Spatially resolved study of backscattering in the quantum spin Hall state
SUPPLEMENTAL MATERIAL

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DEVICE CHARACTERIZATION

The two-terminal configuration used in the SGM experiments is not well-suited for determining the sample properties like carrier density and mobility. For that purpose, we fabricated a Hall bar structure with $L \times W = 50 \, \mu m \times 30 \, \mu m$ from the same material. Like in the SGM device, we used a metallic layer at the bottom of the CdTe substrates (thickness $\sim 800 \, \mu m$) in our samples as a back gate electrode. Details of the fabrication process can be found in Ref. 1. When a voltage is applied to this back gate electrode to tune the Fermi level in the device, the Hall bar enters the QSH regime around $V_{\text{back}} = 0$ and $n$-type bulk conduction occurs for $V_{\text{back}} > 100 \, V$ (Fig. S1a). The conductance value of $G \approx 0.15 e^2/h$ for the QSH state is comparable to values reported for other devices of similar size [2, 3]. In the vicinity of the conductance minimum, several sharp peaks can be observed. These might be caused by resonances in the scattering mechanism as they have been observed in our SGM data, now with the back gate electrode tuning the properties of multiple dephasing regions simultaneously.

The voltage range for which the Hall bar is tuned into the QSH regime is shifted by $\Delta V_{\text{back}} = -150 \, V$ in comparison to the SGM device. Based on a parallel plate capacitor model, this corresponds to a difference in density of approximately $1.5 \times 10^{10} \, \text{cm}^{-2}$ toward lower $p$-type densities in the Hall bar. Such shifts in density have been observed in several devices. They can be explained by fluctuations of the carrier density on length scales much larger than the typical device size and, to a lesser extent, by small variations in the sample properties between different cooldowns.

Magnetotransport measurements were performed to determine the density and mobility in the quantum well (Fig. S1b). In close proximity to the QSH state, the edge states contribute significantly to the transport, making extraction of bulk properties from magnetotransport data difficult. For $V_{\text{back}} = -210 \, V$, the sample has a $p$-type carrier density and mobility of $p = 3.7 \times 10^{10} \, \text{cm}^{-2}$ and $\mu = 5400 \, \text{cm}^2/(\text{Vs})^{-1}$, respectively. The clear reduction of carrier mobility in comparison to n-type quantum wells with comparable thickness [2, 4, 5] can be explained by the large effective mass in the valence band which is about one order of magnitude higher than the effective electron mass in the conduction band.

Based on these results, we estimate the carrier density in the SGM device to be $p \approx 2 \times 10^{10} \, \text{cm}^{-2}$ at

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1}
\caption{(a) The Fermi level in a Hall bar device can be tuned from the valence band through the gap into the conduction band. (b) For a large negative back gate voltage $V_{\text{back}} = -210 \, V$, the device is clearly $p$-type bulk conducting and carrier density and mobility can be determined from magnetotransport measurements.}
\end{figure}
$V_{\text{back}} = 0$. This estimate does not take into account uncertainties based on the large disorder in the sample, or on contributions of the edge state to the Hall resistance even in the presence of substantial bulk conduction at $V_{\text{back}} = -210$ V. The device geometry itself, in particular the short length of the device between the metallic contacts to the device, may also result in a gate response different than expected from the assumed parallel plate model. However, as the discussion in the main paper does not rely on the exact carrier density in the slightly $p$-type device, the given estimate is sufficient for our purposes.

RESULTS FROM OTHER DEVICES

In total, we performed scanning gate experiments on five different devices which were fabricated as described in the main paper. All devices had the same dimensions as the device discussed in detail in the paper, i.e. $L \times W = 5 \mu m \times 150 \mu m$. The nominal quantum well thickness of the two HgTe heterostructures used is 7.0 nm and 8.0 nm, respectively. From all heterostructures, we further fabricated devices in a Hall bar geometry as described above for characterization purposes and to confirm the presence of the QSH state.

