The Levered Equity Risk Premium and Credit Spreads: A Unified Framework* 

Harjoat S. Bhamra  
Sauder School of Business  
University of British Columbia

Lars-Alexander Kühn  
Sauder School of Business  
University of British Columbia

Ilya A. Streubulev  
Graduate School of Business  
Stanford University

First version: November 2005  
This version: June 12, 2007

*We thank Adlai Fisher for many interesting conversations and sharing his understanding of Markov switching models with us. We are also very grateful to Bernard Dumas for suggestions on how to simplify our modelling approach. We would also like to thank Malcolm Baker, Alexander David, Glen Donaldson, Darrell Duffie, Ron Giammarino, Bob Goldstein, Kris Jacobs, Raman Uppal, and participants at the PIMS/Banff Workshop on Optimization Problems in Financial Economics, the CEPR Gerzensee European Summer Symposium in Financial Markets 2006, the NFA 2006 and seminar participants at UBC, the Cass Business School and the University of Calgary for helpful suggestions. Bhamra: 2053 Main Mall, Vancouver BC, Canada V6T 1Z2; Email: harjoat.bhamra@sauder.ubc.ca; Kühn: 2053 Main Mall, Vancouver BC, Canada V6T 1Z2; Email: lars.kuehn@sauder.ubc.ca; Streubulev: Graduate School of Business, Stanford University, 518 Memorial Way, Stanford, CA 94305 United States, Email: istreubulev@stanford.edu.
Abstract

Much empirical work indicates that there are common factors that drive the equity risk premium and credit spreads. In this paper, we build a dynamic consumption-based asset pricing model of equity and default-risky debt that allows us to study the comovement of stock and bond price variables in one single framework. That paves the way for a unified understanding of what drives the equity risk premium and credit spreads. Our key economic assumptions are that the first and second moments of macroeconomic variables, such as earnings and consumption growth, depend on the state of the economy which switches randomly; agents dislike not knowing what future states of the economy will be; they optimally choose capital structure and the timing of defaults. Under these assumptions the model generates comovement between aggregate stock return volatility and credit spreads, which is quantitatively consistent with the data, and resolves the equity risk premium and credit spread puzzles. For relative risk aversion of 7.5, elasticity of intertemporal substitution of 1.5, the model implies a levered equity risk premium of 4.5%, credit spreads for Baa debt of 130 basis points, and a model-implied 5-year default probability of 0.02, which is realistically small.

JEL Classification Numbers: E44, G12, G32, G33
Keywords: Equity Premium, Corporate Bond Yield Spread, Predictability, Macroeconomic Conditions, Jumps
Contents

I Model 6
   I.A Aggregate Consumption and Firm Earnings ............................ 7
   I.B Intertemporal Macroeconomic Risk ...................................... 8
   I.C Booms and Recessions ..................................................... 9
   I.D Short-Run Risk and Long-Run Risk ..................................... 13

II Asset Valuation 15
   II.A Arrow-Debreu Default Claims ............................................ 17
   II.B Abandonment Value ....................................................... 20
   II.C Credit Spreads and the Levered Equity Risk Premium ............... 21
   II.D Optimal Default Boundary and Optimal Capital Structure .......... 24

III Calibration 24
   III.A Parameter Values ....................................................... 25
   III.B Corporate Bond Market ................................................ 26
   III.C Equity Market ........................................................... 28
   III.D Constant Versus Optimal Default Boundary ........................... 29
   III.E Cross-Market Comovement .............................................. 29

IV Stripping Down the Model: What Causes What? 29

V Conclusion 31

Appendix 33
A Appendix: Derivation of the State Price Density 33
B Appendix: Proofs 38

List of Tables
1 Summary of Models in the Literature ........................................ 62
2 Aggregate Parameter Estimates ................................................. 63
3 Calibrated Parameters Values ................................................ 63
4 Corporate Bond Market ......................................................... 64
5 Equity Market ................................................................. 65
6 Corporate Bond Market: Constant Default Boundary ...................... 66
7 Long-Run Risk ............................................................... 67
8 Model Comparison—Corporate Bond Market .................................. 68
9 Model Comparison—Equity Market .......................................... 69

List of Figures
1 The Price of the Fundamental Security in the Structural-Equilibrium Model without Intertemporal Macroeconomic Risk and with Power Utility ........ 55
<table>
<thead>
<tr>
<th></th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>The Price of the Fundamental Security in the Structural-Equilibrium Model without Intertemporal Macroeconomic Risk and with Epstein-Zin-Weil Preferences</td>
<td>56</td>
</tr>
<tr>
<td>3</td>
<td>The Risk-Adjustment Factor, $R = \frac{\hat{E}}{B}$, in the Structural-Equilibrium Model without Intertemporal Macroeconomic Risk</td>
<td>57</td>
</tr>
<tr>
<td>4</td>
<td>The Time-Adjustment Factor, $T = \frac{\hat{E}}{B}$, in the Structural-Equilibrium Model without Intertemporal Macroeconomic Risk</td>
<td>58</td>
</tr>
<tr>
<td>5</td>
<td>The Risk-Free Rate in the Structural-Equilibrium Model without Intertemporal Macroeconomic Risk</td>
<td>59</td>
</tr>
<tr>
<td>6</td>
<td>Baa Credit Spread in the Structural-Equilibrium Model without Intertemporal Macroeconomic Risk</td>
<td>60</td>
</tr>
<tr>
<td>7</td>
<td>Growth Rates of Earnings and Consumption</td>
<td>61</td>
</tr>
</tbody>
</table>
There is a growing body of empirical work indicating that common factors may affect both the equity risk premium and credit spreads on corporate bonds. In particular, there is now substantial evidence that stock returns can be predicted by credit spreads, and that movements in stock-return volatility can explain movements in credit spreads, both at the individual and aggregate levels. For example, Zhang, Zhou, and Zhu (2005) find that a pooled regression of the CDS credit spreads of firms against their stock return volatilities has a beta of 9 and an $R^2$ of 0.5. Tauchen and Zhou (2006) show that a regression of the Moody’s BAA bond spread index against the jump-component in the volatility of returns on the S&P500 has a beta of just under 2 and an $R^2$ of 0.7. In essence, all these results demonstrate that there is “overlap between the stochastic processes for bond and stock returns” (Fama and French (1993, p. 26, our emphasis)).

The aim of this paper is to investigate two important ramifications of these results. First, the existence of common factors indicate that the two well-known puzzles, the equity risk-premium puzzle and the credit risk puzzle, are inherently linked. Much research has been devoted to finding an explanation of the equity premium puzzle since the seminal paper of Mehra and Prescott (1985) and the credit risk puzzle has been the subject of a great deal of attention since the first empirical evidence that contingent-claim models of defaultable debt underpredict credit spreads, but there has been limited research on linking the two puzzles. Second, empirical evidence suggests that common factors are likely to be related to fundamental macroeconomic risks. For the equity risk premium, for example, Adrian and Rosenberg (2006) provide evidence that a substantial proportion of the equity risk premium stems from the risk of exposure to long-run fluctuations in returns that are linked to business cycle variations. In credit risk, for example, Collin-Dufresne, Goldstein, and Martin (2001) show that credit spread changes across firms are driven by a single factor.

Yet none of these papers provides an economic mechanism that pins down what this common factor is and whether it can explain the two puzzles simultaneously. In this paper, we propose an economically intuitive macroeconomic mechanism related to the business cycle which generates a common factor linking both stock returns and credit spreads. We

---


2The credit risk puzzle refers to the finding that structural models of credit risk generate credit spreads smaller than those observed in the data when calibrated to observed default frequencies. Jones, Mason, and Rosenfeld (1984) was the first paper to show that Merton (1974) substantially overprices corporate debt. Recent evidence is presented in Eom, Helwege, and Huang (1999), Ericsson and Reneby (2003), Huang and Huang (2003), and Schaefer and Streubelaeve (2005).

3See Bansal and Yaron (2004) for a model of the impact of long-run fluctuations in consumption and dividends on the equity risk premium. See Cochrane (2005) for a survey on research linking financial markets to the macroeconomy.
then use this mechanism to explain why stock-return volatility comoves with credit spreads and resolve both the equity risk premium and credit spread puzzles.

In a nutshell, two main ideas underpin our approach. The first is that any claim, equity and debt alike, can be priced in a consumption-based asset pricing model. The second is *intertemporal macroeconomic risk*: the expected values and volatilities (first and second moments) of fundamental economic growth rates vary with the business cycle.

We use the first idea to price corporate bonds in a consumption-based asset pricing model with a representative agent. In particular, we assume that aggregate consumption consists of wages paid to labor and firms’ earnings, and the division between wages and earnings is exogenous. Earnings are divided into coupon payments to bondholders and dividends to equityholders. Capital structure is chosen optimally by equityholders to maximize firm value which implies the endogeneity of both coupons and dividends. In addition, equityholders choose a default boundary to maximize the value of their default option so that the default boundary is also endogenous. Thus, in our model, the prices of equity and debt are not only linked by a common state-price density, but they are also affected by the optimal leverage and default decisions. Essentially, we embed the contingent-claim models of Fischer, Heinkel, and Zechner (1989) and Leland (1994) inside an equilibrium consumption-based model. We call the resulting framework a *structural-equilibrium* model.\(^5\)

We then use the second idea and introduce intertemporal macroeconomic risk into our structural-equilibrium model to capture a common macroeconomic factor that underlies both expected stock and bond returns. Modelling of intertemporal macroeconomic risk hinges on several critical and intuitive features. Firstly, the properties of firms’ earnings growth change with the state of the economy, with expected growth lower in recessions and volatility lower in booms. Secondly, the properties of consumption growth also change with the state of the economy. As expected, first moments are lower in recessions, whereas second moments are higher. We model switches in the state of the economy via a Markov chain.\(^6\) Thirdly, we assume that the representative agent cares about the intertemporal composition of risk. In particular, she prefers uncertainty about which state the economy will be in at future dates to be resolved sooner rather than later.\(^7\) In essence, she is averse to uncertainty

---

\(^4\)Since in contingent-claim models the state-price density is not linked to consumption, the asset prices they produce are completely divorced from macroeconomic variables, such as aggregate consumption. Consequently, these models alone cannot be used to find a macroeconomic explanation for a common factor behind stock and bond returns.

\(^5\)The germ of this idea is contained within Goldstein, Ju, and Leland (2001). They state that their EBIT-based model can be embedded inside a consumption-based model, where the representative agent has power utility, though they do not investigate how credit spreads depend on the agent’s risk aversion.

\(^6\)An example of this modelling of intertemporal macroeconomic risk in a discrete-time setting is Calvet and Fisher (2005a) and an example in a continuous-time setting is Calvet and Fisher (2005b).

\(^7\)Kreps and Porteus (1978, p. 186) explain the intuition for modelling preferences in this way via a coin-flipping example: “If . . . the coin flip determines your income for the next two years, you probably prefer to have the coin flipped now, so that you are better able to budget your income for consumption purposes.”
about the future state of the economy. We model this by assuming that the representative agent has Epstein-Zin-Weil preferences.

As in any asset-pricing model, the representative agent does not use actual probabilities to compute prices. Instead she uses risk-neutral probabilities. It is well-known that for a risk-averse agent, the risk-neutral neutral probability of a bad event occurring exceeds its actual probability. In the context of our model, asset prices will depend on the risk-neutral probability (per-unit time) of the economy moving from boom to recession. Increasing the risk-neutral probability of entering a recession increases the average duration of recessions in the risk-neutral world. When the average time spent in recessions in the risk-neutral world increases, it is intuitive that risk premia will go up. These ideas tell us that one route to increased risk premia, is to conjure up a model where the risk-neutral probability of entering a recession is sufficiently larger than the empirically rather low actual probability. We could try to invent such a model by using an agent with power utility inside our structural-equilibrium model. But this attempt would be doomed to failure. An agent with power utility is indifferent about when she receives information about the future state of the economy. Therefore, the risk-neutral probability of entering a recession equals the actual probability. This restriction implies that the average time spent in a recession in the risk-neutral world is the same as under the actual measure. We we know that empirically recessions are much shorter than booms making their average duration rather short. Given that power utility does not help us, we turn to Epstein-Zin-Weil preferences. The agent now dislikes not knowing what the future state of the economy will be, so the risk-neutral probability of entering a recession exceeds the actual probability. Consequently, the agent prices assets as if recessions last longer than is actually the case, which raises risk premia.

The same mechanism delivers high credit spreads. To see the intuition, observe that the credit spread on default risky debt is given by the standard expression

\[ s = r \frac{lq_D}{1 - lq_D}, \]

where \( r \) is the risk-free rate, \( l \) is the loss ratio for the bond (which gives the proportional loss in value if default occurs) and \( q_D \) is the price of the Arrow-Debreu security which pays out 1 unit of consumption at default. Empirically, both the risk-free rate and loss ratio are low. The expression for the credit spread then tells us that there is only one avenue left open for increasing credit spreads: devising a model, where the price of the Arrow-Debreu default claim, \( q_D \) is high. One of the novel results of this paper shows how \( q_D \) can be decomposed into three factors, each with an economically intuitive meaning:

\[ q_D = pTR. \]
where $p$ is the actual probability of default, $T$ is a downward adjustment for the time value of money and $R$ is an adjustment for risk. Actual default probabilities are small. Our decomposition then tells us that the value of the Arrow-Debreu default claim will be high if the risk-adjustment, $R$ and the time-adjustment, $T$ are high. So, why are they high in our model?

It is well known from Weil (1989) that using Epstein-Zin-Weil preferences makes it possible to obtain a low risk-free rate, simply by increasing the elasticity of intertemporal substitution. When the risk-free rate is low, the discount factor associated with the time-value of money will be high. Therefore, the time-adjustment factor, $T$ is high. This happens even if there is no intertemporal macroeconomic risk. Combining intertemporal macroeconomic risk with Epstein-Zin-Weil preferences increases the risk-neutral probability of entering a recession, which increases the risk-adjustment factor, $R$. Thus, our model can generate high credit spreads, while keeping the actual probability of default low, as observed in the data.

The same economic mechanism increases both the risk-neutral probability of entering a recession and the risk-adjustment, $R$. Comovement then naturally arises between equity and corporate bond market values. In particular, our model generates comovement between credit spreads and stock return volatility as observed by Tauchen and Zhou (2006).

In the remainder of the introduction we discuss the relationship between our paper and the existing literature. The most closely related paper to ours is Chen, Collin-Dufresne, and Goldstein (2006). They study a pure consumption-based model and use two distinct mechanisms to resolve the equity risk premium and credit spread puzzles. The first mechanism is habit formation, which makes the marginal utility of wealth high enough in bad states so that the equity risk premium puzzle is resolved. This does not resolve the credit spread puzzle, since the time and risk-adjustments are not sufficiently high, particularly in recessions and the credit spread is procyclical. To remedy this, Chen, Collin-Dufresne, and Goldstein use a second mechanism: they force the default boundary to be exogenously countercyclical.

This paper differs from Chen, Collin-Dufresne, and Goldstein in using just one economic mechanism to make both marginal utility and the time- and risk-adjustments high in recessions. Furthermore, the default boundary is determined endogenously by maximizing the value of the default option held by equityholders. Because expected earnings growth is lower and the volatility of earnings growth is higher in recessions, the default option held by equityholders is less valuable in recessions, leading to lower credit spreads and higher stock return volatility.
equityholders is more valuable in bad states. Consequently, the optimal default boundary is *endogenously* countercyclical.

But more importantly, in our paper the endogenous countercyclical default boundary is not necessary: our model can generate high, countercyclical credit spreads with an exogenous constant default boundary, in contrast with Chen, Collin-Dufresne, and Goldstein. The decomposition of the price of the Arrow-Debreu default claim into three factors is the key, which opens the door to understanding why this is happens. In this paper, the time-adjustment factor and the actual default probability are countercyclical, whereas the risk-adjustment is procyclical. Overall, the countercyclical factors dominate, so \( q_D \) and hence the credit spread are countercyclical. With Campbell-Cochrane habit formation and a constant default boundary, as in Chen, Collin-Dufresne, and Goldstein, the risk-free rate is constant, so the time-adjustment factor is acyclical and the actual default probability and risk-adjustment are procyclical. Therefore \( q_D \) and hence the credit spread are procyclical. Procyclicality also reduces the size of the credit spread size, because it reduces credit-risk is in bad times. So why are the actual default probabilities and time adjustment countercyclical in our model? The answer lies in our assumptions about intertemporal macroeconomic risk. Firstly, we assume expected earnings growth/earnings growth volatility are smaller/larger in recessions, so our model generates a countercyclical default probability. The second assumption that consumption growth behaves in same way, creates a procyclical risk-free rate. Hence, the time-adjustment is countercyclical.

The second paper closely related to our paper is Hackbarth, Miao, and Morellec (2006). They price corporate debt directly under the risk-neutral measure, without using a state-price density linked to consumption and assume that firms’ earnings levels jump down in recessions. They too endogenize the default decision to obtain a countercyclical default boundary. However, because they rely on a pure structural model, Hackbarth, Miao, and Morellec cannot check the size of actual default probabilities, while our structural-equilibrium model allows us to do so. We can actually strip down our model into 3 basic models. Model 1 has no switching in earnings and consumption growth, but the representative agent has Epstein-Zin-Weil preferences. Model 2 is obtained by adding switching in earnings growth rates to Model 1, and is similar to Hackbarth, Miao, and Morellec—the major difference being the use a representative agent. We find that the credit spreads in Model 2 are very close in magnitude to those in Model 1. The only difference is that in Model 2, credit spreads are countercyclical, whereas in Model 1 they are acyclical. Thus, when one is able to check the size of actual default probabilities of a model like Hackbarth, Miao, and Morellec, introducing switching in earnings growth does not increase credit spreads. Rather
it makes them countercyclical. To significantly increase credit spreads while keeping actual default probabilities down, one must add switching in consumption growth.

David (2007) prices corporate debt in a framework where the means of earnings growth rates follow a Markov switching process and are are unobservable. Our paper differs in both focus in approach. We focus on pricing both corporate bonds and levered equity, whereas David (2007) focuses on corporate bonds alone. David (2007) does not endogenize firms’ corporate financing decisions.

Our technique for modelling intertemporal macroeconomic risk is used in a discrete-time setting by Calvet and Fisher (2005a). While Calvet and Fisher (2005a) consider only unlevered equity, we price corporate debt and levered equity, with both default and capital structure decisions determined optimally. By working in continuous-time and avoiding using a multifrequency process, we can obtain simple closed-form solutions for asset prices, which are natural extensions of the formulae in Leland (1994) and those in the continuous-time analogue of Lucas (1978).

Our paper is not the first to consider default in a consumption-based model (see e.g. Alvarez and Jermann (2000), and Kehoe and Levine (1993)). These papers focus on default from the viewpoint of households. They assume households have identical preferences, but are subject to idiosyncratic income shocks. Households can default on payments in the same way that people cannot always pay back credit card debt or a mortgage. Chan and Sundaresan (2005) consider the bankruptcy of individuals in a production framework, looking at its impact on the equity risk premium and the term structure of risk-free bonds. Unlike the above papers, which look at personal bankruptcy, we look at firm bankruptcy and the pricing of corporate debt.

The remainder of the paper is organized as follows. Section I describes the structural-equilibrium model with intertemporal macroeconomic risk and Epstein-Zin-Weil preferences. Section II explores the implications of the model for pricing corporate debt and levered equity and develops an intuitive decomposition for the Arrow-Debreu default claim. Section III builds on Section II by calibrating the model. In Section IV, we strip down the model to see which assumptions drive which results. We conclude in Section V.

I Model

In this section we introduce the structural-equilibrium model with intertemporal macroeconomic risk. The basic idea is simple: we embed a structural model inside a representative

---

9Because David (2007) restricts the state-price density to one that is obtained from a representative agent with power utility, the shifts in growth rates are not priced, making it impossible to get a large equity risk premia unless risk aversion is very high.
agent consumption-based model. That allows us to price debt and levered equity using the state price density of the representative agent.

Two consequences of this modelling approach are worth noting. Credit spreads depend on the agent’s preferences and aggregate consumption, which is not the case in pure structural models, such as Leland (1994). The equity risk premium is affected by default risk, which is not the case in pure consumption-based models.

It is also important to mention what the structural-equilibrium model does not do. It does not account for the impact of default on consumption, because we model consumption as an exogenous process. Furthermore, our model ignores the impact of agency conflicts on the state-price density, because the state-price density in our model is the marginal utility of wealth of a representative agent. Incorporating these two important effects is beyond the scope of this paper.

We start by describing how firm earnings and aggregate consumption are modelled. Then we explain how we introduce intertemporal macroeconomic risk by making the first and second moments of firm earnings and aggregate consumption growth stochastic. We also give a brief description of the state-price density, which arises from our choice to use a representative agent with Epstein-Zin-Weil preferences. The main result of this section is Proposition 1, which explains how intertemporal macroeconomic risk combined with Epstein-Zin-Weil preferences causes the agent to price securities as if recessions were of longer duration that is actually case. Intuitively, one would expect risk premia to be larger in an economy, where recessions last longer, so Proposition 1 provides a natural explanation of how our model can generate large risk premia. Finally, we give precise quantitative definitions of short-run and long-run risk.

I.A Aggregate Consumption and Firm Earnings

There are $N$ firms in the economy. The output of firm $n$, $Y_n$, is divided between earnings, $X_n$, and wages, $W_n$, paid to workers. Aggregate consumption, $C$, is equal to aggregate output. Therefore,

$$C = \sum_{n=1}^{N} Y_n = \sum_{n=1}^{N} X_n + \sum_{n=1}^{N} W_n.$$  \hspace{1cm} (1)

We model aggregate consumption and individual firm earnings directly, and aggregate wages, $\sum_{n=1}^{N} W_n$, are just the difference between aggregate consumption and aggregate earnings.\(^{10}\)

\(^{10}\)In assuming so we follow such papers as Cecchetti, Lam, and Mark (1993), Campbell and Cochrane (1999), Brennan and Xia (2001) and Bansal and Yaron (2004).
Aggregate consumption, $C$, is given by

$$\frac{dC_t}{C_t} = g_t dt + \sigma_{C,t} dB_{C,t},$$

where $B_{C,t}$ is a standard Brownian motion.

The earnings process for firm $n$ is given by

$$\frac{dX_{n,t}}{X_{n,t}} = \theta_{n,t} dt + \sigma_{id}^{X,n} dB_{id}^{X,n,t} + \sigma_{s}^{X,n,t} dB_{s}^{X,t}.$$  (3)

The quantity $\theta_n$ is the expected earnings growth rate of firm $n$, and $\sigma_{id}^{X,n}$ and $\sigma_{s}^{X,n,t}$ are, respectively, the idiosyncratic and systematic volatilities of the firm’s earnings growth rate. Total risk, $\sigma_{X,n}$, is given by $\sigma_{X,n} = \sqrt{(\sigma_{id}^{X,n})^2 + (\sigma_{s}^{X,n})^2}$. The standard Brownian motion $B_{s}^{X,t}$ is the systematic shock to the firm’s earnings growth, which is correlated with aggregate consumption growth:

$$dB_{s}^{X,t} dB_{C,t} = \rho_{XC} dt,$$  (4)

where $\rho_{XC}$ is the constant correlation coefficient. The standard Brownian motion $B_{id}^{X,n,t}$ is the idiosyncratic shock to firm earnings, which is correlated with neither $B_{s}^{X,t}$ nor $B_{C,t}$.

