Can Ambiguity Aversion explain the Equity Home Bias?

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Abstract

This paper examines how ambiguity about the distribution of asset returns affects equilibrium prices and equity holdings in a two-country CARA-normal setting. All investors possess the same information about the set of possible states and the corresponding returns distribution in each state, but they have different beliefs about the likelihood of these states. Optimism and overconfidence refer to the distorted beliefs about expected mean and dispersion of the asset returns distributions, respectively. I analyze and quantify the effects of optimism and overconfidence on asset prices and asset holdings when investors are ambiguity averse. Furthermore, I show that the equity home bias is larger in countries with smaller market capitalization. I investigate whether the equity home bias observed in data can be explained by intermediate degrees of ambiguity aversion.

JEL classification: G11, G15, D53, D81

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1 Introduction

Equity home bias is a well known puzzle in international finance, referring to a wide disparity between the actual portfolio weights and the weights recommended by the international equity portfolio theory. Under ideal conditions, the international capital asset market model predicts that investors should hold equities from around the world in proportion to their market capitalization. However, according to the empirical findings of French and Poterba [19] and Tesar and Werner [42], investors hold a substantially larger proportion of their wealth in domestic assets: US investors hold 92.2% of their equity portfolio in domestic stocks; Japanese investors - 95.7%; UK investors - 92%; German investors - 79%; French investors - 89.4%, and Canadian investors - 93.4%. This observed high concentration in domestic equity has become known as "equity home bias".

There have been various attempts to explain this puzzle. The first approach is based on information asymmetries\(^1\), hedging possibilities against domestic risk\(^2\), and barriers to international investment such as restrictions on international capital flows\(^3\), withholding taxes, and transactions costs\(^4\). Another approach focuses on investors behavioral biases, e.g., optimism about their domestic markets\(^5\) and preference for the familiar\(^6\). Lewis [35] and Strong and Xin [40] provide an extensive review of proposed explanations. Empirical studies\(^7\) find that home bias is caused by both institutional and behavioral factors.

A more recent research direction explains home bias by means of ambiguity aversion. According to the standard expected utility theory, agents are assumed to make decisions under uncertainty as if they have a prior belief about probability distribution over the set of possible states of the world and then maximize the expected utility according to this distribution. However, individuals often fail to accurately assess such probabilities. Knight [33] suggests that there is an important difference between events with objectively (or subjectively) known probabilities, and events where probabilities are unknown. Uncertainty of the first kind is called risk, and uncertainty of the second kind is called ambiguity or Knightian uncertainty. Ellsberg [16] demonstrates the significance of this distinction by showing that individuals may prefer gambles with specified probabilities over gambles with unknown odds. In the experiment, two urns are given: one contains 50 red balls and 50 black balls, and the other contains 100 red and black balls in unknown proportion. One ball is drawn at random from each urn. In gamble A, the payoff is $100 if a red ball is drawn and $0 if a black ball is drawn. In

\(^3\)Black (1974) [5], Stulz (1981) [41]
\(^4\)Tesar and Werner (1995) [42], Warnock (2002) [47], Obstfeld (2000) [38]
\(^5\)French and Poterba (1991) [19]
\(^6\)Huberman (2001) [26], Coval and Moskowitz (1999) [13]
\(^7\)Bailey, Kumar, and Ng (2005) [3], Karlsson and Norden (2004) [29], Kyrychenko, Shum (2006) [34]
gamble B, the payoff is $100 if a black ball is drawn and $0 if a red ball is drawn. When surveyed, many people choose to draw from first urn in both gambles. Such behavior contradicts the standard expected utility paradigm according to which participants form subjective beliefs in the form of a single probability distribution over the composition of balls in the second urn. This experiment has motivated various generalizations of subjective expected utility theory that incorporate ambiguity. One of the most popular approaches is the maxmin multiple prior model of Gilboa and Schmeidler where agents make decisions based on the worst among the many possible probability distributions for any given choice.

This paper develops a two-country model that illustrates how ambiguity about asset payoffs affects asset prices and portfolio holdings. Agents live in a Lucas pure-exchange economy with a safe asset and two country-specific risky assets. There is ambiguity about assets’ payoffs, i.e., the agents are uncertain about the exact probability distribution. Similarly to the model developed by Easley and O’Hara [14], ambiguity averse investors act as if they have a set of distributions on payoffs, and select a portfolio in order to maximize their utility over this set of distributions. Agents preferences are characterized by the smooth model of decision making under ambiguity that has been axiomatized by Klibanoff, Marinacci, and Mukerji [32]. The advantage of using the smooth model is that it allows for intermediate values of ambiguity aversion coefficients rather than the extreme cases of minimal expected utility and standard expected utility maximizing agents. Moreover, it also simplifies the analysis due to the smoothness conditions, which makes the model analytically tractable.

All investors possess the same information about the set of possible states and the corresponding returns distribution in each state, but have different beliefs about the likelihood of these states. Optimism and overconfidence refer to the distorted beliefs about expected mean and dispersion of the asset returns distributions, respectively. I show that the difference in beliefs about perceived uncertainty leads to the bias in portfolio holdings. The equilibrium portfolio allocation depends on the degree of ambiguity aversion as well as parameters that characterize uncertainty.

To see whether the equity home bias observed in data can be explained by a less extreme degree of ambiguity aversion, I analyze a numerical example using stylized facts about asset returns. I find that when investors are ambiguity averse then it is possible that even small difference in beliefs about perceived uncertainty may generate a home bias in portfolio holdings that is close to the data.

The two most closely related papers are Epstein and Miao [17] and Uppal and Wang [45]. Epstein and Miao use a recursive multiple-prior model, a multi-period extension of Gilboa and Schmeidler (1989) maxmin model. They consider agents (countries) who are equally ambiguity averse but have different sets of multiple priors, and hence do not agree on which states are ambiguous. Uppal and
Wang [45] study the portfolio choice when an investor accounts for model misspecification. They follow the robust control approach introduced by Hansen, Sargent and Tallarini [24] and Anderson, Hansen, and Sargent [2] where agents use a reference model to differentiate among the priors and maximize the minimum expected utility (minimize the worst case loss) over the set of possible models. Hansen, Sargent, Turmuhambetova, and Williams (2006) established that the model set of robust control can be viewed of as a particular specification of Gilboa and Schmeidler's set of priors. In their paper, Uppal and Wang show that if the confidence about joint stock distribution is low then small differences in the degree of confidence for the marginal payoff distribution will result in a significant under diversification relative to the standard mean-variance portfolio.

However, the notion of maxmin ambiguity aversion can be viewed as overly pessimistic and may not accurately reflect actual beliefs and preferences. In particular, Bossaerts, Guarnaschelli, Ghirardato and Zame [6] have shown that the attitude toward ambiguity varies across individuals. This suggests that modeling investors’ decisions by the maxmin rule may significantly overestimate the effects of ambiguity on asset holding and asset prices. Moreover, Condie [11] shows that in an economy where some agents are ambiguity averse (in the maxmin sense), and some are standard expected utility maximizers (in the Bayesian sense), the former are unlikely to survive if there is an aggregate risk. This suggests that agents who exhibit extreme ambiguity aversion may decide not to participate in the market, i.e. not to hold any foreign asset at all. Easley and O’Hara [14] study the non-participation of ambiguity averse individuals and examine its implications for the regulation of financial markets.

In contrast to the robust control approach, the preference representation by Klibanoff, Marinacci, and Mukerji has an axiomatic foundation and stays within the state-independent utility framework. Their model allows to smoothly aggregate the decision maker’s information about the subjective relevance of each possible probability measure as the true probability measure. This makes it similar to the Bayesian approach. Unlike in Uppal and Wang, in my model the degree of ambiguity aversion is the same for all assets, but investors perceive uncertainty differently for home and foreign assets. Also, my model examines the effect of the ambiguity on the asset prices and derives the upper bound on the degree of ambiguity aversion for participation in financial markets.

