Plural in Lexical Resource Semantics

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Abstract

The paper shows how the plural semantic ideas of (Sternefeld, 1998) can be captured in Lexical Resource Semantics, a system of underspecified semantics. It is argued that Sternefeld's original approach, which allows for the unrestricted insertion of pluralisation into Logical Form, suffers from a problem originally pointed out by Lasersohn (1989) with respect to the analysis offered by Gillon (1987). The problem is shown to stem from repeated pluralisation of the same verbal argument and to be amenable to a simple solution in the proposed lexical analysis, which allows for restricting the pluralisations that can be inserted. The paper further develops an account of maximalisation of pluralities as needed to obtain the correct readings for sentences with quantifiers that are not upward monotone. Such an account is absent in the orginal system in (Sternefeld, 1998). The present account makes crucial use of the possibility to have distinct constituents contribute identical semantic material offered by LRS and employs it in an analysis of maximalisation in terms of polyadic quantification

1 Introduction

We propose a treatment of plural semantics in Lexical Resource Semantics (LRS) (Richter & Sailer, 2004; Kallmeyer & Richter, 2007) by developing a lexical implementation of the analysis proposed by Sternefeld (1998). Sternefeld (1998) proposes to treat pluralisation as semantic glue freely insertible into logical forms, an approach that will be refered to as *Augmented Logical Form* (ALF). ALF allows for straightforward derivations of a wide range of conceivable sentence meanings.

An approach that allows for freely inserting semantic material in the derivation of a sentence is *prima facie* at odds with a basic tenet of LRS, namely that every part of an utterance's meaning must be contributed by some lexical element in that utterance. But the combinatory system of LRS will be seen to be flexible enough to achieve very similar results by purely lexical means.

The resulting approach will then be seen to allow for a straightforward solution of an overgeneration problem of ALF. The approach predicts meanings for certain sentences that Lasersohn (1989) discusses as problems for the approach of (Gillon, 1987). Gillon's approach predicts sentence (1) to be true if each of three TAs received \$7,000.

(1) The TAs were paid exactly \$14,000.

The same prediction is made by the system developed by Sternefeld (1998). Its reformulation in LRS will however allow for the formulation of a straightforward lexical constraint that rules it out. All that is required is to rule out more than one

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pluralisation of the same argument position of any verb. In the present proposal this can easily be achieved lexically, while implementing the same idea in ALF should require constraints on logical forms of a highly non-local nature.

The paper furthermore shows how Sternefeld's original proposal can be extended with the maximalisation operations needed to deal with other than upward-monotone quantifiers. Sternefeld's original system predicts the equivalence of *exactly three girls slept* and *at least three girls slept*, which can be overcome by demanding the existence of a set of girls who slept that is a maximal set of girls who slept. Since maximalisations of pluralities introduced by different NPs should sometimes not take scope with respect to one another an analysis is proposed that harnesses the ability of LRS to fuse meaning components of different constituents into a single polyadic quantifier in order to prevent this unwanted result.

The paper will proceed as follows: Section 2 gives an introduction to Sternefeld's analysis. Section 3 introduces the problematic data, which are shown to be actual problems for the ALF account in section 4. Section 5 develops the basic LRS analysis and section 6 extends it with maximalisation operations. Section 8 concludes the paper.

2 Cumulative Predication and Augmented Logical Form

2.1 The System of Cumulative Predication

Sternefeld (1998) employs the pluralisation operations *, familiar from the work of Link, and **. The definitions are given in (2).[1]

(2) a. $^*S = \{\bigcup X \mid X \in \mathcal{P}(S)\backslash\{\emptyset\}\}$

b. $^{**}R = \left\{ \langle X, Y\rangle \;\middle|\; \begin{array}{l} X = \bigcup\{U \subseteq X \mid \exists V \subseteq Y : R(U,V)\} \wedge \\ Y = \bigcup\{V \subseteq Y \mid \exists U \subseteq X : R(U,V)\} \end{array} \right\}$

Basic decisions underlying the system and adhered to in the present paper are that pluralities are represented as sets of non-empty subsets of the universe of discourse D (i.e. subsets of $^*D$) and individual *urlements* are counted as pluralities by assuming $\{x\} = x$ for $x \in D$. There is a distinction between sets in the sense of elements of $^*D$ and expressions of type $\langle e, t\rangle$. In particular, all elements of $^*D$ have type e. (These assumptions are identical with those in (Schwarzschild, 1996)).

According to (2-a), $^*S$ is the set of all unions over sets of subsets of S. Given the equality $\{x\} = x$, $^*S = \mathcal{P}(S)\backslash\{\emptyset\}$ if S is a set of individuals, i.e. if it does not contain any non-singleton sets. So ***sleep**, for instance, is the set of all nonempty sets of sleepers, given that *sleep* contains only individuals. $[\![$*the children*$]\!] \in {}^*$**sleep** thus expresses that the children slept, i.e. are a set of sleepers.

But * also works for sets of non-singleton sets, which are taken to be the extensions of collective verbs like *gather*. The extension of *gather* consists of sets

[1] These original definitions were recursive and did not play well with infinite sets. The present definitions are suitable for the general case.

of individuals that gathered as a single group; different members of the verb extension represent different groups. Then $[\![$*the children*$]\!] \in$ ***gather** expresses that the children gathered, but without any presumption that there was only one group, because $[\![$*the children*$]\!]$ may be the union of an arbitrary set of such groups. By leaving the pluralisation out, a reading can be expressed that expresses that the children gathered as a group. $[\![$*the children*$]\!] \in$ **gather**.

The somewhat involved definition of ** expresses that ${}^{**}R(X, Y)$ holds if (i) the subsets U of X such that $R(U, V)$ holds for some subset V of Y cover X, i.e. every member of X is present in some U, and (ii) the same holds *vice versa*. So every member of X belongs to a subset of X that is related by R to some subset of Y, and *vice versa*.

Sentences of the kind of (3-a), as discussed in (Scha, 1981), can now conveniently be represented as in (3-b). Each of the 500 firms belongs to a (probably singleton) subset of the set of firms that is related to a subset of the 2000 computers, and likewise for each of the 2000 computers.

(3) a. 500 Dutch firms use 2000 Japanese computers.
b. $\exists X(\mathbf{500}(X) \wedge {}^{*}\mathbf{DF}(X) \wedge \exists Y(\mathbf{2,000}(Y) \wedge {}^{*}\mathbf{JC}(Y) \wedge \langle X, Y\rangle \in {}^{**}U))$

As intended by Sternefeld (1998), definition ** also is robust with regard to collective verbs. This can be seen by replacing *use* in (3-a) with *own*. While using arguably takes place on the individual level, computers can certainly be jointly owned by more than one firm. But then this firm will be a member of a subset of the firms which jointly own the computer. So definition (2-b) and the formalisation in (3-b) can handle this case without further ado.