Importantly, to study credit spreads, we consider corporate bonds issued by individual firms, but to study the aggregate equity premium we consider the levered equity claim for the aggregate firm, whose earnings is equal to aggregate firm earnings.

I.B  Intertemporal Macroeconomic Risk

To introduce intertemporal macroeconomic risk into the structural-equilibrium model we assume that the first and second moments of macroeconomic growth rates are stochastic. Specifically, we assume that $g_t$, $\theta_t$, $\sigma_{C,t}$ and $\sigma_{X,t}$ depend on the state of the economy, which follows a 2-state continuous-time Markov chain.\footnote{For simplicity of exposition and to save space, we present the model with two states. The extension to $L > 2$ states does not provide any further economic intuition, but is straightforward and available upon request.} Hence, the conditional expected growth rate of consumption, $g_t$, can take two values, $g_1$ and $g_2$, where $g_i$ is the expected growth rate when the economy is in state $i$ and switches with the state of the economy. Similarly, for $\theta_t$, $\sigma_{C,t}$ and $\sigma_{X,t}$.\footnote{To ensure idiosyncratic earnings volatility, $\sigma_{id}^{X,n}$, is truly idiosyncratic, we assume it is independent of the state of the economy. Therefore $\sigma_{id}^{X,n}$ is a constant.} State 1 is a recession and state 2 is a boom. Since the first moments of fundamental growth rates are procyclical and second moments are countercyclical, we assume that $g_1 < g_2$, $\theta_1 < \theta_2$, $\sigma_{C,1} > \sigma_{C,2}$ and $\sigma_{X,1} > \sigma_{X,2}$.  

\footnote{For simplicity of exposition and to save space, we present the model with two states. The extension to $L > 2$ states does not provide any further economic intuition, but is straightforward and available upon request.}
The above set of assumptions introduces time variation into the expected values and volatilities of cash flow and consumption growth rates. Random switches in the moments of consumption growth will only impact the state price density if the representative agent has a preference for how uncertainty about future growth rates is resolved over time. To ensure this, we assume the representative agent has the continuous-time analog of Epstein-Zin-Weil preferences. Consequently, the representative agent’s state-price density at time-$t$, $\pi_t$, is given by

$$\pi_t = (\beta e^{-\beta t})^{\frac{1-\gamma}{1-\psi}} C_t^{-\gamma} \left( p_C t e^{\int_0^t p_C^{-1} ds} \right)^{\frac{\gamma-\frac{\psi}{1-\psi}}{1-\psi}},$$

where $\beta$ is the rate of time preference, $\gamma$ is the coefficient of relative risk aversion (RRA), and $\psi$ is the elasticity of intertemporal substitution (EIS) under certainty. Unlike the power-utility representative agent, the Epstein-Zin-Weil representative agent’s state price density depends on the value of the claim to aggregate consumption per unit consumption, i.e. the price-consumption ratio, $p_C$.

I.C Booms and Recessions

The state of the economy is described by the vector $\nu_t$, which can take 2 values: $\{1, 2\}$. The evolution of $\nu_t$ is given by a 2-state Markov chain. The Markov chain is defined by $\lambda_{12}$ and $\lambda_{21}$, where $\lambda_{ij}$, $i \neq j$ is the probability per unit time of switching from state $i$ to state $j$. This implies that the average duration of a recession is $1/\lambda_{21}$ and the average duration of a boom is $1/\lambda_{12}$. The intuition is simple: if recessions are shorter than booms ($1/\lambda_{21} < 1/\lambda_{12}$), the probability per unit time of switching from recession to boom must be higher than the probability per unit time of switching from boom to recession ($\lambda_{12} > \lambda_{21}$). This is in contrast with Bansal and Yaron (2004). Bansal and Yaron assume growth rates follow an AR(1) process. But this process and its continuous-time counterpart, the Ornstein-Uhlenbeck process, have symmetric transition probabilities. That forces the probability of switching from a recession to a boom to equal the probability of switching from a boom to recession ($\lambda_{12} = \lambda_{21}$). Historical evidence suggests otherwise. In fact, booms tend to last longer than recessions. Assuming recessions are longer than observed inflates risk premia. Using a Markov chain allows us to make recessions longer than booms.

Intuitively, one would expect that in a model where booms are longer than recessions, asset risk-premia would be lower. While this is indeed the case, the following new insight

---

$^{13}$The continuous-time version of the recursive preferences introduced by Epstein and Zin (1989) and Weil (1990) is known as stochastic differential utility, and is derived in Duffie and Epstein (1992).

$^{14}$Schroder and Skiadas (1999) provide a proof of existence and uniqueness for an equivalent specification of stochastic differential utility.
explains why our model can still generate realistically high risk premia. The switching probabilities per unit time, $\lambda_{12}$ and $\lambda_{21}$, are not relevant for valuing securities. Because we must account for risk, we use the risk-neutral switching probabilities per unit time, $\hat{\lambda}_{12}$ and $\hat{\lambda}_{21}$ to value securities. Intuitively, one would expect the risk-neutral probability per unit time of switching from a boom to a recession to be higher than the actual probability, i.e. $\hat{\lambda}_{21} > \lambda_{21}$. Similarly, when considering the probability of moving from recession to boom, $\hat{\lambda}_{12} < \lambda_{12}$. Using risk-neutral probabilities instead of actual probabilities means securities are priced as if recessions last longer and booms finish earlier than they actually do, which leads to a significant increase in credit spreads and the equity risk premium.

To compute $\hat{\lambda}_{12}$ and $\hat{\lambda}_{21}$ from $\lambda_{12}$ and $\lambda_{21}$, we need a state-price density to define a mapping from the actual measure, $P$, to the risk-neutral measure, $Q$. When the representative agent has Epstein-Zin-Weil utility, her state-price density is given by Equation (5). We use Equation (5) to consider what happens to the state-price density when the state of the economy, $\nu$, jumps from $i$ to $j \neq i$ at time $t$. To distinguish between the state of the economy before and after the jump, denote the time just before the jump occurs by $t-$, and the time at which the jump occurs by $t$. Therefore $\nu_{t-} = i$, whereas $\nu_t = j$. When the economy changes state, the price-consumption ratio jumps, because the first and second moments of consumption growth change. Therefore, from Equation (5), the state-price density also jumps, i.e. $\pi_t \neq \pi_{t-}$. The size of this jump links the risk-neutral switching probabilities per unit time to the actual switching probabilities per unit time, as shown in the proposition below.

**Proposition 1** The risk-neutral switching probabilities per unit time are related to the actual switching probabilities per unit time by the risk-distortion factor, $\omega$,

$$\hat{\lambda}_{12} = \lambda_{12} \omega^{-1},$$

$$\hat{\lambda}_{21} = \lambda_{21} \omega,$$

where $\omega$ measures the size of the jump in the state-price density when the economy shifts from boom to recession, i.e.

$$\omega = \left. \frac{\pi_t}{\pi_{t-}} \right|_{\nu_{t-} = 2, \nu_t = 1}.$$

\(^{15}\) To be more precise, suppose that during the small time-interval $[t - \Delta t, t)$, the economy is in state $i$ and that at time $t$, the state changes, so that during the next small time interval $[t, t + \Delta t)$, the economy is in state $j$. The state of the economy, $\nu$, jumps from $i$ to $j$ at time $t$. To distinguish between the state of the economy before and after the jump, we define the left-limit of $\nu$ at time $t$ as

$$\nu_{t-} = \lim_{\Delta t \to 0} \nu_{t-\Delta t},$$

and the right-limit as

$$\nu_t = \lim_{\Delta t \to 0} \nu_{t+\Delta t}.$$  

Because $\nu$ jumps from $i$ to $j$, the left and right-limits are not equal.
The size of the risk-distortion factor depends on the representative agent’s preferences for resolving intertemporal risk:

1. \( \omega > 1 \), if the agent is averse to intertemporal macroeconomic risk \( (\gamma > 1/\psi) \),

2. \( \omega < 1 \), if the agent likes intertemporal macroeconomic risk \( (\gamma < 1/\psi) \), and

3. \( \omega = 1 \), if the agent is indifferent to intertemporal macroeconomic risk \( (\gamma = 1/\psi) \).

\( \omega \) is given by the solution of the nonlinear algebraic equation:

\[
g(\omega) = 0,
\]

where

\[
g(x) = \begin{cases} 
\frac{1}{x^{\gamma-1} - \frac{1}{\psi}} - \frac{\tau + \gamma \sigma_v^2 - g_2 + \lambda_{21} 1_{x=1} \left( x^\frac{\gamma-1}{\psi} - 1 \right)}{\tau + \gamma \sigma_v^2 - g_1 + \lambda_{12} 1_{x=1} \left( x^\frac{\gamma-1}{\psi} - 1 \right)}, & \psi \neq 1 \\
\ln x^{\frac{1}{\psi-1}} - \frac{g_2 - \frac{1}{2} \gamma \sigma_v^2 + \lambda_{21} (x-1)}{g_1 - \frac{1}{2} \gamma \sigma_v^2 + \lambda_{12} (x-1)}, & \psi = 1
\end{cases}
\]

and

\[
\tau_i = \beta + \frac{1}{\psi} g_i - \frac{1}{2} \gamma \left( 1 + \frac{1}{\psi} \right) \sigma_v^2, \quad (13)
\]

is the risk-free rate when there is no intertemporal macroeconomic risk and the economy is always in state \( i \).

Proposition 1 tells us that when the representative agent prefers intertemporal macroeconomic risk to be resolved sooner than later (she has Epstein-Zin-Weil preferences with \( \gamma > 1/\psi \)), then \( \omega > 1 \), and the duration of recessions under the risk-neutral measure is longer than their actual duration. The effect of this on the state-price density can be seen from Equation (10): the state-price density jumps up in recessions. At the same time as the state-price density jumps upward, the second moment of earnings growth jump upward, while the first moment jumps downward. Therefore, asset returns contain a premium for jump-risk. The premium for jump risk is present in both credit spreads and equity risk premia. Stock-return volatility is a measure of total risk, not just priced risk and as such contains a jump component even when jump-risk is not priced. The presence of jump components forces the stochastic processes for bond and stock returns to overlap, a feature of the data observed by Fama and French (1993). As long as jump-risk is priced, there is a jump-risk component in credit spreads, which comoves with the jump component in stock-return volatility, as documented in Tauchen and Zhou (2006).
When the representative agent does not have a preference for how intertemporal macroeconomic risk is resolved (she has power utility, i.e. $\gamma = 1/\psi$), then $\omega = 1$, so there will be no jump risk-premia. In this case, there is still a jump component in stock return volatility, but not in credit spreads or equity risk premia. When the agent prefers intertemporal risk to resolved later rather than sooner, i.e. she likes not knowing what the future mean and volatility of consumption growth will be ($\gamma < 1/\psi$), then $\omega < 1$ and jump-risk premia will be negative.

We can compute the price-consumption ratio, the locally risk-free rate and the market price of risk in terms of the risk-distortion factor, $\omega$. In the Appendix, we show that the price-consumption ratio is given by

$$p_{C,i} = \begin{cases} \frac{1}{r_1 + \gamma \sigma_{C,1}^2 - g_1 + \frac{1}{\psi} \lambda_{12} \left( \omega^{-\frac{1}{\psi}} - 1 \right)} , & i = 1, \\ \frac{1}{r_2 + \gamma \sigma_{C,2}^2 - g_2 + \frac{1}{\psi} \lambda_{21} \left( \omega^{-\frac{1}{\psi}} - 1 \right)} , & i = 2, \end{cases}$$

(14)

Note that when $\gamma = 1/\psi$, the risk-distortion factor does not impact the price-consumption ratio, but the price-consumption ratio still jumps when the economy changes state. Because the agent is indifferent about not knowing what the future state of the economy will be, these jumps are not priced.

In the Appendix, we show that the state-price density satisfies the stochastic differential equation

$$\frac{d\pi_t}{\pi_{t-} \mid_{\nu_t = \nu_t}} = -r_i dt - \Theta^B_i dB_{C,t} + \sum_{j \neq i} \Theta^P_{ij} dN^P_{ij,t},$$

(15)

where $r_i$ is the locally risk-free rate when the economy is in state $i$, given by

$$r_i = \begin{cases} r_1 - \left( \gamma - \frac{1}{\psi} \right) \lambda_{12} \left( \omega^{-\frac{1}{\psi}} - 1 \right) + \lambda_{12} \left( 1 - \omega^{-1} \right) , & i = 1 \\ r_2 - \left( \gamma - \frac{1}{\psi} \right) \lambda_{21} \left( \omega^{-\frac{1}{\psi}} - 1 \right) + \lambda_{21} \left( 1 - \omega \right) , & i = 2 \end{cases}$$

(16)

$\Theta^B_i$ is the market price of risk from Brownian shocks, when the economy is in state $i$, given by

$$\Theta^B_i = \gamma \sigma_{C,i},$$

(17)

$\Theta^P_{ij}$ is the market price of risk from Poisson shocks, when the economy switches from state $i$ to $j \neq i$, given by

$$\Theta^P_{ij} = \begin{cases} \omega^{-1} - 1 , & i = 1, j = 2 \\ \omega - 1 , & i = 2, j = 1 \end{cases}$$

(18)
and $N_{ij,t}^P$ is the compensated Poisson process, given by

$$N_{ij,t}^P = N_{ij,t} - \lambda_{ij} t,$$  \hspace{1cm} (19)

where $N_{ij,t}$ is the Poisson process which jumps up by one whenever the economy switches from state $i$ to state $j$. The total market price of consumption risk in state $i$ accounts for both Brownian and Poisson shocks, and is given by:

$$\Theta_i = \begin{cases} \sqrt{(\Theta_{B1}^P)^2 + \lambda_{12} (\Theta_{12}^P)^2}, & i = 1. \\ \sqrt{(\Theta_{B2}^P)^2 + \lambda_{21} (\Theta_{21}^P)^2}, & i = 2. \end{cases}$$  \hspace{1cm} (20)

### I.D Short-Run Risk and Long-Run Risk

We introduced intertemporal macroeconomic risk by making the means and volatilities of fundamental growth rates depend on the state of the economy, which changes stochastically. Because the state of the economy follows a Markov chain, we can give precise quantitative measurements of how important intertemporal macroeconomic risk is in both the short-run and the long-run.

In particular, we define short-run risk via the switching probabilities per unit time of the Markov chain under the risk-neutral measure, i.e. $\hat{\lambda}_{12}$ and $\hat{\lambda}_{21}$. The risk-neutral switching probabilities per unit time are used to define the probability that the state of the economy can change in a small time-interval, i.e. the short-run. To have a quantity which measures risk, it is essential to work under the risk-neutral measure.

As time tends to infinity, the Markov chain converges exponentially to a long-run distribution, i.e. the probability of being in a given state becomes constant. The time it will take for the distance of the current distribution from the long-run distribution to halve, is the half-life of the state of the economy, $t_{1/2}$. To adjust this measure for risk, we just compute the half-life of the state of the economy under the risk-neutral measure. This risk-neutral half-life of the state of the economy, $\hat{t}_{1/2}$, is our measure of long-run risk.

To understand the intuition underlying our chosen measure of long-run risk, the risk-neutral half-life, we need to understand how the distribution of the state of economy evolves over time. The risk-neutral probability of being in state $j$ at date $s > t$ (i.e. $\nu_s = \nu_j$), when the current state is $i$ (i.e. $\nu_t = \nu_i$) is given by the $ij$'th element of the matrix exponential $e^{\hat{\lambda}(s-t)}$, i.e.

$$\Pr(\nu_s = \nu_j | \nu_t = \nu_i) = \left[ \hat{P}_{s-t} \right]_{ij} = \left[ e^{\hat{\lambda}(s-t)} \right]_{ij},$$  \hspace{1cm} (21)
where

\[ \hat{\Lambda} = \begin{pmatrix} -\hat{\lambda}_{12} & \hat{\lambda}_{12} \\ \hat{\lambda}_{21} & -\hat{\lambda}_{21} \end{pmatrix} \]  \quad (22)

is the generator matrix of the Markov chain under the risk-neutral measure. We can show that

\[ \hat{P}_{s-t} = \begin{bmatrix} \hat{f}_1 & \hat{f}_2 \\ \hat{f}_1 & \hat{f}_2 \end{bmatrix} + \begin{bmatrix} \hat{f}_2 & -\hat{f}_2 \\ -\hat{f}_1 & \hat{f}_1 \end{bmatrix} e^{-\hat{p}(s-t)}, \]  \quad (23)

where \( \hat{p} = \hat{\lambda}_{12} + \hat{\lambda}_{21} \) and \( (\hat{f}_1, \hat{f}_2) = \left( \frac{\hat{\lambda}_{21}}{\hat{p}}, \frac{\hat{\lambda}_{12}}{\hat{p}} \right) \). From (23) we can deduce the long-run behaviour of the state of the economy under the risk-neutral measure, by letting \( s \to \infty \), to obtain

\[ \lim_{s \to \infty} \hat{P}_{s-t} = \begin{bmatrix} \hat{f}_1 & \hat{f}_2 \\ \hat{f}_1 & \hat{f}_2 \end{bmatrix}. \]  \quad (24)

Hence, in the long-run, the risk-neutral probability of remaining in state \( i \) is \( \hat{f}_i \) and the risk-neutral probability of switching from state \( i \) to \( j \neq i \) is \( \hat{f}_j \). Note that \( \hat{f}_1 + \hat{f}_2 = 1 \). The vector \( (\hat{f}_1, \hat{f}_2) \) is the long-run risk-neutral distribution of the economy. The parameter \( \hat{p} \) tells us how how quickly the state of the economy approaches its long-run risk-neutral distribution. To be precise, convergence to the long-run is exponential at a rate of \( \hat{p} \). The time it will take for the distance of the current distribution from the long-run risk-neutral distribution to halve, is the risk-neutral half-life of the state of the economy, which is given by

\[ \hat{t}_{1/2} = \frac{\ln 2}{\hat{p}}. \]  \quad (25)

The longer the risk-neutral half-life, the longer it takes for the state of the economy to converge to its long-run risk-neutral distribution, and the more long-run risk there is in the economy.\(^{16}\)

\(^{16}\)For the general case, when the number of states, \( L \), in the Markov chain is strictly greater than 2, the long-run risk-neutral distribution \( (\hat{f}_1, \ldots, \hat{f}_L) \) is the left-eigenvector of the risk-neutral generator matrix, \( \hat{\Lambda} \), which has eigenvalue of zero. Convergence to the risk-neutral long-run distribution is exponential and the convergence rate is given by the size of the maximal eigenvalue of \( \hat{\Lambda} \), i.e. the one with the largest modulus. This is a well-known result from the theory of Markov chains, and is just an application of the Perron-Frobenius Theorem from matrix analysis. See Bremaud (1999) for details and Hansen and Scheinkman (2006) for applications to asset pricing.
II Asset Valuation

In this section we derive the prices of all assets in the economy and investigate the properties of credit spreads and the equity premium. The proposition below is an implication of Proposition 1 and considerably simplifies the computation of asset prices.

**Proposition 2** Suppose the first and second moments of consumption growth do not switch, i.e. $g_1 = g_2 = g$ and $\sigma_{C,1} = \sigma_{C,2} = \sigma_C$, but the first and second moments of earnings growth do switch. Then the price of an asset when the economy is in state $i$ is given by

$$P_i(\lambda_{12}, \lambda_{21}, g, \sigma_C).$$

(26)

If we now introduce switching into the first and second moments of consumption growth, then the price of an asset when the economy is in state $i$ is given by

$$P_i(\hat{\lambda}_{12}, \hat{\lambda}_{21}, g_i, \sigma_{C,i}).$$

(27)

The above proposition shows that prices in an economy with switching in the expected value and/or volatility of the consumption growth rate can be obtained from prices in an economy where there is no switching in the expected value or volatility of the consumption growth rate merely by adjusting the probability that the economy changes state by the risk-distortion factor to get risk-neutral probabilities and replacing the constant expected consumption growth rate and volatility by the relevant state-dependent quantities.

We use the state-price density in Equation (5) to value the corporate debt and equity issued by firms. As in EBIT-based models of capital structure (see Goldstein, Ju, and Leland (2001)), the earnings (or EBIT cash flow), $X_t$, of a firm is split between a constant coupon, $c$, paid to debtholders and a risky dividend, $X - c$, paid to equityholders. Because of taxes paid at the rate $\eta$, equityholders actually receive the amount $(1 - \eta)(X - c)$. If the firm defaults, bondholders no longer receive coupons, but receive what can be recovered of the firm’s assets, i.e. a fraction $\alpha_t$ of the after-tax value of the firm’s earnings at default. The recovery rate $\alpha_t$ is assumed to be procyclical: $\alpha_t \in \{\alpha_1, \alpha_2\}$, where $\alpha_1 < \alpha_2$, consistent with empirical findings in Thorburn (2000), Altman, Brady, Resti, and Sironi (2002) and Acharya, Bharath, and Srinivasan (2002). Equityholders no longer receive dividends if default occurs, so equity value is levered, in the sense that is affected by default risk.