The idea that investors have different beliefs about uncertainty is supported by surveys and empirical studies. Several papers in the home bias literature have identified a systematic bias in investors’ payoff expectations. French and Poterba (1991) show that observed portfolio holdings could be explained by domestic investors having more optimistic expectations about domestic stocks than about foreign stocks. This has been confirmed by empirical studies for Japan (Shiller, Kon-Ya, and Tsutsui [39] and China (Chen, Kim, Nofsinger, Rui [7]), experimental studies for Germany (Kilka
and Weber [31]), and surveys of fund-managers (Strong, Xin [40]). Graham, Harvey and Huang [23] also study the link between competence and investor behavior where investor competence is measured through survey responses. They argue that the competence effect contributes to home bias. Tourani-Rad and Kirkby [43] investigate investor overconfidence, socialization and the familiarity effect, using a sample of New Zealand investors. They find support for the investor overconfidence theory, using characteristics such as past success, optimism, confidence in one’s abilities, investment experience and investment-related knowledge. Lutje and Menkhoff [36] find that belief in an informational advantage and relative payoff optimism towards home assets are the driving forces of home bias. They argue that informational advantage often appears to be a perceived advantage, as fund managers with a home preference do not forecast stock indices better than others, and they rely less on fundamental analysis. Christoffersen and Sarkissian [9] relate geographic location and investor behavior by comparing the performance of U.S. equity mutual funds located in and outside of financial centers. They argue that fund managers in financial centers tend to be more overconfident because of their proximity to private information. Furthermore, Van Nieuwerburgh and Veldkamp [46] argue that even when home investors can learn what foreigners know, they choose not to. They show that learning amplifies information asymmetry since investors profit more from knowing information others do not know.

The remainder of the paper is organized as follows: the model environment is described next in section 2. In section 3, I consider three cases of distorted beliefs: in case 1, there is no uncertainty about the home asset, in case 2, investors are overconfident about the home asset, and in case 3, investors are more optimistic about the home asset. Section 4 provides the equilibrium characterization. Section 5 describes the equilibrium properties. Section 6 presents the numerical results, and section 7 concludes. All proofs are delegated to the Appendix.

### 2 Model

I consider a model with two countries: A and B. One can think about country A as one particular country and about country B as the rest of the world. The total number of agents in both countries is \( I \), \( \lambda I \) living in country A and \( (1 - \lambda)I \) living in country B, where \( \lambda \) is between zero and one. Agents live in a Lucas pure-exchange economy. There are three assets in the economy: one safe asset \( m \) (bond) and two risky assets which are country-specific. The safe asset is the same in both countries and its price and payoff are normalized to one. In addition, each country has a risky asset, which yields a stochastic dividend in every period. The holdings of risky asset in country
$k \in \{a, b\}$ by an investor $i$ is denoted by $x_i$. All agents are identical within the country they live in, and each agent from country $k$ is endowed with $m$ units of money and $x_k$ holdings of the home asset. Hence, the total endowment in the economy is $(I m, \lambda I x_a, (1 - \lambda) I x_b)$. All assets are traded on the international market so investors from both country have access to the market. The price of risky asset of country $k$ is $p_k$ and the payoff is $r_k$.

There are infinitely many possible (hidden) states; in each state $s$, asset payoff $r_k$ is normally distributed with mean $\bar{r}_k(s)$ and variance $\sigma^2_k(s)$. I assume that there is uncertainty only about the mean $\bar{r}_k(s)$ of asset’ payoffs, the asset payoff variance is the same in all states: $\sigma_k(s) = \sigma_k$. Furthermore, the payoffs on both assets are independent. Investors do not know which state will be realized, so they form beliefs about a set of possible realizations of mean payoffs for each asset. The investors’ beliefs are modeled as second-order priors over the first order probability distributions of asset payoffs. The set of priors represent the ambiguity about asset payoffs. This partition into first and second order distributions captures the separation between (objective) information and (subjective) beliefs.

The wealth of each investor $i$ from country $k$ is equal to $w_i = (r_a - p_a)x_{ai} + (r_b - p_b)x_{bi} + m + p_i x_k$. Investors choose their optimal portfolio $(x_{ai}, x_{bi})$ to maximize their utility function.

The utility function is adapted from the smooth model of decision making under ambiguity by Klibanoff, Marinacci, and Mukerji (2005). The individual preferences are represented by

$$U(w) = E_{\mu} [\phi(E_{\pi_s}[u(w)|s_n])]$$

where $u(\cdot)$ is a von Neumann-Morgenstern utility function, $\pi_s$ is a known probability distribution in each state $s$, and $\mu$ is subjective probability distribution over the possible probabilities $\pi_s$. The subjective prior $\mu$ weights the importance of each distribution $\pi_s$ reflecting an investor’s beliefs about the likelihood of each state. The increasing function $\phi$ characterizes the attitudes towards

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8For typographical convenience subscripts $a$ and $b$ refer to country $A$ and $B$, respectively.

9Agents only observe the realization of assets’ returns, the realization of states is not observed.
ambiguity. The degree of ambiguity aversion is defined as

\[ \alpha(y) = -\phi''(y)/\phi'(y). \]

If function \( \phi \) is concave then it characterizes ambiguity aversion, which is defined as an aversion to mean preserving spreads in \( \mu \). If function \( \phi \) is linear then the reduction of compound lotteries can be applied and it becomes equivalent to the standard subjective expected utility. The model of maxmin expected utility:

\[ U(\cdot) = \min_\pi E_\pi [u(\cdot)] \]

may be seen as an extreme case of my model with infinite degree of ambiguity aversion.

The smooth model allows the separation between ambiguity (a decision maker’s subjective beliefs \( \mu \)) and ambiguity attitude (a characteristic of the decision maker’s preferences \( \alpha \)). It smoothly aggregates the decision maker’s information about likelihood of each possible probability distribution, consequently, the indifference curves are smooth rather than kinked. Note that in maxmin models, the decision maker only looks at the the worst value.

I assume \( u(w) = -e^{-\gamma w} \) is a CARA utility function where \( \gamma \) is the degree of risk aversion. If investors are ambiguity neutral then \( \phi \) is linear: \( \phi(y) = y \); if investors are ambiguity averse then \( \phi(y) = -e^{-\alpha y} \) where \( \alpha \) is the degree of ambiguity aversion. These assumptions on investors preferences together with the normally distributed payoffs allow to derive results for prices and asset holdings in closed-form.

3 Overconfidence and Optimism

Investors believe that possible mean payoffs \((\tau_a(s), \tau_b(s))\) are jointly normally distributed with mean

\[ (\tau_a, \tau_b) \]

and covariance matrix

\[ \begin{bmatrix} \delta_a^2 & \delta_{ab} \\ \delta_{ab} & \delta_b^2 \end{bmatrix}, \]

where \( \delta_a^2 \) characterizes the dispersion of possible distributions for each asset and \( \delta_{ab} \) characterizes the correlation between states. This correlation reflects the investors’ beliefs that if more favorable state is realized for one country than it is more likely to be realized for the other one as well. The correlation is based on investors’ expectations rather than fundamentals, allowing to capture a possible contagion effect between the two countries. The investors’ expectations (beliefs) are the driving force of at least some episodes of financial market contagion. If investors expect the asset returns in different countries to be correlated then their investment decisions create links between otherwise separate markets that may lead to financial contagion. (Goldstein and Pauzner [21] and Keister[30])

The beliefs about the dispersion of payoffs distributions depend on whether the asset is domestic or foreign. Investors believe that there is less uncertainty about the home asset than about the foreign asset, and they are more optimistic about the payoffs on the home asset. These assumptions are supported by the findings of Kilka and Weber [31]. They conduct a cross-country study in Germany and the U.S. to investigate whether people’s subjective probability distributions on average
exhibit systematic differences in location and in dispersion. Their results show that people consider themselves to be on average more competent in forecasting domestic stock prices than in forecasting foreign stocks prices. Subjective probability distributions of stock payoffs are significantly less dispersed for domestic stocks (associated with high confidence levels) than for foreign stocks (associated with low confidence levels). Furthermore, domestic stocks are judged significantly more optimistically than foreign stocks. These observed patterns are consistent with biases in individual judgment documented by psychological research (Heath and Tversky [25]).

