The Dynamics of Metropolitan Communities

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Preliminary Draft

July 21, 2008

*We would like to thank Steve Durlauf, Francois Ortalo-Mange, Antonio Merlo, Sven Rady, Karl Scholz, Steve Slutsky and seminar participants at the ASSA meeting in Chicago, the University of Wisconsin, and the SED meeting in Prague. We would also like to thank Jason Imbrogno for research assistance. Financial support for this research is provided by the National Science Foundation (SBR-0617844).
Abstract

In this paper we study the life cycle locational choices of heterogeneous households and characterize the dynamics of metropolitan areas. We develop an overlapping generations model for households in a system of multiple jurisdictions. At each point of time households choose among the finite number of jurisdictions, pick consumption and housing plans, and vote over public good and tax policies. A household’s composition changes over the life cycle as children grow up and leave the household. These changes in turn impact the household’s need for housing and for local public services, particularly education. Households face frictions in relocating. A household may relocate as its needs change, bearing the associated financial and psychic costs of moving. Alternatively, the household may choose to remain in a community that suited its initial needs, finding the costs of relocation to be greater than the potential benefits of moving to a community better suited to its changed housing and public good needs. Our theoretical and quantitative analysis shows that the presence of mobility costs is likely to have a large impact on household sorting patterns. Mobility costs also impact the political decisions that determine local tax and expenditure policies.
1 Introduction

A fundamental premise in modeling local jurisdictions is that households make their location decisions taking account of the public good bundles available in alternative jurisdictions. This hypothesis, first proposed by Tiebout (1956), has been the subject of extensive formal modeling and empirical analysis. Early empirical work, pioneered by Oates (1969), investigated the extent to which differentials in housing prices across jurisdictions reflect differentials in quality of local public goods and property tax rates.\(^1\) Much recent empirical work has focused directly on the extent to which households stratify based on differences in the quality of local public goods.\(^2\) Both research on capitalization and research on stratification of households across jurisdictions supports the hypothesis that households do in fact take account of differences in local public good bundles in making location choices.

In research to date, both theoretical models and empirical research have largely focused on static equilibrium models or cross-sectional empirical studies.\(^3\) However the same logic that suggests households sort based on tastes for local public goods implies that households have incentives to change location over the life cycle. For example, a household’s consumption of local public education begins when its first child enters kindergarten and ends when its last child leaves high school. Thus, one would expect that households with school-age children would place weight on the quality of local public schools when considering location choices, but that those same households would place little weight on quality of local public schools when their children have left school. Indeed, one would expect households would tend to

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\(^1\)Black (1999), Epple and Romano (2003), Figlio and Lucas (2004) and Bayer, Ferreira, and McMillan (2007) have extended this analysis to investigate whether differences within jurisdictions in the quality of local public goods are capitalized into house prices.


move to locations with good quality public schools when they have school-age children while moving to locations with lower housing prices and lower quality public schools when their children have left school. Departure of children from high school presages their departure from the household, and the associated decrease in need for housing reinforces the incentives for relocation that accompany the decrease in need for local public services. These incentives for relocation over the life cycle in turn create incentives for young households to make their initial location choices and housing purchases taking account of the likelihood that they will relocate in the future. A dynamic equilibrium model embodying household life cycle choices offers the potential to improve understanding both of community characteristics and of housing markets.

The main contribution of this paper is a new life cycle model of community formation with limited household mobility. Our model captures three important dimensions by which households differ: income, moving cost, and age. Income is clearly a key factor influencing a household’s ability and willingness to pay the moving cost and housing price premium to relocate to a community with higher quality public services. Moving costs, both financial and psychic, are important factors in decision process. In addition to transactions costs, relocation often entails costs associated with moving away from friends, neighbors, and familiar surroundings and the associated costs of becoming established in a new neighborhood. While financial costs will typically be roughly proportional to house value, psychic costs are likely to exhibit greater variation across households. Finally, our model also captures the fact that relocation incentives vary over the life cycle. These incentives are largely driven by the presence or absence of children at various points during the life cycle.

We derive the stationary equilibrium of our model and characterize its properties. In our model, adults live for two periods and thus can live in at most two different locations. One important property of equilibrium is that many community pairs are strictly dominated by other pairs in equilibrium. This result is important since it reduces the dimensionality of the choice set. Restricting our attention to community pairs in the effective choice set, we can

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4 Family size is also clearly important. For example, a household with no children would have little incentive to move to a community with high-quality public education.

5 Children live with their parents until they reach adulthood.
provide conditions that guarantee that households stratify by wealth (conditional on moving costs) in equilibrium. Old households have few incentives to move to a community that has higher levels of public good provision than the community chosen when young. We show that this conjecture is correct if the relative weight placed on the local public good is higher when young than when old.

Equilibria do not have analytical solutions. Nevertheless, we can provide a general characterization of the partition of household types into residential plans. Moreover, we develop an algorithm that can be used to compute equilibria numerically. To illustrate some of the quantitative implications of our model, we calibrate our model and compute equilibria for economies with two and three communities. We find that our model can generate equilibria in which a reasonable fraction of households relocates to a different community when old in equilibrium. This property of equilibrium is consistent with evidence on turnover in local housing markets. This feature of our model has important implications for the political decisions made in the communities. We find that older households are typically in the majority in communities with low levels of public good provisions, while young households dominate in communities with high levels of expenditures.

The rest of the paper is organized as follows. Section 2 presents some stylized facts that characterize various dynamic aspects of community formation using Census data for the Boston metropolitan area. Section 3 develops our theoretical model. Section 4 defines equilibrium, discusses its properties and develops an algorithm to compute equilibria. In Section 5 we examine some quantitative properties of our model. Section 6 offers conclusions.

2 Some Stylized Facts

To gain some insights into the quantitative importance of mobility across the life cycle and the persistence of community dynamics, we consider the Boston Metropolitan Area. The first part of the analysis is based on data for the census years 1970, 1980, 1990 and 2000. A distinctive feature of the Boston metropolitan area is that the population was virtually the same in 1970,
1980 and 1990. Thus, the Boston metropolitan area allows us to investigate interjurisdictional mobility in an environment in which the overall community population was unchanging. Of course, real incomes were growing over this period of time and family size was declining.

A striking feature of the data for those three census years is not only that the metropolitan population remained unchanged but that individual community populations remained virtually unchanged as well, despite the growth in income and the decline in family size. For example, the correlation of the logarithms of community populations in 1970 and 2000 was .981, revealing an extraordinary degree of stability of sizes of communities over that thirty-year period. From 1990 to 2000, the Boston metropolitan area population grew by approximately 8%. The comparison of year 2000 to the prior years thus provides a basis for evaluating the impact of metropolitan growth on interjurisdictional mobility.

In addition to stability in population size, the data also reveal strong persistence in community incomes. This is illustrated in Table 1 which shows the correlations of median community incomes for each decade from 1970 to 2000. For example, median community incomes in 1970 and median community incomes in the year 2000 have a correlation of .93. We thus conclude that community level incomes are highly persistent across decades.

