Selling Substitute Goods to Loss-Averse Consumers: Limited Availability, Bargains and Rip-offs

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Abstract

I characterize the profit-maximizing pricing and product-availability strategies for a retailer selling two substitute goods to loss-averse consumers. Consumers have unit demand, are interested in buying at most one good, and their reference point is given by their recent rational expectations about what consumption value they would receive and what price they would pay. If the goods are close substitutes, the seller maximizes profits by creating an “attachment effect” for the consumers through a tempting discount on a good available only in limited supply (the bargain) and cashing in with a high price on the other good (the rip-off). Consumers are enticed by the possibility of getting a bargain, but, if it is not there, they buy a substitute good as a means of minimizing disappointment. The seller tends to use the more valuable product as a bargain because consumers feel a larger loss, in terms of forgone consumption, if this item is not available and are hence willing to pay a larger premium to reduce uncertainty in their consumption. I also show that the bargain can be priced below marginal cost, that the seller’s product mix differs from the welfare-maximizing one and that she might supply a socially wasteful product.

JEL classification: D11; D42; L11.

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1 Introduction

Retailers frequently use low prices and offer deals to attract consumers. In many cases, these deals apply only to a subset of a store’s product line, and it is not uncommon for them to be subject to a “limited-availability” condition. Some shops, for example, offer deals that are valid only “while supplies last,” or they might offer price reductions on sale items only to the very first customers of the day. Limited-time offers can also be interpreted as an example of limited availability. Hence, among the consumers who go shopping with the intention of getting a deal, some fulfill their goal and get a bargain; others, however, might not find what they were looking for and might end up buying a different item, maybe one that is not even on sale and for which they would have not gone to the store in the first place. Consider the two following examples:

Example 1  A retailer in Berkeley California has offered the following:
Converse All Star high-top in black for just $24.99 (offer valid while supplies last).
Any other color for $54.99.\footnote{At the same retailer, Bancroft Clothing Co., the regular price during the “non-deal” weeks is $44.99, independent of color. The manufacturer online price is $50 plus shipping fees.}

Example 2  On Black Friday 2011, Best Buy offered, among other items, the following:
Panasonic 50” Class / Plasma / 1080p / 600Hz / Smart HDTV for $599.99.
Panasonic 50” Class / Plasma / 720p / 600Hz / HDTV for $799.99.\footnote{Black Friday is the day following American Thanksgiving and traditionally marks the beginning of the Christmas shopping season. The 1080p TV first appeared at Best Buy on March 20th 2011 for $1,000 and its price has been constant until Thanksgiving Day of the same year. The 720p TV first appeared at Best Buy on March 28th for $719.99 and its price was reduced to $649.99 on August 9 2011 and raised again up to $799.99 on November 10th 2011, two weeks before Thanksgiving. These data have been collected using camelbuy.com, a website that provides a price tracker and price history charts for products sold online at Amazon.com and Best Buy.com.}

In the first example, the store is offering a deal on black shoes — $20 less than the regular price. There is, however, no deal on other colors; indeed their price is $10 higher than the regular price. The $30 difference between the price of black and non-black shoes is unlikely explained by differences in cost or demand. Furthermore, the deal on black shoes is valid only while current supplies last and the price could well be higher once the store restocks.

In the second example, the store is selling two very similar TVs for very different prices; moreover, somewhat puzzlingly, the TV with the higher-resolution screen, universally preferred, is offered at a lower price. The original Best Buy ad specified that the one on the superior TV was an online-only deal, that availability was “limited to warehouse quantity,” and no rainchecks would be offered to consumers.

The goods in these examples are substitutes and consumers normally buy at most one unit. Why, then, do stores discount only a few items heavily, and why is there so much dispersion, within
the same store, in the price of similar goods? And how do stores select which items to offer for a discount?

Most economic models of sales pertain mainly to retailers supplying only one product and are concerned with explaining price dispersion either across different stores (as in Salop and Stiglitz, 1977) or across different time periods (as in Varian, 1980), but do not study the issue of within-store price dispersion across similar items. Furthermore, these models consider sales as a means to separate those consumers who are informed about prices from those who are not. As such, this argument does not work for Black Friday or similar widely advertised promotions when virtually all consumers know about the sales.

In this paper, I provide a new, different explanation for the above retailing strategy. I do so by introducing consumer loss aversion into an otherwise classical model of linear pricing: a risk-neutral profit-maximizing monopolist sells two substitutable goods to homogeneous consumers who demand at most one unit of either good and whose reference point for evaluating a purchase, following the model of Kőszegi and Rabin (2006), is given by their recent rational expectations about the purchase itself. With these preferences, a consumer’s willingness to pay for a good is determined not only by his intrinsic value for it, but also endogenously by the market conditions and his own anticipated behavior, and the monopolist can directly affect a consumer’s expectations by making announcements regarding prices or availability. For example, if a consumer expects to buy with high probability, he experiences a loss if he fails to buy. This, in turn, increases his willingness to pay. On the other hand, compared to the possibility of getting a deal, paying a high price is assessed by the consumer as a loss, which in turn decreases his willingness to pay. Since expectations are the reference point and because expectations are (also) about own future behavior, the reference point is determined endogenously by requiring that the (possibly stochastic) outcome implied by optimizing behavior conditional on expectations coincides with expectations.

The main result of the paper is that, when two goods are close substitutes, the monopolist maximizes profits by offering a limited-availability deal on one of the two goods to lure consumers and then cashes in with a high price on the other good. Crucially, consumers perceive this limited-availability sale as equivalent to a lottery on both which good they will end up with and how much they will pay. The price of the good on sale (the bargain) is chosen such that it is not credible for the consumers to expect not to buy it. Thus, the limited-availability deal works as a bait in luring consumers into the store. Then, because the consumers expect to make a purchase with positive probability and dislike the uncertainty in their consumption outcomes, in the event that

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3 Rhodes (2011) and Zhou (2012) study multi-product search models with complements. A notable exception is provided by Konishi and Sandfort (2002). In their paper a multi-product store can increase its profits by discounting only some of its products, even when they are substitutes. However, consumers in this model shop for a “search good” and hence they learn their tastes only once they arrive at the store and discounts on few items are a way to increase store traffic. The logic in my model is quite different.

4 There is a reason why in Black Friday jargon these deals are called “doorbusters.”
the bargain is not available, they prefer to buy the substitute good, even at a higher price (the rip-off). In other words, consumers go to the store enticed by the possibility of the bargain, but if it is not there they buy a substitute good as a means of reducing their disappointment.\footnote{Because of loss aversion, consumers are willing to pay a premium in order to avoid the feeling of loss resulting from not getting the bargain. So, the seller is not exploiting a cognitive bias of the consumers. This is in contrast to several models with boundedly rational or naïve consumers where firms design prices and tariffs to exploit consumers’ cognitive biases, as in DellaVigna and Malmendier (2004), Eliaz and Spiegler (2006, 2008), Gabaix and Laibson (2006), Grubb (2009) and Spiegler (2006). See Spiegler (2011) for a survey.}

The limited-availability aspect of the deal is critical and the degree of availability of the bargain is publicly announced by the seller.\footnote{This differentiates the strategy the seller employs in my model from classical \textit{bait-and-switch}, a form of false advertising in which the seller advertises a low-price good but then replaces it with a different, more expensive one at the showroom.} When doing this, the seller faces the following trade-off. On the one hand, choosing a high likelihood of availability for the bargain makes the consumers more attached to the idea of buying and this in turn allows the seller to charge a higher mark-up on the rip-off. On the other hand, a greater availability of the bargain necessarily means fewer sales of the rip-off. At the optimum, these two effects offset each other.

Interestingly, with this strategy, the seller is able to extract, in expectation, more than the maximum of consumers’ intrinsic valuations for the goods. The intuition is as follows: if a consumer finds a product he likes available for a very low price, he will definitely buy it. The attachment to the good induced by realizing that he will do so, however, changes his attitudes toward the purchasing decision. If the store runs out of the good on sale for a low price, but still has a similar one available for a higher price, the consumer must now choose between a loss of money from paying a higher price and a loss of consumption from returning home empty-handed. While, in equilibrium, buying is indeed the best response to his expectations, it is still worse than if he could have avoided the feeling of loss by avoiding the expectation of getting the good for a low price in the first place.\footnote{More generally, because a loss-averse consumer does not internalize the effect of his ex-post behavior on ex-ante expectations, the strategy that maximizes ex-ante expected utility is often not a credible plan.} Moreover, consumers are hurt also by the uncertainty about which item they will get to consume and how much they will pay. Therefore, despite the fact that, with some probability, they get a good deal, on average consumers are made worse off by this combination of limited availability, bargains, and rip-offs.

When the goods are vertically differentiated, the seller tends to use the more valuable item as the bargain. This may, at first, seem odd, given that consumers are (intrinsically) willing to pay a higher price for the superior good. Yet, exactly because consumers value the superior item more, the possibility of a bargain causes them to feel a larger loss, in terms of forgone consumption, when this item is not available; hence, they are willing to pay an even bigger premium to reduce the uncertainty over their consumption outcome, which, in turn, allows the seller to charge an even higher price for the rip-off. So my model predicts that more valuable items should be more likely
to be used as bait, as in Example 2 above.

A related implication is that the seller, in order to effectively induce uncertainty into the consumers’ purchasing plans, might even introduce a less socially desirable or, worse, socially wasteful product. If consumers have common taste for quality and unit demand, then, with classically assumed preferences, the monopolist’s incentives would be perfectly aligned with the social ones: the monopolist selects the product that maximizes total welfare, and extracts all the surplus. With reference-dependent preferences, however, the incentives are misaligned and the profit-maximizing product mix need not be the socially optimal one.\(^8\) Although this implication appears also in models of second-degree price discrimination via quality distortion (i.e., Deneckere and McAfee, 1996), the motive is not screening and my result holds even if consumers have identical tastes.

Nothing precludes the bargain item from being a “loss leader” (i.e., being priced below cost). Standard models of consumer behavior in industrial organization can explain the use of loss leaders for complementary goods, but not substitutes (see Ambrus and Weinstein, 2008). My model can rationalize the use of loss leaders for substitutes. With classically assumed preferences, the scope for loss leaders is to increase store traffic; however, for this increase in store traffic to be profitable, consumers must buy other items as well \textit{in addition} to the loss leader. Here, instead, loss leaders have the purpose of luring consumers into the store, but their profitability stems from the fact that, if the seller has run out of the loss-leading product, consumers will buy another item \textit{instead} of the loss leader in order to minimize their disappointment. Furthermore, while in traditional IO models those products with lower consumer value are more natural candidates for loss-leading pricing, my model can explain the use of highly valuable products as loss leaders. This is consistent with the fact that, on Black Friday, Best Buy offers a below-cost large-screen flat TV to the first ten people who buy one.

The remainder of this paper proceeds as follows. Section 2 briefly summarizes the key empirical evidence on sales and limited availability. Section 3 describes the baseline model with homogeneous consumers and the features of market demand when consumers have reference-dependent preferences. Section 4 presents the main result about the seller’s optimal pricing and availability with homogeneous consumers. Section 5 deals with two extensions of the baseline model: endogenous product lines and heterogeneous consumers. Section 6 discusses the related literature. Section 7 concludes by recapping the results of the model and pointing out some of its limitations.

\(^8\)Klemperer and Padilla (1997) obtain a similar result in an oligopoly model where consumers have classical preferences and multi-unit demand. For this environment they show that a firm might want to introduce an additional, socially wasteful variety, because of a profitable business stealing effect.
2 Summary of Evidence

Sales, in the sense of periodic, temporary price reductions, are a ubiquitous feature of retail pricing.\textsuperscript{9} Not only are sales frequent, but they also represent a significant portion — between 20\% and 50\% — of all price variation within a given product category (Hosken and Reiffen, 2004a and Nakamura and Steinsson, 2008). Aguirregabiria (1999) finds that price promotions are very frequent and account for 22\% of total sales.

However, among all the items supermarkets and other retailers carry, usually only a small fraction are offered at a low sale price each week and, within categories, retailers seem to systematically place some products on sale more often then others, with more popular items — those appealing to a wider range of customers — being more likely to go on sale (Hosken and Reiffen, 2004b). Relatedly, Nakamura (2008) finds that only a small fraction (19\%) of price variation is common to all products in a category at a given retail store.\textsuperscript{10}

Chevalier, Kashyap and Rossi (2003) find that the majority of sales are not caused by changes in wholesale pricing, implying therefore that sales are primarily due to changes in retailers’ margins. Similarly, Anderson, Nakamura, Simester and Steinsson (2012) report that while regular prices react strongly to costs and wholesale price movements, the frequency and depth of sales is largely unresponsive.

While not as ubiquitous as sales, stockouts are also prevalent in retail markets. Gruen, Corsten and Braradwaj (2002) report a 8.3\% of out-of-stock rate worldwide, rising to even 25\% for some promoted items. A survey about retail stockouts in the U.S., conducted by Andersen Consulting in 1996, reports that 8.2\% of the items were out of stock on a typical day in a typical supermarket (15\% considering only advertised items) and that nearly half of the items tracked (48\%) were out-of-stock at least once a month. Hess and Gerstner (1987) sampled two general merchandise stores and found that stockouts occurred more often for products on sale than for similar products not on sale. Using data from a supermarket chain in Spain, Aguirregabiria (2005) documents a significant amount of heterogeneity across items in the frequency of stockouts; most of this heterogeneity is within-product (i.e., among brands of the same product line) and not among products. Bils (2004) presents evidence on temporary stockouts for durable consumer goods using data from the CPI Commodity and Services Survey and finds that from January 1988 to June 2004 the temporary stockout rate averaged between 8.8\% and 9.2\%.

Empirical evidence on consumers’ response to product unavailability suggests that buyers are often willing to buy substitute items when faced with stockouts (Emmelhainz, Stock and Em-

\textsuperscript{9}Sales might also refer to systematic reductions in the price of fashion items; see Lazear (1986), Pashigian (1988) and Pashigian and Bowen (1991).

\textsuperscript{10}In other words, when the price of Diet Coke drops, chances are that price of the Pepsi Max down the aisle will not change.
melhainz, 1991; and Anupindi, Dada and Gupta, 1998). Conlon and Mortimer (2011) conducted a field experiment by exogenously removing top-selling products from a set of vending machines and tracking subsequent consumer responses. Their results show that most consumers purchase another good when a top-selling product is removed. Moreover, some product removals increase the vendor’s profits as consumers substitute toward products with higher margins. Finally, Ozcan (2008) realized a survey study in a grocery store where the manager had previously agreed to create stockouts artificially by removing some items entirely from the shelves. Of all the consumers who replied to the survey by saying that they had experienced a stockout, 11% said they cancelled or postponed the purchase, 49% decided to switch store (there are two other supermarkets within a 4 minute walking distance from the treated store), and 40% said they bought a substitute item for the one that was not available.

3 Model

In the first part of this section, I introduce the environment by describing the consumers’ preferences, the seller’s strategies and the timing of the interaction between the seller and the consumers. In the second part, I define the solution concepts and describe the features of market demand when consumers are expectations-based loss-averse.

3.1 Environment

There is a unit mass of identical consumers whose intrinsic valuation for good $i$ is $v_i$, $i = 1, 2$. Assume $v_1 \geq v_2 > 0$. The goods are substitutes and each consumer is interested in buying at most one of the two goods.\textsuperscript{11}

Consumers have expectations-based reference-dependent preferences as formulated by Kőszegi and Rabin (2006). In this formulation, a consumer’s (his) utility function has two components. First, when buying item $i$ at price $p_i$, a consumer experiences consumption utility $v_i - p_i$. Consumption utility can be thought of as the classical notion of outcome-based utility. Second, a consumer also derives utility from the comparison of his actual consumption to a reference point given by his recent expectations (probabilistic beliefs).\textsuperscript{12} For a riskless consumption outcome

\textsuperscript{11}Alternatively, this situation can be thought as one of vertical differentiation in which there are two versions of the same item, with good 2 being the “basic” version and good 1 being the “advanced” version. All consumers agree on the vertical ranking of the two goods.

and riskless expectations \((\bar{v}_i, \bar{p}_i)\), a consumer’s total utility is given by

\[
U [(v_i, p_i) | (\bar{v}_i, \bar{p}_i)] = v_i - p_i + \mu (v_i - \bar{v}_i) + \mu (\bar{p}_i - p_i)
\]

(1)

where

\[
\mu(x) = \begin{cases} 
\eta x & \text{if } x \geq 0 \\
\eta \lambda x & \text{if } x < 0 
\end{cases}
\]

is gain-loss utility.\(^{13}\)

I assume \(\eta > 0\) and \(\lambda > 1\). By positing a constant marginal utility from gains and a constant, but larger marginal disutility from losses, this formulation captures prospect theory’s (Kahneman and Tversky 1979, Tversky and Kahneman 1991) loss aversion, but without its diminishing sensitivity. The parameter \(\eta\) can be seen as the relative weight a consumer attaches to gain-loss utility, and \(\lambda\) can be seen as the coefficient of loss aversion.

According to (1), a consumer assesses gains and losses over product’s quality and payment, separately. Hence, if his reference point is that he will not get the product — and thus pay nothing — for instance, then he evaluates getting the product and paying for it as a gain in the item dimension and a loss in the money dimension rather than as a single gain or loss depending on total consumption utility relative to his reference point. This feature of the Kőszegi-Rabin’s model is what allows the monopolist to extract more than the consumer’s intrinsic valuation for the good.\(^{14}\) Furthermore, this is consistent with much experimental evidence commonly interpreted in terms of loss aversion.\(^{15}\)

Because expectations are rational, and in many situations such rational expectations are stochastic, Kőszegi and Rabin (2006) extend the utility function in (1) to allow for the reference point to be a pair of probability distribution \(F = (F^v, F^p)\) over the two dimensions of consumption

\(^{13}\)The model of Kőszegi and Rabin (2006) assumes that the gain-loss utility function \(\mu\) is the same across all dimensions. In principle, one could also allow for this function to differ across the item and the money dimension. For example, Novemsky and Kahneman (2005) and Kőszegi and Rabin (2009) argue that reference dependence and loss aversion are weaker in the money than in the item dimension.

\(^{14}\)The other crucial feature of these preferences, which is that the reference point is determined by the decision maker’s forward-looking expectations, is shared with the disappointment-aversion models of Bell (1985), Loomes and Sugden (1986), and Gul (1991). However, because in these models gains and losses are assessed along only one dimension, the consumer’s intrinsic utility \((v_i - p_i,\) in this paper), they are unable to predict the type of pricing schemes that is the subject of this paper.