In this paper, I refer to optimism as distorted beliefs about the expected mean and overconfidence as distorted beliefs about the variance of mean returns distribution. The optimistic investors believe the expected mean is larger than the true value, in particular, beliefs about the home asset payoffs first order stochastically dominates beliefs about the foreign asset payoffs. The overconfident investors overestimate the precision of probability distribution of asset returns, in particular, beliefs about the home asset payoffs second order stochastically dominates beliefs about the foreign asset payoffs.

First, I will consider the economy with an extreme version of overconfidence in which investors completely ignore uncertainty about the home asset, and consequently they behave as standard expected utility maximizers with respect to home asset. In the next case, investors believe there is less uncertainty about the home asset, i.e. the dispersion of possible distributions is smaller for the home asset than for the foreign asset. Third, I consider the model where investors face the same uncertainty about home and foreign assets but they are more optimistic about payoffs on the home asset.

3.1 Case 1: No uncertainty about the home asset

First, consider the extreme case when investors completely ignore uncertainty about the home asset but not about the foreign asset. Investors form a single prior about payoffs on the home asset, i.e., instead of considering all possible distributions they put a mass point weight on one average distribution with mean $\tau_k$ and variance $\sigma_k^2$. In this case investors can exhibit any degree of ambiguity aversion $\alpha$ with respect to the home asset but it is irrelevant since their beliefs about asset payoffs consist of a single prior. Consequently, results are equivalent to having a linear $\phi$- function with respect to the asset payoffs, and therefore, it is equivalent to ambiguity neutrality with respect to that asset. For foreign asset, investors take into consideration all possible distributions and, therefore, the degree of ambiguity aversion $\alpha$ matters. Effectively, investors are ambiguity neutral with respect to domestic asset and ambiguity averse with respect to the foreign asset. The asset payoffs are normally distributed with some mean $\tau_k(s)$ and variance $\sigma_k^2$: $r_k \sim N(\tau_k(s), \sigma_k^2)$, $k = a, b$. 
Investors believe that the possible mean payoffs are equal to the average of mean payoffs if it is a home asset, or normally distributed with mean $r_k$ and variance $\delta_k^2$, if it is a foreign asset,

$$\begin{align*}
\bar{r}_k(s) &= r_k \quad \text{if} \quad k \text{ is home asset} \\
&\sim N(\bar{r}_k, \delta_k^2) \quad \text{if} \quad k \text{ is foreign asset}
\end{align*}$$

In the competitive equilibrium, investors choose portfolio holdings to maximize their expected utility, and prices are determined such that markets clear. Since asset payoffs are assumed to be distributed normally and investors have a CARA utility function, maximization problem can be expressed in terms of mean and variance.

An investor $i$ from country $k$ solve the following optimization problem:

$$\max_{x_{hi}, x_{fi}} \left\{ (\bar{r}_h - p_h)x_{hi} + (\bar{r}_f - p_f)x_{fi} - \frac{1}{2} \left( \sigma_h^2 x_{hi}^2 + \sigma_f^2 x_{fi}^2 + \alpha \delta_k^4 x_{hi}^2 x_{fi} \right) \right\}$$

where $h, f \in \{a, b\}$ denote respectively the home country and the foreign country for investor $i$.

From now on, denote investors from country $B$ by $j$ and investors from country $A$ by $i$. Then the optimal demands for the home and the foreign assets are given by

$$\begin{align*}
\text{country A investors } i: & \quad x_{ai} = \frac{r_a - p_a}{\gamma \sigma_a^2}; \quad x_{bi} = \frac{r_a - p_a}{\gamma (\sigma_a^2 + \alpha \delta_k^2)}; \\
\text{country B investors } j: & \quad x_{aj} = \frac{r_a - p_a}{\gamma (\sigma_k^2 + \alpha \delta_k^2)}; \quad x_{bj} = \frac{r_a - p_a}{\gamma \sigma_a^2}.
\end{align*}$$

Note that the difference in the demand functions for home and foreign assets depends on $\alpha \delta_k^2$ where $\alpha$ represents the ambiguity attitude and $\delta_k^2$ - the difference in beliefs about uncertainty of asset payoffs. If investors consider asset payoffs to be more uncertain then they demand less of that asset.

In equilibrium, the demand for optimal asset holdings should satisfy the market clearing conditions: the aggregate demand for optimal asset holdings should be equal to the total endowment,

$$\begin{align*}
\lambda x_{ai} + (1 - \lambda)x_{aj} &= \lambda \bar{r}_a; \\
\lambda x_{bi} + (1 - \lambda)x_{bj} &= (1 - \lambda)\bar{r}_b.
\end{align*}$$
For investor $i$ from country A, the equilibrium portfolio holdings $(x_{ai}, x_{bi})$ of asset $a$ and $b$ are given by

$$x_{ai} = \frac{\lambda \pi_a}{\sigma_a^2 + \alpha \delta_a^2}; \quad (6)$$

$$x_{bi} = (1 - \lambda) \frac{\sigma_b^2}{\sigma_b^2 + (1 - \lambda) \lambda \delta_b^2}.$$  

For investor $j$ from country B, the equilibrium portfolio holdings $(x_{aj}, x_{bj})$ of asset $a$ and $b$ are given by

$$x_{aj} = \frac{\lambda \pi_a}{\sigma_a^2 + \alpha \delta_a^2}; \quad (7)$$

$$x_{bj} = (1 - \lambda) \frac{\sigma_b^2}{\sigma_b^2 + \alpha (1 - \lambda) \delta_b^2}.$$  

If there is no ambiguity then the holding of each country’s asset is the same for investors from both countries, and should be equal to the per capita supply of that asset, i.e., $x_{ki} = x_{kj} = \lambda_k \pi_k$ where $\lambda_a = \lambda$ and $\lambda_b = 1 - \lambda$. However, if there is a difference in perceived uncertainty about the home and the foreign asset then portfolio holdings are biased towards the home asset: the holdings of home asset $x_h$ is larger than its market capitalization $\lambda_h \pi_h$ and the holdings of foreign asset $x_f$ is smaller than its market capitalization $\lambda_f \pi_f$ where $h, f \in \{a, b\}$.

Note it is not the ambiguity by itself that causes the bias in portfolio holdings but the distortion in beliefs. If investors perceive both assets as equally ambiguous then their asset holdings are proportional to the market capitalization. Moreover, Gollier [22] identified the conditions when the increase in ambiguity aversion can lead to the increase in demand for the ambiguous risky asset.

