Direct numerical simulations of turbulent convection with a variable gravity and Keplerian rotation

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1. Motivation

Thermal convection has been proposed as a possible mechanism for generation and maintenance of turbulence in the inner accretion disk regime of the primordial solar nebula. Resulting Reynolds stresses would produce a torque on the disk, causing "viscous" spreading and the eventual dispersal of the gas component of the disk; this process was coeval with and coupled to the formation of the planetesimals and, ultimately, planets. Turbulent convection under solar nebula conditions has been described to date by *ad hoc* and mostly untested models of thermal convection and turbulence. The models of Lin & Papaloizou (1980) and Cabot *et al* (1987), in particular, give vastly different results in terms of the vertical distribution and intensity of turbulence, the efficiency of convection, and structural stability properties. There are yet few, if any, conclusive astronomical observations of solar nebula analogues, nor does indirect evidence from planetary compositions provide clear constraints on these models. It is, therefore, of fundamental interest to design experiments with the basic physical features of the solar nebula in order to constrain old and new models. Although solar nebula conditions cannot be reproduced in the laboratory, numerical simulations of hydrodynamic flows, which have been very successful in describing aerodynamic flows, can be suitably modified to provide "experimental" data for solar nebula modelling.

2. Objectives

The goals of this project are (1) to modify an extant, "proven" hydrodynamics code with the most important features of the solar nebula and other thin accretion disks: buoyancy terms to generate convection, internal heating representing the release of gravitational potential energy, a variable gravity linearly proportional to the distance from the vertical midplane due to centrifugal balance, rapid rotation with axis aligned with gravity, and Keplerian rotational shear; (2) to determine the effect that these features have on the turbulent convection by introducing them individually and to determine the cumulative nature of the turbulent convection for accretion disk conditions; and (3) to model the convection (viz., the convective heat flux) and the turbulence (viz., turbulence intensities, heat dissipation, and Reynolds stresses). In this manner, prior solar nebula models can be tested and their deficiencies rectified.

3. Work to Date

Uniform rotation. The code for direct numerical simulations of a incompressible flow in a semi-infinite channel (Kim, Moin & Moser, 1987) was modified to include buoyancy in the Navier-Stokes equation in the Boussinesq approximation, the variable gravity, and uniform rotation. Both no-slip and no-stress boundary conditions on the vertical walls were implemented (the latter being more realistic for the solar nebula). Sequences of simulations with varying rotation rates were performed, from no rotation to rates approaching the limit of stability. A linear analysis was performed to determine theoretical critical rotation rates for marginal stability. It was found that the convective fluxes in the numerical simulations become negligible between the marginal limits of stationary and oscillatory convection, and there may also be finite amplitude effects occurring in this rotation regime.

The turbulence intensities and efficiency of the convection were found to be virtually undiminished in the midchannel region where buoyancy vanishes; this region posed a major uncertainty in prior solar nebula modelling, and this result not only clarifies the nature of the turbulence there but allows some simplification in modelling. An extension of the standard stellar mixing length model for convective heat fluxes and speeds was developed to include viscosity and rotation and was found to agree qualitatively with the rotational dependence of the simulation results, although convective fluxes and speeds in the model were found to be a factor of 3 too low for standard values of coefficients. Canuto & Goldman's (1985) turbulence model, used in Cabot *et al*'s (1987) solar nebula model, also gave a qualitatively similar dependence of convective heat flux and speed on rotation, but gave convective fluxes that were *30 times too low.*

The vertical distribution of convective buoyancy production of turbulence kinetic energy has a bimodal shape with peaks on either side of midchannel and vanishing at midchannel. For low rotation rates, nonlinear diffusion (primarily by convective transport) redistributes the turbulence so that it is dissipated with a flat interior profile. For high rotation rates, the overall diffusion is suppressed (with diffusion by pressure fluctuations now dominating) such that dissipation near midchannel is depressed. This result further supports the assumption made by Cabot *et al* (1987) that heat dissipation is evenly distributed throughout the convective region, than it does the model by Lin & Papaloizou (1980), in which the turbulence intensities and dissipation is sharply peaked at the midplane. Since the heat dissipation is ultimately the heat source in the solar nebula, it is important to assess the sensitivity of the heat dissipation distribution to that of the internal heat source. Tests with centrally concentrated internal heating show that the interior heat dissipation profile is very insensitive to that of the heat source, which is again consistent with the assumption that the dissipation will have a relatively flat interior distribution.