2.2 Augmented Logical Form

In (Sternefeld, 1998), the operators introduced in the previous section can freely be inserted into logical forms without needing to be contributed by some lexical item. More precisely, while pluralised nouns typically carry * as the semantic contribution of the plural 'morpheme', morphological pluralisation of verbs (which, in English, is mostly redundant anyway and only realises agreement with a single argument) is, as such, semantically vacuous. Argument slots of verbs are instead pluralised by inserting * or ** as 'semantic glue', which may happen in any place, given that the types permit it.

For a sentence like (4), among others, the readings illustrated in (5) are thus predicted.[2]

(4) Five men lifted two pianos.

[2]To enhance legibility, I follow Sternefeld (1998) in using $x \in S$ as a notational variant of $S(x)$ and in using uncapitalised letters for variables that are subject to a pluralisation operation. But capitalisation has no bearing on the identity of variables, i.e. X and x are merely notational variants of the same variable. Variables are (also following Sternefeld (1998)) reused 'after' pluralisation, i.e. ${}^{*}\lambda x.\phi$ typically will be applied to the variable x (then written X) again.

(5) a. $\exists X(\mathbf{5}(X) \wedge {}^{*}\mathbf{M}(X) \wedge \exists Y(\mathbf{2}(Y) \wedge {}^{*}\mathbf{P}(Y) \wedge \mathbf{L}(X,Y)))$
b. $\exists X(\mathbf{5}(X) \wedge {}^{*}\mathbf{M}(X) \wedge$
$X \in {}^{*}\lambda x.\exists Y(\mathbf{2}(Y) \wedge {}^{*}\mathbf{P}(Y) \wedge \mathbf{L}(x,Y)))$
c. $\exists X(\mathbf{5}(X) \wedge {}^{*}\mathbf{M}(X) \wedge \exists Y(\mathbf{2}(Y) \wedge {}^{*}\mathbf{P}(Y) \wedge Y \in {}^{*}\lambda y.\mathbf{L}(X,y)))$
d. $\exists X(\mathbf{5}(X) \wedge {}^{*}\mathbf{M}(X) \wedge \exists Y(\mathbf{2}(Y) \wedge {}^{*}\mathbf{P}(Y) \wedge X \in {}^{*}\lambda x.\mathbf{L}(x,Y)$
e. $\exists X(\mathbf{5}(X) \wedge {}^{*}\mathbf{M}(X) \wedge \exists Y(\mathbf{2}(Y) \wedge {}^{*}\mathbf{P}(Y) \wedge X \in {}^{*}\lambda x.Y \in {}^{*}\lambda y.\mathbf{L}(x,y)))$
f. $\exists X(\mathbf{5}(X) \wedge {}^{*}\mathbf{M}(X) \wedge \exists Y(\mathbf{2}(Y) \wedge {}^{*}\mathbf{P}(Y) \wedge \langle X,Y\rangle \in {}^{**}\lambda x.\lambda y.\mathbf{L}(x,y)))$

(5-a) means that five men, together, lifted two pianos, at once. Generally, an unpluralised lexical predicate is supposed to relate only objects that stand in some given relation, e.g. lifting or being lifted, *together*. (5-b) means that there are five men who can, in some way, be split into subgroups, each of which, together, lifted two pianos at once. (5-c) means that there are five men and two pianos and that the men, together, lifted the pianos, either at once or separately. (5-d) means that the five men can be divided into subgroups, all of which lifted the same two pianos at once. According to (5-e), there are five men and two pianos and subsets of the five men exist who lifted the pianos together, but perhaps not all of them at once. (5-f) means that the five men lifted the two pianos in some arbitrary configuration. All that is required is that every man took part in a lifting and that every piano was lifted.

These examples illustrate that, on the verb, the numbers and types of pluralisation operations may vary freely (while plural noun denotations always involve *). Furthermore their scope need not be the verb meaning alone but may also involve arguments of the verb, as shown by (5-b). This is achieved by the treatment of pluralisation as 'semantic glue': it is not a part of the lexical meanings of plural verbs but inserted into logical forms in appropriate places.

Sternefeld (1998) points out that, while there is significant overlap and even entailment between the different propositions expressed by the readings shown in (5), all of these readings are conceivable and there is thus no harm in assuming their existence.

3 Lasersohn's criticism of (Gillon 1987)

Having introduced the system of (Sternefeld, 1998), I now turn to the examples that Lasersohn (1989) put forth against the plural semantics advocated by Gillon (1987). It will turn out that Lasersohn's criticism is also applicable to Sternefeld's system. The reason will turn out to be that (Sternefeld, 1998) allows for pluralising the same verbal argument position more than once, which is a direct consequence of the treatment of pluralisation as semantic glue.

Gillon (1987) argues that the readings of a plural sentence like (6) correspond to the *minimal covers* of its subject.

(6) The TAs wrote papers.

A minimal cover of a set S is a subset C of $\mathcal{P}(S)$ such that $\bigcup C = S$ and for no $C' \subset C$ is it the case that $\bigcup C' = S$.[3] So a minimal cover of S splits S into (not necessarily disjoint) groups such that every member of S is in some group and there is no redundant group that could be dispensed with while retaining all members of S as members of one of the remaining groups.

Given a set of TAs comprising Alice, Bob and Ludwig, Gillon's proposal then predicts, among others, the following readings.

- $\{\{a, b, l\}\}$: All TAs wrote papers together.
- $\{\{a, b\}, \{l\}\}$: Alice and Bob wrote papers together and Ludwig wrote papers alone.
- $\{\{a\}, \{b\}, \{l\}\}$: Each of the TAs wrote papers alone.

Clearly, the examples given do not exhaust the possible readings of (6) under the analysis advocated by Gillon (1987), and it is clear that the number of readings will grow exponentially with the cardinality of the subject's extension.

The criticism put forth by Lasersohn (1989) is twofold: for one thing, he claims that Gillon's very concept of a *reading* is misguided: the number of readings should not be inflated in the manner indicated and, most importantly, not in a way that makes the readings that exist depend on contigent facts about the world. This criticism seems well justified; in a world in which Bob is not a TA or there is a fourth TA, the class of readings assigned to (6) would not only be different from the one assigned to that sentence in the situation considered above, but the classes would actually be disjoint. So in two utterance situations in which there are different sets of TAs, the meanings of utterances of (6) would have *nothing in common at all*, it seems.

More importantly, regarding our present concerns, Lasersohn (1989) points out that the account of the ambiguity of plural sentences offered by Gillon (1987) predicts that the sentences in (7) all have true readings in a situation in which each of three TAs got paid (exactly) \$7,000. (7-a) is true under a distributive reading and (7-b) under a collective readig in this situation. But (7-c) should not have a true reading.

