The Zoom strategy for accelerating and warm-starting interior methods

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Abstract

Interior methods using iterative solvers for each search direction can require drastically increasing work per iteration as higher accuracy is sought.

The Zoom strategy solves first to low accuracy, and then solves for a correction to both primal and dual variables, again to low accuracy. We “zoom in” on the correction by scaling it up, thus permitting a cold start for the correction.

The same strategy applies to warm-starting in general.
Outline

1 Background on IPMs
2 PDCO
3 Zoom
4 Conclusions and Next Steps
The Problems

**CP**

minimize \( \phi(x) \)

subject to \( c_i(x) \geq 0, i = 1, \ldots, m \)

\( \phi(x), c_i \in C^2, \) convex

**NP**

minimize \( \phi(x) \)

subject to \( Ax = b, \ell \leq x \leq u \)

\( \phi(x) \) convex, separable

**LP-Primal**

minimize \( c^T x \)

subject to \( Ax = b, x \geq 0 \)

\( A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m, c \in \mathbb{R}^n \)
Basic Elements (cont.)

Duality

| LP-Dual | maximize \( b^T y \) \\
|---------|----------------|
|         | \( y \in \mathbb{R}^m \) \\
|         | subject to \( A^T y \leq c \).

Weak duality:

\[ c^T x \geq b^T y \]

Strong duality:

\[ c^T x = b^T y \iff x \text{ and } y \text{ are optimal} \]
Basic Elements (cont.)

KKT

\( x^* \) is a KKT point for (CP) if there exists an \( m \)-vector \( \lambda^* \), such that

\[(i) \quad c_i(x^*) \geq 0 \text{ (feasibility)} \]
\[(ii) \quad g(x^*) = J(x^*)^T \lambda^* \text{ (optimality)} \]
\[(iii) \quad \lambda^* \geq 0 \]
\[(iv) \quad c_i(x^*) \lambda_i^* = 0, \; i = 1, \ldots, m \text{ (complementarity)} \]

- No duality gap \( \rightarrow \) Lagrange multipliers are the dual solutions
- Non-zero duality gap \( \rightarrow \) no Lagrange multipliers exist
Basic Elements (cont.)

Constraint Qualifications

Constraint qualifications ensure linear approximation at a point captures the essential geometric information of the true feasible set in a neighborhood.

CQs are therefore required to ensure a KKT point is a local minimizer.

- **LICQ**: linear problem or $J_A(\bar{x})$ has full row rank
- **MFCQ**: $c_i(\bar{x}) > 0, \forall i$ or there exists $p; J_A(\bar{x})p > 0$
- **SLCQ**: $c_i(\bar{x}) > 0, \forall i$
Necessary Optimality Conditions

First order
If \( x^* \) is a local minimizer of (CP) at which the MFCQ holds then \( x^* \) must be a KKT point

Second order
If \( x^* \) is a local constrained minimizer of (CP) at which the LICQ holds. Then there exists \( \lambda^*; \lambda^* \geq 0, \ c^T \lambda^* = 0, \ g^* = J^T \lambda^* \), and

\[
p^T H(x^*, \lambda^*) p \geq 0 \text{ for all } p \text{ satisfying } J_A p = 0
\]
Basic Elements (cont.)

Sufficient Optimality Conditions

Sufficient conditions for an isolated constrained minimizer to (CP)

(i) $x^*$ is a KKT point, i.e. $c^* \geq 0$ and there exists a nonempty set $M_\lambda$ of multipliers $\lambda$ satisfying $\lambda \geq 0$, $c^T \lambda = 0$, and $g^* = J^T p$;
(ii) the MFCQ holds at $x^*$
(iii) for all $\lambda \in M_\lambda$ and all nonzero $p$ satisfying $g^T p = 0$ and $J^*_A p \geq 0$, there exists $\omega > 0$ such that $p^T H(x^*, \lambda)p \geq \omega \|p\|^2$
IPM strategies

Strategy categories

- **Algorithm type**: affine-scaling, potential-reduction, and path-following
- **Iterate space**: primal, dual, primal-dual
- **Iterate type**: feasible and infeasible (equality constraints)
- **Step type**: short-step, long-step
Affine Scaling

LP'
\[
\begin{align*}
\text{minimize} & \quad c^T x \\
\text{subject to} & \quad Ax = b, \quad \|X^{-1} x - e\| \leq 1,
\end{align*}
\]

Algorithm scheme is \(x^{k+1} = x^k + \alpha \Delta x^k\) \((0 < \alpha \leq 1)\), where

\[
\Delta x^k = -\frac{X^k P_{AX^k} X^k c}{\|P_{AX^k} X^k c\|},
\]

and \(P_{AX^k} = I - (AX^k)(AX^k(AX^k)^T)^{-1}(AX^k)^T\) is the projection matrix onto the null space of \(AX^k\).