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Next we analyze the persistence of community compositions by age and family size. We consider 119 communities as part of the greater Boston metropolitan area. This list of communities includes almost all communities that were considered to be part of the MSA using the definitions for 1970, 1980, and 1990. The Census drastically changed the definition of the Boston MSA in 2000 and we do not include the communities that were added in 2000. A list of all communities in our sample is available from the authors.
have calculated the correlation between median community income and the fraction of the population aged 19 or younger in each of the four census years that we study. We find the following correlations .33, .42, .22, and .54. These correlations might simply reflect variation in family size with income. However, data for the U.S. population reveal that the proportion of individuals under age 19 is relatively constant across household incomes. This suggests that the positive correlations between income and population below age 19 reported above are a result of household sorting across communities based on the presence of children. The data also reveal much persistence in the population age distribution over time. This finding is illustrated in Table 2 which correlates the fraction of community populations aged 19 or younger for the four census years.

Table 2: Correlations of fraction of community populations aged 19 or younger

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<tr>
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<td>0.66</td>
<td>0.79</td>
<td>0.84</td>
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We are also interested in characterizing the persistence of public policies. We have assembled a comprehensive data base that contains the main fiscal and tax variables for all communities in the Boston MSA during the past 25 years. We find that the main patterns of expenditures and tax revenues are similar among the communities in our sample. To illustrate some of the main features, we pick two communities, Norton and Woburn, and plot personal property tax rates, total levies, and total educational expenditures. The plots are illustrated in Figure 1. We find that government policies are more volatile than income sorting patterns. For example, we find a decrease in property tax taxes in the first part of the 1980’s. This decrease is a direct consequence of Proposition 2 1/2 – a law that restricted property taxation. This law, which was passed in 1981, limited property tax rates to two-and-a-half percent (after some adjustment period). Since many jurisdictions had property taxes in the period leading
up to 1981 that were higher than the limits set in Proposition 2\(\frac{1}{2}\), the law imposed for all practical purposes a binding constraint on these communities (Calabrese, Epple, and Romano, 2007). Both tax levies and educational expenditures are more stable than tax rates and largely track income increases during that time period.

Some suggestive evidence regarding intra-metropolitan migration can also be obtained by studying a given cohort over time. If individuals in a given cohort did not move, the number of individuals in a community in a given cohort would be the same as the number of individuals 10 years older in that community 10 years later.\(^7\) Recall that community populations in the metropolitan area of Boston were essentially unchanged from 1970 through 1990. Hence, we compare the proportions of the population in various cohorts over this period in Table 3.\(^8\)

The correlations reveal that there is a high degree of mobility of individuals in the 20-24 and 25-29 age groups. For example, the correlation between those aged 20 to 24 in 1970 and those aged 30 to 34 in 1980 is only .11. The correlation between those aged 25 to 29 in 1970 and those aged 35 to 39 in 1980 is -.01. By contrast, individuals aged 35 to 39 and individuals age 40 to 44 exhibit relatively low mobility.\(^9\) The correlation between those aged 35 to 39 in 1980 and those aged 45 to 49 in 1990 is .88. The correlation between those aged 40 to 44 in 1980 and those aged 50 to 54 in 1990 is also .88. Mobility with age then begins to rise again. The correlation between those aged 45 to 49 in 1980 and those aged 55 to 59 in 1990 is .70. Similarly, the correlation between those aged 50 to 54 in 1980 and those aged 60 to 64 in 1990 is .69.\(^10\)

These results suggest that cross-community mobility is very high for those aged 20 to 29, and quite low for those aged 35 to 45. Mobility then begins to rise for those aged 45 to 54, but the correlations for that age group suggest that mobility for those households is substantially lower than for young households.

\(^7\)This discussion neglects mortality rates.  
\(^8\)Correlations mentioned specifically in the text are marked with bold in Table 3.  
\(^9\)We should note that these comparisons relate to net mobility. We expect that similar results would be obtained with the measures of gross mobility of different age groups.  
\(^10\)Again, these are not adjusted for mortality.
Figure 1: Tax Rates, Tax Levy, and Educational Expenditures
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<td>-0.06</td>
<td>0.41</td>
<td>0.69</td>
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</table>
3 An Overlapping Generations Model

We now develop an overlapping generations model to study the life cycle location choices of heterogeneous households, the associated demographic composition and public good provision of communities, and the dynamics of metropolitan areas.

3.1 Communities

Consider a local economy in which activity occurs at discrete points of time $t = 1, 2, \ldots$. The economy consists of $J$ communities. At each point of time, each community provides a local public good $g$ which is financed by property taxes $\tau$. Each community has a fixed supply of land and thus a housing supply that is not perfectly elastic. Let $p^h$ denote the net of tax price of a unit of housing services, $h$, in a community and $p = (1 + \tau)p^h$ the gross of tax price.

3.2 Households

There is a continuum of households that live for two adult periods. Thus at each point of time households in the economy consist of two over-lapping generations, denoted young (y) and old (o). Each household is characterized by a lifetime income level denoted by $w$. Households have a current period utility function which is defined over housing $h$, a local public good, and a numeraire good $b$.

**Assumption 1** The current period utility function of a young (old) household is denoted by $U^y(b, h, g)$ ($U^o(b, h, g)$) and increasing, twice differentiable, and concave in $(b, h, g)$.

Implicitly we assume that young households have children at home and thus may have different preferences for public goods and housing than old households whose children are

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11We often suppress the time subscripts for notational simplicity. Subscripts have the obvious ranges unless we state otherwise.

12Municipal boundaries, once drawn, rarely change. For example, municipal boundaries in Massachusetts were drawn in the 1930’s and have remained unchanged since that time.
assumed to have moved out.\footnote{An extension of our model incorporates explicit differences in family. An appendix that discusses how to model differences in family size is available upon request from the authors.} Lifetime utility is separable over the two periods. Since there is no uncertainty about future outcomes in this model, we make the following assumption:

**Assumption 2** *Households behave as price takers and have perfect foresight about current and future prices, tax rates, and levels of local public goods.*

### 3.3 Mobility Costs

When they leave their parents’ homes, young households are not endowed with a place of residence. We assume that they can pick any community of residence in the first period without facing mobility costs. Old households have already established a residence when they were young. If they decide to relocate in the second period, they face mobility costs, i.e. mobility costs are only born by old households if they decide to relocate in the second period of their life. Mobility costs are denoted by $mc$. We assume that households are heterogeneous in income and mobility costs.

**Assumption 3** *The distribution of lifetime income and mobility costs, denoted by $F(w, mc)$, is stationary and continuous with support $S \subseteq R^2_+$ and joint density $f(w, mc)$ with $f(\cdot)$ everywhere positive on its support.*

The assumption that $f(\cdot)$ everywhere positive on its support simplifies some of the arguments, but is not crucial for the main results. The assumption that the distribution is stationary implies that we consider a model without income growth.

### 3.4 The Decision Problem of Households

Households are forward-looking and maximize life-time utility which is time separable with constant discount factor $\beta$. Households recognize that locational and housing choices in the first period will have an impact on their well-being in the second period. Households must
choose a community of residence in each period. Let \( d_{jt} \in \{0,1\} \) denote an indicator that is equal to one if a young household lives in community \( j \) at time \( t \) and zero otherwise. Similarly define \( d_{jt} \in \{0,1\} \) for old households.

