Patent Breadth in an International Setting\footnote{We would like to thank Miguel Ballester, Elias Dinopoulos, Edwin Lai, Jee-Hyeong Park, Katheryn Russ, Tobias Seidel, Alan Spearot, Stefano Trento and Xuebing Yang for useful comments and conversations about this paper. We also appreciated comments by seminar participants at Universitat Autònoma de Barcelona, the Econometric Society Meetings at IMPA, Rio de Janeiro, the Midwest International Meetings at the University of Iowa, and the Asia Pacific Economic Association meetings at UC Santa Cruz. Financial support by the Center for the Americas at Vanderbilt University is gratefully acknowledged.}

Eric Bond\footnote{E-mail: eric.bond@vanderbilt.edu}

Ben Zissimos\footnote{Dept. of Economics, Vanderbilt University, Nashville, TN 37235.
Tel: ++1 615 322 3339.
Fax: ++1 615 343 8495.
E-mail: ben.zissimos@vanderbilt.edu}

\textit{Vanderbilt University}

First draft: September 27th 2008
This draft: July 14th, 2010

\textbf{Abstract:} This paper shows how patent breadth can vary across countries (in asymmetric equilibrium) even if countries are identical. Our two-country model has governments setting patent breadths and firms competing to develop and produce a new product. If a firm does not innovate then it is allowed to produce an imitation providing this does not infringe on a patent. It has long been recognized that patent setting varies between North and South to reflect stages of development. The present paper addresses this issue, but also the more puzzling question of why countries at a similar stage of development should set patents differently.

\textbf{Keywords:} Asymmetric equilibrium, innovation, patent breadth, patent race, R&D.

\textit{JEL Classification Numbers:} F02, F13, O3, O31, 032
1 Introduction

The significant variation between developed and developing countries in their breadths of patent protection has already been widely documented, and is understood to derive from the differing extents to which innovation drives their growth processes. Yet there is also a surprising degree of such variation across developed countries all of whom are understood to rely on innovation to drive economic growth. For example, there is relatively broad patent protection in the US, and comparatively narrow protection in Japan, while the breadth of patent protection in the UK is set at an intermediate level. US patent law embodies the doctrine of equivalents, which allows a patentee to claim infringement even if a patent has not literally been infringed, if the competing product provides essentially the same service or outcome. This provides substantial latitude to the patent holder in claims of infringement against competing products. The United Kingdom patent law, on the other hand, does not have a doctrine of equivalents and calls for a balance between a literal interpretation of the patent and a more liberal interpretation based on claims of infringement. In Japan, patent officials are encouraged to interpret patents very literally. The first main objective of this paper is to explain this substantial variation across developed countries in the breadth of patent protection.

The variation in patent protection across countries has prompted questions over whether countries would benefit from a harmonization of these policies. The most familiar question, both in the academic debate and in policy circles, is whether there are gains from agreements to set minimum levels on the length (i.e. the life) and the breadth of a patent. Coordination over a minimum length and breadth of patent protection has already been established in the Trade-Related Aspects of Intellectual Property Rights (TRIPS) in the World Trade Organization (WTO). The novel idea which we will bring to light in this regard is that there may be an incentive for such agreements to impose a maximum allowable patent breadth instead. This addresses a situation that has been

---

4 The breadth of patent protection determines how different a product must be in terms of its characteristics in order not to infringe on a patented product. We will discuss this idea below.

5 “If two devices do the same work in substantially the same way, and accomplish substantially the same result, they are the same, even though they differ in name, form or shape.” Union Paper-Bag Machine Co. v. Murphy, 97 U. S. 120, 97 U. S. 125.

observed at the European Patent Office (EPO). Not only has the EPO upheld patents that some member countries refused to grant, an example of coordination across Europe that raises the breadth of patent protection. But the EPO has also been observed to revoke patents that have been granted by member countries, an example of coordination that reduces the breadth of patent protection.\(^7\) Thus the second main objective of this paper is to evaluate the incentives for governments in different countries to coordinate over patent breadth.

To explore these issues, we develop a model of a patent race in which governments set the breadth of patent protection strategically.\(^8\) We set up a two-country duopoly model, with one firm in each country, in which each firm must decide whether or not to enter a patent race to develop a new product. The winner of the patent race obtains a monopoly right to produce that good, which we will refer to as the ‘innovation’. The loser must decide whether or not to produce an ‘imitation’ that competes with the patented product (an option that is available whether or not it participates in the patent race). The profitability of the imitation depends on the breadth of patent protection provided by governments in each country. The narrowest possible patent breadth will allow the imitating firm to produce a product that is a perfect substitute for the innovation, whereas a very broad patent will preclude entry by the imitator because the imitation is not valued at all by consumers.\(^9\)

Our modeling of patent breadth works as follows. We think of a product as being characterized by a number of product attributes. A patent then protects the technology that delivers a particular attribute of the product. The patent breadth in a country determines how similar a product’s technology is allowed to be in terms of the specific attribute protected by the patent. We assume that firms know of an attribute that consumers would like to be developed. For example, in terms of outdoor clothing consumers want the attribute that a garment to keeps them dry. Firms may engage in a patent

\(^7\)The Economist (2009) discusses examples of such occurrences.

\(^8\)Maskus and McDaniel (1999) provide support for the idea that patent breadths are set strategically with specific reference to Japan’s choice of a narrow patent breadth policy. They argue that this choice was made in order to encourage domestic firms to develop their own versions of new innovations by limiting the range of protection for existing patent holders.

\(^9\)Our modeling of patent races draws on the work of Loury (1979) and Reinganum (1982). Our approach to patent breadth is similar to that of Klemperer (1990) as explained below.
race to develop the technology required to deliver the attribute. If one firm successfully
develops a technology that engineers the attribute into a product and receives a patent
for it, then patent breadth imposes a limit on how similar the technology is allowed to be
in an imitation. Since the innovation is assumed to provide the attribute that consumers
want, and since patent breadth determines how different the technology must be that
delivers the attribute, patent breadth effectively determines how much lower is the level
of utility provided to the consumer by the imitation.\footnote{The effectiveness of outdoor clothing at keeping the wearer dry serves as a useful motivating example. In 1976, W.L. Gore & Associates were awarded a patent for Gore-tex. The key innovation incorporated in this fabric was its ability to ‘breathe’; to allow water vapors to escape in one direction while being waterproof in the other. Waterproof fabrics had existed for a long time beforehand but, because they could not breathe, the consumer would become damp in such fabrics from their own perspiration even when it did not rain. Throughout the life of the patent, imitators were allowed to copy other unpatented attributes of outdoor clothing made by Gore (the company) such as the color and cut of the garment. But patent breadth in each country determined how close an imitation could be in terms of the similarity of the material used to produce the garment. In simple terms, the broader the patent, the more different the material had to be in order not to infringe on the patent, the less waterproof it would be, and so the lower the utility it would yield to the consumer all else equal. An alternative concern, which we will not explore in the present paper, is by how much a new innovation must improve upon existing technology in order not to infringe on the patent; see our literature review for further discussion of this. And there may be some attributes that can be achieved by more than one technology, so that a patent race may give rise to horizontal product differentiation, but we will rule that out by assumption here.}

The choice of patent breadth involves a trade-off to each of the national govern-
ments. A broader patent will make innovative activity more profitable and thus make it
more likely that there is a successful innovation. However, broader patent protection will
also result in a greater static deadweight loss due to less intense competition in output
markets. For a national government, the importance of these two effects will depend on
the likelihood that its firm is successful in the patent race. To highlight the role of the
country’s stage of development (and hence its likelihood of winning the race), we focus on
two cases. The first case, which we refer to as the North-South model, is one in which the
home (Northern) firm has a positive probability of successful innovation and the foreign
(Southern) firm has a zero probability of success. The second case, referred to as the
North-North model, is one in which the home and foreign firms have an equal probability
of success.

We show that in the North-South model, the profit of the Northern (Southern) firm is
monotonically increasing (decreasing) in the breadth of patent protection in each country,
since it is always the innovating (imitating) firm.\(^{11}\) (This means that a patent race cannot arise in the North-South model since the Southern firm cannot compete to make an innovation.) For the Northern government, the profit shifting effect of increasing patent breadth dominates the effect on consumer welfare, so the dominant strategy is to make patents as broad as possible. For the Southern government, whose firm is an imitator, welfare is monotonically decreasing in the breadth of the patent as long as the Northern firm chooses to undertake R&D. However, circumstances may arise in which the Northern firm cannot cover the costs of R&D even if it successfully innovates. Thus, the North-South model generates an outcome in which the North chooses the broadest protection and the South chooses the narrowest protection that is consistent with innovative effort by the Northern firm.

A natural question in the North-South model is whether an agreement to harmonize patent breadth across North and South would increase the level of world welfare. We will show that in fact harmonization reduces world welfare because of the strict convexity of world welfare in patent breadth, which means that the marginal deadweight loss from increasing patent breadth declines as patent breadth is increased. Thus, world welfare is maximized at the non-cooperative Nash equilibrium in this case.

For the North-North model, in contrast, firms and governments in each country face symmetric incentives due to the fact that firms have an equal probability of being a successful innovator. Firms must decide whether or not they will enter into a patent race. Profits from innovation for a firm are increasing in the breadth of patent protection in each country, so the model yields a threshold locus of worldwide patent protection such that only one firm undertakes R&D and a (higher) threshold locus such that both firms enter into a patent race. If each firm undertakes R&D then each has the same probability of being the innovator and each has the same probability of being the imitator. Therefore, each country gains from increased patent breadth when it is the innovator and loses when it is the imitator. All else equal, consumers always prefer narrower patent breadth because this allows an imitation that is more valuable. Thus narrower patent breadth gives rise to more intense competition in the product market. For a given number of firms

\(^{11}\)We assume national treatment of patent breadth; within a country, patent breadth applies equally to both domestic and foreign firms.
undertaking R&D, the effects of patent breadth on consumer surplus dominate those on producer profits. Therefore, perhaps surprisingly, in our North-North model national welfare is decreasing in the breadth of the patent. So each government is driven to reduce patent breadth just to the point where the desired number of firms undertakes R&D, taking as given the patent breadth set by the other government.