Convergence proof exists when \(\alpha = 1/8\), but no complexity results
Potential Reduction

\[ \text{minimize} \quad P(x, z) = q \log(x^Tz) + I(x, z) \]
\[ \text{subject to} \quad Ax = b \]
\[ x > 0, \]
\[ x^T y + z = c \]
\[ z \geq 0, \]

\( q \) is a potential function parameter (Karmarkar used \( q = m \))
\( I(x) \) is usually logarithmic interior function or Tanabe-Todd-Ye:

\[ I(x) = - \sum_{i=1}^{m} \ln(x_j z_j) \]

Assuming \( \min \delta \) reduction per iteration yields \( \max N \) iterations:

\[ N = \frac{q}{\delta} \left( \log\left( \frac{x^T z}{\epsilon} \right) + C_2 \right) \]
Path-following

\[ \text{LP} (\mu) \begin{array}{l}
\text{minimize} \\
\quad x \quad B (x, \mu) = \phi (x) + \mu I (x)
\end{array} \]
subject to \[ c_i (x) \geq 0, \]

where \( I (x) \) satisfies:

- \( I (x) \) depends only on \( x \geq 0 \)
- \( I (x) \) preserves continuity
- For any feasible sequence converging to boundary, \( I (x) \to \infty \)

Two frequently used interior functions:

\[ I (x) = \sum_{i=1}^{m} \frac{1}{x_j}, \quad I (x) = - \sum_{i=1}^{m} \ln (x_j) \]
Formulating the Primal-Dual Equations

\[ F^\mu(x, y) = \begin{pmatrix} g(x) - J(x)^T y \\ C(x)y - \mu e \end{pmatrix} = 0 \]

Newton step:

\[ F^\mu(x, y)'(\Delta x, \Delta y) = -F^\mu(x, y) \]

Collecting terms on the right-hand side:

\[
\begin{pmatrix}
H(x, y) & -J(x)^T \\
Y J(x) & C(x)
\end{pmatrix}
\begin{pmatrix}
\Delta x \\
\Delta y
\end{pmatrix}
= -
\begin{pmatrix}
g(x) - J(x)^T y \\
C(x)(y - \pi(x, \mu))
\end{pmatrix},
\]

\[ \pi_i = \mu/c_i(x) \text{ for } i = 1, \ldots, m, \text{ and we assume } C \succ 0, Y \succ 0 \]
One option: Pre-conditioned CG on P-D Newton Equations

Block elimination allows for $LL^T$ Cholesky factorization to

$$(H(x, y) + J(x)^T Y^{-1} C(x) J(x)) \Delta x = -(g(x) - J(x)^T \pi(x, \mu))$$

Pre-multiplying by $Y^{-1/2}$ allows for $PMP^T = LDL^T$ Cholesky factorization to

$$
\begin{pmatrix}
H(x, y) & J(x)^T Y^{1/2} \\
Y^{1/2} J(x) & -C(x)
\end{pmatrix}
\begin{pmatrix}
\Delta x \\
Y^{-1/2} \Delta y
\end{pmatrix}
= -
\begin{pmatrix}
g(x) - y J(x)^T \\
Y^{-1/2} C(x)(y - \pi)
\end{pmatrix}
$$
Line Search

**Exact**: find argmin of merit function along Newton step direction

**Inexact**: find step size reducing merit function by a threshold

**Backtracking** is most popular inexact line search:

Given direction $\Delta x$ and $\alpha \in (0, 0.5)$, $\beta \in (0, 1)$, $t^0 = 1$, $t^{k+1} = \beta t^k$

while

$$f(x + t\Delta x) > f(x) + \alpha t \nabla f(x)^T \Delta x$$

Consider residual based on actual newton step $p_k$

$$r^k = \nabla^2 f(x^k)p_k + \nabla f(x^k)$$

Inexact methods are locally convergent if

$$\|r^k\|/\|\nabla f(x^k)\| \leq \eta_k$$

and $\{\eta_k\}$ is uniformly less than 1
Handling $\mu$

- Too large $\mu$ yields small step sizes (slow)
- Too small $\mu$ yields steps far from central path

