MATHEMATICS

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Professors: Gregory Brumfiel, Daniel Bump, Gunnar Carlsson, Paul J. Cohen, Ralph L. Cohen, Persi Diaconis, Yakov Eliashberg, Solomon Feferman, Yitzhak Katznelson, Steven Kerckhoff, Jun Li, Tai-Ping Liu, Rafe Mazzeo, R. James Milgram, Donald S. Ornstein, George Papanicolaou, Karl Rubin, Richard Schoen, Leon Simon, Brian White
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The Department of Mathematics offers programs leading to the degrees of Bachelor of Science, Master of Science, and Doctor of Philosophy in Mathematics and participates in the program leading to the B.S. in Mathematical and Computational Science. The department also participates in the M.S. and Ph.D. degree programs in Scientific Computing and Computational Mathematics.

ADVANCED PLACEMENT FOR FRESHMEN

Students of unusual ability in mathematics often take one or more semesters of college-equivalent courses in mathematics while they are still in high school. Under certain circumstances, it is possible for such students to secure both advanced placement and credit toward the bachelor's degree. A decision as to placement and credit is made by the department after consideration of the student's performance on the Advanced Placement Examination in Mathematics (forms AB or BC) of the College Entrance Examination Board. This examination is the only one used for granting credit.

The department does not give its own advanced placement examination. Students can receive either 5 or 10 units of advanced placement credit, depending on their scores on the Advanced Placement Examination. Entering students who have credit for two quarters of single variable calculus (10 units) are encouraged to enroll in Mathematics 51-53 in multivariable mathematics, or the honors version 51H-53H. These three-course sequences, which can be completed during the freshman year, supply the necessary mathematics background for most majors in science and engineering. They also serve as excellent background for major or minor degrees in Mathematics, or in Mathematical and Computational Science. Students who have credit for one quarter of single variable calculus should take Mathematics 42 in the Autumn Quarter and 51 in Winter Quarter. Options available in the Spring Quarter include Mathematics 52, 53, 103, or 130. For proper placement, contact the Department of Mathematics.

UNDERGRADUATE PROGRAMS

BACHELOR OF SCIENCE

The following department requirements are in addition to the University's basic requirements for the bachelor's degree:

MAJORS

Students wishing to major in Mathematics must satisfy the following requirements:

1. Department of Mathematics courses totaling at least 49 units credit; such courses must be taken for a letter grade. For the purposes of this requirement, Statistics 116 counts as a Department of Mathematics course.
2. Additional courses taken from Department of Mathematics courses numbered 100 and above or from approved courses in other disciplines with significant mathematical content, totaling at least 15 units credit.
3. A Department of Mathematics adviser must be selected, and the courses selected under items '1' and '2' above must be approved by that adviser, acting under guidelines laid down by the department's Committee for Undergraduate Affairs. The Department of Mathematics adviser can be any member of the department's faculty.
4. To receive the department's recommendation for graduation, a student must have been enrolled as a major in the Department of Mathematics for a minimum of two full quarters, including the quarter immediately before graduation. In any case students are strongly encouraged to declare as early as possible, preferably by the end of the sophomore year.

Students are normally expected to complete either the sequence 19, 20, 21 or the sequence 41, 42 (but not both). Students with an Advanced Placement score of at least 4 in BC math or 5 in AB math may, if they so choose, take 10 units credit and fulfill requirement '1' by taking at least 39 units of Department of Mathematics courses numbered 51 and above. Students with an Advanced Placement score of at least 3 in BC math or at least 4 in AB math may, if they so choose, take 5 units credit and fulfill requirement '1' by taking at least 44 units of Department of Mathematics courses numbered 42 and above.

It is recommended that the choice of courses under item '1' above should include at least the following: the sequence 51, 52, 53 or the sequence 51H, 52H, 53H (but not both); 104 or 114; 106 or 116; 115 or 171; 120 or both 109 and 110. In addition, it is recommended that students make a reasonably broad selection of courses from the department's algebra and analysis courses 121, 131, 132, 152, 153, 155, 156, 173, 174A,B (formerly 134A,B), 175, 176, the geometry/topology courses 141, 143, 145, 147, 148 and foundation courses 160A, 160B, 161. The probability course Statistics 116 is also recommended.

Some of the additional courses mentioned under item '2' above could, for example, be taken from the physics series 41, 43, 45, 47, or the physics series 61, 63, 65. A variety of other courses may be allowed under item '2', including some courses taken from the departments of Engineering, Computer Science, Statistics, and Economics.

Subject to approval of the department's Committee for Undergraduate Affairs, it may, in some cases, be permitted to count freshmen and sophomore seminar courses as part of a choice of courses under item '1'. Other variations of the course requirements laid down above (under items '1' and '2') may, in some circumstances, be allowed (for example, students transferring from other universities may be allowed credit for some courses completed before their arrival in Stanford). In all cases, approval must be obtained from the department's Committee for Undergraduate Affairs. Application for such approval should be made through the student's Department of Mathematics adviser.

It is to be emphasized that the above regulations are minimum requirements for the major—students contemplating graduate work in mathematics are strongly encouraged to include the courses 114, 120, and 171 in their selection of courses, and in addition, take at least three Department of Mathematics courses over and above the minimum requirements laid out under items '1' and '2' above, including at least one 200-level course. Such students are also encouraged to consider the possibility of taking the honors program, discussed below.

To help develop a sense of the type of course selection (under items '1' and '2' above) which would be recommended for math majors with various backgrounds and interests, see the following examples. These represent only a few of a very large number of possible combinations of courses which could be taken in fulfillment of the Mathematics major requirements:

Example 1—A "General" program (a balanced program of both pure and applied components, without any particular emphasis on any one field of mathematics or applications) as follows:

Either Mathematics 19, 20 and 21, or 41 and 42 (or satisfactory Advanced Placement credit); 51, 52, 53; 104; 106; 109; 110; 115

Plus any selection of at least eight of the following courses, including three Department of Mathematics courses (or two math courses and Statistics 116): Mathematics 131, 132, 143, 147, 152, 153, 161, 173; Statistics 116; Physics 41, 43, 45; Computer Science 137; Economics 50

It would be common in this sequence to also include Mathematics 103 or 113 in the selection of Department of Mathematics courses here, especially for students wishing to supplement their knowledge of linear algebra, gained in Mathematics 51 and 53, before attempting some of the other courses listed. However, Mathematics 51 and 53 would generally be deemed sufficient introduction to linear algebra for these other courses. Students could take 103 (or 113) and 130 in lieu of 53.