Households also determine consumption choices for housing and the composite private good. A young household with characteristics \((w_t, mc_t)\) maximizes lifetime utility in period \( t \):

\[
\max_{d_{yt}^y, h_{yt}^y, b_{yt}^y, d_{ot}^{o+1}, h_{ot}^{o+1}, b_{ot}^{o+1}} \sum_{k=1}^{J} d_{kt}^y U_y(b_{kt}^y, h_{kt}^y, g_{kt}) + \beta \sum_{l=1}^{J} d_{lt+1}^o U_o(b_{lt+1}^o, h_{lt+1}^o, g_{lt+1})
\]

subject to the lifetime budget constraint

\[
\sum_{k=1}^{J} d_{kt}^y (p_{kt} h_{kt}^y + b_{kt}^y) + \sum_{l=1}^{J} d_{lt+1}^o (p_{lt+1} h_{lt+1}^o + b_{kt+1}^o) = w_t - \sum_{k=1}^{J} \sum_{l \neq k} 1\{d_{kt}^y = d_{lt+1}^o = 1\} mc_t
\]

and residential constraints:

\[
\sum_{k=1}^{J} d_{kt}^y = 1
\]

\[
\sum_{l=1}^{J} d_{lt+1}^o = 1
\]

The last two constraints in (3) impose the requirement that the household lives in one and only one community at each point of time. Since there is no uncertainty in the model, the planned choices by young households for the future correspond to the optimal future choices in equilibrium. Also, \( w_t \) is the present value of lifetime income, thus assuming perfect capital markets.\(^{14}\) Finally, we have abstracted from discounting of future prices just for simplicity of exposition.

It is often convenient to express this decision problem using a conditional indirect utility (or value) function. Given a household with wealth, \( w \), moving cost, \( mc \), and community choice \( k \) when young and \( l \) when old, we can solve for the optimal demand for housing and other goods in both periods. Substituting these demand functions into the lifetime utility function yields the conditional indirect utility function which can be written:

\[
V_{kl}^y = V(w - \delta_{kl} mc, g_k, p_k, g_l, p_l)
\]

\(^{14}\)Abstracting from uncertainties is obviously a strong assumption, and the consequences of uncertainties are of interest to study.
where $\delta_{kl} = 1$ if $k \neq l$ and zero otherwise. Similarly, the conditional utility function of an old household that occupied community $k$ when young and is occupying community $l$ when old is:

$$V^o(w^o_n, g_l, p_l) = \max_{h_l} U^o(w^o_n - p_l h_l, h_l, g_l)$$

where $w^o_n = w - \delta_{kl} mc - p_k h^y_k$.

Define the set of young households living in community $j$ at time $t$ as follows:

$$C^y_{jt} = \{(w_t, mc_t) \mid d^y_{jt} = 1\}$$

The number of young households living in community $j$ at time $t$ is given by:

$$n^y_{jt} = \int \int_{C^y_{jt}} f(w_t, mc_t) \, dw_t \, dmc_t$$

Similarly define the set of old households living in community $j$ at time $t$ as follows:

$$C^o_{jt} = \{(w_{t-1}, mc_{t-1}) \mid d^o_{jt} = 1\}$$

The number of old households living in community $j$ at time $t$ is given by:

$$n^o_{jt} = \int \int_{C^o_{jt}} f(w_{t-1}, mc_{t-1}) \, dw_{t-1} \, dmc_{t-1}$$

3.5 Housing Markets

In this model all households are renters.\textsuperscript{15} Housing demand functions $h^y_j(\cdot)$ and $h^o_j(\cdot)$ can be derived by solving the decision problems characterized above. Below we introduce subscripts $t$ to indicate the dependence of housing demands on prices young and old households confront at date $t$. Aggregate housing demand in community $j$ at time $t$ is then defined as the sum of the demand of young and old households:

$$H^d_{jt} = H^y_{jt} + H^o_{jt}$$

where

$$H^y_{jt} = \int \int_{C^y_{jt}} h^y_j(w_t, mc_t) \, f(w_t, mc_t) \, dw_t \, dmc_t$$

$$H^o_{jt} = \int \int_{C^o_{jt}} h^o_j(w_{t-1}, mc_{t-1}) \, f(w^y_{t-1}, mc_{t-1}) \, dw_{t-1} \, dmc_{t-1}$$

\textsuperscript{15}We discuss the implications of housing ownership in the conclusions.
To characterize housing market equilibria, we need to specify housing supply in each community.

**Assumption 4** Housing is owned by absentee landlords. Housing supply is stationary and exogenously given by $H_j^s(p_{jt}^h)$.

This assumption is primarily imposed for convenience. The alternative would be to assign property rights over land. Households would then obtain revenues from rental income. While this extension is feasible, it adds little. We avoid the additional notational complexity by assuming absentee owners of land.

The housing market in community $j$ is in equilibrium at time $t$ if:

$$H_{jt}^d = H_j^s(p_{jt}^h) \tag{11}$$

### 3.6 Community Budget Constraints

We assume that each community provides a congested local public good $g_{jt}$ that is financed by property taxes $\tau_{jt}$. Community budgets must be balanced at each point of time. We assume that the public good primarily reflects expenditures per student on education. Hence we can express the community specific budget constrained as:

$$\tau_{jt} p_{jt}^h H_{jt} = g_{jt} n_{jt}^y \tag{12}$$

The right hand side of equation (12) equals public expenditure on young households, consistent with young households having one child in public education.

### 3.7 Voting

Next consider public choice of the tax rate and public good provision in a community. Households have the opportunity to vote twice in their lives, once when they are young and again when they are old. Households vote to maximize their lifetime utility from that point forward. To fully specify a voting model, we need a) to describe the set of alternatives that are
considered to be feasible outcomes by the voters; b) define preference orderings over feasible outcomes; and c) define a majority rule equilibrium.

Defining the set of feasible outcomes requires specifying the timing of decisions. We assume the following timing structure.

**Assumption 5** Each young household chooses their initial community of residence and rents a home there. The young household also commits to their old-aged community of residence. Housing markets then clear. Young households then vote taking as given the net housing price, which have already been established, but also \( g \) and \( p \) in their future community of residence. Numeraire and public good consumption take place. The old household then occupies the community planned when young and consumes housing. The old household votes, and, last, old-aged numeraire and public good consumption occur.

The timing and voter beliefs incorporated in Assumption 5 make the problem tractable. The key simplification is that young voters take their future community choices and the variables that characterize their old-aged community as given when voting. Of course, these variables will be those that arise in equilibrium and the future choice of community will be optimal given the equilibrium values. For example, a household that has committed to move to another community when it becomes old will in fact find it optimal to do so. But voters will not account for changes that would become optimal out of equilibrium, tremendously simplifying the voting problem.