Our results exhibit two types of multiplicity of equilibria that arise in the North-North model. The first is that there may be a continuum of Nash equilibria that differ in the pattern of patent breadth across countries, but not in the number of firms that engage in R&D. This forms the basis of our novel explanation for how identical countries can have different choices of patent breadth in equilibrium, with neither country wanting to change its patent breadth because its choice is essential to keeping the desired number of firms engaged in R&D. This in turn presents one possible way to explain how the US, UK and Japan each set different breadths of patent protection despite being at a similar stage of development. The second type of multiplicity arises when there are equilibria in which only one firm is engaged in R&D as well as equilibria where there is a patent race between two firms. In this case, international coordination on patent breadth could raise the welfare of both countries by moving to a more socially desirable level of innovative activity. A key contribution of our paper is to show that the non-cooperative equilibrium could involve either too much or too little innovative activity, so coordination might either raise or lower patent breadth. Either way, in contrast to the North-South model, in the North-North model harmonization does allow scope to increase efficiency. This is how we explain that the EPO sometimes upholds patents when individual country governments refuse to do so and at other times revokes patents previously upheld by individual governments.

We will now explain how our work relates to the existing literature on international trade and the protection of intellectual property. As mentioned earlier, one strand of the literature has focused on the choice of the length of patent protection; the approach taken has been to follow the classic analysis of Nordhaus (1969). Deardorff (1991) uses this approach to examine the impact of extending patent protection in technology importing countries, and identifies a trade-off between a worsening terms of trade and a higher world rate of innovation. Grossman and Lai (2004) solve for the non-cooperative equilibrium in the setting of patent life (or equivalently the strictness of enforcement) when governments
set policy to maximize national welfare and the industry structure is one of monopolistic competition. They find that patent protection in the non-cooperative equilibrium will be lower in countries that do less R&D, and that the non-cooperative equilibrium will involve patent lives that are too short relative to the cooperative level. We obtain similar findings for our North-South model but in the context of patent breadth as opposed to length. However, our North-North model differs in that the Nash equilibrium may have either more or less R&D than the optimal level. Furthermore, we have the possibility that similar countries have different levels of protection in the Nash equilibrium.

A second strand of the literature has explicitly addressed the choice of patent breadth. Klemperer (1990) and Gilbert and Shapiro (1990) compare the effects of expanding patent breadth with the effects of extending patent life in a closed economy model. Their formulation, in which the breadth of the patent determines how close the patent holder can get to obtaining the monopoly profit level, is similar to the one we adopt here. However, our work differs in that we treat the imitator as having market power, and we extend the analysis to a two country setting.\textsuperscript{12}

The type of patent breadth we consider can be thought of as specifying how much lower the quality of an imitation must be in order not to infringe on a patent. An alternative approach to patent breadth is to examine the amount of quality improvement that a new product must represent over an existing product in order to receive a patent.\textsuperscript{13} Chor and Lai (2009) examine the strategic incentives of governments in setting patent breadth of the latter type using an endogenous growth model. Extending patent breadth results in a higher rate of innovation at the cost of a longer period of monopoly pricing. They find that the non-cooperative equilibrium involves under-investment in R&D, which parallels the basic characterization of equilibrium in our North-South model but in an environment where the competing firm can improve on product quality in order not to infringe on the patent.

\textsuperscript{12}We simplify by treating the patent life as fixed. The exogeneity of patent life in our framework can be interpreted as resulting from the fact that the product may have a finite life during which it is in demand, in which case extensions of the patent life beyond that point will be of no value to the firm. In practice, the useful life of patents is often less than the statutory life, as discussed by O’Donoghue, Scotchmer and Thisse (1998).

\textsuperscript{13}Donoghue et al (1998) define the former concept as lagging patent breadth, and the latter as leading patent breadth. They consider a model of sequential innovation with both leading and lagging breadth in a closed economy.
The paper proceeds as follows. Section 2 sets out the basic model and derives the efficient solution. Section 3 defines the patent-breadth game and characterizes equilibrium of the game. It is in this section that the scope for international coordination over patent breadth is considered. Conclusions are drawn in Section 4.

2 The Model

We consider a model in which two governments, a home government and a foreign government, set the breadth of their patent protection on a newly developed product. Two firms, one in the home country and the other in the foreign country, compete to develop a new product then compete in production and sales. Home variables are denoted with “no-∗” while foreign variables are denoted with a “∗”. Since we will use backwards induction to study patent setting, we will first specify the behavior of firms after which we will turn to the governments.

2.1 Product Innovation and Imitation

Firms take patent breadth as given. In order to participate in the competition to develop a new product, a firm must pay a fixed cost of R&D, r (assumed to be the same for both firms), which results in the successful development of a new product with probability \( \theta \in [0, 1] \) for the home firm and \( \theta^* \in [0, 1] \) for the foreign firm. Without loss of generality, assume \( \theta \geq \theta^* \). If both firms engage in R&D then the event that one firm innovates successfully is statistically independent of that of the other. The probability that a new product is discovered is then calculated as

\[
1 - (1 - \theta) (1 - \theta^*) = \theta + \theta^* - \theta \theta^*.
\]

In the event that only one firm makes a discovery, it receives the patent on the product. If both make a discovery there is a probability of \(1/2\) that the patent will be received by the home firm. So if both firms engage in R&D then the probability that the home firm is awarded the patent is \( \theta (1 - \theta^*/2) \). We will denote the newly discovered good, i.e. the innovation, with \( n \) to reflect the fact that it is an innovation and we will refer to the firm that is awarded the patent as the innovator.

\[14\] For simplicity we assume that \( r, \theta \) and \( \theta^* \) are parametric. Essentially the same model characteristics would be obtained, at the cost of greater complexity, if \( \theta \) and \( \theta^* \) were made to depend on \( r \).
Once the innovating firm (also denoted by $n$) has been awarded the patent, the other firm (denoted by $m$) can develop a competing product that imitates the innovator’s good as long as it does not infringe on the patent. We will assume that the fixed cost of developing the imitation is arbitrarily small, and that a firm can imitate whether or not it chose to engage in R&D. If the imitator does not engage in the initial stage R&D, then it free-rides on the R&D activity of the innovator.

The innovator produces $q_n$ for the home market and $q_n^*$ for the foreign market while the imitator produces $q_m$ for the home market and $q_m^*$ for the foreign market. For later use, $q$ will refer to the vector of all outputs; $q = (q_n, q_n^*, q_m, q_m^*)$.

We capture the effect of patent breadth through its effect on preferences. As argued in the introduction, the patent protects the technology that delivers a particular attribute of the product. Therefore, the broader the patent, the more different the technology that delivers the attribute of the imitator must be from that of the innovator; hence the less effective the technology is, and the less the imitation is valued by consumers relative to the innovation. We will adopt a particularly tractable functional form, wherein the product in question only has a single attribute. We formalize this notion by expressing the preferences of the home consumer as

$$u = e (q_n + q_m) - \frac{1}{2} (q_n + q_m)^2 - wq_m + x,$$

where $x$ is the numeraire in the model and captures the basket of other goods consumed and $w$ is the breadth of the home country patent. If $w = 0$ then patent protection is so narrow that the imitator can introduce a product that consumers view as equally valuable without violating the patent. If $w = e$, the protection is so broad that the imitation would be worthless to the consumers if it were produced. Each country is endowed with a sufficient quantity of $x$ to balance trade. An analogous expression is used

---

15 The framework intentionally leaves unspecified the nationality of each firm. The reason is that in the North-South model the nationality is known ex-ante while in the North-North model the government sets policy from behind the veil of ignorance regarding whether its firm is the innovator or the imitator. The details will be developed in due course.

16 Differentiation across another product attribute such as color or design may be captured by extending (1) to the form

$$u = e (q_n + q_m) - \frac{2}{2} (q_n + q_m)^2 - \frac{1}{2} (q_n)^2 + (q_m)^2 - wq_m + x,$$

where $\gamma \in [0, 1]$ is a parameter that determines substitutability between the innovation and the imitation in the attribute not covered by the patent. We have carried out all of the analysis for this extended functional form and it makes little difference to our results, so we will not elaborate further on this in the paper.
to represent foreign preferences.\textsuperscript{17}

\section*{2.2 Patent Breadth and Firm Profits}

In production, each firm perceives the home market and the foreign market as being segmented. Based on this perception, each firm chooses its level of output in each market under the (Cournot) conjecture that the other firm will hold its output in that market constant. The marginal cost of producing an additional unit is $c$ for each firm in either market. From (1), the inverse demand function in the home market for $n$ and $m$ will be $p_n = e - q_n - q_m$ and $p_m = e - q_n - q_m - w$ respectively. Although we assume that the innovator and imitator have the same marginal cost, production of the imitation is less profitable in the home market when $w > 0$ because of the fact that it is less valuable to consumers. If $w$ is so large that the imitator cannot make non-negative profits then we assume that its output and profits are zero, yielding the entire market to the innovating firm. Analogous expressions give inverse demands in the foreign market.\textsuperscript{18}

Profits made by the innovator in the home and foreign markets are calculated in the usual way as $\pi_n = (p_n - c) q_n$ and $\pi_n^* = (p_n^* - c) q_n^*$ respectively, while profits made by the imitator at home and abroad are calculated as $\pi_m = (p_m - c) q_m$ and $\pi_m^* = (p_m^* - c) q_m^*$. Total profits for $n$ and $m$, denoted by $\Pi_n$ and $\Pi_m$ respectively, are then calculated as follows:

\begin{align*}
\Pi_n (w, q) &= \pi_n (w, q_n, q_m) + \pi_n^* (w^*, q_n^*, q_m^*) ; \\
\Pi_m (w, q) &= \pi_m (w, q_n, q_m) + \pi_m^* (w^*, q_n^*, q_m^*) ,
\end{align*}

where $w = (w, w^*)$ is the vector of patent breadths.

We are now able to evaluate the payoffs to firms of investment in R&D or otherwise. There are four cases to consider: only the home firm invests in R&D; only the foreign firm

\textsuperscript{17}Klemperer’s (1990) formalization of patent breadth is similar, except that he assumes imitations are provided by a competitive fringe of suppliers, and that the reduction of welfare from consuming an imitation relative to the innovation differs across consumers. We assume that the innovator and imitator are Cournot duopolists, and that the relative reduction in welfare from consuming the imitation is the same for all consumers. This simplification enables us to focus on strategic interactions over patent breadths by governments, which Klemperer does not consider.