Example 2—A "Theoretical" program (recommended for those contemplating possible later graduate work (see, also, the discussion of the honors program below), providing an introduction to the main areas of mathematics both broader and deeper than the general program outlined above).

Either Mathematics 19, 20, and 21, or 41 and 42 (or satisfactory Advanced Placement credit)

Either the sequence 51, 52, 53, or the sequence 51H, 52H, 53H; 106 or 116; 114; 120; 171

Plus at least nine of the following courses: 121, 131, 132, 141, 143, 145, 147, 148, 152, 153, 155, 156, 160A,B, 161, 173, 174A,B (formerly 134A.B), 175, 176; Statistics 116

In addition, those contemplating eventual graduate work in mathematics should seriously consider including at least one graduate-level math course such as Mathematics 205A or 206A. Such students should also consider the possibility of entering the honors program.

(Students taking 51, 52, 53 rather than 51H, 52H, 53H should consider taking 113 before attempting 114.)

*Example 3**—An "Applied Mathematics" program:

Either Mathematics 19, 20 and 21; or 41 and 42 (or satisfactory Advanced Placement credit); 51, 52, 53; 104; 106; 109; 110; 115; 131; 132; Statistics 116

Plus at least 15 units of courses in Applied Mathematics, including, for example, suitable courses from the departments of Computer Science, Economics, Engineering, and Statistics

* Students with interests in applied mathematics, but desiring a broader-based program than the type of program suggested in Example 3, including significant computational and/or financial and/or statistical components, are encouraged to also consider the Mathematics and Computational Sciences program.

MINORS

To qualify for the minor in mathematics, a student should successfully complete, for a letter grade, at least six Department of Mathematics courses numbered 51 or higher, totaling a minimum of 24 units. It is recommended that these courses include either the sequence 51, 52, 53 or the sequence 51H, 52H, 53H.

HONORS PROGRAM

The honors program is intended for students who have strong theoretical interests and abilities in mathematics. The goal of the program is to give students a thorough introduction to the main branches of mathematics, especially analysis, algebra, and geometry. Through the honors thesis, students may be introduced to a current or recent research topic, although occasionally more classical projects are encouraged. The program provides an excellent background with which to enter a master's or Ph.D. program in mathematics. Students successfully completing the program are awarded a B.S. in Mathematics with Honors.

It is recommended that the sequence 51H, 52H, 53H be taken in the freshman year. Students who have instead taken the sequence 51, 52, 53 in their freshman year may be permitted to enter the honors program, but such entry must be approved by the Department of Mathematics Committee for Undergraduate Affairs.

To graduate with a B.S. in Mathematics with Honors, the following conditions apply in addition to the usual requirements for math majors:

1. The selection of courses under items '1' and '2' above must include all the math courses 106 or 116, 114, 120, 171 and also must include at least two courses from the analysis sequence 131, 132, 173, 174A, 174B (formerly 134A, 134B), 175, 176; at least two courses from the algebra sequence 121, 152, 153, 155, 156; at least one course in the geometry/topology sequence 141, 143, 145, 147, 148; and at least one course in logic or set theory (from 160A, 160B, 161). The probability course Statistics 116 is also strongly recommended.
2. Students in the honors program must write a senior thesis. In order to facilitate this, the student must, by the end of the junior year, choose an undergraduate thesis adviser from the Department of Mathematics faculty, and map out a concentrated reading program under the direction and guidance of the adviser. During the senior year, the student must enroll in Mathematics 197 for a total of 6 units (typically spread over two quarters), and work toward completion of the thesis under the direction and guidance of the thesis adviser. The thesis may contain original material, or be a synthesis of work in current or recent research literature. The 6 units of credit for Mathematics 197 are required in addition to the course requirements laid out under items '1' and '2' above of the usual requirements for math majors.

In addition to the minimum requirements laid out above, it is strongly recommended that students take at least one graduate-level course (that is, at least one course in the 200 plus range). Mathematics 205A, 206A are especially recommended in this context.

Students with questions about the honors program should see the Director of Undergraduate Advising.

BACHELOR OF SCIENCE IN MATHEMATICAL AND COMPUTATIONAL SCIENCE

The Department of Mathematics participates with the departments of Computer Science, Management Science and Engineering, and Statistics in a program leading to a B.S. in Mathematical and Computational Science. See the "Department of Mathematical and Computational Science" section of this bulletin.

GRADUATE PROGRAMS

MASTER OF SCIENCE

The University's basic requirements for the master's degree are discussed in the "Graduate Degrees" section of this bulletin. The following are additional department requirements:

Candidates must complete an approved course program of 36 units beyond the department requirements for the B.S. degree. It must include 18 units in courses numbered 200 or above. The candidate must have a grade point average (GPA) of 'B' over all course work taken in mathematics, and a GPA of 'B' in the 200-level courses considered separately. Course work for the M.S. degree must be approved during the first quarter of enrollment in the program by the department's Director of Graduate Studies.

For the degree of M.S. in Computer Science, see the "Computer Science" section of this bulletin.

TEACHING CREDENTIALS

For information concerning the requirements for teaching credentials, see the "School of Education" section of this bulletin or address inquiries to Credential Secretary, School of Education.

MASTER OF ARTS IN TEACHING (MATHEMATICS)

In cooperation with the School of Education, the department offers a program leading to a Master of Arts in Teaching (Mathematics). It is intended for candidates who have a teaching credential or relevant teaching experience and wish to strengthen their academic preparation. Detailed requirements are outlined under the "School of Education, Master of Arts in Teaching" section of this bulletin.

