

# MATHEMATICS

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Courses given in Mathematics have the subject code MATH. For a complete list of subject codes, see Appendix.

The Department of Mathematics offers programs leading to the degrees of Bachelor of Science, Master of Science, and Doctor of Philosophy in Mathematics, and participates in the program leading to the B.S. in Mathematical and Computational Science. The department also participates in the M.S. and Ph.D. degree programs in Scientific Computing and Computational Mathematics and the M.S. degree program in Financial Mathematics.

## ADVANCED PLACEMENT FOR FRESHMEN

Students of unusual ability in mathematics often take one or more semesters of college-equivalent courses in mathematics while they are still in high school. Under certain circumstances, it is possible for such students to secure both advanced placement and credit toward the bachelor's degree. A decision as to placement and credit is made by the department after consideration of the student's performance on the Advanced Placement Examination in Mathematics (forms AB or BC) of the College Entrance Examination Board, and also after consideration of transfer credit in mathematics from other colleges and universities.

The department does not give its own advanced placement examination. Students can receive either 5 or 10 units of advanced placement credit, depending on their scores on the CEEB Advanced Placement Examination. Entering students who have credit for two quarters of single variable calculus (10 units) are encouraged to enroll in MATH 51-53 in multivariable mathematics, or the honors version 51H-53H. These three-course sequences, which can be completed during the freshman year, supply the necessary mathematics background for most majors in science and engineering. They also serve as excellent background for the major or minor in Mathematics, or in Mathematical and Computational Science. Students who have credit for one quarter of single variable calculus (5 units) should take MATH 42 in the Autumn Quarter and 51 in Winter Quarter. Options available in the Spring Quarter include MATH 52, 53, or 103. For proper placement, contact the Department of Mathematics.

## UNDERGRADUATE PROGRAMS BACHELOR OF SCIENCE

The following department requirements are in addition to the University's basic requirements for the bachelor's degree:

## MAJORS

Students wishing to major in Mathematics must satisfy the following requirements:

1. Department of Mathematics courses (other than MATH 100) totaling at least 49 units credit; such courses must be taken for a letter grade. For the purposes of this requirement, courses crosslisted with another department, such as MATH 105 (STATS 116) and MATH 160A and 160B (PHIL 151 and 152), count as Department of Mathematics courses.
2. Additional courses taken from Department of Mathematics courses numbered 101 and above or from approved courses in other disciplines with significant mathematical content, totaling at least 15 units credit. At least 9 of these units must be taken for a letter grade.
3. A Department of Mathematics adviser must be selected, and the courses selected under items '1' and '2' above must be approved by the department's director of undergraduate study, acting under guidelines laid down by the department's Committee for Undergraduate Affairs. The Department of Mathematics adviser can be any member of the department's faculty.
4. To receive the department's recommendation for graduation, a student must have been enrolled as a major in the Department of Mathematics for a minimum of two full quarters, including the quarter immediately before graduation. In any case, students are strongly encouraged to declare as early as possible, preferably by the end of the sophomore year.

Students are normally expected to complete either the sequence 19, 20, 21 or the sequence 41, 42 (but not both). Students with an Advanced Placement score of at least 4 in BC math or 5 in AB math may receive 10 units credit and fulfill requirement '1' by taking at least 39 units of Department of Mathematics courses numbered 51 and above. Students with an Advanced Placement score of at least 3 in BC math or at least 4 in AB math may receive 5 units credit and fulfill requirement '1' by taking at least 44 units of Department of Mathematics courses numbered 42 and above.

Sophomore seminar courses may be counted among the choice of courses under item '1'. Other variations of the course requirements laid down above (under items '1' and '2') may, in some circumstances, be allowed. For example, students transferring from other universities may be allowed credit for some courses completed before their arrival at Stanford. However, at least 24 units of the 49 units under item '1' above and 9 of the units under item '2' above must be taken at Stanford. In all cases, approval for variations in the degree requirements must be obtained from the department's Committee for Undergraduate Affairs. Application for such approval should be made through the department's director of undergraduate study.

It is to be emphasized that the above regulations are minimum requirements for the major; students contemplating graduate work in mathematics are strongly encouraged to include the courses 116, 120, 121, 147 or 148, and 171 in their selection of courses, and in addition, take at least three Department of Mathematics courses over and above the minimum requirements laid out under items '1' and '2' above, including at least one 200-level course. Such students are also encouraged to consider the possibility of taking the honors program, discussed below.

To help develop a sense of the type of course selection (under items '1' and '2' above) which would be recommended for math majors with various backgrounds and interests, see the following examples. These represent only a few of a very large number of possible combinations of courses that could be taken in fulfillment of the Mathematics major requirements:

*Example 1*—A general program (a balanced program of both pure and applied components, without any particular emphasis on any one field of mathematics or applications) as follows:

- a) either MATH 19, 20, and 21, or 41 and 42 (or satisfactory Advanced Placement credit); 51, 52, 53; 103; 106; 109; 110; 111; 115
- b) plus any selection of at least eight of the following courses, including three Department of Mathematics courses: MATH 105 (STATS 116), 108, 131, 132, 143, 146, 147, 148, 152, 161; PHYSICS 41, 43, 45; CS 137; ECON 50. (These specific courses from other departments are

only meant as an example. There are many suitable courses in several departments which can be taken to fulfill part or all of requirement 2.)

*Example 2*—A theoretical program (recommended for those contemplating possible later graduate work (see, also, the discussion of the honors program below), providing an introduction to the main areas of mathematics both broader and deeper than the general program outlined above):

- either MATH 19, 20 and 21, or 41 and 42 (or satisfactory Advanced Placement credit)
- either the sequence 51, 52, 53, or the sequence 51H, 52H, 53H; 106 or 116; 113; 120; 171
- plus nine or more of the following courses, including at least one from each group: algebra sequence 114, 121, 152, 156; analysis sequence 131, 132, 135, 151, 174A,B, 175; geometry/topology sequence 143, 145, 146, 147, 148; logic and set theory sequence 160A,B, 161.

In addition, those contemplating eventual graduate work in Mathematics should seriously consider including at least one graduate-level math course such as MATH 205A, 210A, or 215A or B. Such students should also consider the possibility of entering the honors program.

(Students taking 51, 52, 53 rather than 51H, 52H, 53H should consider taking 113 before attempting 114.)

