Diverse Beliefs and Time Variability of Asset Risk Premia

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Defining A Risk Premium on An Asset

- **Realized Premium**

\[ \pi_{t+1} = \frac{p_{t+1} + D_{t+1} - R_t p_t}{p_t} \]

**Assume:** Much past data available to compute empirical moments. No one knows true probability distributions.

- \( m = \) probability implied by the empirical distribution
- \( m \) is a unique and stationary probability.
- All agree on \( m \)
- **The Premium** is the conditional expectations under \( m \)

\[ E_t^m [\pi_{t+1} \mid I_t] = \frac{1}{p_t} E_t^m [p_{t+1} + D_{t+1} - R_t p_t \mid I_t] \]
The Issue

The questions we aim to answer are:

What are the factors determining the Risk Premium?
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What are the factors determining the Risk Premium?

Why does it fluctuate over time?
Some Answers

**Bond Market**: macroeconomic Variables

- Fama and Bliss (1987) Internal dynamics of past yields

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**Some Answers**

**Bond Market: macroeconomic Variables**

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**Bond Market: macroeconomic Variables**

Fama and Bliss (1987)  
Internal dynamics of past yields

Campbell and Shiller (1991)  
Unexplained shocks to the bond market

Federal Reserve policy shocks
Some Answers

**Bond Market: macroeconomic Variables**

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- Cochrane and Piazzesi (2005) Past yields and Business Cycles
Some Answers

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**Macroeconomic Variables**

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**Bond Market: macroeconomic Variables**

- **Fama and Bliss (1987)**: Internal dynamics of past yields
- **Campbell and Shiller (1991)**: Unexplained shocks to the bond market
- **Bernanke and Kuttner (2003)**: Federal Reserve policy shocks
- **Cochrane and Piazzesi (2005)**: Past yields and Business Cycles
- **Piazzesi and Swanson (2004)**: Recessions forecasts, i.e. Non-Farm Payroll
- **Kurz and Motoolese (2008)**: The importance of Market Beliefs
Some More Answers

Stock Market: framed as a problem of forecasting returns.

Using Belief Variables some studies focus on earning forecasts dispersion:
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Miller (1977),
Diether et al. (2002),
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Park (2005),
Baker and Wurgler (2006),
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▶ Virtually no theory: most start from *Noise Trading* conception
▶ Hence no real hypotheses to test
This Paper

- Effects of diverse beliefs on the risk premium
- Restrictions diverse beliefs rationality imposes
- Measuring market belief and testing hypotheses
- Conclusions complementary to Kurz and Motolese (2008) on the Bond Market
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The Approach:
1. Outline basic model of belief
2. Formulate an Asset Pricing model
3. Deduce conclusion about risk premium
4. Test empirically using a reference model from literature
A Sketch of the Basic Model of Belief

- Risky asset payoff \( \{D_t, t = 1, 2, \ldots \} \) with true probability \( \Pi \).
- Non stationary with structural breaks. \( \Pi \) is unknown.
- True process is Stable:
  - Relative frequencies converge
  - Has empirical distribution
- Substantial past data.
- All compute empirical probability \( m \): Common knowledge.
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- All compute empirical probability \( m \): *Common knowledge*.

Assume: under \( m \) process is Markov with mean \( \mu \) and transition \( F \)

\[
d_{t+1} = \lambda_d d_t + \rho_{t+1}^d, \quad \rho_{t+1}^d \sim N(0, \sigma_d^2) \quad \text{where} \quad d_t = D_t - \mu
\]
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Assume: under \( m \) process is Markov with mean \( \mu \) and transition \( F \)

\[
d_{t+1} = \lambda_d d_t + \rho^d_{t+1}, \quad \rho^d_{t+1} \sim N(0, \sigma^2_d) \quad \text{where} \quad d_t = D_t - \mu
\]

- The truth is [I tell you, agents do not know it, and it does not matter]

\[
d_{t+1} = \lambda_d d_t + b_t + \xi^d_{t+1}, \quad \xi^d_{t+1} \sim N(0, \sigma^2_\xi) \quad \text{where} \quad b_t \text{ sequence of regimes.}
\]
A Sketch of the Basic Model of Belief (cont.)

Note \( \Pi \neq m \).

**Theorem:** \( m \) is stationary, unique and expressed by \( F \).
A Sketch of the Basic Model of Belief (cont.)

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**Theorem:** \( m \) is stationary, unique and expressed by \( F \).