(7) a. The TAs were paid exactly \$7,000.
b. The TAs were paid exactly \$21,000.
c. The TAs were paid exactly \$14,000.

But if the TAs are again Alice, Bob and Ludwig, then $\{\{a, b\}, \{a, l\}\}$ is a minimal cover. But then each element of this cover fulfills *were paid exactly \$14,000* and *the TAs* thus also should, contrary to fact.

While one might guess that the non-empty intersection of the elements of the cover is to blame and that partitions should be used instead of covers, allowing non-empty intersections is actually a feature of Gillon's analysis, motivated by (8).

[3] $\subset$ denotes *proper* subsethood.

(8) The men wrote musicals.

If *the men* is taken to refer to the set comprising Rodgers (*r*), Hammerstein (*hs*) and Hart (*ht*), it seems that the sentence would be judged true by those familiar with these men. But none of them wrote any musicals alone and likewise no musical was written by all three of them collaboratively. The minimal cover that corresponds to the true reading of (8) is $\{\{r, ht\}, \{r, hs\}\}$, the set of subsets of these men who collaboratively wrote musicals. This cover has just the same shape as the minimal cover of the TAs that proved problematic above. Lasersohn (1989) suggests that a meaning postulate like (9) may be used to guarantee the truth of (8) in the pertinent situation.

(9) $W(x, y) \,\&\, W(u, v) \Rightarrow W(x \cup u, y \cup v)$

This clearly defies Gillon's aim to treat (8) as ambiguous between a collaborative (i.e. simple collective) and a distributive reading and further ones that are neither fully distributive nor collective. While the scepticism expressed by Lasersohn (1989) regarding the readings licensed by Gillon's account seems very justified, obliterating the distinction between collective and non-collective for the predicate *write* might be going too far.

4 Applying Sternefeld's semantics

(8) can be analysed in Sternefeld's system as in (10).

(10) $[\![\textit{the men}]\!] \in {}^{*}\mathbf{WM}$

Given that $\{\{r, ht\}, \{r, hs\}\} \subseteq \mathbf{WM}$, $\bigcup\{\{r, ht\}, \{r, hs\}\} = \{r, ht, hs\} \in {}^{*}\mathbf{WM}$.

Since * need not be inserted into the logical form, analysis (11) is also possible. As a set is an element of an unpluralised predicate if its members fulfill the predicate as a group, this reading would only be true if the three composers had collaborated, which is not the case.

(11) $[\![\textit{the men}]\!] \in \mathbf{WM}$

The sentences in (7) can receive the representations in (12).

(12) a. $\mathbf{TA} \in {}^{*}\lambda x.\exists Y({}^{*}\mathbf{\$}(Y) \wedge \mathbf{7,000}(Y) \wedge Y \in {}^{*}\lambda y.\mathbf{PAID}(x, y))$
b. $\exists Y({}^{*}\mathbf{\$}(Y) \wedge \mathbf{21,000}(Y) \wedge \langle \mathbf{TA}, Y\rangle \in {}^{**}\mathbf{PAID})$
c. $\exists Y({}^{*}\mathbf{\$}(Y) \wedge \mathbf{14,000}(Y) \wedge \langle \mathbf{TA}, Y\rangle \in {}^{**}\mathbf{PAID})$

As things stand, all of these will be true. But (12-c) only is because the meaning of *exactly* cannot be correctly represented so far, which requires that Y be the unique maximal set of dollars that fulfills the scope (i.e. what follows the part stating that Y is a certain amount of dollars), and issue that will be taken up in section 6 below. When this maximization operation is put in place, (12-c) will come out false in the pertinent situation. But there is a further possible rendering of (7-c), shown in (13).

(13) $\textbf{TA} \in {}^{*}\lambda x.\exists Y({}^{*}\textbf{\$}(Y) \wedge \mathbf{14,000}(Y) \wedge \langle x, Y\rangle \in {}^{**}\textbf{PAID})$

In (13), the expression beginning with λx denotes the set of all sets of individuals who received \$14,000 in total. In the situation considered above, both $\{a, b\}$ and $\{a, l\}$ – for instance – are such sets. Pluralising $\lambda x. \cdots$ then yields a set that contains all unions of sets of this kind, and the set **TA**$= \bigcup\{\{a, b\}, \{a, l\}\}$ is such a set. Thus it appears that (13) is a predicted reading of (7-c) that should be true in the situation considered, while in fact (7-c) is not true. This parallels the situation found in (Gillon, 1987): under both accounts, finding groups who received a total of \$14,000 is enough to make (7-c) true.[4]

In Sternefeld's system, it is essential for (13) to be obtained that the subject position of **PAID** be pluralised twice, once using ** and then again using *. Leaving out the latter operation yields (14). It is easily seen that this formula – also a predicted reading of (7-c) under Sternefeld's approach – is not true in the given situation. It expresses that there is a sum of (exactly) \$14,000 that the TAs received, without any implications as to who of them received how much.

(14) $\textbf{TA} \in \lambda x.\exists Y({}^{*}\textbf{\$}(Y) \wedge \mathbf{14,000}(Y) \wedge \langle x, Y\rangle \in {}^{**}\textbf{PAID})$

Since the ALF framework allows for free insertion of pluralisation, it is not clear how it could rule out (13), which is just (14) with an additional pluralisation operator inserted, without imposing restrictions of a decidedly non-local nature on LF. In the next section, a solution to this problem is developed in LRS.

5 Recasting the system in Lexical Resource Semantics

The present proposal addresses the problem discussed above by capturing the essential ideas of (Sternefeld, 1998) about *where* pluralisation should be insertible while taking a different stance with respect to *how* pluralisation should be inserted. The locus of pluralisation will be strictly lexical. At the same time, pluralisation can occur in different places, not directly tied to the core meaning of the verb, i.e. with material contributed by other expressions intervening. This is achieved using Lexical Resource Semantics.

[4]In a reply to (Lasersohn, 1989), Gillon (1990) describes a situation in which two departments employ two TAs each. \$14,000 are paid for each pair of TAs, which they may divide among themselves as they deem fit. It then seems that (i-a) would be judged true. But – disregarding the role of "their" and ignoring the temporal modifier – this can be formalised as in (i-b) under the present approach.

(i) a. The TAs were paid their \$14,000 last year.
b. $\textbf{TA} \in {}^{*}\lambda x.\exists Y({}^{*}\textbf{\$}(Y) \wedge \mathbf{14,000}(Y) \wedge \textbf{PAID}(x, Y))$

Since each pair of TAs was paid as a team, the sets of respective team members will each be related to \$14,000, but the individual members will not. Under these circumstances, (i-b) is true.