*Example 3\**—An Applied Mathematics program:

- either MATH 19, 20, and 21; or 41 and 42 (or satisfactory Advanced Placement credit); 51, 52, 53; 103; 105 (Statistics 116); 106; 108; 109; 110; 111; 115; 131
- plus at least 15 units of additional courses in Applied Mathematics, including, for example, suitable courses from the departments of Physics, Computer Science, Economics, Engineering, and Statistics.

\* Students with interests in applied mathematics, but desiring a broader-based program than the type of program suggested in Example 3, including significant computational and/or financial and/or statistical components, are encouraged to also consider the Mathematics and Computational Science program.

## MINORS

To qualify for the minor in Mathematics, a student should successfully complete, for a letter grade, at least six Department of Mathematics courses (other than MATH 100) numbered 51 or higher, totaling a minimum of 24 units. It is recommended that these courses include either the sequence 51, 52, 53 or the sequence 51H, 52H, 53H. At least 12 of the units applied toward the minor in Mathematics must be taken at Stanford. The policy of the Mathematics Department is that no courses other than the MATH 50 series and below may be double-counted toward any other University major or minor.

## HONORS PROGRAM

The honors program is intended for students who have strong theoretical interests and abilities in mathematics. The goal of the program is to give students a thorough introduction to the main branches of mathematics, especially analysis, algebra, and geometry. Through the honors thesis, students may be introduced to a current or recent research topic, although occasionally more classical projects are encouraged. The program provides an excellent background with which to enter a master's or Ph.D. program in Mathematics. Students successfully completing the program are awarded a B.S. in Mathematics with Honors.

It is recommended that the sequence 51H, 52H, 53H be taken in the freshman year. Students who have instead taken the sequence 51, 52, 53 in their freshman year may be permitted to enter the honors program, but such entry must be approved by the Department of Mathematics Committee for Undergraduate Affairs.

To graduate with a B.S. in Mathematics with Honors, the following conditions apply in addition to the usual requirements for math majors:

- The selection of courses under items '1' and '2' above must include all the math courses 106 or 116, 120, 171 and also must include seven or more additional courses, with at least one from each of the groups: algebra sequence 114, 121, 152, 153, 156; analysis sequence 131, 132, 135, 136, 151, 174A, 174B, 175, 176; geometry/topology sequence 143, 145, 146, 147, 148; logic and set theory sequence 160A, 160B, 161.

- Students in the honors program must write a senior thesis. In order to facilitate this, the student must, by the end of the junior year, choose an undergraduate thesis adviser from the Department of Mathematics faculty, and map out a concentrated reading program under the direction and guidance of the adviser. During the senior year, the student must enroll in MATH 197 for a total of 6 units (typically spread over two quarters), and work toward completion of the thesis under the direction and guidance of the thesis adviser. The thesis may contain original material, or be a synthesis of work in current or recent research literature. The 6 units of credit for MATH 197 are required in addition to the course requirements laid out under items '1' and '2' above and in addition to all other requirements for math majors.

In addition to the minimum requirements laid out above, it is strongly recommended that students take at least one graduate-level course (that is, at least one course in the 200 plus range). MATH 205A, 210A, and 215A or B are especially recommended in this context.

Students with questions about the honors program should see the Director of Undergraduate Advising.

## BACHELOR OF SCIENCE IN MATHEMATICAL AND COMPUTATIONAL SCIENCE

The Department of Mathematics participates with the departments of Computer Science, Management Science and Engineering, and Statistics in a program leading to a B.S. in Mathematical and Computational Science. See the "Mathematical and Computational Science" section of this bulletin.

## GRADUATE PROGRAMS MASTER OF SCIENCE

The University's basic requirements for the master's degree are discussed in the "Graduate Degrees" section of this bulletin. Students entering Stanford in 2001 or later should pay particular attention to the University's course requirements for graduate degrees. The following are specific departmental requirements:

Candidates must complete an approved course program of 45 units of courses beyond the department requirements for the B.S. degree, of which at least 36 units must be Mathematics Department courses, taken for a letter grade. The Mathematics courses must include at least 18 units numbered 200 or above. The candidate must have a grade point average (GPA) of 3.0 (B) over all course work taken in Mathematics, and a GPA of 3.0 (B) in the 200-level courses considered separately. Course work for the M.S. degree must be approved during the first quarter of enrollment in the program by the department's Director of Graduate Studies.

For the M.S. degree in Financial Mathematics, see the "Financial Mathematics" section of this bulletin.

## TEACHING CREDENTIALS

For information concerning the requirements for teaching credentials, see the "School of Education" section of this bulletin or address inquiries to Credential Secretary, School of Education.

## DOCTOR OF PHILOSOPHY

The University's basic requirements for the doctorate (residence, dissertation, examinations, etc.) are discussed in the "Graduate Degrees" section of this bulletin. The following are specific departmental requirements.

To be admitted to candidacy, the student must have successfully completed 27 units of graduate courses (that is, courses numbered 200 and above). In addition, the student must pass qualifying examinations given by the department.

Beyond the requirements for candidacy, the student must complete a course of study approved by the Graduate Affairs Committee of the Department of Mathematics and submit an acceptable dissertation. In accordance with University requirements, Ph.D. students must complete a total of 135 course units beyond the bachelor's degree. These courses should be Department of Mathematics courses or approved courses from other departments. The course program should display substantial breadth

in mathematics outside the student's field of application. The student must receive a grade point average (GPA) of 3.0 (B) or better in courses used to satisfy the Ph.D. requirement. In addition, the student must pass the Department area examination and the University oral examination and pass a reading examination in one foreign language, chosen from French, German, or Russian.

Experience in teaching is emphasized in the Ph.D. program. Each student is required to complete nine quarters of such experience. The nature of the teaching assignment for each of those quarters is determined by the department in consultation with the student. Typical assignments include teaching or assisting in teaching an undergraduate course or lecturing in an advanced seminar.

For further information concerning degree programs, fellowships, and assistantships, inquire of the academic associate of the department.

## PH.D. MINOR

The student should complete both of the following:\*

1. MATH 106 or 116, 131, 132
2. MATH 113, 114, 120 or 152

These courses may have been completed during undergraduate study, and their equivalents from other universities are acceptable.

In addition, the student should complete 21 units of 200-level courses in Mathematics. These must be taken at Stanford and approved by the Department of Mathematics' Ph.D. minor adviser.

\* A third coherent sequence designed by the student, subject to the approval of the graduate committee, may be considered as a substitute for items '1' or '2'.