\(^{15}\)This feature is able to predict the endowment effect observed in many laboratory experiments (see Kahneman, Knetsch, and Thaler 1990, 1991). The common explanation of the endowment effect is that owners feel giving up the object as a painful loss that counts more than money they receive in exchange, so that they demand a lot of money for the object. But if gains and losses were defined over the value of the entire transaction, owners would not be more sensitive to giving up the object than to receiving money in exchange. Heфetz and List (2011), however, find no evidence that expectations alone play a part in the endowment effect.
utility. In this case a consumer’s total utility from the outcome \((v_i, p_i)\) can be written as

\[
U [(v_i, p_i) | (F^v, F^p)] = v_i - p_i + \int_{v_i}^{\tilde{v}_i} \mu (v_i - \tilde{v}_i) dF^v (\tilde{v}_i) + \int_{p_i}^{\tilde{p}_i} \mu (\tilde{p}_i - p_i) dF^p (\tilde{p}_i)
\] (2)

In words, when evaluating \((v_i, p_i)\) a consumer compares it to each possible outcome in the reference lottery. For example, if he had been expecting to buy good 1 for $15, then buying good 2 for $10 feels like a loss of \(v_1 - v_2\) on the quality dimension and a gain of $5 on the money dimension.\(^{16}\) Similarly, if a consumer had been expecting to buy good 1 for either $10 or $20, then paying $15 for it feels like a loss of $5 relative to the possibility of paying $10, and like a gain of $5 relative to the possibility of paying $20. In addition, the weight on the loss (gain) in the overall experience is equal to the probability with which he had been expecting to pay $10 ($20).

To complete this theory of consumer behavior with the above belief-dependent preferences, Kőszegi and Rabin (2006) assume that beliefs must be consistent with rationality: a consumer correctly anticipates the implications of his plans, and makes the best plan he knows he will carry through. Notice that any plan of behavior — which in my setting amounts simply to a price-contingent strategy of which item to buy — induces some expectations. If, given these expectations, the consumer is not willing to follow the plan, then he could not have rationally formulated the plan in the first place. Hence, a credible plan must have the property that it is optimal given the expectations it generates. Following the original definitions in Kőszegi and Rabin (2006) and Kőszegi (2010), I call such a credible plan a personal equilibrium (PE). If there exist multiple credible plans, a rational consumer chooses the one that maximizes his expected utility from an ex-ante perspective. I call such a favorite credible plan a preferred personal equilibrium (PPE).\(^{17}\)

The seller (she) is a monopolist supplying good 1 and good 2 at a unit cost of \(c_1 \geq 0\) and \(c_2 \geq 0\), respectively (these could be the wholesale prices). The seller does not experience economies of scale or scope in supplying these goods. For \(i = 1, 2\), let \(q_i\) denote the amount or degree of availability of good \(i\) offered by the monopolist. If \(q_i < 1\), then good \(i\) is subject to “limited availability” so that only a fraction \(q_i\) of the consumers can purchase it. I assume that, in the event of a stockout, rationing is proportional: each consumer has the same ex-ante probability of obtaining the good.\(^{18}\)

The interaction between the monopolist and the consumers lasts two periods, 0 and 1. In period 0, the seller announces (and commits to) a price vector \((p_1, p_2) \in \mathbb{R}_+^2\) and a quantity vector

\(^{16}\)Therefore, the two goods are substitute not only in the usual sense, but also in the sense of being evaluated along the same hedonic dimension.

\(^{17}\)In the simple environment considered in this paper, a PPE always exists and is generically unique. Kőszegi (2010) discusses conditions for existence and uniqueness of PPE in more general environments.

\(^{18}\)Gilbert and Klemperer (2000) show that rationing can be a profitable strategy if consumers must make sunk investments to enter the market, and Nocke and Peitz (2007) show that rationing across periods can be profitable in a model of intertemporal monopoly pricing under demand uncertainty.
after observing the seller’s choice of quantities and prices, consumers pick the plan that is consistent and that maximizes their expected reference-dependent preferences (PPE).\footnote{While retailers frequently advertise their good deals, it is rather uncommon to see a store publicizing its high prices. However, in appendix C I show that the main results of the paper are unchanged if the seller commits only to the price and the degree of availability of the bargain. This partial commitment on the seller’s side can be interpreted as advertising. In fact, much like in a model with search costs, consumers here have rational expectations about prices and therefore would correctly infer the price of the rip-off even if it was not publicly advertised.} In period 1, consumers execute their purchasing plans and payments are made. Finally, I assume that when indifferent between a plan that involves buying and another plan that involves not buying, consumers always break the indifference in favor of the first of these plans.

3.2 Consumers’ Demand

Let \( H \in \Delta ([0,1]^2 \times \mathbb{R}_+^2) \) denote a consumer’s expectations, induced by the seller’s strategy, about the degree of availability and the prices he might face. For a given seller’s choice of prices and degree of availability, a consumer chooses among five possible plans: (i) “never buy,” (ii) “buy item 1 if available and don’t buy otherwise,” (iii) “buy item 2 if available and don’t buy otherwise,” (iv) “buy item 1 if available and otherwise buy item 2 if available” and (v) “buy item 2 if available and otherwise buy item 1 if available.”\footnote{Mixing between plans on the consumers’ side can easily be ruled out by the fact that the seller would never choose a price-pair inducing a buyer to buy with probability less than 1. Moreover, the assumption that any indifference on the consumers’ side is broken in favor of the seller guarantees also that the latter would never find it profitable to mix between prices.} Let \( \sigma \in \{\{\emptyset\} , \{1, \emptyset\} , \{2, \emptyset\} , \{1, 2\} , \{2, 1\}\} \) denote a consumer’s plan and let \( \Gamma_{H,\sigma} \) denote the distribution over final consumption outcomes induced jointly by \( H \) and \( \sigma \). In a personal equilibrium the behavior generating expectations must be optimal given the expectations:

**Definition 1** \( \sigma \) is a Personal Equilibrium (PE) if

\[
U[\sigma | \Gamma_{H,\sigma}] \geq U[\sigma' | \Gamma_{H,\sigma}]
\]

for any \( \sigma' \neq \sigma \).

Utility maximization in period 0 implies that the consumer chooses the PE plan that maximizes his expected utility:

**Definition 2** \( \sigma \) is a Preferred Personal Equilibrium (PPE) if it is a PE and

\[
EU_{\Gamma_{H,\sigma}} [\sigma | \Gamma_{H,\sigma}] \geq EU_{\Gamma_{H,\sigma'}} [\sigma' | \Gamma_{H,\sigma'}]
\]

for any \( \sigma' \) such that \( \sigma' \) is a PE.
In the remainder of this section, I analyze the conditions for when plans (i), (ii) and (iv) constitute a PE or a PPE. This allows me to both illustrate the logic of PE and PPE, as well as to start developing the intuition for my main result on the optimality of limited-availability schemes. Specifically, a central element of the seller’s strategy is to make sure that plan (i) is not a PE and I start by analyzing conditions for this.

For plan (i) to be a PE, the consumer must expect never to buy. Suppose a buyer enters the store with the expectation of not buying; in this case his reference point is to consume nothing and pay nothing. Let the price of good 1 be \( p_1 \) and suppose the consumer sticks to his plan. Then, his overall utility is

\[
U[(0, 0) | \{\emptyset\}] = 0.
\]

What if instead the consumer decides to deviate from his plan and buys item 1? In this case his overall utility equals

\[
U[(v_1, p_1) | \{\emptyset\}] = v_1 - p_1 + \eta v_1 - \eta \lambda p_1
\]

where \( v_1 - p_1 \) is his intrinsic consumption utility from buying item 1 at price \( p_1 \), the term \( \eta v_1 \) captures the gain he feels from consuming item 1 when he was expecting to consume nothing and \( -\eta \lambda p_1 \) captures the loss he feels from paying \( p_1 \) when he was expecting to pay nothing. Thus, the consumer will not deviate in this way from the plan to never buy if

\[
U[(0, 0) | \{\emptyset\}] \geq U[(v_1, p_1) | \{\emptyset\}] \Leftrightarrow p_1 > \frac{1 + \eta}{1 + \eta \lambda} v_1.
\]

A similar threshold can be derived for the case in which the consumer considers deviating from his original plan and buy item 2 at price \( p_2 \). Therefore, the plan to never buy is a PE if and only if

\[
p_1 > \frac{1 + \eta}{1 + \eta \lambda} v_1 \equiv p_1^{\text{min}} \quad \text{and} \quad p_2 > \frac{1 + \eta}{1 + \eta \lambda} v_2 = p_2^{\text{min}}.
\]

The expected utility associated with the plan to never buy is

\[
EU[\{\emptyset\} | \{\emptyset\}] = 0
\]

as the expected utility from planning to consume nothing and pay nothing and expecting to follow this plan is of course zero.

Therefore, if either \( p_1 \leq p_1^{\text{min}} \) or \( p_2 \leq p_2^{\text{min}} \) plan (i) cannot be a PE and consumers must select a plan that involves buying at least one item with positive probability. As I will show in the next section, it turns out that (unsurprisingly) it is optimal for the seller to induce consumers to select plan (iv) and thus to expect to never leave the store empty-handed whenever an item is available; however, (less obviously) it is not optimal for that to be the only PE plan. Hence, the seller would

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21 The relevant conditions for plans (iii) and (v) are analogous to the ones for plans (ii) and (iv), respectively; hence, I do not show them here. In appendix A, I thoroughly derive the conditions for when each plan constitutes a PE.
like the consumer to prefer plan (iv) over plan (ii) ex-ante.

Suppose a buyer enters the store with plan (ii) in mind. In this case his reference point on the item dimension is to enjoy \( v_1 \) with probability \( q_1 \) and to consume nothing with probability \( 1 - q_1 \); similarly, on the price dimension he expects to pay \( p_1 \) with probability \( q_1 \) and to pay nothing with probability \( 1 - q_1 \). If the consumer follows this plan his total utility if item 1 is indeed available is

\[
U [(v_1, p_1) \mid \{1, \varnothing\}] = v_1 - p_1 + \eta (1 - q_1) v_1 - \eta \lambda (1 - q_1) p_1
\]

where \( v_1 - p_1 \) is his intrinsic consumption utility from buying item 1 at price \( p_1 \), the term \( \eta (1 - q_1) v_1 \) captures the gain he feels from consuming item 1 when he was expecting to consume nothing with probability \( 1 - q_1 \) and \(- \eta \lambda (1 - q_1) p_1 \) captures the loss he feels from paying \( p_1 \) when he was expecting to pay nothing with probability \( 1 - q_1 \). Suppose that item 1 is available but the buyer instead deviates and does not buy. In this case his overall utility equals

\[
U [(0, 0) \mid \{1, \varnothing\}] = 0 - \eta \lambda q_1 v_1 + \eta q_1 p_1
\]

where 0 is his intrinsic consumption utility, \(- \eta \lambda q_1 v_1 \) captures the loss he feels from consuming nothing when he was expecting to consume item 1 with probability \( q_1 \), and \( \eta q_1 p_1 \) captures the gain from paying nothing instead of \( p_1 \) which he was expecting to pay with probability \( q_1 \). Thus, the consumer will not deviate in this way from his plan if

\[
U [(v_1, p_1) \mid \{1, \varnothing\}] \geq U [(0, 0) \mid \{1, \varnothing\}] \iff p_1 \leq \frac{1 + \eta (1 - q_1) + \eta \lambda q_1}{1 + \eta q_1 + \eta \lambda (1 - q_1)} v_1.
\]

On the other hand, consider the case in which item 1 is not available. If the buyer follows his plan, his overall utility is \( U [(0, 0) \mid \{1, \varnothing\}] \). If instead he deviates and buys item 2, for \( p_1 \geq p_2 \) his overall utility is

\[
U [(v_2, p_2) \mid \{1, \varnothing\}] = v_2 - p_2 + \eta (1 - q_1) v_2 - \eta \lambda q_1 (v_1 - v_2) + \eta q_1 (p_1 - p_2) - \eta \lambda (1 - q_1) p_2
\]

where \( v_2 - p_2 \) is his intrinsic consumption utility from buying item 2 at price \( p_2 \), the term \( \eta (1 - q_1) v_2 \) captures the gain he feels from consuming item 2 compared to the expectation of consuming nothing with probability \( (1 - q_1) \), the term \(- \eta \lambda q_1 (v_1 - v_2) \) captures the loss he feels from consuming item 2 instead of item 1 when he was expecting to consume item 1 with probability \( q_1 \) (recall that \( v_1 \geq v_2 \)), the term \( \eta q_1 (p_1 - p_2) \) captures the gain from paying \( p_2 \) instead of \( p_1 \) which he was expecting to pay with probability \( q_1 \) and, finally, \(- \eta \lambda (1 - q_1) p_2 \) captures the loss from paying \( p_2 \) when he was expecting to pay nothing with probability \( 1 - q_1 \). Thus, the consumer will
not deviate in this way from his plan if

\[ U \{(0, 0) \mid \{1, \emptyset\}\} > U \{(v_2, p_2) \mid \{1, \emptyset\}\} \iff p_2 > \frac{1 + \eta(1 - q_1) + \eta\lambda q_1}{1 + \eta q_1 + \eta\lambda (1 - q_1)} v_2. \]  

(4)

Notice that conditions (3) and (4) together imply that \( U \{(v_1, p_1) \mid \{1, \emptyset\}\} > U \{(v_2, p_2) \mid \{1, \emptyset\}\} \) so that there is no need to check that a consumer does not want to deviate and buy item 2 when item 1 is available. Therefore, for \( p_1 \geq p_2, \{1, \emptyset\}\) is a PE if and only if \( p_2 > \frac{1 + \eta(1 - q_1) + \eta\lambda q_1}{1 + \eta q_1 + \eta\lambda (1 - q_1)} v_2 \) and \( p_1 \leq \frac{1 + \eta(1 - q_1) + \eta\lambda q_1}{1 + \eta q_1 + \eta\lambda (1 - q_1)} v_1. \) The expected utility associated with this plan is

\[ EU \{\{1, \emptyset\} \mid \{1, \emptyset\}\} = q_1 (v_1 - p_1) - q_1 (1 - q_1) \eta (\lambda - 1) (v_1 + p_1). \]  

(5)

The first term in (5), \( q_1 (v_1 - p_1), \) captures standard expected consumption utility. The second term, \( -q_1 (1 - q_1) \eta (\lambda - 1) (v_1 + p_1), \) captures expected gain-loss utility and it is derived as follows. On the consumption dimension, the consumer compares the outcome in which with probability \( q_1 \) he consumes the item and to enjoy \( v_1 \) with the outcome in which with probability \( 1 - q_1 \) he does not consume and gets 0. Similarly, on the price dimension he compares paying the price \( p_1 \) with probability \( q_1 \) with paying 0 with probability \( 1 - q_1 \). Notice that the expected gain-loss component of the consumer’s overall expected utility is always negative as, because of \( \lambda > 1 \), losses are felt more heavily than equal-size gains. Also, notice that uncertainty in the product and uncertainty in the money dimension are “added up” so that the expected gain-loss term is proportional to \( v_1 + p_1 \).

For plan (iv), a consumer’s reference point in the item dimension is to consume good 1 and enjoy \( v_1 \) with probability \( q_1 \), to consume good 2 and enjoy \( v_2 \) with probability \( q_2 \) and to consume nothing with probability \( 1 - q_1 - q_2 \); similarly, in the price dimension, a buyer expects to pay \( p_1 \) with probability \( q_1, p_2 \) with probability \( q_2 \) and to pay nothing with probability \( 1 - q_1 - q_2 \). Then, if he follows his plan and buys item 1 his overall utility is

\[ U \{(v_1, p_1) \mid \{1, 2\}\} = v_1 - p_1 + \eta q_2 (v_1 - v_2) + \eta (1 - q_1 - q_2) v_1 - \eta\lambda q_2 (p_1 - p_2) - \eta\lambda (1 - q_1 - q_2) p_1. \]

If instead he deviates and buys item 2, for \( p_1 \geq p_2 \) his overall utility equals

\[ U \{(v_2, p_2) \mid \{1, 2\}\} = v_2 - p_2 - \eta\lambda q_1 (v_1 - v_2) + \eta (1 - q_1 - q_2) v_2 + \eta q_1 (p_1 - p_2) - \eta\lambda (1 - q_1 - q_2) p_2. \]

Thus, the consumer will not deviate in this way from his plan if

\[ U \{(v_1, p_1) \mid \{1, 2\}\} \geq U \{(v_2, p_2) \mid \{1, 2\}\} \iff p_1 \leq p_2 + \frac{1 + \eta(1 - q_1) + \eta\lambda q_1}{1 + \eta q_1 + \eta\lambda (1 - q_1)} (v_1 - v_2). \]  

(6)

\[ ^{22}\text{Similarly, for } p_1 < p_2, \{1, \emptyset\}\text{ is a PE if and only if } p_1 < \frac{1 + \eta(1 - q_1) + \eta\lambda q_1}{1 + \eta q_1 + \eta\lambda (1 - q_1)} v_1 \text{ and } p_2 > \frac{v_2 [1 + \eta(1 - q_1) + \eta\lambda q_1] + q_1 (\lambda - 1) p_1}{1 + \eta\lambda}.\]
Now, instead, suppose that once the buyer arrives at the store, item 2 is everything that is left. If he follows his plan and buys item 2 his overall utility is \( U [(v_2, p_2) \mid \{1, 2\}] \). If instead he deviates and does not buy his utility is

\[
U [(0, 0) \mid \{1, 2\}] = 0 - \eta \lambda q_1 v_1 - \eta \lambda q_2 v_2 + \eta q_1 p_1 + \eta q_2 p_2.
\]

Thus, the consumer will not deviate in this way from his plan if

\[
U [(v_2, p_2) \mid \{1, 2\}] \geq U [(0, 0) \mid \{1, 2\}] \Leftrightarrow p_2 \leq \frac{1 + \eta \lambda (q_1 + q_2) + \eta (1 - q_1 - q_2)}{1 + \eta (q_1 + q_2) + \eta \lambda (1 - q_1 - q_2)} v_2.
\]

Notice that conditions (6) and (7) together imply that \( U [(v_1, p_1) \mid \{1, 2\}] > U [(0, 0) \mid \{1, 2\}] \). Therefore, for \( p_1 \geq p_2, \{1, 2\} \) is a PE if and only if \( p_2 \leq \frac{1 + \eta \lambda (q_1 + q_2) + \eta (1 - q_1 - q_2)}{1 + \eta (q_1 + q_2) + \eta \lambda (1 - q_1 - q_2)} v_2 \). 23 The expected utility associated with this plan is

\[
EU \{\{1, 2\} \mid \{1, 2\} \} = q_1 (v_1 - p_1) + q_2 (v_2 - p_2)
- q_1 (1 - q_1 - q_2) \eta (\lambda - 1) (v_1 + p_1)
- q_2 (1 - q_1 - q_2) \eta (\lambda - 1) (v_2 + p_2)
- q_1 q_2 \eta (\lambda - 1) (v_1 - v_2)
- q_1 q_2 \eta (\lambda - 1) (\max \{p_1, p_2\} - \min \{p_1, p_2\}).
\]

The first and second terms in (8), \( q_1 (v_1 - p_1) + q_2 (v_2 - p_2) \) are the standard expected consumption utility terms. The third term, \( q_1 (1 - q_1 - q_2) \eta (\lambda - 1) (v_1 + p_1) \), is negative and captures expected gain-loss utility in both the item and the money dimensions from comparing the outcome in which the consumer buys item 1 and pays \( p_1 \) with the outcome of returning home empty-handed. Similarly, the fourth term captures expected gain-loss utility in both dimensions from comparing the outcome of buying item 2 with the outcome of returning home empty-handed. The fifth term, \( -q_1 q_2 \eta (\lambda - 1) (v_1 - v_2) \), captures expected gain-loss utility in the consumption dimension when comparing the two outcomes in which he buys something; with probability \( q_1 \) the consumer expects to buy good 1 and with probability \( q_2 \) he expects to buy good 2. Notice again that this term is negative, but it is proportional to \( (v_1 - v_2) \). This is because with this plan, the consumer is “guaranteeing” himself to enjoy at least the item he values \( v_2 \) and the expected gain-loss utility is therefore related only to by how much more he would prefer to consume the other good (or, the degree of substitutability between the two goods). The sixth term, \( -q_1 q_2 \eta (\lambda - 1) (\max \{p_1, p_2\} - \min \{p_1, p_2\}) \), captures expected gain-loss utility in the money dimension and can be explained in a similar fashion.