DOCTOR OF PHILOSOPHY

The University's basic requirements for the doctorate (residence, dissertation, examination, and so on) are discussed in the "Graduate Degrees" section of this bulletin. The following are additional department requirements.

To be admitted to candidacy, the student must have successfully completed 27 units of graduate courses (that is, courses numbered 200 and above). In addition, the student must pass qualifying examinations given by the department.

Beyond the requirements for candidacy, the student must complete a course of study of at least 48 units approved by the Graduate Affairs Committee of the Department of Mathematics and submit an acceptable dissertation. The course program should display substantial breadth in mathematics outside the student's field of application. The student must receive a grade point average (GPA) of 'B' or better in courses used to satisfy the Ph.D. requirement. In addition, the student must pass the University oral examination and pass a reading examination in two foreign languages, chosen from French, German, or Russian.

Experience in teaching is emphasized in the Ph.D. program. Each student is required to complete nine quarters of such experience. The nature of the teaching assignment for each of those quarters is determined by the department in consultation with the student. Typical assignments include teaching or assisting in teaching an undergraduate course or lecturing in an advanced seminar.

For the Ph.D. degree in Computer Science, see the "Computer Science" section of this bulletin.

For further information concerning degree programs, fellowships, and assistantships, inquire of the academic associate of the department.

APPLIED MATHEMATICS OPTION

This option differs from the standard Ph.D. program in that qualifying examinations in more applied areas are substituted for the regular qualifying examinations. Also, the courses Mathematics 220 (basic methods in partial differential equations) and Computer Science 237 (numerical methods) are a required part of the curriculum in the first year. Students are required to take 18 units of graduate-level courses in computer science and applied areas such as financial mathematics, fluid mechanics, operations research, or statistics.

Ph.D. MINOR

The student should complete both of the following:*

1. Mathematics 106 or 116, 131, 132
2. Mathematics 113, 114, 120 or 152

These courses may have been completed during undergraduate study, and their equivalents from other universities are acceptable.

In addition, the student should complete 21 units of 200-level courses in mathematics. These must be taken at Stanford and approved by the Department of Mathematics' Ph.D. minor adviser.

* A third coherent sequence designed by the student, subject to the approval of the graduate committee, may be considered as a substitute for items '1' or '2'.

COURSES

(WIM) indicates that the course meets the Writing in the Major requirements.

INTRODUCTORY AND UNDERGRADUATE

The department offers two sequences of introductory courses in single variable calculus.

1. Mathematics 41, 42 present single variable calculus. Differential calculus is covered in the first quarter, integral calculus in the second.
2. Mathematics 19, 20, 21 cover the material in 41, 42 in three quarters instead of two.

There are options for studying multivariable mathematics:

1. Mathematics 51, 52, 53 cover differential and integral calculus in several variables, linear algebra, and ordinary differential equations. These topics are taught in an integrated fashion and emphasize application. Mathematics 51 covers differential calculus in several variables and introduces matrix theory and linear algebra, 52 covers integral calculus in several variables and vector analysis, 53 studies further topics in linear algebra and applies them to the study of ordinary differential equations. This sequence is strongly recommended for incoming freshmen with 10 units of Advanced Placement credit.
2. Mathematics 51H, 52H, 53H cover the same material as 51, 52, 53, but with more emphasis on theory and rigor.

The introductory course in modern algebra is Linear Algebra (103 or 113). There are no formal prerequisites for these courses, but appropriate mathematical maturity is expected. The material in 103 is covered in the sequence 51, 52, 53.

19,20,21. Calculus—The content is the same as the sequence 41 and 42 described below, over three quarters rather than two.

19. Calculus—GER:2c (DR:4)
3 units, Aut (Moore)
Win (Staff)

20. Calculus—Continuation of 19. Prerequisite: 19. GER:2c (DR:4)
3 units, Win (Moore)
Spr (Staff)

21. Calculus—Continuation of 20. Prerequisite: 20. GER:2c (DR:4)
4 units, Spr (Moore)

41,42.—Three large lecture classes per week plus two classes in small sections.

41. Single Variable Calculus—Introduction to differential and integral calculus of functions of one variable. Topics: review of elementary functions including exponentials and logarithms, rates of change, and the derivative. Prerequisites: algebra, trigonometry. GER:2c (DR:4)
5 units, Aut (Bray)

42. Single Variable Calculus—Continuation of 41. Methods of symbolic and numerical integration, applications of the definite integral, introduction to differential equations. Infinite series. Prerequisite: 41 or equivalent. GER:2c (DR:4)
5 units, Aut (Ni)
Win (Bray)

51,52,53. Multivariable Mathematics—Recommended for incoming freshmen with 10 units of Advanced Placement credit, and for those interested in science, engineering, or economics. Provides an integrated treatment of multivariable calculus, linear algebra, and ordinary differential equations involving numerical algorithms and computer experiments. Applications are stressed.

51. Linear Algebra and Differential Calculus of Several Variables—Geometry and algebra of vectors, systems of linear equations, matrices, vector valued functions and functions of several variables, partial derivatives, gradients, chain rule in several variables, vector fields, optimization. Prerequisite: completion of 21, 42, or a score of at least 4 on the BC Advanced Placement Examination or 5 on the AB Advanced Placement Examination, or consent of the instructor. GER:2c (DR:4)
5 units, Aut (Brumfiel, Hutchings, S. Levandosky, Staffalani, White)
Win (Kiem, Rubin)
Spr (Bray)
Sum (Staff)

52. Integral Calculus of Several Variables—Iterated integrals, line and surface integrals, vector analysis with applications to vector potentials and conservative vector fields, physical interpretations. Divergence theorem and the theorems of Green, Gauss, and Stokes. Prerequisite: 51.
5 units, Aut (Wang)
Win (Cieliebak, S. Levandosky)
Spr (Bump)

53. Ordinary Differential Equations with Linear Algebra—Linear ordinary differential equations, applications to oscillations, matrix methods including determinants, eigenvalues and eigenvectors, matrix exponentials, solving of systems of linear differential equations with constant coefficients, numerical methods, and Laplace transforms. Similar to 130, integrated with topics from linear algebra (103, 104). Prerequisite: 51.