## COURSES

WIM indicates that the course satisfies the Writing in the Major requirements.

### INTRODUCTORY AND UNDERGRADUATE

The department offers two sequences of introductory courses in single variable calculus.

1. MATH 41, 42 present single variable calculus. Differential calculus is covered in the first quarter, integral calculus in the second.
2. MATH 19, 20, 21 cover the material in 41, 42 in three quarters instead of two.

There are options for studying multivariable mathematics:

1. MATH 51, 52, 53 cover differential and integral calculus in several variables, linear algebra, and ordinary differential equations. These topics are taught in an integrated fashion and emphasize application. MATH 51 covers differential calculus in several variables and introduces matrix theory and linear algebra, 52 covers integral calculus in several variables and vector analysis, 53 studies further topics in linear algebra and applies them to the study of ordinary differential equations. This sequence is strongly recommended for incoming freshmen with 10 units of advanced placement credit.
2. MATH 51H, 52H, 53H cover the same material as 51, 52, 53, but with more emphasis on theory and rigor.

The introductory course in modern algebra is Linear Algebra (103 or 113). There are no formal prerequisites for these courses, but appropriate mathematical maturity is expected. Much of the material in 103 is covered in the sequence 51, 52, 53.

**MATH 19. Calculus**—The content of MATH 19, 20, 21 is the same as the sequence MATH 41, 42 described below, but covered in three quarters, rather than two. GER:DB-Math  
3 units, Aut (*Lucianovic*), Win (*Staff*), Sum (*Staff*)

**MATH 20. Calculus**—Continuation of 19. Prerequisite: 19. GER:DB-Math  
3 units, Win (*Lucianovic*), Spr (*Staff*)

**MATH 21. Calculus**—Continuation of 20. Prerequisite: 20. GER:DB-Math  
4 units, Spr (*Lucianovic*)

**MATH 41. Calculus**—Introduction to differential and integral calculus of functions of one variable. Topics: review of elementary functions including exponentials and logarithms, rates of change, and the derivative. Introduction to the definite integral and integration. Prerequisites: algebra, trigonometry. GER:DB-Math  
5 units, Aut (*Meckes*)

**MATH 42. Calculus**—Continuation of 41. Methods of symbolic and numerical integration, applications of the definite integral, introduction to differential equations. Infinite series. Prerequisite: 41 or equivalent. GER:DB-Math  
5 units, Aut (*Gromoll*), Win (*Meckes*)

**MATH 51. Linear Algebra and Differential Calculus of Several Variables**—Geometry and algebra of vectors, systems of linear equations, matrices, vector valued functions and functions of several variables, partial derivatives, gradients, chain rule in several variables, vector fields, optimization. Prerequisite: 21, 42, or a score of 4 on the BC Advanced Placement exam or 5 on the AB Advanced Placement exam, or consent of instructor. GER:DB-Math  
5 units, Aut (*Brubaker, Blumberg, Clingher, Milanov, Oprea*), Win (*Thiem, White, Young*), Spr (*Meckes*), Sum (*Staff*)

**MATH 51A. Linear Algebra and Differential Calculus of Several Variables, ACE**— GER:DB-Math  
5-6 units, Aut, Win, Spr (*Staff*)

**MATH 51H. Honors Multivariable Mathematics**—For prospective Mathematics majors in the honors program and students from other areas of science or engineering who have a strong mathematics background. Three quarter sequence covers the material of 51, 52, 53, and additional advanced calculus and ordinary and partial differential equations. Unified treatment of multivariable calculus, linear algebra, and differential equations with a different order of topics and emphasis from standard courses. Students should know one-variable calculus and have an interest in a theoretical approach to the subject. Prerequisite: score of 5 on BC Advanced Placement exam, or consent of instructor. GER:DB-Math  
5 units, Aut (*Kerckhoff*)

**MATH 52. Integral Calculus of Several Variables**—Iterated integrals, line and surface integrals, vector analysis with applications to vector potentials and conservative vector fields, physical interpretations. Divergence theorem and the theorems of Green, Gauss, and Stokes. Prerequisite: 51.  
5 units, Aut (*Wieczorek*), Win (*Papikian, Wieczorek*), Spr (*Papikian*)

**MATH 52H. Honors Multivariable Mathematics**—Continuation of 51H. Prerequisite: 51H.  
5 units, Win (*Simon*)

**MATH 53. Ordinary Differential Equations with Linear Algebra**—Linear ordinary differential equations, applications to oscillations, matrix methods including determinants, eigenvalues and eigenvectors, matrix exponentials, systems of linear differential equations with constant coefficients, stability of non-linear systems and phase plane analysis, numerical methods, Laplace transforms. Integrated with topics from linear algebra (103). Prerequisite: 51.  
5 units, Aut (*Bump*), Win (*T. Li*), Spr (*Ionel, Shirokova*), Sum (*Staff*)

**MATH 53H. Honors Multivariable Mathematics**—Continuation of 52H. Prerequisite: 52H.  
5 units, Spr (*Brendle*)

**MATH 80Q. Capillary Surfaces: Explored and Unexplored Territory**—Stanford Introductory Seminar. Preference to sophomores. Capillary surfaces: the interfaces between fluids that are adjacent to each other and do not mix. Recently discovered phenomena, predicted mathematically and subsequently confirmed by experiments, some done in space shuttles. Interested students may participate in ongoing investigations with affinity between mathematics and physics.  
3-5 units, Win (*Finn*)

**MATH 87Q. Mathematics of Knots, Braids, Links, and Tangles**—Stanford Introductory Seminar. Preference to sophomores. Types of knots and how knots can be distinguished from one another by means of numerical or polynomial invariants. The geometry and algebra of braids, including their relationships to knots. Topology of surfaces. Brief summary of applications to biology, chemistry, and physics.

3 units, Spr (Brumfiel)

**MATH 88N. Mathematics and Magic Tricks**—Stanford Introductory Seminar. How a magic trick works is more amazing than the trick itself. Tricks that use mathematics in a hidden way: permutations, parity, finite fields, and graphs. History of magic; performance and invention of magic and new theorems.

3 units, Win (Diaconis)

**MATH 88Q. The Mathematics of the Rubik's Cube**—Stanford Introductory Seminar. Preference to sophomores.

3 units, Win (Bump)

## UNDERGRADUATE AND GRADUATE

Unless stated, there are no prerequisites for the courses listed below. Where a prerequisite is stated, it may be waived by the instructor.