23Similarly, for \( p_1 < p_2, \{1, 2\} \) is a PE if and only if \( p_2 \leq \frac{1 + \eta \lambda (q_1 + q_2) + \eta (1 - q_1 - q_2)}{1 + \eta q_2 + \eta \lambda (1 - q_2)} v_2 + \eta (\lambda - 1) q_1 p_1 \).
4 Optimal Availability and Pricing

For given prices \((p_1, p_2)\) and “quantities” \((q_1, q_2)\), the monopolist’s profit is

\[
\pi(p_1, p_2, q; c_1, c_2) = q_1 (p_1 - c_1) + q_2 (p_2 - c_2).
\]

If consumers were not loss-averse, the profit-maximizing strategy for the seller would be to just set \(p_i = v_i\), for \(i = 1, 2\) and \(q_1 = 1\) (resp. \(q_2 = 1\)) if \(v_1 - c_1 \geq v_2 - c_2\) (resp. if \(v_1 - c_1 < v_2 - c_2\)). Consumers would get zero surplus and the seller’s profit would be exactly \(v_1 - c_1\) (resp. \(v_2 - c_2\)).

The first lemma of this section shows that with loss-averse consumers, if the monopolist supplies only one good, the above mentioned strategy remains the profit-maximizing one.\(^{24}\)

**Lemma 1** With perfect availability the monopolist cannot extract more than \(v_1\) from the consumers.

In general, however, this strategy need not be the profit-maximizing one when consumers are loss-averse as the seller instead can achieve a higher profit by reducing the availability of some goods and thus inducing uncertainty into the buyers’ plans.

The next lemma states that even though she might reduce the degree of availability of some goods, it is in the seller best interest that all consumers get to buy one good for sure and the uncertainty is only about which good they will buy.\(^{25}\) The intuition for this result relies on the seller’s intent to mitigate the “comparison effect” and simultaneously magnify the “attachment effect” for the consumers (Kőszegi and Rabin, 2006). An increase in the likelihood of buying increases a consumer’s sense of loss if he does not buy, creating an “attachment effect” that increases his willingness to pay. On the other hand, for a fixed probability of buying, a decrease in the price a consumer expects to pay makes paying a higher price feel like more of a loss, creating a “comparison effect” that lowers his willingness to pay the high price.

**Lemma 2** The market is always fully covered: \(q_1 + q_2 = 1\).

With \(q_1 + q_2 = 1\), if a consumer plans to always buy, he is guaranteed to get at least the less preferred item \((v_2)\) and thus he is not exposed anymore to the possibility of returning home empty-handed; this increases the consumer’s willingness to pay through the attachment effect. At the same time, because the possibility of buying nothing has disappeared, the consumer expects to always spend some money; this also increases the consumer’s willingness to pay through reducing the comparison effect.

\(^{24}\)All proofs are relegated to appendix B.

\(^{25}\)A similar result is provided by Pavlov (2011) and Balestrieri and Leao (2011) for the case of a monopolist selling substitutes to risk-neutral consumers.
Given the result in Lemma 2, from this point forward I am going to use $q$ and $1-q$ to denote the quantities of good 1 and 2, respectively. The lemma below shows that with limited availability, the monopolist must offer at least one good at a discounted price.

**Lemma 3** If $q \in (0, 1)$ then either $p_1 < v_1$ or $p_2 < v_2$.

With limited availability, a consumer faces uncertainty about his consumption outcome before arriving at the store and because losses are felt more heavily than gains, if he expects to buy with positive probability, his expected gain-loss utility is negative. Therefore, for a consumer to be willing to plan to buy, the seller must guarantee him a strictly positive intrinsic surplus on at least one item, otherwise he would be better off by planning to not buy.

Having established that the monopolist can sell a strictly positive quantity of both goods only if one of them is priced at a discount, the next question is how big this discount must be. The next lemma states that the seller must offer a bargain on this good; in other words, its price must be so low that it is not credible for the consumers planning of not buying it.

**Lemma 4** If $q \in (0, 1)$ the seller chooses prices such that the plan to never buy is not a PE.

Since, for a given product $i$, the highest price the seller can charge to make not buying a noncredible plan is $p_{\text{min}}^i = \frac{1+\eta}{1+\eta\lambda}v_i$, then it must be that if the seller is producing both goods in strictly positive quantity, one of them is priced at this “forcing price.”

What about the price of the other item? If she produces a strictly positive quantity of both goods, the seller wants the buyers to plan to always buy. However, as the lemma below shows, it is not optimal for the seller to choose the other price such that always buying is the unique consistent plan. Instead, the optimal price pair is such that consumers are indifferent, ex-ante, between the plan of always buying and the plan of buying only the discounted item.

**Lemma 5** For $q \in (0, 1)$, if the seller uses item 2 as the bargain (i.e., $p_2 = p_{\text{min}}^2$), then the optimal price for item 1 is

$$p_1^* = v_1 + \frac{2(1-q)\eta(\lambda-1)}{(1+\eta\lambda)[1+\eta(\lambda-1)(1-q)]} > v_1.$$

If instead she uses item 1 as the bargain (i.e., $p_1 = p_{\text{min}}^1$), then the optimal price for item 2 is

$$p_2^* = v_2 + \frac{2qv_1\eta(\lambda-1)(1+\eta)}{(1+\eta\lambda)[1+\eta(\lambda-1)q]} > v_2.$$

---

26 This result is akin to the single-product one in Heidhues and Köszegi (2010) from which I borrowed the terms “forcing price.”
This last lemma implies that consumers are willing to pay a premium, in the form of a higher price on the item that is not on sale (and therefore in the form of a higher expected expenditure), to avoid ex-ante the disappointment from the possibility of returning home from shopping empty-handed. Furthermore, \( p_i^* \) is the highest price such that consumers (weakly) prefer, from an ex-ante point of view, the plan of buying item \( j \) if available and item \( i \) otherwise to the plan of buying item \( j \) if available and nothing otherwise, when item \( j \) is sold at its “forcing price.” To gain intuition on why a consumer might find it optimal to plan to buy at \( p_i^* > v_i \), suppose the seller were to use item 1 as a bait, by pricing it at \( p_{min}^1 \). If a consumer plans to buy only item 1 and nothing otherwise his expected utility is equal to

\[
q (v_1 - p_{min}^1) - \eta (\lambda - 1) q (1 - q) (v_1 + p_{min}^1).
\]

While the term relating to consumption utility in the above expression is strictly positive, the expected gain-loss utility term is strictly negative. If instead the consumer plans to always buy, then his expected utility is

\[
q (v_1 - p_{min}^1) + (1 - q) (v_2 - p^*_2) - \eta (\lambda - 1) q (1 - q) (v_1 - v_2 + p^*_2 - p_{min}^1).
\]

In the above expression the expected gain-loss utility is still negative, but now its magnitude is \( (v_1 - v_2 + p^*_2 - p_{min}^1) \). Therefore, as long as \( p^*_2 - v_2 < 2p_{min}^1 \), by planning to always buy the buyer is subject to a smaller expected gain-loss disutility and this allows the seller to raise \( p^*_2 \) even above \( v_2 \).\(^{27}\) Furthermore, notice that fixing \( v_1 \) (and therefore \( p_{min}^1 \)), the closer \( v_2 \) is to \( v_1 \), the more freedom the seller has in raising \( p^*_2 \), implying that dispersion in prices and dispersion in valuations are inversely related.

Having established what are the optimal prices for the bait and the rip-off, the next question is what is the optimal degree of availability for each item. For example, consider the case in which the seller uses item 2 as the bait. Then, she is going to choose the \( q \) that solves the following maximization problem:

\[
\max q (p^*_1 - c_1) + (1 - q) (p_{min}^2 - c_2).
\]

The first-order condition yields

\[
q \frac{\partial p^*_1}{\partial q} + p^*_1 - c_1 - (p_{min}^2 - c_2) = 0. \tag{9}
\]

Notice that \( q \frac{\partial p^*_1}{\partial q} < 0 \): the higher the degree of availability of the bargain, \( 1 - q \) in this case,\(^{27}\) more generally, as shown in Kőszegi and Rabin (2007), a decisionmaker with rational-expectations-based loss aversion dislikes uncertainty in consumption utility because he dislikes the possibility of a resulting loss more than he likes the possibility of a resulting gain.
the more optimistic the consumers’ beliefs about making a deal. This in turn, allows the seller to charge a higher mark-up on the rip-off. On the other hand, a greater availability of the bargain necessarily means fewer sales of the rip-off as captured by the fact that $p_1^* - c_1 - (p_2^{\min} - c_2) > 0$. At the optimal degree of availability these two effects completely offset each other at the margin.

**Lemma 6** If the seller uses item 2 as the bait, the optimal degree of availability for item 1 is $q = \arg \max_q \pi(p_1^*, p_2^{\min}, q; c_1, c_2)$. If instead she uses item 1 as the bait, the optimal degree of availability for item 1 is $q = \arg \max_q \pi(p_1^{\min}, p_2^*, q; c_1, c_2)$. Furthermore, $q \in \left(\frac{1}{2}, 1\right)$ and $q \in \left(0, \frac{1}{2}\right)$, with $q > 1 - q$.

According to the above lemma, the seller always supplies more units of the rip-off item than the bargain. That is, even if a high degree of availability for the bargain allows her, via the attachment effect, to increase the price of the rip-off, the effect is not strong enough for the seller to be willing to sell the bargain more often than the rip-off. The intuition can be seen most easily in the case in which the two items are perfect substitutes ($v_1 = v_2 = v$) and have zero costs. For this case (9) reduces to:

$$1 + \frac{2\eta(\lambda - 1)(1 - q)}{1 + \eta(\lambda - 1)(1 - q)} \frac{1 + \eta}{1 + \eta \lambda} = \frac{1 + \eta}{1 + \eta \lambda} + \frac{2\eta(\lambda - 1)q}{[1 + \eta(\lambda - 1)(1 - q)]^2} \frac{1 + \eta}{1 + \eta \lambda}. \quad (10)$$

The left-hand-side of (10) captures the seller’s marginal gain from an increase in $q$; similarly, the right-hand-side captures the seller’s marginal loss. The following is a necessary condition for (10) to hold:

$$\frac{2\eta(\lambda - 1)q}{[1 + \eta(\lambda - 1)(1 - q)]^2} > \frac{2\eta(\lambda - 1)(1 - q)}{1 + \eta(\lambda - 1)(1 - q)}$$

$$\Leftrightarrow \frac{q}{1 - q} > 1 + \eta(\lambda - 1)(1 - q).$$

The above inequality can be satisfied only for $q > \frac{1}{2}$. Furthermore, as $q > 1 - q$, the seller chooses a higher degree of availability for the bait when this is the superior item. Intuitively, when the seller uses the superior item as the bait, some consumers will end up paying a very high price for the item they like the least; in order to convince them to do so, the seller must compensate the consumers with a higher ex-ante chance of making a deal.

The above analysis does not specify which item the seller would prefer to use as the bait. To determine whether the seller would prefer to use item 1 or 2 as the bait, we must compare her profits in the two cases. Unfortunately, these are complex non-linear functions of $v_1$ and $v_2$, which are difficult to sign even in the simplest cases and are intractable in general. To overcome this difficulty, I employ comparative statics techniques based on the envelope theorem; but the downside of this approach is that some of the results in the following lemma apply only for small
changes in the relevant parameters.\textsuperscript{28}

\textbf{Lemma 7} If the two goods are perfect substitutes (i.e., \( v_1 = v_2 \)) the seller prefers to use as the bait the one with the higher marginal cost and is indifferent if the two goods have the same marginal cost (i.e., \( c_1 = c_2 \)). For \( v_1 > v_2 \), the seller uses item 2 as the bait only if \( v_1 - c_1 + c_2 > v_2 > \frac{2(1+\eta \lambda) (c_1-c_2)}{1+2\eta} \) and \( v_1 \geq \tilde{v}_1 \), where \( \tilde{v}_1 \) is implicitly defined by:

\[
\frac{1 - \eta (\lambda - 1) (1-q)}{1+\eta (\lambda - 1) (1-q)} - \frac{1 + \eta q}{1 + \eta \lambda} q - (1-q) \frac{1 + \eta \frac{2q \eta (\lambda - 1)}{1+q \eta (\lambda - 1)}}{1 + \eta \lambda 1 + q \eta (\lambda - 1)} (v_1 - v_2) \geq (q - q) (c_1 - c_2) .
\]

Otherwise, she prefers using item 1 as the bait.

Interestingly, the seller might prefer to use the superior item as the bait even when this is the item with the greater social surplus (i.e., \( v_1 - c_1 > v_2 - c_2 \)). Although this might seem counterintuitive, the seller has two very good reasons for doing so. First, as \( v_1 > v_2 \) it follows that \( p_1^\text{min} > p_2^\text{min} \) and this implies that \( p_2^* > p_1^* \) through the comparison effect: the higher the “forcing price” at which a consumer expects to buy the bait, the higher the price that the seller can charge for the rip-off. Second, recall that, by lemma 5, at the preferred personal equilibrium plan, the consumers are indifferent between the plan to buy only the bargain and the plan to always buy. And since \( v_1 + p_1^\text{min} > v_2 + p_2^\text{min} \), it follows that the expected gain-loss disutility from the plan of buying only the bargain is bigger when this is the superior good than when it is the inferior one; this in turn implies that when the superior item is offered for a discount, consumers are willing to pay an even higher premium to reduce the uncertainty in their consumption outcome, thus allowing the seller to extract more surplus from them. On the other hand, if \( v_1 - c_1 \leq v_2 - c_2 \), the seller never uses item 2 as the bait.

The following proposition, which constitutes the main result of this section, identifies the necessary and sufficient conditions for a combination of bargains and rip-offs to be profit-maximizing.

\textbf{Proposition 1} Fix any \( \eta > 0 \) and \( \lambda > 1 \). The seller’s profit-maximizing strategy is as follows:

(i) for \( v_1 \leq v_2 - c_2 + c_1 \) there exists a \( \alpha (v_2, c_1, c_2, \eta, \lambda) \) such that if \( v_1 \geq \alpha \) then item 1 is used as the bargain item and item 2 is used as the rip-off item;

(ii) for \( \tilde{v}_1 > v_1 > v_2 - c_2 + c_1 \) there exists a \( \beta (v_2, c_1, c_2, \eta, \lambda) \) such that if \( v_1 \leq \beta \) then item 1 is used as the bargain item and item 2 is used as the rip-off item;

(iii) for \( v_1 > \tilde{v}_1 \) there exists a \( \gamma (v_1, c_1, c_2, \eta, \lambda) \) such that if \( v_2 \geq \gamma \) then item 2 is used as the bargain item and item 1 is used as the rip-off item;

\textsuperscript{28}The results apply only for small changes because comparative statics techniques linearize profits around the maximum. Klemperer and Padilla (1997) use the same approach in a similar context.
(iv) otherwise, the seller supplies only the item with the larger social surplus and prices it at its intrinsic value.

Furthermore, \( \pi(p_1, p_2, q; c_1, c_2) \geq \max \{v_1 - c_1, v_2 - c_2\} \) and the inequality is strict for cases (i), (ii), (iii).

The exact expressions for \( \alpha, \beta \) and \( \gamma \) are derived in the proof of the proposition in Appendix B. What they imply is that, if the two goods are close substitutes or have similar marginal costs, the seller’s profit-maximizing strategy consists of luring the consumers with a tempting discount on one good which is available only in short supply \( (p_i^{\min} < v_i) \) and ripping them off on the other \( (p_j > v_j) \). On the other hand, if the two goods are quite different in their degree of substitutability or in their marginal costs (or both), the seller’s profit-maximizing strategy is to supply only the good with the larger social surplus and price it at its intrinsic value (as with non loss-averse consumers). Moreover, by offering both products and inducing uncertainty into the buyers’ plans through this type of limited-availability deals the seller is able to achieve a profit higher than \( \max \{v_1 - c_1, v_2 - c_2\} \).