The debt and levered equity values for a firm can be written in terms of the prices of a set of Arrow-Debreu default claims and unlevered firm value (after taxes). To see the intuition behind this, note that when there is no intertemporal macroeconomic risk, the debt value is given by the standard expression

$$B_t = \frac{c}{\bar{r}} (1 - qD, t),$$

(28)
where \( r \) is the risk-free rate, \( l \) is the loss ratio at default, given by

\[
l = \frac{c_r}{r} - \alpha \frac{A(X_D)}{r},
\]

(29)

and \( \alpha \) is the constant recovery rate. Observe that \( q_{D,t} \) is the value of the Arrow-Debreu default claim which pays out one unit of consumption, if default occurs (i.e. \( X \) hits or falls below the default boundary \( X_D \)). \( A(X_D) \) is the after-tax abandonment value at default. The equity value also depends on the value of the Arrow-Debreu default claim and the firm’s after-tax abandonment value \( A(X) \):

\[
S_t = A(X_t) - (1 - \eta) \frac{c}{r} + q_{D,t} \left( (1 - \eta) \frac{c}{r} - A(X_D) \right).
\]

(30)

When there is intertemporal macroeconomic risk, debt and equity values become state-dependent and we obtain simple generalizations of (29) and (30). Debt value in state-\( i \) is

\[
B_{i,t} = \frac{c}{r_{P,i}} \left( 1 - \sum_{j=1}^{2} l_{ij,t} q_{D,ij,t} \right),
\]

(31)

where

\[
l_{ij,t} = \frac{c_{r_{P,j}}}{r_{P,i}} - \alpha_j A_j (X_{D,j})
\]

(32)

is the loss ratio at default, when the current state is \( i \) and default occurs in state \( j \). \( X_{D,i} \) is the state-\( i \) default boundary and \( A_i (X_{D,i}) \) is the after-tax abandonment value at default when the current state is \( i \). \( r_{P,i} \) is the discount rate for a risk-free perpetuity when the current state is \( i \). Equity value in state-\( i \) is given by

\[
S_{i,t} = A_i (X_t) - (1 - \eta) \frac{c}{r_{P,i}} + \sum_{j=1}^{2} q_{D,ij} \left( (1 - \eta) \frac{c}{r_{P,j}} - A_j (X_{D,j}) \right), i \in \{1, 2\}.
\]

(33)

Observe that with intertemporal macroeconomic risk, we no longer have just one Arrow-Debreu default claim. Instead we have four: \( \{q_{D,ij,t}\}_{i,j \in \{1,2\}} \), where \( q_{D,ij,t} \) is the time-\( t \) value of a claim in state-\( i \) that pays 1 unit of consumption at default, conditional on default occurring in state-\( j \). In other words, if the current date is \( t \) and earnings hits the boundary \( X_{D,j} \) from above for the first time in state \( j \), one unit of consumption will be paid that instant. If not, the security does not pay anything and expires worthless if default happens in any other state. In the following section, we link the Arrow-Debreu default claims to both risk-neutral and actual default probabilities.
II.A Arrow-Debreu Default Claims

Each Arrow-Debreu default claim is a perpetual digital-put, so we can derive their values by solving the following system of ordinary differential equations:

\[
\frac{\partial q_{D,ij,t}}{\partial X_t} \hat{\theta}_i X_t + \frac{1}{2} \frac{\partial^2 q_{D,ij,t}}{\partial X_t^2} \sigma^2_{\lambda i,X} X_t^2 + \sum_{k \neq i} \lambda_{ik} (q_{D,ik,t} - q_{D,ij,t}) = r_t q_{D,ij,t}, \quad i,j \in \{1,2\},
\]

where

\[
\hat{\theta}_i = \theta_i - \gamma \rho_{XC,i} \sigma^2_X \sigma_{C,i}
\]

is the risk-neutral earnings growth rate in state-\(i\). The definitions of the payoffs of the Arrow-Debreu default claims give us the following boundary conditions: for \(i,j \in \{1,2\}\)

\[
q_{D,ij}(X) = \begin{cases} 
1, & j = i, \ X \leq X_{D,i} \\
0, & j \neq i, \ X \leq X_{D,i}.
\end{cases}
\]

Value-matching and smooth-pasting give us the remaining boundary conditions: for \(j \in \{1,2\}\)

\[
\begin{align*}
\lim_{X \downarrow X_{D,j}} q_{D,2j} &= \lim_{X \uparrow X_{D,j}} q_{D,2j}, \\
\lim_{X \downarrow X_{D,j}} q_{D,2j}' &= \lim_{X \uparrow X_{D,j}} q_{D,2j}'.
\end{align*}
\]

To link the Arrow-Debreu default claims to the actual probability of default, we decompose the value of the claim into three factors: a time-adjustment, a risk-adjustment and the actual default probability, as shown in the proposition below.

**Proposition 3** The price of the Arrow-Debreu default claim, which pays out one unit of consumption if default occurs in state-\(j\) and the current state is \(i\), is given by

\[
q_{D,ij} = p_{D,ij} T_{ij} R_{ij},
\]

where \(p_{D,ij}\) is the actual probability of default occurring in state-\(j\), conditional on the current state being \(i\), \(T_{ij}\), is a time-adjustment factor, and \(R_{ij}\), is a risk-adjustment factor.

The risk-neutral probability of default occurring in state-\(j\), conditional on the current state being \(i\), \(\hat{p}_{D,ij}\), is given by

\[
\hat{p}_{D,ij} = p_{D,ij} R_{ij}.
\]

Proposition 3 tells us that the price of the Arrow-Debreu default claim is not equal to the risk-neutral default probability, a fact not explicitly noted in the previous literature. Chen, Collin-Dufresne, and Goldstein (2006) note that to resolve the credit spread puzzle
risk-default probabilities must be high, while actual default probabilities. In fact, Arrow-Debreu default claims must have high prices, while actual default probabilities are low. The decomposition in (39) tells us this is achieved via high time- and risk-adjustments.

The time- and risk-adjustments are computed by solving for the actual and risk-neutral default probabilities, because

\[ R_{ij} = \frac{\hat{p}_{D,ij}}{p_{D,ij}}, \]  

\[ (41) \]

and

\[ T_{ij} = \frac{q_{D,ij}}{\hat{p}_{D,ij}}. \]  

\[ (42) \]

The set of actual default probabilities is the solution of (34), but with the risk-free rate set to equal zero and the risk-distortion factor set equal to one. Similarly, risk-neutral default probabilities are the solution of (34), but with just the risk-free rate set to zero.

To gain more intuition about the decomposition in (39), we compute the actual default probability, the time- and risk-adjustments for the case when there is no intertemporal macroeconomic risk. We can show that

\[ p_D(X_t) = \left( \frac{X_D}{X_t} \right)^{\frac{\theta - \frac{1}{2} \sigma^2_X}{\sigma^2_X}} \]  

\[ (43) \]

is the actual probability of default,

\[ R(X_t) = \left( \frac{X_D}{X_t} \right)^{-\frac{\gamma \sigma_X \sigma^2_C \sigma_C}{\sigma^2_C X^2}} \]  

\[ (44) \]

is the risk-adjustment factor, and

\[ T(X_t) = \left( \frac{X_D}{X_t} \right)^{(\theta - \frac{1}{2} \sigma^2_X) \left( \frac{1}{\theta - \frac{1}{2} \sigma^2_X} \right)^{-1}} \]  

\[ (45) \]

is the time-adjustment factor.

The risk-adjustment factor is always greater than or equal to one and, as expected, is increasing in relative risk aversion, \( \gamma \) (see Figure 3). In particular, when the representative agent is risk-neutral (\( \gamma = 0 \)), the risk associated with not knowing the time of default is no longer priced, and the risk-adjustment factor reduces to unity. The risk-adjustment also increases with systematic earnings volatility, and the volatility of consumption growth. But the risk-adjustment factor is lower when idiosyncratic risk is a higher proportion of total risk.
The time-adjustment factor is a downward adjustment, reflecting the time-value of money. Consistent with this interpretation, the time-adjustment is decreasing in the risk-free rate and reduces to one when the riskfree rate is zero. Consequently, the time-adjustment factor decreases with the elasticity of intertemporal substitution (consumption smoothing) and increases with relative risk aversion (precautionary savings), as shown in Figure 4. Higher earnings volatility makes earlier default more likely, which increases the time-adjustment factor. At first glance it may seem counterintuitive that the time-adjustment factor depends on the risk-neutral expected earnings growth rate, \( \hat{\theta} \), rather than the actual expected earnings growth rate, \( \theta \). But a more risk-averse agent puts more weight on the likelihood of early default, making the time-adjustment larger.

It follows that \( q_D \) is high when both relative risk aversion and the elasticity of intertemporal substitution are high, as shown in Figure 2.

Because we have closed-form expressions for \( R \) and \( T \) in terms of preference parameters, we can gauge how large relative risk aversion and the elasticity of intertemporal substitution must be to resolve the credit spread puzzle. Thus, we can analyze the credit-spread puzzle in the same way Mehra and Prescott (1985) analyzed the equity risk-premium puzzle. To do this explicitly for Baa bonds, which according to Huang and Huang (2003) have spreads over treasuries of 158 bp, we choose the coupon, \( c \), default boundary, \( X_D \) and idiosyncratic earnings growth volatility to match the historical 5-year default probability of 2.222%, the historical loss rate of 1.241% and historical book leverage of 50.21% for given relative risk aversion, \( \gamma \), and elasticity of intertemporal substitution, \( \psi \). We compute the resulting model-implied credit spread for different values of \( \gamma \) and \( \psi \) as illustrated in Figure 6. We make two initial observations. Firstly, even for very high risk aversion, the model-implied Baa credit spreads is much less than in the data. Secondly, using Epstein-Zin-Weil preferences instead of power utility makes it easier to generate higher credit spreads. This is because separating relative risk aversion from the elasticity of intertemporal substitution makes it possible to get lower risk-free rate, and hence a larger time-adjustment factor. But the risk-adjustment factor is still not large enough to make \( q_D \) large while keeping \( p_D \) small.

Introducing intertemporal macroeconomic risk allows us to increase the risk-adjustment factor, \( R \), without lowering the time-adjustment, \( T \). The increase in the risk-adjustment, \( R \), stems directly from the risk-distortion factor, \( \omega \), from Proposition 1. When there is a greater aversion to intertemporal macroeconomic, the risk-distortion factor increases, which leads to an increase in the risk-adjustment factor.

---

17 The standard condition, \( \frac{\theta}{2} \sigma^2 > 0 \), ensures that \( T_t \) is less than one when \( r > 0 \).
18 The historical default probabilities and loss rates are taken from Cantor, Hamilton, Ou, and Varma (2006). In lieu of figures for market leverage we use the the sample average book-leverage for BBB+, BBB and BBB- firms in Kisgen (2003).
In summary, two assumptions lie behind the model’s ability to generate high prices for Arrow-Debreu default claims, without increasing actual default probabilities. The first is the use of Epstein-Zin-Weil preferences, which increases the time-adjustment factor by lowering the risk-free rate. The second is the assumption of switching in the first and second moments of both earnings and consumption growth rates (intertemporal macroeconomic risk). When the representative agent is averse to the delayed resolution of intertemporal risk \( \gamma > 1/\psi \), she dislikes intertemporal macroeconomic risk, which is reflected in an increased risk-adjustment factor.

II.B Abandonment Value

The firm’s state-conditional liquidation, or abandonment, value, denoted by \( A_{i,t} \) is the after-tax value of the future unlevered firm’s earnings in state \( i \) which is given by the following formula:

\[
A_{i,t} = (1 - \eta)X_tE_t \left[ \int_t^\infty \frac{\pi_sX_s}{\pi_tX_t} ds \right]_{\nu_t = i}, \text{ for } i \in \{1, 2\}. \tag{46}
\]

The liquidation value in (46) is a function of the current earnings level and is time-independent, \( A_{i,t} = A_i(X_t) \). The next proposition derives the value of \( A_i \) in terms of fundamentals of the economy:

**Proposition 4** The liquidation value in state \( i \in \{1, 2\} \), is given by

\[
A_i(X_t) = \frac{(1 - \eta)X_t}{r_{A,i}}, \tag{47}
\]

where

\[
r_{A,i} = \bar{\rho}_i - \theta_i + \left( \bar{\rho}_j - \theta_j \right) - \left( \bar{\rho}_i - \theta_i \right) \hat{\rho}_{ij}, j \neq i, \tag{48}
\]

and

\[
\bar{\rho}_i = r_i + \gamma_0 \chi C_i \sigma X_i \sigma C_i, \tag{49}
\]

is the discount rate in the standard Gordon growth model.

To understand the intuition behind (47), suppose the economy is currently in state \( i \). Then, the risk-neutral probability of the economy switching into state \( j \neq i \) during a small time interval \( \Delta t \) is \( \hat{\lambda}_{ij} \Delta t \) and the risk-neutral probability of not switching is \( 1 - \hat{\lambda}_{ij} \Delta t \). We can therefore write the unlevered firm value in state \( i \) as

\[
A_i = (1 - \eta)X \Delta t + e^{-(\bar{\rho}_i - \theta_i)\Delta t} \left[ 1 - \hat{\lambda}_{ij} \Delta t \right] A_i + \hat{\lambda}_{ij} \Delta t A_j, i \in \{1, 2\}. \tag{50}
\]
The first term in (50) is the after-tax cash flow received in the next instant and the second term is the discounted continuation value. The continuation value depends on the future state of the economy. For example, with risk-neutral probability \( \tilde{\lambda}_{ij} \Delta t \) the economy will be in state \( j \) and the continuation value is \( A_j \). This continuation value is discounted back at a rate reflecting the discount rate \( \bar{\mu}_i \) and the expected earnings growth rate over that instant which is \( \theta_i \). This gives us a set of 2 linear simultaneous equations in 2 unknowns, the solution of which produces \( A_1 \) and \( A_2 \).

To gauge the intuition behind the risk-adjusted discount rate, note that if the economy stays in state \( i \) forever, the discount rate in the perpetuity formula (47) is simply

\[
   r_{A,i} = \bar{\mu}_i - \theta_i. \tag{51}
\]

To be concrete, assume we are in state 1. The discount rate is adjusted downwards to account for time spent in state 2. This adjustment increases with the growth rate in the high state, \( \theta_2 \), and the risk-neutral probability per unit time of switching into state 2, \( \tilde{\lambda}_{12} \). Note that as the economy is more likely to switch to another state (i.e. \( \tilde{p} \) becomes bigger), the adjustment approaches \([ (\bar{\mu}_2 - \theta_2) - (\bar{\mu}_1 - \theta_1) ] \tilde{f}_2 \) (where \( \tilde{f}_i \) is the long-run risk-neutral probability of being in state \( i \)) and the discount rate approaches \((\bar{\mu}_1 - \theta_1) \tilde{f}_1 + (\bar{\mu}_2 - \theta_2) \tilde{f}_2 \), which is the long-run risk-neutral mean of the difference between the discount rate and the expected earnings growth rate.

### II.C Credit Spreads and the Levered Equity Risk Premium

Turning now to corporate debt, the generic value of debt at time \( t \), conditional on the state being \( i \) denoted by \( B_{i,t} \), is given by

\[
   B_{i,t} = E_t \left[ \int_t^{\tau_D} \frac{\pi_{i} \text{cds}}{\pi_t} \nu_t = \nu_i \right] + E_t \left[ \frac{\pi_{\tau_D}}{\pi_t} \alpha_{\tau_D} A_{\tau_D} \mid \nu_t = i \right], i \in \{1, 2\}. \tag{52}
\]

The first term in (52) is the present value of a perpetual coupon stream until default occurs at a random stopping time \( \tau_D \). The second term is the present value at time \( t \) of the asset recovery value the debtholders successfully claim upon default, where \( \alpha_t \in \{\alpha_1, \alpha_2\} \) is the date-\( t \) recovery rate. We can show that (52) reduces to (31), which can be decomposed into the product of two factors. The first factor is the price of the equivalent riskless consol bond, \( c/r_{P,i} \), and the second factor is a downward adjustment for default risk. The discount rate for a riskless perpetuity when the current state is \( i \), is given by

\[
   r_{P,i} = r_i + \frac{r_j - r_i}{\tilde{p} + r_j} \tilde{p} \tilde{f}_j, j \neq i. \tag{53}
\]
$r_{P,i}$ is not equal to the risk-free rate in state $i$, $r_i$, because the risk-free rate is expected to change in the future whenever the state of the economy switches.\textsuperscript{19} We now explain the form of the default risk-adjustment factor, $1 - \sum_{j=1}^{2} l_{ij,t} q_{D,ij,t} < 1$. Observe that if the current state is $i$ and default occurs in state $j$, then the proportional loss at default is $l_{ij,t}$. The time-adjusted risk-neutral probability of this loss occurring is $q_{D,ij,t}$. Default can occur in 2 possible states, so the time-adjusted risk-neutral expected loss rate is $\sum_{j=1}^{2} l_{ij,t} q_{D,ij,t}$.

The next proposition gives the corporate bond yield spread in terms of the discount rate for a risk-free perpetuity, loss ratios and Arrow-Debreu default claims.

**Proposition 5** The credit spread in state $i$, $s_{i,t}$, is given by

\begin{align}
  s_{i,t} &= \frac{c}{B_{i,t}} - r_{P,i} \\
  &= r_{P,i} \frac{\sum_{j=1}^{2} l_{ij,t} q_{D,ij,t}}{1 - \sum_{j=1}^{2} l_{ij,t} q_{D,ij,t}}. 
\end{align}

The above proposition tells us that we can generate high credit spreads in three ways: a high risk-free rate, high loss rates or high prices for Arrow-Debreu default claims. Empirically, we know both risk-free rates and loss rates are low. Therefore, the only way to generate high credit spreads is via high prices for Arrow-Debreu default claims. But we know from Proposition 3 that Arrow-Debreu default claims are just actual default probabilities adjusted for time and risk. Because actual default probabilities are low, the time and risk-adjustments need to be large, which confirms our earlier intuition.

Current levered equity value is given by the expected present value of future cashflows less coupon payments up until bankruptcy, conditional on the current state:

\[ S_{i,t} = (1 - \eta) E_t \left[ \int_{t}^{\tau_D} \pi_s \frac{\pi_t}{\pi_i} (X_s - c) \, ds \bigg| \nu_t = i \right], \quad i \in \{1, 2\}. \]  

where $\tau_D$ is the first passage time of earnings to the date-$t$ default boundary. Recall that the default boundary depends on the state of the economy at date-$t$, so when the current state is $i$, the default boundary is $X_{D,i}$. We can show that the above equation simplifies to give (33). The first two terms in Equation (33) are the present after-tax value of future cashflows less coupon payments, if the firm were never to default. The terms in the square brackets represent the adjustment to the equity value if default occurs in state $j$. Upon default shareholders no longer have to pay coupons to bondholders and at the same time they lose the rights to any future cash flows from owning the firm’s assets.

\textsuperscript{19}Note that (53) can also be intuitively obtained from the formula for the discount rate for a stochastically growing cash flow, (48), by replacing the Gordon growth model discount rate, $\pi_i$, with the risk-free rate, $r_i$ and setting the expected growth rate of earnings, $\theta_i$, equal to zero.
In the next proposition we derive the levered equity risk-premium and levered stock-market return volatility of an individual firm.

**Proposition 6** The levered stock market return in state $i$ is given by

$$dR_t|_{\nu_t=\nu_i} = \frac{dS_t + (1 - \eta)(X_t - c)dt}{S_t} = \mu_{R,i}dt + \sigma_{R,i}^{id}d\beta^id + \sigma_{R,i}^{s}d\beta^sd_X + \sum_{j \neq i} \sigma_{R,ij}^PdN_{ij,t}^P,$$

(57)

where

$$\sigma_{R,i}^{id} = \frac{\partial \ln S_t}{\partial \ln X_t} \sigma_{X,i}^{id},$$

(58)

$$\sigma_{R,i}^{s} = \frac{\partial \ln S_t}{\partial \ln X_t} \sigma_{X,i}^s,$$

(59)

$$\sigma_{R,ij}^P = \frac{S_j}{S_i} - 1,$$

(60)

and

$$\frac{\partial \ln S_{t,t-}}{\partial \ln X_t} = \frac{\Lambda_i(X_t)}{S_t} + \sum_{j=1}^2 q_{D,ij}^i \left[ (1 - \eta) \frac{\omega}{r_{P,j}} - A_j(X_{D,j}) \right].$$

(61)

The conditional levered equity risk premium in state $i$ is

$$\mu_{R,i} - r_i = \gamma \rho_{XC,i} \sigma_{R,i}^{s} \sigma_{C,i}^s + \Lambda_i,$$

(62)

where $\Lambda_i$ is the jump risk-premium in state $i$:

$$\Lambda_i = \left\{ \begin{array}{ll} (1 - \omega^{-1})\sigma_{R,12}^P \lambda_{12}, & i = 1. \\ (1 - \omega)\sigma_{R,21}^P \lambda_{21}, & i = 2. \end{array} \right.$$  

(63)

Conditional levered stock return volatility in state $i$ is

$$\sigma_{R,i} = \sqrt{\left(\sigma_{R,i}^{id}\right)^2 + \left(\sigma_{R,i}^{s}\right)^2 + \sum_{j \neq i} \left(\sigma_{R,ij}^P\right)^2 \lambda_{ij}}.$$  

(64)

At first blush, one might expect the levered equity risk premium is larger than the unlevered risk premium, because paying coupons reduces the dividends that can be paid so equityholders. But introducing leverage into a firm also brings in default risk. And Equations (61) and (62), tell us that default risk decreases the risk premium, because the price of the Arrow-Debreu default claim, $q_{D,ij}$, is decreasing in $X$, implying that $q_{D,ij}^i < 0$. The same argument applies to stock-return volatility.
Equations (62) and (63) tell us that jumps in the value of levered equity are priced if and only if the risk-distortion factor, \( \omega \), is not equal to 1. This result is intuitive because the risk-distortion factor is defined as the jump in the state-price density that occurs when the state changes (see Equation 10). When \( \omega \), the state-price density does not jump, so jumps in the levels of prices do not appear in risk-premia. When the representative agent prefers intertemporal risk to be resolved sooner rather than later, \( \gamma > 1/\psi \), and \( \omega > 1 \), so in recessions, the levered equity risk premium is increased. The opposite is the case when the agent likes intertemporal risk: \( \gamma < 1/\psi \), making \( \omega < 1 \).

Stock return volatility is increased by jumps in the equity value, regardless of the agent’s preferences for the timing of the resolution of intertemporal risk, as is seen in Equation (64). This is intuitive, because volatility includes all risk, not just risks which are priced.

II.D Optimal Default Boundary and Optimal Capital Structure

The default boundary depends on the current state state of the economy, i.e. there is a set of default boundaries \( X_{D,i} \), \( i \in \{1, 2\} \), where \( X_{D,i} \) is the default boundary when the economy is in state \( i \). We can prove that the default boundary must be weakly countercyclical, i.e. \( X_{D,1} \geq X_{D,2} \) (see Appendix). The optimal default boundaries are chosen to maximize levered equity value in each state. Therefore they are obtained by solving 2 simultaneous equations:

\[
\left. \frac{\partial S_i(X)}{\partial X} \right|_{X=X_{D,i}} = 0, \quad i \in \{1, 2\}, \tag{65}
\]

which are just the standard smooth-pasting conditions. Because the optimal default boundary switches with the state of the economy, default can take place even if earnings does not change. To see why, suppose that we are in state 2, with default boundary \( X_{D,2} \) and that \( X > X_{D,2} \), but \( X < X_{D,1} \). If the economy stays in a boom, the firm will stay solvent. But as soon as the economy goes into recession, the firm will go bankrupt.

The optimal coupon paid to debtholders will depend on the state of the economy at date-0, so we label it as \( c_{\nu_0} \), where \( \nu_0 \) is the state of the economy at date-0. We find \( c_{\nu_0} \) by maximizing date-0 levered firm value, \( F_{\nu_0,0} = B_{\nu_0,0} + S_{\nu_0,0} \). The choice of optimal default boundaries will depend on the coupon choice, so each \( X_{D,i} \) also depends on the initial state of the economy, but for simplicity we deliberately exclude this from the notation.

III Calibration

We use aggregate data for the US economy. Consumption is real non-durables plus service consumption expenditures from the Bureau of Economic Analysis. Earnings data are from
S&P and provided on Robert J. Shiller’s website. We delete monthly interpolated values and obtain a time-series at quarterly frequency. The personal consumption expenditure chain-type price index is used to deflate the earnings time-series. Figure 7 shows the growth rate of real earnings and real consumption expenditures. All data is at quarterly frequency for the period 1947Q1-2005Q4.