Consider a community \( j \) which is characterized by a pair of housing prices and public good provision denoted by \((p_{jt}, g_{jt})\). Combining the equation relating net and gross housing prices, \( p_{jt} = p_{jt}^b (1 + \tau_{jt}) \), and the community budget constraint, we obtain:

\[
p_{jt} = p_{jt}^b + \frac{g_{jt}}{H_{jt}^y/n_{jt}^y + (n_{jt}^o/n_{jt}^y) (H_{jt}^o/n_{jt}^o)}
\]

Given our timing assumptions, all variables in this expression except \((p_{jt}, g_{jt})\) have been determined prior to voting. Thus the set of feasible alternatives yields a linear relationship between the choice of \( g_{jt} \) and the resulting gross-of-tax housing price \( p_{jt} \).
In each community $j$, there are two types of voters. The first type of voter is an old household. The second type of voter is a young household. Given the beliefs of each voter about feasible alternatives in equation (13), we can characterize each voter’s decision problem and thus characterize the voter’s behavior.

First consider an old household that has chosen to live in community $j$ after living in community $i$ when young. The household’s old age income is given by:

$$w_{nt}^o = w_{t-1} - p_{t-1}h_{nt-1}^o - b_{nt-1}^o - \delta_{ij}mc_{t-1}.$$  

The household’s budget constraint when old is given by:

$$w_{nt}^o = ph_{jt}^o + b_{jt}^o.$$  

Let $h_{jt}^o$ be the amount of housing the household has chosen. The quantity $h_{jt}^o$ is then fixed at the time that voting occurs. Substituting the community budget constraint that prevails at the time of voting into the voter’s budget constraint, we obtain:

$$w_{nt}^o = ph_{jt}^o + g_{jt}n_{jt}^y + b_{jt}^o$$  

The voter’s utility function is $U^o(g_{jt}, h_{jt}^o, b_{jt}^o)$. At the time of voting, all elements of the preceding budget constraint and utility function have been determined except $(g_{jt}, b_{jt}^o)$. Quasi-concavity of the utility function and convexity of the budget constraint imply that the voter’s induced preference over $g_{jt}$ is single-peaked (Denzau and Mackay, 1976).

Next consider a young voter that lives in community $j$ at $t$ and plans to live in community $k$ in $t+1$. The development is analogous to that for old voters, and we thus summarize briefly. At the time of voting in community $j$, this household will have purchased housing $h_{jt}^y$. The budget constraint of the young voter is then:

$$w_t = ph_{jt}^y + \frac{g_{jt}}{H_{jt}/n_{jt}^y + (n_{jt}^o/n_{jt}^y)} h_{jt}^y + b_{jt}^y + p_{kt+1}h_{kt+1}^o + b_{kt+1} + \delta_{jk} mc_t$$

The young voters utility function is: $U^y(b_{jt}^y, h_{jt}^y, g_{jt}) + \beta U^o(b_{kt+1}^o, h_{kt+1}^o, g_{kt+1})$. At the time of voting, the community tax base, $H_{jt}/n_{jt}^y + (n_{jt}^o/n_{jt}^y)$, the voter’s housing consumption, $h_{jt}^y$, have been determined. The voter takes current and future prices ($p_{jt}^y, p_{kt+1}$) and future government provision, $g_{kt+1}$, as given. Quasi-concavity of the voter’s utility function, $U^y + \beta U^o$, and convexity of the budget constraint then imply that induced preferences over $g_{jt}$ are single-peaked (Slutsky, 1975).
**Definition 1** A majority voting equilibrium is a public good provision level $g_{jt}$ that defeats all alternative feasible public good provision levels in pairwise majority voting.

We have the following result:

**Lemma 1** Voting equilibrium exists in all communities.

Lemma 1 follows from single-peakedness of preferences of all voters. Note that the median voter is not necessarily the household with median income. In general, the identity of the median voter will vary not only with income but also with the age composition of the community.

**4 Equilibrium Analysis**

**4.1 Definition of Equilibrium**

**Definition 2** An equilibrium for this economy is defined as an allocation that consists of a vector of prices, taxes, and public goods denoted by $\{p_{1t}, \tau_{1t}, g_{1t}, \ldots, p_{Jt}, \tau_{Jt}, g_{Jt}\}_{t=1}^{\infty}$, consumption plans for each household type, and a distribution of households among communities, $\{C_{1t}^{y}, \ldots, C_{Jt}^{y}, C_{1t}^{o}, \ldots, C_{Jt}^{o}\}_{t=1}^{\infty}$, such that:

1. Households maximize lifetime utility and live in their preferred communities.
2. Housing markets clear in every community at each point of time.
3. Community budgets are balanced at each point of time.
4. There is a majority voting equilibrium in each community at each point of time.

A stationary equilibrium is an equilibrium that satisfies the following additional conditions:

1. Constant prices, tax rates and levels of public good provision, i.e. for each community $j$,
   
   $p_{jt} = p_j$, $\tau_{jt} = \tau_j$, and $g_{jt} = g_j \ \forall t$.  

2. A stationary distribution of households among communities, i.e. for each community \( j \), we have \( C^o_{jt} = C^o_j \) and \( C^y_{jt} = C^y_j \) \( \forall t \).

The remainder of the paper focuses on properties of stationary equilibria.

### 4.2 Equilibrium Residential Choices

Upon entering adulthood, young households choose an initial and an old-age community of residence, correctly anticipating housing prices and local public good provision. Let \( k \) and \( l \) denote, respectively, the initial and old-age communities, \( k, l \in \{1, 2, ..., J\} \). If \( k \neq l \), then the household bears moving cost with present value of \( mc \). We adopt the convention of numbering the communities so that \( g_{j+1} > g_j \). Since households correctly anticipate \( g \)'s and \( p \)'s, gross housing prices will also ascend with the community number.\(^{16}\)

We now place some restrictions on the form of the household utility function that greatly facilitate the analysis.

**Assumption 6** The utility function

\[
U^a(b,h,g) = u^a_g(g) + u^a(b,h), \ a \in \{y,o\},
\]

is separable and \( u^a(b,y) \) is homogeneous of degree \( \rho \).

Let \( V^y(g_k, g_l, p_k, p_l, \tilde{w}) \) denote indirect lifetime utility of a young household choosing residential plan \( kl \), where \( \tilde{w} = w - \delta_{kl}mc \) is lifetime wealth adjusted for any moving cost. Given the separability assumption, the indirect utility can be written as:

\(^{16}\)We do not examine cases where communities have the same value of \( g \) and thus \( p \). If two communities had the same values, then households would be indifferent between them, and we assume they would randomize so that the distributions of \( (w, mc) \) would be the same in the two communities. In turn, this would imply there is no difference between the two communities, so they could be treated as one community (with the usual aggregation of housing supplies). Thus there is no loss in generality in requiring that communities be different (as a case with two clone communities is equivalent to another case with one fewer distinct communities).
Lemma 2

\[ V^y(g_k, g_l, p_k, p_l, \tilde{w}) = G(g_k, g_l) + \tilde{w}^{-\rho}W(p_k, p_l); \]  \hspace{1cm} (17)

with \( G \) an increasing function of \((g_k, g_l)\) and \( W \) a decreasing function of \((p_k, p_l)\).