\textsuperscript{18}We simply assume that an imitator produces a good that is as close as allowed by the patent to the innovation. This could be derived as the outcome of a profit maximizing decision.
invests in R&D; both firms invest in R&D and thus enter into a patent race; neither firm invests in R&D. Formally, let the home firm have an action $a \in \{d, f\}$, where $d$ represents ‘development’ through R&D of a new product (whether successful or not) and $f$ stands for ‘free riding’ on the R&D activities of the other firm. The foreign firm has analogous actions $a^* \in \{d^*, f^*\}$. Letting $\psi(a, a^*, w, w^*, q)$ be the expected payoff to the home firm when it chooses $a$ and the foreign firm chooses $a^*$, we have

$$
\begin{align*}
\psi(d, f^*, w, q; r) & = \theta \Pi_n(w, q) - r \\
\psi(f, d^*, w, q; r) & = \theta^* \Pi_m(w, q) \\
\psi(d, d^*, w, q; r) & = \theta (1 - \theta^*/2) \Pi_n(w, q) + \theta^* (1 - \theta/2) \Pi_m(w, q) - r \\
\psi(f, f^*, w, q; r) & = 0.
\end{align*}
$$

(2)

The expected payoff to the foreign firm of its action $a^*$, denoted by $\psi^*(a, a^*, w, q; r)$, is defined in a similar way.

2.3 Patent Breadth and National Welfare

In the patent-breadth game, we treat $w$ as the choice variable of the home government and $w^*$ as the choice variable of the foreign government. Each government chooses its patent breadth to maximize its own nation’s social welfare where social welfare is, as standard, the sum of consumer surplus and profits made by the domestic firm. Next we will consider how patent breadth affects national welfare.

National welfare at home is calculated (in terms of expectations) as the sum of two components: the home firm’s expected profits and the home consumers’ expected surplus. The home firm’s expected profits are given by $\psi(a, a^*, w, q; r)$ from (2). Since $p_n = p_m + w$ in any equilibrium where imitated goods are consumed, consumer surplus can be expressed as

$$
S(Q) = \frac{1}{2} Q^2, \quad \text{where } Q \equiv q_n + q_m
$$

Letting $\Phi(a, a^*) = \Pr[\text{innovation} | (a, a^*)]$, expected home welfare takes the form

$$
v(a, a^*, w, q; r) = \psi(a, a^*, w, q; r) + \Phi(a, a^*) S(Q). \quad (3)
$$

---

19 Equivalently, a firm plays $d$ in order to enter the patent race and $f$ in order not to. We are using the term ‘free ride’ loosely here as a label because if the other firm does not undertake R&D then there is no activity on which to free ride.
This function will be taken to represent the home government’s objective function in the patent-breadth game. An analogous function represents the foreign government’s objective function.\(^{20}\)

From the point of view of the representative consumer at home, R&D by the home firm is always beneficial whether the foreign firm engages in R&D or not. If the foreign firm does not engage in R&D then the domestic consumer benefit from the home firm’s R&D is \(\Phi(d, f^*) - \Phi(f, f^*)\) \(S(Q) = \frac{\theta}{2} Q^2\), while if the foreign firm does engage in R&D then the domestic consumer benefit from the home firm’s R&D is \(\Phi(d, d^*) - \Phi(f, d^*)\) \(S(Q) = \frac{\theta(1-\theta^*)}{2} Q^2\). Note that \(S(Q)\) is unambiguously decreasing in \(w\), reflecting that fact that for a given number of firms undertaking R&D citizens favor the narrowest possible patent, allowing the imitation to be closer to the innovation.

### 2.4 Efficiency

In this section we provide a benchmark by solving for the policies that would be chosen by a planner whose objective is to maximize the sum of home and foreign welfare:

\[
\Omega(a, a^*, w, q; r) \equiv v(a, a^*, w, q; r) + v^*(a, a^*, w, q; r).
\]

The planner is assumed to be able to choose the actions of firms concerning R&D, and the output levels of firms in the event that R&D is successful, with lump sum taxes available to finance R&D and to make transfers between countries.

**Proposition 1.** World welfare, \(\Omega(a, a^*, w, q; r)\), is maximized by choosing actions as follows.

(a) \(a = f, a^* = f^*\) for \(r > \bar{r}_1\), where \(\bar{r}_1 = \theta (e - c)^2\).

(b) \(a = d, a^* = f^*\) for \(r \in (\bar{r}_2, \bar{r}_1]\).

(c) \(a = d, a^* = d^*\) for \(\bar{r}_2 = (1 - \theta) \theta^* (e - c)^2 \geq r\).

In the event of a successful innovation, any set of outputs satisfying \(p_n = p_m = c\) in home and foreign markets with \(w = w^* = 0\) will yield the maximum surplus.

\(^{20}\)From now on, unless stated otherwise, it will be understood that when we refer to profits and consumer surplus these are calculated in terms of their expectations.
Since she has access to lump sum taxes, the planner does not need to implement policies that create distortions in output markets (either by choosing a price exceeding marginal cost or choosing \( w, w^* > 0 \) when the imitator is undertaking production) in order to finance R&D. Therefore, marginal cost pricing is optimal and any allocation of output between the two firms that yields the perfectly competitive output level in each market will yield a consumer surplus of \( \frac{1}{2} (e - c)^2 \) in each country. Expected world surplus is \( \Omega (d, f^*, w, q; r) = \theta (e - c)^2 - r \) when only the home firm undertakes R&D and \( \Omega (d, d^*, w, q; r) = (\theta + \theta^* - \theta \theta^*) (e - c)^2 - 2r \) when both do so. The optimal R&D policy, presented in the proposition for different ranges of \( r \), then follows immediately from a comparison of these payoffs.

3 The Patent-Breadth Game

In this section we examine the outcome of the patent-breadth game. We model the patent-breadth game as having three stages. In the first stage, the home and foreign governments, simultaneously and without communicating, set \( w \) and \( w^* \) respectively. In the second stage, the home and foreign firms choose their R&D actions, \( a \) and \( a^* \). In the third stage, the firms compete as Cournot duopolists if at least one of the firms has been successful in R&D. If only one of the firms is successful, then the unsuccessful firm competes as the imitator. If both firms are successful in R&D, we assume that each firm has an equal probability of obtaining the patent and competing as the innovator (with the loser competing as the imitator). We refer to this whole process as the patent-breadth game. Using backwards induction, we can solve for an equilibrium in patent breadths, actions and quantities.

3.1 Production

We begin by solving for the equilibrium in the production game in the home country market, with the identity of the firms as innovator and imitator already having been determined by the outcome of the previous stage. Since firms are symmetric in terms of production cost, the equilibrium does not depend on the identity of the innovator. The equilibrium in the foreign market will be symmetric. The breadth of the respective
patents, \( w \), is known to firms.

For given patent breadth, \( w \), a production level by the innovator is a best response quantity against a production level \( q_m \) when it maximizes \( \pi_n(w, q_n, q_m) \). Firm \( n \)'s best response function is written as \( R_n(q_m; w) \) and firm \( m \)'s best response function is written as \( R_m(q_n; w) \). A Nash equilibrium in quantities is a pair \((\hat{q}_n, \hat{q}_m)\) for which \( \hat{q}_n \) is a best response to \( \hat{q}_m \) and vice versa. We are now able to obtain the following result, which gives Nash equilibrium quantities and profit levels for the home market.

**Proposition 2.** For \( w \leq w_{\text{max}} = (e - c)/2 \), the Nash equilibrium output and price levels are given by:

\[
\begin{align*}
\hat{q}_n(w) &= \frac{e - c + w}{3}; & \hat{q}_m(w) &= \frac{e - c - 2w}{3}; & \hat{Q}(w) &= \frac{2(e - c) - w}{3} \\
\hat{p}_n(w) &= \frac{ew + (w + 1)c}{3}; & \hat{p}_m(w) &= \hat{p}_n - w.
\end{align*}
\]

For \( w > w_{\text{max}} \), output levels are:

\[
\hat{Q} = \hat{q}_n = \frac{e - c}{2}; & \hat{q}_m = 0.
\]

**Profits in equilibrium** are given by:

\[
\hat{\pi}_i(w) = (\hat{q}_i(w))^2 \text{ for } i \in \{n, m\}.
\]

Proof: See appendix.

For \( 0 \leq w < w_{\text{max}} \), the best-response functions are illustrated in Figure 1. From Proposition 2 and from the figure we see that an increase in patent breadth has the effect of increasing \( \hat{q}_n \) and decreasing \( \hat{q}_m \). These effects on quantities are reflected in turn in an increase of \( \hat{\pi}_n \) and decrease of \( \hat{\pi}_m \). This stands to reason. As the patent is broadened, the imitator is forced to produce a product that is less valuable to consumers. This induces consumers to increase their demand for the innovation at the expense of the imitation, with reciprocal impact on quantities produced and profits. The overall effect on consumer welfare of an increase in \( w \) is negative, as reflected by the decrease in \( \hat{Q} \). Characterization of the effect of changes in \( w^* \) is analogous.
3.2 Competition for the Patent

We will now examine the conditions under which, in equilibrium, one firm undertakes R&D or both firms enter into a patent race. Our goal is to characterize the industry structure. We will achieve this by studying the game of actions (still taking \( w \) and \( w^* \) as given). Note from Proposition 2 that, for \( w \geq w_{\text{max}} \), the innovator has the entire home market and the imitator does not produce; moreover, quantities produced are invariant over this range of policies. Therefore, without loss of generality, we may restrict attention to \((w, w^*) \in W \equiv [0, w_{\text{max}}] \times [0, w_{\text{max}}]\).

It will be helpful at this point to formalize our concepts of the North-South and North-North models. In the North-South model, we assume \( \theta^* = 0 \), placing the foreign firm in the South; it can be an imitator but not an innovator. In the North-North model, the home and foreign firms have equal probabilities of making a successful innovation, \( \theta^* = \theta \), and are equally adept at imitating if the other firm receives the patent.

3.2.1 The North-South Model

In the North-South model, the foreign firm never chooses to innovate since it has zero probability of success. Since \( \psi(f, f^*, w; r) = 0 \), the optimal action of the home firm is to undertake R&D if and only if \( \psi(d, f^*, w; r) \geq 0 \). Recall that \( \psi(d, f^*, w; r) \) is increasing in both \( w \) and \( w^* \) on the interior of \( W \), because an increase in the breadth of the patent in either market favors the innovator at the expense of the imitator. Figure 2 illustrates the \( \psi(d, f^*, w; r) = 0 \) locus, which is negatively sloped and concave to the origin due to the properties of the profit functions established in Proposition 2. We refer to this locus as the just profitable protection (jp) locus, because the home firm will expect to make a profit and therefore undertake R&D for any values of \( w \) lying on or above the jp-locus.