III.A Parameter Values

The unconditional parameter estimates are summarized in Table 2. The growth rate of real consumption during the post-war period was on average 3.33% with a standard deviation of 0.99%. Not surprisingly, the mean growth of real earnings was similar with 3.43% and also more volatile with a standard deviation of 10.72%.

Our parameter choices are summarized in Panel A of Table 3. We restrict the model to 2 states so that state 1 can be interpreted as recession and state 2 as boom. To calibrate the Markov chain, we have to pin down two free parameters: $\lambda_{12}$ and $\lambda_{21}$. The average durations of recessions and booms are $\lambda_{21}^{-1}$ and $\lambda_{12}^{-1}$, respectively. Using NBER data on recessions and booms thus gives as an intuitive way of selecting $\lambda_{12}$ and $\lambda_{21}$. A recession is a period of falling economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales.\(^{20}\) Between 1854-2001, the average length of booms and recessions was 38 and 17 months, respectively. Because the NBER definition of recessions is very narrow and pertains to part of the economy we do not explicitly model, we set the average duration of booms and recessions equal to 4 and 2 years, respectively. Therefore, $\lambda_{12} = 1/2$ and $\lambda_{21} = 1/4$. Hence $p = \lambda_{12} + \lambda_{21} = 3/4$ and $f_1 = \lambda_{21}/p = 1/3$.

Following Hamilton (1989), Markov switching models are estimated by assuming that the state is unobservable to the econometrician. Since in our model the state is observable, we do not follow this approach but a much simpler one. To calibrate the state-dependent first and second moments of consumption and earnings growth, we split each sample of growth rates into two sub-samples along their respective unconditional means. Within each sub-sample, we then compute the average and standard deviation of growth rates.

By construction, we compute a truncated mean and standard deviation of growth rates in each sub-sample—specially, the boom sample has a lower bound and the recession sample an upper one. As a result, the long-run mean and standard deviation are not equal with their unconditional value. For instance, $f_1\overline{g}_1 + f_2\overline{g}_2 \neq \overline{g}$ where $\overline{g}_i$ denotes the consumption growth rates in sub-sample $i$ and $\overline{g}$ the unconditional consumption growth rate. By multiplying the

\(^{20}\)Source: Announcement of NBER’s Business Cycle Dating Committee on how to choose turning points in the economy, dated 07/17/03.
left-hand side of the equation by a constant, we scale $g_1$ and $g_2$ up or down so that the equation holds with equality. The same correction applies for the conditional standard deviations.

It is well known that the estimated correlation between consumption and dividends growth decreases with the frequency of the data. Using quarterly data the correlation between consumption and earnings growth is 17% but increases to 36% using annual data. We choose a value in between of 30%. For parsimony, we assume the conditional correlation is identical across states. The resulting calibration values are presented in Panel A of Table 3.

The last parameter of the calibration is the idiosyncratic earnings volatility. Moody’s reports that the average cumulative issuer-weighted corporate default rate between 1970 and 2005 for Baa bonds was and 2.037%. We match the actual long-run 5 year default probability for risk aversion of 10 and EIS of 1.5 by setting setting idiosyncratic earnings volatility to 33.5%. Another possibility to calibrate the model to Baa firms would be to scale the drift and standard deviation of earnings growth rates. To keep the calibration as simple as possible, we set the drift and standard deviation of earnings growth rates of Baa firms equal to the aggregate values.

To calibrate the levered economy, we further have to pick parameter values for the corporate tax rate $\eta$, recovery rates $\alpha_1$ and $\alpha_2$, and total firm value volatility. Our parameter choice is summarized in Panel B of Table 3.

Huang and Huang (2003) report recovery rates of 51%. Since the recovery rate for corporate bond holders is higher in a boom than in a recession of the economy, we assume $\alpha_1 = 0.4$ and $\alpha_2 = 0.6$ so that the average recovery rate is roughly 51%. Following Hackbarth, Miao, and Morellec (2006), we set the tax rate $\eta$ equal to 15%. For all tables, the annual rate of time preference $\beta$ equals 0.01.

### III.B Corporate Bond Market

Using the parameter values chosen above, we now compute corporate bond market variables, with an optimal default boundary and capital structure. We restrict relative risk aversion and the elasticity of intertemporal substitution to the values 7.5, 10 and 0.1, 0.5, 1.5, respectively.\[21\]

Note two special cases: $\gamma = 10$ and $\psi = 0$, imply power utility; $\gamma = 7.5$ and $\psi = 0.1$ imply a preference for late resolution of uncertainty. All other combinations of $\gamma$ and $\psi$ imply a preference for early resolution of uncertainty—the natural assumption regarding preferences.
Table 4 summarizes the results. The first two columns apply when the economy is in a recession (state 1) and the last two column when the economy is in a boom (state 2).

The credit spread increases with relative risk aversion and falls with the elasticity of intertemporal substitution. To understand the intuition behind the effect of risk aversion, note that it affects the discount rate in two ways: First, higher risk aversion increases the demand for precautionary savings which reduces the risk-free rate. Second, higher risk aversion increases the risk adjustment $\gamma \sigma_{C,i} \sigma_{X,i}$. Both effects increase the risk-neutral default probabilities (Panel B) and thus credit spreads.

With higher risk aversion, the agent optimally chooses a smaller coupon (Panel E), an intuitive result since the agent would like to lower leverage to reduce the chance of default. A smaller coupon reduces the bond value which has a positive effect on credit spreads. At the same time, a smaller coupon lowers the default boundary (Panel D) since the agent is willing to pay a smaller coupon for a longer time period. While this last effect tends to reduce the credit spread, it is dominated by the first effect.

Credit spreads fall with EIS. The intuition is as follows: when EIS increases, the risk-free rate falls which increases the risk-neutral default probabilities. This effect increases the credit spread. However, EIS also affects the optimal coupon choice. As a result, leverage dramatically falls with EIS, explaining the lower credit spread.

Since credit spreads fall with EIS, we generate the highest credit spread when EIS equals 0.1. In this case, however, the optimal coupon generates leverage ratios of above 40% in state 1 and more than 50% in state 2. Empirically, leverage is around 35% (for example, see Huang and Huang (2003). High leverage is also reflected in large actual default probabilities (Panel D), which are above 13% in state 1 and 2. Moody’s reports actual five-year default probabilities for Baa bonds of 2.037%. To be in line with empirically observed values for leverage and actual default probabilities requires high EIS of at least 0.5.

The crucial determinant of credit spreads is the price of an Arrow-Debreu default claim which pays one unit in the case of default. As shown in Proposition 3, this price can be decomposed into three components: the actual default probability (Panel D), the risk-adjustment $R$ (Panel E), and the time-adjustment $T$ (Panel F). Even though the actual default probabilities fall with risk aversion and EIS—as a result of the optimal default boundary—the price of the Arrow-Debreu default claim rises with risk aversion and EIS.

---

In Panel A and B, we report credit spreads and credit spread volatilities, respectively, in basis points for different combinations of risk aversion and EIS. Panel C contains the price of Arrow-Debreu default claims in percent. Its three components are reported in Panel D to F for a 5-year horizon: actual default probability (Panel D), the risk-adjustment $R_5$ (Panel E), and the time-adjustment $T_5$ (Panel F). Panel G shows optimal default boundaries and Panel H optimal coupon levels. Panel I contains leverage ratios in state 1 ($B_1/F_1$) and state 2 ($B_2/F_2$).
The reason is the positive effect of risk aversion and EIS on the risk-adjustment and time-adjustment.

Two more facts about our model are noteworthy: First, as an endogenous outcome of our model, credit spreads and credit spread volatilities are countercyclical. This fact fits well with empirical regularities. Second, credit spread volatilities increase with risk aversion and fall with EIS. This result

III.C Equity Market

Table 5 summarizes the equity market variables. All numbers in this table are based on zero idiosyncratic volatility. Panel A shows the unlevered excess return, Panel B the semi-levered excess return (the excess return resulting from the payment of coupons, but ignoring default risk) and Panel C the levered excess return (the fully levered excess return, accounting for default risk). The unlevered and levered stock return volatilities are given in Panel D and Panel E. Panel F and G contain unlevered and levered price-earnings ratios. The risk-free rate of this economy is presented in Panel H.

Leverage has two effects on excess returns. First, the dividend payment to stock holders is reduced by the coupon. Second, the probability of bankruptcy shifts value from bond holders to stock holders. The first effect increases the excess returns in most cases which can be seen by comparing the unlevered return (Panel A) with the semi-levered one (Panel B). But the second effect tends to reduce excess returns.

The unlevered, semi-levered and levered excess return increase with risk aversion and EIS. The rationale for risk aversion is obvious. To understand the intuition for the effect of EIS, note that a switching model with power utility \((\gamma = 1, \psi = 10)\) generates a tiny risk premium of 0.03%. Holding risk aversion constant but increasing EIS leads to a huge increase in the risk premium because the agent prefers the early resolution of uncertainty about the long run state of the economy. Interestingly, the excess return is negative in state 2 when the agent has preference for the late resolution of uncertainty. The effect of leverage on the risk premium is smaller than the effect of switching. For instance, with risk aversion of 10 and EIS of 1.5, the long-run unlevered risk premium is 4.55% and the levered one is 6.58%.

The unlevered and levered stock return volatility fall with risk aversion but increase in EIS. Here leverage helps to generate realistic values because the levered stock return volatility is lower than the unlevered one and closer to empirical observed values. The levered and unlevered price-earnings ratios fall with risk aversion and increase with EIS. Leverage reduces the price-earnings ratio and helps to generate realistic values.
III.D Constant Versus Optimal Default Boundary

A important insight of Chen, Collin-Dufresne, and Goldstein (2006) is that a model with habit formation preferences generates only a realistic credit spread when the default boundary is counter-cyclical. To see whether this feature is also crucial in an economy with Epstein-Zin preferences, we solve our model with a constant default boundary.

In Table 6, we present the solution to our model when the default boundary is flat. We take the optimal coupon from Table 4 and set the default boundary so that the full model and this version have the same actual 5 year default probabilities. In contrast to Chen, Collin-Dufresne, and Goldstein (2006), we find that a flat default boundary barely reduces the credit spread.

III.E Cross-Market Comovement

Both earlier work by Fama and French (1993) and more recent work by Tauchen and Zhou (2006) finds evidence of comovement between bond and stock market variables. For instance, Tauchen and Zhou regress Moody’s Aaa and Baa yield spread on the jump-component of aggregate stock return volatility and find significant regression coefficients of 23.71 and 30.33, respectively.

To replicate Tauchen and Zhou’s findings we simulate 100 panels each containing 1,000 firms. The representative agent has risk aversion of 10 and EIS of 1.5. We simulate 20 years of monthly data and then regress the equally-weighted credit spread on the value-weighted jump component of stock return volatility for each panel following their empirical exercise. Our model generates a mean regression coefficient of 22.07 with a standard deviation of 1.382 across panels, which is very to close to the results Tauchen and Zhou obtain from the actual data.

IV Stripping Down the Model: What Causes What?

In this section, we show how each of our modelling assumptions impacts the credit spread and the equity risk premium. We strip down the model, by removing intertemporal macroeconomic risk in the first and second moments of earnings and consumption growth and the representative agent’s preference about the resolution of that risk over time. This leaves us with Model 1a, where aggregate consumption growth and earnings growth is i.i.d. and the representative agent has power utility. We then rebuild the model, step-by-step. In Model 1b, we introduce Epstein-Zin-Weil preferences. In Model 2, we add Markov switching

\[^{23}\text{Tauchen and Zhou (2006) run regressions using daily data—see their Table 5. We annualize their regression coefficients by multiplying by the square root of 250.}\]
in the first and second moments of earnings growth, but not to consumption growth. And finally, in Model 3, we rebuild the model fully by having Markov switching in the first and second moments of earnings growth. To achieve a fair comparison across models, we calibrate them all to the same data on earnings and consumption and use an optimal default boundary with a coupon chosen so leverage is the same in each model.

Table 8 summarizes our results for the corporate bond market. First we look at the size of credit spreads. When there is no intertemporal macroeconomic risk, using Epstein-Zin-Weil preferences instead of power utility (moving from Model 1a to Model 1b) increases credit spreads. This increase stems purely from an increase in the time-adjustment factor in the Arrow-Debreu default claim: separating relative risk aversion from the elasticity of intertemporal substitution allows us to reduce the risk-free rate. Adding Markov switching to the first and second moments of earnings growth, but not consumption growth (moving from Model 1b to Model 2) does not impact the size of the credit spread much at all, because these switches are not correlated with the state-price density, and are hence not priced. This is summarized by the fact that the risk-distortion factor, $\omega$ is 1. But introducing switching in the moments of consumption growth (moving from Model 2 to Model 3) leads to a significant increase in spread. Switches in the moments of earnings growth are now correlated with the state-price density ($\omega=1.35$, so the state-price density jumps up whenever expected earnings growth/earnings growth volatility jumps down/up), so they are priced into credit spreads. This is reflected by an increase in the risk-adjustment factor.

We now look at the cyclicality of credit spreads. Understanding this will provide an intuitive explanation for why our model can generate countercyclical credit spreads with a flat default boundary, whereas the habit-formation model used in Chen, Collin-Dufresne, and Goldstein (2006) cannot. First, we note that in models 1a and 1b, credit spreads are not cyclical. That is because there is no intertemporal macroeconomic risk. Introducing switches in the moments of earnings growth alone (Model 2) leads to countercyclical credit spreads. This countercyclicality is driven by countercyclical Arrow-Debreu default claims, which are in turn countercyclical because actual default probabilities are. Cyclicality in the time and risk-adjustment factors is negligible. Introducing switching in the moments of consumption growth (Model 3) makes the time-adjustment procyclical, but the risk-adjustment countercyclical. Overall, the Arrow-Debreu default claims remain countercyclical. This is in contrast with Chen, Collin-Dufresne, and Goldstein (2006), where the credit spread is procyclical, unless the default boundary is countercyclical. The reason for this stems from the time-adjustment factor. Chen, Collin-Dufresne, and Goldstein (2006) use Campbell-Cochrane habit formation to model preferences. Therefore, the risk-free rate is constant, implying that the time-adjustment is constant. Furthermore, the risk-adjustment is very
procyclical. Consequently the Arrow-Debreu default claims and hence the credit spread are procyclical, unless an exogenously countercyclical default boundary is imposed.

We study the impact of our modelling assumptions on equity market variables and the risk-free rate in Table 9. Introducing Epstein-Zin-Weil preferences when there is no intertemporal macroeconomic risk allows us to decrease the risk-free rate, thus increasing the price-earnings ratio. But this has no impact on risk premia or return volatilities. Switching in the moments of earnings growth makes the price-earnings ratio procyclical, which implies risk premia and volatilities are countercyclical. Cross-state mean risk-premia and volatilities remain small. Introducing switching in the moments of consumption growth makes risk premia and volatilities higher. This happens because upward jumps in marginal utility are now correlated with downward jumps in price-earnings ratios, creating a jump-risk premium. We can see this clearly in the behavior of the risk-distortion factor, which is now greater than 1. That implies that the risk-neutral probability of switching from a boom to a recession is now higher than the actual probability, which is reflected in higher risk premia and volatilities.

V Conclusion

We develop a model which can explain the following features of the data:

1. Comovement between credit spreads and the jump component of stock market return volatility,
2. a high equity risk premium,
3. high credit spreads, and
4. low probabilities of default,

using relative risk aversion ($\gamma$) parameters in the range 7-10 and with an elasticity of intertemporal substitution ($\psi$) of 1.5.

The key features of this model are intertemporal macroeconomic risk (Markov regime-shifts in the drifts and volatilities of earnings and aggregate consumption growth rates) and a representative agent who dislikes these regime-shifts (Epstein-Zin-Weil preferences with $\gamma > 1/\psi$). Because of the representative agent’s dislike of regime-shifts, the regime-shifts are priced, leading to jump-risk premia in asset returns. These jump-risk components are common to both corporate bond and stock returns, generating comovement between credit spreads and the jump component of stock market return volatility. The stock market risk premium increases as the agent’s dislike for regime-shifts increases. Our model can generate
realistically high credit spreads without raising actual default probabilities and leverage. This is crucial, because in the data expected default frequencies are very small and leverage is low. Our model can drive a wedge between the value of of the Arrow-Debreu default claim and actual default probabilities, because the agent’s aversion to random regime-shifts increases the risk-adjustment factor and increasing $\psi$ increases the time-adjustment factor.
A Appendix: Derivation of the State Price Density

To show that the results in this section are valid for more general models than the one considered in this paper, we assume that aggregate consumption is given by

\[
\frac{dC_t}{C_t} = g_t dt + \sigma_{C,t} dB_{C,t},
\]  

(A1)

where \(g_t\), \(\sigma_{C,t}\) depend on the current state of the economy, \(\nu_t\), and the current value of some continuous state variable, \(z_t\), which evolves according to

\[
dz_t = \mu_{z,t} dt + \sigma_{z,t} dB_{z,t},
\]  

(A2)

where

\[
dB_{z,t} dB_{C,t} = \rho_{zC,t} dt.
\]  

(A3)

The quantities \(\mu_{z,t}, \sigma_{z,t}\) may depend on \(z_t\) and the current state of the economy, \(\nu_t\), but not on consumption. If the economy changes state, then the variable \(\nu\) jumps. To model these jumps, we assume \(\nu\) is right continuous with left limits, i.e. RCLL (or to use the French abbreviation, càdlàg). Thus, if the state of the economy changes from \(i\) to \(j \neq i\) at date \(t\), then the left limit of \(\nu\) at time \(t\) is

\[
\nu_{t-} = \lim_{s \uparrow t} \nu_s = i,
\]  

(A4)

and the right limit of \(\nu\) at time \(t\) is

\[
\nu_t = \lim_{s \downarrow t} \nu_s = j.
\]  

(A5)

If we define

\[
\Delta \nu_t = \nu_t - \nu_{t-},
\]  

(A6)

then we know that \(\Delta \nu_t = j - i\). In light of this discussion, we must replace the drift and diffusion coefficients in (A1) and (A1) with their left limits to ensure that they are predictable processes.

The value function, \(J\), depends on the state of the economy, implying that the value function is also a jump process, which is RCLL. If a jump from state \(i\) to \(j \neq i\) occurs at date \(t\), then we abuse notation slightly and denote the left limit of \(J\) at time \(t\) by \(J_i\), where \(i\) is the index for the state. i.e. \(J_{t-} = \lim_{s \uparrow t} J_s = J_i\). Similarly \(J_t = \lim_{s \downarrow t} J_s = J_j\). We shall use the same notation for all processes that jump, because of their dependence on the state of the economy.

Before stating our result, we rewrite the normalized Kreps-Porteus aggregator in the following more compact, but less familiar form:

\[
f (c, v) = \beta \left(h^{-1} (v)\right)^{1-\gamma} u \left(c/h^{-1} (v)\right),
\]  

(A7)

where

\[
u (x) = \frac{x^{1-\phi} - 1}{1 - \psi}, \psi > 0,
\]  

(A8)

\[
h (x) = \left\{ \begin{array}{ll}
\frac{1-\gamma}{\ln x}, & \gamma \geq 0, \gamma \neq 1, \\
\frac{1-\gamma}{\ln x}, & \gamma = 1.
\end{array} \right.
\]  

(A9)
Theorem 1 When \( \psi \neq 1 \), the state-price density of a representative agent with the continuous-time version of Epstein-Zin-Weil preferences is given by

\[
\pi_t = \begin{cases} 
(\beta e^{-\beta t})^{1+\frac{\gamma-1}{\psi}} C_t^{-\gamma} \left( \frac{\gamma - \frac{\beta}{1+\gamma}}{\psi} \right), & \psi \neq 1, \\
\beta e^{\beta \int_0^t (V_s - (\gamma - 1) \ln(V_s^{-1}) ds)) C_t^{-\gamma} V_t^{\gamma - (\gamma - 1)}, & \psi = 1
\end{cases}
\] (A10)

and \( p_C \) satisfies the nonlinear ordinary differential-equation system:

\[
p_{C,t^-}^{-1} = \tau_{t^-} + \gamma \sigma_{C,t^-}^2 - g_{t^-} - \left( \mu_{z,t^-} - (\gamma - 1) \rho_{zC,t^-} \sigma_{z,t^-} \sigma_{C,t^-} \right) \frac{p_{C,t^-}}{p_{C,t^-} + \frac{1}{2} \sigma_{C,t^-}^2} \left( \frac{p_{C,t^-} + \frac{1}{2} \sigma_{C,t^-}^2}{p_{C,t^-}} - \frac{(\gamma - \frac{1}{2})}{\frac{1}{2} \gamma} \right) \left( \frac{p_{C,t^-} + \frac{1}{2} \sigma_{C,t^-}^2}{p_{C,t^-}} \right)^2
\]

\[
- \left( 1 - \frac{1}{\psi} \right) \sum_{j \neq i} \lambda_{V_{t^-},j} \left( \frac{(p_{C,t^-}/p_{C,t^-} + \frac{1}{2} \sigma_{C,t^-}^2)^{1+\frac{\gamma-1}{\psi}} - 1}{1 - \gamma} \right), \quad \nu_{t^-} \in \{1, 2\},
\] (A11)

where

\[
\tau_t = \beta + \frac{1}{\psi} \ln - \frac{1}{2} \gamma \left( 1 + \frac{1}{\psi} \right) \sigma_{C,t^-}^2,
\] (A12)

and \( V \) satisfies the nonlinear ordinary differential-equation system:

\[
\beta \ln V_{t^-} = g_{t^-} - \gamma \frac{\sigma_{C,t^-}^2}{2} + \sum_{j \neq i} \lambda_{V_{t^-},j} \left( \frac{V_j}{V_{t^-}} \right)^{1-\gamma} - 1
\]

\[
+ \left( \mu_{z,t^-} - (\gamma - 1) \rho_{zC,t^-} \sigma_{z,t^-} \sigma_{C,t^-} \right) \frac{V_{t^-} - z}{V_{t^-}} + \frac{1}{2} \sigma_{z,t^-}^2 \left[ \frac{V_{t^-} - z}{V_{t^-}} - \gamma \left( \frac{V_{t^-} - z}{V_{t^-}} \right)^2 \right],
\] (A13)

Proof of Theorem 1

Duffie and Skiadas (1994) show that the state-price density for a general normalized aggregator \( f \) is given by

\[
\pi_t = e^{\int_0^t f_e(C_t, J_t) dt} f_e(C_t, J_t).
\] (A14)

where \( f_e(\cdot, \cdot) \) and \( f_e(\cdot, \cdot) \) are the partial derivatives of \( f \) with respect to its first and second arguments, respectively, and \( J \) is the value function is given by

\[
J_t = E_t \int_t^\infty f(C_t, J_t) dt.
\] (A15)

The Feynman-Kac Theorem implies

\[
f(C_t, J_t^-)|_{\nu_{t^-} = i} dt + E_t [dJ_t|_{\nu_{t^-} = i}] = 0, \quad i \in \{1, 2\}.
\] (A16)

Using Ito's Lemma we can rewrite the above equation as

\[
f(C, J_i) + CJ_{i,C}g_t + \frac{1}{2} C^2 J_{i,CC} \sigma_{C,i}^2 + \mu_{z,i} J_{i,z} + \frac{1}{2} \sigma_{z,i}^2 J_{i,zz} + CJ_{i,z} \rho_{zC,i} \sigma_{z,i} \sigma_{C,i} + \sum_{j \neq i} \lambda_{ij} (J_j - J_i) = 0, \quad i \in \{1, 2\}.
\] (A17)
We guess and verify that the value function is given by

\[ J_t = h(C_t V_t), \tag{A18} \]

where \( V_t = V(\nu_t, z_t) \) and \( V \) satisfies the nonlinear ordinary differential-equation system

\[
\begin{align*}
\beta u \left( V_t^{\gamma - 1} \right) + g_t - \frac{1}{2} \gamma \sigma^2_{C,t} \\
+ \left( \mu_{x,t} - (\gamma - 1) \rho_{x,C,t} \sigma_{x,t} \sigma_{C,t} \right) \frac{V_{i,t}^{\gamma}}{V_t} + \frac{1}{2} \sigma^2_{x,t} \left[ \frac{V_{i,t}^{\gamma}}{V_t} - \gamma \left( \frac{V_{i,t}^{\gamma}}{V_t} \right)^2 \right] \\
+ \sum_{j \neq i} \lambda_{ij} \left( \frac{(V_j/V_t)^{1-\gamma} - 1}{1 - \gamma} \right) = 0, \ i \in \{1, 2\}. \tag{A19}
\end{align*}
\]

Substituting (A18) into (A14) and simplifying gives

\[ \pi_t = \beta e^{-\beta} J'_0 \left[ 1 + \left( \gamma - \frac{1}{\psi} \right) u(V_t^{\gamma - 1}) \right] dt C_t^{-\gamma} V_t^{\gamma - \gamma} \tag{A20} \]

From (A19) it follows that

\[
\beta \left[ 1 + \left( \gamma - \frac{1}{\psi} \right) u \left( V_t^{\gamma - 1} \right) \right] = \tau_t - \left( \gamma - \frac{1}{\psi} \right) \sum_{j \neq i} \lambda_{ij} \left( \frac{(V_j/V_t)^{1-\gamma} - 1}{1 - \gamma} \right) \\
- \left( \gamma - \frac{1}{\psi} \right) \left[ \mu_{x,t} - (\gamma - 1) \rho_{x,C,t} \sigma_{x,t} \sigma_{C,t} \right] \frac{V_{i,t}^{\gamma}}{V_t} + \frac{1}{2} \sigma^2_{x,t} \left[ \frac{V_{i,t}^{\gamma}}{V_t} - \gamma \left( \frac{V_{i,t}^{\gamma}}{V_t} \right)^2 \right] \\
- \left( \gamma g_t - \frac{1}{2} \gamma \left( 1 + \gamma \right) \sigma^2_{C,t} \right), \tag{A21}
\]

where

\[ \tau_t = \beta + \frac{1}{\psi} g_t - \frac{1}{2} \gamma \left( 1 + \frac{1}{\psi} \right) \sigma^2_{C,t}. \tag{A22} \]

Setting \( \psi = 1 \), we obtain the second expression in (A10) and (A13).