Proof of Lemma 2: Indirect utility is given by:\(^{17}\)

\[ V^y = \max_{h_k, h_l} [u^y_g(g_k) + u^o_g(g_l) + u^y(b_k, h_k) + u^o(b_l, h_l)] \]
\[ \text{s.t. } p_k h_k + b_k + p_l h_l + b_l \leq \tilde{w} \]
\[ = G(g_k, g_l) + \max_{h_k, h_l} [u^y(b_k, h_k) + u^o(b_l, h_l)] \]
\[ \text{s.t. } p_k h_k + b_k + p_l h_l + b_l \leq \tilde{w}; \]  \hspace{1cm} (18)

where \( G(g_k, g_l) \equiv u^y_g(g_k) + u^o_g(g_l) \) is an increasing function of \((g_k, g_l)\). Since \( u^a(b, h) \) is homogeneous of degree \( \rho \), it follows from Theorem I in (Lau, 1970) (p. 376) that the maximand in the lower line of (18) equals \( \tilde{W}(p_k, p_l) \), a function homogeneous of degree \(-\rho\) and decreasing in its arguments. Then: \( V^y = G(p_k, p_l) + \tilde{w}^{-\rho}\tilde{W}(p_k, p_l) \). Q.E.D.

To simplify notation, define \( V^y_{kl} = V^y(g_k, g_l, p_k, p_l, \tilde{w}) \). The optimal residential choice plan of young adults maximizes \( V^y_{kl} \) over \((k, l)\) taking anticipated \( p's \) and \( g's \) as given. It is also convenient to adopt a notation in which locational choices can be characterized by a single index subscript \( i \). Let \( i \in I_{kl} \), \( I_{kl} = \{kl|k, l = 1, 2, ..., J\} \), indicate a residential plan. Let \( P_i = -W(p_k, p_l) \) for \( i = kl \), which we refer to as the composite price of residential plan \( i \). Note that \( P_i \) is increasing in \((p_k, p_l)\). Using this definition, we have that indirect utility from residential plan \( i \) is given by:

\[ V^y_i = G_i - (w - \delta_i mc)^{-\rho}P_i, \]  \hspace{1cm} (19)

where \( G_i \equiv G(g_k, g_l) \) for \( i = kl \). As a final step, let \( T \equiv mc/w \) and again rewrite indirect utility using type-dependent price \( P^T_i \).

\[ V^y_i = G_i - w^{-\rho}P^T_i; \]  \hspace{1cm} (20)

\(^{17}\)The discount factor \( \beta \) is subsumed in the old age utility function with no loss of generality.
where

\[ P_i^T = \begin{cases} P_i & \text{if } i \text{ does not move } (k = l) \\ P_i(1 - T)^{-\rho} & \text{if } i \text{ moves } (k \neq l) \end{cases} \] \quad (21)

Household type \((w, T)\) then chooses a residential plan \(i\) to maximize \(V_i^y\) in (20) taking \((G_i, P_i^T)\), \(i \in I_{kl}\), as given.

Immediate properties of the choice problem are summarized in the following lemma:

**Lemma 3** The choice problem must satisfy the following conditions:

(P1) Indifference curves \(V_i^y = \text{const.}\) in the \((G_i, P_i^T)\) plane are linear with slope \(w^\rho\).

(P2) Indifference curve satisfy single crossing, with “slope increasing in income (SII).”

(P3) \(dP_i^T/dT > 0\) for \(k \neq l\); choices with moving are effectively more expensive as \(mc\) rises.

Properties (P1) – (P3) are intuitive and simply confirmed. (P1) will greatly simplify the analysis that follows. (P2) and (P3) are keys to the character of sorting over communities over the life cycle.

**An Example.** In our computational analysis below, we adopt a CES lifetime utility function:

\[
U = \frac{1}{\rho} \left[ \alpha_g g^\rho_k + \alpha_h h^\rho_k + \alpha_b b^\rho_k + \beta_g g^\rho_l + \beta_h h^\rho_l + \beta_b b^\rho_l \right], \rho < 0; \quad (22)
\]

which satisfies Assumption 6 and where we have assumed again residential plan \(i = kl\). After some manipulation one obtains indirect utility:\(^{18}\)

\[
V_i^y = G_i - (w - \delta mc)^\rho P_i; \quad (23)
\]

where:

\[
P_i = -\frac{1}{\rho} z_{kl}^{-\rho} \left[ \alpha_h \left( \frac{\alpha_b}{\alpha_h} p_k \right)^{-\frac{1}{1-\rho}} + \alpha_b + \beta_h \left( \frac{\alpha_b}{\beta_h} p_l \right)^{-\frac{1}{1-\rho}} + \beta_b \left( \frac{\alpha_b}{\beta_b} \right)^{-\frac{1}{1-\rho}} \right];
\]

\[
z_{kl} = \left[ p_k \left( \frac{\alpha_b}{\alpha_h} \right)^{-\frac{1}{1-\rho}} + 1 + p_l \left( \frac{\alpha_b}{\beta_h} \right)^{-\frac{1}{1-\rho}} + \left( \frac{\alpha_b}{\beta_b} \right)^{-\frac{1}{1-\rho}} \right]; \quad (24)
\]

\[
G_i = \frac{1}{\rho} \left[ \alpha_g g^\rho_k + \beta_g g^\rho_l \right].
\]

\(^{18}\)A minus sign does not arise with the exponent \(\rho\) in (22), in contrast to the general development leading to (20), because \(\rho\) is negative in this CES example.
Figure 2:
Indifference curve of household with $w_2$

Indifference curves of household with $w_1$

utility increasing
Figure 2 depicts some indifference curves in the \((P, G)\) space for households of two wealth levels \(w_2 > w_1\).^{19}

Keeping in mind that \(\rho < 0\), one can see that all the properties of the preceding more general case are satisfied. In particular the composite public good \(G_i\) is increasing in the \(g\)'s and the composite price \(P_i\) is increasing in the \(p\)'s.

With \(J\) communities, there are \(J^2\) residential plans that could feasibly be chosen. Using properties of the choice problem, we can develop restrictions on the set of plans that are actually chosen and then develop an algorithm for mapping household types into their equilibrium residential plans. Let \(B^0 \equiv \{G_i, P_i^T \mid i \in I_k\}\) denote the set of bundles, corresponding to residential plans, that are feasible for households with \(T = mc/w\). Let \(H^T\) denote the convex hull of \(B^0\) and let \(B^0(T)\) denote the set of residential plans \((G_i, P_i^T)\) on the lower boundary of \(H^T\). Formally, \(B^0(T)\) is defined:

\[
B^0(T) \equiv \{ (G_i, P_i^T) \in B^0 \mid \text{no distinct} (\tilde{G}_i, \tilde{P}_i^T) \in H^T \text{exist with } \tilde{G}_i \geq G_i \text{ and } \tilde{P}_i^T \leq P_i^T \} \quad (25)
\]

Figure 3 shows an example of these concepts for a case with \(J = 3\). We have:

**Proposition 1** (i) Any and all non-moving residential plans chosen by households with the maximum \(T\) comprise the non-moving residential plans chosen by all households.

(ii) Any and all moving plans chosen by households with the minimum \(T\) comprise the moving residential plans chosen by all households.

First, prove Lemma 4.

**Lemma 4** Households with relative moving cost \(T\) choose in equilibrium any and all residential plans in \(B^0(T)\).