Adaptive updates are typically used:

$$\mu^k = \sigma^k \frac{(x^k)^T z}{n}$$

$n$ is the dimension of $x$ and $\sigma^k \in (0, 1)$, since iterates are assumed not to be on central path
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Nominal problem:

\[
\begin{align*}
\text{NP} & \quad \text{minimize} \quad \phi(x) \\
\text{subject to} & \quad Ax = b, \quad \ell \leq x \leq u
\end{align*}
\]

\[\phi(x) \text{ convex, separable}\]

Regularized problem:

\[
\begin{align*}
\text{NP}(D_1, D_2) & \quad \text{minimize} \quad \phi(x) + \frac{1}{2}\|D_1 x\|^2 + \frac{1}{2}\|r\|^2 \\
\text{subject to} & \quad Ax + D_2 r = b, \quad \ell \leq x \leq u
\end{align*}
\]
Introduce slack variables and replace nonnegativity constraints by the log barrier:

\[
\begin{align*}
\text{NP}(\mu) \quad &\text{minimize} & \quad \phi(x) + \frac{1}{2} \| D_1 x \|^2 + \frac{1}{2} \| r \|^2 - \mu \sum_j \ln([x_1]_j [x_2]_j) \\
\text{subject to} & \quad A x + D_2 r = b & : y \\
& \quad x - x_1 = \ell & : z_1 \\
& \quad -x - x_2 = -u, & : z_2
\end{align*}
\]
Eliminate \( r = D_2 y \) and apply Newton’s method:

\[
\begin{pmatrix}
\Delta x - \Delta x_1 \\
-\Delta x - \Delta x_2 \\
X_1 \Delta z_1 + Z_1 \Delta x_1 \\
X_2 \Delta z_2 + Z_2 \Delta x_2
\end{pmatrix}
= \begin{pmatrix}
(\Delta x - \Delta x_1) \\
(\Delta x - \Delta x_2) \\
X_1 \Delta z_1 + Z_1 \Delta x_1 \\
X_2 \Delta z_2 + Z_2 \Delta x_2
\end{pmatrix}
= \begin{pmatrix}
\ell - x + x_1 \\
-\ell + x + x_2 \\
\mu e - X_1 z_1 \\
\mu e - X_2 z_2
\end{pmatrix},
\]

where \( H_1 = H + D_2^2 \).
Substitute the first 2 equations into the third:

\[
\begin{pmatrix}
-H_2 & A^T \\
A & D_2^2
\end{pmatrix}
\begin{pmatrix}
\Delta x \\
\Delta y
\end{pmatrix}
= \begin{pmatrix}
w \\
r_1
\end{pmatrix},
\]

where

\[
H_2 \equiv H + D_1^2 + X_1^{-1}Z_1 + X_2^{-1}Z_2
\]
\[
w \equiv r_2 - X_1^{-1}(c_\ell + Z_1 r_\ell) + X_2^{-1}(c_u + Z_2 r_u)
\]
PDCO search directions

3 methods for computing $\Delta y$:

- Cholesky on $(AD^2A^T + D_2^2I)\Delta y = AD^2w + D_2r_1$

- Sparse QR on $\min \left\| \begin{pmatrix} DA^T \\ D_2I \end{pmatrix} \Delta y - \begin{pmatrix} Dw \\ r_1 \end{pmatrix} \right\|$

- LSQR on same LS problem (iterative solver)

Must use LSQR when $A$ is an operator
Scaling inside PDCO

PDCO allows inputs $\beta$ and $\zeta$ to scale problem data

Guiding principle:

\[
\beta = \text{input estimate of } \|x\|_{\infty} \\
\zeta = \text{input estimate of } \|z\|_{\infty}
\]

Typical choices: $\beta = \|b\|_{\infty}$ and $\zeta = \|\tilde{c}\|_{\infty} = \|\beta c\|_{\infty}$

Final scaling becomes:

\[
\bar{A} = A, \quad \bar{b} = b/\beta, \quad \bar{c} = \beta c/\zeta, \\
\bar{l} = l/\beta, \quad \bar{u} = u/\beta, \quad \bar{x} = x/\beta, \\
\bar{y} = \beta y/\zeta, \quad \bar{z}_1 = \beta z_1/\zeta, \quad \bar{z}_2 = \beta z_2/\zeta, \\
\bar{D}_1 = \beta D_1/\sqrt{\zeta}, \quad \bar{D}_2 = \sqrt{\zeta} D_2/\beta, \quad \bar{r} = r/\sqrt{\zeta}.
\]
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Motivation