5 units, Aut (Carlton)
Win (Schlichtkrull)
Spr (J. Levandosky, Etnyre)

51H,52H,53H. Honors Calculus—For prospective math majors in the honors program and students from other areas of science or engineering who have a strong mathematics background. Three-quarter sequence covers the material of 51, 52, 53, and additional advanced calculus and ordinary and partial differential equations. Provides a unified treatment of multi-variable calculus, linear algebra, and differential equations with a different order of topics and emphasis from standard courses. Students should know one-variable calculus and have an interest in a theoretical approach to the subject. Prerequisite: score of 5 on BC Advanced Placement Exam, or consent of the instructor. Recommended: complete at least the first two quarters. 51H satisfies GER:2c (DR:4)

51H. *5 units, Aut (Eliashberg)*
52H. *5 units, Win (Kerckhoff)*
53H. *5 units, Spr (R. Cohen)*

83Q. Stanford Introductory Seminar: Introduction to Contact Geometry—Preference to sophomores.

3 units, Win (Eliashberg)

84Q. Stanford Introductory Seminar: Finite Mathematics, Codes, and Cryptography—Preference to sophomores.

3 units, Win (Carlsson)

85Q. Stanford Introductory Seminar: Mathematical Stability and Asymptotics—Preference to sophomores.

3 units, Win (Simon)

86Q. Stanford Introductory Seminar: Problem Seminar—Preference to sophomores.

3 units, Aut (Katznelson)

87Q. Stanford Introductory Seminar: The Mathematics of Knots, Braids, and Tangles—Preference to sophomores.

3 units, Spr (R. Cohen)

UNDERGRADUATE AND GRADUATE

Unless explicitly stated, there are no prerequisites for the courses listed below. Where a prerequisite is stated, it may be waived by the instructor.

103. Matrix Theory and Its Applications—Linear algebra and matrices, emphasizing the computational and algorithmic aspects and the scientific problems in which matrix theory is applied. Solution of linear equations. Linear spaces and matrices. Orthogonal projection and least squares. Determinants, eigenvalues, and eigenvectors. GER:2c (DR:4)

3 units, Aut (Kiem, Mohnke)
Win (Levy, Ni)
Spr (Kim, Schlichtkrull)
Sum (Staff)

104. Matrix Theory and Its Applications—Continuation of 103. Positive definite matrices, extremum problems, computations with matrices, elements of linear programming, and Markov chains.

3 units, Spr (Ni)

106. Introduction to Theory of Functions of a Complex Variable—Complex numbers, analytic functions, Cauchy-Riemann equations, complex integration, Cauchy formula; elementary conformal mappings. Prerequisite: 52 or 52H.

3 units, Aut (Levy)
Sum (Staff)

108. Introduction to Combinatorics and Its Applications—Topics: graphs, trees (Cayley's Theorem, application to phylogony), eigenvalues, basic enumeration (permutations, Stirling and Bell numbers), recurrences, generating functions, basic asymptotics. Prerequisites: Mathematics 51 and 103, or equivalent.

3 units, Aut (Diaconis)

109. Applied Modern Algebra—Applications of the theory of groups. Topics: elements of groups theory, groups of symmetries, matrix groups, group actions, and applications to combinatorics and computing. Applications: rotational symmetry groups, the study of the Platonic solids, crystallographic groups and their applications in chemistry and physics. (WIM)

3 units, Win (R. Cohen)

110. Applied Number Theory and Field Theory—Introduction to number theory and its applications to modern cryptography. Topics: congruences, finite fields, primality testing and factorization, public key cryptography, error correcting codes, and elliptic curves, with emphasis throughout on algorithms. (WIM)

3 units, Spr (Rubin)

112. Symmetric Functions and Algebraic Combinatorics—Unified treatment of topics in classical enumeration via the study of symmetric polynomials. Classical symmetric functions, Schur functions, Young tableaux, Schensted correspondence, character theory of the symmetric group, introduction to random matrix theory. Prerequisite: 109 or 120, or equivalent.

3 units, Win (Diaconis)

113. Linear Algebra and Matrix Theory—Algebraic properties of matrices and their interpretation in geometric terms. The relationship between the algebraic and geometric points of view and matters fundamental to the study and solution of linear equations. Topics: linear equations, vector spaces, linear dependence, bases and coordinate systems; linear transformations and matrices; similarity; eigenvectors and eigenvalues; diagonalization.

3 units, Aut (Katznelson)
Win (Hutchings)

114. Linear Algebra and Matrix Theory—Continuation of 113. Deeper study of 113 topics plus additional topics from invariant subspaces, canonical forms of matrices; minimal polynomials and elementary divisors; vector spaces over arbitrary fields; inner products; Jordan normal forms; Hermitian and unitary matrices; multilinear algebra.

3 units, Win (Milgram)

115. Fundamental Concepts of Analysis—The development of real analysis in Euclidean space: sequences and series, limits, continuous functions, derivatives. Basic point set topology. Honors math majors and students who intend to do graduate work in mathematics should take 171. Prerequisite: 51.

3 units, Aut (S. Levandosky)
Win (Ornstein)
Sum (Staff)

116. Complex Analysis—Analytic functions, Cauchy integral formula, power series and Laurent series, calculus of residues and applications, conformal mapping, analytic continuation, introduction to Riemann surfaces, Fourier series and integrals. Applications of complex analysis to electrostatics, hydrodynamics, and theoretical physics. Prerequisite: 52.

