Low-dimensional chaos in turbulence

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Abstract

Direct numerical simulations are being performed on two different fluid flows in an attempt to discover the mechanisms underlying the transition to turbulence in each. The first system is Taylor-Couette flow; the second, two-dimensional flow over an airfoil. Both flows exhibit a gradual transition to high-dimensional turbulence through low-dimensional chaos. The hope is that the instabilities leading to chaos will be easier to relate to physical processes in this case, and that the new understanding of these mechanisms can then be applied to a wider array of turbulent systems.

In the past decade a new understanding of nonlinear processes in nature has been provided by the application of mathematical ideas on chaos and bifurcation theory (Ott 1981, Swinney 1983). An important result for turbulence researchers has been the finding that some systems undergo a transition to turbulence by first becoming low-dimensionally chaotic (Brandstater and Swinney 1987, Keefe 1987, Sreenivasan 1985). Turbulence models should benefit from a better understanding of weakly turbulent cases in which low-dimensional models can in principle capture *all* of the system dynamics.

The purpose of this research project is to study instability mechanisms in two model cases that have exhibited a gradual transition to turbulence, i. e., a transition to turbulence via low-dimensional chaos. The hope is that the instabilities leading to chaos will be easier to relate to physical processes in these models, and that the new understanding of these mechanisms can then be applied to a wide array of turbulent systems.

The first of the two projects, undertaken with the advice and assistance of Drs. Robert Moser (NASA-Ames) and Lawrence Keefe (CTR), is the computation of the Lyapunov exponent spectrum for a model of Taylor-Couette flow, the flow between concentric rotating cylinders (Brandstater and Swinney 1987, Moser 1983). Experiments on moderate aspect ratio Taylor-Couette systems have observed a transition from laminar, time-independent Couette flow to periodic, quasiperiodic, and finally to low-dimensional chaotic motion. The physical mechanisms driving the low-order transitions have been found, but the transition to chaos is not well understood. The numerical method that we will use was developed by Moser (1983) to study curved channel flow and assumes axial periodicity. The method recovers the low-order transitions well, but the axial constraint may significantly affect the chaotic states. The experimental evidence in the chaotic regime includes measurements of wavespeeds associated with dominant frequency components in the flow as well as estimates of the fractal dimension for the chaotic attractor. We can, therefore, verify the accuracy of our simulation: the wavespeeds can be measured just as they are in experiment, and the fractal dimension can be computed directly from a knowledge of the Lyapunov exponent spectrum for the flow (Keefe 1987). If our numerical results differ greatly from the experimental results on wavespeed, we may need to consider a finite axial extent model. Given an good numerical model of the flow, our dimension estimate will be more accurate (as accurate as computer resources allow) than dimension estimates obtained indirectly from experimental time series data.

Although the dimension estimate we obtain from the Lyapunov exponent spectrum is necessary to validate the model (or, perhaps, the experiment), there is a great deal more to be learned from the exponents. The Lyapunov exponents for a system measure the exponential rates of growth or decay for infinitesimal perturbations in each phase space direction. There are, therefore, an infinite number of Lyapunov exponents for any spatially-extended system, but for dissipative systems almost all of the exponents are large and negative, signifying the decay of transients to some attractor. The number of positive Lyapunov exponents is a rough indication of the number of distinct mechanisms destabilizing the flow. Moreover, the time-varying contributions to the long-time average exponents will indicate *when* in the system evolution the chaos-generating mechanisms are operating. The spatial structure of the eigenvectors associated with the positive Lyapunov exponents at those times may indicate the nature of that mechanism by showing *where* in the flow it is operating.

Progress on this work since 1 Sept 1988 includes familiarization with the NAS operating environment and the Vectoral programming language, and full implementation of the curved channel code on the Cray 2. The additions to the code necessary to compute the Lyapunov exponents have been inserted and tested on periodic and quasiperiodic Taylor vortex flow. The basic code (without the exponent calculations) is being run in order to estimate the minimum resolution needed to model chaotic Taylor-Couette flow. Moser's previous results showed, for example, that a $32 \times 16 \times 32$ $(R - \theta - Z)$ simulation of a periodic (wavy Taylor vortex) flow reproduced the azimuthal wave speed found in experiments to within the experimental uncertainty. Lowering the resolution to $16 \times 16 \times 16$ only changes the wave speed by 1%, while a resolution of $16 \times 16 \times 8$ gives a 5% deviation. Moser attained good agreement with experiments on quasiperiodic (modulated wavy Taylor vortex) flows using a $64 \times 32 \times 64$ model. We are currently checking the effects of lowered resolution on this case. It seems likely that only moderately-high resolution will be needed for chaotic solutions at Reynolds numbers close to the quasiperiodic-chaotic transition.

A second, related research project is being conducted in collaboration with

Dr. Thomas Pulliam (NASA-Ames) (Pulliam 1989, Pulliam and Vastano). This is a study of the transition to turbulence in two-dimensional flow over airfoils at high angles of attack. Experiments on forced flow over airfoils have found that a transition to a chaotic state can be produced (Stuber and Gharib). Numerical simulations of two-dimensional unforced flow had found a transition for increasing Reynolds number from periodic flow to aperiodicity that included one period-doubling (Pulliam 1989). We have now seen a period-doubling cascade of bifurcations and a transition to low-dimensional chaos past a period-doubling accumulation point. We are currently attempting to characterize the observed chaos using Poincaré sections and Lyapunov exponent estimates from time series data. As in the Taylor-Couette research, the goal here is to identify the physical mechanisms causing the period-doubling to chaos, rather than simply establishing that such a transition exists.

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