Since \( \psi(d, f^*, w; 0) > 0 \) and \( \psi \) is decreasing in \( r \), for any given \( w \in W \) there is a unique threshold value of \( r \) for which \( \psi(d, f^*, w; \rho_1) = 0 \). We will denote this threshold value of \( r \) by \( \rho_1 \), and write the functional relationship between \( w \) and \( \rho_1 \) as \( \rho_1(w) \). Assuming as a tie-breaking rule that the firm chooses to engage in R&D if \( \psi(d, f^*, w; r) = 0 \), it follows that the home firm will engage in R&D iff \( r \leq \rho_1(w) \). This threshold value \( \rho_1(w) \) must be increasing in \( w \) and \( w^* \) because broader protection makes entry
into the patent race more profitable. Using the payoff functions of Proposition 2, we can
determine how the action of the home firm at stage 2 varies with government policy and
the technology parameters.

**Proposition 3.** Assume the North-South model.

(a) If \( r > \rho_1 (w_{\text{max}}, w_{\text{max}}) = \frac{\theta}{2} (e - c)^2 \) then in equilibrium the home firm does not
undertake R&D for any feasible choices of \( w \) and \( w^* \).

(b) If \( \rho_1 (w_{\text{max}}, w_{\text{max}}) \geq r \geq \rho_1 (0, 0) \) then there exists a non-empty set of patent
breadths \( \{w\} \) for which, in equilibrium, the home firm undertakes R&D if and only if \( w \) lies on or above the \( j\)-p-locus.

(c) If \( \rho_1 (0, 0) = \frac{26}{9} (e - c)^2 \geq r \) then in equilibrium the home firm undertakes R&D
for all feasible values of \( w \) and \( w^* \).

Note that for the value \( r = \frac{139}{36} (e - c)^2 \) illustrated in Figure 2, the \( j\)-p-locus goes through
the points \((w, w^*) = (0, w_{\text{max}})\) as well as \((w, w^*) = (w_{\text{max}}, 0)\). (Ignore the \( b\)-p-locus for
now.) At this level of \( r \), providing that the home government sets maximum patent
breadth \( w = w_{\text{max}} \), it is profitable for the home firm to undertake R&D whatever value
the foreign government chooses for \( w^* \). It follows that, for \( r \leq \frac{139}{36} (e - c)^2 \), the home
government can ensure unilaterally that it is profitable for the home firm to undertake R&D. On the other hand, for \( r > \frac{139}{36} (e - c)^2 \), the question of whether R&D by the home
firm is profitable depends on both \( w \) and \( w^* \). For any given \( r \in (\frac{139}{36} (e - c)^2, \frac{26}{9} (e - c)^2] \),
the \( j\)-p-locus would lie everywhere above the one pictured in Figure 2. In such cases, even
if \( w = w_{\text{max}} \), the profitability of R&D is not assured; the foreign government must set \( w^* \)
sufficiently high in order for the home firm to find it profitable to undertake R&D.

Proposition 3 also reveals that patent protection gives the home firm less than the
socially optimal incentive to undertake R&D. To see this, observe that \( \rho_1 (w_{\text{max}}, w_{\text{max}}) = \frac{1}{2} r_1 = \frac{\theta}{2} (e - c)^2 \). This means that, for \( \rho_1 (w_{\text{max}}, w_{\text{max}}) < r \leq r_1 = \theta (e - c)^2 \), R&D will
not be undertaken even when both governments set patent breadth as broad as possible,
the fact that R&D would be socially valuable notwithstanding. Moreover, the level of
\( r \) at which R&D will cease to be profitable even though it would be socially valuable is
further reduced if patent breadth is less than the maximum level in either country.
Recall that, in the North-North model, each firm has the same probability of successfully discovering a new product and obtaining the patent. This leads to the possibility that there are Nash equilibria in which both firms enter into a patent race, as well as equilibria in which only one firm finds it profitable to undertake R&D. In the latter case, symmetry ensures that there be two Nash equilibria, \((d, f^*)\) and \((f, d^*)\). In order to provide a unique outcome for the game we will assume that at the beginning of the second stage, nature randomly selects one of the firms (with equal probability) to move first in the R&D decision. This assumption is made to provide a unique equilibrium to the game for all values of \(r\), as well as to maintain the symmetry between the home and foreign firms.\(^{21}\) Since the home and foreign firms are symmetric, we can derive the best-response functions by considering the decision of the home firm only.

If the foreign firm does not undertake R&D then the best-response criteria for undertaking R&D are the same as in Proposition 3. If the foreign firm does undertake R&D but the home firm does not then the home firm earns the return of an imitator: \(\psi(f, d^*, w; r) \geq 0\). If the foreign firm undertakes R&D and the home firm chooses to enter into a patent race, it earns \(\psi(d, d^*, w; r)\). Entry by the home firm will be a best-response to entry by the foreign firm if and only if\(^{22}\)

\[
\beta(w; r) \equiv \psi(d, d^*, w; r) - \psi(f, d^*, w; r) \\
= \theta \left(1 - \frac{\theta}{2}\right) \Pi_n(w) - \frac{\theta^2}{2} \Pi_m(w) - r \geq 0.
\]

This equation is increasing in \(w\) and \(w^*\) for all \(w \in W\), because broadening patent protection in either country makes innovation more profitable and imitation less so.

Since \(\psi(d, f^*, w; r) > \beta(w; r)\), for any given \(r\) and \(w \in W\), the best responses of the home firm can be divided into three cases. If patent breadth is sufficiently broad that \(\psi(d, f^*, w; r) > \beta(w; r) > 0\), then it is a dominant strategy for the home firm to enter the patent race. If \(\psi(d, f^*, w; r) > 0 > \beta(w; r)\), then the home firm undertakes R&D

---

\(^{21}\)This can be thought of as reflecting randomness in the R&D process that allows one of the firms to be slightly ahead in the process of innovation at the point where a significant resource commitment (\(r\)) must be made.

\(^{22}\)When we refer to ‘entry’ we mean entry into a patent race.
iff the foreign firm does not. Finally, if $\psi(d, f^*, w; r) < 0$ the home firm does not find it profitable to undertake R&D.

These three regions for home best responses give rise to three possible types of equilibrium outcome, as illustrated in Figure 2. Since $\psi(d, f^*, w; r) > \beta(w; r)$ the $\beta(w; r) = 0$ locus, which will be referred to as the both profitable protection (bp) locus, must lie to the right of the jp-locus. For values of $w$ above the bp-locus, entry is a dominant strategy for the home firm. Since the foreign firm is symmetric, the Nash equilibrium involves a patent race between the home and foreign firms. For values of $w$ between the jp- and bp-loci, patent protection is only broad enough to give one firm an incentive to undertake R&D. Therefore, the firm that is randomly selected by nature to move first undertakes R&D but the other firm does not. Finally, for $w$ lying below the jp-locus neither firm chooses to undertake R&D.

It should be noted that the value of $r$ determines which of these three market structures can arise as an equilibrium. Define $\rho_2$ as the value of $r$ that solves $\beta(w; r) = 0$. Then $\rho_2(w) < \rho_1(w)$ because a lower value of $r$ is required for each firm to make positive expected profits when both enter. We can then divide the values of $r$ into five intervals, each differing in which of the possible equilibrium market structures can arise.

**Proposition 4.** Assume the North-North model. The relationship between R&D cost and the number of firms undertaking R&D in equilibrium is characterized as follows.

(a) If $r > \rho_1(w_{\text{max}}, w_{\text{max}}) = \frac{\theta}{2}(e - c)^2$ neither firm undertakes R&D for any feasible choices of $w$ and $w^*$.

(b) If $\rho_1(w_{\text{max}}, w_{\text{max}}) \geq r > \rho_2(w_{\text{max}}, w_{\text{max}}) = \frac{\theta(2-\theta)}{4}(e - c)^2$, one firm undertakes R&D iff $w$ lies on or above the jp-locus and no firms otherwise.

(c) If $\rho_2(w_{\text{max}}, w_{\text{max}}) \geq r > \rho_1(0, 0) = \frac{2\theta}{9}(e - c)^2$, no firms undertake R&D iff $w$ lies below the jp-locus, one firm undertakes R&D if and only if $w$ lies on or above the jp-locus but below the bp-locus, and both firms enter into a patent race if and only if $w$ lies on or above the bp-locus.

(d) If $\rho_1(0, 0) \geq r > \rho_2(0, 0) = \frac{2\theta(1-\theta)}{9}(e - c)^2 \geq r$, one firm undertakes R&D iff $w$ lies below the bp-locus, and both firms enter into a patent race if and only if $w$ lies on
or above the bp-locus.

(e) If \( \rho_2 (0, 0) \geq r \) then both firms enter into a patent race for all feasible values of \( w \) and \( w^* \).

Parts (a) and (e) cover the parameter range of \( r \) over which \( w \) and \( w^* \) do not affect the number of firms that undertake R&D; the former because no firms ever undertake R&D and the latter because they always do. For values of \( r \) in between, as illustrated in Figure 2, the value of \( r \) fixes the location of the jp-locus and the bp-locus in \( (w, w^*) \)-space and then the specific values of \( w \) and \( w^* \) determine how many firms undertake R&D. It is worth clarifying that for values of \( r \) approaching \( \rho_1 (0, 0) \), the jp-locus is located towards the upper bound \( (w, w^*) = (w_{\text{max}}, w_{\text{max}}) \) and there is no bp-locus. Conversely, for values of \( r \) approaching \( \rho_2 (0, 0) \), the bp-locus is located towards the lower bound of the parameter space \( (w, w^*) = (0, 0) \) and there is no jp-locus. The critical range of \( r \) over which both the jp-locus and the bp-locus appear in Figure 2 is \( r \in [\rho_1 (0, 0), \rho_2 (w_{\text{max}}, w_{\text{max}})] \). (Note that \( \rho_2 (w_{\text{max}}, w_{\text{max}}) > \rho_1 (0, 0) \) for all \( \theta \in [0, 1] \).