3 units, Win (Eliashberg)

120. Modern Algebra I—Basic structures in algebra: groups, rings, and fields. Elements of group theory: permutation groups, finite Abelian groups, p-groups, Sylow theorems. Polynomial rings, principal ideal domains, unique factorization domains. (WIM)
3 units, Aut (J. Li)

121. Modern Algebra II—Continuation of 120. Fields of fractions. Solvable and simple groups. Elements of field theory and Galois theory. Prerequisite: 120.
3 units, Win (J. Li)

130. Ordinary Differential Equations—Special, exact, and linear equations; series solutions, numerical solution; Laplace transform; systems of equations. Students with some background in matrix theory should take 53, which integrates linear algebra with differential equations. Pre- or corequisite: 51 or consent of instructor.
3 units, Aut (Milgram)
Win (Yang)
Sum (Staff)

131. Partial Differential Equations I—First-order equations, classification of second-order equations. Initial-boundary value problems for heat, wave, and related equations. Separation of variables, eigenvalue problems, Fourier series, existence and uniqueness questions. Prerequisite: 53 or 130 or equivalent.
3 units, Win (Papanicolaou)
Spr (S. Levandosky)

132. Partial Differential Equations II—Initial and initial-boundary value problems in infinite domains. Fourier transforms. Boundary value problems for Laplace equation. Bessel functions and Legendre polynomials. Prerequisite: 131 or equivalent.
3 units, Spr (Kim)

143. Differential Geometry—Geometry of curves and surfaces in three-space and higher dimensional manifolds. Parallel transport, curvature, and geodesics. Surfaces with constant curvature. Minimal surfaces.
3 units, Aut (Schoen)

145. Algebraic Geometry—Affine and projective spaces, plane curves, Bezout's theorem, singularities and genus of a plane curve, applications of commutative algebra to geometry. Prerequisites: 106 and either 120 or 110.
not given 2000-01

147. Differential Topology—Smooth manifolds, transversality, Sards' theorem, embeddings, degree of a map, Borsuk-Ulsm theorem, Hopf degree theorem, Jordan Curve Theorem. Prerequisite: 115 or 171.
not given 2000-01

148. Algebraic Topology—Fundamental group, covering spaces, Euler characteristic, classification of surfaces, knots. Prerequisite: 109 or 120.
3 units, Win (Kerckhoff)

151. Introduction to Probability Theory—Probability spaces as models for uncertainty, and introduction to the corresponding mathematical analysis. Combinatorial analysis for discrete spaces (binomial, hypergeometric, Poisson). Conditional probability and stochastic independence. Random variables and expectation. Law of large numbers, Normal and Poisson approximation. Continuous spaces (normal, exponential, uniform) and densities. Prerequisite: 51. Corequisite: 115 or consent of instructor.
3 units, Win (Lee)

152. Elementary Theory of Numbers—Euclid's algorithm, fundamental theorems on divisibility; prime numbers, congruence of numbers; theorems of Fermat, Euler, Wilson; congruence of first and higher degrees; Lagrange's theorem, its applications; residues of power; quadratic residues; introduction to the theory of binary quadratic forms.
3 units, Spr (Carlton)

153. Combinatorics—Topics in Ramsey's theorem, generating functions, partition functions, and in number theory (sums of integers and van der Waerden's theorem). Recommended: general background in algebra, analysis, and some number theory.
not given 2000-01

156. Group Representations—Designed for undergraduates. Experimental, primarily examining symmetries on objects such as vector spaces ("group representations"), geometric objects ("geometric group actions"), and discrete sets (combinatorics). Topics: group representations and their characters, classification of permutation representations using partitions and Young tableaus, group actions on sets and the Burnside ring, and spherical space forms. Prerequisites: basic knowledge of linear algebra (51-53, 103 or 113) and Group Theory (109, 110 or 120).
not given 2000-01

160A. First-Order Logic—(Enroll in Philosophy 160A.)
4 units, Win (Mints)

160B. Computability and Logic—(Enroll in Philosophy 160B.)
4 units, Spr (Mints)

161. Set Theory—Informal and axiomatic set theory: sets, relations, functions and set-theoretical operations. The Zermelo-Fraenkel axiom system and the special role of the axiom of choice and its various equivalents. Well-orderings and ordinal numbers; transfinite induction and transfinite recursion. Equinumerosity and cardinal numbers; Cantor's Alephs and cardinal arithmetic. Open problems in set theory.
3 units, Spr (White)

162. Philosophy of Mathematics—(Enroll in Philosophy 162.)
4 units, Win (Feferman)

171. Fundamental Concepts of Analysis—Recommended for math majors and required of honors math majors. Similar to 115 but altered contents and more theoretical orientation. Properties of Riemann integrals, continuous functions and convergence in metric spaces; compact metric spaces, basic point set topology. Prerequisites: either 51, 52, 53; or 51H, 52H, 53H. (WIM)
3 units, Aut (Kerckhoff)

173. Analysis on Manifolds—Differentiable manifolds, tangent space, submanifolds, implicit function theorem, differential forms, vector and tensor fields. Frobenius' theorem, DeRham theory. Prerequisite: 52 or 52H.
3 units, Win (R. Cohen)

174A,B. Honors Analysis—(Formerly 134A,B) Primarily for students planning graduate work in mathematics or physics who would normally enroll in an honors sequence. Required of honors math majors, but of use and interest to other majors at ease with rigorous proofs and qualitative discussion. Coherent, mathematically sophisticated presentation of the basic areas in classical real analysis. Emphasis is on ordinary and partial differential equations. Prerequisite: 53H or 171, or consent of instructor.
174A. *3 units, Win (P. Cohen)*
174B. *3 units, Spr (P. Cohen)*

175. Elementary Functional Analysis—Linear operators on Hilbert space. Spectral theory of compact operators; applications to integral equations. Elements of Banach space theory. Prerequisite: 115 or 171.
3 units, Spr (Yang)

180. Introduction to Financial Mathematics—Basic theory of interest and fixed-income securities. Preferences and risk aversion, stochastic dominance. Mathematics of efficient portfolio frontier. Capital Asset

Pricing Model, arbitrage pricing theory. Utility-based optimization.
3 units, Aut (Dembo)

195. Teaching Practicum—Students assist in an undergraduate course, lead problems sessions, and tutor. Some reading in topics in mathematics education is required.
1 unit, Aut, Win, Spr

197. Senior Honors Thesis
1-6 units, Aut, Win, Spr (Staff)

199. Independent Work—Undergraduates pursue a reading program; topics limited to those not in regular department course offerings. Credit can fulfill the elective requirement for math majors. Approval of Undergraduate Affairs Committee is required to use credit for department's area requirement. Consult the academic associate for help in finding an adviser.
(Staff)