- \( Q \) is a Rational Belief if data generated under it reproduces \( m \)
- We assume agents’ beliefs \( Q^i \) are Markov
- Disagreement persists:
  - Data shows diverse forecasts
  - Must hold diverse transitions \( F^i_t \)

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A Sketch of the Basic Model of Belief (cont.)

Note $\Pi \neq m$.

**Theorem:** $m$ is stationary, unique and expressed by $F$.

- $Q$ is a Rational Belief if data generated under it reproduces $m$
- We assume agents’ beliefs $Q^i$ are Markov
- Disagreement persists:
  - Data shows diverse forecasts
  - Must hold diverse transitions $F^i_t$

Rationality implies (as a minimum):

(A) A Rational agent cannot hold a constant $F^i \neq F$
(B) $F^i_t$ fluctuate over time with the restriction
(C) $1/N \sum_{t=1}^{N} (F^i_t - F) \rightarrow 0$ for all $i$: Rational Agents are right on average

Rationality $\implies$ Dynamics
Definition: A Belief State $g^i_t$ pins down i’s perceived transition of all state variables. In the case of dividend, it takes the form

$$d^i_{t+1} = \lambda_d d_t + \lambda^g_d g^i_t + \rho^i_{t+1}, \quad \rho^i_{t+1} \sim N(0, \hat{\sigma}_d^2)$$
Definition: A Belief State $g^i_t$ pins down i’s perceived transition of all state variables. In the case of dividend, it takes the form

$$d_{t+1}^i = \lambda_d d_t + \lambda_d^g g_t^i + \rho_{t+1}^{id}, \quad \rho_{t+1}^{id} \sim N(0, \hat{\sigma}_d^2)$$

- $g_t^i$ is observable

$$E_t^i [d_{t+1}^i | l_t, g_t^i] - E_t^m [d_{t+1} | l_t] = \lambda_d^g g_t^i$$

- Persistent Diversity $\implies g_t^i$ are different across i
**In Sum**: if rational, $g_t$ must fluctuate and have a zero mean.
A Sketch of the Basic Model of Belief (cont.)

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We represent state of belief with:

$$g_{t+1}^i = \lambda z g_t^i + \rho_{t+1}^{ig}, \quad \rho_{t+1}^{ig} \sim N(0, \sigma_g^2) \quad (1)$$
**A Sketch of the Basic Model of Belief (cont.)**

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Why:

(I) It is Compatible with empirical evidence in survey data

(II) Can give an analytic-Bayesian justification. See Kurz (2006) and will be discussed later in the conference.
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Dynamics of Market Belief

**Definition:** Market belief is the distribution \((g^1_t, g^2_t, \ldots, g^N_t)\) observed by sampling hence with known moments.
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- Define mean market state of belief by \(Z_t = \frac{1}{N} \sum_{i=1}^{N} g_i^t\)
- Average (1) \(\frac{1}{N} \sum_{i=1}^{N} g_{i+1}^t = \lambda Z \frac{1}{N} \sum_{i=1}^{N} g_i^t + \frac{1}{N} \sum_{i=1}^{N} \rho_{i+1}^g\)
- Key condition: \(\rho_{i+1}^g\) are correlated hence \(\frac{1}{N} \sum_{i=1}^{N} \rho_{i+1}^g = \rho^Z_{t+1} \neq 0\)
- \(Z_t\) is a state variable with empirical distribution

\[Z_{t+1} = \lambda_Z Z_t + \rho^Z_{t+1}, \quad \rho^Z_{t+1} \sim \mathcal{N}(0, \sigma_Z^2)\]
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- Define mean market state of belief by \(Z_t = \frac{1}{N} \sum_{i=1}^{N} g^i_t\)

- Average (1) \(\frac{1}{N} \sum_{i=1}^{N} g^i_{t+1} = \lambda Z_t \frac{1}{N} \sum_{i=1}^{N} g^i_t + \frac{1}{N} \sum_{i=1}^{N} \rho^i_{gt+1}\)

- Key condition: \(\rho^i_{gt+1}\) are correlated hence

\[
\frac{1}{N} \sum_{i=1}^{N} \rho^i_{gt+1} = \rho^Z_{t+1} \neq 0
\]

- \(Z_t\) is a state variable with empirical distribution

\[
Z_{t+1} = \lambda Z_t + \rho^Z_{t+1}, \quad \rho^Z_{t+1} \sim N(0, \sigma^2_Z)
\]