**MATH 100. Mathematics for Elementary School Teachers**—Mathematics and pedagogical strategies. Core mathematical content in grades K-6, classroom presentation, how to handle student errors, and mathematical issues that come up during instruction.

4 units, Spr (Ma, Milgram)

**MATH 103. Matrix Theory and its Applications**—Linear algebra and matrices, emphasizing the computational and algorithmic aspects and the scientific problems in which matrix theory is applied. Solution of linear equations. Linear spaces and matrices. Orthogonal projection and least squares. Determinants, eigenvalues, and eigenvectors. GER:DB-Math

3 units, Aut (Dembo, Khuri), Win (Goel, Shirokova),  
Spr (T. Li), Sum (Staff)

**MATH 105. Theory of Probability**—(Enroll in STATS 116.)

3-5 units (Staff)

**MATH 106. Functions of a Complex Variable**—Complex numbers, analytic functions, Cauchy-Riemann equations, complex integration, Cauchy integral formula, residues, elementary conformal mappings. Prerequisite: 52.

3 units, Aut (Galatius), Sum (Staff)

**MATH 108. Introduction to Combinatorics and its Applications**—Topics: graphs, trees (Cayley's Theorem, application to phylogeny), eigenvalues, basic enumeration (permutations, Stirling and Bell numbers), recurrences, generating functions, basic asymptotics. Prerequisites: 51 or 103 or equivalent.

3 units, Aut (Thiem)

**MATH 109. Applied Group Theory**—Applications of the theory of groups. Topics: elements of group theory, groups of symmetries, matrix groups, group actions, and applications to combinatorics and computing. Applications: rotational symmetry groups, the study of the Platonic solids, crystallographic groups and their applications in chemistry and physics. WIM

3 units, Aut (Carlsson)

**MATH 110. Applied Number Theory and Field Theory**—Number theory and its applications to modern cryptography. Topics: congruences, finite fields, primality testing and factorization, public key cryptography, error correcting codes, and elliptic curves, emphasizing algorithms. WIM

3 units, Spr (Brumfiel)

**MATH 111. Computational Commutative Algebra**—Introduction to the theory of commutative rings, ideals, and modules. Systems of polynomial equations in several variables from the algorithmic viewpoint. Groebner bases, Buchberger's algorithm, elimination theory. Applications to algebraic geometry and to geometric problems.

3 units, Win (Lucianovic)

**MATH 113. Linear Algebra and Matrix Theory**—Algebraic properties of matrices and their interpretation in geometric terms. The relationship between the algebraic and geometric points of view and matters fundamental to the study and solution of linear equations. Topics: linear equations, vector spaces, linear dependence, bases and coordinate systems; linear transformations and matrices; similarity; eigenvectors and eigenvalues; diagonalization.

3 units, Aut (Katznelson), Win (Vakil)

**MATH 114. Linear Algebra and Matrix Theory II**—Continuation of 113. Deeper study of 113 topics plus additional topics from invariant subspaces, canonical forms of matrices; minimal polynomials and elementary divisors; vector spaces over arbitrary fields; inner products; Jordan normal forms; Hermitian and unitary matrices; multilinear algebra; applications.

3 units, Win (Katznelson), Spr (Milgram)

**MATH 115. Functions of a Real Variable**—The development of real analysis in Euclidean space: sequences and series, limits, continuous functions, derivatives, integrals. Basic point set topology. Honors math majors and students who intend to do graduate work in mathematics should take 171. Prerequisite: 51.

3 units, Aut (Liu), Win (Brumfiel), Sum (Staff).

**MATH 116. Complex Analysis**—Analytic functions, Cauchy integral formula, power series and Laurent series, calculus of residues and applications, conformal mapping, analytic continuation, introduction to Riemann surfaces, Fourier series and integrals. Applications of complex analysis to electrostatics, hydrodynamics, and theoretical physics. Prerequisite: 52.

3 units, Spr (Clingher)

**MATH 118. Numerical Analysis**—Topics: iterative methods for nonlinear equations, topics from numerical linear algebra, interpolation theory, splines, approximation of functions, and numerical differentiation and integration.

3 units, Aut (Fetecau)

**MATH 120. Modern Algebra**—Basic structures in algebra: groups, rings, and fields. Elements of group theory: permutation groups, finite Abelian groups, p-groups, Sylow theorems. Polynomial rings, principal ideal domains, unique factorization domains. WIM

3 units, Aut (Shirokova), Spr (Clingher)

**MATH 121. Modern Algebra II**—Continuation of 120. Fields of fractions. Solvable and simple groups. Elements of field theory and Galois theory. Prerequisite: 120.

3 units, Win (J. Li)

**MATH 131. Partial Differential Equations I**—First-order equations, classification of second-order equations. Initial-boundary value problems for heat, wave, and related equations. Separation of variables, eigenvalue problems, Fourier series, existence and uniqueness questions. Prerequisite: 53 or equivalent.

3 units, Aut (Elgart), Win (Schoen)

**MATH 132. Partial Differential Equations II**—Laplace's equation and properties of harmonic functions. Green's functions. Distributions and Fourier transforms. Eigenvalue problems and generalized Fourier series. Numerical solutions. Prerequisite: 131.

3 units, Spr (Schoen)

**MATH 135. Nonlinear Dynamics and Chaos**—Topics: one- and two-dimensional flows, bifurcations, phase plane analysis, limit cycles and their bifurcations. Lorenz equations, fractals and strange attractors. Prerequisite: 51 and 53 or equivalent.

3 units, Spr (Fetecau)

**MATH 136. Stochastic Processes**—Introduction to measure theory, Lp spaces and Hilbert spaces. Random variables, expectation, conditional expectation, conditional distribution. Uniform integrability, almost sure and Lp convergence. Stochastic processes: definition, stationarity, sample path continuity. Examples: random walk, Markov chains, Gaussian

processes, Poisson processes, Martingales. Construction and basic properties of Brownian motion. Prerequisites: STATS 116, or MATH 151 or equivalent. Recommended: 115 or equivalent.

3 units, Aut (Dembo)

**MATH 143. Differential Geometry**—Geometry of curves and surfaces in three-space and higher dimensional manifolds. Parallel transport, curvature, and geodesics. Surfaces with constant curvature. Minimal surfaces.