When the back gate electrode was used to tune the SGM devices into the QSH state, the devices showed either a minimum in $G$ when the Fermi level in the sample could be tuned from the valence into the conduction band, as is the case for particularly low residual carrier densities at $V_{\text{back}} = 0$, or a saturation in $G$ as shown in Fig. 1b if the QSH state could be reached but the voltage applied to the back gate did not suffice to tune the Fermi level through the entire bulk gap.

Some of the SGM devices showed a minimum conductance above $2e^2/h$, i.e. larger than expected for the QSH state. We attribute this excess conductance to a bulk contribution caused by potential fluctuations. As the mesa is much wider than it is long, only $1/30$ of a square, substantial bulk conduction, probably via hopping or percolative transport judging from the low conductivity, can already happen when bulk density and mobility are very low. Our assumption of disorder-based bulk transport is corroborated by the observation that the excess conductance is particularly strong for devices fabricated from heterostructures with a quantum well thickness of 7 nm which possess a smaller bulk energy gap than 8 nm quantum wells. Thus, potential fluctuations in the device are more likely to result in conductive channels through the bulk of the device.

As demonstrated in the main paper, the QSH edge states not only can coexist with low density bulk carriers, but actually contribute significantly to the total conductance in this regime. Thus, while some of the devices do not exhibit a clean QSH state with pure edge state transport, the results obtained on those devices can still be used for further demonstration of resonant backscattering affecting transport in the QSH edge states.

Fig. S2 shows SGM results obtained on a device ($d_{\text{QW}} = 8.0$ nm) other than the one presented in the main paper; it is tuned to its conductance minimum. The conductance clearly larger than $2e^2/h$ points toward sizable bulk transport in this device. At the same time, ring patterns centered near the mesa edge (similar to the features shown in Figs. 2 and 3) indicate that edge states also contribute significantly to the total conductance. Again, three scattering sites can be identified by the presence of distinct sets of concentric rings and their spacing is around 2 microns, comparable to the value determined in the main paper. Similar to Fig. 3, where bulk conductance is intentionally induced via the back gate electrode, signatures of tip-controlled bulk transport are also visible, as expected for a device with coexisting bulk and edge conductance.

BAND STRUCTURE CALCULATIONS

The typical analysis of subband energies in HgTe structures as a function of the quantum well thickness [2, 6] considers the energies of the subbands at $k = 0$. For narrow HgTe quantum well layers, the band structure shows a direct gap at $k = 0$ so that the given energies for the respective subbands represent the bulk gap correctly. However, already for $d_{\text{QW}} = 8.0$ nm the band structure shows an indirect gap with a valence band maximum at $k \neq 0$ [5]. Thus, the separation of valence and conduction band at $k = 0$ overestimates the bulk gap for wider quantum wells. We performed band structure calculations for HgTe quantum wells within a $8 \times 8 \mathbf{k} \cdot \mathbf{p}$ model [7] to determine the exact gap size in HgTe QW structures with a layer sequence identical to our samples (Fig. S3a). For QW layers with a thickness up to $d_{\text{QW}} \lesssim 7.0$ nm, the band structure has a direct gap at $k = 0$. For wider QW layers, the inverted gap becomes indirect and reaches its maximum size of $E_{\text{gap}} = -13$ meV at $d_{\text{QW}} = 8.0$ nm (Fig. S3b). For even wider quantum wells, the gap size decreases continuously until a transition to a semimetallic band structure occurs around $d_{\text{QW}} = 15$ nm. The semimetallic behavior was recently observed experimentally in 20 nm wide quantum wells [8].