### 3.2 Case 2: Overconfidence about the home asset

In this section I relax the assumption that investors completely ignore uncertainty about the home asset, i.e., they behave as if they know the true distribution. Suppose investors are now effectively ambiguity averse with respect to both assets, home and foreign, but they believe there is less uncertainty about the home asset. In their beliefs they put more weight on distributions that are close to the mean distribution, i.e., the dispersion of possible distributions is smaller for the home asset than for the foreign asset. The payoffs of asset from country $k$ are normally distributed with some mean $\pi_k(s)$ and variance $\sigma_k^2$ : $\tau_k \sim N(\pi_k(s), \sigma_k^2)$, $k = a, b$. Investors believe that possible mean payoffs are normally distributed with mean $\pi_k$ and perceived variance $\delta_{kh}^2$ if $k$ is a home asset and perceived
variance $\delta_{kf}^2$ if $k$ is a home asset, where $\delta_{kh} < \delta_{kf}$.\(^{10}\)

\[
\begin{align*}
\tau_k(s) &\sim \begin{cases} 
N(\tau_k, \delta_{kh}^2) & \text{if } k \text{ is a home asset} \\
N(\tau_k, \delta_{kf}^2) & \text{if } k \text{ is a foreign asset}
\end{cases}
\end{align*}
\]

If the interstate correlation between assets is zero ($\delta_{ab} = 0$)\(^{11}\), then the equilibrium prices are given by

\[
\begin{align*}
p_a &= \tau_a - \lambda \pi_a \gamma \left( \sigma_a^2 + \alpha \delta_{ah}^2 \right) \left( \sigma_a^2 + \alpha \delta_{af}^2 \right) \frac{1}{\sigma_a^2 + \alpha (\lambda \delta_{af}^2 + (1 - \lambda) \delta_{ah}^2)}; \\
p_b &= \tau_b - (1 - \lambda) \pi_b \gamma \left( \sigma_b^2 + \alpha \delta_{bh}^2 \right) \left( \sigma_b^2 + \alpha \delta_{bf}^2 \right) \frac{1}{\sigma_b^2 + \alpha (\lambda \delta_{bf}^2 + (1 - \lambda) \delta_{bh}^2)}.
\end{align*}
\]

For investor $i$ from country A, the equilibrium portfolio holdings $(x_{ai}, x_{bi})$ of asset $a$ and $b$ are given by

\[
\begin{align*}
x_{ai} &= \lambda \pi_a \gamma \left( \sigma_a^2 + \alpha \delta_{ah}^2 \right) \frac{\sigma_a^2 + \alpha \delta_{af}^2}{\sigma_a^2 + \alpha (\lambda \delta_{af}^2 + (1 - \lambda) \delta_{ah}^2)}; \\
x_{bi} &= (1 - \lambda) \pi_b \gamma \left( \sigma_b^2 + \alpha \delta_{bh}^2 \right) \frac{\sigma_b^2 + \alpha \delta_{bf}^2}{\sigma_b^2 + \alpha (\lambda \delta_{bf}^2 + (1 - \lambda) \delta_{bh}^2)}.
\end{align*}
\]

For investor $j$ from country B, the equilibrium portfolio holdings $(x_{aj}, x_{bj})$ of asset $a$ and $b$ are given by

\[
\begin{align*}
x_{aj} &= \lambda \pi_a \gamma \left( \sigma_a^2 + \alpha \delta_{ah}^2 \right) \frac{\sigma_a^2 + \alpha \delta_{ah}^2}{\sigma_a^2 + \alpha (\lambda \delta_{ah}^2 + (1 - \lambda) \delta_{ah}^2)}; \\
x_{bj} &= (1 - \lambda) \pi_b \gamma \left( \sigma_b^2 + \alpha \delta_{bf}^2 \right) \frac{\sigma_b^2 + \alpha \delta_{bf}^2}{\sigma_b^2 + \alpha (\lambda \delta_{bf}^2 + (1 - \lambda) \delta_{bf}^2)}.
\end{align*}
\]

As in the previous case, the portfolio holdings are biased towards the home asset. For the home asset, equilibrium holdings are larger than the market capitalization: $x_{ai} > \lambda \pi_a$ and $x_{bj} > (1 - \lambda) \pi_b$.

\(^{10}\)That is, $\delta_{ah}^2$ denotes the perceived variance of asset $a$ by investors from country $A$ and $\delta_{af}^2$ denotes the perceived variance of asset $a$ by investors from country $B$. Similarly for the perceived variance of the asset $b$.

\(^{11}\)See section 7.2 of the Appendix for the solution when $\delta_{ab} \neq 0$. 

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For the foreign asset, equilibrium holdings are smaller than the market capitalization: \( x_{bi} < \lambda \pi_b \) and \( x_{aj} < (1 - \lambda) \pi_a \). When investors face uncertainty (ambiguity) about both, home and foreign, assets, their portfolio holdings will be biased towards the home asset if they are overconfident about the home asset relative to the foreign. The extent of the bias depends on the difference in perceived uncertainty about two assets: \( \delta_{kf}^2 > \delta_{kh}^2 \).

### 3.3 Case 3: Optimism about the home asset

Now suppose investors face the same uncertainty about the home and the foreign asset but they are more optimistic about payoffs on the home asset. Investors have distorted beliefs about the second-order distributions over states, with respect to the home asset they are optimistic about the realization of states with realization of payoffs mean above average, and with respect to the foreign asset, investors think that the states with realization of payoffs mean which is below average is more likely.

The asset payoffs are normally distributed with some mean \( \pi_k(s) \) and variance \( \sigma_k^2 : r_k \sim N(\pi_k(s), \sigma_k^2), k = a, b \). Investors believe that possible mean payoffs are normally distributed with mean \( \pi_k \) and variance \( \delta_k^2 \); i.e., \( \pi_k(s) \sim \begin{cases} N(\pi_{kh}, \delta_k^2) & \text{if } k \text{ is a home asset} \\ N(\pi_{kf}, \delta_k^2) & \text{if } k \text{ is a foreign asset} \end{cases} \) where \( \pi_{kh} > \pi_{kf} \).

![Optimism about home asset](image-url)

In this case, the equilibrium prices are given by

\[
\begin{align*}
p_a &= (\lambda \pi_{ah} + (1 - \lambda) \pi_{af}) - \lambda \pi_a \gamma (\sigma_a^2 + \alpha \delta_a^2) - (1 - \lambda) \pi_b \gamma \alpha \delta_{ab}; \\
p_b &= (\lambda \pi_{bf} + (1 - \lambda) \pi_{bh}) - (1 - \lambda) \pi_b \gamma (\sigma_b + \alpha \delta_b) - \lambda \pi_a \gamma \alpha \delta_{ab}.
\end{align*}
\]

For investor \( i \) from country A, the equilibrium portfolio holdings \( (x_{ai}, x_{bi}) \) of asset \( a \) and \( b \) are
given by

\[ x_{ai} = \frac{\lambda \pi_a}{1 + \frac{(1 - \lambda)(\tau_{ah} - \tau_{af})((\sigma_a^2 + \alpha \delta_a^2) + (1 - \lambda)(\tau_{bh} - \tau_{bf})\alpha \delta_{ab})}{\gamma ((\sigma_a^2 + \alpha \delta_a^2)(\sigma_b^2 + \alpha \delta_b^2) - (\alpha \delta_{ab})^2)}}; \]

\[ x_{bi} = (1 - \lambda)\pi_b \left(1 - \frac{(1 - \lambda)(\tau_{bh} - \tau_{bf})((\sigma_a^2 + \alpha \delta_a^2) + (1 - \lambda)(\tau_{ah} - \tau_{af})\alpha \delta_{ab})}{\gamma ((\sigma_a^2 + \alpha \delta_a^2)(\sigma_b^2 + \alpha \delta_b^2) - (\alpha \delta_{ab})^2)}\right). \]

For investor \( j \) from country B, the equilibrium portfolio holdings \((x_{aj}, x_{bj})\) of asset \( a \) and \( b \) are given by

\[ x_{aj} = \frac{\lambda \pi_a}{1 - \frac{\lambda(\tau_{ah} - \tau_{af})((\sigma_a^2 + \alpha \delta_a^2)) + \lambda(\tau_{bh} - \tau_{bf})\alpha \delta_{ab}}{\gamma ((\sigma_a^2 + \alpha \delta_a^2)(\sigma_b^2 + \alpha \delta_b^2) - (\alpha \delta_{ab})^2)}; \]

\[ x_{bj} = (1 - \lambda)\pi_b \left(1 + \frac{\lambda(\tau_{bh} - \tau_{bf})((\sigma_a^2 + \alpha \delta_a^2)) + \lambda(\tau_{ah} - \tau_{af})\alpha \delta_{ab}}{\gamma ((\sigma_a^2 + \alpha \delta_a^2)(\sigma_b^2 + \alpha \delta_b^2) - (\alpha \delta_{ab})^2)}\right). \]

As expected, there is a bias towards the home asset in portfolio holdings. The holdings of home asset is larger than its market capitalization \((x_h > \lambda_h \pi_h)\) and the holdings of foreign asset is smaller than its market capitalization \((x_f < \lambda_f \pi_f)\).