In all models: this correlation is the crucial factor
We thus expand the empirical distribution to

\[(d_{t+1}, Z_{t+1}), \ t = 1, 2, \ldots\] 

\[d_{t+1} = \lambda_d d_t + \rho^d_{t+1} \begin{pmatrix} \rho^d_{t+1} \\ \rho^z_{t+1} \end{pmatrix} \sim N(0, \tilde{\Sigma}), \ i.i.d.\]

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where

\[\tilde{\Sigma} = \begin{bmatrix} \sigma^2_d & 0 \\ 0 & \sigma^2_Z \end{bmatrix} .\]
Individual i’s perception model (together with (1)) takes the form:

\[ d_{t+1}^i = \lambda_d d_t + \lambda^g d g_t^i + \rho_{t+1}^{id} \]

\[ Z_{t+1}^i = \lambda_Z Z_t + \lambda_Z g_t^i + \rho_{t+1}^{iZ} \]

where

\[ \Sigma^i = \begin{bmatrix} \hat{\sigma}_d^2 & \hat{\sigma}_{dZ} \\ \hat{\sigma}_{dZ} & \hat{\sigma}_Z^2 \end{bmatrix} \]

Parameter sign \( \lambda^g_d \geq 0 \) and \( \lambda^g_Z \geq 0 \) orient the model: When \( g_t^i > 0 \), agent i believes t+1 dividend and market belief will persist above normal.
An Infinite Horizon Model

Assumptions:

- Large number of agents.
- A single commodity – “consumption”.
- Riskless technology producing $R > 1$ at $t + 1$ with 1 unit of input at $t$.
- A single aggregate risky asset with supply $S=1$.
- The sequence of dividends $\{D_t, t = 1, 2, \ldots\}$ with unknown distribution $\Pi$.
- Under $m\{D_t, t = 1, 2, \ldots\}$ is Markov with transition

$$d_{t+1} = \lambda d_t + \rho_{t+1}^d, \quad \rho_{t+1}^d \sim N\left(0, \sigma_d^2\right)$$

where $d_t = D_t - \mu$. 
An Infinite Horizon Model (Cont.)

Some Notation:

- $\theta^i_t =$ date $t$ stock purchases of agent $i$.
- $B^i_t =$ amount invested in the riskless asset.
- $p_t =$ price of the stock. Think of it as the S&P500.

Agent $i$ optimization problem is:

$$\max_{(\theta^i_t, B^i_t)} E^i_t \left[ \sum_{k=t}^{\infty} -\beta^{k-1} e^{-\frac{c^i_k}{\tau}} \right] I_t$$

Subject to:

$$c^i_t = \theta^i_{t-1} (p_t + d_t + \mu) + B^i_{t-1} R - \theta^i_t p_t - B^i_t$$

Initial values $(\theta^i_0, B^i_0)$

i’s belief as specified in (2)

Advantage: only mean market belief matters.
An Infinite Horizon Model (a remark)

The Exponential utility model is common in studies of asset pricing. See for example

Singleton (1987)
Brown and Jennings (1989)
Grundy and McNichols (1989)
Wang (1994)
He and Wang (1995)
Duffie (2002)
Dai and Singleton (2002)
Allen, Morris and Shin (2005) and many others
Asset Demand Functions

- Assume for a moment: agents believe $p_{t+1} + d_{t+1}$ is conditionally normal.
- Define:
  Excess return per share $V_{t+1} = p_{t+1} + (d_{t+1} + \mu) - Rp_t$.
- Define the state variables in the optimization:

$$\psi^i_t = (1, d_t, Z_t, g_t^i)$$
$$u = (u_0, u_1, u_2, u_3)$$
Asset Demand Functions

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- Define the state variables in the optimization:

  $\psi_i^t = (1, d_t, Z_t, g_t)$

  $u = (u_0, u_1, u_2, u_3)$

We show that the demand function of agent $i$ is:

$$\theta_i^t(p_t, \psi_t^i) = \frac{R_T}{r\hat{\sigma}_V^2} \left[ E_t^i (V_{t+1} | I_t, g_t^i) + u\psi_i^t \right]$$

where

$$\hat{\sigma}_V^2 = adjusted \ variance \ of \ V_t \ assumed \ constant \ for \ all \ i.$$