5.1 Lexical Resource Semantics

LRS (Richter & Sailer, 2004; Kallmeyer & Richter, 2007) is a flavour of underspecified semantics that makes use of the descriptive means of HPSG and uses its constraint language, which is assumed here to be Relational Speciate Reentrant Language (RSRL; Richter, 2004), as the locus of underspecification. Disregarding the treatment of local semantics (Sailer, 2004a), the semantic representation connected to a sign (i.e. a syntactic object) is an object of a sort *lrs* to which three features are appropriate: INCONT, EXCONT and PARTS. For each word, the value of the INCONT feature is this word's scopally lowest semantic contribution, i.e. that part of its semantics over which every other operator in the word's maximal projection takes scope. The EXCONT value roughly corresponds to the meaning of the maximal projection of a word. Both INCONT and EXCONT project strictly along head lines.

The value of PARTS is a list that contains the lexical resources that a sign contributes. For words, they are lexically specified. For phrases, they always are the concatenation of the PARTS lists of the daughters. In an utterance, – an unembedded sign – each element of the PARTS list must occur in the EXCONT value and everything that occurs in it must be on the PARTS list. The EXCONT value of an utterance is regarded as its meaning.

The values of the three attributes are related by a small set of core constraints. In addition to these, the SEMANTICS PRINCIPLE provides further more or less construction-specific constraints that ensure that they are also related in a way that correctly represents how meaning is composed in different syntactic configurations. For example, in *every dog*, the INCONT of *dog*, $D(x)$, must be a subexpression of the restrictor of the universal quantifier. Since the NP contains no further material that combines with *dog*, this will actually result in identity. Similarly, if a quantified NP combines with a verb, the verb's INCONT must be found in the NP quantifier's scope. *Every dog barks* thus receives the desired interpretation $\forall x(D(x), B(x))$.

5.2 The analysis

Almost everything that needs to happen for the present approach to work happens on the PARTS list. Manipulating this list gives the opportunity to furnish lexical items with semantic material that must occur in the utterance they are used in, but the places in which this material can occur are not subject to any restriction that is not explicitly stated. By placing pluralisations on this list, it is thus possible to achieve an effect similar to their treatment as freely insertible glue in (Sternefeld, 1998). At the same time, lexically constraining the PARTS list to disallow repeated pluralisation of the same variable makes it possible to rule out readings like (13).

To illustrate the approach, it will be shown how the system accounts for the readings of *five men lifted two pianos* in (5).

The general shape of the LRS semantics of verbs like *lift* is as follows.[5]

$$\begin{bmatrix} \text{INCONT} & \boxed{1}\mathbf{L}(y)(x) \\ \text{EXCONT} & [\boxed{1}] \\ \text{PARTS} & \langle \boxed{1}, (\mathbf{L}y), \mathbf{L} \rangle \oplus \boxed{P} \end{bmatrix}$$

The PARTS list contains the INCONT (as required by a fundamental principle of LRS) and those of its subexpressions that *lift* needs to contribute as lexical resources (the variables are contributed by the NPs). In addition, it contains all elements of the list $\boxed{P}$, which is where pluralisation operations enter. $\boxed{P}$ is subject to the following conditions.[6,7]

(15) a. Every variable that is associated with a plural nominal argument of the verb may be subject to at most one pluralisation operation on $\boxed{P}$.
b. Only variables that are associated with a plural nominal argument of the verb may be subject to a pluralisation operation on $\boxed{P}$.

In the sense of (15), a variable x_i, $1 \leq i \leq n$, is subject to a pluralisation operation on $\boxed{P}$ if $\boxed{P}$ contains ${}^{*n}\lambda x_1 \ldots \lambda x_n.\phi$.[8] It is the restriction (15-a) that will prevent the unwanted reading of Lasersohn's example sentence. Since at most one pluralisation is allowed, the kind of double pluralisation that was identified as problematic above is ruled out. Formalising (15) in RSRL is a tedious but straightforward task.

Now consider again the lexical entry for *lift*. $\boxed{P}$ may be empty. This will give the reading of (4) in (5-a). Five men jointly lifted two pianos at once. Further admissible lists are shown in (16).[9]

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[5] Tags like $\boxed{1}$ are variables used to indicate token identity. So $\boxed{1}$ in the entry for *lift* always denotes the expression $\mathbf{L}(y)(x)$. $[\boxed{1}]$ may be any expression that contains $\boxed{1}$.

[6] Readers have raised questions regarding independent motivation of this constraint. But I think that (while it would of course be welcome) demanding such makes the possibility of repeated pluralisation, as in ALF, the null hypothesis. But since being allowed to enrich meanings with unlimited amounts of material is not the established standard in semantics. I fail to see any better motivation for allowing multiple pluralisations of the same argument than for not doing so, especially if the latter approach makes more accurate predictions.

[7] It must be possible to isolate $\boxed{P}$ from the idiosyncratic contributions of the verb, i.e. the parts on the list that $\boxed{P}$ is appended to. The most straightforward solution is to introduce a new attribute PLURALISATIONS whose value is $\boxed{P}$. Then the following AVM would describe all verbal lexial items.

$$\begin{bmatrix} \text{PARTS} & \langle \cdots \rangle \oplus \boxed{P} \\ \text{PLURALISATIONS} & \boxed{P} \end{bmatrix}$$

Constraining the pluralisations that may be introduced is then possible by formulating the appropriate constraints with regard to the value of PLURALISATIONS.

[8] I.e. an operator of n stars applied to an n-place relation. In this paper, $n \leq 2$.

[9] The dots on the list stand for lexical resources that are needed in addition to those explicitly shown (namely parts of these). For list (16-a), for instance, these would be ${}^{*}\lambda x.\boxed{1}$ and $\lambda x.\boxed{1}$.

(16) a. $\langle (^*\lambda x.[\boxed{1}])(x), \ldots \rangle$
b. $\langle (^*\lambda y.[\boxed{1}])(y), \ldots \rangle$
c. $\langle (^*\lambda x.[\boxed{1}])(x), (^*\lambda y.[\boxed{1}])(y), \ldots \rangle$
d. $\langle (^{**}\lambda x.\lambda y.[\boxed{1}])(x)(y), \ldots \rangle$

It is easily seen that all lists in (16) conform to (15): In (16-a), only x is subject to pluralisation and only pluralised once. The same is true for y regarding list (16-b). In (16-c), both are pluralised once, independently of each other. In (16-d), both variables are pluralised together once using **.

Importantly, now, $[\boxed{1}]$ in the expressions above may stand for any expression that has $\boxed{1}$ (the verb's INCONT) as a subexpression.[10] All that is thus said about the scope of the pluralisations is that they contain $\mathbf{L}(y)(x)$. Unless constrained further, they may thus occur anywhere in the meaning representation of a complete sentence, provided that the types fit. While the number of possible pluralisations is thus limited and while they enter into the semantics from the lexicon, their distribution will in other respects be as under the ALF approach.