To derive the first expression in (A10) from (A20) we shall now prove that

\[ V_t = (\beta p_{C,t})^{1 - \frac{1}{\psi} \gamma}, \ \psi \neq 1. \tag{A23} \]

We start by considering the optimization problem for the representative agent:

\[ J_t = \sup_{C^*, \varphi} \mathbb{E}_t \int_t^{\infty} f \left( C^*_t, J_t \right) dt, \tag{A24} \]

\( W \) is the financial wealth of the representative agent and she chooses her optimal consumption \( C^* \) and risky asset portfolio \( \varphi \) to maximize her expected utility subject to the dynamic budget constraint, which we now describe. If she consumes at the rate, \( C^* \), invests a proportion, \( \varphi \), of her remaining financial wealth in the claim on aggregate consumption (the risky asset), and puts the remainder in the locally risk-free asset, then her financial wealth evolves according to the dynamic budget constraint:

\[ \frac{dW_t}{W_{t-}} = \varphi_{t-} \left( dR_{C,t} - r_{t-} dt \right) + r_{t-} dt - \frac{C^*_t}{W_{t-}} dt, \tag{A25} \]
We define $N_{ij,t}$ as the Poisson process which jumps upward by one whenever the state of the economy switches from $i$ to $j \neq i$. The compensated version of this process is the Poisson martingale

$$N^p_{ij,t} = N_{ij,t} - \lambda_{ij} t.$$  

(A26)

It follows from applying Ito’s Lemma to $P = pC_i$, that the cumulative return on the claim to aggregate consumption is

$$dR_{C,i,t} = \frac{dP_i + C_i dt}{P_i} = \mu_{R_{C,i},t} dt + \sigma^B_{R_{C,i},t} dB_{R_{C,i},t} + \sum_{j \neq i} \sigma^p_{R_{C,i},t-j} dN^p_{t-j,i},$$  

(A27)

where

$$\mu_{R_{C,i},t}\bigg|_{t-i} = \mu_{R_{C,i}} = \gamma_i + \frac{1}{pC_i} \frac{\partial pC_i}{\partial z_i} \mu_{z,i} + \frac{1}{2 pC_i} \frac{\partial^2 pC_i}{\partial z_i^2} \sigma^2_{z,i} + \frac{1}{pC_i} \sum_{j \neq i} (p_{C,j} - p_{C,i}) \lambda_{ij} + \frac{1}{pC_i} \frac{\partial pC_i}{\partial z_i} \rho_{z,i} \sigma_{z,i} \sigma_{C,i} dt + \frac{1}{pC_i},$$  

(A28)

$B_{R_{C,i},t}$ is the standard Brownian motion defined by

$$dB_{R_{C,i},t} = \sigma_{R_{C,i},t} dB_{C,i,t} + \frac{1}{pC_i} \frac{\partial pC_i}{\partial z_i} \sigma_{z,i} dB_{z,i,t},$$  

(A29)

and

$$\sigma^B_{R_{C,i},t}\bigg|_{t-i} = \sigma^B_{R_{C,i}} = \sqrt{\sigma^2_{C,i} + \frac{1}{2 pC_i} \frac{\partial^2 pC_i}{\partial z_i^2} \rho_{z,i}^2 \sigma^2_{z,i} \sigma^2_{C,i} + \left( \frac{1}{pC_i} \frac{\partial^2 pC_i}{\partial z_i} \sigma^2_{z,i} \right)^2},$$  

(A30)

$$\sigma^p_{R_{C,i},t-j}\bigg|_{t-i} = \sigma^p_{R_{C,i},j} = \frac{p_{C,j}}{pC_i} - 1.$$  

(A31)

The total volatility of returns to holding the consumption claim, when the current state is $i$ is given by

$$\sigma_{R_{C,i}} = \sqrt{(\sigma^B_{R_{C,i}})^2 + \sum_{j \neq i} (\sigma^p_{R_{C,i},j})^2 \lambda_{ij}},$$  

(A32)

Note that $C^*$ is the consumption to be chosen by the agent, i.e. it is a control, and at this stage we cannot rule out the possibility that it jumps with the state of the economy. In contrast, $C$ is aggregate consumption, i.e. the dividend received by an investor who holds the claim to aggregate consumption. Because aggregate consumption, $C$, is continuous, it’s left and right limits are equal, i.e. $C_t^- = C_t^+$.

The system of Hamilton-Jacobi-Bellman partial differential equations for the agent’s optimization problem is

$$\sup_{C^*,\varphi} f(C^*_{t-i},J_{t-i}) |_{\nu_{t-i}} dt + \mathbb{E}_t [dJ_t | \nu_{t-i} = i] = 0, \ i \in \{1,2\}.$$  

(A33)

There is one equation for each state of the economy. Applying Ito’s Lemma to $J_t = J_t (W_t, \nu_t, z_t)$ allows us to write (A33) as

$$0 = \sup_{C^*,\varphi} f(C^*_{t-i},J_i) + W_t J_i W \left( \varphi_i (\mu_{R_{C,i}} - r_i) + r_i - \frac{C^*}{W_t} \right) + \frac{1}{2} W_t^2 J_i WW \varphi_i^2 \sigma^2_{R_{C,i}} + \mu_{z,i} J_{i,z} + \frac{1}{2} \sigma^2_{z,i} J_{i,zz}$$  

$$+ W_t J_{i,z} W \varphi_i \rho_{z,R_{C,i}} \sigma_{z,i} \sigma_{R_{C,i}} + \sum_{j \neq i} \lambda_{ij} (J_j - J_i), \ i \in \{1,2\}.$$  

(A34)
We guess and verify that the value function is given by
\[ J_t = h(W_t F_t), \]  
where \( F_t = F(\nu_t, z_t) \) and \( F \) satisfies the nonlinear ordinary differential-equation system:

\[
0 = \sup_{C_t, \varphi_t} \beta u \left( \frac{C_t^*}{W_t F_t} \right) + \left( \varphi_t \left( \mu_{RC,i} - r_i \right) + \frac{C_t^*}{W_t} \right) - \frac{1}{2} \gamma \varphi_t^2 \sigma_{RC,i}^2 \\
+ \mu_{s,i} F_{t,i} + \frac{1}{2} \sigma_{s,i}^2 \left( \frac{F_{t,i}'}{F_t} - \gamma \left( \frac{F_{t,i}'}{F_t} \right)^2 \right) \\
+ (1 - \gamma) \frac{F_{t,i}}{F_t} \varphi_t \sigma_{z,i} B_{RC,i} + \sum_{j \neq i} \lambda_{ij} \left( \frac{(F_{t,j}/F_t)^{1-\gamma} - 1}{1-\gamma} \right), i \in \{1, 2\}. \tag{A36}
\]

From the above equations, we obtain the first order conditions:

\[
\beta u' \left( \frac{C_t^*}{W_t F_t} \right) - F_i = 0, i \in \{1, 2\}, \tag{A37}
\]

\[
\left( \mu_{RC,i} - r_i \right) - \gamma \varphi_i \sigma_{RC,i}^2 + (1 - \gamma) \frac{F_{t,i}}{F_t} \rho_{zRC,i} \sigma_{z,i} B_{RC,i} = 0, i \in \{1, 2\}. \tag{A38}
\]

Hence, we obtain the optimal consumption and portfolio policies:

\[
C_t^* = \beta^{-\psi} F_t^{-(\psi-1)} W_t, i \in \{1, 2\}, \tag{A39}
\]

\[
\varphi_i = \frac{1}{\gamma} \frac{\mu_{RC,i} - r_i}{\sigma_{RC,i}^2} + \left( \frac{1}{\gamma} - 1 \right) \frac{F_{t,i}}{F_t} \rho_{zRC,i} \sigma_{z,i} B_{RC,i}, i \in \{1, 2\}. \tag{A40}
\]

The market for the consumption good must clear, so \( \varphi_1 = 1, W_1 = P_t \) and \( C_t^* = C \). Note that this forces the optimal portfolio proportion to be one and the optimal consumption policy to be continuous. Hence

\[
\mu_{RC,i} - r_i = \gamma \sigma_{RC,i}^2 - (1 - \gamma) \frac{F_{t,i}}{F_t} \rho_{zRC,i} \sigma_{z,i} B_{RC,i}, \tag{A41}
\]

and

\[
p_{C,i} = \beta^{-\psi} F_t^{1-\psi} = \frac{V_i}{F_t}. \tag{A42}
\]

The above equation implies that for \( \psi = 1 \),

\[
p_{C,i} = 1/\beta. \tag{A43}
\]

For \( \psi \neq 1 \),

\[
V_i = (\beta p_{C,i})^{1-\psi}. \tag{A44}
\]

Substituting the above equation into (A20) and (A21) gives the first expression in (A10) and (A11), respectively, after some algebra.
B Appendix: Proofs

Proof of Proposition 1

We start by proving that intertemporal marginal rate of substitution between dates \( t \) and \( t+s \), \( \pi_{t+s}/\pi_t \), is given by

\[
\frac{\pi_{t+s}}{\pi_t} = e^{-\int_t^{t+s} r_u \, du} \frac{M_{t+s}}{M_t},
\]

where

\[
M_t = M^B_t M^P_t,
\]

and

\[
M^B_t = e^{-\int_t^0 \Theta B_{i,t} \tau_u \, du - \int_t^0 \Theta B_{i,t} dB_{C,u}},
\]

\[
M^P_t = e^{-\int_t^0 \sum_{j \neq i} \lambda_{i,t} \omega_{i,j} (\omega_{i,j} - 1) \, du} \omega_{i,t},
\]

where

\[
\Theta^B_{i,t} \big|_{\nu_t = i} = \Theta^B_{i,t} = \gamma \sigma_{C,i}.
\]

The quantity \( \omega_{i,j} \) is equal to one when \( j = i \), and for \( j \neq i \),

\[
\omega_{21} = \omega_{12} = \omega,
\]

where \( \omega \) is the solution of Equation (11). The risk-free rate in state \( i \), \( r_i \big|_{\nu_t} = r_i \) is given by

\[
\begin{align*}
\frac{\pi}{\pi_t} & \big|_{\nu_t = i, \nu_t = j} = \frac{\pi_{i,j}}{\pi_{i,t}}, \\
\end{align*}
\]

First, we note that if

\[
\omega_{i,j} = \frac{\pi_{i,j}}{\pi_{i,t}} \big|_{\nu_t = i, \nu_t = j},
\]

then Equation (A10) implies that

\[
\omega_{i,j} = \begin{cases} 
\left( \frac{p_{C,i}}{p_{C,1}} \right)^{-\frac{\gamma-1}{\gamma}}, & i = 1, j \neq i, \\
\left( \frac{p_{C,2}}{p_{C,1}} \right)^{-\frac{\gamma-1}{\gamma}}, & i = 2, j \neq i, \\
(\gamma-1)^{1-\psi}, & i = 1, j \neq i.
\end{cases}
\]

The above equation implies that \( \omega_{21} = \omega_{12}^{-1} \), so we can set \( \omega_{21} = \omega_{12}^{-1} = \omega \), where \( \omega \) is to be determined.

We set \( \mu = \sigma = 0 \), in Equations (A11) and (A13) and use (B53) to obtain

\[
\frac{p_{C,i}}{p_{C,1}} = \frac{1}{\frac{p_{C,1}}{p_{C,2}}}, \quad i \in \{1, 2\}, j \neq i,
\]

38
\[ \beta \ln V_i = g_i - \frac{1}{2} \sigma_{C,i}^2 + \sum_{j \neq i} \lambda_{ij} \frac{\omega_{ij} - 1}{1 - \gamma}, i \in \{1, 2\}, j \neq i. \]  
(B55)

Therefore, from (B53) and the above two equations it follows that \( \omega \) is the solution of Equation (11).

Ito’s Lemma implies that the state-price density satisfies the following stochastic differential equation

\[
\begin{align*}
\frac{d\pi_t}{\pi_t} &= -\frac{1}{\pi_t} \frac{\partial \pi_t}{\partial t} + \frac{1}{\pi_t} \frac{\partial \pi_t}{\partial C_t} dC_t \frac{\partial^2 \pi_t}{\partial C_t^2} \left( \frac{dC_t}{C_t} \right)^2 + \sum_{j \neq i} \lambda_{ij} \frac{\Delta \pi_t}{\pi_t} dt + \sum_{j \neq i} \frac{\Delta \pi_t}{\pi_t} dN^P_{ij,t}. \\
&= \left( \kappa_i + \gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 + \sum_{j \neq i} \lambda_{ij} (1 - \omega_{ij}) \right) dt - \gamma \sigma_{C,i} dB_{C,t} \\
&\quad + \sum_{j \neq i} (\omega_{ij} - 1) dN^P_{ij,t}. \\
\end{align*}
\]  
(B56)

where

\[ \Delta \pi_t = \pi_t - \pi_{t-}. \]  
(B57)

We know that

\[ \frac{\Delta \pi_t}{\pi_t} \bigg|_{\nu_{t-} = i, \nu_t = j} = \omega_{ij} - 1. \]  
(B58)

Together with some standard algebra that allows us to rewrite (B56) as

\[
\begin{align*}
\frac{d\pi_t}{\pi_t} \bigg|_{\nu_{t-} = i, \nu_t = i} &= -\left( \kappa_i + \gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 + \sum_{j \neq i} \lambda_{ij} (1 - \omega_{ij}) \right) dt - \gamma \sigma_{C,i} dB_{C,t} \\
&\quad + \sum_{j \neq i} (\omega_{ij} - 1) dN^P_{ij,t}. \\
\end{align*}
\]  
(B59)

The state-price density satisfies the stochastic differential equation

\[
\begin{align*}
\frac{d\pi_t}{\pi_t} \bigg|_{\nu_{t-} = i, \nu_t = i} &= -r_i dt - \Theta^B_i dB_{C,t} + \sum_{j \neq i} \Theta^P_{ij} dN^P_{ij,t}, \\
\end{align*}
\]  
(B60)

where \( r_i \) is the locally risk-free rate when the economy is in state \( i \), \( \Theta^B_i \) is the market price of risk from Brownian shocks, when the economy is in state \( i \), \( \Theta^P_{ij} \) is the market price of risk from Poisson shocks, when the economy switches from state \( i \) to \( j \neq i \). Therefore, we obtain

\[ r_i = \kappa_i + \gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 + \sum_{j \neq i} \lambda_{ij} (1 - \omega_{ij}), \]  
(B61)

where

\[
\kappa_i = \left\{ \begin{array}{ll}
\beta \left[ 1 + \left( \frac{\gamma}{1 - \psi} \right) \left( \frac{\psi_{C,i}}{1 - \frac{1}{\psi}} \right)^{-1} \right], & \psi \neq 1 \\
\beta \left[ 1 + (\gamma - 1) \ln \left( V_i^{-1} \right) \right], & \psi = 1.
\end{array} \right.
\]  
(B62)

and

\[
\begin{align*}
\Theta^B_i &= \gamma \sigma_{C,i}, \\
\Theta^P_{ij} &= \omega_{ij} - 1. \\
\end{align*}
\]  
(B63, B64)
We use Equations (B54) and (B55) to eliminate $p_{C,i}$ and $V_i$ from (B62) to obtain

$$\kappa_i = \left\{ \begin{array}{ll}
\tau_t - \left( \gamma - \frac{1}{\psi} \right) \sum_{j \neq i} \lambda_{ij} \left( \frac{\psi}{1 - \gamma} \right) - \left[ \gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 \right], & \psi \neq 1, \\
\tau_t + \sum_{j \neq i} \lambda_{ij} (\omega_{ij} - 1) - \left[ \gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 \right], & \psi = 1
\end{array} \right. ,$$

(B65)

so

$$\pi_t = \left\{ \begin{array}{ll}
\pi_t - \left( \gamma - \frac{1}{\psi} \right) \sum_{j \neq i} \lambda_{ij} \left( \frac{\psi}{1 - \gamma} \right) - \sum_{j \neq i} \lambda_{ij} (1 - \omega_{ij}), & \psi \neq 1, \\
\tau_t, & \psi = 1
\end{array} \right. ,$$

(B66)

Now,

$$\pi_t = \left\{ \begin{array}{ll}
-\beta e^{-\beta \int_0^t 1 + \left( \gamma - \frac{1}{\psi} \right) \left( \frac{\psi - 1}{1 - \gamma} \right) ds} C_t^{\gamma - \frac{1}{\psi} \lambda - \gamma - 1}, & \psi \neq 1, \\
\beta e^{-\beta \int_0^t \left[ 1 + (\gamma - 1) \ln \left( V^{-1} \right) \right] ds} C_t^{\gamma V^{-1} - (\gamma - 1)}, & \psi = 1.
\end{array} \right. ,$$

(B67)

From (B65) and (B66), we obtain

$$\kappa_i = \pi_t + \sum_{j \neq i} \lambda_{ij} (\omega_{ij} - 1) - \left[ \gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 \right],$$

(B68)

for all $\psi > 0$. Therefore, from (A10), we obtain

$$\frac{\pi_{t+s}}{\pi_t} = e^{-\int_{t}^{t+s} \mu_u du \frac{M^B_t}{M^F_t}} e^{-\int_{t}^{t+s} \sum_{j \neq i} \lambda_{ij} (\omega_{ij} - 1) du \omega_u \nu_{t+s}},$$

(B69)

where $M^B$ is defined in (B47). Because the Poisson and Brownian shocks are independent, $M$ is a martingale under the actual measure $P$, and satisfies

$$\frac{dM^B_t}{M^B_t} \bigg|_{\nu_{-i}} = -\Theta^B dB_t + \sum_{j \neq i} \Theta^B_{ij} dN^B_{ij,t}.$$

(B70)

The martingale $M$ defines the Radon-Nikodym derivative $\frac{dQ}{dP}$ via

$$M_t = E_t \left[ \frac{dQ}{dP} \right].$$

(B71)

If $A$ is an event which is observable at date $t + s$, conditional on information available at date $t$, the risk-neutral probability of $A$ occurring is

$$Q(A | \mathcal{F}_t) = E_t \left[ 1_A \frac{M_{t+s}}{M_t} \right].$$

(B72)

The risk-neutral probability per unit time of the state of the economy switching from $i$ to $j \neq i$ is given by

$$\hat{\lambda}_{ij} = \lim_{\Delta t \to 0} \frac{1}{2\Delta t} Q(\nu_{t+\Delta t} = j | \nu_{t} = i) = \lim_{\Delta t \to 0} \frac{1}{2\Delta t} E_t - \Delta t \left[ 1_{(\nu_{t+\Delta t} = j)} \frac{M_{t+\Delta t}}{M_{t-\Delta t}} | \nu_{t} = i \right],$$

(B73)
Now,
\[
\frac{M_{t+\Delta t}}{M_{t-\Delta t}} \big| \nu_t-\Delta t=1, \nu_{t+\Delta t}=2 \big) = \left\{ \begin{array}{ll}
\frac{M_{t+\Delta t}}{M_{t-\Delta t}} e^{-\left(\omega^{-1}-1\right)\lambda_{12}2\Delta t}, & i = 1, j = 2 \\
\frac{M_{t+\Delta t}}{M_{t-\Delta t}} e^{-\left(\omega^{-1}-1\right)\lambda_{21}2\Delta t}, & i = 2, j = 1 
\end{array} \right.,
\]
where
\[
\frac{M_{t+\Delta t}}{M_{t-\Delta t}} = e^{-\frac{1}{2}(\theta^T)^2 x - \theta^T (B_{t+s} - B_t)}.
\]

Hence,
\[
E_{t-\Delta t} \left[ 1_{(\nu_t+\Delta t=2) \frac{M_{t+\Delta t}}{M_{t-\Delta t}} \nu_t-\Delta t = 1} \right] = E_{t-\Delta t} \left[ 2|\nu_t-\Delta t = 1 \right] e^{-\frac{1}{2}(\theta^T)^2 2\Delta t - \theta^T (B_{t+s} + B_t) - \theta^T (B_{t+s} - B_t)} e^{-\left(\omega^{-1}-1\right)\lambda_{12}2\Delta t} \omega^{-1} \right]
\]
\[
= \omega^{-1} P (\nu_t+\Delta t = 2 \| \nu_t-\Delta t = 1) E_{t-\Delta t} \left[ e^{-\frac{1}{2}(\theta^T)^2 2\Delta t - \theta^T (B_{t+s} + B_t) - \theta^T (B_{t+s} - B_t)} \right] E_{t-\Delta t} \left[ e^{-\left(\omega^{-1}-1\right)\lambda_{12}2\Delta t} \right],
\]
where the last step is valid, because the increments in the Brownian motion are independent of the increments in the Poisson process. Therefore,
\[
E_{t-\Delta t} \left[ 1_{(\nu_t+\Delta t=2) \frac{M_{t+\Delta t}}{M_{t-\Delta t}} \nu_t-\Delta t = 1} \right] = \omega^{-1} (\lambda_{12}2\Delta t + o (\Delta t)) E_{t-\Delta t} \left[ e^{-\left(\omega^{-1}-1\right)\lambda_{12}2\Delta t} \right],
\]
and we obtain
\[
\hat{\lambda}_{12} = \lim_{\Delta t \to 0} \frac{1}{2\Delta t} \left( \nu_{t+\Delta t} = 1 \| \nu_{t-\Delta t} = 2 \right) = \lambda_{12} \omega^{-1},
\]
which gives Equation (8). Similarly, we can prove Equation (9) holds.