Stability Conditions:

$$R = 1 + r > 1, \ 0 < \lambda_d < 1, \ \lambda_Z < 1, \ 0 < \lambda_Z + \lambda_Z^g < 1.$$
Equilibrium Asset Prices and Risk Premium

**Theorem:** There is a unique equilibrium price function which takes the form $p_t = a_d d_t + a_z Z_t + P_0$ with parameters

$$a_d = \frac{\lambda_d + u_1}{R - \lambda_d} > 0 \quad a_z = \frac{(a_d + 1) \lambda_d^g + (u_2 + u_3)}{R - (\lambda Z + \lambda Z^g)} > 0$$

$$P_0 = \frac{(\mu + u_0)}{r} - \frac{\hat{\sigma}_V^2}{R_T}.$$
Equilibrium Asset Prices and Risk Premium

**Theorem**: There is a unique equilibrium price function which takes the form

\[ p_t = a_d d_t + a_z Z_t + P_0 \]

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\[ a_z = \frac{(a_d + 1) \lambda^g_d + (u_2 + u_3)}{R - (\lambda_Z + \lambda^g_Z)} > 0 \]
\[ P_0 = \frac{(\mu + u_0)}{r} - \frac{\hat{\sigma}_V^2}{R_T}. \]

- The Theorem above confirms equilibrium price \( p_t \) is conditionally normal
- Price exhibits “excess” volatility due to beliefs
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$$P_0 = \frac{(\mu + u_0)}{r} - \frac{\hat{\sigma}^2_v}{R^\tau}.$$

- The Theorem above confirms equilibrium price $p_t$ is conditionally normal
- Price exhibits “excess” volatility due to beliefs
- Diverse perceived premia:
  $$E^i_t [\pi_{t+1} | l_t] = \frac{1}{p_t} E^i_t [p_{t+1} + d_{t+1} + \mu - Rp_t | l_t]$$
- Compute the Premium:
  $$E^m_t [\pi_{t+1} | l_t] = \frac{1}{p_t} E^m_t [p_{t+1} + D_{t+1} - R_t p_t | l_t]$$
We compute:

\[ E^m_t [\pi_{t+1} \mid I_t] = \frac{1}{\rho_t} E^m_t \left[ \left( \frac{r \hat{\sigma}^2}{R^2} - u_1 d_t - u_0 \right) - a_z (R - \lambda Z) Z_t \right] \]
Structure of the Risk Premium: Main Result

We compute:

\[ E_t^m [\pi_{t+1} | I_t] = \frac{1}{p_t^m} E_t^m \left[ \left( \frac{r \hat{\sigma}_V^2}{R_T} - u_1 d_t - u_0 \right) - a_z (R - \lambda Z) Z_t \right] \]

Main Theorem: Since \( a_z (R - \lambda Z) > 0 \), the Risk Premium increases with \( \hat{\sigma}_V^2 \) and declines with market belief \( Z_t \)

Also, \( \hat{\sigma}_V^2 \approx (a_d + 1)^2 \hat{\sigma}_d^2 + a_Z^2 \hat{\sigma}_Z^2 \)
Structure of the Risk Premium: Main Result

We compute:

\[
E^m_t [\pi_{t+1} \mid I_t] = \frac{1}{p_t} E^m_t \left[ \left( \frac{r \hat{\sigma}_V^2}{R^T} - u_1 d_t - u_0 \right) - a_z (R - \lambda_Z) Z_t \right]
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Also, \( \hat{\sigma}_V^2 \approx (a_d + 1)^2 \hat{\sigma}_d^2 + a_Z^2 \hat{\sigma}_Z^2 \)

(I) Mean Premium increases with variance of \( d \) and \( Z \): belief state volatility increases mean premium

(II) Time Changes in Market Belief reflected in

\[-a_z (R - \lambda_Z) > 0.\]

The risk premium on a long position

- lower when market belief about the future is favorable
- higher when market belief about the future is unfavorable
Effects of Belief on Premium

What It Does Not Say:

- Agents are on their demand functions
- Not “optimal” to “choose” to be a contrarian; may be short when wants to be long and long when wants to be short
- Analogous to why we do not adopt a log utility
- Note: you don’t change forecast of $d_{t+1}$ when you find $Z_t < 0$!