From Proposition 4 we can also see that, in the North-North model, patent protection can give firms more than the socially optimal incentive to undertake R&D; that is, firms may enter into a patent race when it would be socially optimal for only one firm to undertake R&D. To examine this possibility, we need to work out the highest value of \( r \) at which both firms undertake R&D in equilibrium, \( \rho_2 (w_{\text{max}}, w_{\text{max}}) \), which is the level of \( r \) at which R&D is profitable by both firms given the broadest possible patent protection. Comparing \( \rho_2 (w_{\text{max}}, w_{\text{max}}) \) with the maximum value of \( r \) at which it is socially efficient for both firms to undertake R&D, \( r_2 = (1 - \theta) \theta (e - c)^2 \), we have that

\[
\rho_2 (w_{\text{max}}, w_{\text{max}}) \leq r_2 \text{ iff } \theta \leq \frac{2}{3}.
\]

If \( \theta > \frac{2}{3} \) then, for \( w = w^* = w_{\text{max}} \), in equilibrium firms enter into a patent race where it would be socially optimal for only one firm to undertake R&D. This contrasts markedly with the North-South model where over-investment in R&D is not a possibility. Our interpretation of this result is as follows. In the North-North model, when the probability of success is high, the social return to a patent race adds relatively little to the aggregate probability of obtaining the innovation. However, the private incentive to enter into a patent race is still high because, in the (likely) event that both firms are successful, each
firm has an equal chance of getting the patent. Thus, part of the private benefit to the innovator is the possibility of obtaining the market at the other firm’s expense. The possibility of undertaking too much R&D becomes less likely as the patent breadth is reduced but it does not disappear completely for \( w \in (0, w_{\text{max}}) \).

### 3.3 Patent Breadth

Our aim now is to characterize the choice of patent breadth at home and abroad through strategic interaction between the governments. In contrast to the efficient solution in Proposition 1, we assume that the government is unable to control quantity choices of firms or to use lump sum taxes to finance R&D investments. Following from (3) then, for \( w \in W \), welfare of the home government can be expressed as \( \tilde{v}(w; r) = v(a(w), a^*(w), w, q(w); r) \).

A patent breadth \( \hat{w} \in [0, w_{\text{max}}] \) for the home government is a best response against a patent breadth \( w^* \) when it maximizes \( \tilde{v}(w; r) \). A Nash equilibrium in patent breadths is a pair \( \hat{w} = (\hat{w}, \hat{w}^*) \) where \( \hat{w} \) is a best response against \( \hat{w}^* \) and vice-versa. We will follow the format of the previous subsection by considering equilibrium first in the North-South model and then in the North-North model.

#### 3.3.1 The North-South Model

The fact that only the home (Northern) firm can innovate creates an asymmetry between the welfare functions of the respective countries. For the home country, welfare is given by the sum of the innovating firm’s profits in the two markets and consumer surplus at home when evaluated at the Nash equilibrium output levels:

\[
\tilde{v}(w; r) = \begin{cases} 
\theta \left[ \Pi_n(w, \hat{q}(w)) + S(\hat{Q}(w)) \right] - r & \text{for } \psi(d, f^*, w, r) \geq 0 \\
0 & \text{for } \psi(d, f^*, w, r) < 0
\end{cases}
\]

Broader patent breadth will shift profits from the foreign imitator to the home innovator, while reducing consumer surplus at home. To see how home welfare is affected by a change in patent breadth, differentiate (4) with respect to \( w \) and \( w^* \). This yields

\[
\frac{d\tilde{v}(w; r)}{dw} = \frac{w\theta}{3} \geq 0, \quad \frac{d\tilde{v}(w; r)}{dw^*} = \frac{2\theta(e - c + w^*)}{9} > 0 \text{ for } \psi(d, f^*, w, r) \geq 0.
\]

The gain in profits from increasing patent breadth dominates the loss of consumer surplus for all \( w \in [0, w_{\text{max}}] \) and \( \psi(d, f^*, w, r) \geq 0 \), so \( \hat{w}(w^*) = w_{\text{max}} \) in the North. Since welfare
is independent of $w$ for $\psi(d, f^*, w, r) < 0$, setting $w = w_{\text{max}}$ is a weakly dominant strategy for the home country. Increases in the breadth of foreign patents has a favorable spillover effect on the home country, because it raises the return from innovation.

For the foreign country, welfare is the sum of imitator profits and consumer surplus in the domestic market when worldwide protection is sufficiently broad that the home firm undertakes R&D:

$$\tilde{v}^*(w; r) = \begin{cases} \theta \left( \Pi_m(w, \hat{q}(w)) + S \left( \hat{Q}^*(w^*) \right) \right) & \text{if } \psi(d, f^*, w, r) \geq 0 \\ 0 & \text{if } \psi(d, f^*, w, r) < 0 \end{cases}$$

Differentiating with respect to $w$ and $w^*$ yields

$$\frac{d\tilde{v}^*(w; r)}{dw^*} = -\frac{\theta (2(e - c) - 3w)}{3} < 0,$$
$$\frac{d\tilde{v}^*(w; r)}{dw^*} = -\frac{4\theta ((e - c - 2w)}{9} < 0(6)$$

Making patent protection broader in the foreign country will transfer surplus from foreign consumers and the imitating (foreign) firm to the innovating (home) firm, which reduces foreign welfare. Increases in the home patent breadth will reduce foreign welfare similarly. Foreign welfare is independent of $w$ and $w^*$ if $\psi(d, f^*, w, r) < 0$, since the home firm does not undertake R&D. If $\psi(d, f^*, w, w_{\text{max}}; r) \geq 0$, then there exists an interval $J^*(w; r) = [j^*(w; r), w_{\text{max}}] \in [0, w_{\text{max}}]$ such that $\psi(d, f^*, w, w^*, r) \geq 0$ iff $w^* \in J^*(w; r)$ and $\psi(d, f^*, w, w^*, r) = 0$ if $w^* = j^*(w; r)$. The best response correspondence for the foreign country can then be expressed as follows:

$$\hat{w}^*(w) = \begin{cases} j^*(w; r) & \text{if } \psi(d, f^*, w, w_{\text{max}}; r) \geq 0 \\ w^* \in [0, w_{\text{max}}] & \text{if } \psi(d, f^*, w, w_{\text{max}}; r) < 0 \end{cases}$$

The foreign country will always choose the lowest value of $w^*$ that is consistent with the home firm undertaking R&D, so $w^* = 0$ if foreign patent protection is not essential for the home firm to undertake R&D. If $\psi(d, f^*, w, w_{\text{max}}; r) < 0$, then the foreign country cannot influence the R&D decision and the foreign country is indifferent over all feasible values of $w^*$.

Since the home country has a weakly dominant strategy of setting $w = w_{\text{max}}$, if $\psi(d, f^*, w_{\text{max}}, w_{\text{max}}; r) \geq 0$ then the pair $\{w_{\text{max}}, j^*(w_{\text{max}}; r)\}$ is a Nash equilibrium in which the home firm undertakes R&D. If we assume that the home government does
not play weakly dominated strategies, then this is the unique equilibrium.\footnote{If the home government is allowed to play weakly dominated strategies, then there will also be Nash equilibria in which no firm undertakes R&D if $\psi(d, f^*, w_{\max}; r) < 0$. For example, $(0, 0)$ would be an equilibrium in that case.} If on the other hand $\psi(d, f^*, w_{\max}, w_{\max}; r) < 0$, the home firm will not undertake R&D in Nash equilibrium. We can use this characterization of the Nash equilibrium to identify how the equilibrium government behavior varies with the value of $r$. Since Proposition 3 established that government choices of patent breadth can have no effect on firm decisions for $r > \rho_1(w_{\max}, w_{\max})$, we restrict attention to $r \leq \rho_1(w_{\max}, w_{\max})$.

**Proposition 5.** Assume the North-South model.

(a) If $\rho_1(w_{\max}, w_{\max}) \geq r > \rho_1(w_{\max}, 0) = \frac{13\theta}{36} (e - c)^2$ then in Nash equilibrium the home government sets patent breadth $w = w_{\max}$ and the foreign government sets patent breadth $w^* = j^*(w_{\max}; r) > 0$.

(b) If $\rho_1(w_{\max}, 0) \geq r$ then in Nash equilibrium the home government sets patent breadth $w = w_{\max}$ and the foreign government sets patent breadth $w^* = 0$.

The Nash equilibrium is highly asymmetric with regard to patent breadth. The North chooses the maximum level of patent breadth while, since the Southern firm never enters into a patent race in the North-South model, the South only provides the minimum level of patent protection required to ensure that the home firm undertakes R&D.

A natural question to ask is whether world welfare could be raised by the harmonization of patent breadth across countries. A movement in the direction of harmonization involves an increase in $w^*$ accompanied by a reduction in $w$ to maintain $\psi(d, f^*, w, w^*; r) = 0$. This requires $dw/dw^* = -(e - c + w^*)/(e - c + w)$. Using (5) and (6), the effect of this change on world welfare is

$$\frac{d(\tilde{v} + \tilde{v}^*)}{dw^*} = \frac{5\theta (e - c) (w^* - w)}{3 (e - c + w)}.$$  

An increase in $w^*$ will reduce world welfare if $w^* < w$, which means that the harmonization of patent breadth from Nash equilibrium in the North-South model will reduce world welfare. This yields the following result.
Proposition 6. Assume the North-South model. An increase in $w^*$, with a corresponding reduction in $w$ to maintain zero profits, will reduce world welfare iff $w^* < w$.

The intuition behind the result is as follows. Harmonization makes Northern consumers better off by narrowing patent protection while holding the profits of the Northern firm constant. Southern consumers are hurt by a broadening of patent protection, and the profits of the Southern firm are reduced. The positive effect on Northern welfare is not sufficient to compensate for the negative effect on Southern welfare and so overall world welfare is reduced by harmonization. This result is driven by the property of the model that welfare is convex in patent breadth. Thus, in the North-South model, world welfare is maximized at the non-cooperative Nash equilibrium.