### 4 Portfolio Holdings and Ambiguity

The Proposition 1 summarizes and generalizes results considered in the previously analyzed cases.

**Proposition 1** If investors are ambiguity averse with respect to both assets and they are overconfident about the home asset relative to the foreign asset: \( \delta_{kh} < \delta_{kf}, \ k = a, b \), or they are optimistic about the home asset relative to the foreign asset: \( \tau_{kh} > \tau_{kf}, \ k = a, b \) then investors will choose their portfolio so that the proportion of the home asset is larger than its market share and the proportion of the foreign asset is smaller than its market share:

\[
\frac{x_{ai}}{x_{ai} + x_{bi}} > \frac{\lambda \pi_a}{\lambda \pi_a + (1 - \lambda)\pi_b}; \\
\frac{x_{aj}}{x_{aj} + x_{bj}} < \frac{\lambda \pi_a}{\lambda \pi_a + (1 - \lambda)\pi_b}.
\]

In equilibrium, the price for the home asset is higher than the expected price for the home asset, and the price for the foreign asset is lower than the expected price for the foreign asset based on the investor’s perceived ambiguity.

For simplicity, consider again the case 1 when investors completely ignore the uncertainty about the home asset. The investors from country A believe that the price for their home asset should
be \( p_{ah} = \tau_a - \lambda \sigma_a \gamma \sigma_a^2 \) which is the equilibrium price when all investors ignore uncertainty about the asset. However, since foreign investors view the asset \( a \) as ambiguous the equilibrium price is given by

\[
p_a = \tau_a - \lambda \sigma_a \gamma \sigma_a^2 \frac{\sigma_a^2 + \alpha \delta_a^2}{\sigma_a^2 + \alpha \lambda \delta_a^2}
\]  

(14)

The equilibrium price is higher than the price expected by home investors. Therefore, country A believe that asset \( a \) is overpriced, so they have incentive to hold more of the home asset. Similarly, country B investors believe that the price for asset \( a \) should be \( p_{af} = \tau_a - \lambda \sigma_a \gamma (\sigma_a + \alpha \delta_a^2) \), which is the equilibrium price when all investors view the asset \( a \) as ambiguous. This expected price \( p_{af} \) is lower than the equilibrium price \( p_a \). Since country B investors believe that foreign asset is underpriced, they hold less of it in their portfolio.

Therefore, investors believe that asset \( a \) is overpriced if it is home asset, and underpriced if it is a foreign asset. The same conclusions hold for asset \( b \) due to the symmetry. Hence, investors see the arbitrage opportunities and as a result hold more of the home asset and less of the foreign asset relative to their respective market capitalization weights. Therefore, the equity home bias arises as consequences of investors difference in beliefs about uncertainty of the asset returns. The same conclusions apply for both asset in more general cases when investors are optimistic and overconfident about the home asset relative the foreign asset.

Unlike in models with asymmetric information, in this framework prices are not informative. If prices are informative than informational advantage should eventually be arbitraged away through the active trading. In this model, the difference in actual vs expected asset price reflect the investors’ difference in beliefs. When an investor thinks that others have wrong beliefs then he has no incentive to adjust his portfolio allocation after observing prices different from what is expected.

5 Equilibrium Properties

5.1 Comparative statics

Next proposition summarizes the effects of change in the degree of ambiguity aversion, difference in the perceived mean returns and perceived dispersions of mean asset returns, correlation of asset returns, and market capitalization.

**Proposition 2.** The equity home bias is larger if (i) market capitalization \( \lambda \) is smaller; (ii) degree of ambiguity aversion \( \alpha \) is higher; (iii) difference in the perceived mean returns \( \Delta \tau_k = \tau_{kh} - \tau_{kf} \); (iv) difference in the perceived dispersions of mean asset returns \( \Delta \delta_k = \delta_{kf} - \delta_{kh} \) are larger; and (v) correlation of asset returns \( \rho_{ab} = \frac{\delta_{ab}}{\delta_a \delta_b} \) is positive.

\(^{12}\)It is equivalent to the case where all investors behave as standard expected utility maximizers.
The first result explains why countries with small market capitalization (like Canada or Scandinavian countries) exhibit significantly larger home bias relative to their market capitalization share. If investors from one country dominate the market then they have a large impact on equilibrium asset prices. So the deviation between equilibrium prices and the expected prices is smaller, therefore, the portfolio holdings are closer to the market capitalization weights. Similarly, if the proportion of investors from one country is relatively small than their asset holding will be strongly biased towards the home asset.

The next three results are intuitive: these parameters \((\alpha, \Delta \delta_k, \Delta \tau_k)\) directly contribute to the difference in perceived uncertainty about two assets. If any of these parameters increase it will lead to the increase of the home bias. If the degree of ambiguity aversion increases then the prices of both assets go down, the holding of the home asset may increase or decrease and the foreign asset holding decreases. Overall, the equity home bias becomes larger. If for a given asset the difference in the perceived dispersions increases then its equilibrium price goes down. The holding of this asset decreases if it is a foreign asset, and increases if it is a home asset. Therefore, the equity home bias becomes larger. If for a given asset the difference in the perceived mean returns increases then its equilibrium price goes up. The holding of this asset decreases if it is a foreign asset, and increases if it is a home asset. Therefore, the equity home bias becomes larger.

If investors believe that the state realization of asset return distributions are correlated then they will have incentive to hedge. If the correlation is negative then investors will diversify more due to hedging motives, hence, the equity home bias is smaller. On the other hand, the positive correlation reduces benefits from the diversification and leads to the larger home bias.

In the following table all effects of possible changes in parameters of asset \(a\) are summarized:

<table>
<thead>
<tr>
<th>()</th>
<th>(\alpha)</th>
<th>(\Delta \delta_a)</th>
<th>(\Delta \tau_a)</th>
<th>(\rho_{ab})</th>
<th>(\lambda)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>↓</td>
<td>↑</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>(x_{ah})</td>
<td>?</td>
<td>↑</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>(x_{af})</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>↑</td>
<td>↑</td>
</tr>
</tbody>
</table>

5.2 Non-participation

Another implication of my model is that there is an upper bound on the degree of ambiguity aversion that comes from the requirement of the asset price to be non-negative. Investors will choose to participate in the market only if they believe that the price for the foreign asset is positive. This means that if investors have a degree of ambiguity aversion \(\alpha\) such that
\[
\alpha \geq \frac{\tau_f - \lambda_f \beta_f \gamma \sigma_f^2}{\lambda_f \beta_f \gamma \delta_{k_f}^2}, \quad (15)
\]

they will choose not hold any of the foreign asset. If the degree of ambiguity aversion is too large, investor may prefer to hold on to their endowment of home asset, rather than bear ambiguity associated with the foreign asset. This upper bound out the participation of agents with maxmin type of preferences if there are no restrictions on the set of possible means for asset returns. The upper bound on the degree of ambiguity aversion is inversely related to the perceived ambiguity about the foreign asset characterized by \( \delta_{k_f}^2 \). Therefore, reducing ambiguity about the foreign asset will increase its portfolio share and, hence, decrease the home bias. Easley and O’Hara [15] and [14] demonstrate the potential benefits from reducing ambiguity, and examine the implications of the presence of ambiguity averse traders for market regulations such as deposit insurance and securities regulation.