The flat dispersion of the valence band near its maximum at finite $k$ allows for large carrier densities already for a Fermi level only a few meV below the valence band maximum whereas much higher Fermi energies, now with respect to the minimum of the conduction band, are required to obtain comparable $n$-type densities (Fig. S3b).
FIG. S2: (Color) SGM data taken on another device. As in the other SGM maps, the mesa is located above the edge indicated by the dashed line: (a)-(f) Conductance maps recorded with (a) $V_{\text{tip}} = -10.0$ V, (b) -6.0 V, (c) -3.0 V, (d) +3.0 V, (e) +6.0 V, and (f) +10.0 V. (g)-(l) show the corresponding maps of the conductance gradient.

**FULL SUPPRESSION OF EDGE CONDUCTANCE**

In the main text, we showed that the conductance of the edge state can be fully suppressed by local gating (see Fig. 2). However, as we have seen such a strong effect of the tip potential on the conductance only in one case, our data is not sufficient to extract details regarding the underlying mechanism. In the following, we will discuss several proposed models that in principle can result in a full suppression of transport in a QSH edge state and assess their applicability in our samples.

The first possible scenario relies on coupling between the QSH edge and a nearby magnetic impurity [9]. While conventional magnetic impurities are not expected to be present in our samples, an accidentally-formed quantum dot could play the same role. If the dot is occupied by an odd number of electrons, one electron spin remains unpaired and serves as a magnetic impurity. In ordinary 1D “Luttinger liquids”, coupling to a local site should fully suppress conductance as $T \to 0$, even for weak electron-electron interaction. In contrast, in a QSH “holographic liquid” at the 1D boundary of a 2D system, for weak electron-electron interaction a magnetic impurity should be screened by the formation of a Kondo singlet, restoring conductance of the QSH edge state at low temperatures. For strong electron-electron interactions, two-particle backscattering is predicted to result in the formation of a “Luttinger liquid insulator” characterized by full suppression of edge state transport for $T \to 0$. The strength of electron-electron interaction can be defined by the Luttinger parameter $K$ for the helical liquid [9, 10] with $0 \leq K \leq 1$ where lower $K$ connotes stronger interaction. While it is difficult to determine $K$ for a given system exactly, it can be estimated when the relevant interactions are taken into account [10]. In our specific case, the strength of Coulomb interaction in the HgTe devices depends on the device geometry: $K \approx 0.55$ for the devices with a top-gate electrode as used in earlier transport experiments [2–5], whereas the absence of such an electrode in our SGM devices leads to a reduced screening of electron-electron interaction and the Luttinger parameter decreases to $K \approx 0.35$. This value is close to the regime of $K < 0.25$ required for the “Luttinger liquid insulator” and a small error - either inherent to the approximation or due to a slight deviation of the sample parameters used in calculating $K$ - might result in an overestimation of $K$ in our devices. Thus, interactions in our devices might be stronger than anticipated. However, it is not clear whether our experimental temperature should be low enough to see nearly full suppression under this scenario.

Rashba spin-orbit interaction has also been proposed to lead to localization of QSH edge states if it is spatially nonuniform along the edge [11]. HgTe quantum well structures show very strong spin-orbit interaction, and a Rashba splitting in the conduction band of up to 30 meV [12–14]. In addition, the Rashba interaction can be tuned externally by an electric field, making the proposed model a seemingly good candidate for an explanation of the observed behavior. However, the proposed Rashba-induced localization length shows a strong dependence on the electron-electron interaction strength [11]. For $K \approx 0.35$, the localization length would exceed...
FIG. S3: (a) The evolution of the bulk gap in HgTe quantum wells as a function of the QW width was determined by $8 \times 8 k \cdot p$ band structure calculations. (b) Dispersion for a HgTe structure with $d_{\text{QW}} = 8$ nm as it was used in our SGM experiments. The dashed horizontal lines indicate the Fermi energy for $n = 2.5 \times 10^{11}$ cm$^{-2}$ (green) and $p = 2.5 \times 10^{11}$ cm$^{-2}$ (blue), respectively. Inset: Valence band maximum and Fermi level for $p = 2.5 \times 10^{11}$ cm$^{-2}$.