As remarked above, plural nouns are always pluralised using *. For *pianos*, e.g., the semantics is as follows.[11]

$$\begin{bmatrix} \text{INCONT} & \boxed{3}\,{}^*\mathbf{P}(X) \\ \text{EXCONT} & \boxed{2}\,\exists X([\boxed{3}], \boxed{4}) \\ \text{PARTS} & \langle \boxed{2}, \boxed{3}, {}^*\mathbf{P}, \mathbf{P}, X \rangle \end{bmatrix}$$

For cardinals, an analysis as higher-order intersective modifiers is assumed here, where intersective modifiers are analysed along the lines outlined in (Sailer, 2004b), although nothing hinges on this decision. A cardinal like *three* will have the following LRS semantics.

$$\begin{bmatrix} \text{INCONT} & \boxed{5}\,3(X) \\ \text{EXCONT} & \boxed{6}([\boxed{5}] \wedge \boxed{7}) \\ \text{PARTS} & \langle \boxed{5}, 3, \boxed{6} \rangle \end{bmatrix}$$

The INCONT expresses that X is a set of (at least) three individuals. It is required to be a part of the first conjunct of the EXCONT, which conjoins it with an underspecified expression represented as $\boxed{7}$. The clauses of the SEMANTICS PRINCIPLE responsible for adjective-noun-constructions will then require the INCONT of the nominal head to be a part of this second conjunct. The variable X is embedded in the agreement index that is shared between noun and adjective: the adjective modifies a noun with an INDEX value token-identical with its own. So the adjectival and nominal INCONT involve the same variable. Hence *three pianos*

[10] Each occurrence of $[\boxed{1}]$, even on the same list, may stand for a different such expression.

[11] The existential quantifier is now taken to combine with the variable it binds and two expressions of type t and to state that the sets formed by abstracting over the bound variable have a non-empty intersection. This slightly reduces syntactic complexity since $\wedge$ can be eliminated. The representation of the existential quantifier will be revised further below.

will contribute the part $[3(X)] \wedge [{}^*\mathbf{P}(X)]$, and in the absence of additional material, this will resolve to $3(X) \wedge {}^*\mathbf{P}(X)$.

The combinatorial behaviour of plural NPs is as dictated by the SEMANTICS PRINCIPLE and discussed above: the INCONT of a verbal projection they combine with needs to be a part of their scope, i.e. $\boxed{4}$ above.

The system can be illustrated by comparing (5-b) and (5-d).

(5-b) $\exists X(\mathbf{5}(X) \wedge {}^*\mathbf{M}(X), ({}^*\lambda x.\exists Y(\mathbf{2}(Y) \wedge {}^*\mathbf{P}(Y), \mathbf{L}(Y)(x)))(X))$

(5-d) $\exists X(\mathbf{5}(X) \wedge {}^*\mathbf{M}(X), \exists Y(\mathbf{2}(Y) \wedge {}^*\mathbf{P}(Y), ({}^*\lambda x.\mathbf{L}(Y)(x))(X)))$

Both expressions are predicted to represent possible meanings of *five men lifted two pianos*. They are only distinguished by the place in which x is pluralised. The variable y is not pluralised. As required by the SEMANTICS PRINCIPLE,

- $\boxed{1}$ (i.e. $\mathbf{L}(y)(x)$) is in the scope of both quantifiers in both (5-b) and (5-d),
- the INCONT values of the nouns are in the restrictors of the existential quantifiers,
- the INCONT values of the nouns further are the second conjuncts of the conjunctions in these restrictors and
- the INCONT values of the adjectives are the first conjuncts of these conjunctions.

The basic requirements for components other than pluralisation are thus fulfilled. The pluralisation remains to be considered.

Since only x is pluralised, the pluralisation list (16-a) needs to be assumed in both cases, but with different expressions as values of $[\boxed{1}\mathbf{L}(y)(x)]$. In (5-b), this expression includes the expression $\exists Y \ldots$, in (5-d) it is identical with $\boxed{1}$. Both conform to the requirement of (16-a) that $\boxed{1}$ be in the scope of the pluralisation operator. Since both readings fulfill all pertinent constraints, they both are predicted to be possible, as desired.

The problematic reading (13) of (7-c) is ruled out since it would require a list like (17) in order for the two pluralisations found in that formula to be available as lexical resources.

(17) $* \langle \boxed{1}, ({}^*\lambda x.[\boxed{1}])(x), ({}^{**}\lambda x.\lambda y.[\boxed{1}])(x)(y), \ldots \rangle$

But since x is pluralised twice, (17) violates (15-a) and is hence not a possible pluralisation list. (14) only requires list (16-d) and thus remains a possible reading.

6 Maximalisation

As remarked above, the system as it stands cannot deal with non-upward-monotone quantifiers. The predicted reading (18-b) of (18-a) is true even if 10 children were at the party. (${}^{=}2$ denotes the set of all sets of entities of cardinality *exactly* 2.)

(18) a. Exactly two children were at the party.

b. $\exists X({}^{=}2(X) \wedge {}^{*}\mathbf{C}(X), {}^{*}\mathbf{P}(X))$

If there were ten children at the party, it is possible to single out a set of exactly two of them, which is enough to make (18-b) true, under the plausible assumption that **P** itself is a set of individuals, i.e. having been at the party is lexically a property of individuals, not pluralities (cf. van Benthem, 1986, 52f.). This issue can be addressed by requiring the set of exactly two children to be a maximal set of children who were at the party, as shown in (19).

(19) $\exists X({}^{=}2(X) \wedge {}^{*}\mathbf{C}(X), \mathit{max}(X)(\lambda X.{}^{=}2(X) \wedge {}^{*}\mathbf{C}(X))(\lambda X.{}^{*}\mathbf{P}(X)))$

The meaning of *max* is defined in (20).

(20) $\mathit{max}(X)(P)(Q) := X \in Q \wedge \forall Y \in {}^{*}P \cap Q\colon X \not\subset Y$

$\mathit{max}(X)(P)(Q)$ is true if X is in Q (the extension of *were at the party* in the case of (18-a)) and no set that is in both ${}^{*}P$ and Q is a superset of X. In this second condition, pluralising P is necessary in order to cancel out the cardinality restriction that the elements of P obey, like being sets of exactly two children. If this cardinality restriction remained in place (if P were used instead of ${}^{*}P$), then each set considered in (18-b) would be maximal in the sense of *max* because $\forall$ would only quantify over sets of exactly two children. Hence, even if there were ten children at the party, (18-a) would still come out as true because the set of ten children would be no element of the restrictor. The use of * ascertains that the quantification is over the set of all sets of at least two children. So (18-b) is true if there is a set of exactly two children which is a maximal set of children who were at the party.