5.3 Equity Premium

Define the equity premium as \( EP \equiv E[r_k]/p - 1 \). If all investors are ambiguity neutral (SEU) then equity premium is

\[
EP_{SEU} = \frac{\tau_k}{\tau_k - \lambda_k \beta_k \gamma \sigma_k^2} - 1. \quad (16)
\]

If all investors are ambiguity averse (AA) then equity premium becomes

\[
EP_{AA} = \frac{\tau_k}{\tau_k - \lambda_k \beta_k \gamma (\sigma_k^2 + \alpha \delta_k^2)} - 1. \quad (17)
\]

The equity premium is higher under ambiguity, and as degree of ambiguity aversion or the dispersion of possible distribution increases, the premium becomes larger. In the presence of ambiguity, risk sharing opportunities offered by financial markets become less complete which could lead to a no-trade equilibrium (Mukurji and Tallon [37]). The positive effect of ambiguity on the equity premium has been addressed as an application by several papers on decision theory under uncertainty (Epstein and Wang [18], Chen and Epstein [8]).

6 Numerical Results

In this section I will investigate the quantitative joint effect\(^{13}\) of optimism and overconfidence on asset holdings. The asset returns are normally distributed with some mean \( \tau_k(s) \) and variance \( \sigma_k^2 : r_k \sim N(\tau_k(s), \sigma_k^2), k = a, b \). Investors believe that the possible mean returns are normally

\(^{13}\) The theoretical results are presented in the Appendix in the proof of Proposition 1.
distributed with mean $\tau_k$ and variance $\delta_k^2$, i.e., $\tau_k(s) \sim \begin{cases} N(\tau_{kh}, \delta_{kh}^2) & \text{if } k \text{ is a home asset} \\ N(\tau_{kf}, \delta_{kf}^2) & \text{if } k \text{ is a foreign asset} \end{cases}$ where $\delta_{kh} < \delta_{kf}$ and $\tau_{kh} > \tau_{kf}$.

I assume the following stylized facts: expected asset return $r_m^t$ is 9%, asset standard deviation $\sigma(r_m^t)$ is 16%, coefficient of risk aversion is equal to 2. According to Ahearne, Griever, and Warnock (2004), the US market capitalization is about 48.3% ($\lambda \simeq 0.5$) and the estimated home asset holding is about 89.9%.

Table 1 presents the home asset holdings for several values of the difference in perceived mean returns $\Delta \tau_k$ and perceived dispersions $\Delta \delta_k$, for different degrees of ambiguity aversion. The perceived mean returns for the home asset is $\tau_{kh} = 1.09 + \Delta \tau_k/2$, and for the foreign asset it is $\tau_{kf} = 1.09 - \Delta \tau_k/2$. The exact values of dispersions are chosen to match the equity premium of 8%; these values are presented in Table 1b.

As the degree of ambiguity aversion increases, the bias toward the home asset becomes larger for any given difference in perceived mean returns $\Delta \tau_k$ and perceived dispersions $\Delta \delta_k$. The difference in mean returns $\Delta \tau_k$ contributes more to the bias than the difference in perceived dispersions $\Delta \delta_k$. The significant portion of the bias can be explained with relatively a small degree of ambiguity aversion and differences in beliefs within 5%. It is possible to match exactly the US domestic asset holding observed in data but it requires large (but still finite) degree of ambiguity aversion or large differences in beliefs. The dispersion levels required to match the equity premium is smaller for a higher degree of ambiguity aversion.

---

14 Cochrane [10]
Table 1a. Optimal home asset holdings

<table>
<thead>
<tr>
<th>$\Delta \delta_k \setminus \Delta \bar{r}_k$</th>
<th>0%</th>
<th>1%</th>
<th>2%</th>
<th>5%</th>
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</tr>
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</tr>
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<td>5%</td>
<td>56.45</td>
<td>59.45</td>
<td>62.41</td>
<td>71.03</td>
</tr>
<tr>
<td>10%</td>
<td>62.56</td>
<td>65.37</td>
<td>68.10</td>
<td>75.83</td>
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</tbody>
</table>

Table 1b. Dispersion of the foreign asset

<table>
<thead>
<tr>
<th>$\Delta \delta_k \setminus \Delta \bar{r}_k$</th>
<th>0%</th>
<th>1%</th>
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<th>5%</th>
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<tbody>
<tr>
<td>0%</td>
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<td>23.48</td>
<td>23.48</td>
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<tr>
<td>1%</td>
<td>23.99</td>
<td>24.02</td>
<td>24.05</td>
<td>24.08</td>
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<td>30.24</td>
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</table>

Table 1b. Dispersion of the foreign asset

<table>
<thead>
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<th>2%</th>
<th>5%</th>
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</thead>
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<td>10.5</td>
<td>10.5</td>
</tr>
<tr>
<td>1%</td>
<td>11.02</td>
<td>11.17</td>
<td>11.08</td>
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<tr>
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<td>11.70</td>
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</tr>
<tr>
<td>10%</td>
<td>17.32</td>
<td>18.49</td>
<td>17.82</td>
<td>18.49</td>
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</table>

Figure 1 presents the home asset holdings as a function of the difference in perceived mean returns $\Delta \bar{r}_k$ and dispersion $\Delta \delta_k$ when $\alpha = 1$ (ambiguity neutrality), $\alpha = 5$ and $\alpha = 10$ when $\lambda = 0.5$. The difference in perceived mean returns $\Delta \bar{r}_k$ ranges from 0% to 5% and perceived dispersions $\Delta \delta_k$
ranges from 0% to 10%

Figure 1. Portfolio holdings of home asset for $\alpha = 1, 5, 10$ and $\lambda = 0.5$.

As proportion $\lambda$ of country A investors decreases, the equity home bias becomes larger. The intuition is the following: if investors from one country dominate the market then they have a large impact on equilibrium asset prices. So the deviation between equilibrium prices and the expected prices is smaller, hence, the portfolio holdings are closer to the market capitalization weights. Similarly, if the proportion of investors from one country is relatively small than their asset holdings will be strongly biased towards the home asset. Table 2 presents the home asset holdings when the market capitalization is 10%.