3.3.2 The North-North Model

Let us return now to the situation where both firms have an equal probability of innovation; both countries are in the North. In this setting, Proposition 4 established that there are three possible outcomes: \{d, d^*\} if $\psi(d, f^*, w, r) > \beta(w; r) \geq 0$; \{d, f^*\} or \{f, d^*\} if $\psi(d, f^*, w, r) \geq 0 > \beta(w; r)$; and \{f, f^*\} if $0 > \psi(d, f^*, w, r)$. For the North-North model we will use the notation $\tilde{v}^{NN}$ to denote home welfare. Since the North-North model is symmetrical, we will not need notation for the foreign country.\(^{24}\) Under our assumption that \{d, f^*\} and \{f, d^*\} occur with equal probability when R&D is profitable for only one firm, the payoff function of the home country is as follows:\(^{25}\)

$$
\tilde{v}^{NN}(w; r) = \begin{cases} 
\theta(2 - \theta)V(w) - r & \text{if } \psi(d, f^*, w, r) > \beta(w; r) \geq 0 \\
\theta V(w) - r/2 & \text{if } \psi(d, f^*, w, r) \geq 0 > \beta(w; r) \\
0 & \text{if } 0 > \psi(d, f^*, w; r)
\end{cases}
$$

where $V(w) = \frac{1}{2}\Pi_n(w, \hat{q}(w)) + \frac{1}{2}\Pi_m(w, \hat{q}(w)) + \Phi(a, a^*)S(\hat{Q}(w))$ is the sum of (expected) consumer and producer surplus in the event that an innovation is made. The home country

\(^{24}\)We focus on a symmetrical model deliberately to show that an asymmetric equilibrium can arise in such an environment. Such an outcome would be less surprising in an asymmetrical environment and such an analytical framework would be more cumbersome to work with. The results of our North-South model give a clear indication of the forces introduced by asymmetry and how the results change.

\(^{25}\)The reason that, in $\tilde{v}^{NN}(w; r)$, $r$ is divided by 2 if $\psi(d, f^*, w; r) \geq 0 > \beta(w; r)$ is because under the strategy profile \{d, f^*\} there is, ex ante, a fifty percent probability of the home firm being selected to choose first whether or not it wants to undertake R&D.
expects a higher surplus from an innovation when both firms enter the patent race, but
must pay a higher expected R&D cost. The symmetry of the probability that either firm
makes an innovation means that each country’s firm is equally likely to serve in the role of
imitator and innovator, both in the outcome where one firm undertakes R&D and when
both firms enter into a patent race. Differentiating $V(w)$ with respect to $w$ yields

$$\frac{\partial V(w)}{\partial w} = -\hat{q}_m(w) > 0. \tag{7}$$

Expected surplus is decreasing in $w$ because the profit gains to the home firm from broader
patent protection if it ends up being the innovator are outweighed by the loss of profit if it
ends up being the imitator added to the resultant loss of consumer surplus.

In order to derive the best-response function of the home government, we first define
intervals $J(w^*; r) = [j(w^*; r), w_{max}] \in [0, w_{max}]$ and $B(w^*; r) = [b(w^*; r), w_{max}] \in [0, w_{max}]$
such that $\psi(d, f^*, w, r) \geq 0$ iff $w \in J(w^*; r)$ and $\beta(w, r) \geq 0$ iff $w \in B(w^*; r)$. Figure 3
illustrates an example of the home welfare function for a case in which $J(w^*; r), B(w^*; r) \neq$
$\emptyset$. These sets must be connected because the expected profit functions with one firm
undertaking R&D and two firms entering into a patent race - $\psi(d, f^*, w, r)$ and $\beta(w, r)$
respectively - are both increasing in $w$ and $w^*$. Home welfare (labeled simply as $v$), is
decreasing on the intervals $[j(w^*; r), b(w^*; r))$ and $[b(w^*; r), w_{max}]$ as indicated by (7), with
a discontinuity occurring where the change in $w$ causes a change in the number of firms
that undertake R&D. The maximum value of $\hat{v}^{NN}(w^*; r)$ must be either at $b(w^*; r)$ or
at $j(w^*; r)$, with the former being preferred if the gain in expected surplus from having a
second firm undertake R&D exceeds the additional R&D cost that results. Applying this
same reasoning to the remaining cases characterizes the home best response.

**Lemma 1.** Assume the North-North model. The best-response correspondence of the
home country, $\hat{w}^{NN}(w^*; r)$, has the following properties.

(a) If $J(w^*; r), B(w^*; r) \neq \emptyset$ and $\hat{v}^{NN}(b(w^*; r), w^*; r) \geq \hat{v}^{NN}(j(w^*; r), w^*; r)$ then
$\hat{w}(w^*; r) = b(w^*; r)$;
if $J(w^*; r), B(w^*; r) \neq \emptyset$ and $\hat{v}^{NN}(j(w^*; r), w^*; r) > \hat{v}^{NN}(b(w^*; r), w^*; r)$ then $\hat{w}(w^*; r) =$
$j(w^*; r)$.

(b) If $J(w^*; r) \neq \emptyset$ and $B(w^*; r) = \emptyset$, then $\hat{w}(w^*; r) = j(w^*; r)$. 

23
Lemma 1 shows that the best response of the home country will always be at a boundary value of patent breadth if R&D activity takes place, in the sense that it is the narrowest patent protection that supports a given number of firms engaging in R&D. If for given \( r \) it is not possible to sustain R&D activity, then any value of \( w \) will yield a payoff of 0. The best-response function of the foreign government takes the same form.

Lemma 1 can be used to characterize the Nash equilibrium in patent breadth for the North-North model. Recall that Proposition 4 identifies five possible configurations of the \( \text{jp} - \) and \( \text{bp} - \) loci that can be observed, depending on the value of \( r \). In two of those cases, (a) and (e), the R&D costs are at such extreme values that government policies have no impact on R&D decisions. Therefore, we will focus our discussion on the remaining cases where government policies do play a role. We will begin with case (b) of Proposition 4, which is the simplest case. We will then consider case (d). As shall become clear, case (c) can be thought of as a combination of cases (b) and (d) and so therefore we will not need to analyze it explicitly.

**Case (b): No firms or one firm.** Recall that in case (b) of Proposition 4, the equilibrium outcome must have either no firms or one firm undertaking R&D. Since \( B(w^*; r) = \emptyset \) for all \( w^* \) when \( r \in (\rho_2(w_{\max}, w_{\max}), \rho_1(w_{\max}, w_{\max})) \), Lemma 1 establishes that the the best response of the home government will be to choose \( j(w^*, r) \) if \( J(w^*; r) \neq \emptyset \) and \( w \in [0, w_{\max}] \) otherwise. If the foreign government chooses \( w^* \) such that \( J(w^*; r) \neq \emptyset \), then by Lemma 1b the home best-response is \( j(w^*; r) \). The symmetry of the home and foreign countries will ensure that this is a Nash equilibrium, since it is the maximum value of \( w^* \) consistent with \( \psi(d, f^*, w, w^*, r) \geq 0 \). Since this holds for any \( w^* \) such that \( J(w^*; r) \neq \emptyset \), we obtain the following result:

**Proposition 7.** Assume the North-North model and \( r \in (\rho_2(w_{\max}, w_{\max}), \rho_1(w_{\max}, w_{\max})) \). Any \( (w, w^*) \) satisfying \( \psi(d, f^*, w, w^*, r) = 0 \) is a Nash equilibrium. If governments do not play weakly dominated strategies, these are the only Nash equilibria.

Proposition 7 shows that in this range of \( r \), there is a continuum of Nash equilibria in which identical countries choose different levels of patent breadth. Each govern-
ment chooses its level of patent breadth to ensure that expected profits are zero, given the patent breadth set by the other government. It follows that any \((w, w^*)\) satisfying \(\psi(d, f^*, w, w^*, r) = 0\) will be a Nash equilibrium. Among the Nash equilibria there is one in which both countries set the same patent breadth. In addition, either side of this symmetric Nash equilibrium, there is a continuum of Nash equilibria in which one country sets successively broader patent breadths and the other country sets patent breadth correspondingly narrower. Therefore, identical Northern countries can choose different levels of patent breadth in equilibrium and thus have different equilibrium payoffs. Although the countries set different patent breadths neither has an incentive to change its policy, given that of the other government. Specifically, neither government has an incentive to reduce patent breadth because this level of patent breadth is required to sustain R&D by one firm; neither government has an incentive to increase patent breadth either because, by (7), this would reduce national welfare as well.

Finally, note the contrast with Proposition 5 for the North-South model, where the equilibrium is unique. In that case the home government has the incentive to provide maximum patent breadth because it knows its firm will be the innovator.

**Case (d): One firm or two firms.** Next consider case (d) of Proposition 4, where the equilibrium outcome must have either one firm undertaking R&D or a patent race involving both firms. This case is associated with an R&D cost of \(r \in (\rho_2(0, 0), \rho_1(0, 0))\), which is sufficiently low that at least one firm will find it profitable to undertake R&D regardless of the countries’ choices of patent breadth (i.e. \(J(w^*; r) = [0, w_{\text{max}}]\) for all \(w^*)\).

In this situation equilibrium characterization depends on \(r\) as well as \(\theta\) because the value of having a second firm enter relative to the first depends on the probability that each will make an innovation. Therefore, our approach will be to present a characterization of equilibrium on a diagram drawn in \((r, \theta)\)-space. This characterization is developed in Figures 4a and 5 as we will now show.

The region of \((r, \theta)\)-space consistent with case (d), or the ‘case (d)-region,’ is illustrated in Figure 4a as the area between the \(\rho_1(0, 0)\) line and the \(\rho_2(0, 0)\) locus. Within this region there is the possibility of Nash equilibria in which patent protection is so narrow that only one firm undertakes R&D as well as patent protection that is sufficiently
broad as to induce both firms to enter. Above this region we have the case (c)-region of Proposition 4 wherein the relatively high level of $r$ may prohibit any firm from profitably undertaking R&D for certain $w \in W$; below the case (d)-region we have the case (e)-region where both firms undertake R&D and government policy is irrelevant.

We begin by showing that there is a sub-region within the case (d)-region for which, in Nash equilibrium, patent protection is such that only one firm undertakes R&D. Since it would be the case that $j(w^*) = 0$ for all $w^*$ in this region, the only possible candidate for equilibrium with one firm undertaking R&D is the symmetric equilibrium $\hat{w} = (0,0)$. This will be a Nash equilibrium if it is not possible for one country to gain by broadening its patent protection to induce a patent race among the two firms. Essentially, there are two possible situations in which this will be the case. The first and most straight-forward is where, if $w^* = 0$, there is no feasible $w \in [0,w_{\text{max}}]$ that will result in a patent race. We will say that $(r,\theta)$ in this range are sufficient for $\hat{w} = (0,0)$ to be a Nash equilibrium. The second is where there does exist a range of $w$ that would result in a patent race but no best-response to $w^* = 0$ exists in this range. We will now discuss each in turn. The basic intuition in each case is that in these regions of $(r,\theta)$-space, the relatively high R&D cost of inducing the second firm to enter into a patent race is large relative to the benefit it provides in the form of the increased likelihood of innovation.