Although we apply no magnetic field in our measurements and use no magnetic materials in the fabrication of the devices, for completeness we note that backscattering becomes possible when time-reversal symmetry is broken. Several theoretical proposals have considered the effect of a magnetic field, applied either locally or globally, with a resulting gap in the linear Dirac spectrum of the QSH edge states. Two similar models [15, 16] considered magnetic field applied locally along the edge to form a quantum dot in the QSH edge channel, suppressing conductance when the dot is tuned into Coulomb blockade. More recently, a local potential perturbation was predicted to suppress transport in the QSH state in the presence of an external magnetic field [17]. However, with no magnetic field in our experiments, these mechanisms cannot explain the observed full suppression either.

FIG. S4: Defects on the mesa surface are clearly visible in a high-resolution in-situ AFM scan. The scale bar in the AFM on the left hand side (same as Fig. 1a in the main paper) is 1 micron.

While existing theoretical models do not appear to account for the full suppression of edge state transport, the observed behavior might be related to the presence of strong defects in our samples. As can be seen in Fig. S4, multiple defects of a characteristic shape can be found within an area of tens of $\mu$m$^2$ on the mesa surface. These defects typically have a circular shape with a diameter of approximately 1 micron and a height of a few nanometers. Several of these defects feature a central ridge, occasionally elongated along one of the $<110>$ directions, with an additional height of approximately 10 nm. One of these particularly tall defects can be found at the center of the region of strong suppression shown in Fig. 2 in the main paper. We believe that these defects originate at the surface of the substrate and subsequently propagate through the MBE-grown layers of the heterostructure. The resulting lattice distortion can affect the layer structure in multiple ways. If the quantum well thickness locally is reduced below the critical value $d_{\text{QW,crit}} \approx 6.3$ nm, the affected region would constitute a topologically-trivial insulator. A trivially insulating region could also occur if the quantum well is completely interrupted by the defect (which is taller than $d_{\text{QW}} = 8$ nm) or if the well and barrier material, respectively, get intermixed [18] due to imperfect growth conditions in the vicinity of the defect. In either case, the QSH edge state should propagate along the boundary between topologically trivial and non-trivial insulator. The creation of trivially-insulating regions may also cause a strong deformation of the edge states which could form loops in the edge state [19] or antidots of helical edge states coupled to the QSH state at the edge of the mesa. Such perturbations of the helical edge state likely will cause some degree of backscattering, but it is expected to be a weak effect, not full suppression [19]. On the other hand, a strong local increase in the QW thickness could result in a semimetallic band structure (see above
and Ref. 8), resulting in a bulk-conducting region coupling to the QSH state. Finally, the defect could also cause a local shift of the chemical potential relative to the band edge, possibly into the bulk bands. In either case, the resulting metallic puddle could couple to the edge state and cause a suppression by up to $0.5e^2/h$, but again no full suppression [4].

In summary, it is not clear that the models already discussed theoretically suffice to explain our observed full suppression (and some can even be ruled out completely as explanations in our case). Nor do the obvious consequences of a strong defect appear to give rise to a full suppression. Thus, it will require further effort to understand the observed full suppression caused by local gating near the edge.

**SIMULATING THE TIP-INDUCED POTENTIAL PROFILE**

We used COMSOL [20] to calculate the electrostatic potential profile of the tip and the resulting induced modulation of the carrier density. We simulated the effect of a conical tip over a quantum well structure with a lateral extension exceeding the expected long-range effect of the tip. An axially symmetric layout was chosen to reduce the computational complexity. This simplified configuration will not provide quantitatively exact results as it does not take into account details of our device geometry such as the edge of the heterostructure, giving rise to a spatially varying dielectric environment, and the nearby Ohmic contacts which might shield the tip potential. Nonetheless, the simulations can give a rather accurate representation of the tip-induced density profile. Fig. S5a shows the density induced in the quantum well layer for $V_{\text{tip}} = -10$ V. At the center, i.e., directly beneath the tip, a hole density of $2.6 \times 10^{11} \text{ cm}^{-2}$ is induced. The density profile decays rapidly with a full width at half maximum of 520 nm.