The idea of stores using baits to attract consumers is not new in industrial organization and it can also be explained with consumers’ search or shopping costs as shown by Gerstner and Hess (1990) and Lazear (1995). However, for the environment considered here, reference-dependent preferences and loss aversion are necessary for the seller to prefer this combination of limited-availability bargains and rip-offs over classical monopoly prices. Indeed, as I show in Appendix C, a combination of bargains and rip-offs would not be profit-maximizing for a monopolist selling to risk-neutral consumers who have unit demand and suffer search costs to learn the prices or shopping costs to reach the store.

It is possible for the seller to find this strategy profit-maximizing even when the bait is a loss leader, as the following example shows.

**Example 3 (Loss Leader)** Let \( \eta = 1, \lambda = 3, v_1 = 60, v_2 = 40, c_1 = 35 \) and \( c_2 = 22 \). For these parameters’ values the seller profit-maximizing strategy is given by: \( q = \frac{3\sqrt{2}}{\sqrt{83}} - \frac{1}{2} \), \( p_1^{\min} = 30 \) and \( p_2^* = \frac{200q + 40}{2q + 1} = 59.26 \). Item 1 is used as a loss leader and the seller’s profit is \( \pi = \$27.27 \).

By combining the results in Proposition 1 with the condition for the bait to be a loss leader (i.e., \( p_i^{\min} < c_i \)) we immediately obtain the following result.

**Corollary 1** Item 1 is a loss leader if either \( \frac{1 + \eta \lambda}{1 + \eta} c_1 > v_1 \geq \alpha \) or \( v_1 < \min \left\{ \left( \frac{1 + \eta \lambda}{1 + \eta} \right) c_1, \beta \right\} \). Similarly, item 2 is a loss leader if \( \frac{1 + \eta \lambda}{1 + \eta} c_2 > v_2 \geq \gamma \).

As shown by Ambrus and Weinstein (2008), classical models of consumers behavior can rationalize the use of loss leaders when the goods are complements but not when they are substitutes.
The reason is that with classical preferences a store might benefit from using a loss-leading strategy only if consumers buy other items together with the loss leader. In my model, instead, the presence of loss leaders still attracts consumers into the store but, because the loss-leading product is in shortage, in equilibrium some consumers end up buying a different, more expensive product.

Despite the consumers being homogeneous in terms of tastes for both items, the bargains and rip-offs strategy described above endogenously separates them into two different groups. A group of consumers ends up purchasing the good that is offered at a discount, making a bargain indeed; the others instead end up purchasing the other good and paying for it even more than their intrinsic valuation. The next result shows that on average consumers are hurt by this strategy.

**Proposition 2** For any $\eta > 0$ and $\lambda > 1$ a consumer’s expected surplus is at most zero and therefore he would be better off expecting and following through a strategy of never buying than expecting and following through his actual equilibrium strategy of always buying.

As with the similar result obtained in Heidhues and Kőszegi (2010), Proposition 2 suggests that firms’ sales are “manipulative” in the sense that they lead the consumers to go to the store even though ex-ante they would prefer not to. Consumers enter the store with the expectations — induced by the seller — of making a bargain by purchasing a good on sale and then might end up buying something else at an even higher price. Of course, this rather extreme result relies on the assumption that the seller has perfect information about the consumers’ preferences.

In addition to the consumers being worse off with limited availability, also social welfare is below its first-best level, as shown by the following corollary.

**Remark 1** With limited availability, if $v_1 - c_1 \neq v_2 - c_2$, the monopolist’s profit-maximizing product mix does not maximize welfare.

Therefore, except for the non-generic case in which the two goods contribute equally to social surplus ($v_1 - c_1 = v_2 - c_2$), by employing a limited-availability strategy, the seller is reducing welfare compared to first-best, according to which only the item with the larger social value should be produced.

The monopolist, however, can make matters even worse and bring into the market a socially wasteful product, as the following examples show.

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29 An implicit premise of the model is that consumers cannot commit not to go shopping or, equivalently, cannot learn whether the seller has run out of the bargain item before they visit the store. However, there exist some real-life situations in which this assumption is not too restrictive after all. For example, around Christmas consumers “have to” go shopping in order to buy gifts for their friends and relatives. Furthermore, while many papers in IO emphasize consumers’ “shopping costs,” it is also possible that some consumers might actually enjoy to go shopping.
Example 4 (Wasteful Product 1) Let $\eta = 1$, $\lambda = 3$, $v_1 = 20$, $v_2 = 15$, $c_1 = 21$ and $c_2 = 10$. For these parameters’ values the seller profit-maximizing strategy is given by: $q = \sqrt{15-3} \cdot \frac{7q+15}{2q+1} = 35 - 4\sqrt{15}$ for a total profit of $\pi = $6.52.

Example 5 (Wasteful Product 2) Let $\eta = 1$, $\lambda = 3$, $v_1 = 30$, $v_2 = 24$, $c_1 = 28$ and $c_2 = 25$. For these parameters’ values the seller profit-maximizing strategy is given by: $q = \sqrt{105-7} \cdot \frac{108q+24}{2q+1} = 54 - 2\sqrt{105}$ for a total profit of $\pi = $3.52.

The intuition in Example 4 is that, albeit socially wasteful, item 1 is highly valuable to the consumers and this makes it an ideal candidate for a bait. The intuition is somewhat different for Example 5 because the seller is now introducing an item that is socially wasteful as well as inferior for the consumers; the key here is that item 2 has a lower marginal cost than item 1 and therefore the seller can reduce her average marginal cost by introducing such a wasteful item. Average revenue also decreases, but as the example shows the cost-saving effect might outweigh the decrease in revenue. Furthermore, by comparing Example 4 with Example 5, we see also that the socially wasteful product can be either the bargain or the rip-off.

By combining the results in Proposition 1 with the condition for an item to be socially wasteful (i.e., $v_i < c_i$) we immediately obtain the following result.

Corollary 2 The seller supplies a socially wasteful product only if item 1 is used as a bait. She supplies a socially wasteful item 1 if and only if $v_2 - c_2 \geq 0 \Rightarrow v_1 - c_1$ and $v_1 \geq \alpha$. She supplies a socially wasteful item 2 if and only if $v_1 - c_1 \geq 0 \Rightarrow v_2 - c_2$ and $\beta \geq v_1$.

Moreover, with limited availability the seller could even supply two socially wasteful products and still obtain strictly positive profits.\(^{30}\)

Example 6 (Wasteful Products) Let $\eta = 1$, $\lambda = 3$, $v_1 = 20$, $v_2 = 9$, $c_1 = 21$ and $c_2 = 10$. For these parameters’ values the seller profit-maximizing strategy is given by: $q = \sqrt{7-1} \cdot \frac{58q+9}{2q+1} = 29 - 10\sqrt{2}$ for a total profit of $\pi = $1.57.

This last example shows how the seller can simultaneously exploit the aforementioned effects and supply two socially wasteful products at the same time: item 1 is highly valuable and thus allows the seller to increase her revenue whereas item 2 has a strong cost-saving effect.

\(^{30}\)A similar implication arises also in the paper of Heidhues and Kőszegi (2010), where a single-product monopolist sells an item valued at $v > 0$ by the consumers. Because the monopolist is able to extract, in expectation, more than $v$ from the consumer, she can still attain strictly positive profits for $c > v$. 

21
5 Extensions

To investigate the robustness of the result concerning the optimality of bargains and rip-offs, I here analyze two extensions of the main model. In the first subsection I study a variation of the previous model in which the seller is able to create perfect substitutes for a given product through a cosmetic change and with no additional cost. In the second subsection I consider a variant of the main model in which consumers have heterogeneous consumption values for both goods and I show that even in this more general case the seller’s profit-maximizing strategy is to reduce availability and use a combination of bargains and rip-offs.

5.1 Endogenous Product Line

In the model of the previous section, the seller was exogenously endowed with two different products that the consumers regarded as imperfect substitutes. However, retailers can often create almost-perfect substitutes of a given product through a small cosmetic change that does not affect consumers’ valuations. For example, two TVs might share the same technology and have the same screen-size and number of pixels, thus providing consumers with the same picture quality, and just differ in their frame’s color. An alternative interpretation is that the seller is able to charge different prices for some units of the same item. This happens, for example, when a retailer offers a price reduction on a particular item only for the first units sold on given day.

To formally model this idea, I consider a situation in which the seller can create an artificial perfect substitute for a product without incurring any additional cost and I am going to allow her to price these \textit{de facto} identical products differently. Therefore, the seller now has the choice between supplying two substitutable but distinct items or just supplying two slightly different versions of the same item. In either case, the seller has the option of reducing the availability of one of the items, just like in the model of the previous section.

Let $v_i > 0$ and $c_i \geq 0$, $i \in \{1,2\}$ and, as before, assume $v_1 > v_2$. Also, let $p_i^{\min}$, $i \in \{1,2\}$, $\bar{q}(\eta, \lambda, v_1, v_2, c_1, c_2)$ and $\bar{q}(\eta, \lambda, v_1, v_2, c_1, c_2)$ be defined as in the previous section. Because now the seller can supply two different versions of the same product, let $p^*_i$ be the price of the rip-off item $i$, when item $j$ is the bargain, $i, j \in \{1,2\}$. The following proposition characterizes the seller’s profit-maximizing strategy.

\textbf{Proposition 3} Fix any $\eta > 0$ and $\lambda > 1$. If $v_1 - c_1 > v_2 - c_2$, the seller maximizes profits by supplying two different versions of item 1: the bargain version is priced at $p_1^{\min}$, with degree of availability $1 - \bar{q}(\eta, \lambda, v_1, v_1, c_1, c_1)$ and the rip-off version is priced at $p^*_{1,1}$, with degree of availability $\bar{q}(\eta, \lambda, v_1, v_1, c_1, c_1)$. If $v_1 - c_1 \leq v_2 - c_2$, there exists a $\tilde{v}_2 < v_1$ such that: (i) for $v_2 \leq \tilde{v}_2$ the seller maximizes profits by using item 1 as a bargain, with price $p_1^{\min}$ and degree of
availability \( q(\eta, \lambda, v_1, v_2, c_1, c_2) \) and item 2 as a rip-off, with price \( p^*_{2,1} \) and degree of availability \( 1 - q(\eta, \lambda, v_1, v_2, c_1, c_2) \); (ii) for \( v_2 > \bar{v}_2 \) the seller maximizes profits by supplying two different versions of item 2: the bargain version is priced at \( p^\text{min}_{2} \), with degree of availability \( q(\eta, \lambda, v_2, v_2, c_2, c_2) \) and the rip-off version is priced at \( p^*_{2,2} \), with degree of availability \( 1 - q(\eta, \lambda, v_2, v_2, c_2, c_2) \).

Proposition 3 delivers several interesting results. First, if the seller can easily create perfect substitutes of the same item that are valued equally by consumers, the profit-maximizing strategy is always a combination of limited availability, bargains and rip-offs. This result can be interpreted as a foundation for the analysis in Heidhues and Köszegi (2010): although it might not be possible for the seller to credibly commit to a stochastic pricing strategy, she could achieve the same goal by introducing many slightly different — but equivalent from the consumers’ point of view — versions of the same product. Second, if the socially superior product is the most preferred by the consumers, the seller prefers to create perfect substitutes of this product instead of introducing another, inferior, one. On the other hand, if the socially superior item is the one consumers value the least, the seller might want to supply both products, even if she could create a perfect substitute for either product at no additional cost. The intuition is that, albeit socially inferior, item 1 is highly valuable to the consumers and this makes it an ideal candidate for a bait because it allows the seller to charge an even higher price for the rip-off, therefore increasing average revenue; although average cost also increases, the former effect might dominate. In this case the consumers’ most preferred item is used as a bait and the seller’s product line is not welfare-maximizing. Finally, it is easy to see that the results from the previous section about loss leaders and socially wasteful products still apply in this context.

5.2 Heterogeneous values (incomplete)

As in Section 4, there is a unit mass of consumers and one seller. The seller (she) is a monopolist supplying good 1 and good 2 at zero cost. The goods are substitutes and consumers demand at most one unit of one good. The distribution of consumers’ valuations for the two items is as follows. A fraction \( \rho \) of the consumers has \( (v_1, v_2) = (v + k, v) \), with \( k > 0 \): these consumers strictly prefer item 1 over item 2. Similarly, a fraction \( \sigma \) of them has \( (v_1, v_2) = (v, v + k) \). For the remaining \( 1 - \rho - \sigma \) consumers the two goods are perfect substitutes and \( (v_1, v_2) = (v, v) \). This distribution is common knowledge, whereas a consumer’s individual valuations are his own private information.

A non loss-averse consumer buys item \( i \) if \( v_i - p_i \geq \max \{v_j - p_j, 0\}, i, j \in \{1, 2\} \) and \( i \neq j \). Furthermore, suppose that when indifferent, a consumer always buys the item he likes the most. To simplify the number of cases to be considered, but without any significant loss of generality, I assume that \( \sigma \geq \rho \). The following lemma, whose proof is straightforward and hence omitted,
characterizes the seller’s profit-maximizing strategy with risk-neutral consumers.

**Lemma 8** The seller maximizes profits by setting $p_1 = v$ and $p_2 = v + k$ if $k \leq \left(\frac{1-\rho-\sigma}{\rho}\right)v$ and by setting $p_1 = p_2 = v + k$ otherwise.

Therefore, if $k$ is high enough, with risk-neutral consumers the seller prefers to exclude the lowest segment of the market for the well-known screening motive.

With loss-averse consumers, suppose the seller uses the limited-availability strategy that, according to proposition 1, is profit-maximizing for a type-$(v,v)$ consumer (there is also the possibility of the seller playing the limited-availability strategy that is profit-maximizing for a type-$(v,v+k)$ consumer, but I’ll check this case later). First, we state an intermediate result.

**Lemma 9** In the profit-maximizing limited-availability strategy for type $(v,v)$ the seller uses item 1 as the bargain.

By the above lemma we have that $q = q$, $p_1 = p^{\min}$ and $p_2 = \frac{1+q \eta (\lambda-1)(1+\frac{2(1+\lambda)}{1+2\lambda})}{1+q \eta (\lambda-1)}v$. With these prices and availability levels, consumers of type $(v,v)$ are indifferent between the plan of buying item 1 if available and nothing otherwise and the plan of buying item 1 if available and item 2 otherwise. Furthermore, it is easy to see that the same indifference holds for type $(v+k,v)$ as well. For consumers of type $(v,v+k)$, however, we must distinguish three separate cases:

- $1 - q \eta (\lambda-1) > 1 - (1-q) \eta (\lambda-1) > 0$ (low gain-loss utility);
- $1 - q \eta (\lambda-1) > 0 > 1 - (1-q) \eta (\lambda-1)$ (intermediate gain-loss utility);
- $0 > 1 - q \eta (\lambda-1) > 1 - (1-q) \eta (\lambda-1)$ (high gain-loss utility).\(^{31}\)

For now, let’s focus on the first two cases, which seems more likely. Then, type-$(v,v+k)$ consumers can plan to buy the bargain if available and the rip-off otherwise, like the other two types, in which case I say we have a pooling equilibrium with limited-availability or, they could plan to buy only the rip-off (which is always available), in which case we have a separating equilibrium with limited-availability.

Let $\Sigma$ be such that if $k \geq \Sigma$ type-$(v,v+k)$ consumers prefer to buy the rip-off item for sure. Then, if $k \geq \Sigma$ the seller’s profit with limited availability is

\[ (1-\sigma) \left[ qp_1 + (1-q) p_2 \right] + \sigma p_2. \]

\(^{31}\)A sufficient condition for being in the low gain-loss utility range is that $\eta (\lambda-1) < 1$. A necessary condition for being in the high gain-loss utility range is that $\eta > 1$. Therefore, the most common empirical estimates of the gain-loss utility parameters, $\eta = 1$ and $\lambda = 3$, fall in the intermediate gain-loss utility range.
On the other hand, if \( k < \Sigma \) the seller’s profit with limited availability is just \( q p_1 + (1 - q) p_2 \) because type-(\( v, v + k \)) consumers prefer to pool with the other types and plan to buy the bargain if available and the rip-off otherwise. Furthermore, let \( m \) and \( n \), with \( m > n \) be such that

\[
(1 - \sigma) [q p_1 + (1 - q) p_2] + \sigma p_2 > (\rho + \sigma)(v + k) \iff k < m
\]

and

\[
q p_1 + (1 - q) p_2 > (\rho + \sigma)(v + k) \iff k < n.
\]

The whole thing is a mess and I am still working out all the different cases. However, there exist parameters values for \( \rho, \sigma, \eta \) and \( \lambda \) such that \( m > \Sigma > \frac{1 - \rho - \sigma}{\rho} \) or such that \( \Sigma > n > \frac{1 - \rho - \sigma}{\rho} \).

In the first case, if \( k \in [\Sigma, m] \) the seller maximizes profits with a limited-availability strategy that separates types \( (v, v + k) \) from the other types, but with no exclusion of types \( (v, v) \), whereas with risk-neutrality these types would be excluded. On the other hand, if \( k \in \left[ \frac{1 - \rho - \sigma}{\rho}, n \right] \), the seller maximizes profits with a limited-availability strategy in which all consumers pool and select the same plan (the lower types would be excluded under risk neutrality). To be continued...

### 6 Related Literature

This paper belongs to a recent and growing literature on how firms respond to consumer loss aversion. Heidhues and Köszegi (2008), Karle and Peitz (2012) and Zhou (2011) study the implications of reference-dependent preferences and loss aversion in an oligopolistic environment with differentiated goods. In a monopolistic-screening setting, Carbajal and Ely (2012), Hahn, Kim, Kim and Lee (2012) and Herweg and Mierendorff (forthcoming) analyze the implications of reference-dependent preferences and loss aversion for the design of profit-maximizing menus and tariffs.\(^{32}\)

My paper is most related to Heidhues and Köszegi (2010), which provides an explanation for why regular prices are sticky, but sales prices are variable, based on expectations-based loss aversion. In their model, a monopolist sells only one good and maximizes profits by employing a stochastic-price strategy made of low, variable sales prices and a high, sticky regular price. Their result and mine share a similar intuition: low prices work as baits to lure consumers who, once in the store, are willing to pay a price even above their intrinsic valuation for the item. However, in my model the monopolist sells two goods and uses one of them as bait to attract the consumers and the other to exploit them. Also, in Heidhues and Köszegi (2010) consumers face uncertainty about the price whereas in my case the uncertainty stems from the limited availability of the

\(^{32}\)For other applications of the Köszegi and Rabin’s models of reference-dependent preferences outside the field of IO, see Herweg, Muller and Weinschenk (2010) and Macera (2011) on agency contracts, Lange and Ratan (2010) and Eisenhuth and Ewers (2012) on sealed-bid auctions, and Daido and Murooka (2012) on team incentives.
deal. If consumers value the two goods equally and the goods have the same production cost, my
model coincides with a special case of theirs in which the monopolist uses a two-price distribution.
I consider my model of limited-availability deals to be more compelling and realistic than their
assumption of a seller being able to credibly commit to an entire price distribution. Furthermore,
my result about the optimality of bargains and rip-offs holds also if the seller can credibly commit
to only one price — the one of the bargain. Therefore, compared to theirs, my result requires
less prior knowledge of the prices on the part of the consumers as well as less commitment on the
seller’s side.