We now consider $(r,\theta)$ that are sufficient for $\hat{w} = (0,0)$. Lemma 1b establishes that a sufficient condition for $\hat{w} = (0,0)$ to be an equilibrium is $B(0;r) = \emptyset$, since there is no feasible home patent breadth that will result in a patent race. This is illustrated by the bp’-locus in Figure 4b, for which $B(0;r) = \emptyset$ because $\beta(w_{\text{max}},0;r) < 0$. Of course this would hold for any bp-locus whose end-points lie above $w = w_{\text{max}}$ and $w^* = w_{\text{max}}$. The reason is that, for zero patent breadth set by one government, there is no patent breadth set by the other government sufficient to induce a patent race. This case will arise if $r \geq \rho_2(w_{\text{max}},0) = \theta(e-c)^2(26 - 17\theta)/72$, which is shown in Figure 4a as the shaded region $A$ above the $\rho_2(w_{\text{max}},0)$ locus and below the $\rho_1(0,0)$ line. Therefore, $\hat{w} = (0,0)$ for the values of $(r,\theta)$ in region $A$. Note that $\rho_1(0,0) < \rho_2(w_{\text{max}},0)$ if $\theta < 10/17$ so, for all possible values of $c > c$, $\theta = 10/17$ is the lowest value of $\theta$ for which the condition $r \geq \rho_2(w_{\text{max}},0)$ holds.

We will now consider $(r,\theta)$ outside of region $A$ and identify where in this region
\( \hat{\mathbf{w}} = (0,0) \). Formally, the region of \((r,\theta)\) under consideration is the region satisfying
\[
(\rho_2(0,0), \min \{(\rho_1(0,0), \rho_2(w_{\text{max}},0))\}].
\]
In this range of \( r \) we have \( B(0;r) \neq \emptyset \). To consider this range, it will be helpful to define \( H(r) \equiv \tilde{\nu}^{NN}(0,0;r) - \tilde{\nu}^{NN}(b(0;r),0;r) \).
The key thing is that, by Lemma 1a, \( \hat{\mathbf{w}} = (0,0) \) if \( H(r) \geq 0 \). This case is shown by the bp-locus in Figure 4b, where \( \hat{\mathbf{w}} = (0,0) \) if \( w = 0 \) yields a higher payoff than is obtained at the pair \((b(0;r),0)\). Note that \( H(r) \) is increasing in \( r \), because home welfare is decreasing in \( w \) and higher values of \( r \) require the home country to choose higher values of \( w \) in order to induce a patent race.\(^{26}\)

For \( \theta \geq 10/17 \) and \( H(\rho_2(w_{\text{max}},0)) > 0 \) there exists a value \( h(\theta) \in (\rho_2(0,0),\rho_2(w_{\text{max}},0)) \) such that \( j(0;h(\theta)) = 0 \) and \( \hat{\mathbf{w}} = (0,0) \) for \( r > h(\theta) \). For \( \theta < 10/17 \), there are two possibilities. If \( H(\rho_1(0,0)) > 0 \), then there exists a value \( h(\theta) \in (\rho_2(0,0),\rho_1(0,0)) \) such that \( j(0;h(\theta)) = 0 \), and so \( \hat{\mathbf{w}} = (0,0) \) for \( r > h(\theta) \). If \( H(\rho_1(0,0)) \leq 0 \), then \( \mathbf{w} = (0,0) \) will not be an equilibrium for any \( r \in (\rho_2(0,0),\rho_1(0,0)) \).

In the proof of Proposition 8, we show that the latter case must hold for \( \theta \) sufficiently close to 0.

Figure 5 illustrates the set of \((r,\theta)\) combinations for which \( \hat{\mathbf{w}} = (0,0) \) is an equilibrium. Note that Figure 5 provides a general characterization of equilibria because the model is invariant to a re-scaling of \( e \) relative to \( c \), where \( e \) and \( c \) are the only two other relevant parameters. The fg-locus is the locus of values at which the patent race and the equilibrium with one firm undertaking R&D yield the same payoff (i.e. \( H(r) = 0 \)). The policy pair \( \mathbf{w} = (0,0) \) is an equilibrium for the region of \((r,\theta)\) on or above the fg locus. Therefore, \( \hat{\mathbf{w}} = (0,0) \) in regions A, B and C of Figure 5.

We will next consider the conditions under which there can be an equilibrium with a patent race. Note that if \( \hat{\mathbf{w}} \neq (0,0) \), then the equilibrium may be asymmetric, i.e. \( \hat{\mathbf{w}} = (b(0,r),0) \), in which case the two firms will engage in a patent race. For the home government, we established above that if \( H(r) \leq 0 \) then, by Lemma 1a, \( \hat{\mathbf{w}} = b(0,r) \).

For the foreign government, all of its policy choices result in having a patent race, so \( \hat{\mathbf{w}}^* = 0 \). Our formal statement of the result in Proposition 8 below focuses on the extreme asymmetric equilibria \( \hat{\mathbf{w}} = (0,1) \) and \( \hat{\mathbf{w}} = (1,0) \). However, as in case (b), there is the potential for a continuum of equilibria in which there is a patent race. Such asymmetric equilibria \( \hat{\mathbf{w}} = (b(0,r),0) \) exist for values of \((r,\theta)\) in region D of Figure 5. Finally, we have

\(^{26}\) This property of \( H(r) \) is demonstrated formally in the proof of Proposition 8; see the Appendix.
now established that, for all \( r \in (\rho_2(0,0), \rho_1(0,0)] \), an equilibrium of the patent breadth game must exist.

Now we examine the possibility of a different type of multiplicity of equilibria, and it is this that gives rise to the scope for a beneficial agreement between governments to coordinate over patent breadth. We have already established conditions under which \( \hat{w} = (0,0) \) with only one firm undertaking R&D. Now let us focus on the conditions under which there will be a second symmetric equilibrium \( \hat{w} = (\hat{w}, b(\hat{w};r)) \) where \( \hat{w} = b(\hat{w};r) \) and the two firms engage in a patent race; this is illustrated by point Z in Figure 4b. Applying Lemma 1a, this symmetric pair will be an equilibrium if it yields higher welfare to the home country than could be obtained from a deviation by the home government to \( j(w;r) = 0 \). (By symmetry, we could check the same incentive to deviate by the foreign government.) Under this deviation, only one firm would undertake R&D. Such a deviation is illustrated by point Y in Figure 4b. The deviation involves a trade-off of the loss from the lower probability of successful innovation against the gain from shifting the cost of patent protection to support innovation onto foreign consumers. Point Y becomes more attractive relative to Z as \( r \) increases because a higher value of \( r \) brings about a lower expected payoff to a patent race; the corresponding rise in required patent breadth makes shifting the burden of paying for innovations onto the other country’s consumers more attractive. For \( r \) sufficiently close to \( \rho_2(0,0) \), point Z must be preferred to Y. Thus, we have two possibilities. Either the patent race equilibrium with symmetric patent breadth exists for all \( r \in [\rho_2(0,0), \rho_1(0,0)] \) or there is a critical value, which we will call \( k(\theta) \), such that the patent race equilibrium exists for \( r \leq k(\theta) \). Point Z will fail to be an equilibrium for values of \( \theta \) on the \( \rho_1(0,0) \) locus sufficiently close to 1. However, point Z will be an equilibrium for values of \( \theta \) on the \( \rho_1(0,0) \) locus that are sufficiently low. Thus, low values of \( \theta \) and \( r \) will be more favorable to the patent race equilibrium with symmetric patent breadth. The dotted eg locus in Figure 5 illustrates the \((r,\theta)\) combinations for which point Z yields the same payoff as point Y. For values below (above) the eg locus point Z yields a higher (lower) payoff and is (is not) an equilibrium to the patent breadth game. Therefore, symmetric equilibria \( \hat{w} = (\hat{w}, b(\hat{w};r)) \) exist for values of \((r,\theta)\) in regions C and D of Figure 5. We can summarize this discussion formally as follows.
Proposition 8. Assume the North-North model and \( r \in (\rho_2(0,0), \rho_1(0,0)) \).

(a) There exists a value \( h(\theta) \in [\rho_2(0,0), \rho_1(0,0)] \) such that:

(i) For any \( r \in [\rho_2(0,0), h(\theta)] \), in Nash equilibrium, \( \hat{w} = (0,1) \) or \( \hat{w} = (1,0) \) and two firms engage in a patent race;

(ii) For any \( r \in [h(\theta), \rho_1(0,0)] \), in Nash equilibrium \( \hat{w} = (0,0) \) and one firm undertakes R&D.

(b) There exists a value \( k(\theta) \in (\rho_2(0,0), \rho_1(0,0)) \) such that for any \( r \in [\rho_2(0,0), k(\theta)] \), in Nash equilibrium \( w = b(w; r) \) and two firms engage in a patent race.

Proof: See appendix.

Note several interesting points that come to light in Figure 5. First, in region C (between the eg- and fg-loci) there are multiple symmetric equilibria. Thus, for values of \((r, \theta)\) in this region there is the potential for coordination failure in the setting of patent breadth since, whether a country adopts narrow or broad patent protection, there is no incentive for the other country to deviate from the same level. Second, the region of \((r, \theta)\) values for which there is a symmetric equilibrium with a patent race - regions C and D - contains the region in which there is an extreme asymmetric equilibrium with a patent race - region D only. This suggests that the symmetric equilibrium of a patent race is more likely to be an equilibrium than the asymmetric equilibrium, in the sense that it is an equilibrium for a larger set of parameter values. Although we have not examined the conditions under which the various points on the bp-locus are equilibria, these results lead us to conjecture that the points on the bp-locus that are closer to the diagonal (i.e. point Z in Figure 5) are more likely to be equilibria than the points further away from the diagonal.