3 units, Win (P. Cohen)

**MATH 145. Algebraic Geometry**—Real algebraic curves, Hilbert's nullstellensatz, complex affine and projective curves, Bezout's theorem, the degree/genus formula, Riemann surfaces, Riemann-Roch theorem. Prerequisites: 106 or 116, and 109 or 120. Recommended: familiarity with surfaces equivalent to 143, 146, 147, or 148.

3 units (Staff) not given 2005-06

**MATH 146. Analysis on Manifolds**—(Formerly 173.) Differentiable manifolds, tangent space, submanifolds, implicit function theorem, differential forms, vector and tensor fields. Frobenius' theorem, DeRham theory. Prerequisite: 52 or 52H.

3 units, Aut (Galatius)

**MATH 147. Differential Topology**—Smooth manifolds, transversality, Sard's theorem, embeddings, degree of a map, Borsuk-Ulam theorem, Hopf degree theorem, Jordan curve theorem. Prerequisite: 115 or 171.

3 units, Spr (Munson) alternate years, not given 2006-07

**MATH 148. Algebraic Topology**—Fundamental group, covering spaces, Euler characteristic, homology, classification of surfaces, knots. Prerequisite: 109 or 120.

3 units (Staff) alternate years, given 2006-07

**MATH 151. Introduction to Probability Theory**—Counting; axioms of probability; conditioning and independence; expectation and variance; discrete and continuous random variables and distributions; joint distributions and dependence; central limit theorem and laws of large numbers. Prerequisite: 52 or consent of instructor.

3 units, Win (Liu)

**MATH 152. Elementary Theory of Numbers**—Euclid's algorithm, fundamental theorems on divisibility; prime numbers, congruence of numbers; theorems of Fermat, Euler, Wilson; congruences of first and higher degrees; Lagrange's theorem and its applications; quadratic residues; introduction to the theory of binary quadratic forms.

3 units, Spr (Brubaker)

**MATH 156. Group Representations**—Experimental, primarily examining symmetries on objects such as vector spaces (group representations), geometric objects (geometric group actions), and discrete sets (combinatorics). Topics: group representations and their characters, classification of permutation group representations using partitions and Young tableaux, group actions on sets and the Burnside ring, and spherical space forms. Prerequisites: linear algebra (51 and 53, or 103 or 113) and group theory (109 or 120).

3 units, Aut (Thiem)

**MATH 160A. First-Order Logic**—(Enroll in PHIL 151/251.)

4 units, Win (Pauly)

**MATH 160B. Computability and Logic**—(Enroll in PHIL 152/252.)

4 units, Spr (Pauly)

**MATH 161. Set Theory**—Informal and axiomatic set theory: sets, relations, functions, and set-theoretical operations. The Zermelo-Fraenkel axiom system and the special role of the axiom of choice and its various equivalents. Well-orderings and ordinal numbers; transfinite induction and transfinite recursion. Equinumerosity and cardinal numbers; Cantor's Alephs and cardinal arithmetic. Open problems in set theory.

3 units, Win (de Queiroz)

**MATH 162. Philosophy of Mathematics**—(Enroll in PHIL 162/262.)

4 units (Staff) not given 2005-06

**MATH 171. Fundamental Concepts of Analysis**—Recommended for math majors and required of honors math majors. Similar to 115 but altered content and more theoretical orientation. Properties of Riemann integrals, continuous functions and convergence in metric spaces; compact metric spaces, basic point set topology. Prerequisites: 51 and 52, or 51H and 52H. WIM

3 units, Aut, Spr (Fetecau)

**MATH 174A. Topics in Analysis and Differential Equations with Applications**—For students planning graduate work in mathematics or physics, and for honors math majors and other students at ease with rigorous proofs and qualitative discussion. 174A topics may include: geometric theory of ODE's with applications to dynamics; mathematical foundations of classical mechanics including variational principles, Lagrangian and Hamiltonian formalisms, theory of integrable systems; theorems of existence and uniqueness; Sturm-Liouville theory. 174B topics may include: introduction to PDEs including transport equations, Laplace, wave, and heat equations; techniques of solution including separation of variables and Green's functions; Fourier series and integrals; introduction to the theory of distributions; mathematical foundations of quantum mechanics. Prerequisite: 53H or 171, or consent of instructor.

3 units, Win (Guth)

**MATH 174B. Honors Analysis**—Continuation of 174A. Prerequisite: 174A.

3 units, Spr (Guth)

**MATH 175. Elementary Functional Analysis**—Linear operators on Hilbert space. Spectral theory of compact operators; applications to integral equations. Elements of Banach space theory. Prerequisite: 115 or 171.

3 units, Spr (Simon)

**MATH 180. Introduction to Financial Mathematics**—Financial derivatives: contracts and options. Hedging and risk management. Arbitrage, interest rate, and discounted value. Geometric random walk and Brownian motion as models of risky assets. Initial boundary value problems for the heat and related partial differential equations. Self-financing replicating portfolio. Black-Scholes pricing of European options. Dividends. Implied volatility. Optimal stopping and American options. Prerequisite: 53. Corequisites: 131, 151 or STATS 116.

3 units, Aut (Durreleman)

**MATH 197. Senior Honors Thesis**

1-6 units, Aut, Win, Spr (Staff)

**MATH 199. Independent Work**—Undergraduates pursue a reading program; topics limited to those not in regular department course offerings. Credit can fulfill the elective requirement for math majors. Approval of Undergraduate Affairs Committee is required to use credit for honors majors area requirement.

1-3 units, Aut, Win, Spr (Staff)

**PRIMARILY FOR GRADUATE STUDENTS**

**MATH 202A. Advising and Mentoring in Financial Mathematics**—Students advise and mentor those enrolled in 236 or 238 to enhance professional experience consistent with the M.S. in Financial Mathematics program. May be repeated for credit. Prerequisite: 236 or 238, stochastic calculus and mathematical finance, and consent of instructor.

1-3 units, Win (Papanicolaou)

**MATH 202B. Advising and Mentoring in Financial Mathematics**—Students advise and mentor those enrolled in 239 or 240. Goal is to enhance professional experience consistent with the Financial Mathematics master's program. Prerequisites: 239 or 240, computation and simulation in finance or fixed income modeling, and consent of instructor.

1-3 units, Spr (Durreleman)

**MATH 205A. Real Analysis**—Basic measure theory and the theory of Lebesgue integration. Prerequisite: 171 or equivalent.

3 units, Aut (White)

**MATH 205B. Real Analysis**—Point set topology, basic functional analysis, Fourier series, and Fourier transform. Prerequisites: 171 and 205A or equivalent.