We deduce the properties of the risk distortion factor, \( \omega \), from the properties of the function \( g \) defined in (12). We restrict the domain of \( g \) to \( x > 0 \). First we consider the case where \( \psi \neq 1 \).

We assume that the price-consumption ratios, \( p_{C,i}, i \in \{1, 2\} \) are strictly positive. Therefore, \( g \) is continuous. We observe that if \( g \) is monotonic, then by continuity, \( g(1) \) and \( g'(1) \) are of the same sign iff \( \omega < 1 \) and \( g(1) \) and \( g'(1) \) are of different signs iff \( \omega > 1 \). Clearly, in both cases, \( \omega \) is unique. To establish monotonicity note that
\[
g'(x) = \frac{1 - \frac{1}{\gamma} - \frac{x}{\gamma}}{\frac{x}{\gamma}} \left( \frac{\tau_1 + \gamma \sigma^2_{C,1} - g_1 + \lambda_{12} \frac{1}{\gamma} x - \frac{\gamma - 1}{\gamma}}{\frac{x}{\gamma}} - 1 \right) \left( \lambda_{12} x \frac{1}{\gamma - 1} \right) \left( x - \frac{\gamma - 1}{\gamma} \right)
\]
\[
+ \left( \frac{\tau_2 + \gamma \sigma^2_{C,2} - g_2 + \lambda_{21} \frac{1}{\gamma} x - \frac{\gamma - 1}{\gamma}}{\frac{x}{\gamma}} - 1 \right) \left( \lambda_{21} x \frac{1}{\gamma - 1} \right) \left( x - \frac{\gamma - 1}{\gamma} \right)
\]
\[
+ \left( \frac{\tau_1 + \gamma \sigma^2_{C,1} - g_1 + \lambda_{12} \frac{1}{\gamma} x - \frac{\gamma - 1}{\gamma}}{\frac{x}{\gamma}} - 1 \right) \left( \lambda_{12} x \frac{1}{\gamma - 1} \right) \left( x - \frac{\gamma - 1}{\gamma} \right)
\]
\[
\]
and
\[ g'(1) = -\frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} \left[ 1 + \frac{(\tau_1 + \gamma \sigma^2_{C,1} - g_1)\lambda_1 + (\tau_2 + \gamma \sigma^2_{C,2} - g_2)\lambda_2}{(\tau_1 + \gamma \sigma^2_{C,1} - g_1)^2} \right], \] (B83)
to relate the signs of \( g(1) \) and \( g'(1) \) to the properties of the agent’s preferences. Note that \( g'(1) < 0 \), \( g'(1) > 0 \) iff \( \frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} > 0 \), \( \frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} < 0 \). We assume that \( \tau_i + \gamma \sigma^2_{C,i} - g_i > 0 \) for \( i \in \{1, 2\} \), which is equivalent to assuming that if the economy were always in state \( i \), then the price-consumption ratio would be positive. Simple algebra tells us that \( \tau_i + \gamma \sigma^2_{C,i} - g_i = \beta + \left( \frac{1}{\psi} - 1 \right) \left( g_i - \frac{1}{\gamma} \sigma^2_{C,i} \right) \). We know that \( g_1 - \frac{1}{\gamma} \sigma^2_{C,1} < g_2 - \frac{1}{\gamma} \sigma^2_{C,2} \). Therefore \( g(1) < 0 \), \( g(1) > 0 \) iff \( \psi > 1 \), \( \psi < 1 \). Consequently, \( g(1) \) and \( g'(1) \) are of the same sign iff \( \gamma < 1/\psi \) and \( g(1) \) and \( g'(1) \) are of different signs iff \( \gamma > 1/\psi \). It then follows that \( \omega > 1 \) iff \( \gamma > 1/\psi \) and \( \omega < 1 \) iff \( \gamma < 1/\psi \), assuming that \( \psi \neq 1 \).

Similarly, when \( \psi = 1 \), if we assume that \( V_i > 0 \) for \( i \in \{1, 2\} \), then we can prove that: \( \omega > 1 \) if \( \gamma > 1 \) and \( g_i - \frac{1}{2} \gamma \sigma^2_{C,i}, i \in \{1, 2\} \) are of the sign and \( \omega > 1 \) if \( \gamma < 1 \) and \( g_i - \frac{1}{2} \gamma \sigma^2_{C,i}, i \in \{1, 2\} \) are of opposite sign. Now, if \( \gamma < 1 \), then \( \tau_i + \gamma \sigma^2_{C,i} - g_i > 0 \) implies \( g_i - \frac{1}{2} \gamma \sigma^2_{C,i} > 0 \), which means \( g_i - \frac{1}{2} \gamma \sigma^2_{C,i}, i \in \{1, 2\} \) cannot be of opposite sign. Therefore, \( \omega > 1 \) iff \( \gamma > 1 \).

So, for \( \psi > 0 \), \( \omega > 1 \) iff \( \gamma > 1/\psi \) and \( \omega < 1 \) iff \( \gamma < 1/\psi \). It follows that \( \omega = 1 \) iff \( \gamma = 1/\psi \).

**Proof of Proposition 2**

Proposition 2 follows immediately from Proposition 1.

**Derivations of Equations (43), (44) and (45)**

The actual probability of default occurring within the time interval \([t, T]\), with current earnings being equal to \( X_t \) is given by
\[ p_{D,T-t}(X_t) = \mathbb{P} \left( \inf_{s \in [t, T]} X_s \leq X_D \right). \] (B84)
It is a standard result that
\[ p_{D,T-t}(X_t) = N(-d_1(\theta)) + \left( \frac{X_D}{X_t} \right)^{\theta - \frac{1}{2} \sigma^2_X} \frac{\sigma^2_X}{\sigma^2_X} \mathbb{P}(-d_2(\theta)), \] (B85)
where
\[ \sigma_X = \sqrt{(\sigma^2_X)^2 + (\sigma^2_X)^2}, \] (B86)
and
\[ d_1(\theta) = \frac{\ln \left( \frac{X_D}{X_t} \right) + \left( \theta - \frac{1}{2} \sigma^2_X \right) (T-t)}{\sigma_X \sqrt{T-t}}, \] (B87)
\[ d_2(\theta) = \frac{\ln \left( \frac{X_D}{X_t} \right) - \left( \theta - \frac{1}{2} \sigma^2_X \right) (T-t)}{\sigma_X \sqrt{T-t}}. \] (B88)
The risk-neutral probability of default occurring within the time interval \([t, T]\), with current earnings being equal to \(X_t\) is given by
\[
\hat{p}_{D,T-t}(X_t) = \mathcal{Q}\left(\inf_{s \in [t,T]} X_s \leq X_D\right).
\] (B89)

Therefore,
\[
\hat{p}_{D,T-t}(X_t) = N\left(-d_1\left(\hat{\theta}\right)\right) + \left(\frac{X_D}{X_t}\right)^{\frac{\hat{\theta} - \frac{1}{2}\sigma^2_t}{\frac{\sigma^2_t}{2}}} N\left(-d_2\left(\hat{\theta}\right)\right),
\] (B90)

Taking the limits of (B85) and (B90) as \(T \to \infty\) gives the actual and risk-neutral default probabilities
\[
p_D(X_t) = \left(\frac{X_D}{X_t}\right)^{\frac{\theta - \frac{1}{2}\sigma^2_t}{\frac{\sigma^2_t}{2}}},
\] (B91)
and
\[
\hat{p}_D(X_t) = \left(\frac{X_D}{X_t}\right)^{\frac{\hat{\theta} - \frac{1}{2}\sigma^2_t}{\frac{\sigma^2_t}{2}}},
\] (B92)
respectively. The ratio of the risk-neutral to actual default probability, \(\frac{\hat{p}_D}{p_D}\), is the risk-adjustment factor, \(R(X_t)\), i.e. Equation (44). The time-adjustment factor is given by the ratio \(q_D/\hat{q}_D\), i.e. Equation (45).

**Proof of Proposition 3**

The principle of no-arbitrage implies that the Arrow-Debreu default claims satisfy the following ordinary differential-equation system:
\[
\left(\frac{1}{2} \left[ \begin{array}{cc} \sigma^2_t \sigma^2_t \sigma^2_t \sigma^2_t X^2 \sigma^2_t d^2 \sigma^2_t dX^2 \end{array} \right] + \left[ \begin{array}{cc} \hat{\theta}_1 & 0 \\ 0 & \hat{\theta}_2 \end{array} \right] \right) \frac{d}{dX} \left[ \begin{array}{c} r_1 \\ r_2 \end{array} \right] + \left[ \begin{array}{cc} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{array} \right] \left[ \begin{array}{cc} q_D,11 & q_D,12 \\ q_D,21 & q_D,22 \end{array} \right] = \left[ \begin{array}{c} 0 \\ 0 \end{array} \right].
\] (B93)

To solve this system of ordinary differential equations, we need boundary conditions. We know that \(B_i(X_t) = \alpha_i A_i(X_t)\) for \(X \leq X_i\). When the current state is \(i\) and earnings is at or below the default boundary for that state, \(X_{D,i}\), we know that the risk-neutral probability of defaulting in state \(i\) is 1 and the risk-neutral probability of defaulting in any other state is zero. Hence,
\[
q_{D,ij}(X) = \left\{ \begin{array}{cl} 1, & j = i, X \leq X_{D,i} \\ 0, & j \neq i, X \leq X_{D,i} \end{array} \right..
\] (B94)

Therefore, in particular
\[
q_{D,ij}\vert_{X = X_i} = \left\{ \begin{array}{cl} 1, & j = i \\ 0, & j \neq i \end{array} \right..
\] (B95)

We also need values and derivatives to be continuous at \(X = X_{D,1}\):
\[
\lim_{X \downarrow X_1} q_{D,21} = \lim_{X \uparrow X_1} q_{D,21},
\] (B96)
\[
\lim_{X \downarrow X_1} q'_{D,21} = \lim_{X \uparrow X_1} q'_{D,21}.
\] (B97)
Hence,

\[
\lim_{X \downarrow X_1} q_{D,22} = \lim_{X \downarrow X_1} q_{D,22}.
\]

Therefore,

\[
\lim_{X \downarrow X_1} q_{D,22} = \lim_{X \downarrow X_1} q_{D,22}'.
\]

We shall first solve (B93) subject to the boundary conditions above for the region \( X > X_{D,1} \). We seek solutions of the form

\[ q_{D,ij} = h_{ij} X^k, \quad i, j \in \{ 1, 2 \}. \]

Hence,

\[
\left( \frac{1}{2} \begin{bmatrix} \sigma_{X,1}^2 & 0 \\ 0 & \sigma_{X,2}^2 \end{bmatrix} k (k - 1) + \begin{bmatrix} \tilde{\theta}_1 & 0 \\ 0 & \tilde{\theta}_2 \end{bmatrix} \right) k + \begin{bmatrix} -\lambda_{12} - r_1 & \tilde{\lambda}_{12} \\ \lambda_{21} & -\lambda_{21} - r_2 \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{11} & h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.
\]

(A solution of the above equation exists if)

\[
\det \left( \frac{1}{2} \begin{bmatrix} \sigma_{X,1}^2 & 0 \\ 0 & \sigma_{X,2}^2 \end{bmatrix} k (k - 1) + \begin{bmatrix} \tilde{\theta}_1 & 0 \\ 0 & \tilde{\theta}_2 \end{bmatrix} \right) k + \begin{bmatrix} -\lambda_{12} - r_1 & \tilde{\lambda}_{12} \\ \lambda_{21} & -\lambda_{21} - r_2 \end{bmatrix} = 0.
\]

Therefore, \( k \) is a root of the quartic polynomial

\[
\left[ \frac{1}{2} \sigma_{X,1}^2 k (k - 1) + \tilde{\theta}_1 k + (-\lambda_{12} - r_1) \right] \left[ \frac{1}{2} \sigma_{X,2}^2 k (k - 1) + \tilde{\theta}_2 k + (-\lambda_{21} - r_2) \right] - \tilde{\lambda}_{12} \tilde{\lambda}_{12} = 0,
\]

which is the characteristic function for (B93). The above quartic has 4 distinct real roots, two of which are positive, provided that \( \sigma_{X,i}, r_i, \tilde{\lambda}_{ij} > 0 \) for \( i \in \{ 1, 2 \} \) and \( j \neq i \) (see Guo (2001)). Therefore the general solution of is

\[ q_{D,ij} = \sum_{m=1}^{4} h_{ij,m} X^{k_m}, \]

where \( k_m \) is the \( m \)th root (ranked in order of increasing size, accounting for sign) of (B102). To ensure that \( q_{D,ij}, i, j \in \{ 1, 2 \} \) are finite as \( X \rightarrow \infty \), we set \( h_{ij,3} = h_{ij,4} = 0, i, j \in \{ 1, 2 \} \), so we use only the two negative roots: \( k_1 < k_2 < 0 \). From equation (B100), it follows that

\[
\frac{h_{21,m}}{h_{11,m}} = \frac{h_{22,m}}{h_{12,m}} = \epsilon(k_m), \quad m \in \{ 1, 2 \},
\]

where

\[
\epsilon(k) = -\frac{\tilde{\lambda}_{12}}{\frac{1}{2} \sigma_{X,2}^2 k (k - 1) + \tilde{\theta}_2 k + (-\lambda_{21} - r_2)} = -\frac{1}{2} \sigma_{X,1}^2 k (k - 1) + \tilde{\theta}_1 k + (-\lambda_{12} - r_1). \]

Therefore

\[ q_{D,1j} = \sum_{m=1}^{2} h_{1j,m} X^{k_m}, \quad j \in \{ 1, 2 \}, \]

\[ q_{D,2j} = \sum_{m=1}^{2} h_{1j,m} \epsilon(k_m) X^{k_m}, \quad j \in \{ 1, 2 \}. \]
We now solve (B93) subject to the relevant boundary conditions for the region $X_2 < X \leq X_1$. We know

\[
q_{D,11} = 1, \quad q_{D,12} = 0. \tag{B107}
\]

Therefore

\[
\frac{\partial q_{D,21,t}}{\partial X_t} \hat{\theta}_2 X_t + \frac{1}{2} \frac{\partial^2 q_{D,21,t}}{\partial X_t^2} \sigma_{2,X}^2 X_t^2 + \tilde{\lambda}_{21} (1 - q_{D,21,t}) = r_2, \tag{B109}
\]

\[
\frac{\partial q_{D,22,t}}{\partial X_t} \hat{\theta}_2 X_t + \frac{1}{2} \frac{\partial^2 q_{D,22,t}}{\partial X_t^2} \sigma_{2,X}^2 X_t^2 - \tilde{\lambda}_{21} q_{D,22,t} = r_2 q_{D,22,t}, \tag{B110}
\]

which can written in matrix form as

\[
\left( \frac{1}{2} \sigma_{X,2}^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{d^2}{dX^2} + \hat{\theta}_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{d}{dX} - \left( \tilde{\lambda}_{21} + r_2 \right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} q_{D,21,t} \\ q_{D,22,t} \end{bmatrix} + \begin{bmatrix} \tilde{\lambda}_{21} \\ 0 \end{bmatrix} = 0 \tag{B111}
\]

First we find the complementary function, i.e. the solution of

\[
\left( \frac{1}{2} \sigma_{X,2}^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{d^2}{dX^2} + \hat{\theta}_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{d}{dX} - \left( \tilde{\lambda}_{21} + r_2 \right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} q_{D,21,t} \\ q_{D,22,t} \end{bmatrix} = 0 \tag{B112}
\]

Set

\[
q_{D,21} = s_1 X^k, \quad q_{D,22} = s_2 X^k. \tag{B113}
\]

Therefore

\[
\left( \frac{1}{2} \sigma_{X,2}^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} k (k - 1) + \hat{\theta}_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} k - \left( \tilde{\lambda}_{21} + r_2 \right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = 0, \tag{B115}
\]

the characteristic equation of which is the quadratic

\[
\frac{1}{2} \sigma_{X,2}^2 k (k - 1) + \hat{\theta}_2 k - \left( \tilde{\lambda}_{21} + r_2 \right) = 0, \tag{B116}
\]

so we get

\[
q_{D,21} = s_{1,1} X^{j_1} + s_{1,2} X^{j_2}, \quad q_{D,22} = s_{2,1} X^{j_1} + s_{2,2} X^{j_2}. \tag{B117}
\]

For $q_{D,21}$, we need a particular solution, given by

\[
\tilde{\lambda}_{21} (1 - q_{D,21}) = r_2 q_{D,21}, \tag{B119}
\]

\[
q_{D,21} = \frac{\tilde{\lambda}_{21}}{r_2 + \lambda_{21}}. \tag{B120}
\]
Then, the general solution is

\[ q_{D,21} = \frac{\tilde{\lambda}_{21}}{r_2 + \lambda_{21}} + s_{1,1}X^{11} + s_{1,2}X^{12}, \quad (B121) \]
\[ q_{D,22} = s_{2,1}X^{21} + s_{2,2}X^{22}. \quad (B122) \]

In summary

\[ q_{D,11} = \begin{dcases} 
\sum_{m=1}^{2} h_{11,m} X^{km}, & X > X_{D,1}, \\
1 & X \leq X_{D,1}.
\end{dcases} \quad \text{and} \quad q_{D,12} = \begin{dcases} 
\sum_{m=1}^{2} h_{12,m} X^{km}, & X > X_{D,1}, \\
0 & X \leq X_{D,1}.
\end{dcases} \]
\[ q_{D,21} = \begin{dcases} 
\sum_{m=1}^{2} h_{21,m} X^{km}, & X > X_{D,1}, \\
\sum_{m=1}^{2} s_{1,m} X^{jm}, & X \leq X_{D,1}.
\end{dcases} \quad \text{and} \quad q_{D,22} = \begin{dcases} 
\sum_{m=1}^{2} h_{22,m} X^{km}, & X > X_{D,1}, \\
\sum_{m=1}^{2} s_{2,m} X^{jm}, & X \leq X_{D,1}.
\end{dcases} \quad (B123) \]

To find the 8 constants: \( h_{11,1}, h_{11,2}, h_{12,1}, h_{12,2}, s_{1,1}, s_{1,2}, s_{2,1}, s_{2,2} \), we use the following 8 boundary conditions:

\[ q_{D,11} \big|_{X=X_{D,1}} = 1, \quad q_{D,12} \big|_{X=X_{D,1}} = 0, \quad (B124) \]
\[ \lim_{X \downarrow X_{D,1}} q_{D,21} = \lim_{X \downarrow X_{D,1}} q_{D,21}, \quad \lim_{X \downarrow X_{D,1}} q_{D,22} = \lim_{X \downarrow X_{D,1}} q_{D,22}. \quad (B125) \]
\[ \lim_{X \uparrow X_{D,1}} q_{D,21} = \lim_{X \uparrow X_{D,1}} q_{D,21}, \quad \lim_{X \uparrow X_{D,1}} q_{D,22} = \lim_{X \uparrow X_{D,1}} q_{D,22}. \quad (B126) \]

and

\[ q_{D,21} \big|_{X=X_{D,2}} = 0, \quad q_{D,22} \big|_{X=X_{D,2}} = 1. \quad (B127) \]

The first set being applied at \( X = X_{D,1} \) and the second set at \( X = X_{D,2} \). The 8 boundary conditions give 8 linear equations.

