Quantum phase is not directly observable. We determined a method to map complete electron wave functions, including internal quantum phase, from measured single-state probability densities. Using the mathematical discovery of drum-like manifolds having different shapes but identical resonances, we constructed quantum isospectral nanostructures (upper panels in Figure) possessing matching electronic structure but divergent physical structure. Quantum measurement (scanning tunneling microscopy) of these “quantum drums” (degenerate two-dimensional electron states on the Cu(111) surface confined by individually positioned CO molecules) revealed that isospectrality provides an extra topological degree of freedom enabling robust quantum phase extraction.

Quantum Phase Extraction in Isospectral Nanostructures
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