The problem that started it all

Image reconstruction

Image Reconstruction

\[
\begin{align*}
\min & \quad \lambda e^T x + \frac{1}{2} \| r \|^2 \\
\text{st} & \quad A x + r = b, \quad x \geq 0
\end{align*}
\]

NNLS: Non-negative least squares \( \lambda = 10^{-4} \)

\( A \) is an expensive operator \( 2\)-D DFT

\( 65K \times 65K \)

PDCO uses LSQR for each dual search direction \( \Delta y \):

\[
\min \left\| \begin{pmatrix} DA^T \\ I \end{pmatrix} \Delta y - \begin{pmatrix} Dw \\ r_1 \end{pmatrix} \right\|
\]
Motivation

**LSQR iterations increase exponentially with requested accuracy**

The graph shows the relationship between the iteration of the interior method PDCO and the corresponding LSQR iterations. It indicates that:

- 3 digits are achieved at iteration 11.
- 6 digits are achieved at iteration 22.

The graph highlights the exponential increase in LSQR iterations with increasing requested accuracy.
**Zoom strategy: Accelerating IPMs**

- Solve to 3 digits: cheap approximation to $x, y, z$
- Define new problem for correction $dx, dy, dz$
- **Zoom in** (scale up correction)
- Solve to 3 digits: cheap approximation to $dx, dy, dz$

---

**Cold start** for both solves
Regularized LP:

\[
\begin{align*}
\text{minimize} \quad & c^T x + \frac{1}{2} \|D_1 x\|^2 + d^T r + \frac{1}{2} \|r\|^2 + c_1^T x_1 + c_2^T x_2 \\
\text{subject to} \quad & A x + D_2 r = b \\
& x - x_1 = \ell \\
& -x - x_2 = -u \\
& x_1, x_2 \geq 0
\end{align*}
\]

Suppose \((\tilde{x}, \tilde{y}, \tilde{z}_1, \tilde{z}_2, \tilde{x}_1, \tilde{x}_2, \tilde{r})\) is an approximate solution

Redefine problem with

\[
\begin{align*}
x &= \tilde{x} + dx \\
r &= \tilde{r} + dr
\end{align*}
\]
Zoom theory (cont.)

\[
\begin{align*}
\text{minimize} & \quad c^Tdx + \frac{1}{2}\|D_1dx\|^2 + \cdots \\
\text{subject to} & \quad Adx + D_2dr = \tilde{b} : y \\
& \quad dx - x_1 = \tilde{\ell} : z_1 \\
& \quad -dx - x_2 = -\tilde{u} : z_2 \\
& \quad x_1, x_2 \geq 0
\end{align*}
\]

where

\[
\begin{align*}
\tilde{b} &= b - A\tilde{x} - \delta\tilde{r} \\
\tilde{\ell} &= \ell - \tilde{x} \\
\tilde{u} &= u - \tilde{x}
\end{align*}
\]
Zoom theory (cont.)

Add Lagrangian terms

\[ \tilde{y}^T (\tilde{b} - A dx - D_2 dr) \quad \tilde{z}_1^T (\tilde{\ell} - dx + x_1) \quad \tilde{z}_2^T (-\tilde{u} + dx + x_2) \]

to objective:

\[
\begin{align*}
\text{minimize} \quad & c^T dx + \frac{1}{2} \| D_1 dx \|^2 + d^T dr + \frac{1}{2} \| dr \|^2 + c_1^T x_1 + c_2^T x_2 \\
\text{subject to} \quad & A dx + D_2 dr = \tilde{b} \\
& dx - x_1 = \tilde{\ell} \\
& -dx - x_2 = -\tilde{u} \\
& x_1, x_2 \geq 0
\end{align*}
\]

Same form as original RLP
Primal and dual variables are small
Hence, scale up and use cold start
Revisions to PDCO