The meaning representations can be simplified somewhat. Note that *max* needs to know the restrictor of the existential quantifier in order to determine the correct subclass of sets in which to maximise – (18-a) is not false if there were twenty adults at the party in addition to the two children. But then *max* can also state itself that the quantified variable takes a value from the restrictor. At this point, then, the meaning of *max* can also be incorporated directly into the existential quantifier, defining

(21) a. $\mathit{max}(X)(P)(Q) := X \in P \cap Q \wedge \forall Y \in {}^{*}P \cap Q\colon X \not\subset Y$

b. $\exists^{\circ} := \lambda R.\lambda S.\exists X : \mathit{max}(X)(R)(S)$

$\exists^{\circ}$ will be called a *maximalisation quantifier*. Using this quantifier, the representation of the meaning of (18-a) becomes (22).

(22) $\exists^{\circ}(\lambda X.{}^{=}2(X) \wedge {}^{*}\mathbf{C}(X))(\lambda X.{}^{*}\mathbf{P}(X))$

What are the predictions of this account in cases involving more than one plural noun phrase?

For (23-a), the reading (23-b) is predicted, among others.[12] Readings without pair maximalisation are ruled out since they will not be able to use the *max_me* expression contributed by the verb.

(23) a. Exactly three boys invited exactly four girls.
b. $\exists^{\circ}(\lambda X.{}^{=}3(X) \wedge {}^{*}B(X))(\lambda X.\exists^{\circ}(\lambda Y.{}^{=}4(Y) \wedge {}^{*}G(Y))(\lambda Y.{}^{**}\mathbf{I}(X,Y)))$

This formula is verified by fig. 1, where black circles stand for boys and white circles stand for girls: there is a set of three boys X (those on the left) for whom a set of four girls Y exists such that ${}^{**}\mathbf{I}(X,Y)$ and such that no larger set of girls exists such that the same holds. X also is the largest set of this kind, i.e. no superset of X is also related to a (maximal) set of four girls.

Figure 1: A situation that verifies (23-b).

This prediction will be discussed below. First note that there is a problem with the current approach due to the fact that one maximalisation quantifier must outscope above the other. This is illustrated by fig. 2. Considering sentence (24), does fig. 2 verify it or not?

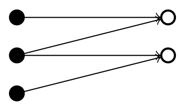

Figure 2: Figure that verifies (24).

(24) Exactly one boy invited exactly one girl.[13]

This depends on the scope relations. If the scope is as in (25-a), then (24) is true: there is a set of one boy such that there is a set of one girl that is a maximal set of girls he invited; the sets may be any set containing a boy who invited just one girl and the girl he invited, respectively.

But (25-b) is false in the same situation: there is no set of one girl such that the maximal set of boys who invited her has just one member.

(25) a. $\exists X(1\mathbf{B}(X) \wedge \mathit{max}(X)(\lambda X.\exists Y(1\mathbf{G}(Y) \wedge \mathit{max}(Y)(\lambda Y.{}^{**}\mathbf{I}(X,Y)))))$

[12] Predications are written as $R(x,y)$ instead of $R(y)(x)$ from now on. This is only a shorthand of no further significance.

[13] There is no claim made here that a sentence with singular noun phrases should really be analysed in the way indicated. The only purpose for using this sentence is that it allows to keep the illustration small. In fact, the more complex fig. 1 could also be used to illustrate the same fact.

b. $\exists Y(1\mathbf{G}(Y) \wedge max(Y)(\lambda Y.\exists X(1\mathbf{B}(X) \wedge max(X)(\lambda X.^{**}\mathbf{I}(X,Y)))))$

According to my intuitions, there is no such ambiguity involved in sentences like (24) (or actual plural sentences of the same kind) and (24) is not verified by the situation in fig. 2. So it seems that the approach is in need of modification.

Before such a modification is actually introduced, let us return to the prediction about (23-a) and fig. 1; the former was predicted to be true in the situation depicted in the latter. This prediction differs from that of the theory laid out in (Landman, 2000). According to (Landman, 2000), (23-a) should be false for fig. 1 since all boys who invited girls and all girls invited by boys are taken into account by determining the maximal event of boys inviting girls. Only if this event has three boys as its agent and four girls as its patient will (23-a) come out true, but in fig. 1 the agent of this event would consist of five boys and the patient of six girls.

Robaldo (2010, 260), offers evidence against the approach advocated by Landman (2000) and an analysis according to which (23-a) in fact has a reading that is true given fig. 1. Note that this implies that (23-a) is not incompatible with e.g. *exactly five boys invited excatly six girls*, which is also verified by fig. 1. My own (non-native) intuitions on the issue are equivocal, but I tend to side with the predictions of (Robaldo, 2010). This also holds for the corresponding sentence in my native German.

(26) Genau drei Jungen haben genau vier Mädchen eingeladen.

Unlike the account developed so far, that of (Robaldo, 2010) does not exhibit false scope ambiguities. Taking its departure from (Sher, 1997), it is based on maximalisation of pairs of sets similarly to what is shown in (27).[14]. Robaldo (2010) argues that this approach should be used for all plural quantificational expressions, regardless of their monotonicity properties, giving examples corroborating the approach even for downward monotone quantifiers.[15]

(27)
$$\begin{aligned}&\mathit{MAX}(\langle P,Q\rangle, N_1, N_2, R) :=\\ &{}^{**}R(P,Q) \wedge \forall P'Q'\\ &((P \subseteq P' \wedge P' \in {}^{*}N_1 \wedge {}^{**}R(P',Q) \to P' \subseteq P)\wedge\\ &\ (Q \subseteq Q' \wedge Q' \in {}^{*}N_2 \wedge {}^{**}R(P,Q') \to Q' \subseteq Q))\end{aligned}$$

A pair is maximal if no component of it can be made any larger while keeping the other fixed.[16] The representation of (23-a) then becomes

[14] The formulation in Robaldo (2010) employs quantification over covers to achieve the effect of **, and a contextually determined cover variable as advocated by Schwarzschild (1996). Furthermore, Robaldo's definition does not incorporate the restrictors N_1 and N_2. This omission is an error: if three children watched two movies and one of their grandparents also watched one of the movies, *exactly three children watched exactly two movies* will be predicted to be false due to the larger set that includes the grandparent.

[15] Robaldo (2010) suggests that apparent failures of the approach for such quantifiers in other cases should be explained by pragmatics. In this paper, I follow these assumptions.