Within the realm of industrial organization, this paper is also closely related to the literature
on advertising, bait-and-switch and loss leaders. Lazear (1995) studies a model of oligopoly with
differentiated goods in which each firm produces only one good and consumers pay search costs
to visit a firm, and derives the conditions under which bait-and-switch is a profitable strategy.
Although consumers have rational expectations and understand that a firm might engage in bait-
and-switch, this strategy can be profitable if the goods sold by different firms are similar and if
search is costly. However, bait-and-switch is a form of false advertising in which a firm claims
to sell a different good than the one it actually produces. In my model, instead, the firm is not
lying to the consumers but is using a truthful version of the bait and switch strategy through
endogenously reducing the availability of the goods.\footnote{Gerstner and Hess (1990) present a model of bait-and-switch in which retailers advertise only selected brands, low-priced advertised brands are understocked and in-store promotions are biased towards more expensive substitute brands. In their model consumers are rational and foresee stock outages. However, the authors assume that in-store promotions can create a permanent utility increase for consumers and this is the reason why in equilibrium some consumers will switch to more expensive brands.}

Ellison (2005) presents a model of competitive price discrimination with horizontal and vertical
taste differences across consumers in which firms advertise a base price for a product and then try
to sell “add-ons” or more sophisticated versions of the product for a higher price at the point of
sale. However, apart from the result that the “basic” version of the product can be a loss leader,
our models are quite different; furthermore, the pricing strategy described in Example 2 in the
Introduction, where the more sophisticated version of the product is offered at a lower price and
is in shortage, can be rationalized by my model but not his.

Hess and Gerstner (1987) develop a model in which multi-product firms might stock out of
advertised products and offer rain checks to consumers, and Lal and Matutes (1994) consider
multi-product firms competing for consumers that are initially unaware of prices. In both of these
papers firms might advertise loss leaders in order to increase store traffic. The profitability of this
strategy, then, stems from the fact that once they arrive at the store, consumers will buy also
other complementary items that are priced at a high mark-up; that is, each firm enjoys a form
of monopoly on the other items once a consumer is attracted into the store by the loss leader.\footnote{A somewhat different explanation for the use of loss leaders is the one advanced by Chen and Rey (forthcoming).}
As mentioned above, my model is different as I consider a monopolist selling substitutable goods to consumers who demand at most one unit of one good and therefore loss-leading is not aimed at increasing store traffic in order to boost demand for complementary products. Furthermore, in these models the products with lower consumer value are the more natural candidates for loss-leading pricing; my model instead can also rationalize the use of more valuable or popular products as loss leaders.

Finally, Thanassoulis (2004) studies the problem of a multi-product monopolist selling two substitute goods to risk-neutral consumers with unit demand, and derives generic conditions such that the optimal tariff includes lotteries.\textsuperscript{35} In my model, when the seller endogenously reduces the availability of the goods, from the consumers’ point of view this is equivalent to taking a lottery on both which good they will end up with and how much they will have to pay.\textsuperscript{36} Nevertheless, there are several differences between his model and mine. First, my result on the optimality of limited-availability deals holds also when consumers have homogeneous tastes, whereas his result on the optimality of lotteries does not. Second, in his lotteries there is uncertainty only on the item dimension but not on the price one, whereas in my case the uncertainty is on both dimensions. Last, in his model a lottery is offered in addition to each good being offered in isolation with its own posted price; in my model instead each good is offered in isolation with its own price, but because the items are in short supply, consumers are uncertain about their consumption outcomes.

\section*{7 Conclusion}

Limited-availability offers are a common selling technique for many stores selling durable goods such as electronics, computer appliances or clothes. Sometimes a store offers promotions that are only valid “while supplies last” whereas for other popular sales, like Black Friday, the limited-availability of a deal stems from the fact that the volume of consumers shopping exceeds the store’s supplies.

In this paper, I have provided a novel explanation, based on consumer loss aversion, for why a monopolist selling substitute goods might find it profitable to use limited-availability deals. The optimal pricing strategy for the monopolist consists of a combination of bargains and rip-offs: she lures the consumers with a limited-availability tempting deal on one good and exploit them with a

\textsuperscript{35}Pavlov (2011) solves for the optimal mechanism when selling two substitutable goods and generalizes the analysis in Thanassoulis (2004). Balestrieri and Leao (2011) extend this result to an oligopoly setting where consumers have horizontally differentiated tastes.

\textsuperscript{36}Thanassoulis (2004) makes also the related point that capacity constraints, actual or alleged, are an indirect way to implement lotteries.
high price on another one. Furthermore, this strategy allows the monopolist to extract more than
the maximum of consumers’ intrinsic valuations for the goods. The model also predicts that more
valuable or popular items are more likely to be used as baits and that the monopolist’s product line
might feature too many varieties compared to what social efficiency dictates. Finally, the model
provides a possible explanation also for why a seller might find it profitable to use loss leaders even
when selling substitute goods.

While my model provides a potential explanation for several retail-pricing patterns, it suffers
from some limitations. First, I have followed Kőszegi and Rabin (2006) in assuming that consumers
have rational expectations and fully understand the firm’s pricing strategy. However, consumers
need not have correct beliefs about the probability of getting the deal. Furthermore, I have
assumed that all consumers show up at the store at the same time and are served randomly with
equal probability. In reality, however, especially during popular promotions like Black Friday,
consumers line up outside stores before they open. This suggests that consumers’ heterogeneity
in waiting costs is likely to play a role. Also, consumers at the end of the line would most likely
update their chances of getting the deal based on how many consumers they have ahead.

Finally, as my model is static, once a consumer arrives at the store and realizes there are no
items left for a discounted price anymore, he has to choose between the feeling of loss on the item
dimension by returning home empty handed or the feeling of loss by paying a higher price for a
substitute. In reality, the consumer could decide to wait and return to the store some time later.
More generally, sales and promotions appear to be periodic and inter-temporal price discrimination
on the part of firms is a big part of the story. The exploration of a dynamic model of sales with
reference-dependent preferences is left for future research.
A Buyers’ Plans

For each plan, below I first derive the necessary and sufficient conditions for it to be a PE and then I compute its expected reference-dependent utility at time 0. For any \((p_1, p_2)\) and \((q, 1 - q)\), suppose a buyer enters the store with the expectation of not buying; in this case his reference point is to consume nothing and pay nothing. Let the price of good 1 be \(p_1\) and suppose the consumer sticks to his plan. Then, his overall utility is

\[
U [(0, 0) | \{\emptyset\}] = 0.
\]

What if instead the consumer decides to deviate from his plan and buys item 1? In this case his overall utility equals

\[
U [(v_1, p_1) | \{\emptyset\}] = v_1 - p_1 + \eta v_1 - \eta \lambda p_1
\]

where \(v_1 - p_1\) is his intrinsic consumption utility from buying item 1 at price \(p_1\), the term \(\eta v_1\) captures the gain he feels from consuming item 1 when he was expecting to consume nothing and \(-\eta \lambda p_1\) captures the loss he feels from paying \(p_1\) when he was expecting to pay nothing. Thus, the consumer will not deviate in this way from the plan to never buy if

\[
U [(0, 0) | \{\emptyset\}] \geq U [(v_1, p_1) | \{\emptyset\}] \iff p_1 > \frac{1 + \eta}{1 + \eta \lambda} v_1.
\]

A similar threshold can be derived for the case in which the consumer considers deviating from his original plan and buy item 2 at price \(p_2\). Therefore, the plan to never buy is a PE if and only if \(p_1 > \frac{1 + \eta}{1 + \eta \lambda} v_1\) and \(p_2 > \frac{1 + \eta}{1 + \eta \lambda} v_2\). The expected reference-dependent utility associated with the plan to never buy is

\[
EU [\{\emptyset\} | \{\emptyset\}] = 0
\]

as the expected utility from planning to consume nothing and pay nothing and expecting to follow this plan is zero.

Now suppose a buyer enters the store planning to buy item 1 if available and nothing otherwise. In this case his reference point on the item dimension is to enjoy \(v_1\) with probability \(q\) and to consume nothing with probability \(1 - q\); similarly, on the price dimension he expects to pay \(p_1\) with probability \(q\) and to pay nothing with probability \(1 - q\). If the consumer follows this plan his total utility if item 1 is indeed available is

\[
U [(v_1, p_1) | \{1, \emptyset\}] = v_1 - p_1 + \eta (1 - q) v_1 - \eta \lambda q p_1
\]

where \(v_1 - p_1\) is his intrinsic consumption utility from buying item 1 at price \(p_1\), the term \(\eta (1 - q) v_1\) captures the gain he feels from consuming item 1 when he was expecting to consume nothing with probability \((1 - q)\) and \(-\eta \lambda q p_1\) captures the loss he feels from paying \(p_1\) when he was expecting to pay nothing with probability \(q\). Suppose that item 1 is available but the buyer instead deviates and does not buy. In this case his overall utility equals

\[
U [(0, 0) | \{1, \emptyset\}] = 0 - \eta \lambda q v_1 + \eta q p_1
\]

where 0 is his intrinsic consumption utility, \(-\eta \lambda q v_1\) captures the loss he feels from consuming
nothing when he was expecting to consume item 1 with probability \( q \), and \( \eta p_1 \) captures the gain from paying nothing instead of \( p_1 \) which he was expecting to pay with probability \( q \). Thus, the consumer will not deviate in this way from his plan if

\[
U [(v_1, p_1) \mid \{1, \emptyset\}] \geq U [(0, 0) \mid \{1, \emptyset\}] \iff p_1 \leq \frac{1 + \eta (1 - q) + \eta \lambda q}{1 + \eta q + \eta \lambda (1 - q)} v_1. \quad (11)
\]

On the other hand, consider the case in which item 1 is not available. If the buyer follows his plan, his overall utility is \( U [(0, 0) \mid \{1, \emptyset\}] \). If instead he deviates and buys item 2, for \( p_1 \geq p_2 \) his overall utility is

\[
U [(v_2, p_2) \mid \{1, \emptyset\}] = v_2 - p_2 + \eta (1 - q) v_2 - \eta \lambda q (v_1 - v_2) + \eta q (p_1 - p_2) - \eta \lambda (1 - q) p_2
\]

where \( v_2 - p_2 \) is his intrinsic consumption utility from from buying item 2 at price \( p_2 \), the term \( \eta (1 - q) v_2 \) captures the gain he feels from consuming item 2 compared to the expectation of consuming nothing with probability \( (1 - q) \), the term \( -\eta \lambda q (v_1 - v_2) \) captures the loss he feels from consuming item 2 instead of item 1 when he was expecting to consume item 1 with probability \( q \) (recall that \( v_1 \geq v_2 \)), the term \( \eta q (p_1 - p_2) \) captures the gain from paying \( p_2 \) instead of \( p_1 \) which he was expecting to pay with probability \( q \) and, finally, \( -\eta \lambda (1 - q) p_2 \) captures the loss from paying \( p_2 \) when he was expecting to pay nothing with probability \( 1 - q \). Thus, the consumer will not deviate in this way from his plan if

\[
U [(0, 0) \mid \{1, \emptyset\}] > U [(v_2, p_2) \mid \{1, \emptyset\}] \iff p_2 > \frac{1 + \eta (1 - q) + \eta \lambda q}{1 + \eta q + \eta \lambda (1 - q)} v_2. \quad (12)
\]

Notice that conditions (11) and (12) together imply that \( U [(v_1, p_1) \mid \{1, \emptyset\}] > U [(v_2, p_2) \mid \{1, \emptyset\}] \) so that there is no need to check that a consumer does not want to deviate and buy item 2 when item 1 is available. Therefore, for \( p_1 \geq p_2 \), \( \{1, \emptyset\} \) is a PE if and only if \( p_2 > \frac{1 + \eta (1 - q) + \eta \lambda q}{1 + \eta q + \eta \lambda (1 - q)} v_1 \). Similarly, for \( p_1 < p_2 \), \( \{1, \emptyset\} \) is a PE if and only if \( p_1 < \frac{1 + \eta (1 - q) + \eta \lambda q}{1 + \eta q + \eta \lambda (1 - q)} v_1 \) and

\[
p_2 > \frac{v_2 (1 + \eta (1 - q) + \eta \lambda q) + \eta \lambda (\lambda - 1) p_2}{1 + \eta \lambda}.
\]

The expected reference-dependent utility associated with this plan is

\[
EU [(\{1, \emptyset\} \mid \{1, \emptyset\}] = q (v_1 - p_1) - q (1 - q) \eta (\lambda - 1) (v_1 + p_1). \quad (13)
\]

The first term in (13), \( q (v_1 - p_1) \), captures standard expected consumption utility. The second term, \( -q (1 - q) \eta (\lambda - 1) (v_1 + p_1) \), captures expected gain-loss utility and it is derived as follows. On the consumption dimension, the consumer compares the outcome in which with probability \( q \) he gets to consume the good and to enjoy \( v_1 \) with the outcome in which with probability \( 1 - q \) he does not consume and gets 0. Similarly, on the price dimension he compares paying the price \( p_1 \) with probability \( q \) with paying 0 with probability \( 1 - q \). Notice that the expected gain-loss component of the consumer’s overall expected utility is always negative as, because of \( \lambda > 1 \), losses are felt more heavily than equal-size gains. Also, notice that uncertainty in the product and uncertainty in the money dimension are "added up" so that the expected gain-loss term is proportional to \( v_1 + p_1 \).

Similarly, suppose a buyer enters the store with the plan of buying item 2 if available and nothing otherwise. In this case his reference point on the item dimension is to enjoy \( v_2 \) with probability \( 1 - q \) and to consume nothing with probability \( q \); similarly, on the price dimension he
expects to pay \( p_2 \) with probability \( 1 - q \) and to pay nothing with probability \( q \). If the consumer follows his plan his total utility if item 2 is indeed available is

\[
U [(v_2, p_2) | \{2, \varnothing\}] = v_2 - p_2 + \eta q v_2 - \eta \lambda (1 - q) p_2.
\]

Suppose that item 2 is available but the buyer instead deviates and does not buy. In this case his overall utility equals

\[
U [(0, 0) | \{2, \varnothing\}] = 0 - \eta \lambda (1 - q) v_2 + \eta (1 - q) p_2.
\]

Thus, the consumer will not deviate in this way from his plan to buy only good 2 if

\[
U [(v_2, p_2) | \{2, \varnothing\}] \geq U [(0, 0) | \{2, \varnothing\}] \iff p_2 \leq \frac{1 + \eta q + \eta \lambda (1 - q)}{1 + \eta (1 - q) + \eta \lambda q} v_2.
\]

(14)

On the other hand, consider the case in which item 2 is not available. If the buyer follows his plan, his overall utility is \( U [(0, 0) | \{2, \varnothing\}] \). If instead he deviates and buys item 1, for \( p_1 \geq p_2 \) his overall utility is

\[
U [(v_1, p_1) | \{2, \varnothing\}] = v_1 - p_1 + \eta q v_1 + \eta (1 - q) (v_1 - v_2) - \eta \lambda q p_1 - \eta \lambda (1 - q) (p_1 - p_2).
\]

Thus, the consumer will not deviate in this way from his plan if

\[
U [(0, 0) | \{2, \varnothing\}] > U [(v_1, p_1) | \{2, \varnothing\}] \iff p_1 > \frac{(1 + \eta) v_1 + \eta (\lambda - 1) (1 - q) (v_2 + p_2)}{1 + \eta \lambda}.
\]

(15)

Notice that conditions (14) and (15) together imply that \( U [(v_2, p_2) | \{2, \varnothing\}] > U [(v_1, p_1) | \{2, \varnothing\}] \). Therefore, for \( p_1 \geq p_2 \), \( \{2, \varnothing\} \) is a PE if and only if \( p_1 > \frac{(1 + \eta) v_1 + \eta (\lambda - 1) (1 - q) (v_2 + p_2)}{1 + \eta \lambda} \) and \( p_2 \leq \frac{1 + \eta q + \eta \lambda (1 - q)}{1 + \eta (1 - q) + \eta \lambda q} v_2 \).\(^{37}\) The expected reference-dependent utility associated with this plan is

\[
EU [(2, \varnothing) | \{2, \varnothing\}] = (1 - q) (v_2 - p_2) + \eta (1 - \lambda) q (1 - q) (v_2 + p_2).
\]

(16)

The above expression is the obvious analogous of (13) for the plan of buying only good 2 if available and nothing otherwise.

