Bursting Neuron

Bursting (thalamic reticular neuron modified from Steriade et al. 2003) due to M-current when neuron is bistable.

Interspike intervals \( T_i \) get longer with time

Frequency \( \frac{1}{T_i} \) drops accordingly—the cell adapts

Bistability maintains spiking—until M-current terminates it

Membrane-voltage equation

After a spike \( t_{spk} \) the neuron is reset to \( V_{reset}, (>0) \) which is at a higher potential than the unstable point, \( V_{unstable} \).
Once the neuron spikes it continues to spike until it is pulled below $V_{\text{unstable}}$; it expresses hysteresis.

Bistable neuron model

\[
\tau_m \frac{dx}{dt} = x - \left( 1 + \frac{g_K}{g_L} \right) x + \frac{1}{3} x^3
\]

$x(t_{\text{spk}}) \rightarrow x_{\text{reset}} > x_{\text{unstable}}$
$g_K$ adapts the spike frequency

Interspike intervals lengthen (left) and sensitivity decreases (right)

We previously computed $g_K(t_n)$. What happens if $g_K(t_n)$ (or $g_K(t_{n-1})$) becomes large enough to pull $x$ below the unstable equilibrium, pushing $\frac{dx}{dt} < 0$?

Steady-state value ($g_K(t_{\infty})$)

We find $g_K$ necessary to terminate the burst by $g_{RT}$ setting $\frac{dx}{dt} = 0$ for $x_{\text{reset}} = x_{\text{unstable}}$ and solving for $g_K$:

$$
\tau_n \frac{dx}{dt} = r - \left(1 + \frac{g_{RT}}{g_L}\right) x_{\text{reset}} + \frac{1}{3} x_{\text{reset}}^3 = 0
$$

$$
g_{RT} = \frac{r + \frac{1}{3} x_{\text{reset}}^3}{x_{\text{reset}}} - 1
$$

if $g_K(t_n) > g_{RT}$ then the burst ends, otherwise the spike rate slows but does not burst

Bursting termination
Dynamics of bursting
Bursting Neuron

Bursting (Aplysia abdominal ganglion $R_{15}$ neuron) due to slow inward current

Interspike intervals ($T_i$) decrease and then increase (parabolic)

Interburst intervals ...

Spikes per burst...

Membrane-voltage equation

Slow voltage-dependent, high-threshold $\text{Ca}^{2+}$ current ($I_{\text{Ca}}$) added

Like the $\text{K}^+$ channels, these $\text{Ca}^{2+}$ channels open only during a spike:

$$C_m \frac{dV_m}{dt} + g_L V_m + g_K V_m = I_{\text{in}} + I_{\text{Ca}} + \frac{1}{3} \left( \frac{V_m}{V_{\text{th}}} \right)^2 g_L V_m \quad \text{where} \quad I_{\text{Ca}} = f \Delta Q_{\text{Ca}}$$
Here $\Delta Q_{Ca}$ is to the charge carried by the brief pulse of $Ca^{2+}$-current each spike evokes and $f$ is the frequency of spikes.

Effect of $Ca^{2+}$ current

Ca current lowers input current required for a given frequency.

The $f(r)$ curve shifts to the left by

$$r_{Ca} = \frac{f \Delta Q_{Ca}}{g_L v_{th}}$$

at the frequency $f$. This leads to a negative slope (dashed line) where

$$\frac{dr_{Ca}}{df} > \frac{1}{f'[r]} \iff f'[r] < \frac{g_L v_{th}}{\Delta Q_{Ca}}$$

In this region, the current required to sustain firing is lower than that required to start it (dotted lines).
Phase plot

No Ca

Unstable point

Bursting behavior

Bursting

Adding a Ca$^{2+}$ current leads to bursting