We obtain \( \{ \tilde{h}_{D,ij} \}_{i,j \in \{1,2\}} \) and \( \{ p_{D,ij} \}_{i,j \in \{1,2\}} \), by setting \( r_1 = r_2 = 0 \), and \( r_1 = r_2 = 0, \gamma = 1/\psi = 0 \), respectively. Then we can compute the risk- and time-adjustments via

\[ \mathcal{R}_{ij} = \frac{p_{D,ij}}{\tilde{h}_{D,ij}}, \quad (B128) \]

and

\[ T_{ij} = \frac{q_{D,ij}}{p_{D,ij}}. \quad (B129) \]

**Proof of Proposition 4**

We take the limits of (50) as \( \Delta t \to 0 \), to obtain

\[ 0 = (1 - \eta) X - (\theta_i - \theta_i) A_i + \tilde{\lambda}_{ij} (A_j - A_i) \quad \forall i \in \{1,2\}. \quad (B130) \]

which is system of 2 linear simultaneous equations in 2 unknowns, the solution of which produces \( A_1 \) and \( A_2 \). To obtain the solution, we define

\[ p_i = \frac{1}{(1 - \eta)} A_i, \quad (B131) \]

46
the before price-earnings ratio in state $i$. Therefore

$$
\left( \text{diag} (\mu_1 - \theta_1, \mu_2 - \theta_2) - \tilde{A} \right) \left( \begin{array}{c} \eta_1 \\ \eta_2 \end{array} \right) = 1_2,
$$

(B132)

where $\text{diag} (\mu_1 - \theta_1, \mu_2 - \theta_2)$ is a $2 \times 2$ diagonal matrix, with the quantities $\mu_1 - \theta_1$ and $\mu_2 - \theta_2$ along the diagonal. Solving the above equation gives (48). We now define

$$
P_i^X = p_iX,
$$

(B133)

the before tax value of the claim to the earnings stream $X$ in state $i$. Hence, from the basic asset pricing equation:

$$
E_t \left[ \frac{dP^X}{P^X} + \frac{Xdt}{P^X} \right] \nu_t = i = -E_t \left[ \frac{dM}{M} \frac{dP^X}{P^X} \right] \nu_t = i,
$$

(B134)

we obtain the risk premium:

$$
E_t \left[ \frac{dP^X + Xdt}{P^X} - rdt \nu_t = i \right] = \gamma_{C,t} \sigma_{C,t} \sigma_{X,t} dt - \sum_{j \neq i} \left( \tilde{\lambda}_{ij} - \lambda_{ij} \right) \left( \frac{p_j}{p_i} - 1 \right) dt.
$$

(B135)

The unlevered volatility of returns on equity in state $i$ is given by

$$
\sigma_{R,i} = \sqrt{\sigma_{X,i}^2 + \sum_{j=1}^{2} \left( \frac{p_j}{p_i} - 1 \right)^2 \lambda_{ij}},
$$

(B138)

where

$$
\sigma_{X,i}^2 = (\sigma_{X,i}^d)^2 + (\sigma_{X,i}^s)^2.
$$

(B139)

**Proof of Proposition 5**

The central part of our proof consists of proving that

$$
E_t \left[ \int_1^{r_{D,t}} \pi_s ds \nu_t = i \right] = \frac{1}{r_{P,t}} - \sum_{j=1}^{2} \frac{q_{D,i,j}}{r_{P,j}},
$$

(B140)

where

$$
r_{P,t} = \left( E_t \left[ \int_1^{\infty} \pi_s ds \nu_t = i \right] \right)^{-1},
$$

(B141)

and

$$
E_t \left[ \frac{\pi_{D,t} \alpha_{D,t} A_{rD}(X_{rD})}{\pi_t} \nu_t = i \right] = \sum_{j=1}^{2} \alpha_j A_j (X_{rD}) q_{D,i,j}.
$$

(B142)
Using the above result, (31) follows immediately from (52). First, we observe that

\[ q_{D,ij,t} = E_t \left[ \Pr (\nu_t = i | \nu_{r_D} = j) \frac{\pi_{r_D}}{\pi_t} | \nu_t = i, \nu_{r_D} = j \right]. \]  

(B143)

To prove (B140), we note that

\[ E_t \left[ \int_t^{\tau_D} \frac{\pi_s}{\pi_t} ds \bigg| \nu_t = i \right] = E_t \left[ \int_t^{\infty} \frac{\pi_s}{\pi_t} ds \bigg| \nu_t = i \right] - E_t \left[ \int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{r_D}} ds \bigg| \nu_t = i \right], \]  

and conditioning on the event \( \{ \nu_t = i, \nu_{r_D} = j \} \), we obtain

\[ E_t \left[ \frac{\pi_{r_D}}{\pi_t} \int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{r_D}} ds \bigg| \nu_t = i \right] \]

\[ = \sum_{j=1}^{2} E_t \left[ \Pr (\nu_{r_D} = j | \nu_t = i) \frac{\pi_{r_D}}{\pi_t} \int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{r_D}} ds \bigg| \nu_t = i, \nu_{r_D} = j \right]. \]  

(B145)

What happens from date \( \tau_D \) onwards is independent of what happened before, so

\[ E_t \left[ \Pr (\nu_{r_D} = j | \nu_t = i) \frac{\pi_{r_D}}{\pi_t} \int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{r_D}} ds \bigg| \nu_t = i, \nu_{r_D} = j \right] \]

\[ = E_t \left[ \Pr (\nu_{r_D} = j | \nu_t = i) \frac{\pi_{r_D}}{\pi_t} \nu_t = i, \nu_{r_D} = j \right] E_t \left[ \int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{r_D}} ds \bigg| \nu_{r_D} = j \right]. \]  

(B146)

Therefore

\[ E_t \left[ \int_t^{\tau_D} \frac{\pi_s}{\pi_t} ds \bigg| \nu_t = i \right] \]

\[ = E_t \left[ \int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_t} ds \bigg| \nu_t = i \right] - \sum_{j=1}^{2} E_t \left[ \Pr (\nu_{r_D} = j | \nu_t = i) \frac{\pi_{r_D}}{\pi_t} \nu_t = i, \nu_{r_D} = j \right] E_t \left[ \int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{r_D}} ds \bigg| \nu_{r_D} = j \right]. \]  

(B147)

Conditional on being in state \( i \), the value of a claim which pays out one risk-free unit of consumption in perpetuity is \( E_t \left[ \int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{r_D}} ds \bigg| \nu_t = i \right] \), so the discount rate for this perpetuity, \( r_{P,i} \), is given by (B141). Consequently,

\[ E_t \left[ \int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_t} ds \bigg| \nu_t = i \right] = \frac{1}{r_{P,i}} - \sum_{j=1}^{2} \frac{E_t \left[ \Pr (\nu_t = i | \nu_{r_D} = j) \frac{\pi_{r_D}}{\pi_t} \nu_t = i, \nu_{r_D} = j \right]}{r_{P,j}}. \]  

(B148)

Using the definition of the Arrow-Debreu default claim, \( q_{D,ij} \), given in (B143), (B140) follows. We do not have to evaluate \( r_{P,i} \) from scratch based on (B141), because we can infer its value from (48), by setting \( \theta_i = \sigma_{X,i} = \rho_{X,i} \), \( \forall i \in \{1, 2\} \) to obtain (53).

To prove (B142), we condition on the event \( \{ \nu_t = i, \nu_{r_D} = j \} \) to obtain

\[ E_t \left[ \frac{\pi_{r_D}}{\pi_t} \alpha_{r_D} A_{r_D} (X_{r_D}) \bigg| \nu_t = i \right] = \sum_{j=1}^{2} \alpha_j A_j (X_j) E_t \left[ \Pr (\nu_{r_D} = j | \nu_t = i) \frac{\pi_{r_D}}{\pi_t} \nu_t = i, \nu_{r_D} = j \right]. \]  

(B149)

From (B143) we then obtain (B142).
The corporate bond yield in state \( i \) of the economy is
\[
y_i = \frac{c}{B_i},
\] (B150)
and the yield spread in state \( i \) is
\[
s_i = y_i - r_i = \frac{c}{B_i} - r_i.
\] (B151)
Substituting (31) into the above equation and simplifying gives (54).

We now compute the volatility of shocks to the yield. The corporate bond yield in state \( i \) of the economy is
\[
y_i = \frac{c}{B_i}.
\] (B152)
Therefore, from Ito’s Lemma
\[
dy_i = \frac{\partial y_i}{\partial X} dX + \frac{1}{2} \frac{\partial^2 y_i}{\partial X^2} (dX)^2 + \sum_{j \neq i} (y_j - y_i) dN_{ij,t},
\] (B153)
which simplifies to
\[
dy_i = -y_i \frac{\partial \ln B_i}{\partial \ln X} dX + \frac{1}{2} y_i \left[ \left( \frac{\partial \ln B_i}{\partial \ln X} \right)^2 - \frac{X^2}{B_i} \frac{\partial^2 B_i}{\partial X^2} \right] \left( \frac{dX}{X} \right)^2 + \sum_{j \neq i} (y_j - y_i) (dN_{ij,t} + \lambda_{ij} dt).\] (B154)
Therefore, corporate bond yield volatility in state \( i \) is given by
\[
\text{Var}_{t} [dy_t | \nu_{t-} = i]^{1/2} = \sqrt{\left( \frac{\partial \ln B_i}{\partial \ln X} \right)^2 \left( (\sigma_X)^2 + (\sigma_{\ln B_i})^2 \right) + \sum_{j \neq i} (y_j - y_i)^2 \lambda_{ij}}.
\] (B155)
Similarly, yield spread volatility in state \( i \) is given by
\[
\text{Var}_{t} [ds_t | \nu_{t-} = i]^{1/2} = \sqrt{\left( \frac{\partial \ln B_i}{\partial \ln X} \right)^2 \left( (\sigma_X)^2 + (\sigma_{\ln B_i})^2 \right) + \sum_{j \neq i} (s_j - s_i)^2 \lambda_{ij}}.
\] (B156)
From (31), we can show that in state \( i \), the elasticity of the bond price with respect to earnings is given by
\[
\frac{\partial \ln B_i}{\partial \ln X} = -y_i \sum_{j=1}^{2} \left( \frac{\partial \ln q_{D,ij}}{\partial \ln X} \right) q_{D,ij}.\] (B157)

**Proof of Proposition 6**

The value of levered equity in state \( i \) is given by
\[
S_{i,t} = A_i (X_t) - (1 - \eta) \frac{c}{r_{P,i}} + \sum_{j=1}^{2} q_{D,ij} \left( (1 - \eta) \frac{c}{r_{P,j}} - A_j (X_{D,j}) \right).
\] (B158)
From Ito’s Lemma we obtain
\[
dR_{t} |_{\nu_{t-} = i} = \left. \frac{dS_t + (1 - \eta) (X_t - c) dt}{S_{t-}} \right|_{\nu_{t-} = i} = \mu_{R,i} dt + \sigma_{R,i} dB_{X,t} + \sigma_{R,i}^* dB_{X,t}^* + \sum_{j \neq i} \sigma_{R,i}^P dN_{ij,t},\] (B159)
where

\[
\mu_{R,i} = \frac{A_i(X) + \sum_{j=1}^{2} X q'_{D,i,j} \theta_i + \frac{1}{2} X^2 q''_{D,i,j} \sigma_{X,i}^2 \left[(1 - \eta) \frac{c}{T_{D,j}} - A_j(X_{D,j})\right]}{S_i} + \sum_{j \neq i} \left( \frac{S_j}{S_i} - 1 \right) \lambda_{ij} + \frac{A_i(X)}{S_i} .
\]  
(B160)

and \( \sigma_{B,R,i}^2, \sigma_{P,R,i}^2 \) are given in (58) and (60), respectively. Therefore,

\[-E_t \left[ \frac{dS}{S} + \frac{dM}{M} \bigg| \nu_t = i \right] = \left\{ \gamma \rho_{X_C,i} \sigma_{B,R,i}^2 \sigma_{C} - \sum_{j \neq i} \sigma_{P,R,ij} \left[ \omega_{ij} - 1 \right] \lambda_{ij} \right\} dt \]

The levered equity risk premium is given by

\[-E_t \left[ \frac{dS}{S} + (1 - \eta)(X - c) dt \bigg| \nu_t = i \right] = -E_t \left[ \frac{dS}{S} + \frac{dM}{M} \bigg| \nu_t = i \right]. \]  
(B162)

Consequently, we obtain (57). Overall levered stock return volatility in state \( i \) is given by combining the variances from Brownian and Poisson shocks to obtain (64).

**Proof that the Default Boundary is Weakly Countercyclical**

We assume the default boundary is strongly procyclical, i.e. \( X_{D,1} < X_{D,2} \). Then the 8 linear equations used to find \( h_{11,1}, h_{11,2}, h_{12,1}, h_{12,2}, s_{1,1}, s_{1,2}, s_{2,1}, s_{2,2} \) are not linearly independent (this is easy to see by writing the linear equations in matrix form and checking that the determinant is zero), implying that the Arrow-Debreu default claims are not unique. Hence, by reductio ad absurdum, the default boundary must be weakly countercyclical.
References


Ericsson, Jan, and Joel Reneby, 2003, Valuing Corporate Liabilities, Unpublished working paper.


Huang, J., and Ming Huang, 2003, How much of the corporate-treasury yield spread is due to credit risk? A new calibration approach, Unpublished working paper.


Zhang, Benjamin Yibin, Hao Zhou, and Haibin Zhu, 2005, Explaining Credit Default Swap Spreads with the Equity Volatility and Jump Risks of Individual Firms, Unpublished working paper.
Figure 1: The Price of the Fundamental Security in the Structural-Equilibrium Model without Intertemporal Macroeconomic Risk and with Power Utility

This figure shows the price of the fundamental security, $q_D$, as a function of relative risk aversion, $\gamma$, when there is no intertemporal macroeconomic risk and the representative agent has power utility. The rate of time preference is $\beta = 0.01$. The mean growth rate of firm earnings is $\theta = 0.0333$, systematic earnings growth volatility is $\sigma_X = 0.0115$, idiosyncratic earnings growth volatility is $\sigma_{id}^X = 0.1700$. The mean growth rate of consumption is $g = 0.0333$, consumption growth volatility is $\sigma_C = 0.0099$ and the correlation between consumption growth and firm earnings growth is $\rho_X C = 0.1759$. The recovery rate is $\alpha = 0.5$ and the per annum tax rate on firm earnings is $\eta = 0.15$. The default boundary is $X_D = 0.48$. 
Figure 2: The Price of the Fundamental Security in the Structural-Equilibrium Model without Intertemporal Macroeconomic Risk and with Epstein-Zin-Weil Preferences

This figure shows the price of the fundamental security, $q_D$, as a function of relative risk aversion, $\gamma$, when there is no intertemporal macroeconomic risk and the representative agent has the continuous-time version of Epstein-Zin-Weil preferences. The elasticity of intertemporal substitution takes the values $\psi = \{1, 2, 4, 6, 8, 1.0, 1.2, \frac{1}{\gamma}\}$. When $\psi = \frac{1}{\gamma}$, the Epstein-Zin-Weil utility function reduces to power utility. The rate of time preference is $\beta = 0.01$. The mean growth rate of firm earnings is $\theta = 0.0333$, systematic earnings growth volatility is $\sigma_N^2 = 0.0115$, idiosyncratic earnings growth volatility is $\sigma_{id}^2 = 0.1700$. The mean growth rate of consumption is $g = 0.0333$, consumption growth volatility is $\sigma_C = 0.0099$ and the correlation between consumption growth and firm earnings growth is $\rho_{XC} = 0.1759$. The recovery rate is $\alpha = 0.5$ and the per annum tax rate on firm earnings is $\eta = 0.15$. The default boundary is $X_D = 0.48$. 

![Graph of the Price of the Fundamental Security](image-url)
This figure shows the risk-adjustment factor, $R = \frac{\hat{P}_D}{P_D}$, as a function of relative risk aversion, $\gamma$. Idiosyncratic earnings growth volatility takes the values $\sigma_{X, id} = \{0.11, 0.17\}$. The rate of time preference is $\beta = 0.01$. The mean growth rate of firm earnings is $\theta = 0.0343$, systematic earnings growth volatility is $\sigma_{X} = 0.0115$. The mean growth rate of consumption is $g = 0.0333$, consumption growth volatility is $\sigma_{C} = 0.0099$ and the correlation between consumption growth and firm earnings growth is $\rho_{XC} = 0.1759$. The recovery rate is $\alpha = 0.5$ and the per annum tax rate on firm earnings is $\eta = 0.15$. The default boundary is $X_D = 0.48$. 

Figure 3: The Risk-Adjustment Factor, $R = \frac{\hat{P}_D}{P_D}$, in the Structural-Equilibrium Model without Intertemporal Macroeconomic Risk
This figure shows the risk-adjustment factor, $T = \frac{q_D}{P_D}$, as a function of relative risk aversion, $\gamma$, when there is no intertemporal macroeconomic risk and the representative agent has the continuous-time version of Epstein-Zin-Weil preferences. The elasticity of intertemporal substitution takes the values $\psi = \{0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, \frac{1}{\gamma}\}$. When $\psi = \frac{1}{\gamma}$, the Epstein-Zin-Weil utility function reduces to power utility. The rate of time preference is $\beta = 0.01$. The mean growth rate of firm earnings is $\theta = 0.0333$, systematic earnings growth volatility is $\sigma_{\Delta Y} = 0.0115$, idiosyncratic earnings growth volatility is $\sigma_{\Delta Y}^d = 0.1700$. The mean growth rate of consumption is $g = 0.0333$, consumption growth volatility is $\sigma_C = 0.0099$ and the correlation between consumption growth and firm earnings growth is $\rho_XC = 0.1759$. The recovery rate is $\alpha = 0.5$ and the per annum tax rate on firm earnings is $\eta = 0.15$. The default boundary is $X_D = 0.48$. 
This figure shows the annualized risk-free rate, $\bar{r}$, in per cent as a function of relative risk aversion, $\gamma$. The elasticity of intertemporal substitution takes the values $\psi = \{0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4\}$. When $\psi = 1/\gamma$, the Epstein-Zin-Weil utility function reduces to power utility. The rate of time preference is $\beta = 0.01$. The mean growth rate of firm earnings is $\theta = 0.0343$, systematic earnings growth volatility is $\sigma_{X}^{s} = 0.0115$, idiosyncratic earnings growth volatility is $\sigma_{X}^{id} = 0.2258$. The mean growth rate of consumption is $g = 0.0333$, consumption growth volatility is $\sigma_{C} = 0.099$ and the correlation between consumption growth and firm earnings growth is $\rho_{XC} = 0.1759$. The recovery rate is $\alpha = 0.5$ and the tax rate on firm earnings is 0.15 per annum.
Figure 6: Baa Credit Spread in the Structural-Equilibrium Model without Intertemporal Macroeconomic Risk

This figure shows the model-implied Baa credit spread as a function of relative risk aversion, $\gamma$, when there is no intertemporal macroeconomic risk and the representative agent has the continuous-time version of Epstein-Zin-Weil preferences. The elasticity of intertemporal substitution takes the values $\psi = \{.1, .2, .4, .6, .8, 1.0, 1.2, 1.4\}$. When $\psi = \frac{1}{2}$, the Epstein-Zin-Weil utility function reduces to power utility. The rate of time preference is $\beta = 0.01$. The mean growth rate of firm earnings is $\theta = 0.0333$, systematic earnings growth volatility is $\sigma_X^\theta = 0.0115$, idiosyncratic earnings growth volatility is $\sigma_X^{id} = 0.1700$. The mean growth rate of consumption is $g = 0.0333$, consumption growth volatility is $\sigma_C = 0.0099$ and the correlation between consumption growth and firm earnings growth is $\rho_{XC} = 0.1759$. The recovery rate is $\alpha = 0.5$ and the per annum tax rate on firm earnings is $\eta = 0.15$. 

![Credit Spread Diagram](image-url)
Figure 7: Growth Rates of Earnings and Consumption

This figure shows the quarterly growth rates of aggregate real earnings and real (non-durable plus service) consumption expenditures.
This table compares features of structural models which are used to price corporate debt: Leland (1994), Goldstein, Ju, and Leland (2001) (GJL), Hackbarth, Miao, and Morellec (2006) (HMM), consumption-based models which are used to price the aggregate stock market: Lucas (1978), Campbell and Cochrane (1999) (CC), Bansal and Yaron (2004) (BY), Calvet and Fisher (2005a) (CF), and models which are used to price both corporate debt and the aggregate stock market: Chen, Collin-Dufresne, and Goldstein (2006) (CDG) and this paper. The comparison table is divided into 3 panels. The first panel focuses on the features of structural models, the second on the features of consumption-based models, while the third focuses on the pricing implications of the various models. The third panel is key to understanding the difference between the implications of our modeling approach relative to Chen, Collin-Dufresne, and Goldstein (2006). In Chen, Collin-Dufresne, and Goldstein default risk does not affect the equity risk premium and the default boundary must be exogenously countercyclical to generate realistic credit spreads. But in our model, default risk affects the equity risk premium and the default boundary can be chosen endogenously.

 genesis  by maximizing equity value.

<table>
<thead>
<tr>
<th>Structural Model Features</th>
<th>Leland</th>
<th>GJL</th>
<th>HMM</th>
<th>Lucas</th>
<th>CC</th>
<th>BY</th>
<th>CF</th>
<th>CDG</th>
<th>This paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>default risky corporate debt</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>endogenous default boundary</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>dynamic capital structure</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>stochastic earnings growth rates</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

| Consumption-based Model           |        |     |     |       |     |     |     |     |            |
| recursive preferences             | ✓      | ✓   | ✓   | ✓     | ✓  | ✓  | ✓  | ✓   | ✓          |
| habit-formation preferences       | ✓      | ✓   | ✓   | ✓     | ✓  | ✓  | ✓  | ✓   | ✓          |
| stochastic dividend growth rate    | ✓      | ✓   | ✓   | ✓     | ✓  | ✓  | ✓  | ✓   | ✓          |
| stochastic consumption growth rate | ✓      | ✓   | ✓   | ✓     | ✓  | ✓  | ✓  | ✓   | ✓          |

| Pricing Implications              |        |     |     |       |     |     |     |     |            |
| credit spread                     | ✓      | ✓   | ✓   | ✓     | ✓  | ✓  | ✓  | ✓   | ✓          |
| equity risk premium               | ✓      | ✓   | ✓   | ✓     | ✓  | ✓  | ✓  | ✓   | ✓          |
| impact of default risk on equity risk premium | ✓      | ✓   | ✓   | ✓     | ✓  | ✓  | ✓  | ✓   | ✓          |
| cross-market predictability       | ✓      | ✓   | ✓   | ✓     | ✓  | ✓  | ✓  | ✓   | ✓          |
Table 2: Aggregate Parameter Estimates
To calibrate the model to the aggregate US economy, we use quarterly real non-durable plus service consumption expenditure from the Bureau of Economic Analysis and quarterly earnings data from Standard and Poor’s, provided by Robert J. Shiller. The personal consumption expenditure chain-type price index is used to deflate nominal earnings. All estimates are annualized and based on quarterly log growth rates for the period 1947-2005.

| Real consumption growth | 0.0333 | 0.0099 |
| Real earnings growth    | 0.0343 | 0.1072 |

Table 3: Calibrated Parameters Values
The table lists parameter values used in the empirical analysis. Panel A contains parameters calculated from market data (Table 2) and Panel B the remaining free parameters.

Panel A

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption growth rate</td>
<td>( g_i )</td>
<td>0.0162</td>
<td>0.0419</td>
</tr>
<tr>
<td>Consumption volatility</td>
<td>( \sigma_{C,i} )</td>
<td>0.0113</td>
<td>0.0092</td>
</tr>
<tr>
<td>Earnings growth rate</td>
<td>( \theta_i )</td>
<td>-0.0475</td>
<td>0.0840</td>
</tr>
<tr>
<td>Earnings volatility</td>
<td>( \sigma_{X,i} )</td>
<td>0.1243</td>
<td>0.0987</td>
</tr>
<tr>
<td>Correlation</td>
<td>( \rho_{XC,i} )</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Long run probabilities</td>
<td>( f_i )</td>
<td>1/3</td>
<td>2/3</td>
</tr>
<tr>
<td>Probability of switching</td>
<td>( p )</td>
<td>3/4</td>
<td>3/4</td>
</tr>
</tbody>
</table>

Panel B

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual discount rate</td>
<td>( \beta )</td>
<td>1%</td>
</tr>
<tr>
<td>Tax rate</td>
<td>( \eta )</td>
<td>15 %</td>
</tr>
<tr>
<td>Recovery rate in the state 1</td>
<td>( \alpha_1 )</td>
<td>40 %</td>
</tr>
<tr>
<td>Recovery rate in the state 2</td>
<td>( \alpha_2 )</td>
<td>60 %</td>
</tr>
<tr>
<td>Baa idiosyn. earnings vola.</td>
<td>( \sigma^d_X )</td>
<td>32%</td>
</tr>
</tbody>
</table>
Table 4: Corporate Bond Market
This Table summarizes the moments of the corporate bond market of Baa firms. The capital structure is static and the optimal coupon is state-dependent maximizing total firm value, \( \max_c F_t \). The first two columns are generated when the economy is state one and the last two columns when it is in state two.