For the plan to buy item 1 if available and item 2 otherwise, a consumer’s reference point in the item dimension is to consume good 1 and enjoy \( v_1 \) with probability \( q \) and to consume good 2 and enjoy \( v_2 \) with probability \( 1 - q \); similarly, in the price dimension, a buyer expects to pay \( p_1 \) with probability \( q \) and \( p_2 \) with probability \( 1 - q \). Let \( p_1 \geq p_2 \) and suppose that when the consumer arrives at the store item 1 is indeed available. Then, if he follows his plan and buys item 1 his overall utility is

\[
U [(v_1, p_1) | \{1, 2\}] = v_1 - p_1 + \eta (1 - q) (v_1 - v_2) - \eta \lambda (1 - q) (p_1 - p_2).
\]

\(^{37}\) It is easy to verify that \( \{2, \varnothing\} \) cannot be a PE for \( p_2 > p_1 \) as a consumer would always deviate and buy item 1 if this is available.
Similarly, for Therefore, for
If he follows his plan and buys item 2 his overall utility is and does not buy his utility is between the two goods). The fourth term, to enjoy at least the item he values is proportional to is a PE if and only if
Thus, the consumer will not deviate in this way from his plan if
$$U[(v_2, p_2) | \{1, 2\}] = v_2 - p_2 - \eta \lambda q (v_1 - v_2) + \eta q (p_1 - p_2).$$

Thus, the consumer will not deviate in this way from his plan if
$$U [(v_1, p_1) | \{1, 2\}] \geq U [(v_2, p_2) | \{1, 2\}] \iff p_1 \leq p_2 + \frac{1 + \eta (1 - q) + \eta \lambda q}{1 + \eta q + \eta \lambda (1 - q)} (v_1 - v_2). \quad (17)$$

Now, instead, suppose that once the buyer arrives at the store, item 2 is everything that is left. If he follows his plan and buys item 2 his overall utility is $U [(v_2, p_2) | \{1, 2\}]$. If instead he deviates and does not buy his utility is
$$U [(0, 0) | \{1, 2\}] = 0 - \eta \lambda q v_1 - \eta \lambda (1 - q) v_2 + \eta q p_1 + \eta (1 - q) p_2.$$

Thus, the consumer will not deviate in this way from his plan if
$$U [(v_2, p_2) | \{1, 2\}] \geq U [(0, 0) | \{1, 2\}] \iff p_2 \leq \frac{1 + \eta \lambda}{1 + \eta} v_2. \quad (18)$$

Notice that conditions (17) and (18) together imply that $U [(v_1, p_1) | \{1, 2\}] > U [(0, 0) | \{1, 2\}]$. Therefore, for $p_1 \geq p_2$, $\{1, 2\}$ is a PE if and only if $p_1 \leq p_2 + \frac{1 + \eta (1 - q) + \eta \lambda q}{1 + \eta q + \eta \lambda (1 - q)} (v_1 - v_2)$ and $p_2 \leq \frac{1 + \eta \lambda}{1 + \eta} v_2$.

Similarly, for $p_1 < p_2$, $\{1, 2\}$ is a PE if and only if $p_2 \leq \frac{(1 + \eta \lambda) v_2 + \eta q (\lambda - 1) p_1}{1 + \eta (1 - q) + \eta \lambda q}$ and $p_1 \leq \frac{1 + \eta (1 - q) + \eta \lambda q}{1 + \eta} v_1 + \frac{\eta (\lambda - 1)(1 - q)}{1 + \eta} v_2$. The expected reference-dependent utility associated with this plan is
$$EU [\{1, 2\} | \{1, 2\}] = q (v_1 - p_1) + (1 - q) (v_2 - p_2) + q (1 - q) \eta (1 - \lambda) (v_1 - v_2) + q (1 - q) \eta (1 - \lambda) (\max \{p_1, p_2\} - \min \{p_1, p_2\}). \quad (19)$$

The first and second terms in (19), $q (v_1 - p_1) + (1 - q) (v_2 - p_2)$ are the standard expected consumption utility terms. The third term, $q (1 - q) \eta (1 - \lambda) (v_1 - v_2)$, captures expected gain-loss utility in the consumption dimension: with probability $q$ the consumer expects to buy good 1 and with probability $1 - q$ he expects to buy good 2. Notice again that this term is negative, but now it is proportional to $(v_1 - v_2)$. This is because with this plan, the consumer is "guaranteeing" himself to enjoy at least the item he values $v_2$ and the expected gain-loss utility is therefore related only to by how much more he would prefer to consume the other good (or, the degree of substitutability between the two goods). The fourth term, $q (1 - q) \eta (1 - \lambda) (\max \{p_1, p_2\} - \min \{p_1, p_2\})$, captures expected gain-loss utility in the money dimension and can be explained in a similar fashion.

Finally, let's consider the plan to buy item 2 if available and item 1 otherwise. The reference point for this plan is the same as for the previous one. Let $p_1 \geq p_2$ and suppose that when the consumer arrives at the store item 2 is indeed available. Then, if he follows his plan and buys item 1 his overall utility is
$$U [(v_2, p_2) | \{2, 1\}] = v_2 - p_2 - \eta \lambda q (v_1 - v_2) + \eta q (p_1 - p_2).$$
If instead he deviates and buys item 1, his overall utility equals

\[ U\left[ (v_1, p_1) \mid \{2, 1\} \right] = v_1 - p_1 + \eta (1 - q) (v_1 - v_2) - \eta \lambda (1 - q) (p_1 - p_2). \]

Thus, the consumer will not deviate in this way from his plan if

\[ U\left[ (v_2, p_2) \mid \{2, 1\} \right] \geq U\left[ (v_1, p_1) \mid \{2, 1\} \right] \iff p_2 \leq p_1 - \frac{1 + \eta (1 - q) + \eta \lambda q}{1 + \eta q + \eta \lambda (1 - q)} (v_1 - v_2). \] (20)

Now, instead, suppose that once the buyer arrives at the store, item 1 is everything that is left. If he follows his plan and buys item 1 his overall utility is \( U\left[ (v_1, p_1) \mid \{2, 1\} \right] \). If instead he deviates and does not buy his utility is

\[ U\left[ (0, 0) \mid \{2, 1\} \right] = 0 - \eta \lambda q v_1 - \eta \lambda (1 - q) v_2 + \eta q p_1 + \eta (1 - q) p_2. \]

Thus, the consumer will not deviate in this way from his plan if

\[ U\left[ (v_1, p_1) \mid \{2, 1\} \right] \geq U\left[ (0, 0) \mid \{2, 1\} \right] \iff p_1 \leq \frac{1 + \eta (1 - q) + \eta \lambda q}{1 + \eta q + \eta \lambda (1 - q)} v_1 + \frac{\eta (\lambda - 1) (1 - q)}{1 + \eta q + \eta \lambda (1 - q)} (v_2 + p_2). \] (21)

Notice that conditions (20) and (21) together imply that \( U\left[ (v_2, p_2) \mid \{2, 1\} \right] \geq U\left[ (v_1, p_1) \mid \{2, 1\} \right] \). Therefore, for \( p_1 \geq p_2 \), \( \{2, 1\} \) is a PE if and only if \( p_1 \leq \frac{1 + \eta (1 - q) + \eta \lambda q}{1 + \eta q + \eta \lambda (1 - q)} v_1 + \frac{\eta (\lambda - 1) (1 - q)}{1 + \eta q + \eta \lambda (1 - q)} (v_2 + p_2) \) and \( p_2 \leq p_1 - \frac{1 + \eta (1 - q) + \eta \lambda q}{1 + \eta q + \eta \lambda (1 - q)} (v_1 - v_2) \). And the expression of the expected reference-dependent utility associated with this plan is the same as in (19).38

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38 It is easy to verify that \( \{2, 1\} \) cannot be a PE for \( p_2 > p_1 \) as a consumer would always deviate and buy item 1 if this is available.
B Proofs

Proof of Lemma 1: As shown in Kőszegi and Rabin (2006), the plan of buying good \( i = 1, 2 \) is a PE if and only if \( p_i \leq \frac{1+\eta}{1+\eta} v_i \equiv p_i^{\max} \) and the plan of not buying good \( i \) is a PE if and only if \( p_i > \frac{1+\eta}{1+\eta} v_i \equiv p_i^{\min} \). Therefore, for \( p_i \in \left( p_i^{\min}, p_i^{\max} \right) \) both plans are consistent. However, the plan of buying good \( i \) at \( p_i \) is the PPE if and only if

\[
EU \left[ \{i\} \mid \{i\} \right] \geq EU \left[ \emptyset \mid \emptyset \right]
\]

\[\Leftrightarrow \quad v_i - p_i \geq 0\]

and this proves the statement. ■

Proof of Lemma 2: The result holds trivially for the case of perfect availability. Then, let \( q_1 > 0, q_2 > 0 \) with \( q_1 + q_2 < 1 \) and suppose the seller charges \( p_1 \) for item 1 and \( p_2 \) for item 2, with \( p_2 \geq p_1 \). The highest price the seller can charge for item 2 is the one that makes the following inequality bind:

\[
EU \left[ \{1, 2\} \mid \{1, 2\} \right] \geq EU \left[ \emptyset \mid \emptyset \right].
\]  

(22)

Substituting and re-arranging yields

\[
p_2 \leq \frac{v_2 [1 + \eta (\lambda - 1) q_1 - \eta (\lambda - 1) (1 - q_1 - q_2)] - 2\eta (\lambda - 1) q_1}{1 + \eta (\lambda - 1) (1 - q_2)}.
\]

It is easy to see that the right-hand-side of the above inequality is increasing in \( q_2 \). Therefore, the seller can raise \( q_2 \) up to \( 1 - q_1 \) and increase her profits without violating condition (22). A similar analysis applies if \( p_2 < p_1 \). ■

Proof of Lemma 3: I prove the result by contradiction. Suppose that \( q \in (0, 1) \) and \( p_i = v_i \) for \( i = 1, 2 \) and that \( v_1 > 2v_2 \); then we have that

\[
EU \left[ \emptyset \mid \emptyset \right] = 0
\]

\[
> -2\eta (\lambda - 1) q (1 - q) v_2 = EU \left[ \{2, \emptyset\} \mid \{2, \emptyset\} \right]
\]

\[
> -2\eta (\lambda - 1) q (1 - q) (v_1 - v_2) = EU \left[ \{1, 2\} \mid \{1, 2\} \right]
\]

\[
> -2\eta (\lambda - 1) q (1 - q) v_1 = EU \left[ \{1, \emptyset\} \mid \{1, \emptyset\} \right].
\]

Furthermore, we know that not buying is a PE when \( p_i = v_i \). Therefore, for this quantity vector and this price vector the buyers would strictly prefer the plan of not buying. The seller would then do better by setting \( p_i = p_i^{\min} \) for at least one good and thus force the consumers to buy it. The same argument applies to the case in which \( v_1 \leq 2v_2 \) (just switch the first and second inequalities). ■

Proof of Lemma 4: I prove the result by contradiction. Suppose that \( q \in (0, 1) \) and \( p_i > p_i^{\min} \) for \( i = 1, 2 \) and that \( v_1 - c_1 \geq v_2 - c_2 \). By producing a strictly positive quantity of both goods, the seller wants the buyers to choose the plan to always buy; however, for this plan to be the PPE it must be that

\[
EU \left[ \{1, 2\} \mid \{1, 2\} \right] \geq EU \left[ \emptyset \mid \emptyset \right]
\]

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\[ q (v_1 - p_1) + (1 - q) (v_2 - p_2) > 0 \]
\[ \iff q p_1 + (1 - q) p_2 < q v_1 + (1 - q) v_2 \]
\[ \Rightarrow q (p_1 - c_1) + (1 - q) (p_2 - c_2) < q (v_1 - c_1) + (1 - q) (v_2 - c_2) \leq v_1 - c_1. \]

But then the seller would prefer to set \( q = 1 \) and \( p_1 = v_1 \) and this contradicts the assumption that seller produces a strictly positive quantity of both goods. The same argument applies to the case in which \( v_1 - c_1 < v_2 - c_2 \).

**Proof of Lemma 5:** Let \( q \in (0, 1) \) From Lemma 4 we know that \( p_i = p_i^{\min} \) for at least one good; let this be good 2. I now show that it is not profitable for the seller to choose \( p_1 \) such that the plan to always buy is the unique credible plan for the consumers. First, we have that, for \( p_2 = p_2^{\min} \), the highest price the seller can use, in order to make the plan to buy only good 2 not credible, is

\[
p_1 \leq \frac{(1 + \eta) v_1 + \eta (\lambda - 1) (1 - q) v_2 \left( 1 + \frac{1 + \eta q}{1 + \eta \lambda} \right)}{1 + \eta \lambda} \equiv \bar{p}_1 (q)
\]

Then, we have that, for \( p_2 = p_2^{\min} \), the plan to always buy is a PE if and only if

\[
p_1 \leq \frac{[1 + \eta (1 - q) + \eta \lambda q] v_1 + \eta (\lambda - 1) (1 - q) v_2 \left( 1 + \frac{1 + \eta q}{1 + \eta \lambda} \right)}{1 + \eta q + \eta \lambda (1 - q)} \equiv \mathcal{P}_1 (q).
\]

It is readily verified that \( \mathcal{P}_1 (q) > \bar{p}_1 (q) \iff q > 0 \). However, for \( \mathcal{P}_1 (q) \geq p_1 > \bar{p}_1 (q) \) both the plan to always buy and the plan to buy only item 2 are personal equilibria; but the plan of always buying is the PPE if and only if

\[
p_1 \leq v_1 + \frac{2 (1 - q) \eta (\lambda - 1) [v_2 (2 + \eta + \eta \lambda) - v_1 (1 + \eta \lambda)]}{(1 + \eta \lambda) [1 + \eta (\lambda - 1) (1 - q)]} \equiv \bar{p}_1 (q).
\]

It is easy to see that \( \bar{p}_1 (q) > \bar{p}_1 (q) \). Therefore, the highest price \( p^*_1 \) at which a buyer prefers the plan to always buy is given by

\[ p^*_1 = \min \{ \mathcal{P}_1 (q), \bar{p}_1 (q) \} \]

and this proves that it is not profit-maximizing for the seller to make always buying the unique consistent plan.

Then, in order to prove that \( p^*_1 = \bar{p}_1 (q) \), notice that

\[
\mathcal{P}_1 (q) < \bar{p}_1 (q) \iff q < \frac{v_2 (1 + 2\eta \lambda) (2 + \eta + \eta \lambda) - \eta v_1 (1 + \lambda) (1 + \eta \lambda)}{2v_2 \eta (\lambda - 1) (2 + \eta + \eta \lambda) - \eta v_1 (1 + \lambda) (1 + \eta \lambda) - \sqrt{A^2 v_2^2 - 2Bv_1 v_2 + C^2 v_2^2}}
\]

where \( A \equiv \eta (1 + \eta \lambda) (1 + \lambda), B \equiv \eta (1 + \eta \lambda) (2 + \eta + \eta \lambda) [3 + 2\eta + 2\eta (\lambda - 1)] \) and \( C \equiv (1 + 2\eta) (2 + \eta + \eta \lambda) \).
It is also easy to verify that 
\[
\frac{v_2 (1-2p\lambda)(2+\eta+\eta\lambda)-\eta v_1 (1+\lambda)(1+\eta\lambda)-\sqrt{A^2 v_1^2 - 2Bv_1 v_2 + C v_2^2}}{2v_2 (\lambda - 1)(1+\eta)} < 1.
\]
However, it is in the seller’s interest to select the \( p_1^* \) that maximizes \( qp_1^* \) and since
\[
\frac{\partial [qp_1^* (q)]}{\partial q} = \frac{(2 + \eta + \eta \lambda) v_2 - (1 + \eta \lambda) v_1}{1 + \eta \lambda} > 0
\]
it follows that \( p_1^* = \tilde{p}_1 (q) \).

Finally, I show that \( p_1^* > v_1 \). Suppose, by contradiction, that \( p_1^* \leq v_1 \). The seller’s profit is
\[
q (p_1^* - c_1) + (1 - q) \left( p_2^\min - c_2 \right).
\]

We have that
\[
p_1^* \leq v_1 \Rightarrow q (p_1^* - c_1) + (1 - q) \left( p_2^\min - c_2 \right) < \max \{ v_1 - c_1, v_2 - c_2 \}.
\]

But then the seller would prefer to choose either \( q = 1 \) or \( q = 0 \), contradicting the hypothesis that she is producing a strictly positive quantity of both goods. Notice also that
\[
p_1^* > v_1 \Leftrightarrow v_2 (2 + \eta + \eta \lambda) - v_1 (1 + \eta \lambda) > 0
\]
\[
\Leftrightarrow v_2 + p_2^\min > v_1.
\]

The same argument applies if the seller uses item 1 as the bait (i.e., \( p_1 = 1 \)).

**Proof of Lemma 6:** Suppose the seller uses item 2 as the bait and thus prices it at \( p_2^\min \). Then, by Lemma 5 we know that the optimal price for item 1 is
\[
p_1^* = v_1 + \frac{2 (1-q) \eta (\lambda - 1) [v_2 (2 + \eta + \eta \lambda) - v_1 (1 + \eta \lambda)]}{(1 + \eta \lambda) [1 + \eta (\lambda - 1) (1-q)]}.
\]

This pair of prices provides the seller with profits equal to
\[
q \left\{ \frac{v_1 [1 - \eta (\lambda - 1) (1-q)] + 2\eta (\lambda - 1) (1-q) \left( 1 + \frac{1+\eta}{1+\eta \lambda} \right) v_2}{1 + \eta (\lambda - 1) (1-q)} - c_1 \right\}
\]
\[
+ (1-q) \left( p_2^\min - c_2 \right).
\]

The above expression is maximized at
\[
q = \frac{1 + \eta \lambda - \eta}{\eta (\lambda - 1)} - \frac{\sqrt{2}}{\eta (\lambda - 1)} \sqrt{\frac{(1 + \eta \lambda - \eta) (-v_1 + 2v_2 + \eta v_2 - \lambda \eta v_1 + \lambda \eta v_2)}{(-c_1 + c_2 - v_1) (1 + \eta) + v_2 (3 + \eta + 2\eta \lambda)}}
\]
\[
\equiv \tilde{q} (\eta, \lambda, v_1, v_2, c_1, c_2).
\]
Notice that for the above expression to be well-defined, it must be that

\((-c_1 + c_2 - v_1) (1 + \eta \lambda) + v_2 (3 + \eta + 2\eta \lambda)\)

since we know that \((2 + \eta + \eta \lambda) v_2 > (1 + \eta \lambda) v_1\) for \(p_1^*\) to be greater than \(v_1\). It is easy to see that \(q > 0\). Furthermore, we have that

\[ q + 1 \Leftrightarrow v_1 \left[ 1 + 3\eta \lambda - 2\eta + 2\eta^2 \lambda (\lambda - 1) \right] < (c_1 - c_2) (1 + \eta \lambda) + v_2 \left[ 1 + 4\eta \lambda - 3\eta + 2\eta^2 (\lambda - 1) (\lambda + 1) \right].\]

Notice that

\[ q(\eta, \lambda, v_1, v_2, c_1, c_2) > \frac{1}{2} \]

since

\[ q(\eta, v_1, v_2, c_1, c_2) > q(\eta, v, v, c, c) \]

which is true for any \(\eta > 0\) and \(\lambda > 1\) provided that \(v_1 - c_1 > v_2 - c_2\) (which, as shown below, is a necessary condition for the seller to want to use item 2 as the bait); and

\[ q(\eta, \lambda, v, v, c, c) > \frac{1}{2} \]

\[ q(\eta, \lambda, v, v, c, c) > \frac{1}{2} \]

\[ q(\eta, \lambda, v, v, c, c) > \frac{1}{2} \]

which is true for any \(\eta > 0\) and \(\lambda > 1\).