We are now in a position to address the question of whether there is scope for an international agreement to harmonize patent breadths in the North-North model and, if so, what it would look like. Recall that, for values of \((r, \theta)\) in region C, there may be a symmetric equilibrium \( \hat{w} = (0,0) \) with one firm undertaking R&D and symmetric equilibrium \( \hat{w} = (\hat{w}, b(\hat{w}; r)) \) with a patent race. The answer to the question of what an agreement to harmonize patent breadths would look like rests on which noncooperative
equilibrium yields higher social welfare. In Figure 5, the hg-locus shows the \( (r, \theta) \) values for which \( \hat{w} = (0, 0) \) yields the same level of national welfare as \( \hat{w} = (\hat{w}, b(\hat{w}; r)) \). For equilibria below the hg-locus and above the fg-locus, \( \hat{w} = (\hat{w}, b(\hat{w}; r)) \) yields the higher level of national welfare and hence is efficient. Thus, in this region we have the familiar situation where an equilibrium \( \hat{w} = (0, 0) \) may arise in which patent breadth is too narrow in both countries. There is an incentive to have an agreement that coordinates on the symmetric equilibrium \( \hat{w} = (\hat{w}, b(\hat{w}; r)) \) wherein patent protection in both countries is broader, thereby maximizing efficiency. For equilibria above the hg-locus and below the eg-locus, by contrast, we have a more surprising case where the opposite may hold; \( \hat{w} = (0, 0) \) yields the higher level of national welfare and hence is efficient, but \( \hat{w} = (\hat{w}, b(\hat{w}; r)) \) may arise. There can thus also arise the incentive to have an agreement that coordinates on the narrower symmetric equilibrium \( \hat{w} = (0, 0) \) which, for this range of \( (r, \theta) \), is efficient. Thus we have one possible rationale for why the EPO may sometimes revoke patents that have been granted by individual national governments as well as vice versa.

Case (c): No firms, one firm or two firms. The remaining range of values of \( r \), case (c) of Proposition 4, is where equilibria with zero, one, or two firms undertaking R&D are all possible. The analysis for this case is a combination of that in the previous two cases (b) and (d), in that it involves the possibility of a continuum of equilibria along both the jp- and bp-loci. Therefore, as mentioned above, we will not discuss it in detail.

These results indicate that, in the North-North model, a multiplicity of equilibria can arise in two ways. One is that for a given number of firms in the industry, there can be a continuum of equilibria which just support that number of firms, but differ in which country offers the broader patent protection. A second type of multiplicity, in which there are equilibria that involve different numbers of firms, can arise as well.

4 Conclusions

We considered two cases: a North-South model in which the Southern firm could imitate but not innovate and a North-North model in which firms in each country had the same probability of success at making an innovation. In the North-South model the equilibrium
level of innovative activity, measured by the number of firms engaged in R&D, had to be less than the socially optimal level. However, in the North-North model the level of innovative activity could exceed the socially optimal level when the probability of success was high. In the North-South model there was no scope for agreement over patent breadths while in the North-North model agreement could be beneficial, involving either an increase or more surprisingly a decrease over the levels set non-cooperatively by national governments.

A promising direction for future work would be, following Bessen and Maskin (2009), to consider the possibility that imitation is a productive activity for the imitating firm and thus raise the likelihood of future innovation by that firm. Bessen and Maskin (2009) show that the equilibrium with patent protection might yield lower social welfare than an equilibrium without patent protection when the act of imitation produces knowledge that may affect the future rate of innovation. They analyze this in a model of sequential innovation where firms produce differentiated products, which results in a complementarity in innovative activity between firms. However, their setting is essentially domestic and does not allow for strategic interaction in the breadth of patent setting across countries. In a model that combined the Bessen-Maskin framework with ours, very broad patent protection in the North could have the adverse effect of reducing the degree of accumulation of knowledge for future innovation, which could add to the benefit that we have demonstrated of an agreement to lower patent breadth.

A Appendix

Proof of Proposition 2. Fix \( w \in [0, (e-c)/2] \). Then a standard solution for Cournot-Nash equilibrium yields the first part of the result. Solving for \( n \)'s best response function in the home market, we obtain

\[
R_n (q_m; w) = \frac{e - c - q_m}{2}.
\]

Solving in the same way for \( m \)'s best response in the home market, we obtain

\[
R_m (q_n; w) = \frac{e - c - w - q_n}{2}.
\]

Using these functions to solve for mutual best responses obtains the equilibrium result.
Using \( w = (e - c)/2 \) in the equilibrium solutions, we obtain \( \tilde{q}_n = (e - c)/2 \) and \( \tilde{q}_m = 0 \). The second part of the result follows. □

**Proof of Proposition 8:**

a) Calculating the critical values yields
\[
\rho_2(w_{\text{max}}, 0) = \theta(e - c)^2(17\theta - 26)/72 \leq \rho_1(0, 0) = 2\theta(e - c)^2/9 \text{ iff } \theta \geq 10/17.
\]
Consider first the case with \( \theta \geq 10/17 \). If \( r > \rho_2(w_{\text{max}}, 0) \) we have \( B(0; r) = \emptyset \) and \( w = 0 \) is a best response for the home country by Lemma 1a. By symmetry, the same hold for the foreign country. Therefore, \( w = (0, 0) \) is a Nash equilibrium in this interval. For \( r \in (\rho_2(0,0), \rho_2(w_{\text{max}},0)] \), we define \( \tilde{H}(w) = \tilde{v}^{NN}(0,0; \rho_2(w,0)) - \tilde{v}^{NN}(w,0; \rho_2(w,0)) \). Since \( w = b(0; \rho_2(w,0)) \), \( \tilde{w}(0) = 0 \) if \( \tilde{H}(w) > 0 \) and \( \tilde{w}(0) = w \) if \( \tilde{H}(w) \leq 0 \) by Lemma 1a. Since \( \partial (\tilde{v}^{NN}(0,0; \rho_2(w,0)) - \tilde{v}^{NN}(w,0; \rho_2(w,0))) / \partial r = 1/2, \partial \rho_2(w,0)/\partial w = \partial \beta(w,0,r)/\partial w > 0 \), and \( \partial \tilde{v}^{NN}(w,0; \rho_2(w,0))/\partial w < 0 \), we have \( \tilde{H}'(w) > 0 \). Evaluation of the objective function at the endpoints yields \( \tilde{H}(0) = -\theta(1 - \theta)(e - c)^2/3 \leq 0 \) and \( \tilde{H}(w_{\text{max}}) = 7\theta(5\theta - 2)(e - c)/144 \). For \( \theta \geq 10/17 \), the latter term must be positive and there exists a unique \( \omega(\theta) \in (0,w_{\text{max}}) \) such that \( \tilde{H}(\omega(\theta)) = 0 \) and \( \rho_2(\omega(\theta),0) \in [0,\rho_2(w_{\text{max}},0)) \). Solving yields

\[
\omega(\theta) = (e - c) \left( \frac{14 - 5\theta - \sqrt{208\theta - 59\theta^2 - 65}}{22 - 7\theta} \right)
\]

which satisfies \( \omega(\theta) < w_{\text{max}} \) for \( \theta \geq 10/17 \). It then follows that \( \tilde{w} = (0,0) \) for \( r > \rho_2(\omega(\theta),0) \).

For \( \theta < 10/17 \), let \( g(\theta) \) solve \( \rho_2(g,0) - \rho_1(0,0) = 0 \), where \( g(\theta) \in [0,w_{\text{max}}] \). The above arguments also ensure that \( \tilde{H}(w) \) is decreasing in \( w \) for \( w \in [0,g(\theta)] \) with \( H(0) < 0 \). There are two possibilities. If \( H(g(\theta)) > 0 \), then there exists a unique \( \omega(\theta) \in (0,g(\theta)) \) such that \( \tilde{H}(\omega(\theta)) = 0 \) and \( \rho_2(\omega(\theta),0) \in [\rho_2(0,0),\rho_1(0,0)) \). In this case \( w = (0,0) \) is an equilibrium for \( r > \rho_2(\omega(\theta),0) \). Equation (8) will be the solution for \( \omega(\theta) \) in this case as well. If \( H(g(\theta)) \leq 0 \), then \( w = (0,0) \) is not an equilibrium for any \( r \in [\rho_2(0,0),\rho_1(0,0)] \).

b) Define \( \tilde{K}(w) = \tilde{v}^{NN}(w,w;\rho_2(w,w)) - \tilde{v}^{NN}(0,w;\rho_2(w,w)) \). It follows from Lemma 1a that \( (w,w) \) is an equilibrium for all \( r = \rho(w,w) \) such that \( \tilde{K}(w) \geq 0 \). Differentiating this expression yields the fact that \( \tilde{K} \) is strictly convex in \( \theta \) for \( \theta \in [0,1] \) with

\[
\frac{d\tilde{K}}{dw} = -(e - c)(3 - \theta) - w(5 - 2\theta)/3,
\]
Since this expression is equal to \(-(e - c)\theta/6 < 0\) when evaluated at \(w = w_{\text{max}}\), it follows that \(\bar{K}(w)\) is decreasing in \(w\). Note also that \(\bar{K}(0) = -\bar{H}(0) > 0\), so \((w, w)\) must be an equilibrium for \(w\) sufficiently small. This yields two possibilities. If \(\bar{K}(w)\) is non-negative for all \(w\) such that \(\rho_2(w, w) < \rho_1(0, 0)\), then \((w, w)\) is always an equilibrium. If there exists \(w\) such that \(\rho_2(w, w) < \rho_1(0, 0)\) and \(\bar{K}(w) = 0\), then \((w, w)\) fails to be an equilibrium for all higher values of \(w\). Solving \(\bar{K}(w) = 0\) yields the critical value for \(w\) to be

\[
\gamma(\theta) = (e - c) \left( \frac{3 - \theta - \sqrt{8\theta - 3\theta^2 - 1}}{5 - 2\theta} \right)
\]

Setting \(k(\theta) = \rho_2(\gamma(\theta), \gamma(\theta))\) if \(\rho_2(\gamma(\theta), \gamma(\theta)) < \rho_1(0, 0)\) and \(k(\theta) = \rho_1(0, 0)\) otherwise yields the desired cutoff for Proposition 8.

References


Figure 1: The effect of patent breadth on equilibrium quantities in the home market

\[ R_n(q_n; w) = \frac{e - c - w}{2} - \frac{1}{2} q_n \]

\[ R_n(q_m; w) = \frac{e - c}{2} - \frac{1}{2} q_m \]
Figure 2: The jp-locus and the bp-locus
Figure 3: Home Welfare Function in the North-North Model

(given foreign policy)
Figure 4a: Feasible $(r, \theta)$ values for case (d)
Figure 4b: bp loci for case (d)
Figure 5: One Firm or Two Firms; Characterization of Equilibrium