Montana (allocated) Montana (accumulated)
 $4106 > 4096$

IV Determine what action, if any, needs to be taken.

a. Is more water needed to meet Montana's demand?



Is $Q(\text{accum.WY}) > Q(\text{alloc.WY})$?



release sufficient
divertable flow across
state line such that
 $Q(\text{accum.WY}) = Q(\text{alloc.WY})$

✓ a. regulate Wyoming Post-1950
use.

b. drawdown storage to meet
extra Wyoming demand.

ALL VALUES ARE
IN ACRE-FT

EXHIBIT
#7 a

ALLOCATION FOR
July 1 to 31st

2nd
iteration

A: inflow to system

B: state line flow

C: flow at gage

$\overleftarrow{\Box}$: agricultural diversion

∇ : post-1950 reservoir

pre-1950 W1 = 2000
post-1950 W2 = 2655

pre-1950 M1 = 2000
post-1950 M2 = 1007

3190

(A)

release
1144

4334

2334

3334

W1

1030

ΔS this month:

$$\begin{array}{r} 1144 \\ -1144 \\ \hline 0 \end{array}$$

679

W2

2655

1328

[1511 direct diversion]
1144 from storage

WYOMING

MONTANA

2007

7

M1

1000

1007

M2

1007

0

0

return flow from
M2 enters stream
below (C).

Wyoming - 60%
Montana - 40%

MT-01052

FALLING LINES
APPROPORTIONMENTEXHIBIT
E-7B

I

Calculate the accumulated diversions $D(WY)$ and $D(MT)$ from Oct 1 through this date for Wyoming and Montana:

$$a. Q(\text{accumWY}) = D(WY) \text{ previous} + W2 \text{ this period}$$

$$6155 = 4644 + 1511$$

$$b. Q(\text{accumMT}) = D(MT) \text{ previous} + M2 \text{ this period}$$

$$4103 = 3096 + 1007$$

II

Calculate the total divertable flow accumulated by both states from October 1 through today:

$$a. Q(\text{accum}) = Q(\text{accumWY}) + Q(\text{accumMT})$$

$$10258 = 6155 + 4103$$

III

Calculate the allocable flow to each state:

$$a. Q(\text{allocWY}) = Q(\text{accum}) \times \%_w$$

$$6155 = 10258 \times .60$$

$$b. Q(\text{allocMT}) = Q(\text{accum}) \times \%_m$$

$$4103 = 10258 \times .40$$

IV

Compare accumulated divertable flow and allocable divertable flow for each state:

$$a. \underline{\text{Wyoming (allocated)}} : \underline{\text{Wyoming (accumulated)}}$$

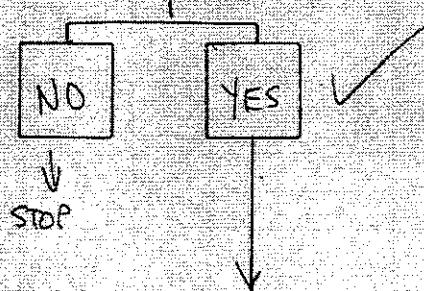
$$6155 = 6155$$

$$b. \underline{\text{Montana (allocated)}} : \underline{\text{Montana (accumulated)}}$$

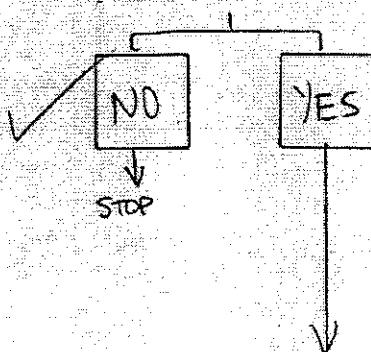
$$4103 = 4103$$

V Determine what action, if any, needs to be taken.

a. Is more water needed to meet Montana's demand?



Is $Q(\text{accumWY}) > Q(\text{allocWY})$?



release sufficient
divertable flow across
state line such that
 $Q(\text{accumWY}) = Q(\text{allocWY})$

a. regulate Wyoming Post-1950
use.

b. drawdown storage to meet
extra Wyoming demand.