Rotational shear. Linear rotational shear is treated in the numerical integration by working in a cosheared reference frame, in which the governing equations remain horizontally homogeneous, and implementing Rogallo's (1981) remeshing transformation in the homogeneous horizontal directions, which allows one to circumvent the tendency of the numerical grid to become overly distorted as it follows the sheared flow. The remeshing transformation was implemented in the channel code and has been extensively tested. The tendency of the streamwise length scales to become elongated and the spanwise length scales (in the direction of the shear) to become compressed requires balancing horizontal box and mesh sizes in the simulations in order to optimize the numerical resolution, and runs are currently under way in order to determine the optimal numerical domain(s). Several simulations were carried out with less than optimal but acceptable resolution, which have provided the preliminary results subsequently quoted.

Linear stability analysis predicts that the rotation profile is centrifugally stable if the specific angular momentum gradient increases with distance from the rotation axis ($S > -2\Omega$, where S is the shear rate and Ω the rotation rate). This limit was tested and verified with the numerical code with no buoyancy forces in effect. This indicates that the rotational shear flow for Keplerian rotation ($S = -3\Omega/2$) is centrifugally stable and, by itself, cannot maintain the turbulence in accretion disks. Another verification of this proposition was found by allowing a convecting, rotationally sheared flow simulation with well developed Reynolds stresses due to the shear to decay when the internal heating was extinguished; both the convective and shear production rates decayed to zero, indicating that it is convection alone that drives the turbulence in this flow and indirectly maintains Reynolds stresses by the "catalytic" effect of the rotational shear.

Preliminary results for Keplerian rotation at a moderate rotation rate with both no-slip and no-stress (coshearing) vertical walls show well developed Reynolds stresses in the horizontal components ($-\overline{uw}$). The corresponding shear correlation coefficient was found to be 0.21 for no-stress walls and 0.23 for no-slip walls, compared to 0.45 for homogenous, unidirectional, plane shear flows (Townsend, 1956); these values were very uniform across the channel, even at the walls. The shear production rate of kinetic energy was about 16% of the convective production rate. Simulations at different rotation rates are needed to determine the robustness of these results. For uniform rotation with the same inertial frequency, the rms vertical velocity (v_{rms}) was found to exceed the equal streamwise (u_{rms}) and spanwise (w_{rms}) components by about 1.4, because it is the component directly receiving the production by convective buoyancy. For the Keplerian rotational shear case, $v_{rms} \approx 1.3\, w_{rms}$ and $w_{rms} \approx 1.4\, u_{rms}$, compared to the result for unidirectional, plane shear flow of $u_{rms} > v_{rms} > w_{rms}$. Because convective buoyancy is driving the turbulence in the numerical simulation, one expects v_{rms} to still be the largest. The discrepancy in the horizontal components may be due to rotation: The production rate in the streamwise component is $-(2\Omega+S)\,\overline{uw} = -\Omega/2\,\overline{uw} < 0$, rather than $-S\,\overline{uw} > 0$ without rotation, and the spanwise production rate is $2\Omega\,\overline{uw} > 0$.

4. Future Work

In upcoming work, numerical simulations using sequences of rotation rates with Keplerian shear will be constructed. The results will be used to test the robustness of the value of the shear correlation coefficient and the ratio of shear to convective production, or to suggest ways to model whatever variations arise. The stability behavior of the rotationally sheared flow at very rapid rotation rates will be compared with uniform rotation results to determine any effects of the shear; also the effect of shear on the modelling of convective fluxes and speeds will be examined. The interior distribution of kinetic energy dissipation with both convective and shear production will be examined for different rotation rates; modelling for the dissipation and the Reynolds stress will be attempted. Simulations with different Rayleigh and Prandtl numbers will be performed in order to assess their effects on the preceding key quantities with the viscosity-independent product of Rayleigh number and Prandtl number held close to solar nebula model values ($\sim 10^4$ to 10^5); lower Prandtl number simulations will allow higher, more realistic rotation rates to be accessed. At this stage, much more physical and realistic models and parametrizations of turbulent convection in thin accretion disk environments, such as the solar nebula, should be attainable than have been available heretofore.

In the realm of the more distant future are a number of modifications that would make the numerical simulations more realistic. The problem of unrealistic, impermeable vertical boundaries and the imposed vertical scale of the convective region can possibly be relaxed by introducing compressibility to the code, at least in the form of a variable vertical density. For coverage of many density scale heights, the vertical boundary region would be buffered by a convective stable region and the convective motions would select their natural vertical scale; however, implementation of this change will require extensive modifications of vertical integration scheme in the present code, probably requiring finite differencing rather than the present Chebyshev polynomial expansion. Since the dissipation of kinetic energy should properly be equated with the internal heat source, ways to make a self-consistent "bootstrap" will be explored; this will provide a self-consistent way to specify the energy source, and will indicate if the coupling between energy source and turbulence leads to any unanticipated behavior.

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