PRIMARILY FOR GRADUATE STUDENTS

200. Graduate Problem Seminar
not given 2000-01

205A,B. Theory of Functions of a Real Variable—Lebesgue measure and integration, LP spaces, and convergence theorems. General Banach and Hilbert spaces and linear operators. Elements of Functional Analysis. Prerequisite: 171 or equivalent.
3 units, Aut (Simon)
Win (P. Cohen)

205C. Topics in Functions of a Real Variable—Introduction to homogenization. Basic theory of elliptic differential equations with oscillating coeffecients, their asymptotic properties, use of multiple scale methods, weak and sharp convergence, and applications in analysis and physics. Prerequisite: 205A or equivalent.
3 units, Spr (Papanicolaou)

206A. Theory of Functions of a Complex Variable—Complex integration. Cauchy's theorem, Residue theorem, argument principle, power series, conformal mapping. Prerequisite: 171.
3 units, Aut (Mazzeo)

206B. Theory of Functions of a Complex Variable—Riemann mapping theorem, product developments, entire functions, elliptic functions, Dirichlet problem, Picard's theorem. Prerequisites: 171, 206A.
3 units, Win (Wang)

210A,B,C. Modern Algebra—Groups, rings, and fields, Galois theory, ideal theory. Introduction to algebraic geometry and algebraic number theory. Representations of groups and non-commutative algebras, multilinear algebra. Prerequisite: 120 or equivalent.
210A. *3 units, Aut (Brumfiel)*
210B. *3 units, Win (Brumfiel)*
210C. *3 units, Spr (Bump)*

216A,B. Introduction to Algebraic Geometry—Affine and projective varieties, schemes, the functor of points, sheaves, sheaf cohomology, Serre duality, Riemann-Roch, algebraic curves. Emphasis is on methods that apply in arithmetic and geometric situations. Prerequisite: 210 or equivalent, or consent of instructor.
216A. *3 units, Win (Carlton)*
216B. *3 units, Spr (Carlton)*

217A,B. Differential Geometry—Differential manifolds and forms, deRham's theorem. Hodge theory. Theory of connections. Riemannian geometry. Curvature tensor. Parallel transport. Holonomy groups. Morse theory of smooth paths, closed geodesics comparison theorems. Manifolds with positive curvature. Manifolds with negative curvature. Symmetric spaces. Prerequisite: 173.
217A. *3 units, Win (Schoen)*
217B. *3 units, Spr (Wang)*

220A,B,C. Partial Differential Equations of Applied Mathematics—Greens functions, integral transforms, variational and distribution theoretic methods for the analysis of differential and integral equations, with illustrative examples. Prerequisite: some familiarity with differential equations and functions of a complex variable.
220A. *3 units, Aut (J. Levandosky)*
220B. *3 units, Win (J. Levandosky)*
220C. *3 units, Spr (Mattingly)*

222A,B. Computational Methods for Fronts, Interfaces, and Waves—High-order methods for multidimensional systems of conservation laws and Hamilton-Jacobi equations (central schemes, discontinuous Galerkin methods, relaxation methods). Level set methods and fast marching methods. Computation of multi-valued solutions. Multi-scale analysis, including wavelet-based methods. Boundary schemes (perfectly matched layers). Examples from (but not limited to) geometrical optics, transport equations, reaction-diffusion equations, imaging, and signal processing.
222A. *3 units, Aut (Levy)*
222B. *3 units, Win (Levy)*

226. Capillary Surfaces—Introduction to the modern theory of capillary free surface interfaces in the contexts of differential geometry and the calculus of variations. Emphasis is on the interactions between the formal mathematics and physical reality. Results from NASA space experiments. Topics: global estimates on configuration of solution surfaces, discontinuous dependence of solutions on boundary data, symmetry breaking, criteria for existence, failure of existence under physical conditions, failure of uniqueness under conditions for which solutions exist, etc.
3 units, Win (Finn)

228. Introduction to Ergodic Theory—Measure preserving transformations and flows, ergodic theorems, mixing properties, spectrum, Kolmogorov automorphisms, entropy theory. Examples. Classical dynamical systems, mostly geodesic and horocycle forms on homogeneous spaces of SL(2,R). Prerequisites: 205A,B.
not given 2000-01

230A. Theory of Probability—(Enroll in Statistics 310A.)
3 units, Aut (Dembo)

230B. Theory of Probability—(Enroll in Statistics 310B.)
3 units, Win (Siegmund)

230C. Theory of Probability—(Enroll in Statistics 310C.)
3 units, Spr (Lai)

231. Point and Spatial Processes—(Enroll in Statistics 317.)
3 units, Win (Diaconis)

233. Probabilistic Methods in Analysis—Proofs and constructions in analysis obtained from basic results in Probability Theory and a "probabilistic way of thinking." Topics: Rademacher functions, Gaussian processes, entropy.
3 units, Win (Katznelson)

234. Large Deviations—(Same as Statistics 374.) Combinatorial estimates and the method of types. Large deviation probabilities for partial sums and for empirical distributions, Cramer's and Sanov's theorems and their Markov extensions. Applications in statistics, information theory, and statistical mechanics. Prerequisite: 230A or Statistics 310.
3 units, Aut (Dembo)

235A,B,C. Selected Topics in Ergodic Theory—Topics from the Kolmogorov-Sinai theory of entropy; the isomorphism theorem for Bernoulli shifts and Bernoulli flows; K-automorphisms applications to mechanical systems, and automorphisms of compact groups.