Table 2a. Optimal home asset holdings

<table>
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<th></th>
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</thead>
<tbody>
<tr>
<td>$\Delta \delta \backslash \Delta \mathcal{r}$</td>
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<td>5%</td>
</tr>
<tr>
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<td>10</td>
<td>15.81</td>
<td>21.56</td>
<td>38.52</td>
</tr>
<tr>
<td>1%</td>
<td>10.99</td>
<td>17.02</td>
<td>22.94</td>
<td>40.18</td>
</tr>
<tr>
<td>2%</td>
<td>12.05</td>
<td>18.29</td>
<td>24.38</td>
<td>41.86</td>
</tr>
<tr>
<td>5%</td>
<td>15.68</td>
<td>22.48</td>
<td>28.98</td>
<td>46.92</td>
</tr>
<tr>
<td>10%</td>
<td>23.12</td>
<td>30.52</td>
<td>37.36</td>
<td>55.14</td>
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<table>
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<td>1%</td>
<td>2%</td>
<td>5%</td>
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<td>15.81</td>
<td>21.56</td>
<td>38.52</td>
</tr>
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<td>11.42</td>
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<td>2%</td>
<td>12.99</td>
<td>19.39</td>
<td>25.61</td>
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<tr>
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</table>

<table>
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<td>26.11</td>
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<tr>
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<td>24.49</td>
<td>31.13</td>
<td>49.14</td>
</tr>
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<td>5%</td>
<td>33.42</td>
<td>40.86</td>
<td>47.50</td>
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Table 2b. Dispersions of the foreign asset

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<td>22.85</td>
<td>23.09</td>
<td>0%</td>
</tr>
<tr>
<td>1%</td>
<td>23.51</td>
<td>23.61</td>
<td>23.71</td>
<td>23.97</td>
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</tr>
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<td>2%</td>
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<td>31.57</td>
<td>32.07</td>
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</tbody>
</table>

Figure 2 presents the home asset holdings as a function of the difference in perceived mean returns \( \Delta \tau_k \) and dispersion \( \Delta \delta_k \) when \( \alpha = 1 \) (ambiguity neutrality) and \( \alpha = 10 \) for \( \lambda = 0.1 \). The difference in perceived mean returns \( \Delta \tau_k \) and perceived dispersions \( \Delta \delta_k \) ranges from 0% to 5%.

![Figure 2](image-url)

Figure 2. Portfolio holdings of home asset for \( \alpha = 1, 5, 10 \) and \( \lambda = 0.1 \).

7 Conclusion

My paper provides a simple theoretical framework that illustrates how differences in investors' beliefs can generate equity home bias. In my model, all investors possess the same information about the set of possible states and the corresponding returns distribution in each state but they have different beliefs about the likelihood of these states. This heterogeneity of beliefs leads to the asymmetry of
portfolio choices. This asymmetry is fundamentally different from information asymmetry in the sense that prices are not informative. The idea that investors have biased beliefs about uncertainty of asset returns is supported by several papers in the literature on home bias.

I quantify the effect of ambiguity and ambiguity attitude on the portfolio holdings and asset prices using the stylized facts. I show that the difference in perceived uncertainty can significantly contribute to the bias towards domestic assets. The extent to which the observed bias can be explained by the differences in beliefs and ambiguity aversion depends on which parameter values one is willing to accept as reasonable. Even though ambiguity does contribute to the explanation of equity home bias, it is unlikely that the observed lack of diversification is entirely due to ambiguity aversion, which leaves room for other explanations based on institutional factors and information asymmetries. This is consistent with empirical findings that equity home bias is caused by both institutional and behavioral factors.
References


8 Appendix

8.1 No uncertainty about asset returns

The investor’s decision problem if there is no uncertainty about distribution of asset returns\(^\text{15}\):

\[
\max \left\{ (\tau_a - p_a)x_a + (\tau_b - p_b)x_b + e - \frac{\gamma}{2}(\sigma_a x_a^2 + \sigma_b x_b^2) \right\}
\]

The optimal demand for risky assets \( k = a, b \):

\[
x_k^* = \frac{\tau_k - p_k}{\gamma \sigma_k^2}
\]

The equilibrium prices resulting from market clearing conditions:

\[
p_k^* = \tau_k - \lambda_k \tau_k \gamma \sigma_k^2
\]

The equilibrium demand for risky asset is equal to its market share

\[
x_k^* = \lambda_k \bar{x}_k
\]

8.2 Uncertainty about both assets returns

The distribution of asset \( k \) returns, there is uncertainty (ambiguity) about mean returns

\[
\tau_k \sim N(\tau_{ks}, \sigma_k^2)
\]

\[
\tau_{ks} \sim N(\tau_k, \delta_k^2)
\]

The investor’s decision problem in the presence of uncertainty about distribution of both asset returns

\[
\max \alpha E \left[ E[w|s] - \frac{1}{2} \text{var}[w|s] \right] - \frac{1}{2} \alpha^2 \text{Var} \left[ E[w|s] - \frac{1}{2} \text{var}[w|s] \right]
\]

(i) \( \text{Cov}(\tau_a, \tau_b) = \delta_{ab} \neq 0 \)

\[
\max \left\{ \begin{array}{c}
\alpha \gamma (\tau_a - p_a)x_a + \alpha \gamma (\tau_b - p_b)x_b + \alpha \gamma e \\
- \frac{\alpha^2}{2} (\sigma_a^2 x_a^2 + \sigma_b^2 x_b^2) - \frac{\alpha^2}{2} (\delta_a^2 x_a^2 + \delta_b^2 x_b^2 + 2 \delta_{ab} x_a x_b)
\end{array} \right\}
\]

Optimal demand for risky assets:

\[
x_a = \frac{(\tau_a - p_a) (\sigma_a^2 + \alpha \delta_b^2) - (\tau_b - p_b) \alpha \delta_{ab}}{\gamma \left[ (\sigma_a^2 \sigma_b^2 + \alpha \sigma_a^2 \delta_b^2 + \alpha \sigma_b^2 \delta_a^2) \right]}
\]

\[
x_b = \frac{(\tau_b - p_b) (\sigma_a^2 + \alpha \delta_a^2) - (\tau_a - p_a) \alpha \delta_{ab}}{\gamma \left[ (\sigma_a^2 \sigma_b^2 + \alpha \sigma_a^2 \delta_b^2 + \alpha \sigma_b^2 \delta_a^2) \right]}
\]

\(^{15}\)Two-asset extension of Grossman and Stiglitz model.
Equilibrium prices:
\[
\bar{p}_a = r_a - \lambda \pi_a \gamma (\sigma_a^2 + \alpha \delta_a^2) - (1 - \lambda) \pi_b \gamma \alpha \delta_{ab}
\]
\[
\bar{p}_b = r_b - (1 - \lambda) \pi_b \gamma (\sigma_b^2 + \alpha \delta_b^2) - \lambda \pi_a \gamma \alpha \delta_{ab}
\]

(ii) returns distributions are independent across states than \(\text{Cov}(\pi_a, \pi_b) = \delta_{ab} = 0\)

The optimal demand for assets:
\[
x_a = \frac{(\pi_a - p_a)(\sigma_a^2 + \alpha \delta_a^2)}{\gamma [(\sigma_a^2 \sigma_b^2 + \alpha \sigma_a^2 \delta_a^2 + \alpha \sigma_b^2 \delta_b^2)]}
\]
\[
x_b = \frac{(\pi_b - p_b)(\sigma_b^2 + \alpha \delta_b^2)}{\gamma [(\sigma_a^2 \sigma_b^2 + \alpha \sigma_a^2 \delta_a^2 + \alpha \sigma_b^2 \delta_b^2)]}
\]

Equilibrium prices:
\[
\bar{p}_a = r_a - \lambda \pi_a \gamma (\sigma_a^2 + \alpha \delta_a^2)
\]
\[
\bar{p}_b = r_b - (1 - \lambda) \pi_b \gamma (\sigma_b^2 + \alpha \delta_b^2)
\]