[16] Specifying a game with two players where, for each $i \in \{1,2\}$, outcome $f_i(\langle x_1, x_2\rangle) \geq$

(28) $\exists X(^{=}3(X) \wedge X \in {}^{*}\mathbf{B} \wedge \exists Y(^{=}4(Y) \wedge Y \in {}^{*}\mathbf{G} \wedge \mathit{MAX}(\langle X, Y\rangle, {}^{*}\mathbf{B}, {}^{*}\mathbf{G}, \mathbf{I})))$

This sentence is true in the situation depicted by fig. 1: the three boys in the group to the left are the maximal set of boys that cumulatively invited the four girls in that group and these four girls are the maximal group of girls they invited. Likewise, (24) is now false in the situation depicted by fig. 2, since for no set of girls is there a set of just one boy that is a maximal set of boys who cumulatively invited the girl. In the sequel I will adopt the idea proposed by Robaldo (2010) and show how it can be implemented in LRS. The implementation does not require adopting Robaldo's propoal, though. With a different definition of the *max* operator below, the essential ideas of (Landman, 2000) could likewise be implemented.

7 Implementation of maximalisation in LRS

In order to implement the proposal by Robaldo (2010), it is necessary to be able to maximalise pairs of sets instead of only one set at a time. In addition, in order to actually rule out the unwanted readings, maximalisation of pairs instead of sets also needs to be enforced. Each of these requirements is addressed in turn.

Maximalisation of pairs

Maximalisation of pairs is achieved by exploiting one of the most notable features of LRS, namely that distinct expressions may contribute identical parts of the semantics, which allows for meaning components to be 'fused'. This feature was put to use in an analysis employing polyadic quantifiers in (Iordăchioaia & Richter, 2015). In the present approach, the maximalisation quantifiers contributed by distinct noun phrases can turn out to be one and the same. This is achieved by analysing maximalisation quantifiers categorematically, instead of as variable binders (cf. Richter, 2016) and ascertaining that the contributions of two distinct NPs can be fused into a single semantic expression that employs a polyadic quantifier to express the desired pair maximalisation.

In order to express pair maximalisation, *max* is renamed max^1 and in addition, pair maximalisation max^2 is introduced as defined in (29).[17]

(29) $\mathit{max}^2_{\langle\langle e,t\rangle,\langle\langle e,t\rangle,\langle\langle e,t\rangle,\langle\langle e,t\rangle,\langle\langle e,\langle e,t\rangle\rangle,t\rangle\rangle\rangle\rangle\rangle} :=$
$\lambda X.\lambda N.\lambda Y.\lambda M.\lambda R.$
$X \in N \wedge Y \in M \wedge R(X,Y) \wedge$
$\forall X \subseteq X' \colon ({}^{*}N(X') \wedge R(X',Y) \rightarrow X' \subseteq X) \wedge$
$\forall Y \subseteq Y' \colon ({}^{*}M(Y') \wedge R(X,Y') \rightarrow Y' \subseteq Y)$

$f_i(\langle x_1', x_2'\rangle)$ iff $x_i \in {}^{*}N_1$, $\langle x_1, x_2\rangle \in R$ and $x_i \supseteq x_i'$, $\mathit{MAX}(\langle x_1, x_2\rangle, N_1, N_2, R)$ states that $\langle x_1, x_2\rangle$ is a Nash Equilibrium.

[17] If needed, max^3 or max^n for even larger n could of course also be introduced. Also note that individuals and pluralities both are of type e in the present system.

me
  max_me
    max_application FUNCTOR *max_me*
    $\exists^{\circ 1}$
    $\exists^{\circ 2}$
  application
    max_application
    non_max_application

Figure 3: Additions to the sort hierarchy for LRS expressions.

(29) encodes the meaning of *MAX* above: X and Y are the sets provided by the existentially bound variables and N and M are the sets denoted by the corresponding nominal expressions. As before, the assertion that the sets belong to the noun denotations and the verbal scope is also encoded in *max*.

$\exists^{\circ}$ is likewise renamed $\exists^{\circ 1}$ and $\exists^{\circ 2}$ is defined analogously:

(30) $\exists^{\circ 2} := \lambda R_1.\lambda R_2.\lambda S.\exists XY: max^2(X)(R_1)(Y)(R_2)(S)$

Next, the sort hierarchy is extended slightly by introducing a new subsort *max_me* of the sort *me* of meaningful expressions.[18] This will make it possible to talk about the quantifiers $\exists^{\circ 1}$ and $\exists^{\circ 2}$ without knowing whether they are the primitive max^1 or max^2 or the result of applying max^2 to some of its arguments.

The sort *max_me* has as its subsorts *max_application*, which is also a subsort of *application*, and $\exists^{\circ 1}$ and $\exists^{\circ 2}$.[19] 'Normal' application of non *max_me* functions is now of the sort *non_max_application*, which is not a subsort of *max_me*. The maximally specific subsorts $\exists^{\circ 1}$ and $\exists^{\circ 2}$ of *max_me* represent the monadic and polyadic maximalisation quantifier, respectively. *max_application* respresents the results of applying an expression of sort *max_me* to an argument. Since it is a subsort of *application*, the constraints that regulate the wellformedness of application expressions with regard to typing affect it as well. But so far, nothing necessitates using *max_application* in applications of *max_me* expressions. The sort hierarchy only rules out using *max_application* to apply anything that is not of this sort. To enforce the converse as well, the following constraint is introduced:

[18]The type-logical language used in LRS is built from the same kind of graph structures that are used to model natural language. All expressions of the formal language have the sort *me* to which the attribute TYPE is appropriate. Constants and variables have the subsorts *constant* and *variable* respectively and are identified by an INDEX value (a natural number, itself encoded in the same way). Complex epressions are represented by structures of, e.g. sort *application* with appropriate attributes FUNCTOR and ARG. Suitable constraints guarantee that, for instance, the TYPE value of an *application* is the type of the FUNCTOR value applied to the ARG value. So if the FUNCTOR value of an *application* object represents an expression ϕ of type $\langle\tau,\sigma\rangle$ and its ARG value represents an expression α of type τ, then the *application* object itself represents $\phi(\alpha)$ of type σ. See (Penn & Richter, 2004) for a concise formal statement.

[19]The quantifiers must of course be further constrained to have the appropriate types.

(31) $\quad [\textit{application}, \text{FUNCTOR } \textit{max\_me}] \rightarrow [\textit{max\_me}]$

This constraint states that every application of a *max_me* functor to an argument must again result in a *max_me*. The modified sort hierarchy and constraint (31) ascertain that max^1, max^2 and whatever results from iteratively applying them to their arguments is a *max_me* expression and that nothing else is.