235A. *3 units, Aut (Ornstein)*
235B. *3 units, Win (Ornstein)*
235C. *3 units, Spr (Ornstein)*

236. Introduction to Stochastic Differential Equations—Brownian motion, stochastic integrals, and diffusions as solutions of stochastic differential equations. Functionals of diffusions and their connection with partial differential equations. Random walk approximation of diffusions. Prerequisite: basic probability and differential equations.
3 units, Win (Mattingly)

237. Asymptotic Analysis of Stochastic Equations—Ergodic properties and stability of solutions of stochastic differential equations. Small parameter asymptotics and connections with partial differential equations. Selected applications from physics, biology, and economics.
3 units, Spr (Papanicolaou)

240. Computation and Simulation in Finance—Finite difference methods for the numerical solution of partial differential equations in finance. Binomial and trinomial tree methods. Classical numerical integration. Random variable generation, variance reduction, statistical analysis of simulation output. Applications to scenario analysis and interest rate modeling. Introduction to high-dimensional integration, Quasi- Monte Carlo and mortgage-backed securities.
3 units, Spr (Lee)

241. Mathematical Finance—(Enroll in Statistics 250.)
3 units, Win (T. Lai)

244. Riemann Surfaces—Compact Riemann surfaces: topological classifications, Hurwitz' formula. Riemann-Roch formula, uniformization theorem. Abel's theorem, Jacobian varieties. Some elements of harmonic analysis are developed with applications. Emphasis is on methods which are generally applicable to algebraic curves.
not given 2000-01

245. Topics in Algebraic Geometry—Moduli spaces of algebraic curves and related topics.
3 units, Aut (J. Li)

248. Number Theory—Introduction to algebraic number theory: the arithmetic of local and number fields, and a brief introduction to class field theory. Prerequisite: 210A,B or equivalent.
not given 2000-01

249A. Representations of GL(n) and Automorphic Forms—The representation theory of GL(n) over p-adic fields, with applications to automorphic forms.
3 units, Win (Bump)

249B. Arithmetic of Elliptic Curves—Topics: elliptic curves over local and global fields, the Mordell-Weil theorem, and the Birch Swinnerton-Dyer conjecture.
3 units, Spr (Rubin)

253. Regularity of Sets and Mappings—For students interested in any area of analysis. Topics: Lipschitz functions; C^∞ functions; Sobolev functions; various regularity and extension theorems including Rademacher, Kirzbraun, Whitney, Sard, C^1-Sard. Critical sets of real-analytic, complex analytic functions. Affine approximation properties of subsets of R^n, including a discussion of rectifiability and non-rectifiability, structure theorem, and Reifenberg's topological disc theorem.
not given 2000-01

254A,B. Ordinary Differential Equations—Qualitative theory of ordinary differential equations, analytic and geometric methods. Topics from the stability and perturbation theory of dynamical systems; Hamiltonian systems; applications to the theory of oscillations and celestal mechanics.
not given 2000-01

256A,B,C. Partial Differential Equations—Introduction to the theory of linear and non-linear partial differtial equations, beginning with linear theory involving use of Fourier transform and Sobolev spaces. Topics: Schauder and L2 estimates for elliptic and parabolic equations; De Giorgi-Nash-Moser theory for elliptic equations; non-linear equations, e.g., the minimal surface equation, geometric flow problems, and non-linear hyperbolic equations.
not given 2000-01

257A,B. Symplectic Geometry and Topology—Linear symplectic geometry and linear Hamiltonian systems. Symplectic manifolds and their Lagrangian submanifolds—local properties. Symplectic geometry and mechanics. Contact geometry and contact manifolds. Relations between symplectic and contact manifolds. Hamiltonian systems with symmetries. Momentum map and its properties.

257A. *3 units, Win (Cieliebak)*
257B. *3 units, Spr (Cieliebak)*

259. Microlocal Analysis—The basic calculus of pseudo-differential operators, focusing on the parametrix construction for elliptic operators, and leading to various applications in geometry (Hodge theorem, index theorem for Dirad operators). Possible topics: pseudo-differential operators on singular and noncompact spaces, the microlocal theory of elliptic boundary value problems, Atiyah-Patodi-Singer index theorem.
not given 2000-01

260A,B. Mathematical Methods of Classical Mechanics—Open to undergraduate students. Differential equations and vector fields. Examples of mechanical systems. Variational principles. Lagrange equations. Hamilton equations. Legeudre transform. Lagrange mechanics on manifolds. Symplectic manifolds and Hamiltonian systems. Hamilton-Jacobi equation. System with symmetries and their integrals. Integrable systems. Examples Action-angle variables. Liouville theorem. Perturbation theory. Geometric optic and contact geometry.
not given 2000-01

261A,B. Functional Analysis—Geometry of linear topological spaces. Linear operators and functionals. Spectral theory. Calculus for vector-valued functions. Operational calculus. Banach algebras. Special topics in functional analysis.
not given 2000-01

263A,B. Lie Groups and Lie Algebras—Definitions, examples, basic properties. Semi-simple Lie algebras, their structure and classification. Cartan decomposition: real Lie algebras. Representation theory: Cartan-Stiefel diagram, weights. Weyl character formula. Orthogonal and symplectic representations. Prerequisite: 210 or equivalent.

263A. *3 units, Win (Milgram)*
263B. *3 units, Spr (P. Cohen)*

266. Time Frequency Analysis and Wavelets—Multiresolution analysis of functions, orthonormal wavelets, their properties and construction. Frames, libraries of bases, and the local cosine transform. Applications in approximation, estimation, and compression of signals.
3 units, Win (Papanicolaou)

267A,B. Topics in Functional and Harmonic Analysis—Topics from functional analysis and from the L^p-theory of harmonic analysis—the singular integral theory of Calderon and Zygmund and its extensions, interpolation of operators, multiplier transformations, and smoothness properties of functions. Sets of uniqueness for trigonometric series, spectral syntheses, thin sets, spectral theory of convolution operators, and applications. Prerequisite: knowledge of the elements of Fourier analysis.