If instead the seller uses item 1 as the bait, then by Lemma 5 we know that the optimal price for item 2 is

\[ p_2^* = v_2 + \frac{2q v_1 \eta (\lambda - 1) (1 + \eta)}{(1 + \eta \lambda) [1 + \eta (\lambda - 1) q]}. \]
This pair of prices provides the seller with profits equal to
\[
q \left( p_1^{\text{min}} - c_1 \right) + (1-q) \left\{ \frac{v_2 \left[ 1 + \eta (\lambda - 1) q \right] + 2 \eta (\lambda - 1) q \left( \frac{1+\eta}{1+\eta \lambda} \right) v_1}{1 + \eta (\lambda - 1)} - c_2 \right\}.
\]

The above expression is maximized at
\[
q = \frac{\sqrt{2}}{\eta (\lambda - 1)} \frac{\sqrt{v_1 (1 + \eta) (1 + \eta \lambda - \eta)}}{\sqrt{(c_1 - c_2 + v_2) (\lambda \eta + 1) + v_1 (1 + \eta)}} - \frac{1}{\eta (\lambda - 1)} \equiv q(\eta, \lambda, v_1, v_2, c_1, c_2).
\]

It is easy to see that \( q < 1 \). Furthermore, we have that
\[
q > 0 \iff v_1 (1 + \eta) (1 + 2 \eta \lambda - 2 \eta) > (c_1 - c_2) (1 + \eta \lambda) + v_2 (1 + \eta \lambda).
\]

Notice that
\[
q(\eta, \lambda, v_1, v_2, c_1, c_2) < \frac{1}{2}
\]

\[
\iff 2 \frac{2v_1 (1 + \eta) (1 + \eta \lambda - \eta)}{(c_1 - c_2 + v_2) (\lambda \eta + 1) + v_1 (1 + \eta)} < \eta (\lambda - 1) + 2
\]

\[
\iff v_1 (1 + \eta) \left( 8(1 + \eta \lambda - \eta) - (\eta (\lambda - 1) + 2)^2 \right) < (v_2 - c_2 + c_1) (1 + \eta \lambda) (\eta (\lambda - 1) + 2)^2.
\]

The above condition is trivially satisfied for any \( \eta > 0 \) and \( \lambda > 1 \) since, as shown below, \( (c_1 - c_2 + v_2) (\lambda \eta + 1) > v_1 (\eta + 1) \) must hold if the seller prefers to use item 1 as the bait.

Finally, we have that
\[
\bar{q} > 1 - \frac{1}{q}
\]

\[
\iff \frac{v_2 (2 + \eta + \eta \lambda) - v_1 (1 + \eta \lambda)}{(-c_1 + c_2 - v_1) (1 + \eta \lambda) + v_2 (3 + \eta + 2 \eta \lambda)} < \frac{v_1 (1 + \eta)}{(c_1 - c_2 + v_2) (1 + \eta \lambda) + v_1 (1 + \eta)}
\]

\[
\iff v_2 (2 + \eta + \eta \lambda) (v_2 - c_1 - c_2) - \eta (\lambda - 1) v_1 (c_1 - c_2) < 0
\]

which is true for any \( \eta > 0 \) and \( \lambda > 1 \) given that \( v_2 (2 + \eta \lambda + \eta) > v_1 (1 + \eta \lambda) \) by Lemma 5 and provided that \( v_1 - c_1 > v_2 - c_2 \) (which, as shown below, is the only case in which \( \bar{q} \) and \( q \) are comparable).

**Proof of Lemma 7**: Define \( \pi_1 \equiv \pi(p_1, p_2^{\text{min}}, q; c_1, c_2) \) and \( \pi_2 \equiv \pi(p_1^{\text{min}}, p_2^*, q; c_1, c_2) \) and recall that \( \bar{q} = \arg \max_p \pi(p_1, p_2^{\text{min}}, q; c_1, c_2) \) and \( q = \arg \max_p \pi(p_1^{\text{min}}, p_2^*, q; c_1, c_2) \).

First, consider the special case with \( v_1 = v_2 \) and \( c_1 = c_2 \). It is easy to see that in this case \( p_1^{\text{min}} = p_2^{\text{min}} \), \( p_1^* = p_2^* \), \( \bar{q} = 1 - q \) so that \( \pi_1 = \pi_2 \). Therefore the seller is indifferent between which item to use as the bait. Furthermore, by the envelope theorem we have that \( \frac{d\pi_1}{dc_1} = -\bar{q} \), \( \frac{d\pi_1}{dc_2} = -(1 - \bar{q}) \), \( \frac{d\pi_2}{dc_1} = -(1 - q) \) and \( \frac{d\pi_2}{dc_2} = -q \). By lemma 6 we know that \( \bar{q} > 1 - q \) and therefore it follows that when the two goods are perfect substitutes, the seller maximizes profits by using the more expensive one as the bait.
Next, suppose to change $v_1$ by $dv_1$ and $c_1$ by $dc_1$ with $dv_1 = dc_1 = \delta > 0$ so that $v_1 > v_2$ but $v_1 - v_2 = c_1 - c_2$. By the envelope theorem the effect of these changes on profits are

$$d\pi_1 \simeq \frac{\partial \pi_1}{\partial v_1} dv_1 + \frac{\partial \pi_1}{\partial c_1} dc_1 = \left( q \frac{\partial p^*_1}{\partial v_1} - \bar{q} \right) \delta$$

and

$$d\pi_2 \simeq \frac{\partial \pi_2}{\partial v_1} dv_1 + \frac{\partial \pi_2}{\partial c_1} dc_1 = \left[ \frac{1 + \eta}{1 + \eta \lambda} + \left( 1 - q \right) \frac{\partial p^*_2}{\partial v_1} \right] \delta.$$

By substituting and re-arranging, it follows that $d\pi_2 > d\pi_1$ if and only if

$$\frac{q (1 - q)}{\eta (\lambda - 1)} \left[ \frac{2 \left( 1 - q \right) (1 + \eta) - q \eta (\lambda - 1) - 1}{q \eta (\lambda - 1) + 1} \right] > \bar{q} \left[ \frac{1 - \eta (\lambda - 1) (1 - \bar{q})}{1 + \eta (\lambda - 1) (1 - \bar{q})} - 1 \right]. \tag{23}$$

As the expression on the right-hand-side of (23) is negative, it suffices to show that

$$2 \left( 1 - q \right) (1 + \eta) - q \eta (\lambda - 1) - 1 > 0 \iff \frac{1 + 2 \eta}{2 + \eta + \eta \lambda} > q.$$

Substituting $v_1 = v_2$ and $c_1 = c_2$ into the expression for $q$ yields

$$\frac{2 + \eta + \eta \lambda + \eta (\lambda - 1) (1 + 2 \eta)}{2 + \eta + \eta \lambda} > \sqrt{\frac{2 + \eta + \eta \lambda + \eta (\lambda - 1) (1 + 2 \eta)}{2 + \eta + \eta \lambda}}$$

which is of course true for any $\eta > 0$ and $\lambda > 1$. Thus, the seller maximizes profits by using item 1 as the bait if $v_1 > v_2$ and $v_1 - v_2 = c_1 - c_2$. Furthermore, it is easy to see that the same result holds also if $dc_1 > dv_1 > 0$ so that $v_1 - c_1 < v_2 - c_2$. Therefore, we have that $\pi_2 \geq \pi_1$ for $v_1 - c_1 \leq v_2 - c_2$.

Finally, consider the case in which $v_1 - c_1 > v_2 - c_2$. Again, let’s start with $v_1 = v_2$ and $c_1 = c_2$ so that $\pi_1 = \pi_2$ and suppose to change $v_1$ by $dv_1$ and $c_1$ by $dc_1$ with either $dv_1 > dc_1 \geq 0$ or $dv_1 \geq 0 > dc_1$. By the envelope theorem the effect of these changes on profits are

$$d\pi_1 \simeq \frac{\partial \pi_1}{\partial v_1} dv_1 + \frac{\partial \pi_1}{\partial c_1} dc_1 = \overline{q} \frac{\partial p^*_1}{\partial v_1} dv_1 - \bar{q} dc_1$$

and

$$d\pi_2 \simeq \frac{\partial \pi_2}{\partial v_1} dv_1 + \frac{\partial \pi_2}{\partial c_1} dc_1 = \left[ \frac{1 + \eta}{1 + \eta \lambda} + \left( 1 - q \right) \frac{\partial p^*_2}{\partial v_1} \right] dv_1 - q dc_1.$$

By substituting and re-arranging, it follows that $d\pi_1 \geq d\pi_2$ if and only if

$$\overline{q} (1 - q) \left[ \frac{1 - \eta (\lambda - 1) (1 - \overline{q})}{1 + \eta (\lambda - 1) (1 - \overline{q})} \right] \overline{q} - \overline{q} \frac{1 + \eta}{1 + \eta \lambda} - \left( 1 - q \right) \frac{1 + \eta}{1 + \eta \lambda} \frac{2 \eta q (\lambda - 1)}{1 + \eta (\lambda - 1) q} \geq (\bar{q} - q) dc_1. \tag{24}$$

We know that for $dv_1 = dc_1 > 0$ condition (24) is violated; but for either $dv_1 > dc_1 \geq 0$ or
\( dv_1 \geq 0 > dc_1 \) it can hold (for example, it is readily satisfied for \( dv_1 = 0 \) and \( dc_1 < 0 \)). Then, let \( \bar{v}_1 \) be the value of \( v_1 \) for which (24) binds; if such a value exists then it is unique because the term on the left-hand-side of (24) is continuous and increasing in \( dv_1 \). Notice also that \( \bar{v}_1 \) increases with \( c_1 - c_2 \).

However, from lemma 6 we know that

\[
q < 1 \iff v_1 < \frac{(c_1 - c_2) (1 + \eta \lambda) + v_2 (1 + 4 \eta \lambda - 3 \eta) + 2 \eta^2 v_2 (\lambda - 1) (\lambda + 1)}{1 + 3 \eta \lambda - 2 \eta + 2 \eta^2 \lambda (\lambda - 1)}.
\]

Therefore, a necessary condition for the seller to use item 2 as the bait when \( v_1 - c_1 > v_2 - c_2 \) is that

\[
\frac{(c_1 - c_2) (1 + \eta \lambda) + v_2 (1 + 4 \eta \lambda - 3 \eta) + 2 \eta^2 v_2 (\lambda - 1) (\lambda + 1)}{1 + 3 \eta \lambda - 2 \eta + 2 \eta^2 \lambda (\lambda - 1)} > v_2 - c_2 + c_1
\]

\[
\iff \eta (\lambda - 1) [2 (1 + \eta \lambda) (c_2 - c_1) + v_2 (1 + 2 \eta)] > 0
\]

\[
\iff 2 (1 + \eta \lambda) (c_2 - c_1) + v_2 (1 + 2 \eta) > 0
\]

\[
\iff v_2 > \frac{2 (1 + \eta \lambda) (c_1 - c_2)}{1 + 2 \eta}.
\]

However, the above condition is not sufficient as it could still be that

\[
\bar{v}_1 > \frac{(c_1 - c_2) (1 + \eta \lambda) + v_2 (1 + 4 \eta \lambda - 3 \eta) + 2 \eta^2 v_2 (\lambda - 1) (\lambda + 1)}{1 + 3 \eta \lambda - 2 \eta + 2 \eta^2 \lambda (\lambda - 1)}.
\]

**Proof of Proposition 1:** For an arbitrary price vector \((p_1, p_2)\) and an arbitrary quantity vector \((q, 1 - q)\) the monopolist’s profit is

\[
\pi (p_1, p_2, q; c_1, c_2) = q (p_1 - c_1) + (1 - q) (p_2 - c_2).
\]

By Lemma 1 we know that if the seller produces only one good, then she will price it at its intrinsic value.

By Lemma 3 and Lemma 4 we know that if the seller produces a strictly positive quantity of both goods then one of them, say good \( i \), must be priced at the discounted price \( p_i^{\text{min}} \). By Lemma 5 we also know that in this case the seller will price good \( j \) at \( p_j^* \). Therefore, the seller has three options:

i) Set \( p_2 = p_2^{\text{min}} \), \( p_1 = p_1^* \) and \( q = \bar{q} \). In this case the seller’s profit is

\[
\bar{q} (p_1^* - c_1) + (1 - \bar{q}) (p_2^{\text{min}} - c_2) \equiv \pi_1.
\]

ii) Set \( p_1 = p_1^{\text{min}} \), \( p_2 = p_2^* \) and \( q = q \). In this case the seller’s profit is

\[
q (p_1^{\text{min}} - c_1) + (1 - q) (p_2^* - c_2) \equiv \pi_2.
\]
iii) Set $p_i = v_i$ for $i = 1, 2$. This pair of prices provides the seller with profits equal to

$$q (v_1 - c_1) + (1 - q) (v_2 - c_2).$$

The above expression is maximized at $q = 1$ (resp. $q = 0$) if $v_1 - c_1 > v_2 - c_2$ (resp. if $v_1 - c_1 \leq v_2 - c_2$).

Depending on the degree of substitutability between the two goods, their marginal costs and the degree of loss aversion, the seller will choose the option that will give her the highest profit. Suppose first that $v_1 - c_1 \leq v_2 - c_2$. By Lemma 7 we know that if she were to produce both goods, the seller would prefer to use item 1 as the bait. Then,

$$\pi \left( p_{1 \text{min}}^{\text{min}}, p_{2}^{*}, q; c_1, c_2 \right) \geq v_2 - c_2$$

$$\Leftrightarrow v_1 \geq \frac{v_2 - c_2 + c_1}{1 + 2\eta} \left( \frac{1 + \eta \lambda}{1 + \eta} \right) \equiv \alpha (v_2, c_1, c_2, \eta, \lambda).$$

Now suppose that $\tilde{v}_1 > v_1 > v_2 - c_2 + c_1$. By Lemma 7 we know that if she were to produce both goods, the seller would again prefer to use item 1 as the bait. Therefore,

$$\pi \left( p_{1 \text{min}}^{\text{min}}, p_{2}^{*}, q; c_1, c_2 \right) \geq v_1 - c_1$$

$$\Leftrightarrow v_1 \leq (v_2 - c_2 + c_1) \left( \frac{1 + \eta \lambda}{1 + \eta} \right) \left[ 1 + \eta (\lambda - 1) \right] \times$$

$$\left[ \frac{3\eta + 4\eta^2 + 2\eta^3 + \eta^2 \lambda^2 (1 + \eta)}{4\eta (1 + \eta^3) + \eta^4 \lambda^4 - 2\eta^2 \lambda^2 (1 + 3\eta^2 + 2\eta) - 2\eta (\lambda - 1) \sqrt{2 (1 + \eta)^3} + 1} \right]$$

$$\equiv \beta (v_2, c_1, c_2, \eta, \lambda).$$

Finally, if $v_1 \geq \tilde{v}_1$ then by Lemma 7 the seller prefers to use item 2 as the bait and we have

$$\pi \left( p_{1}^{*}, p_{2 \text{min}}^{\text{min}}, q; c_1, c_2 \right) \geq v_1 - c_1 \Leftrightarrow v_2 \geq \frac{v_1 - c_1 + c_2 + 2\eta (\lambda - 1) v_1}{1 + \eta (\lambda - 1) \left( \frac{3 + 2\eta \lambda + 2\eta}{1 + \eta \lambda} \right)} \equiv \gamma (v_2, c_1, c_2, \eta, \lambda).$$

To conclude the proof, notice that the seller’s profits, if she chooses to produce only one good, are equal to max $\{ v_1 - c_1, v_2 - c_2 \}$. Since she would choose a different option only if this provides her with at least as much, it thus follows that $\pi \geq \max \{ v_1 - c_1, v_2 - c_2 \}$, and the inequality is strict when either option i) or ii) is profit-maximizing.

**Proof of Proposition 2:** Suppose the seller uses item 2 as the bait. We have:

$$\overline{\pi} (p_{1}^{*} - c_1) + (1 - \overline{\pi}) (p_{2}^{\text{min}} - c_2) > \max \{ v_1 - c_1, v_2 - c_2 \}$$

$$\geq \overline{\pi} (v_1 - c_1) + (1 - \overline{\pi}) (v_2 - c_2)$$

$$\Rightarrow \overline{\pi} p_{1}^{*} + (1 - \overline{\pi}) p_{2}^{\text{min}} > \overline{\pi} v_1 + (1 - \overline{\pi}) v_2.$$
In this case, therefore, a consumer expects to buy with probability one at an expected price strictly greater than his expected valuation. Hence, his consumption utility is negative. Furthermore, in any PE expected gain-loss utility is non-positive. If instead he could commit to the plan of never buying, both his consumption utility and his gain-loss utility would be zero. The same argument applies for the case in which the seller uses item 1 as the bait.

**Proof of Proposition 3:** First, we prove that if the seller can create artificial substitutes, a combination limited availability, bargains and rip-offs always yields higher profits than perfect availability. Let \( v_1 - c_1 > v_2 - c_2 \) so that the maximum level of profits the seller can achieve with perfect availability is \( v_1 - c_1 \). If the seller can create perfect substitutes for item 1, then her profits are equal to

\[
\hat{q} \left( p_{1,1}^* - c_1 \right) + (1 - \hat{q}) \left( p_{1}^\text{min} - c_1 \right)
\]

where \( \hat{q} = \bar{q} (\eta, \lambda, v, c, c) \). Then, it suffices to show that

\[
\hat{q} \left( 1 + \frac{2 (1 - \hat{q}) \eta (\lambda - 1) + 1 + \eta}{1 + (1 - \hat{q}) \eta (\lambda - 1) + 1 + \eta \lambda} \right) + \frac{1 + \eta}{1 + \eta \lambda} (1 - \hat{q}) > 1
\]

\[
\iff \hat{q} > \frac{1 + \eta (\lambda - 1)}{\eta + \lambda \eta + 2}.
\]

Substituting for \( \hat{q} \) yields

\[
\frac{2 + 2 \lambda \eta - 2 \eta^2 + 2 \lambda \eta^2 - \sqrt{2 (\eta + 1) \lambda \eta} + 1 + \eta} {\eta (\lambda - 1) (\eta + \lambda \eta + 2)} > 0
\]

\[
\iff 2 \eta (\lambda - 1) (\eta \lambda - \eta + 1) (2 \eta^2 + 3 \eta + 1) > 0
\]

which is of course true for any \( \eta > 0 \) and \( \lambda > 1 \). A similar argument applies if \( v_1 - c_1 \leq v_2 - c_2 \).