What It Does Say:

- A formal theorem about market overshooting: today’s price adjusts to $Z_t$ more then expected tomorrow’s price
- Rational investing agents must form an opinion about future beliefs of “others” - the market.
- Diverse beliefs are central to this results (topic of conference)
**Definition:** An equilibrium exhibits “Endogenous Uncertainty” if its price map depends upon market belief.

- With Non-Exponential Utility: entire distribution \((g^1_t, \ldots, g^N_t)\) matters but Main Result holds if slope of \(Z_t\) satisfies \(a_Z > 0\).

- Mean market belief \(Z_t = \frac{1}{N} \sum_{i=1}^{N} g^i_t\).

\[
\text{Diversity } \sigma^Z_t = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (g^i_t - Z_t)^2}
\]

- Empirical work considers first two moments of \((Z_t, \sigma^Z_t)\) distribution

- Hypothesis: effect of cross sectional diversity \(\sigma^Z_t\) on risk premium is negative (the "Narrow Door Hypothesis")
Notation and Variables Description

Excess Stock Returns

\[ R_{t,t+6} \]

six-month return on the CRSP value weighted index net of the return on a 90 day T-bill

\[ R_{t,t+12} \]

1 year return on the CRSP value weighted index net of the return on a 90 day T-bill

Livingston Six-Month Growth Forecasts

\[ F^g_{t+6,t+12} \]

Mean forecast of six-month real GDP growth rate from 6 months to 12 month after t computed from individual Livingston survey responses about the level of nominal GDP and the CPI 6 and 12 months after date t
Notation and Variables Description (cont.)

Financial Predictors

\[ DP_t \]
dividend yield on the CRSP value-weighted portfolio

\[ DEF_t \]
the yield spread between a broad corporate bond portfolio and the AAA yield spread

\[ TERM_t \]
the yield spread between a 10 year U.S. Treasury bond and a one-month Treasury bill

Macroeconomic Predictors

\[ CAY_t \]
Lettau and Ludvigson (2001) log consumption-wealth ratio
Beliefs Variables

\( Z_{t+6, t+12} \)

Belief Index of Real GDP Growth Rate between the end of period t+6 and t+12

\( \sigma_{Zg} \)

Cross-sectional standard deviation of individual Livingston forecasts of 6-month growth rate in real GDP between the end of period t+6 and t+12

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$R_{t,t+6}$</th>
<th>$R_{t,t+12}$</th>
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<tr>
<td>$R_{t,t+6}$</td>
<td>6.646</td>
<td>23.376</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$R_{t,t+12}$</td>
<td>5.098</td>
<td>16.668</td>
<td>0.692</td>
<td>1.000</td>
</tr>
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## Descriptive Statistics of Predictors (1971:S1-2007:S2)

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<tr>
<th></th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$F_{g,t+1,t+2}^g$</th>
<th>$DP_t$</th>
<th>$DEF_t$</th>
<th>$TERM_t$</th>
<th>$CAY_t$</th>
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<tr>
<td>$F_{g,t+6,t+12}$</td>
<td>3.090</td>
<td>1.125</td>
<td>1.000</td>
<td></td>
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<tr>
<td>$DP_t$</td>
<td>3.104</td>
<td>1.258</td>
<td>0.143</td>
<td>1.000</td>
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<td></td>
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<tr>
<td>$DEF_t$</td>
<td>1.028</td>
<td>0.376</td>
<td>0.451</td>
<td>0.609</td>
<td>1.000</td>
<td></td>
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</tr>
<tr>
<td>$TERM_t$</td>
<td>1.011</td>
<td>1.182</td>
<td>0.297</td>
<td>-0.236</td>
<td>0.056</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>$CAY_t$</td>
<td>0.000</td>
<td>0.016</td>
<td>-0.327</td>
<td>0.119</td>
<td>-0.172</td>
<td>0.158</td>
<td>1.000</td>
</tr>
</tbody>
</table>
### Descriptive Statistics of Beliefs Variables (1971:S1-2007:S2)

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$Z_{t+6,t+12}^g$</th>
<th>$\sigma Z_g$</th>
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</thead>
<tbody>
<tr>
<td>$Z_{t+6,t+12}^g$</td>
<td>-0.603</td>
<td>1.890</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$\sigma Z_g$</td>
<td>2.319</td>
<td>1.012</td>
<td>0.576</td>
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</table>
Belief Index $Z_{t+6,t+12}^g$
The Cross-sectional standard deviation $\sigma_{Zg}$
### The Reference Model (1971:S1-2007:S2)