267A. *3 units, Aut (Katznelson)*

267B. *3 units, Wiu (Katznelson)*

269. Holomorphic Curves and their Applications in Symplectic Geometry—Holomorphic curves in symplectic manifolds. Gromov's compactness theorem. Invariants of symplectic and contact manifolds, and their Lagrangian and Legendrian submanifolds. Gromov-Witten invariants. Floer homology. Contact homology. Symplectic field theory.

not given 2000-01

272A,B. Topics in Partial Differential Equations—Introduction to PDE methods in an intrinsic geometric setting. Topics: Schauder and DeGiorgi-Nash theory in a geometric setting, Sobolev, Poincare, and isoperimetric inequalities. Discussion of nonlinear methods: Leray-Schauder fixed point, and degree and variational methods. Geometric examples introduce basic nonlinear PDEs of geometry (the harmonic map, Yang-Mills, and mean curvature equations) and equations arising from scalar and Ricci curvature. Prerequisite: knowledge of differential geometry through 217A.

not given 2000-01

277. Mathematical Theory of Relativity—Ricci calculus; variational principles and covariance properties; differential geometry of space-time; Cauchy's problem for the differential equations of gravitation and electromagnetism; relativistic hydrodynamics; unified field theories.

not given 2000-01

281A,B. Introduction to Algebraic and Differential Topology—Fundamental group, covering spaces, embeddings and immersions of manifolds, transversality, homotopy theory, homology and cohomology of complexes, differential forms, Poincare Duality.

281A. *3 units, Aut (Milgram)*

281B. *3 units, Win (Hutchings)*

282A. Low Dimensional Topology—The theory of surfaces and 3-manifolds. Curves on surfaces, the classification of diffeomorphisms of surfaces, and Teichmuller space. The mapping class group and the braid group. Knot theory, including knot invariants. Decomposition of 3-manifolds: triangulations, Heegaard splittings, Dehn surgery. Loop theorem, sphere theorem, incompressible surfaces. Geometric structures, particularly hyperbolic structures on surfaces and 3-manifolds.

3 units, Aut (Kerckhoff)

282B. Homotopy Theory—Homotopy groups, fibrations, spectral sequences, simplicial methods, Dold-Thom theorem, models for loop spaces, homotopy limits and colimits, stable homotopy theory.

3 units, Win (Carlsson)

282C. Fiber Bundles and Cobordism—Possible topics: principal bundles, vector bundles, classifying spaces. Connections on bundles, curvature. Topology of gauge groups and gauge equivalence classes of connections. Characteristic classes and K-theory, including Bott periodicity, algebraic K-theory, and indices of elliptic operators. Spectral sequences of Atiyah-Hirzebruch, Serre, and Adams. Cobordism theory, Pontryagin-Thom theorem, calculation of unoriented and complex cobordism.

3 units, Spr (R. Cohen)

283. Topics in Algebraic Topology—Etale homotopy theory. Cech homotopy theory, etale coverings, Grothendieck's theory of the fundamental group of a scheme, etale homotopy type as a pro-space, comparison with the complex topology. Applications to K-theory, homotopy theory, and algebraic geometry.

3 units, Spr (Carlsson)

285. Geometric Measure Theory—Hausdorff measures and dimensions, area and co-area formulas for Lipschitz maps, integral currents and flat chains, minimal surfaces and their singular sets.

3 units, Spr (White)

286. Topics in Differential Geometry

3 units, Aut (Schoen, Yang)

289. 3-Dimensional Contact Geometry—Contact structures and foliations, geometric meaning of integrability, tight and overtwisted contact structures, classification of overtwisted contact structures, contact convexity, classification and finiteness results for tight contact structures, holomorphic methods in contact geometry, Legendrian and transversal knots.

3 units, Aut (Eliashberg)

290B. Model Theory—Kripke (possible world) semantics of intuitionistic and modal logics. Completeness results and strategies in automated deduction. Algebraic models. Second order systems. May be taken independently of 290A. Prerequisites: 160A,B or equivalent.

not given 2000-01

291A,B. Recursion Theory—Theory of recursive functions and recursively enumerable sets. Turing machines and alternative approaches. Diophantine definability. Definability in formal systems. Gödel's incompleteness theorems. Recursively unsolvable problems in mathematics and logic. Introduction to recursive ordinals and hierarchies. Prerequisites: 160A,B and 162, or equivalents.

291A. *3 units, Aut (Feferman)*

291B. *3 units, Win (Feferman)*

292A,B. Set Theory—The basics of axiomatic set theory; the systems of Zermelo-Fraenkel and Bernays-Gödel. Topics: cardinal and ordinal numbers, the cumulative hierarchy and the role of the axiom of choice. Models of set theory, including the constructible sets and models constructed by the method of forcing. Consistency and independence results for the axiom of choice, the continuum hypothesis and other unsettled mathematical and set-theoretical problems. Prerequisites: 160A,B and 161, or equivalents.

not given 2000-01

293A,B. Proof Theory—Gentzen's natural deduction and/or sequential calculi for first-order predicate logic. Normalization respectively cut-elimination procedures. Extensions to infinitary calculi; ordinal complexity of proof trees. Subsystems of analysis and their reduction to constructive theories. Prerequisites: 160A,B and 162, or equivalents.

293A. *3 units, Aut (Schwichtenberg)*

293B. *3 units, Win (Mints)*

294. Topics in Logic—Hilbert's program and its modern modifications. Brouwer-Heyting-Kolmogorov interpretation, realizability, extraction of programs from proofs. Normalization of proofs as computation of programs. Stability of program extraction. Intensional and extensional identity of programs. Continuous cut-elimination and its use for the study of finite proofs and programs. Prerequisites: 160A,B or equivalent.

3 units, Spr (Mints)

297. Algebraic Logic—(Enroll in Computer Science 353.)

3 units, Aut (Pratt)

360. Advanced Reading and Research

any quarter (Staff)

361. Seminar Participation—Participation in a faculty-led seminar which has no specific course number.

any quarter (Staff)

380. Seminar in Applied Mathematics
by arrangement

381. Seminar in Analysis
by arrangement

383. Seminar in Function Theory
by arrangement

385. Seminar in Abstract Analysis
by arrangement

386. Seminar in Geometry and Topology
by arrangement

387. Seminar in Algebra and Number Theory
by arrangement

388. Seminar in Probability and Stochastic Processes
by arrangement

389. Seminar in Mathematical Biology
by arrangement

391. Seminar in Logic and the Foundations of Mathematics