3 units, Win (Mazzeo)

**MATH 205C. Real Analysis**—Continuation of 205B.

3 units, Spr (Katznelson)

**MATH 210A. Modern Algebra**—Groups, rings, and fields; introduction to Galois theory. Prerequisite: 120 or equivalent.

3 units, Aut (Milgram)

**MATH 210B. Modern Algebra**—Galois theory. Ideal theory, introduction to algebraic geometry and algebraic number theory. Prerequisite: 210A.

3 units, Win (Brumfiel)

**MATH 210C. Modern Algebra**—Continuation of 210B. Representations of groups and noncommutative algebras, multilinear algebra.

3 units, Spr (Bump)

**MATH 215A. Complex Analysis, Geometry, and Topology**—Analytic functions, complex integration, Cauchy's theorem, residue theorem, argument principle, conformal mappings, Riemann mapping theorem, Picard's theorem, elliptic functions, analytic continuation and Riemann surfaces.

3 units, Aut (Coffee)

**MATH 215B. Complex Analysis, Geometry, and Topology**—(Math 215A is not a prerequisite for 215B.) Topics: fundamental group and covering spaces, homology, cohomology, products, basic homotopy theory, and applications. Prerequisites: 113, 120, and 171, or equivalent.

3 units, Win (Kerckhoff)

**MATH 215C. Complex Analysis, Geometry, and Topology**—Differentiable manifolds, transversality, degree of a mapping, vector fields, intersection theory, and Poincaré duality. Differential forms and the DeRham theorem. Prerequisite: 215B or equivalent.

3 units, Spr (R. Cohen)

**MATH 216A. Introduction to Algebraic Geometry**—Algebraic curves, algebraic varieties, sheaves, cohomology, Riemann-Roch theorem. Classification of algebraic surfaces, moduli spaces, deformation theory and obstruction theory, the notion of schemes. May be repeated for credit.

3 units, Aut (Vakil)

**MATH 216B. Introduction to Algebraic Geometry**—Continuation of 216A. May be repeated for credit.

3 units, Win (Vakil)

**MATH 216C. Introduction to Algebraic Geometry**—Continuation of 216B. May be repeated for credit.

3 units, Spr (Vakil)

**MATH 217A. Differential Geometry**—Smooth manifolds and submanifolds, tensors and forms, Lie and exterior derivative, DeRham cohomology, distributions and the Frobenius theorem, vector bundles, connection theory, parallel transport and curvature, affine connections, geodesics and the exponential map, connections on the principal frame bundle. Prerequisite: 173 or equivalent.

3 units, Aut (Khuri)

**MATH 217B. Differential Geometry**—Riemannian manifolds, Levi-Civita connection, Riemann curvature tensor, Riemannian exponential map and geodesic normal coordinates, Jacobi fields, completeness, spaces of constant curvature, bi-invariant metrics on compact Lie groups, symmetric and locally symmetric spaces, equations for Riemannian submanifolds and Riemannian submersions. Prerequisite: 217A.

3 units, Win (Brendle)

**MATH 217C. Differential Geometry**—First and second variation of arc length, index form and variational theory of geodesics, comparison theorems and consequences for manifolds of positive and negative curvature, almost complex manifolds and integrability, Hermitian and Kaehler metrics, connections on complex vector bundles and Chern

classes, Hodge theory, vanishing theorems in the Riemannian and Kaehler settings.

3 units, Spr (Schoen)

**MATH 220A. Partial Differential Equations of Applied Mathematics**—First-order partial differential equations, method of characteristics, weak solutions, conservation laws, hyperbolic equations, separation of variables, Fourier series, Kirchoff's formula, Huygen's principle, and hyperbolic systems. Prerequisite: foundation in multivariable calculus and ordinary differential equations.

3 units, Aut (Liu)

**MATH 220B. Partial Differential Equations of Applied Mathematics**—Parabolic and elliptic partial differential equations. Eigenvalue problems, Green's functions, properties of harmonic functions, potential theory, Fourier series and Fourier transform. Prerequisite: 220A or 131.

3 units, Win (Liu)

**MATH 220C. Partial Differential Equations of Applied Mathematics**—Mathematical methods of imaging: array imaging using Kirchoff migration and beamforming, resolution theory for broad and narrow band array imaging in homogeneous media, topics in high-frequency, variable background imaging with velocity estimation, interferometric imaging methods, the role of noise and inhomogeneities, and variational problems that arise in optimizing the performance of imaging algorithms and the deblurring of images. Prerequisites: 220A and B.

3 units, Spr (Papanicolaou)

**MATH 222A. Computational Methods for Fronts, Interfaces and Waves**—High-order methods for multidimensional systems of conservation laws and Hamilton-Jacobi equations (central schemes, discontinuous Galerkin methods, relaxation methods). Level set methods and fast marching methods. Computation of multi-valued solutions. Multi-scale analysis, including wavelet-based methods. Boundary schemes (perfectly matched layers). Examples from (but not limited to) geometrical optics, transport equations, reaction-diffusion equations, imaging, and signal processing.

3 units (Staff) not given 2005-06

**MATH 224. Topics in Mathematical Biology**—Mathematical models for biological processes based on ordinary and partial differential equations. Topics: population and infectious diseases dynamics, biological oscillators, reaction diffusion models, biological waves, and pattern formation. Prerequisites: 53 and 131, or equivalents.

3 units (Staff) not given 2005-06

**MATH 228A. Ergodic Theory**—Measure preserving transformations and flows, ergodic theorems, mixing properties, spectrum, Kolmogorov automorphisms, entropy theory. Examples. Classical dynamical systems, mostly geodesic and horocycle forms on homogeneous spaces of  $SL(2, \mathbb{R})$ . Prerequisites: 205A, B.

3 units, Spr (Ornstein)

**MATH 228B. Ergodic Theory**—Continuation of 228A.

3 units (Staff) not given 2005-06

**MATH 230A. Theory of Probability**—(Enroll in STATS 310A.)

2-4 units, Aut (Dembo)

**MATH 230B. Theory of Probability**—(Enroll in STATS 310B.)

2-4 units, Win (Dembo)

**MATH 230C. Theory of Probability**—(Enroll in STATS 310C.)

2-4 units, Spr (Lai)

**MATH 232. Topics in Probability: Geometry and Markov Chains**—Dirichlet forms; Nash, Sobolev, and log-Sobolev inequalities; and applications to card shuffling and random walk on graphs. May be repeated for credit.