<table>
<thead>
<tr>
<th></th>
<th>Initial State: 1</th>
<th>Initial State: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State 1</td>
<td>State 2</td>
</tr>
<tr>
<td>( \gamma = 7.5 )</td>
<td>253.31</td>
<td>257.26</td>
</tr>
<tr>
<td>( \gamma = 10 )</td>
<td>194.34</td>
<td>200.13</td>
</tr>
<tr>
<td>( \gamma = 7.5 )</td>
<td>200.13</td>
<td>200.53</td>
</tr>
<tr>
<td>( \gamma = 10 )</td>
<td>200.13</td>
<td>200.53</td>
</tr>
</tbody>
</table>

Panel A: Credit spread (b.p.)

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>189.92</td>
<td>187.49</td>
<td>139.76</td>
<td>139.49</td>
<td>330.87</td>
<td>324.23</td>
<td>239.22</td>
<td>236.57</td>
</tr>
<tr>
<td>0.5</td>
<td>57.78</td>
<td>56.99</td>
<td>47.14</td>
<td>47.00</td>
<td>74.64</td>
<td>72.77</td>
<td>59.90</td>
<td>59.09</td>
</tr>
<tr>
<td>1.5</td>
<td>29.92</td>
<td>29.73</td>
<td>25.11</td>
<td>25.16</td>
<td>35.54</td>
<td>35.00</td>
<td>29.54</td>
<td>29.35</td>
</tr>
</tbody>
</table>

Panel B: Credit spread volatility (b.p.)

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>15.99</td>
<td>15.48</td>
<td>11.88</td>
<td>11.48</td>
<td>23.33</td>
<td>22.67</td>
<td>17.55</td>
<td>17.03</td>
</tr>
<tr>
<td>0.5</td>
<td>5.16</td>
<td>4.64</td>
<td>3.77</td>
<td>3.38</td>
<td>7.25</td>
<td>6.51</td>
<td>5.29</td>
<td>4.75</td>
</tr>
<tr>
<td>1.5</td>
<td>2.14</td>
<td>1.80</td>
<td>1.56</td>
<td>1.31</td>
<td>2.96</td>
<td>2.49</td>
<td>2.16</td>
<td>1.81</td>
</tr>
</tbody>
</table>

Panel C: Arrow-Debreu default claims (%)

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>12.15</td>
<td>12.36</td>
<td>6.13</td>
<td>6.27</td>
<td>18.98</td>
<td>19.15</td>
<td>9.94</td>
<td>10.04</td>
</tr>
<tr>
<td>0.5</td>
<td>23.67</td>
<td>24.70</td>
<td>18.82</td>
<td>19.83</td>
<td>27.96</td>
<td>28.87</td>
<td>22.25</td>
<td>23.19</td>
</tr>
<tr>
<td>1.5</td>
<td>35.59</td>
<td>37.84</td>
<td>31.86</td>
<td>34.16</td>
<td>38.85</td>
<td>40.94</td>
<td>34.78</td>
<td>36.95</td>
</tr>
</tbody>
</table>

Panel D: Actual 5 year default probability (%)

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>15.99</td>
<td>15.48</td>
<td>11.88</td>
<td>11.48</td>
<td>23.33</td>
<td>22.67</td>
<td>17.55</td>
<td>17.03</td>
</tr>
<tr>
<td>0.5</td>
<td>5.16</td>
<td>4.64</td>
<td>3.77</td>
<td>3.38</td>
<td>7.25</td>
<td>6.51</td>
<td>5.29</td>
<td>4.75</td>
</tr>
<tr>
<td>1.5</td>
<td>2.14</td>
<td>1.80</td>
<td>1.56</td>
<td>1.31</td>
<td>2.96</td>
<td>2.49</td>
<td>2.16</td>
<td>1.81</td>
</tr>
</tbody>
</table>

Panel E: 5 year risk-adjustment \( R_5 \)

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.379</td>
<td>0.377</td>
<td>0.236</td>
<td>0.235</td>
<td>0.465</td>
<td>0.462</td>
<td>0.299</td>
<td>0.297</td>
</tr>
<tr>
<td>0.5</td>
<td>0.642</td>
<td>0.639</td>
<td>0.569</td>
<td>0.568</td>
<td>0.676</td>
<td>0.672</td>
<td>0.600</td>
<td>0.598</td>
</tr>
<tr>
<td>1.5</td>
<td>0.776</td>
<td>0.776</td>
<td>0.741</td>
<td>0.742</td>
<td>0.793</td>
<td>0.792</td>
<td>0.757</td>
<td>0.757</td>
</tr>
</tbody>
</table>

Panel F: 5 year time-adjustment \( T_5 \)

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.311</td>
<td>0.305</td>
<td>0.298</td>
<td>0.292</td>
<td>0.393</td>
<td>0.386</td>
<td>0.375</td>
<td>0.369</td>
</tr>
<tr>
<td>0.5</td>
<td>0.161</td>
<td>0.151</td>
<td>0.144</td>
<td>0.135</td>
<td>0.198</td>
<td>0.186</td>
<td>0.177</td>
<td>0.166</td>
</tr>
<tr>
<td>1.5</td>
<td>0.095</td>
<td>0.086</td>
<td>0.084</td>
<td>0.075</td>
<td>0.116</td>
<td>0.104</td>
<td>0.102</td>
<td>0.092</td>
</tr>
</tbody>
</table>

Panel G: Optimal default boundary

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.490</td>
<td>0.478</td>
<td>0.490</td>
<td>0.478</td>
<td>0.618</td>
<td>0.605</td>
</tr>
<tr>
<td>0.5</td>
<td>0.537</td>
<td>0.463</td>
<td>0.537</td>
<td>0.463</td>
<td>0.659</td>
<td>0.659</td>
</tr>
<tr>
<td>1.5</td>
<td>3.838</td>
<td>0.984</td>
<td>3.838</td>
<td>0.984</td>
<td>4.671</td>
<td>1.198</td>
</tr>
</tbody>
</table>

Panel H: Optimal coupon

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>49.23</td>
<td>48.62</td>
<td>44.84</td>
<td>44.20</td>
<td>59.21</td>
<td>58.59</td>
</tr>
<tr>
<td>0.5</td>
<td>33.69</td>
<td>32.36</td>
<td>29.40</td>
<td>28.20</td>
<td>39.66</td>
<td>38.10</td>
</tr>
<tr>
<td>1.5</td>
<td>24.79</td>
<td>23.20</td>
<td>21.42</td>
<td>20.03</td>
<td>28.94</td>
<td>27.09</td>
</tr>
</tbody>
</table>

Panel I: Leverage (%)
Table 5: Equity Market
This Table summarizes the moments of the equity market. The capital structure is static and the optimal coupon is state-dependent maximizing total firm value, max\(_c \ F_i\). The first two columns are generated when the economy is state one and the last two columns when it is in state two. All equity market numbers are computed with zero idiosyncratic volatility.

<table>
<thead>
<tr>
<th></th>
<th>Initial State: 1</th>
<th>Initial State: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State 1</td>
<td>State 2</td>
</tr>
<tr>
<td>(\gamma = 7.5)</td>
<td>(\gamma = 10)</td>
<td></td>
</tr>
<tr>
<td>(\psi = 0.1)</td>
<td>0.69</td>
<td>0.42</td>
</tr>
<tr>
<td>(\psi = 0.5)</td>
<td>1.26</td>
<td>0.71</td>
</tr>
<tr>
<td>(\psi = 1.5)</td>
<td>2.18</td>
<td>1.22</td>
</tr>
</tbody>
</table>

Panel A: Unlevered excess return (%)

<table>
<thead>
<tr>
<th></th>
<th>Initial State: 1</th>
<th>Initial State: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State 1</td>
<td>State 2</td>
</tr>
<tr>
<td>(\gamma = 7.5)</td>
<td>(\gamma = 10)</td>
<td></td>
</tr>
<tr>
<td>(\psi = 0.1)</td>
<td>1.17</td>
<td>0.63</td>
</tr>
<tr>
<td>(\psi = 0.5)</td>
<td>4.33</td>
<td>1.57</td>
</tr>
<tr>
<td>(\psi = 1.5)</td>
<td>5.27</td>
<td>2.08</td>
</tr>
</tbody>
</table>

Panel B: Semi-levered excess return (%)

<table>
<thead>
<tr>
<th></th>
<th>Initial State: 1</th>
<th>Initial State: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State 1</td>
<td>State 2</td>
</tr>
<tr>
<td>(\gamma = 7.5)</td>
<td>(\gamma = 10)</td>
<td></td>
</tr>
<tr>
<td>(\psi = 0.1)</td>
<td>36.19</td>
<td>32.08</td>
</tr>
<tr>
<td>(\psi = 0.5)</td>
<td>36.13</td>
<td>31.81</td>
</tr>
<tr>
<td>(\psi = 1.5)</td>
<td>37.31</td>
<td>32.27</td>
</tr>
</tbody>
</table>

Panel C: Levered excess return (%)

<table>
<thead>
<tr>
<th></th>
<th>Initial State: 1</th>
<th>Initial State: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State 1</td>
<td>State 2</td>
</tr>
<tr>
<td>(\gamma = 7.5)</td>
<td>(\gamma = 10)</td>
<td></td>
</tr>
<tr>
<td>(\psi = 0.1)</td>
<td>35.64</td>
<td>23.66</td>
</tr>
<tr>
<td>(\psi = 0.5)</td>
<td>34.06</td>
<td>21.48</td>
</tr>
<tr>
<td>(\psi = 1.5)</td>
<td>33.30</td>
<td>20.58</td>
</tr>
</tbody>
</table>

Panel D: Unlevered excess volatility (%)

<table>
<thead>
<tr>
<th></th>
<th>Initial State: 1</th>
<th>Initial State: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State 1</td>
<td>State 2</td>
</tr>
<tr>
<td>(\gamma = 7.5)</td>
<td>(\gamma = 10)</td>
<td></td>
</tr>
<tr>
<td>(\psi = 0.1)</td>
<td>3.63</td>
<td>3.21</td>
</tr>
<tr>
<td>(\psi = 0.5)</td>
<td>21.45</td>
<td>23.84</td>
</tr>
<tr>
<td>(\psi = 1.5)</td>
<td>431.54</td>
<td>506.08</td>
</tr>
</tbody>
</table>

Panel F: Unlevered PE-ratio

<table>
<thead>
<tr>
<th></th>
<th>Initial State: 1</th>
<th>Initial State: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State 1</td>
<td>State 2</td>
</tr>
<tr>
<td>(\gamma = 7.5)</td>
<td>(\gamma = 10)</td>
<td></td>
</tr>
<tr>
<td>(\psi = 0.1)</td>
<td>1.07</td>
<td>1.15</td>
</tr>
<tr>
<td>(\psi = 0.5)</td>
<td>8.40</td>
<td>10.98</td>
</tr>
<tr>
<td>(\psi = 1.5)</td>
<td>192.33</td>
<td>257.95</td>
</tr>
</tbody>
</table>

Panel G: Levered PE-ratio

<table>
<thead>
<tr>
<th></th>
<th>Initial State: 1</th>
<th>Initial State: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State 1</td>
<td>State 2</td>
</tr>
<tr>
<td>(\gamma = 7.5)</td>
<td>(\gamma = 10)</td>
<td></td>
</tr>
<tr>
<td>(\psi = 0.1)</td>
<td>16.32</td>
<td>42.36</td>
</tr>
<tr>
<td>(\psi = 0.5)</td>
<td>4.23</td>
<td>9.37</td>
</tr>
<tr>
<td>(\psi = 1.5)</td>
<td>1.94</td>
<td>3.70</td>
</tr>
</tbody>
</table>

Panel H: Risk-free rate (%)
Table 6: Corporate Bond Market: Constant Default Boundary
This Table summarizes the moments of the corporate bond market of Baa firms. The default boundary is constant across states and set at the long-run average of the optimal model, i.e. $f_1X_{D,1} + f_2X_{D,2}$. The coupon is the same as in the optimal model. The first two columns are generated when the economy is state one and the last two columns when it is in state two.

<table>
<thead>
<tr>
<th>Initial State: 1</th>
<th>Initial State: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>State 2</td>
</tr>
<tr>
<td>$\gamma = 7.5$</td>
<td>$\gamma = 10$</td>
</tr>
<tr>
<td>Panel A: Credit spread (b.p.)</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.1$ 246.28 249.78 190.40 195.65 407.36 409.69 313.56 319.03</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.5$ 157.09 163.73 137.54 144.95 193.71 199.63 168.32 175.45</td>
<td></td>
</tr>
<tr>
<td>$\psi = 1.5$ 125.69 134.37 113.81 122.73 142.94 151.44 128.81 137.70</td>
<td></td>
</tr>
<tr>
<td>Panel B: Credit spread volatility (b.p.)</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.1$ 183.71 181.14 137.23 136.65 318.09 311.38 235.84 232.65</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.5$ 55.51 54.68 45.47 45.27 71.46 69.59 57.70 56.81</td>
<td></td>
</tr>
<tr>
<td>$\psi = 1.5$ 28.92 28.73 24.32 24.35 34.28 33.75 28.56 28.36</td>
<td></td>
</tr>
<tr>
<td>Panel C: Arrow-Debreu default claims (%)</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.1$ 12.02 12.20 6.12 6.23 18.75 18.88 9.94 10.02</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.5$ 23.30 24.27 18.54 19.50 27.53 28.37 21.93 22.80</td>
<td></td>
</tr>
<tr>
<td>$\psi = 1.5$ 35.18 37.37 31.49 33.74 38.40 40.43 34.38 36.50</td>
<td></td>
</tr>
<tr>
<td>Panel D: Actual 5 year default probability (%)</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.1$ 15.82 15.31 11.82 11.43 23.05 22.39 17.50 16.98</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.5$ 5.01 4.51 3.67 3.30 7.03 6.32 5.17 4.64</td>
<td></td>
</tr>
<tr>
<td>$\psi = 1.5$ 2.07 1.74 1.51 1.27 2.86 2.40 2.09 1.75</td>
<td></td>
</tr>
<tr>
<td>Panel E: 5 year risk-adjustment $R_5$</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.1$ 0.993 1.030 0.993 1.032 0.996 1.024 0.996 1.025</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.5$ 1.192 1.295 1.207 1.323 1.166 1.255 1.180 1.281</td>
<td></td>
</tr>
<tr>
<td>$\psi = 1.5$ 1.336 1.505 1.362 1.550 1.301 1.452 1.326 1.495</td>
<td></td>
</tr>
<tr>
<td>Panel F: 5 year time-adjustment $T_5$</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.1$ 0.374 0.372 0.234 0.233 0.459 0.455 0.299 0.296</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.5$ 0.636 0.633 0.565 0.563 0.670 0.665 0.596 0.593</td>
<td></td>
</tr>
<tr>
<td>$\psi = 1.5$ 0.772 0.772 0.738 0.738 0.789 0.788 0.754 0.754</td>
<td></td>
</tr>
<tr>
<td>Panel G: Optimal default boundary</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.1$ 0.302 0.296 0.302 0.296 0.381 0.374 0.381 0.374</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.5$ 0.150 0.140 0.150 0.140 0.184 0.173 0.184 0.173</td>
<td></td>
</tr>
<tr>
<td>$\psi = 1.5$ 0.088 0.079 0.088 0.079 0.107 0.096 0.107 0.096</td>
<td></td>
</tr>
<tr>
<td>Panel H: Optimal coupon</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.1$ 0.490 0.478 0.490 0.478 0.618 0.618 0.618 0.618</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.5$ 0.537 0.463 0.537 0.463 0.659 0.659 0.659 0.659</td>
<td></td>
</tr>
<tr>
<td>$\psi = 1.5$ 3.838 0.984 3.838 0.984 4.671 4.671 4.671 4.671</td>
<td></td>
</tr>
<tr>
<td>Panel I: Leverage (%)</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.1$ 49.29 48.68 44.87 44.23 59.31 58.69 54.58 53.91</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.5$ 33.83 32.51 29.51 28.31 39.86 38.30 34.97 33.54</td>
<td></td>
</tr>
<tr>
<td>$\psi = 1.5$ 24.94 23.36 21.54 20.15 29.13 27.28 25.26 23.63</td>
<td></td>
</tr>
</tbody>
</table>
Table 7: Long-Run Risk
This table contains the risk-distortion factor (Panel A), the risk-neutral probability of switching \( \hat{p} \) (Panel B), the half-life of the economy \( \hat{t}_{1/2} \) (Panel C), and the long-run risk-neutral distribution \( (\hat{f}_1, \hat{f}_2) \) (Panel D and E).

<table>
<thead>
<tr>
<th>Panel A: Risk-adjustment factor</th>
<th>( \gamma = 7.5 )</th>
<th>( \gamma = 10 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi = 0.1 )</td>
<td>0.939</td>
<td>1.000</td>
</tr>
<tr>
<td>( \psi = 0.5 )</td>
<td>1.203</td>
<td>1.312</td>
</tr>
<tr>
<td>( \psi = 1.5 )</td>
<td>1.275</td>
<td>1.398</td>
</tr>
</tbody>
</table>

| Panel B: \( \hat{p} \)         |                  |                |
| \( \psi = 0.1 \)              | 0.767           | 0.750          |
| \( \psi = 0.5 \)              | 0.716           | 0.709          |
| \( \psi = 1.5 \)              | 0.711           | 0.707          |

| Panel C: Half-life \( \hat{t}_{1/2} \) |                  |                |
| \( \psi = 0.1 \)              | 0.903           | 0.924          |
| \( \psi = 0.5 \)              | 0.968           | 0.977          |
| \( \psi = 1.5 \)              | 0.975           | 0.980          |

| Panel D: \( \hat{f}_1 \)       |                  |                |
| \( \psi = 0.1 \)              | 0.306           | 0.333          |
| \( \psi = 0.5 \)              | 0.420           | 0.462          |
| \( \psi = 1.5 \)              | 0.448           | 0.494          |

| Panel E: \( \hat{f}_2 \)       |                  |                |
| \( \psi = 0.1 \)              | 0.694           | 0.667          |
| \( \psi = 0.5 \)              | 0.580           | 0.538          |
| \( \psi = 1.5 \)              | 0.552           | 0.506          |
Table 8: Model Comparison—Corporate Bond Market

Model 1a and 1b are single state models whereas Model 2 and 3 are switching state models. Model 1a assumes power utility with risk aversion of 10 and Model 1b, 2 and 3 assume Epstein-Zin utility with preference for early resolution of uncertainty. In Model 2 only the drift and volatility of the earnings process switch but the drift and volatility of the consumption process are constant. In Model 3 both the drift and volatility of the earnings process switch. Note that AD stands for Arrow-Debreu and 5y for 5 year.

<table>
<thead>
<tr>
<th></th>
<th>Initial State</th>
<th>Current State</th>
<th>Model 1a</th>
<th>Model 1b</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
</tr>
<tr>
<td>EIS</td>
<td>0.10</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Credit Spread (b.p.)</td>
<td>80.62</td>
<td>128.08</td>
<td>140.67</td>
<td>191.51</td>
<td>80.62</td>
<td>128.08</td>
</tr>
<tr>
<td>AD default claim (%)</td>
<td>3.30</td>
<td>23.91</td>
<td>25.46</td>
<td>34.06</td>
<td>3.30</td>
<td>23.91</td>
</tr>
<tr>
<td>Actual 5y default prob. (%)</td>
<td>5.75</td>
<td>2.44</td>
<td>5.43</td>
<td>4.96</td>
<td>5.75</td>
<td>2.44</td>
</tr>
<tr>
<td>5y risk-adjustment (%)</td>
<td>1.09</td>
<td>1.10</td>
<td>1.04</td>
<td>1.34</td>
<td>1.09</td>
<td>1.10</td>
</tr>
<tr>
<td>5y time-adjustment (%)</td>
<td>0.36</td>
<td>0.83</td>
<td>0.68</td>
<td>0.73</td>
<td>0.36</td>
<td>0.83</td>
</tr>
<tr>
<td>Leverage (%)</td>
<td>35.00</td>
<td>35.00</td>
<td>35.00</td>
<td>35.00</td>
<td>35.00</td>
<td>35.00</td>
</tr>
<tr>
<td>Default boundary</td>
<td>0.22</td>
<td>0.17</td>
<td>0.17</td>
<td>0.16</td>
<td>0.22</td>
<td>0.17</td>
</tr>
<tr>
<td>Coupon 1</td>
<td>0.36</td>
<td>1.23</td>
<td>1.59</td>
<td>0.67</td>
<td>0.36</td>
<td>1.23</td>
</tr>
<tr>
<td>Coupon 2</td>
<td>0.36</td>
<td>1.23</td>
<td>1.88</td>
<td>0.80</td>
<td>0.36</td>
<td>1.23</td>
</tr>
<tr>
<td>Distortion factor</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.35</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Table 9: Model Comparison—Equity Market
Model 1a and 1b are single state models whereas Model 2 and 3 are switching state models. Model 1a assumes power utility with risk aversion of 10 and Model 1b, 2 and 3 assume Epstein-Zin utility with preference for early resolution of uncertainty. In Model 2 only the drift and volatility of the earnings process switch but the drift and volatility of the consumption process are constant. In Model 3 both the drift and volatility of the earnings and consumption process switch.

<table>
<thead>
<tr>
<th></th>
<th>Initial State</th>
<th>Current State</th>
<th>Model 1a</th>
<th>Model 1b</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EIS</td>
<td>0.10</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unlev. excess return (%)</td>
<td>0.32</td>
<td>0.32</td>
<td>0.37</td>
<td>2.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.32</td>
<td>0.32</td>
<td>0.29</td>
<td>1.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lev. excess return (%)</td>
<td>0.46</td>
<td>0.46</td>
<td>0.54</td>
<td>3.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.46</td>
<td>0.46</td>
<td>0.59</td>
<td>3.96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unlev. stock return vola (%)</td>
<td>10.72</td>
<td>10.72</td>
<td>37.56</td>
<td>36.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.72</td>
<td>10.72</td>
<td>32.36</td>
<td>32.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lev. stock return vola (%)</td>
<td>15.63</td>
<td>15.63</td>
<td>26.15</td>
<td>24.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>15.63</td>
<td>15.63</td>
<td>17.08</td>
<td>16.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unlev. PE-ratio</td>
<td>3.32</td>
<td>60.75</td>
<td>77.20</td>
<td>33.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.32</td>
<td>60.75</td>
<td>91.34</td>
<td>38.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levered PE-ratio</td>
<td>1.94</td>
<td>35.43</td>
<td>44.97</td>
<td>19.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.94</td>
<td>35.43</td>
<td>56.97</td>
<td>23.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk-free rate (%)</td>
<td>33.81</td>
<td>5.33</td>
<td>5.33</td>
<td>3.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>33.81</td>
<td>5.33</td>
<td>5.33</td>
<td>6.54</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>