**Proof of Lemma 4:** Households with \(T\) maximize \(V_i^y\) as defined in (20) – (21). Since \(V_i^y\) is increasing in \(G_i\) and decreasing in \(P_i^T\), households choose among the residential plans in \(B^0(T)\). Since \(w\) ranges from 0 to \(\infty\), the slope of an indifference curve in the \((G_i, P_i^T)\) plane

\(^{19}\)The underlying CES utility function leads the composite public good \(G_i\) to be negative.
Figure 3:

Notation: *'s denotes bundles that could be chosen in equilibrium.
ranges from 0 to $\infty$ as well, implying all plans in $B^0(T)$ are chosen by some households with $T$. Q.E.D.

**Proof of Proposition 1**: (i) Obviously all non-moving residential plans chosen by households with the maximum $T$ are in the set of chosen residential plans by all households. To confirm that only these non-moving plans are equilibrium ones, observe from (21) that, since $P^T_i$ is increasing in $T$ for moving plans and independent of $T$ for non-moving plans, lowering $T$ can eliminate but cannot add non-moving plans to $B^0(T)$. It follows from Lemma 4 that no households with lower $T$ than the maximum choose a non-moving residential plan not chosen by a household with the maximum $T$.

(ii) Let $T_m$ denote the minimum $T$. Obviously all moving plans chosen by such households are in the equilibrium set of moving plans. To confirm only such moving plans are in the equilibrium set of all households, suppose household “2” with $(w_2, T_2)$, $T_2 > T_m$, chooses a moving plan $lk$ in equilibrium that is not chosen by any households with $T_m$. Consider household “1” with $(w_1, T_1) = (w_2 \frac{1-T_2}{1-T_m}, T_m)$. Note that $w_1 < w_2$. Households 1 and 2 obtain the same level of utility from all moving plans (by (20) – (21)). Household 1 obtains lower utility from all non-moving plans than does household 2, since household 1 has lower wealth (and moving costs are irrelevant). But then household 1 would share household 2’s preference for moving plan $lk$, a contradiction. Q.E.D.

We now show that equilibrium is characterized by a conditional wealth stratification property. Let $J_e \leq J^2$ denote the number of residential plans chosen by any household.\(^{20}\) Number these plans 1,2,.., $J_e$ such that $G_1 < G_2 < ... < G_{J_e}$. We make the following assumption:

**Assumption 7** *The maximum $T$ prohibits moving in equilibrium for all wealth types.*

Then:

**Lemma 5** *The plan with $G = G_1$ corresponds to $lk = 11$ and the plan with $G = G_{J_e}$ corresponds to $lk = JJ$.*

\(^{20}\)Later we show that $J_e < J^2$ under reasonable restrictions.
Proof of Lemma 5: The residential plans on the lower boundary of the convex hull of all feasible plans corresponds to just non-moving plans for any types with T that will never move in equilibrium. Plans $lk = 11$ and $lk = JJ$ are the endpoints of the lower boundary of the convex hull for all of these types. The result then follows from Assumption 7 and Lemma 4. Q.E.D.

A simple property of equilibrium residential plans is that there will be “ascending bundles” conditional on type.

Lemma 6 (Ascending bundles.) Given residential plans chosen in equilibrium by household with T satisfying $G_i > G_j, P_{i}^{T} > P_{j}^{T}$.

Proof of Lemma 6: If $P_{j}^{T} \geq P_{i}^{T}$, then choice of plan j would contradict maximization of $V_{y}$ (recall (20)). Q.E.D.

Note that the hierarchical ordering of residential plans is consistent across types T though the composite prices differ and the subset of the $J_e$ plans chosen by different T types vary.

The conditional wealth stratification property is:

Proposition 2 (Conditional wealth stratification) For given T, if $w_2 > w_1$ and household with wealth $w_2$ chooses plan with $G_i$ and household with wealth $w_1$ chooses plan with $G_j (j \neq i)$, then $i > j$.

Proof of Proposition 2: Using that households chose residential plans to maximize $V_{y}$, wealth stratification follows from the ascending bundles property and SII. Q.E.D.

Figure 4 shows an example with $J = 3$ of the partition of young households by type $(w, T)$ across residential plans $kl$. In this example, only one residential plan entailing moving arises in equilibrium, with some households choosing the highest $g$ community while young $(k = 3)$ and then the lowest $g$ community when old $(k = 1)$. There are three no-moving plans and thus $J_e = 4$.

Another type of restriction on equilibrium residential plans derives from limits on the relative values of the parameters of the utility function. For the preference function used in
Figure 4:
our computational analysis (introduced in 22), we provide conditions such that no household will move when old to a community with higher \( g \) for the utility function in (22) – (24). We assume that:

**Assumption 8** \( \alpha_g \alpha_h^{1/(\rho-1)} > \beta_g \beta_h^{1/(\rho-1)} \).

**Proposition 3** No household will choose a community with higher \((p, g)\) pair when old than when young in a stationary equilibrium.

**Proof:** The proof is by contradiction, so suppose a household makes such a choice. Then that choice solves the program:

\[
\max_{h_k, b_k, h_l, b_l} U = \frac{1}{\rho} [\alpha_g g_k^p + \alpha_h h_k^p + \alpha_b b_k^p + \beta_g g_l^p + \beta_h h_l^p + \beta_b b_l^p] \quad \text{(26)}
\]

s.t. \( w - mc = p_k h_k + b_k + p_l h_l + b_l \)

with \((p_k, g_k) < (p_l, g_l)\). Let:

\[
L^* \equiv \frac{1}{\rho} [\alpha_g g_k^p + \alpha_h h_k^p + \alpha_b b_k^p + \beta_g g_l^p + \beta_h h_l^p + \beta_b b_l^p] + \lambda [w - mc - p_k h_k - b_k - p_l h_l - b_l] \quad \text{(27)}
\]

denote the Lagrangian function at the household’s optimum, where \( \lambda \) denotes the multiplier on the budget constraint. Thus, \( V_{kl}^y(p_k, g_k, p_l, g_l) \equiv L^*(p_k, g_k, p_l, g_l) \). Using the latter and (27), compute, respectively, slopes of the indifference curves over \((p, g)\) pairs while young and \((p, g)\) pairs while old:

\[
\left. \frac{dp_k}{dg_k} \right|_{V_{kl}^y=\text{const.}} = -\frac{\partial V_{kl}^y}{\partial\partial g_k^{p_k}} = -\frac{\partial L^*}{\partial\partial g_k} = \frac{\alpha_g g_k^{p_k-1}}{\lambda h_k} ; \quad \text{(28)}
\]

and

\[
\left. \frac{dp_l}{dg_l} \right|_{V_{kl}^y=\text{const.}} = -\frac{\partial V_{kl}^y}{\partial\partial g_l^{p_l}} = -\frac{\partial L^*}{\partial\partial g_l} = \frac{\beta_g g_l^{p_l-1}}{\lambda h_l} ; \quad \text{(29)}
\]

where the last equality in each of (28) and (29) uses the Envelope Theorem. Using the first-order conditions from (26), one obtains:

\[
h_k = \frac{w - mc}{z_{kl}} \left( \frac{\alpha_h}{\alpha_h p_k} \right)^{1/(\rho-1)} ; \quad \text{(30)}
\]
and
\[ h_t = \frac{w - mc}{z_{kl}} \left( \frac{\beta_h}{\beta_h p_t} \right)^{1/(\rho-1)}. \] (31)

