Turbulence modeling: near-wall turbulence and effects of rotation on turbulence

By T. -H. Shih

1. Motivation and objectives

Many Reynolds averaged Navier-Stokes solvers use closure models (including two-equation models and second-order closure models) in conjunction with "the law of the wall", rather than deal with a thin, viscous sublayer near the wall. However, law of the wall functions are based on assumptions of local equilibrium which are not always valid. For example, flows with separation, reattachment, body forces, strong secondary flows, or streamwise pressure gradient can cause the behavior of the near-wall sublayer to depart from the law of the wall. To solve these problems, the modeled turbulence equations must be carried out in the sublayer in order to capture the non-equilibrium characteristics of the near wall region. Non-equilibrium turbulence models of the two equation type include Jones and Launder (1973), Chien (1982), and Lam and Bremhorst(1981). Second order closure models include Hanjalic and Launder (1976) and Launder and Shima (1989). However, as Patel et al. (1985) pointed out, the damping functions used in the existing k-ϵ models need further modification in order to improve their performance. In addition, analysis of the near-wall behavior of the current second order closure models shows that they do not have the proper asymptotic behavior. Predictions of the normal stresses near the wall are quite poor. This work is motivated by the need for better models to compute near-wall turbulent flows. We will use direct numerical simulation of fully developed channel flow and one of three dimensional turbulent boundary layer flow (Kim et al. (1987), Mansour et al. (1988), and Moin et al. (1989)) to develop new models. These direct numerical simulations provide us with detailed data that experimentalists have not been able to measure directly.

Another objective of this work is to examine analytically the effects of rotation on turbulence, using Rapid Distortion Theory (RDT). This work is motivated by the observation (Reynolds, 1989) that the pressure-strain models in all current second order closure models are unable to predict the effects of rotation on turbulence. All current rapid pressure-strain models in the equation for the invariants of anisotropy tensor are insensitive to pure rotation.

One of the objectives of this work is to develop better models (for both two-equation model and full-Reynolds stress type models) for the near-wall turbulence, using direct numerical simulation data and existing methodologies. The models will be tested using data from direct simulations, experiments and analysis. Another objective of this work is to use RDT to obtain an analytical solution for pure rotational turbulence, which will hopefully bring us some new understanding of turbulence physics and provide improved turbulence models for rotational flows. Specifically, the objectives of this work can be summarized as follows:

1. Examine the performance of existing two-equation eddy viscosity models and develop better models for the near-wall turbulence using direct numerical simulations of plane channel flow.

2. Use the asymptotic behavior of turbulence near a wall to examine the problems of current second-order closure models and develop new models with the correct near-wall behavior of these models.

3. Use Rapid Distortion Theory to analytically study the effects of mean deformation (especially due to pure rotation) on turbulence. Obtain analytical solutions for the spectrum tensor, Reynolds stress tensor, Anisotropy tensor, and its invariants. Use these results to develop second order closure models.

2. Work accomplished

2.1 k-ϵ model

The k-ϵ model is still the most widely used model for computing engineering flows. In this work, we first examined the near-wall behavior of various eddy viscosity models proposed by different researchers; we then studied the near-wall behavior of terms in the k-equation budget. We found that the modeled eddy viscosity in many existing k-ϵ models does not have correct near-wall behavior, and the pressure transport term in the k-equation is not appropriately modeled. Based on the near-wall asymptotic behavior of the eddy viscosity and the pressure transport term in the k-equation, we proposed a set of new models for them. In addition, a new model for the dissipation rate is derived more rationally. See Shih (1989) for more details.