If we assume that the carrier density in a finite-sized region translates directly into the strength of the dephasing mechanism, we can estimate how the size of a region with a given dephasing strength evolves as a function of the tip voltage. In Fig. S5b, we plot the evolution of quasi-two-dimensional regions above a given threshold density as a function of the applied tip voltage. As discussed in the main paper, a density on the order of $10^{11} \text{ cm}^{-2}$ is required to induce a measurable amount of backscattering. For the corresponding tip voltage of $V_{\text{tip}} < -10$ V, the diameter of the induced dephasing region is a few 100 nm which is comparable to our size estimate for the pre-existing scattering sites and the size of the metallic region studied theoretically in Ref. 4. Thus, the simulations of the electrostatic potential profile further substantiate our interpretation of the SGM results. For simplicity, we neglect the likely effect that higher density at the center of the puddle will result in an increase of the dephasing strength.

**TIP-INDUCED EMERGENCE OF BULK TRANSPORT**

We already briefly discussed in the main paper and showed in Fig. 2a that conductance values exceeding $G = 2e^2/h$ - the expected upper limit for the conductance in the QSH regime - can be observed when large negative tip voltages $V_{\text{tip}} \leq -12.5$ V are used to manipulate the edge states. We attribute this excess conductance to an emergence of bulk conductance caused by a long-range effect of the tip potential. Simulations show that densities on the order of several $10^9 \text{ cm}^{-2}$ can be induced even microns away from the actual tip position for such large tip voltages (Fig. S6). Our SGM devices are 5 $\mu$m long so that they are affected along their entire length by the tip potential. Thus, the bulk region close to the tip can get tuned into the valence band, resulting in p-type bulk transport. Figs. S7(a)-(d) show additional SGM maps from the device presented in the main paper, taken with $-14.5 \text{ V} \leq V_{\text{tip}} \leq -13.0$ V at $V_{\text{back}} = +200$ V. It can be seen that a more negative tip voltage results in a larger bulk contribution to the total conductance as it
is expected for tip-induced bulk conductance.

![Graph showing the tip-induced density for $V_{\text{tip}} = -10.0$ V](image)

**FIG. S6**: Profile of the tip-induced density for $V_{\text{tip}} = -10.0$ V (same data as Fig. S5a, now plotted log-scale).

In the maps plotting the gradient of the conductance in Figs. S7(e)-(h), parallel lines similar to the ones shown in Fig. 2(e) are visible. These additional maps clearly demonstrate the evolution of the lines as a function of the tip voltage: for more negative tip voltages, the lines are more widely spaced. This confirms that the solely tip-induced conductance modulation in the otherwise unperturbed sections of the mesa edge is a function of the potential at the position of the edge states which is consistent with the picture of dephasing occurring in finite-sized metallic regions.

![Conductance maps for different tip voltages](image)

**FIG. S7**: (a)-(d) SGM conductance maps for (a) $V_{\text{tip}} = -14.5$ V, (b) -14.0 V, (c) -13.5 V and (d) -13.0 V, respectively, and $V_{\text{back}} = +200$ V. (e) - (h) Gradient maps $|\nabla G(x,y)|$ of (a) - (d). The lowest row is section of the respective gradient map directly above where the color scale is adjusted for a better visibility of the weak lines running parallel to the mesa edge.

Future SGM experiments with coaxially shielded tips [21] which produce a much more localized potential perturbation could help with many of these complications. In particular, we expect the long tails in the induced potential to be significantly suppressed relative to the potential directly below the tip. This improvement will consequently eliminate the long-range gating effect responsible for the emergence of bulk conductance and thus should allow for a more quantitative analysis of the observed conductance modulation.

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