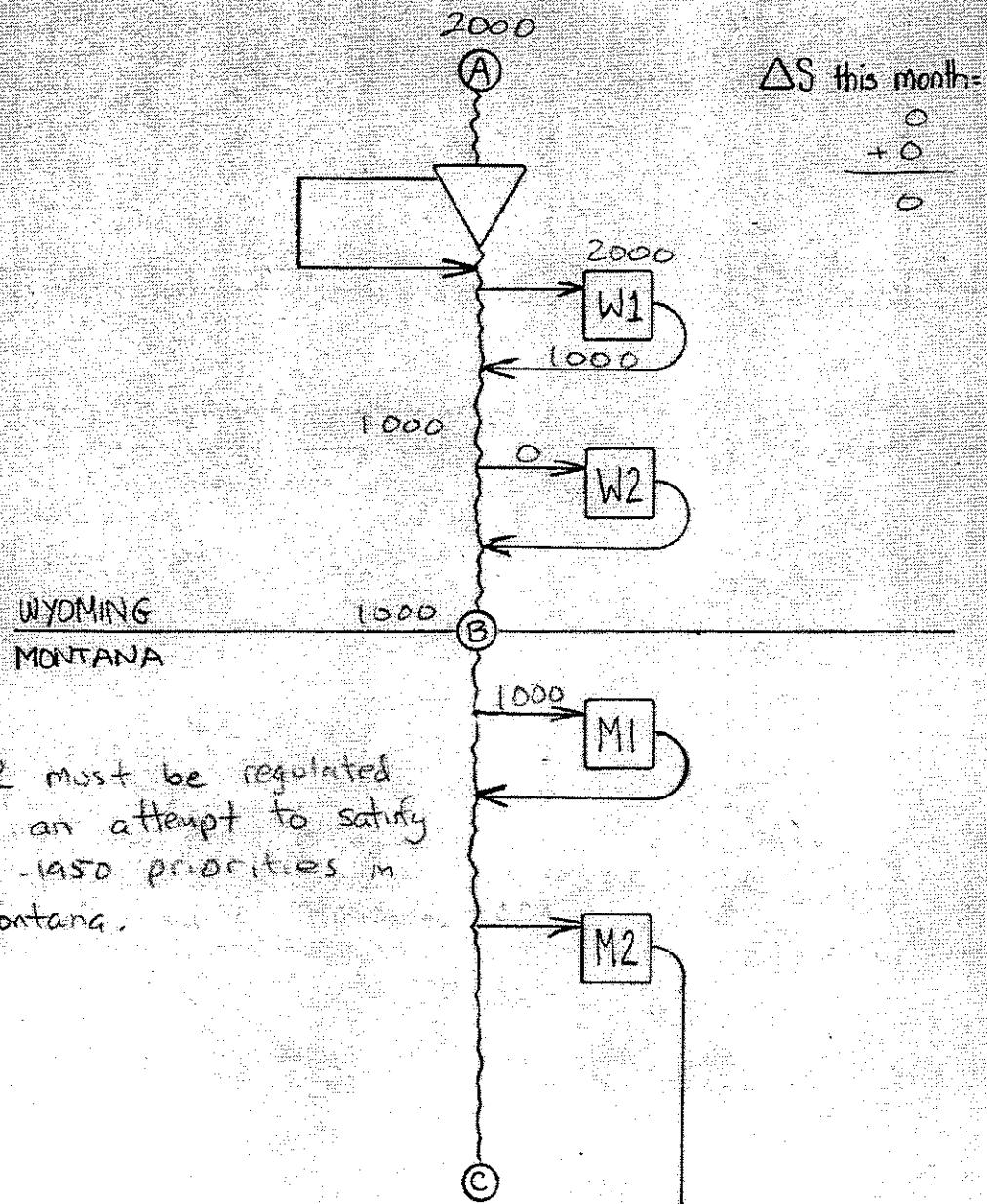
ALL VALUES ARE
IN ACRE-FT

EXHIBIT
#8

ALLOCATION FOR
August - September 30

A = inflow to system
B = state line flow
C = flow at gage
➡ = agricultural diversion
▽ = post-1950 reservoir

pre-1950 W1 =
post-1950 W2 =
pre-1950 M1 =
post-1950 M2 =



ΔS this month:

$$\begin{array}{r} 0 \\ + 0 \\ \hline 0 \end{array}$$

W2 must be regulated
in an attempt to satisfy
pre-1950 priorities in
Montana.

Wyoming - 60%
Montana - 40%

return flow from
M2 enters stream
below C.