<table>
<thead>
<tr>
<th></th>
<th>( R_{t,t+6} )</th>
<th>( R_{t,t+12} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{t+6,t+12}^g )</td>
<td>(-)</td>
<td>-0.254 (0.032)</td>
</tr>
<tr>
<td>( DP_t )</td>
<td>0.086 (0.563)</td>
<td>0.100 (0.474)</td>
</tr>
<tr>
<td>( DEF_t )</td>
<td>0.138 (0.417)</td>
<td>0.225 (0.149)</td>
</tr>
<tr>
<td>( TERM_t )</td>
<td>0.084 (0.489)</td>
<td>0.171 (0.145)</td>
</tr>
<tr>
<td>( CAY_t )</td>
<td>0.244 (0.026)</td>
<td>0.161 (0.188)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.067</td>
<td>0.097</td>
</tr>
<tr>
<td>S.E.</td>
<td>0.966</td>
<td>0.951</td>
</tr>
</tbody>
</table>

P-values in parenthesis. All \( R^2 \) are adjusted
### Test of Theorem for 6-month horizon (1971:S1-2007:S2)

|                | \( R_{t,t+6} \)
|----------------|-----------------
| (A)            | (B)            | (1)  | (2)  |
| \( F_{t+6,t+12}^g \) | –              | -0.254 | –   | –  |
|                | (0.032)        |       |     |     |
| \( Z_{t+6,t+12}^g \) | –              | –      | -0.587 | -0.555 |
|                |                |       | (0.000) | (0.000) |
| \( \sigma Z_{t+6,t+12}^g \) | –            | –      | –      | -0.082 |
|                |                |       |       | (0.506) |
| \( DP_t \)     | 0.086          | 0.100  | 0.469  | 0.499 |
|                | (0.563)        | (0.474) | (0.007) | (0.009) |
| \( DEF_t \)    | 0.138          | 0.225  | 0.143  | 0.142 |
|                | (0.417)        | (0.149) | (0.213) | (0.203) |
| \( TERM_t \)   | 0.084          | 0.171  | -0.078 | -0.085 |
|                | (0.489)        | (0.145) | (0.397) | (0.378) |
| \( CAY_t \)    | 0.244          | 0.161  | 0.000  | 0.010 |
|                | (0.026)        | (0.188) | (0.998) | (0.926) |
| \( R^2 \)      | 0.067          | 0.097  | 0.215  | 0.208 |
| S.E.           | 0.966          | 0.951  | 0.886  | 0.890 |

P-values in parenthesis. All \( R^2 \) are adjusted.
## Test of Theorem for 12-month horizon (1971:S1-2007:S2)

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<th>$R_{t,t+12}$</th>
<th>(A)</th>
<th>(B)</th>
<th>(1)</th>
<th>(2)</th>
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</thead>
<tbody>
<tr>
<td>$F_{t+6,t+12}^g$</td>
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<tr>
<td>$Z_{t+6,t+12}^g$</td>
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<td>-</td>
<td>-0.422</td>
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<td>-0.363</td>
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</tr>
<tr>
<td>$\sigma Z_{t+6,t+12}^g$</td>
<td></td>
<td>-</td>
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<td>-0.150</td>
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<tr>
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<tr>
<td>$DP_t$</td>
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<td>0.498</td>
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<tr>
<td>$DEF_t$</td>
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<td>0.000</td>
<td>0.058</td>
<td>0.016</td>
<td>0.016</td>
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<td></td>
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</tr>
<tr>
<td>$TERM_t$</td>
<td></td>
<td>0.172</td>
<td>0.230</td>
<td>0.053</td>
<td>0.041</td>
</tr>
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<tr>
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</tr>
<tr>
<td>$CAY_t$</td>
<td></td>
<td>0.264</td>
<td>0.209</td>
<td>0.101</td>
<td>0.119</td>
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</tr>
</tbody>
</table>

P-values in parenthesis. All $R^2$ are adjusted.

| $R^2$ | 0.112 | 0.119 | 0.176 | 0.177 |
| S.E.  | 0.942 | 0.939 | 0.908 | 0.907 |
Conclusions

- Data supports theoretical conclusion
- Standard variables $DEF_t$, $TERM_t$ and $CAY_t$ predominantly reflect market beliefs not exogenous conditions
- More empirical proof that markets overshoot

- More evidence the risk of future market belief (i.e. belief of “others”) is a dominant market risk
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THANKS