**Proof of Proposition 1:**

**Proof.** Investors are optimistic \((\pi_{ah} > \pi_{af})\) and overconfident \((\delta_{ah} < \delta_{af})\) about the home asset relative to the foreign. Then the decision problem of investor \(i\) from country \(A\): ■

\[
\max \alpha E \left[ E[w|s_n] - \frac{1}{2} \text{var}[w|s_n] \right] - \frac{1}{2} \sigma^2 \text{var} \left[ E[w|s_n] - \frac{1}{2} \text{var}[w|s_n] \right]
\]

\[
\max \left\{ (\pi_{ah} - p_a)x_a + (\pi_{bf} - p_b)x_b + e - \frac{\gamma}{2} (\sigma_a x_a^2 + \sigma_b x_b^2) - \frac{\alpha \gamma}{2} \left( \delta_{ah} \gamma^2 x_a^2 + \delta_{bf} \gamma^2 x_b^2 \right) \right\}
\]

\[
\pi_{ah} - p_a - \gamma \sigma_a x_a - \alpha \gamma \delta_{ah} x_a = 0
\]
\[
\pi_{bf} - p_b - \gamma \sigma_b x_b - \alpha \gamma \delta_{bf} x_b = 0
\]
\[
x_{ai} = \frac{(\pi_{ah} - p_a)}{\gamma (\sigma_a + \alpha \delta_{ah})}, \quad x_{bi} = \frac{(\pi_{bf} - p_b)}{\gamma (\sigma_b + \alpha \delta_{bf})}
\]
\[
x_{aj} = \frac{(\pi_{af} - p_a)}{\gamma (\sigma_a + \alpha \delta_{af})}, \quad x_{bj} = \frac{(\pi_{bf} - p_b)}{\gamma (\sigma_b + \alpha \delta_{bf})}
\]

Market clearing conditions:
\[
\lambda x_{ai}^* + (1 - \lambda) x_{aj}^* = \lambda \pi_a
\]
\[
\lambda x_{bi}^* + (1 - \lambda) x_{bj}^* = (1 - \lambda) \pi_b
\]

Equilibrium asset prices:
\[
p_{a}^* = \frac{\lambda \pi_{ah} (\sigma_a + \alpha \delta_{af}) + (1 - \lambda) \pi_{af} (\sigma_a + \alpha \delta_{af})}{\left[ \sigma_a + \alpha (\lambda \delta_{af} + (1 - \lambda) \delta_{ah}) \right]} - \lambda \pi_a \gamma \left( \sigma_a + \alpha \delta_{ah} \right) \left( \sigma_a + \alpha \delta_{af} \right)
\]
\[
p_{b}^* = \frac{(1 - \lambda) \pi_{bh} (\sigma_b + \alpha \delta_{bf}) + \lambda \pi_{bf} (\sigma_b + \alpha \delta_{bf})}{\left[ \sigma_b + \alpha (\lambda \delta_{bf} + (1 - \lambda) \delta_{bf}) \right]} - (1 - \lambda) \pi_b \gamma \left( \sigma_b + \alpha \delta_{bf} \right) \left( \sigma_b + \alpha \delta_{bf} \right)
\]

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Equilibrium portfolio holdings for investor $i$ from country $A$:

$$
x_{ai}^* = \frac{\lambda \pi_a}{\sigma_a + \alpha (\lambda \delta_{af} + (1 - \lambda) \delta_{ah})} + \frac{(1 - \lambda) \pi_{ah} - \pi_a}{\gamma [\sigma_a + \alpha (\lambda \delta_{af} + (1 - \lambda) \delta_{ah})]} > \lambda \pi_a
$$

$$
x_{bi}^* = \frac{(1 - \lambda) \pi_b}{\sigma_b + \alpha (\lambda \delta_{bh} + (1 - \lambda) \delta_{bf})} - \frac{\lambda (\pi_{bh} - \pi_b)}{\gamma [\sigma_b + \alpha (\lambda \delta_{bh} + (1 - \lambda) \delta_{bf})]} < (1 - \lambda) \pi_b
$$

Asset Prices

expected price by home investors : 
$$\bar{p}_{ah} = \pi_{ah} - \pi_a \gamma (\sigma_a + \alpha \delta_{ah})$$

expected price by foreign investors : 
$$\bar{p}_{af} = \pi_{af} - \pi_a \gamma (\sigma_a + \alpha \delta_{af})$$

equilibrium price : 
$$p_a^* = \left( \frac{\pi_{ah} (\sigma_a + \alpha \delta_{af}) + \pi_{af} (1 - \lambda) (\sigma_a + \alpha \delta_{ah})}{\pi_a \gamma (\sigma_a + \alpha (\lambda \delta_{af} + (1 - \lambda) \delta_{ah}))} \right)$$

$$\bar{p}_{ah} > p_a^* > \bar{p}_{af}$$

Proof of Proposition 2:

Proof. Equilibrium portfolio holdings for investor $i$ from country $A$:

$$
x_{ai}^* = \frac{\lambda \pi_a}{\sigma_a + \alpha (\lambda \delta_{a} + \delta_{ah})} + \frac{(1 - \lambda) \pi_a}{\gamma [\sigma_a + \alpha (\lambda \delta_{a} + \delta_{ah})]}$$

$$
x_{bi}^* = \frac{(1 - \lambda) \pi_b}{\sigma_b + \alpha (\lambda \delta_{b} + \delta_{bh})} - \frac{\lambda \pi_b}{\gamma [\sigma_b + \alpha (\lambda \delta_{b} + \delta_{bh})]}
$$

degree of ambiguity aversion $\alpha$: If the degree of ambiguity aversion increases then the prices of both assets go down, the holding of the home asset increases and the foreign asset holding decreases. Hence, the equity home bias becomes larger

$$\frac{\partial x_{ai}^*}{\partial \alpha} = \frac{-\lambda \pi_a \gamma^2 (1 - \lambda) \triangle \delta_a + \lambda \Delta \delta_a + \delta_{ah}}{\gamma^2 (\sigma_a + \alpha (\lambda \Delta \delta_a + \delta_{ah}))^2} > 0$$

$$\frac{\partial x_{bi}^*}{\partial \alpha} = \frac{-(1 - \lambda) \pi_b \gamma^2 \Lambda \delta_b - \lambda \Delta \delta_b - \delta_{bh}}{\gamma^2 (\sigma_b + \alpha (\lambda \Delta \delta_b + \delta_{bh}))^2} < 0$$

If the degree of ambiguity aversion increases then the home asset holdings may increase or decrease and the foreign asset holding decreases. Overall, the equity home bias becomes larger.

- difference in perceived dispersion of mean asset returns $\Delta \delta_k = \delta_{kf} - \delta_{kh}$:

  $$\frac{\partial x_{ai}^*}{\partial \Delta \delta_a} = \frac{\alpha \lambda (1 - \lambda) \pi_a}{\gamma^2 (\sigma_a + \alpha (\lambda \Delta \delta_a + \delta_{ah}))^2} > 0$$

  $$\frac{\partial x_{bi}^*}{\partial \Delta \delta_b} = \frac{-\alpha (1 - \lambda) \lambda \pi_b}{\gamma^2 ([\sigma_b + \alpha (\lambda \delta_{bh} + (1 - \lambda) \delta_{bf})]^2} < 0$$

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If for a given asset difference in perceived dispersion of mean asset returns \( \Delta \delta_k \) increases then the holding of this asset decreases if it is a foreign asset, and increases if it is a home asset. Therefore, the equity home bias becomes larger.

- difference in perceived mean returns of mean asset returns \( \Delta \tau_k = \tau_{kh} - \tau_{k,f} \):

\[
\frac{\partial x_{ai}^*}{\partial \Delta \tau_a} = \frac{(1 - \lambda)}{\gamma \left[ \sigma_a + \alpha (\lambda \Delta \delta_a + \delta_{ah}) \right]} > 0 \\
\frac{\partial x_{bi}^*}{\partial \Delta \tau_b} = \frac{-\lambda}{\gamma \left[ \sigma_b + \alpha \left(\lambda \delta_{bh} + (1 - \lambda)\delta_{bf}\right) \right]} < 0
\]

If size of the population in country A decreases relative to country B then the equilibrium price for asset \( a \) goes up and the equilibrium price for asset \( b \) goes down. In country A, the home asset holdings increase and the foreign asset holdings decrease; vice versa for country B. Therefore, the equity home bias becomes larger.