3 units, Win (Diaconis)

**MATH 234. Large Deviations**—(Same as STATS 374.) Combinatorial estimates and the method of types. Large deviation probabilities for partial sums and for empirical distributions, Cramer's and Sanov's theorems and

their Markov extensions. Applications in statistics, information theory, and statistical mechanics. Prerequisite: MATH 230A or STATS 310.

3 units (Staff) not given 2005-06

**MATH 235. Topics in Ergodic Theory**—May be repeated for credit.

3 units, Win (Ornstein)

**MATH 236. Introduction to Stochastic Differential Equations**—Brownian motion, stochastic integrals, and diffusions as solutions of stochastic differential equations. Functionals of diffusions and their connection with partial differential equations. Random walk approximation of diffusions. Prerequisite: 136 or equivalent and differential equations.

3 units, Win (Papanicolaou)

**MATH 237. Topics in Stochastic Analysis: Credit Risk**—The analysis of structural models for credit default starting from the Black-Scholes-Merton theory and continuing with stochastic volatility models. The theory of reduced models of default risk and doubly stochastic, intensity-based, default models. Connections with actuarial insurance problems. Hybrid structural and intensity based models for the pricing of defaultable bonds and credit default swaps. Extensions to multi-asset models with correlated defaults. Prerequisites: 236 and 238, or equivalents.

3 units, Spr (Papanicolaou)

**MATH 238. Mathematical Finance**—(Same as STATS 250; formerly MATH 241.) Stochastic models of financial markets. Forward and futures contracts. European options and equivalent martingale measures. Hedging strategies and management of risk. Term structure models and interest rate derivatives. Optimal stopping and American options. Corequisites: MATH 236 and 220B or equivalent.

3 units, Win (Papanicolaou)

**MATH 239. Computation and Simulation in Finance**—(Formerly 240.) Monte Carlo, finite difference, tree, and transform methods for the numerical solution of partial differential equations in finance. Emphasis is on derivative security pricing. Prerequisite: 238 or equivalent.

3 units, Spr (Durrleman)

**MATH 240. Topics in Financial Mathematics: Fixed Income Models**—Introduction to continuous time models for arbitrage-free pricing of interest rate derivatives. Bonds, yields, and the construction of yield curves. Caps, floors, swaps, swaptions, and bond options. Short rate models. Yield curve models. Forward measures. Forward and futures. LIBOR and swap market models. Prerequisite: MATH 238.

3 units, Aut, Spr (Durrleman)

**MATH 244. Riemann Surfaces**—Compact Riemann surfaces: topological classifications, Hurwitz' formula, Riemann-Roch formula, uniformization theorem. Abel's theorem, Jacobian varieties. Some elements of harmonic analysis are developed with applications. Emphasis is on methods generally applicable to algebraic curves.

3 units, Win (Eliashberg)

**MATH 245. Topics in Algebraic Geometry**—Derived categories and derived functors, triangulated categories and intersection cohomology, and perverse sheaves. May be repeated for credit.

3 units, Spr (Kiem)

**MATH 246. Symmetric Functions and Algebraic Combinatorics**—Unified treatment of topics in classical enumeration via the study of symmetric polynomials. Classical symmetric functions, Schur functions, Young tableaux, Schensted correspondence, character theory of the symmetric group, introduction to random matrix theory. Prerequisite: 210 or equivalent.

3 units (Staff) not given 2005-06

**MATH 247. Topics in Group Theory**—Topics include the Burnside basis theorem, classification of  $p$ -groups, regular and powerful groups, Sylow theorems, the Frattini argument, nilpotent groups, solvable groups, theorems of P. Hall, group cohomology, and the Schur-Zassenhaus theorem. The classical groups and introduction to the classification of finite simple groups and its applications. May be repeated for credit.

3 units (Staff) not given 2005-06

**MATH 248. Algebraic Number Theory**—Introduction to modular forms and  $L$ -functions. May be repeated for credit.

1-3 units, Aut (Brubaker, Bump)

**MATH 249A. Topics in Representation Theory and Number Theory**—Spectral theory of automorphic forms: representation theory of  $SL(2, \mathbb{R})$ , analytic continuation of Eisenstein series using integral operators, spectral decomposition of  $\Gamma \backslash SL(2, \mathbb{R})$  and the Selberg trace formula. May be repeated for credit.

3 units, Win (Bump)

**MATH 249B. Topics in Representation Theory and Number Theory**—249A and B cover the essential tools in the theory of automorphic forms. Representation theory of  $GL(2)$  over a  $p$ -adic field.

3 units, Spr (Brubaker)

**MATH 253. Regularity of Sets and Mapping**—For students interested in any area of analysis. Topics: Lipschitz functions; smooth functions; Sobolev functions; various regularity and extension theorems including Rademacher, Kirzbraun, Whitney, and Sard. Critical sets of real-analytic, complex analytic functions. Affine approximation properties of subsets of  $\mathbb{R}^n$ , including a discussion of rectifiability and non-rectifiability, structure theorem, and Reifenberg's topological disc theorem.

3 units (Staff) not given 2005-06

**MATH 256A. Partial Differential Equations**—Introduction to the theory of linear and non-linear partial differential equations, beginning with linear theory involving use of Fourier transform and Sobolev spaces. Topics: Schauder and  $L^2$  estimates for elliptic and parabolic equations; De Giorgi-Nash-Moser theory for elliptic equations; non-linear equations such as the minimal surface equation, geometric flow problems, and non-linear hyperbolic equations.

3 units, Aut (Brendle)

**MATH 256B. Partial Differential Equations**—Continuation of 256A.

3 units, Win (Mazzeo)

**MATH 257A. Symplectic Geometry and Topology**—Linear symplectic geometry and linear Hamiltonian systems. Symplectic manifolds and their Lagrangian submanifolds, local properties. Symplectic geometry and mechanics. Contact geometry and contact manifolds. Relations between symplectic and contact manifolds. Hamiltonian systems with symmetries. Momentum map and its properties. May be repeated for credit.

3 units (Staff) not given 2005-06

**MATH 257B. Symplectic Geometry and Topology**—Continuation of 257A. May be repeated for credit.

3 units (Eliashberg) not given 2005-06

**MATH 261A. Functional Analysis**—Geometry of linear topological spaces. Linear operators and functionals. Spectral theory. Calculus for vector-valued functions. Operational calculus. Banach algebras. Special topics in functional analysis. May be repeated for credit.