With these preliminaries in place, it is possible to conveniently refer to expressions of sort *max_me* without needing to know about their exact shape in a way that would straightforwardly generalise to quantifiers with even more than two restrictors. This will be the key to fusing the quantifiers contributed by different NPs into a single polyadic quantifier. The next subsection specifies the syntax-semantics interface that will allow for these fusions and enforce them where required.

Fusing maximalisations

The lexical entries of plural nouns are now given the following shape.

$$\begin{bmatrix} \text{INCONT} & \boxed{3}{}^{*}\mathbf{P}(X) \\ \text{EXCONT} & \boxed{2}\,\phi(\boxed{6}\lambda X.\boxed{3}) \\ \text{PARTS} & \left\langle \ \boxed{3}, {}^{*}\mathbf{P}, \mathbf{P}, X, \boxed{6}, \boxed{2}, \phi \ \right\rangle \end{bmatrix}$$

Where ϕ is of sort *max_me*.

ϕ is a *max_me* expression, i.e. one of the primitive quanitifier symbols or the polyadic quantifier applied to the restrictor that comes with some other noun. This is all that is required to allow quantifiers contributed by distinct noun phrases to fuse.

Note that ϕ itself is contributed by the noun on its PARTS list, even if it is of the shape $\exists^{\circ 2}(\cdots)$. One might suspect this fact to result in the possibility of smuggling in arbitrary meaning parts, since such an expression contains parts that are not contributed by the noun itself. But precisely the fact that the noun itself does not contribute these parts prevents such unwelcome results: if the noun itself does not contribute the components of something on its PARTS list, something else needs to – in this case, another noun. Also, leaving ϕ out is not an option since the primitive $\exists^{\circ 1}$ or $\exists^{\circ 2}$ needs to be contributed somewhere, and this is precisely what ϕ will need to be on the PARTS list of at least one noun.

Note that, unlike in the entries above, the lexical entries of nouns do not mention the verbal scope anymore. The EXCONT of a noun now is a quanitfier that still needs to be applied to the verbal scope. This does not only bring the present LRS analysis more in line with mainstream semantics but also allows for enforcing the fusion of quantifiers: the application of the quantifier to its scope will be enforced in the lexical entry of the verb itself. Verbal lexical entries still look as shown below.

$$\begin{bmatrix} \text{INCONT} & \boxed{1}\mathbf{L}(x, y) \\ \text{EXCONT} & \boxed{1} \\ \text{PARTS} & \langle \boxed{1}, (\mathbf{L}y), \mathbf{L} \rangle \oplus \boxed{P} \end{bmatrix}$$

But now the list $\boxed{P}$ of a transitive verb with arguments pluralised by ** should look as shown in (32).

(32) $\langle\boxed{2}({}^{**}\lambda x.\lambda y.[\boxed{1}])(x, y), \phi(\lambda x.\lambda y.[\boxed{2}]) \ldots\rangle$, where ϕ is of sort *max_me*.

ϕ is a *max_me* expression that is applied to the verbal scope. The pluralisation is contained in the argument to the maximalisation operator and in turn contains the INCONT of the verb. What if a list that contains a ** pluralisation is required to also contain such an expression? Then ϕ must result from the application of *max*^{2} to the semantic material of two noun phrases. This is ascertained by the fact that this is the only kind of *max_me* expression that can consume an argument of type $\langle e, \langle e, t\rangle\rangle$. Note that the only part of the *max_me* expression ϕ that the verb contributes is this expression itself and the verbal scope. All its subexpressions need to be collected from somewhere else, so for ϕ to actually appear in the meaning of a full utterance, they must be contributed by appropriate noun phrases.

To guarantee in a principled way that pluralisation lists in fact have the shape in (32), the constaint in (33) is imposed on them.

(33) For each pluralisation p on $\boxed{P}$ there needs to be a maximalisation on $\boxed{P}$ that maximalises exactly the variables pluralised by p, in the same order.

The constraint guarantees that $({}^{**}\lambda x.\lambda y.[\boxed{1}])(x)(y)$, a pluralisation of x and y, is flanked by a maximalisation quantifer like $\boxed{3}(\lambda x.\lambda y.[\boxed{2}])$ of the same variables. A single-star pluralisation ${}^{*}\lambda x.\phi$ will accordingly need to be flanked by a maximalisation quantier $\exists^{\circ 1}(\lambda x.\phi)$. As a consequence, whenever two variables are pluralised seperately, they also need to be maximised separately. The empirical consequences of this fact merit further investigation but are beyond the scope of the present paper.

There still is need for one more constraint: the restricors of $\exists^{\circ 2}$ must be prevented from swapping places: $\exists^{\circ 2}(\lambda x.\phi)(\lambda y.\psi)(\lambda y.\lambda x.\theta)$ must be disallowed. The outer abstraction in the third argument needs to abstract over the same variable that is abstracted over in the first argument and the inner abstraction needs to abstract over that abstracted over in the second. Such a well-formedness constraint is easy to state.

The system now predicts (34) as a reading of (23-a), as desired.

(34) $\exists^{\circ 2}(\lambda X.{}^{=}3(X) \wedge {}^{*}\mathbf{B})(\lambda Y.{}^{=}5(Y) \wedge {}^{*}\mathbf{G})(\lambda X.\lambda Y.{}^{**}\mathbf{I}(X, Y))$

By the same token, the correct reading is now predicted for (7-c), paralleling (34). This reading is no more true in the situation considered in section 3 above. While \$21,000 have plenty of subsets of \$14,000, none of these is a maximal set of dollars the TAs were cumulatively paid. Of course, the problematic reading of sentence (7-c) discussed in section 4 remains unlicensed, as it would still require pluralising the same variable twice.

8 Conclusion

It has been argued that (Sternefeld, 1998) suffers from the same problem of overgeneration that Lasersohn (1989) points out with respect to the analysis proposed by Gillon (1987). The source of the problem was identified as inherent to the syntax-semantics interface Sternefeld (1998) employs. His approach allows for multiple pluralisations of a single verbal argument position. Without this possibility, the overgeneration disappears. A lexicalist reformulation of Sternefeld's system was then offered that puts verbal argument pluralisation into the lexical semantics of the verb. This allowed for restricting the number of pluralisations on any argument to one. The account employs Lexical Resource Semantics (LRS), thereby offering the first approach plural semantics in this framework.

It was further demonstrated that LRS allows for a straightforward implementation of maximalisation operations that, while absent in (Sternefeld, 1998), are needed to get correct results for quantifiers that are not upward monotone. The analysis relies on the possibility of the semantic contributions of distinct constituents to be the same in LRS. This feature was used to fuse quantifiers associated with different plural NPs into a single polyadic quantifier stating the existence of a maximal pair of sets. This way, scoping of maximalisations over each other is avoided and the correct truth conditions for sentences like *exactly three boys invited exactly four girls* are derived.

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