Substituting (31) into (29) and (30) into (28), one finds that the indifference curve over \((p, g)\) pairs while young are everywhere steeper than the indifference curve over \((p, g)\) pairs while old if \(\alpha_g \alpha_h^{1/(\rho-1)} > \beta_g \beta_h^{1/(\rho-1)}\), i.e., under Assumption 8. In a stationary equilibrium, the \((p, g)\) pairs available in each period of life are the same. The steeper curve in Figure 5 shows the indifference curve of the young household that chooses community \(k\) while young given community \(l\) is available (with \((p_k, g_k) < (p_l, g_l)\)). This curve must pass below the point \((p_l, g_l)\) as shown, or the household would prefer community \(l\) while young. (The fact that the household would save moving costs by choosing \(l\) while young, while the indifference curves assume moving costs are paid, only reinforces the claim.) The flatter indifference curve shows that of the household through \((p_h, g_h)\) when old, which implies the household would prefer to choose community \(l\) when old while paying moving costs. The fact that the household would not have to pay moving costs (since it resides initially in community \(l\)) implies a stronger yet preference for community \(l\) when old, hence a contradiction. Q.E.D.

The willingness to pay a higher housing price to live in a community with higher \(g\) increases with the coefficient on \(g\) in the period utility function and decreases with the coefficient on housing. While the presence of children when young indicates that both \(\alpha_g > \beta_g\) and \(\alpha_h > \beta_h\) are to be expected, the condition of Proposition 3 implies that the relatively stronger preference for \(g\) when young outweighs the relatively stronger preference for housing so that moving to a higher \((p, g)\) community when old would not result.

5 Quantitative Analysis

Equilibria of this model can only be computed numerically. We next turn to the quantitative part of the analysis. We first present an algorithm that can be used to compute equilibria. To implement the algorithm, we must fully specify the model choosing functional forms and assigning parameter values. This section specifies a benchmark model and reports some of its
Figure 5:

Indifference curve over communities when young

Indifference curve over communities when old
quantitative properties.

5.1 Computation of Stationary Equilibria

Given the sorting patterns above, a stationary equilibrium in this model is determined fully by a vector \( \{p_j, g_j, \tau_j\}_{j=1}^J \). Computing an equilibrium is equivalent to finding a root to a system of \( 3 \times J \) nonlinear equations. For each community, the three equations of interest are the housing market equilibrium in (11), the balanced budget requirement in (12), and the majority rule equilibrium requirement. An algorithm for computing an equilibrium can be constructed as follows:

1. Given a vector \( (p_j, \tau_j, g_j) \) we can compute \( p^h_j \) from the identity \( p_j = (1 + \tau_j)p^h_j \).

2. For each young household type \( (w, mc) \), we can compute the optimal residential choices for both time periods. Hence we can characterize household sorting across the \( J \) communities.

3. Given the residential decisions, we can characterize total housing demand, as well as total government revenues for each community.

4. Given \( p^h_j \), we can compute housing supply for each community, and check whether the housing market clears in each community.

5. Given \( g_j \), we can check whether the budget in each community is balanced.

6. For each young household and each old household living in community \( j \) we solve for the bliss point by searching over the set of feasible policies given by equation (13). Given the bliss points of each voter and the fact that preferences in this model are single-peaked, it is straight-forward to compute the bliss point of the median voter. We can then check whether the bliss point of the median voter is \( g_j \).

The algorithm thus implies that we need to find a root of \( 3 \times J \) dimensional system of linear equations.
5.2 Implementation (Preliminary)

Given the parametrization of the utility function of young households in equation (22), the indirect utility function for an old household that used to live in $k$ at $t - 1$ and lives in $j$ at time $t$ is given by:

$$V_{jt}^o = \frac{1}{\rho} \left[ \beta_g g_{jt}^\rho + \beta_h \left( \frac{w^o_{jt}}{z^o_t} \right)^\rho \left( \frac{h}{p_{jt} h_b} \right)^{1-\rho} + \beta_b \left( \frac{w^o_{jt}}{z^o_t} \right)^\rho \right]$$

where

$$z^o_t = \left[ p_t \left( \frac{h}{p_{jt} h_b} \right)^{1-\rho} + 1 \right]$$

To calibrate the parameters of the utility function of young and old households, note that $\rho$ needs to be negative for preferences to satisfy single crossing conditions. Furthermore, the border line case of $\rho = 0$ implies Cobb-Douglas preferences. In our numerical exercises, we set $\rho = -0.4$. To calibrate the remaining parameters, note that old households are likely to have – on average – a significantly lower demand for housing and public goods than younger households. Moreover, average housing expenditures for families is approximately 30 percent. These considerations suggest that the following parameter values are reasonable choices: $\alpha_h = 0.3 : \beta_h = 0.2 : \alpha_g = 0.08 : \beta_g = 0.03 : \alpha_b = 0.4 : \beta_b = 0.5$.

We also assume that the logarithm of income and the level of moving costs are normally distributed. In 2005, US mean and median incomes were $63,344 and $46,326. Taking the income distribution to be log-normal, these imply that $\mu_{lny} = 10.743$ and $\sigma_{lny}^2 = .626$. We calibrate the model by treating each of the two periods of adult life in our model as “representative years.” That would permit us to interpret the equilibrium values of variables as typical annualized values for a young and an old household respectively. This would imply that $w$ is twice annual income, and $\mu_w = \ln(2) + \mu_{lny} = \ln(2) + 10.743 = 11.436$. Hence, we assume that $\ln(w) \sim N(11.436, .626)$. this implies that the mean of $w$ is $112,638 with standard deviation $78,018.

The pecuniary costs of a move are a rather small fraction of lifetime income. For example, suppose that the value of a home is 4 times annual income, and moving costs are 10% of home
value. Then, with a 40-year working lifetime, moving costs would be 1% of lifetime income. Psychic costs of moving are presumably the predominant cost of moving. We assume that $mc \sim N(1500, 100)$. Since moving costs and income are likely to be positively correlated, we set the correlation of $mc$ with $\ln(w)$ to be .5.

Finally, we assume that the housing supply has constant elasticity $\tau$ and is given by

$$H^*_{jt} = [p^h_{jt}]^\eta$$

(34)

Note that this assumption implies that the housing supply function is the same in each community and set the supply elasticity, $\eta$, equal to 3 which is a conservative estimate (Epple, Gordon, and Sieg, 2007).

5.3 Quantitative Properties of Equilibrium (Preliminary)

Having fully parametrized and calibrated the model, we can solve the model numerically. Table 4 illustrates an equilibrium with two communities.

