Bursting Neuron

Bursting in Aplysia (left) and in thalamic reticular neuron (right)

Requires two stable states: Rest and spiking.

And mechanism(s) to switch between states.

Model simulation

Adding a \( \text{Ca}^{2+} \) current converts the adapting neuron into a bursting neuron
Membrane-voltage equation

Slow voltage-dependent, high-threshold Ca\(^{2+}\) current (I\(_{Ca}\)) added

Like the K-channels, these Ca-channels open only during a spike:

\[
\frac{dV_m}{dt} = g_{lk} V_m + g_K V_m = I_{in} + I_{Ca} + \frac{1}{3} \left( \frac{V_m}{V_{th}} \right)^2 g_{lk} V_m
\]

where \(I_{Ca} = \Delta I_{Ca} \tau_{Ca} f\)

Here \(\Delta I_{Ca}\) is the increase in Ca-current each spike evokes, \(\tau_{Ca}\) is the time-constant with which the Ca-current decays, and \(f\) is the frequency of spikes.

Effect of Ca-current

Ca-current lowers input current required to spike at a given frequency

The Ca-current shifts the \(f(r)\)-curve to the left by a different amount \(r_{Ca}(f)\) for each frequency:
\[ r_{Ca}[f] = \Delta r_{Ca} \cdot r_{Ca} \cdot f \] where \[ \Delta r_{Ca} = \frac{\Delta I_{Ca}}{g_{lk} \cdot v_{th}} \]

That is, the current required to sustain firing is lower than that required to start it!

**Ca-current produces bistability**

Three distinct spike-rates are possible for this input current \( r_{in} \).

The middle fixed-point (intermediate spike-rate) is unstable. To see this, consider a slight perturbation in spike-rate \( \Delta f \). This will increase the Ca-current by

\[ \Delta r = \frac{dr_{Ca}}{df} \Delta f \]

This additional current will in turn boost the spike-rate by

\[ \Delta f' = \frac{df}{dr} \Delta r = \frac{df}{dr} \cdot \frac{dr_{Ca}}{df} \Delta f \]

Thus, the perturbation will grow if

\[ \frac{dr_{Ca}}{df} > \frac{dr}{df} \]

which is true in the negative-slope region.
M-current \( (g_{K\infty}(f)) \) switches between states

The trajectory (arrows) orbits the unstable-point—a limit-cycle.

As we increase the Ca-current's strength (either \( \Delta I_{Ca} \) or \( \tau_{Ca} \)), the stable-point (gray, No Ca) becomes unstable (white, \( f_{Ca}(g_K, r) \)).

The trajectory follows \( f_{Ca}(g_K, r) \)'s upper (spiking) or lower (resting) parts, approaching \( g_{K\infty}(f) \) in both cases.

The trajectory switches from one part to the other (resting to spiking or vise-versa) when it reaches the unstable region (slope reversal).
Adapts or bursts (red or blue dotted-lines) when $\Delta r_{ca} = 1$ or 6, respectively.

In actuality, the trajectory deviates from $f_{Ca}(g_K, r)$ because $f$ does not respond instantaneously to changes in $g_K$.

The trajectory crosses $g_{K,\infty}(f)$ vertically—$g_{K,\infty}$ is constant briefly—which makes sense since that’s $g_{K,\infty}$’s steady-state value for the value of $f$ at that point.

Similarly, the trajectory crosses $f_{Ca}(g_K, r)$ horizontally—$f$ is constant briefly—since that’s $f$’s correct value for $g_K$’s value at that point.

Next week: Phase-response curve
Current-pulses decrease a cortical neuron's period (Cat, Layer V) [Fetz93]