Next, we prove the first part of the proposition. Define \( \pi_{1,2} \equiv \pi \left( p_{1,1}^*, p_{1,2}^*, \bar{q}; c_1, c_2 \right) \), \( \pi_{2,1} \equiv \pi \left( p_{1,1}^*, p_{2,1}^*, \bar{q}; c_1, c_2 \right) \), \( \pi_{1,1} \equiv \pi \left( p_{1,1}^*, p_{1,1}^*, \bar{q}; c_1, c_1 \right) \) and \( \pi_{2,2} \equiv \pi \left( p_{1,2}^*, p_{2,2}^*, \bar{q}; c_2, c_2 \right) \). Recall that \( \bar{q} = \arg \max \pi \left( p_{1,1}^*, p_{2,1}^*, q; c_1, c_2 \right) \), \( \bar{q} = \arg \max \pi \left( p_{1,1}^*, p_{2,2}^*, q; c_1, c_2 \right) \) and let \( \hat{q} = \arg \max \pi \left( p_{1,1}^*, p_{i,i}^*, q; c_i, c_i \right) \), for \( i \in \{1, 2\} \). If \( v_1 = v_2 \) and \( c_1 = c_2 \), then \( p_{1,2}^* = p_{2,2}^* = p_{1,1}^* = p_{2,2}^* = p_{2,1}^* \) and \( \bar{q} = 1 - \hat{q} = \bar{q} \) so that \( \pi_{1,1} = \pi_{1,2} = \pi_{2,1} = \pi_{2,2} \).

Suppose to change \( v_1 \) by \( dv_1 \) and \( c_1 \) by \( dc_1 \) with either \( dv_1 > dc_1 \geq 0 \) or \( dv_1 \geq 0 > dc_1 \). By the envelope theorem the effect of these changes on profits are

\[
d\pi_{1,2} \approx \frac{\partial \pi_{1,2}}{\partial v_1} dv_1 + \frac{\partial \pi_{1,2}}{\partial c_1} dc_1 = \frac{\partial p_{1,2}^*}{\partial v_1} dv_1 - \bar{q} dc_1
\]

\[
d\pi_{2,1} \approx \frac{\partial \pi_{2,1}}{\partial v_1} dv_1 + \frac{\partial \pi_{2,1}}{\partial c_1} dc_1 = \begin{bmatrix} \frac{1 + \eta}{1 + \eta \lambda} + \frac{1 - \bar{q}}{1 + \eta \lambda} \frac{\partial p_{2,1}^*}{\partial v_1} \end{bmatrix} dv_1 - q dc_1
\]

\[
d\pi_{1,1} \approx \frac{\partial \pi_{1,1}}{\partial v_1} dv_1 + \frac{\partial \pi_{1,1}}{\partial c_1} dc_1 = \hat{q} \left[ \frac{\partial p_{1,1}^*}{\partial v_1} dv_1 - dc_1 \right] + (1 - \hat{q}) \left[ \frac{1 + \eta}{1 + \eta \lambda} dv_1 - dc_1 \right]
\]
and

\[ d\pi_{2,2} = 0. \]

By substituting and re-arranging, we have that \( d\pi_{1,1} > d\pi_{2,1} \) since

\[
\left[ \tilde{q} + \frac{2(1-\tilde{q}) \eta (\lambda - 1) \tilde{q}}{1 + \eta (\lambda - 1) (1 - \tilde{q}) (1 + \eta \lambda)} + (1 - \tilde{q} - \bar{q}) \frac{1 + \eta}{1 + \eta \lambda} - \frac{2q \eta (\lambda - 1) (1 - \bar{q})}{1 + \eta (\lambda - 1) \bar{q}} \right] dv_1 > (1 - \bar{q}) dc_1
\]

\[ \Leftrightarrow dv_1 > dc_1 \]

where the last inequality follows from \( 1 - \bar{q} = \tilde{q} \). Similarly, \( d\pi_{1,1} > d\pi_{1,2} \) since

\[
\left[ \tilde{q} + \frac{2(1-\tilde{q}) \eta (\lambda - 1) \tilde{q}}{1 + \eta (\lambda - 1) (1 - \tilde{q}) (1 + \eta \lambda)} + (1 - \tilde{q} - \bar{q}) \frac{1 + \eta}{1 + \eta \lambda} - \frac{2q \eta (\lambda - 1) (1 - \bar{q})}{1 + \eta (\lambda - 1) (1 - \bar{q})} \right] dv_1 > (1 - \bar{q}) dc_1
\]

\[ \Leftrightarrow \left[ \frac{2q \eta (\lambda - 1) \tilde{q}}{1 + \eta (\lambda - 1) (1 - \tilde{q})} \right] \left( \frac{1 + \frac{1 + \eta}{1 + \eta \lambda}}{1 + \frac{1 + \eta}{1 + \eta \lambda}} \right) dv_1 > dc_1 \]

where the last inequality follows from \( \bar{q} = \tilde{q} \).

Finally, consider the case in which \( v_1 - c_1 \leq v_2 - c_2 \). Again, let’s start with \( v_1 = v_2 \) and \( c_1 = c_2 \) and suppose to change \( v_2 \) by \( dv_2 \) and \( c_2 \) by \( dc_2 \) with \( dc_2 \leq dv_2 < 0 \) so that \( v_1 - c_1 \leq v_2 - c_2 \). By the envelope theorem the effect of these changes on profits are

\[
d\pi_{1,2} \simeq \frac{\partial \pi_{1,2}}{\partial v_2} dv_2 + \frac{\partial \pi_{1,2}}{\partial c_2} dc_2 = \left[ \tilde{q} + \frac{1 + \eta}{1 + \eta \lambda} \right] dv_2 - (1 - \bar{q}) dc_2
\]

\[
d\pi_{2,1} = \frac{\partial \pi_{2,1}}{\partial v_2} dv_2 + \frac{\partial \pi_{2,1}}{\partial c_2} dc_2 = (1 - \tilde{q}) \frac{\partial p_{2,1}}{\partial v_2} dv_2 - (1 - \bar{q}) dc_2
\]

\[ d\pi_{1,1} = 0 \]

and

\[
d\pi_{2,2} \simeq \frac{\partial \pi_{2,2}}{\partial v_2} dv_2 + \frac{\partial \pi_{2,2}}{\partial c_2} dc_2 = \left[ \tilde{q} + \frac{1 + \eta}{1 + \eta \lambda} \right] dv_2 - dc_2.
\]

By substituting and re-arranging, we have that \( d\pi_{2,1} \geq d\pi_{1,2} \) since

\[
(1 - \bar{q}) (dv_2 - dc_2) \geq (1 - \bar{q}) \left\{ \left[ \frac{2q \eta (\lambda - 1) \tilde{q}}{1 + \eta (\lambda - 1) (1 - \tilde{q}) (1 + \eta \lambda)} \right] 
\left[ \frac{2q \eta (\lambda - 1) \tilde{q}}{1 + \eta (\lambda - 1) (1 - \tilde{q}) (1 + \eta \lambda)} \right] + \frac{1 + \eta}{1 + \eta \lambda} \right\} dv_2 - dc_2
\]

\[ \Leftrightarrow \frac{2q \eta (\lambda - 1) \tilde{q}}{1 + \eta (\lambda - 1) (1 - \tilde{q})} \left[ \frac{2q \eta (\lambda - 1) \tilde{q}}{1 + \eta (\lambda - 1) (1 - \tilde{q})} \right] + \frac{1 + \eta}{1 + \eta \lambda} \geq 1 \]

where the last inequality follows from \( \bar{q} = 1 - \bar{q} > \frac{1}{2} \) and \( 0 > dv_2 \geq dc_2 \).
Finally, we have that \( d\pi_{2,1} \geq d\pi_{2,2} \) if and only if

\[
(1 - \hat{q})(dv_2 - dc_2) \geq \hat{q} \left[ 1 + \frac{2(1 - \hat{q})\eta(\lambda - 1)}{1 + \eta(\lambda - 1)(1 - \hat{q})} \frac{1 + \eta}{1 + \eta \lambda} \right] dv_2 + (1 - \hat{q}) \frac{1 + \eta}{1 + \eta \lambda} dv_2 - dc_2
\]

\[
\Leftrightarrow 0 \geq \frac{1 + \eta}{1 + \eta \lambda} \left[ \frac{2\eta(\lambda - 1)\hat{q}}{1 + \eta(\lambda - 1)(1 - \hat{q})} + 1 \right] dv_2 - dc_2
\]

where the last inequality follows from \( \hat{q} = 1 - q \). Notice that, although \( dv_2 - dc_2 > 0 \), condition (25) might hold. Therefore, let \( \bar{v}_2 \) be the value of \( v_2 \) for which condition (25) binds. This completes the proof of the proposition. \( \blacksquare \)
C Partial Commitment

While retailers frequently advertise their good deals, it is rather uncommon to see a store publicizing its high prices. Therefore, consistently with this observation about stores’ advertising patterns, in this section I assume that in period 0 the seller commits only to the price of the bait $p_{i}^{\text{min}}$, $i = 1, 2$, and its degree of availability. In this case, consumers form rational expectations about the price of the item that is not publicly advertised.

Suppose that the seller uses item 1 as a bait by announcing that she has $q$ units of it available for sale at price $p_{1}^{\text{min}}$. Once at the store, a buyer who had planned to buy item 1 if available and item 2 otherwise will follow his plan and buy item 2 when this is the only item left in the store if

\[ U [(v_{2}, p_{2}) \mid \{1, 2\}] \geq U [(0, 0) \mid \{1, 2\}] \]

\[ \Leftrightarrow \quad p_{2} \leq \frac{(1 + \eta \lambda) v_{2} + \eta (\lambda - 1) q \left( \frac{1 + \eta}{1 + \eta \lambda} \right) v_{1}}{1 + \eta q + \eta (1 - q)}. \quad (26) \]

Notice that this price is higher than the one we found under full commitment because now the price of the rip-off is the highest price consumers are willing to pay ex-post. However, for the consumers to be willing to make the plan of always buying to begin with, the seller’s announced degree of availability for the bait must be such that

\[ EU [\{1, 2\} \mid \{1, 2\}] \geq EU [\{2, \varnothing\} \mid \{2, \varnothing\}]. \quad (27) \]

To have an optimum for the seller both conditions (26) and (27) have to bind, defining a system of two non-linear equations in $q$ and $p_{2}$. The relevant solution is

\[ p_{2}^{*} = \frac{v_{1} \left( 1 + \eta \right) \left( 1 + 2 \eta \right) + v_{2} \left( 1 + \eta \lambda \right) \left( 1 + \eta + \eta \lambda \right) - \sqrt{Y}}{2\eta \left( 1 + \eta \lambda \right)} \]

\[ q = \frac{v_{2} \lambda \left( 1 + \eta \lambda \right) - \frac{1 + \eta \lambda}{2\eta \left( 1 + \eta \lambda \right)} \left[ v_{1} \left( 1 + \eta \right) \left( 1 + 2 \eta \right) + v_{2} \left( 1 + \eta \lambda \right) \left( 1 + \eta + \eta \lambda \right) - \sqrt{Y} \right]}{v_{1} \left( 1 + \eta \right) \left( \lambda - 1 \right) + v_{2} \left( 1 + \eta \lambda \right) \left( \lambda - 1 \right)} \]

where

\[ Y \equiv v_{1}^{2} \left( 1 + \eta \right)^{2} \left( 2 \eta + 1 \right)^{2} + v_{2}^{2} \left( 1 - \eta + \eta \lambda \right)^{2} \left( 1 + \eta \lambda \right)^{2} - 2 v_{1} v_{2} \left( 1 + \eta \right) \left( 1 + \eta \lambda \right) \left( -\eta - 2 \eta^{2} - \lambda \eta + 2 \eta^{2} \lambda - 1 \right). \]

Similarly, if the seller uses item 2 as a bait, degree of availability of item 1 and its price are

\[ p_{1}^{*} = \frac{v_{1} \eta \left( \lambda - 1 \right) \left( 1 + \eta \lambda \right) + v_{2} \left( 1 + 2 \eta \right) \left( 2 + \eta + \lambda \eta \right) - \sqrt{Z}}{2\eta \left( 1 + \eta \lambda \right)} \]

\[ q = \frac{v_{2} \left( 2 \lambda - \eta - 2 \eta^{2} - \eta \lambda - 2 \eta^{2} \lambda + \eta \lambda^{2} - 2 \right) - v_{1} \left( \eta - \eta^{2} + \eta \lambda + \eta^{2} \lambda + 1 \right)}{v_{2} \left( \lambda - 1 \right) \left( \eta + \lambda \eta + 2 \right)} \]

\[ + \frac{1 + \eta \lambda}{2\eta \left( 1 + \eta \lambda \right)} \left[ v_{1} \eta \left( \lambda - 1 \right) \left( 1 + \eta \lambda \right) + v_{2} \left( 1 + 2 \eta \right) \left( 2 + \eta + \eta \lambda \right) - \sqrt{Z} \right]. \]
\[ Z \equiv v_1^2 \eta^2 (1 + \lambda)^2 (1 + \eta \lambda)^2 + v_2^2 (1 + 2\eta)^2 (2 + \eta + \lambda \eta)^2 - 2\eta v_1 v_2 (-\lambda + 2\eta + 2\eta \lambda + 3) (1 + \eta \lambda) (2 + \eta + \eta \lambda) . \]

Compared to the situation where she is able to commit in advance to both prices, now the price of the rip-off is higher but the degree of availability of the bargain is higher as well. Intuitively, since the seller is charging a higher price for the rip-off, and the consumers anticipate this, she must compensate them with a higher ex-ante chance of making a deal otherwise they would not plan to always buy. Thus, given both prices, the seller is not choosing the degree of availability that maximizes her profits. This is because by not committing in advance to the price of the rip-off, the seller must use the degree of availability of the bait to induce the consumers to select the to plan to always buy. Furthermore, the optimal degree of availability with full commitment takes into account also the difference in the marginal costs of the two items, whereas with partial commitment it does not. Therefore, the seller’s profits are lower when she cannot commit to both prices.

Unfortunately, in this case it is hard to obtain a full characterization, like the one in proposition 1, for when the seller would find it profitable to use a limited-availability strategy made of bargains and rip-offs. Nevertheless, a combination of bargains and rip-offs might still be profit-maximizing as the following example shows.

**Example 7** Let \( v_1 = 250, v_2 = 230, c_1 = 20 \) and \( c_2 = 10 \). If the seller produces only one good, then she would produce item 1 and price it at \( p_1 = 250 \), obtaining a profit of 230. Let \( \eta = 1 \) and \( \lambda = 2 \) and suppose the seller uses item 1 as a bait by pricing it at \( p_1^{\min} = \frac{500}{3} \). In this case the seller will also commit to sell \( q = \frac{2}{119} \sqrt{3459} - \frac{75}{119} \) units of item 1 and will price item 2 at \( p_2^* = 710 - \frac{20}{3} \sqrt{3459} \), obtaining a profit of 250.15.

Moreover, example 7 shows that also in this case of partial commitment the seller might prefer to use the superior item as the bait, exactly for the same reason as in the analysis with full commitment.\(^{39}\)

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\(^{39}\)For the parameters in example 7, if the seller were to use item 2 as the bait by pricing it at \( p_2^{\min} = \frac{460}{7} \), then the optimal degree of availability of the bait would be \( 1 - \theta = \frac{1}{7} \sqrt{489} - \frac{12}{7} \) and the price of item 1 would be \( p_1^* = 700 - \frac{50}{3} \sqrt{489} \) for a total profit of 237.52. Less than what the seller can obtain by using item 1 as the bait, but still better than what she would make by selling only item 1.
D Search and Shopping Costs

In this section I show that the combination of bargains and rip-offs described in proposition 1 cannot be rationalized by introducing search or shopping costs into a monopoly model where consumers have classically assumed preferences.

Consider a monopolist (she) selling two goods, 1 and 2 and for simplicity suppose the monopolist’s cost of production is 0 for both goods (this does not matter for the point I am making here).

There is a unit mass of risk-neutral consumers that value the two goods $v_1$ and $v_2$, with $v_1 \geq v_2$. The goods are substitutes and consumers demand at most one unit of either good.

The consumers do not know the prices charged by the monopolist unless they visit the store. Visiting the store entails a cost $t > 0$.

In this very simple set-up the consumers would never go to the store. Indeed, once they are at the store they will buy item $i$ if and only if $p_i \leq v_i$, for $i = 1, 2$. The monopolist knows this and indeed she would charge $p_i^* = v_i$ for both items.

The consumers, however, have rational expectations about the prices and fully anticipating this know that their overall utility if they go to the store and buy item $i$ will be $v_i - p_i^* - t = -t < 0$ for $i = 1, 2$. Therefore, they optimally decide to stay home.

The seller then would find it profitable to advertise (and hence commit to) a price strategy that does not exploit the fact that consumers’ search (or shopping) cost is sunk once they arrive at the store.

Could this strategy be a limited-availability one like in proposition 1?

Suppose the seller announces the following lottery: with probability $q$ the consumers will have the opportunity to buy item 1 at $p_1$ and with probability $1 - q$ they will have the opportunity to buy item 2 at $p_2$.

For this announcement to be effective in inducing consumers to go to the store, the following must hold:

$$q (v_1 - p_1) + (1 - q) (v_2 - p_2) - t \geq 0.$$

Of course, again, once a consumer is in the store, he will buy item $i$ if and only if $p_i \leq v_i$, for $i = 1, 2$.

Suppose that $p_2 = v_2$ (resp. $p_1 = v_1$). Then the highest price the store can charge for item 1 (resp. 2) is $v_1 - \frac{t}{q}$ (resp. $v_2 - \frac{t}{1-q}$); the seller profit would be equal to

$$q \left( v_1 - \frac{t}{q} \right) + (1 - q) v_2 = q v_1 + (1 - q) v_2 - t.$$

It is easy to see that the seller would be exactly indifferent between selling only item 1 at $p_1 = v_1 - t$ and offering a lottery consisting of selling item 1 at $p_1 = v_1 - \frac{t}{q}$ with probability $q$ and item 2 at $p_2 = v_2$ with probability $(1 - q)$ if and only if $v_1 = v_2$. If instead $v_1 > v_2$, the seller will strictly prefer to announce $p_1 = v_1 - t$ and sell only item 1.

Therefore, in this simple environment, limited-availability deals cannot be rationalized with search or shopping costs.
References