Table 4: Equilibrium with 2 Communities

<table>
<thead>
<tr>
<th></th>
<th>community 1</th>
<th>community 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>fraction of young</td>
<td>0.599</td>
<td>0.401</td>
</tr>
<tr>
<td>fraction of old</td>
<td>0.653</td>
<td>0.346</td>
</tr>
<tr>
<td>lifetime income young</td>
<td>66521</td>
<td>181398</td>
</tr>
<tr>
<td>average housing demand young</td>
<td>1609</td>
<td>3867</td>
</tr>
<tr>
<td>average housing demand old</td>
<td>1308</td>
<td>3003</td>
</tr>
<tr>
<td>net price of housing</td>
<td>11.54</td>
<td>12.55</td>
</tr>
<tr>
<td>revenue per student</td>
<td>4609</td>
<td>13609</td>
</tr>
<tr>
<td>tax rate</td>
<td>0.124</td>
<td>0.155</td>
</tr>
<tr>
<td>gross of tax housing price</td>
<td>13.72</td>
<td>15.84</td>
</tr>
</tbody>
</table>

We find that poor households live in community 1 for both periods. On average, medium
income level households and high income households with high mobility costs live in the expensive community in both periods. Only the richest households with relatively low mobility costs first live in the expensive community and then move to the cheap community. This result follows from the fact that the gains from moving increase in income because of the positive housing demand elasticity. Young households that live in community 1 are significantly poorer than young households living in community 2. Mean income of young households in community 1 is $66,521 compared to $181,398 in community 2. While the difference in average housing consumption among the two communities is large for young households, it is much smaller for old households. This is due to lack of stratification of old households, i.e. old households living in community 1 are either fairly poor or fairly rich.

### Table 5: Equilibrium with 3 Communities

<table>
<thead>
<tr>
<th></th>
<th>Community 1</th>
<th>Community 2</th>
<th>Community 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of young</td>
<td>0.413</td>
<td>0.336</td>
<td>0.250</td>
</tr>
<tr>
<td>Fraction of old</td>
<td>0.497</td>
<td>0.336</td>
<td>0.166</td>
</tr>
<tr>
<td>Life time income of young</td>
<td>54258</td>
<td>107187</td>
<td>216077</td>
</tr>
<tr>
<td>Average housing demand young</td>
<td>1435</td>
<td>2629</td>
<td>4959</td>
</tr>
<tr>
<td>Average housing demand old</td>
<td>1410</td>
<td>1968</td>
<td>4206</td>
</tr>
<tr>
<td>Net price of housing</td>
<td>12.40</td>
<td>13.51</td>
<td>14.58</td>
</tr>
<tr>
<td>Revenue per student</td>
<td>4704</td>
<td>8984</td>
<td>16297</td>
</tr>
<tr>
<td>Tax rate</td>
<td>0.137</td>
<td>0.169</td>
<td>0.168</td>
</tr>
<tr>
<td>Gross of tax housing price</td>
<td>12.40</td>
<td>13.51</td>
<td>14.58</td>
</tr>
</tbody>
</table>

Community 1 is significantly larger than community 2. It contains 59.9 percent of young households and 65.3 percent of old households; 5.4 percent of households relocate from community 2 to community 1 when becoming old. As a consequence old households are a majority in community 1. Young households are a majority in community 2. Since distributions of bliss points do not overlap in this example, the median voter in community 1 is a rich old
The median voter in community 2 is a relatively poor young household.

Table 5 illustrates an equilibrium with three communities. Here four of nine residential choice plans are used by households in this equilibrium. Most individuals do not move during their life time, 41.3 % of households always live in community 1, 33.6 % always live in community 2 and 16.6 % chose community 3 in both periods. 8.4 % of all households live in community 3 when young and community 1 when old. This result illustrates the findings in Section 4.2. While there are 9 potential residential plans in this model, only 4 of them are used in equilibrium.

Next we consider a slightly different parametrization of our model in which the logarithm of wealth and the logarithm of moving costs relative to wealth are jointly normally distributed. This assumptions simplifies the computational burden associated with computing equilibria. We compute equilibria with three and four communities. The next two figures illustrate the boundaries among adjacent communities and the equilibrium choice plans.

6 Conclusions

In this paper, we provide a new framework for studying the life cycle locational choices of heterogeneous households and the associated dynamics of metropolitan areas. In the context of an overlapping generations framework, we frame the decision problem of households making private market decisions (choosing community of residence and housing consumption over the life cycle), and participating in the collective choice process determining the public education expenditure of their community at each stage of the life cycle. From this characterization of household decisions, we set forth conditions for market equilibrium in each community, and we demonstrate existence of voting equilibrium in all communities. We characterize potential life cycle residential plans of households and the associated patterns of stratification across communities. Finally, we illustrate properties of the model quantitatively, via computed equilibria for two- and three-community settings. These computational results illustrate patterns

\footnote{In this example, the overlap of both distributions is negligible.}
Choice of Residential Plan in Equilibrium with Three Jurisdictions

Moving Cost as a Proportion of Lifetime Wealth

Lifetime Wealth

Moving Cost as a Proportion of Lifetime Wealth

(3,3)
(3,2)
(3,1)
(2,2)
(2,1)
(1,1)
Choice of Residential Plan in Equilibrium with Four Jurisdictions

Moving Cost as a Proportion of Lifetime Wealth

Lifetime Wealth

0.0000 0.0025 0.0050 0.0075 0.0100 0.0125 0.0150

(4,4) (4,3) (4,2) (4,1) (3,3) (3,2) (2,1) (1,1)
of household sorting and relocation over the life cycle, as well as the housing price, tax, and public spending levels that emerge within each of the jurisdictions.

Understand household and community dynamics is an important research area, and there is ample scope for future research. One interesting avenue for future research is to analyze the differences between families with and without children. In our model we have implicitly assumed that all families have children when young. However, a substantial fraction of households never have children. The life-cycle incentives of these individuals are different, since they do not have reason to pay the housing price premia to locate in areas with high quality public education. The presence of such households in the model can be expected to affect the age composition of communities as well as the outcomes that arise from voting over public good levels.

A second important generalization is introduction of peer effects. As demonstrated in Calabrese, Epple, Romer and Sieg (2007), peer effects in education can have very substantial impact on the character of equilibrium outcomes. In particular, econometric evidence in their static framework reveals that income stratification and associated stratification of peers influences expenditure policies, with poorer jurisdictions taxing heavily in order to provide expenditures to compensate for schools that have relatively disadvantaged peer groups. This is clearly an important issue both from the perspective of providing a fuller characterization of equilibrium and from the perspective of policy analysis.

A third important task for further work is to capture more fully the incentives affecting voting for public services, especially education. For example, households with grown children who plan to move have an incentive to support high provision of education to maintain housing demand and property values. These incentives depend on whether the household owns or rents, and on the household’s beliefs about the way in which quality of public services impacts rental prices or the value of the home. Property owners have different preferences over public good provision than renters since owners are affected by capital gains or losses that may arise from changes in public policies. The key complication in such a generalization is in characterizing voting equilibrium. Owner-occupants who anticipate capital gains and losses when voting

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have been incorporated in static models (Epple and Romer, 1991; Calabrese, Epple, Romano, 2007), and those investigations reveal that ownership substantially affects voter incentives and equilibrium outcomes. Introducing ownership into our dynamic framework is a challenging but important task for future research.
References