Define $\tilde{z}_1 = z_1 + c_1, \tilde{z}_1 = z_1 + c_1$

Eliminate $r = D_2y - d$ and apply Newton’s method:

\[
\begin{pmatrix}
\Delta x - \Delta x_1 \\
-\Delta x - \Delta x_2
\end{pmatrix} =
\begin{pmatrix}
r_{\ell} \\
r_u
\end{pmatrix}
\equiv
\begin{pmatrix}
\ell - x + x_1 \\
-u + x + x_2
\end{pmatrix},
\]

\[
\begin{pmatrix}
X_1\Delta z_1 + \bar{Z}_1\Delta x_1 \\
X_2\Delta z_2 + \bar{Z}_2\Delta x_2
\end{pmatrix} =
\begin{pmatrix}
c_{\ell} \\
c_u
\end{pmatrix}
\equiv
\begin{pmatrix}
\mu e - X_1\bar{z} \\
\mu e - X_2\bar{z}_2
\end{pmatrix},
\]

\[
\begin{pmatrix}
A\Delta x + D_2^2\Delta y \\
-H_1\Delta x + A^T\Delta y + \Delta z_1 - \Delta z_2
\end{pmatrix} =
\begin{pmatrix}
r_1 \\
r_2
\end{pmatrix}
\equiv
\begin{pmatrix}
b - Ax - D_2^2y + D_2d \\
g + D_1^2x - A^Ty - z_1 + z_2
\end{pmatrix},
\]

where $H_1 = H + D_1^2$
Revisions to PDCO (cont.)

Substitute the first 2 equations into the third:

$$
\begin{pmatrix}
-H_2 & A^T \\
A & D_2^2
\end{pmatrix}
\begin{pmatrix}
\Delta x \\
\Delta y
\end{pmatrix}
= 
\begin{pmatrix}
w \\
r_1
\end{pmatrix}
$$

as before, where

$$
H_2 \equiv H + D_1^2 + X_1^{-1} \bar{Z}_1 + X_2^{-1} \bar{Z}_2
$$

$$
w \equiv r_2 - X_1^{-1}(c_\ell + \bar{Z}_1 r_\ell) + X_2^{-1}(c_u + \bar{Z}_2 r_u)
$$

Scaling for additional terms:

$$
\bar{c}_1 = \beta c_1 / \zeta, \quad \bar{c}_2 = \beta c_2 / \zeta, \\
\bar{d} = d / \sqrt{\zeta}, \quad \bar{\kappa} = \kappa / \zeta
$$
Scaling Outside PDCO

A well-scaled $A$ improves numerical properties inside PDCO.

Find diagonal matrices $R, C$ such that $\hat{A} = R^{-1}AC^{-1}$ has entries close to 1.

Adjust other terms for problem consistency:

\[
\hat{A} = R^{-1}AC^{-1}, \quad \hat{b} = R^{-1}b, \quad \hat{c} = C^{-1}c, \\
\hat{c}_1 = C^{-1}c_1, \quad \hat{c}_2 = C^{-1}c_2, \quad \hat{l} = Cl, \\
\hat{u} = Cu, \quad \hat{x} = Cx, \quad \hat{y} = Ry, \\
\hat{z}_1 = C^{-1}z_1, \quad \hat{z}_2 = C^{-1}z_2, \quad \hat{D}_1 = C^{-1}D_1, \\
\hat{D}_2 = R^{-1}D_2,
\]

while $d, r, \kappa$ remain unchanged.

**Zooming** is the choice of appropriate $\beta, \zeta$ for PDCO.
Convergence

**Theorem:**
Using a strictly self-concordant barrier, if the sublevel set \( S = \{ x; f(x) \leq f(x^0) \} \) is closed and \( f \) is bounded below, then there exists \( \eta, \gamma > 0 \), with \( 0 < \eta \leq 1/4 \), dependent only on the line search parameters such that:

- If \( \lambda(x^k) > \eta \), then \( f(x^{k+1}) - f(x^k) \leq -\gamma \)
- If \( \lambda(x^k) \leq \eta \), then the line search selects \( t = 1 \) and

\[
2\lambda(x^{k+1}) \leq (2\lambda(x^k))^2.
\]

In the damped phase the objective decreases monotonically by at least \( \gamma \) every iteration, so convergence is guaranteed (with the assumption about \( f \) on the sublevel set \( S \) being bounded below). Convergence is quadratic in the pure Newton phase.
Complexity

Lemma (outer iterations):
Given $\mu^0$ and updates of $\mu^{k+1} = \sigma \mu^k$, with $0 < \sigma < 1$, then after at most

$$\frac{1}{1 - \sigma} \log \left( \frac{n\mu^0}{\varepsilon} \right)$$

iterations we have $n\mu \leq \varepsilon$

Lemma (inner iterations):
For given $\sigma, 0 < \sigma < 1$, the number of iterations between $\mu$ updates is not larger than

$$2 \left( 1 + \sqrt{\frac{(1 - \sigma) \sqrt{n}}{\sigma}} \right)^4$$
Complexity (cont.)