3 units, Aut (Katznelson)

**MATH 261B. Functional Analysis**—Continuation of 261A.

3 units (Staff) not given 2005-06

**MATH 263A. Lie Groups and Lie Algebras**—Definitions, examples, properties. Semi-simple Lie algebras, their structure and classification. Cartan decomposition: real Lie algebras. Representation theory: Cartan-Stiefel diagram, weights. Weyl character formula. Orthogonal and symplectic representations. May be repeated for credit. Prerequisite: 210 or equivalent.

3 units (Staff) not given 2005-06

**MATH 263B. Lie Groups and Lie Algebras**—Continuation of 263A. May be repeated for credit.

3 units (Staff) not given 2005-06

**MATH 266. Computational Signal Processing and Wavelets**—Theoretical and computational aspects of signal processing. Topics: time-frequency transforms; wavelet bases and wavelet packets; linear and

nonlinear multiresolution approximations; estimation and restoration of signals; signal compression. May be repeated for credit.

3 units (Staff) not given 2005-06

**MATH 270. Geometry and Topology of Complex Manifolds**—Complex manifolds, Kahler manifolds, curvature, Hodge theory, Lefschetz theorem, Kahler-Einstein equation, Hermitian-Einstein equations, deformation of complex structures. May be repeated for credit.

3 units, Win (J. Li)

**MATH 275. Topics in Singularity Theory**—Topics may include: Jet spaces; Thom transversality theorem; h-principle type results in singularity theory with applications to topology; Thom-Boardman singularities; local classification problems and results in singularity theory; Picard-Lefschetz theory; theory of oscillating integrals; Frobenius manifolds.

3 units (Staff) not given 2005-06

**MATH 282A. Low Dimensional Topology**—The theory of surfaces and 3-manifolds. Curves on surfaces, the classification of diffeomorphisms of surfaces, and Teichmüller space. The mapping class group and the braid group. Knot theory, including knot invariants. Decomposition of 3-manifolds: triangulations, Heegaard splittings, Dehn surgery. Loop theorem, sphere theorem, incompressible surfaces. Geometric structures, particularly hyperbolic structures on surfaces and 3-manifolds.

3 units, Aut (Kerckhoff)

**MATH 282B. Homotopy Theory**—Homotopy groups, fibrations, spectral sequences, simplicial methods, Dold-Thom theorem, models for loop spaces, homotopy limits and colimits, stable homotopy theory.

3 units, Win (Carlsson)

**MATH 282C. Fiber Bundles and Cobordism**—Possible topics: principal bundles, vector bundles, classifying spaces. Connections on bundles, curvature. Topology of gauge groups and gauge equivalence classes of connections. Characteristic classes and K-theory, including Bott periodicity, algebraic K-theory, and indices of elliptic operators. Spectral sequences of Atiyah-Hirzebruch, Serre, and Adams. Cobordism theory, Pontryagin-Thom theorem, calculation of unoriented and complex cobordism. May be repeated for credit.

3 units, Spr (R. Cohen)

**MATH 283. Topics in Algebraic and Geometric Topology**—May be repeated for credit.

3 units, Aut (Munson), Win (R. Cohen)

**MATH 285. Geometric Measure Theory**—Hausdorff measures and dimensions, area and co-area formulas for Lipschitz maps, integral currents and flat chains, minimal surfaces and their singular sets.

3 units (Staff) not given 2005-06

**MATH 286. Topics in Differential Geometry**—May be repeated for credit.

3 units, Win (Schoen)

**MATH 290A. Model Theory**—(Enroll in PHIL 350A.)

3 units, Win (Mints)

**MATH 290B. Model Theory**—(Enroll in PHIL 350B.)

3 units (Staff) not given 2005-06

**MATH 291A. Recursion Theory**—(Enroll in PHIL 351A.)

3 units, Win (Feferman)

**MATH 291B. Recursion Theory**—(Enroll in PHIL 351B.)

3 units (Staff) not given 2005-06

**MATH 292A,B. Set Theory**—(Same as PHIL 352A,B.) The basics of axiomatic set theory; the systems of Zermelo-Fraenkel and Bernays-Gödel. Topics: cardinal and ordinal numbers, the cumulative hierarchy and the role of the axiom of choice. Models of set theory, including the constructible sets and models constructed by the method of forcing. Consistency and independence results for the axiom of choice, the continuum hypothesis and other unsettled mathematical and set-theoretical problems. Prerequisites: PHIL 160A,B, and MATH 161, or equivalents.

3 units, A: Win, B: Spr (P. Cohen)

**MATH 293A. Proof Theory**—(Enroll in PHIL 353A.)

3 units, Win (Feferman)

**MATH 293B. Proof Theory**—(Enroll in PHIL 353B.)

3 units, Aut (Mints)

**MATH 294. Topics in Logic**—(Enroll in PHIL 354.)

3 units, Spr (Mints)

**MATH 297. Algebraic Logic**—(Enroll in CS 353.)

3 units (Pratt) not given 2005-06

**MATH 355. Graduate Teaching Seminar**—Required of and limited to first-year Mathematics graduate students.

1 unit, Win (Staff)

**MATH 360. Advanced Reading and Research**

1-9 units, Aut, Win, Spr, Sum (Staff)

**MATH 361. Research Seminar Participation**—Participation in a faculty-led seminar which has no specific course number.

1-3 units, Aut, Win, Spr, Sum (Staff)

**MATH 380-391. Graduate Seminars**—By arrangement. May be repeated for credit.

**MATH 380. Seminar in Applied Mathematics**

1-3 units, Aut, Win, Spr (Staff)

**MATH 381. Seminar in Analysis**

1-3 units, Aut, Win, Spr (Staff)

**MATH 384. Seminar in Geometry**

1-3 units, Aut, Win, Spr (Staff)

**MATH 385. Seminar in Topology**

1-3 units, Aut, Win, Spr (Staff)

**MATH 386. Seminar in Algebra**

1-3 units, Aut, Win, Spr (Staff)

**MATH 387. Seminar in Number Theory**

1-3 units, Aut, Win, Spr (Staff)

**MATH 388. Seminar in Probability and Stochastic Processes**

1-3 units, Aut, Win, Spr (Staff)

**MATH 389. Seminar in Mathematical Biology**

1-3 units, Aut, Win, Spr (Staff)

**MATH 391. Seminar in Logic and the Foundations of Mathematics**

1-3 units, Aut, Win, Spr (Staff)