Synapse Model

Synapse formed between axonal button and dendritic spine

Neurotransmitter concentration is modeled as an extended pulse

Binding and unbinding are modeled by first-order kinetics

Dumping neurotransmitter

Neurotransmitter concentration in cleft increases rapidly and then decays slowly

\[ \frac{dT}{dt} = -I_{\text{leak}} \quad \text{with} \quad T[0] = Q \]

\[ \Rightarrow T[t] = Q - I_{\text{leak}} t \]
**Binding to receptors**

Neurotransmitter binds to receptors and ion-channels open

\[
\text{In[1]} := \quad T(t_{\text{bnd}}) = 0 \implies t_{\text{bnd}} = Q / I_{\text{leak}}
\]

**Receptor affinity (K_D)**

Neurotransmitter binds when concentration exceeds \( K_D \)

\[
\text{In[1]} := \quad T(t_{\text{bnd}^*}) = K_D \implies t_{\text{bnd}^*} = (Q - K_D) / I_{\text{leak}} < t_{\text{bnd}}
\]

Requiring that the concentration exceed \( K_D \) reduces first spike's efficacy by the fraction \( K_D / Q \); the second spike is unaffected.
Pulse extension

Experiment 1.3: Colliding pulses (10ms apart)

(Vq=1.97V Vpl=0.022V)

6 of 13
Linearity of pulse extension

![Graph showing the linearity of pulse extension with number of colliding pulses and pulse length.](image)

(Vq1=1.97V Vpl=0.022V)

Binding and unbinding rates

![Diagram illustrating binding and unbinding rates with time.](image)
Channels do not open or close instantaneously; it takes time

\[ \frac{dR}{dt} = \frac{\alpha T (1 - r)}{\alpha T + \beta} - \beta r \]

With neurotransmitter (T) binding to receptors (R) at a rate \( \alpha \) and unbinding from them at a rate \( \beta \), the fraction of channels that are open changes at the rate:

\[ \frac{dr}{dt} = \frac{\alpha T}{\alpha T + \beta} - \beta r \]

Time-constant and steady-state level

The time-constant determines how quickly steady-state is reached

The reaction kinetics are described by a first-order ODE with time-constant and steady-state level:

\[ \tau[T] = \frac{1}{T + \beta} \quad \text{and} \quad r_\infty[T] = \frac{\alpha T}{\alpha T + \beta} \]

1. Half the channels open when the neurotransmitter concentration is \( \beta/\alpha \) — this is defined as \( K_D \).

2. \( r_\infty \) is a saturating function of \( T \) so it is reasonable to assume that \( r_\infty = 1 \) when \( T > K_D \) — this is how \( P(t) \) is defined.
Responses to step changes

The difference between the initial level and the steady-state level decreases exponentially with time.

When $T$ is constant, the solution is:

$$ r[t] = r_\infty + (r[0] - r_\infty) e^{-(t-t_0)/\tau} $$

This holds for $t > t_0$.

Rise-Time

The rise-time equals the pulse-width, or three time-constants, whichever is shorter.

For $0 < t < t_p$ and $r[0] = 0$:

$$ r[t] = r_\infty (1 - e^{-t/\tau}) $$

When $t = t_p$, the conductance reaches its maximum value.

However, it is within 5% of its steady-state value when $t = 3\tau$. 
Decay-Constant

The decay-constant equals the time-constant

For \( t > t_p \) and \( r[t_p] = r_p \):

\[
\tau(t) = \tau_p e^{-(t-t_p)/\tau_p} \quad \text{where } \tau_p = \frac{1}{\beta}
\]

since \( r_\infty = 0 \) when \( T = 0 \).

When \( t = t_p + \tau \), the conductance has decayed by a factor of \( e \) (a 63% decrease).

Temporal integration

Responses summate over time

If \( p_1(t) \) and \( p_2(t) \) produce \( r_1(t) \) and \( r_2(t) \) when presented separately, what happens when they are presented together? \( p_1(t) + p_2(t) \) produces \( r_1(t) + r_2(t) \); these sums satisfy the ODE if the individual pairs satisfy it.
The only requirement is that \( \tau \) cannot change — this is called **linear** behavior.

\[
\frac{1}{\tau} \frac{d}{dt} \left( r_1[t] + r_2[t] \right) + \left( \frac{1}{\tau} \frac{d}{dt} r_1[t] + r_2[t] \right) = p_1[t] + p_2[t]
\]

\[
\Rightarrow \left( \frac{1}{\tau} \frac{d}{dt} r_1[t] + r_1[t] \right) + \left( \frac{1}{\tau} \frac{d}{dt} r_2[t] + r_2[t] \right) = p_1[t] + p_2[t]
\]

\[
\Rightarrow p_1[t] + p_2[t] = p_1[t] + p_2[t]
\]