Theorem:

Upper bound for total iterations of 2-phase zoom and refine technique:

\[ O \left( n \log \left( \frac{n^2 \mu_1^0 \mu_2^0}{\varepsilon} \right) \right), \]

where \( \mu_1^0 \) and \( \mu_2^0 \) are the starting duality gap values for phase 1 and phase 2, respectively.
A Closer Look at the Inner Workings

The typical Newton system for an IPM applied to an LP is of the form

\[
\begin{pmatrix}
-X^{-1}Z & A^T \\
A \\
\end{pmatrix}
\begin{pmatrix}
\Delta x \\
\Delta y \\
\end{pmatrix}
= \begin{pmatrix}
r_4 \\
r_1 \\
\end{pmatrix}
\]

Wright shows that in the degenerate case,

\[
\text{cond}(M) \approx \frac{1}{\mu}
\]

for iterates near the central path and \( \mu \) sufficiently small.
At the end of the first stage with a target precision of $10^{-4}$ we have

$$\text{cond}(M_1) \approx \frac{1}{\mu} \approx 10^4,$$

where $M_1$ represents the Jacobian at the intermediate solution.

At the next stage, by design $1 \sim \mu_2 >> \mu_1$ and hence

$$\text{cond}(M_2) \sim 1,$$

so

$$\text{cond}(M_2) << \text{cond}(M_1),$$

where $M_2$ represents the Jacobian at the starting point for the scaled new problem (at the start of second stage).
Results: Accelerating IPMs
LSQR iterations inside PDCO

Observed image

Approx. sol. (3 digits)

382 itns.

17273 itns.

11612 itns.

Final sol. (6 digits)

True image

The Zoom strategy – p. 42/4
**Netlib Results: Accelerating IPMs**

**LSQR iterations inside PDCO**

*Zoom* strategy applied to Netlib LP problems:

<table>
<thead>
<tr>
<th>criteria (based on single solve)</th>
<th># problems</th>
<th>avg Zoom benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td># LSQR itns &lt; 5,000</td>
<td>16</td>
<td>-1.49%</td>
</tr>
<tr>
<td>5,000 ≤ # LSQR itns &lt; 15,000</td>
<td>12</td>
<td>1.37%</td>
</tr>
<tr>
<td>15,000 ≤ # LSQR itns</td>
<td>12</td>
<td>28.14%</td>
</tr>
</tbody>
</table>
Motivation

Solving several related problems is common in industry

Reusing prior solutions in IPMs (warmstarting) is usually not efficient

- Solutions are close to each other but proximity to central path varies drastically between problems
- Newton steps end up being greatly shortened and backtracking often occurs
- Coldstarts typically take less time than warmstarting

Much renewed interest in research: Benson & Shanno (2005), Gondzio & Grothey (2006), Roos (2006)
**Zoom strategy:** Warm-starting IPMs

- Set solution to original LP as current approximation
- Define new problem for correction $dx, dy, dz$
- **Zoom in** (scale up correction)
- Solve loosely: cheap approximation to $dx, dy, dz$

**Cold start** for loose solve
Warm-starting IPMs

\[
\begin{align*}
\text{LP}(\gamma, \delta) & \quad \text{minimize} \quad c^T x + \frac{1}{2} \|\gamma x\|^2 + \frac{1}{2} \|r\|^2 \\
& \quad \text{subject to} \quad Ax + \delta r = b, \quad \ell \leq x \leq u
\end{align*}
\]

Regularized LP \( \gamma = \delta = 10^{-3} \)
PDCO with Cholesky on \( AD^2A^T + \delta^2 I \)

- LPnetlib problems with 5 random perturbations to \( A, b, \) or \( c \)
  (cf. Benson and Shanno 2005)
- Smaller problems (< 100KB): 45 runs for each problem
- Compare Zoom to single solve
Results: Warm-starting IPMs

PDCO iterations (warm/cold) vs. perturbation to $x, y$
Next steps

• Multiple Zooms?
• Adaptive Zooms?
• How much is attributable to Zoom, to scaling?
• Explain outliers
  (e.g. Check size of residuals to decide Zoom scaling)

Conclusions

• Minor changes to existing primal-dual algorithms
• Zoom time reduced 30–60%