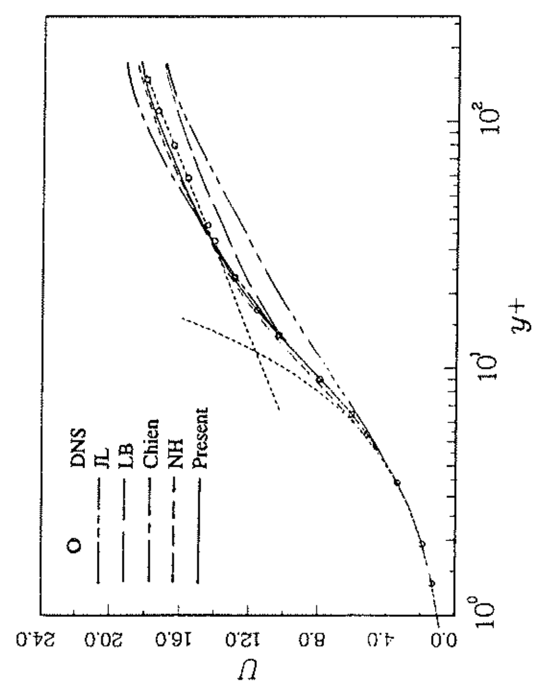

Figure 1. Mean velocity profiles in a channel

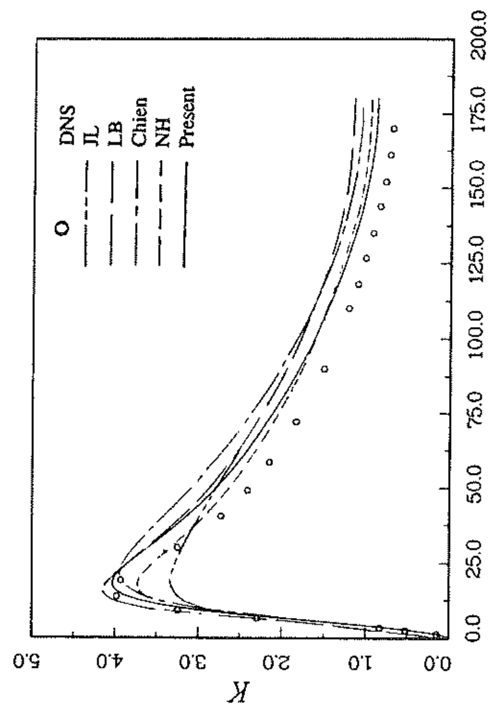

Figure 2. Turbulent energy profiles in a channel

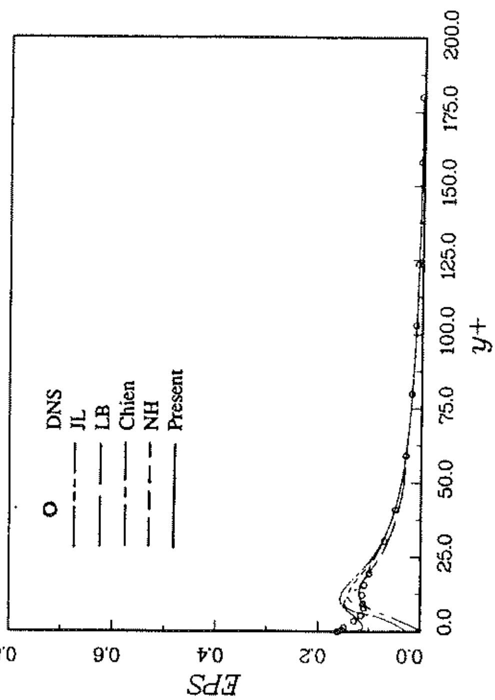

Figure 3. Dissipation rate profiles in a channel

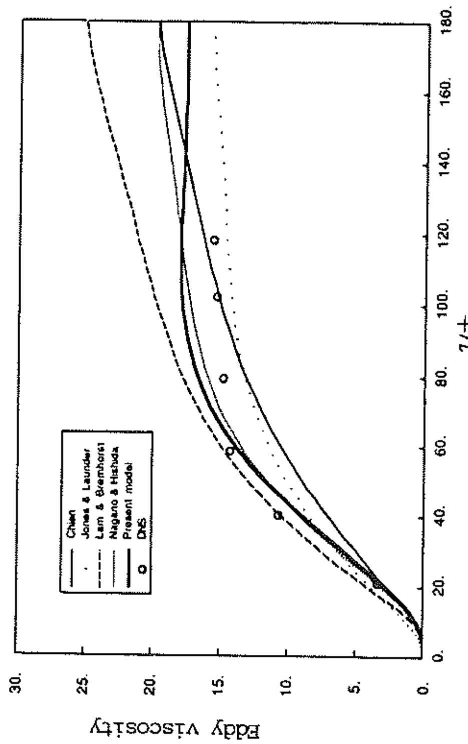

Figure 4. Eddy viscosity profiles in a channel

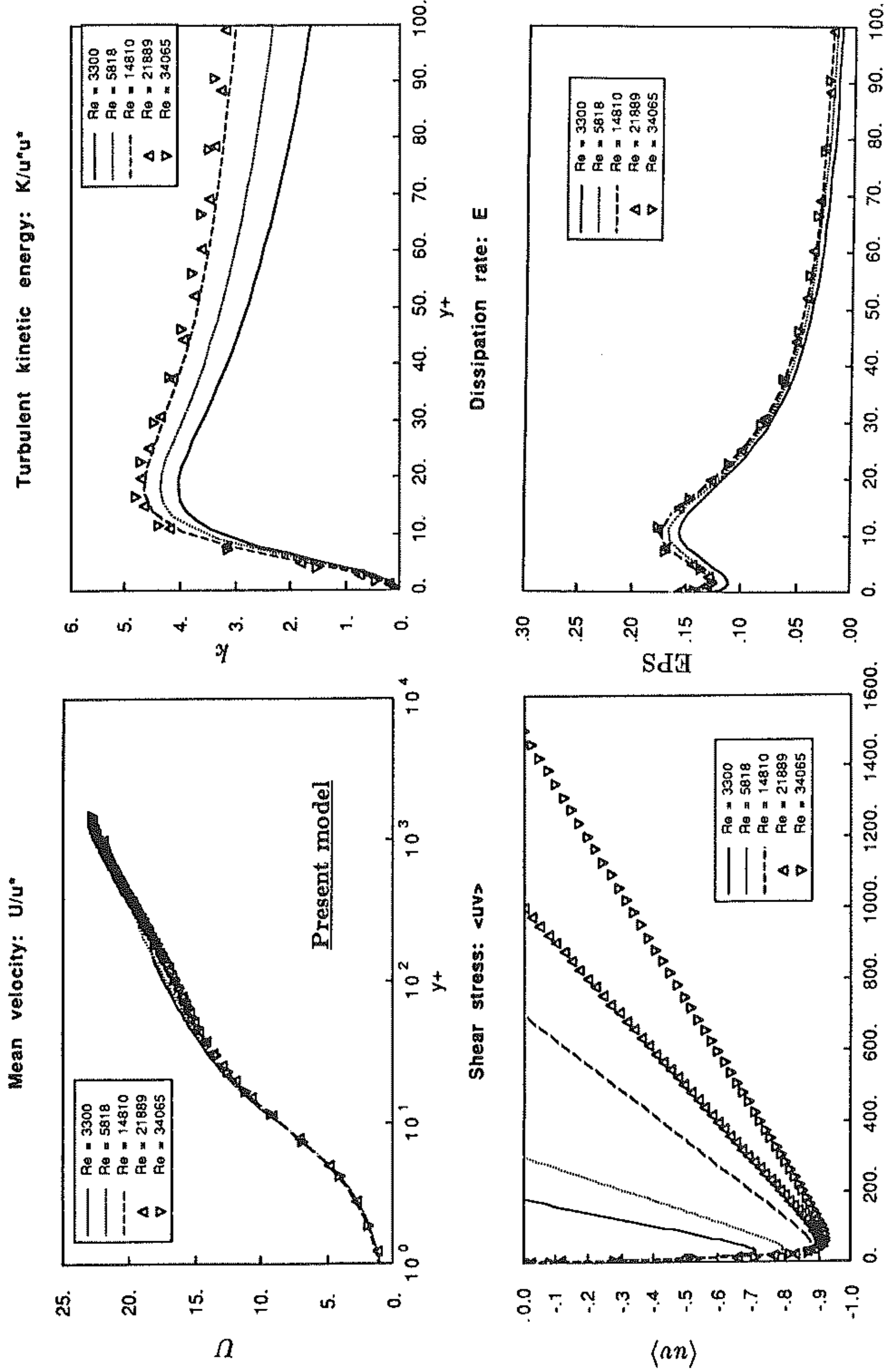

Figure 5. Profiles from present model for channel flows with different Reynolds numbers

The proposed k-ϵ model for the near-wall turbulence has been tested against direct numerical simulations of Kim et al. and compared with other k-ϵ models. The results show that the new model produces significant improvements over existing models, see figures 1 – 5. The modeled equations are given as follows:

$$k_{,t} + U_j k_{,j} = \left[\left((1+C)\frac{\nu_T}{\sigma_k} + \nu\right) k_{,j}\right]_{,j} + \nu_T S_{ij} S_{ij} - \epsilon$$

$$\epsilon_{,t} + U_j \epsilon_{,j} = \left[\left(\frac{\nu_T}{\sigma_\epsilon} + \nu\right) \epsilon_{,j}\right]_{,j} + C_1 \frac{\epsilon}{k} \nu_T S_{ij} S_{ij} - C_2 f_\epsilon \frac{\epsilon\tilde{\epsilon}}{k} + \nu\nu_T U_{i,jm} U_{i,jm}$$

$$\begin{aligned}
\sigma_k &= 1.3 \\
\sigma_\epsilon &= 1.3 \\
C_1 &= 1.45 \\
C_2 &= 2.0 \\
\tilde{\epsilon} &= \epsilon - \phi \\
\phi &= \nu k_{,j} k_{,j} / (2k) \\
\nu_T &= C_\mu f_\mu k^2 / \tilde{\epsilon} \\
C_\mu &= 0.09 \\
f_\mu &= 1 - \exp(-a_1 y^+ - a_2 {y^+}^2 - a_3 {y^+}^3 - a_4 {y^+}^4) \\
a_1 &= 6 \times 10^{-3}, a_2 = 4 \times 10^{-4}, a_3 = -2.5 \times 10^{-6}, a_4 = 4 \times 10^{-9} \\
f_\epsilon &= 1 - \frac{0.4}{1.8} \exp\left(-\left(\frac{k^2}{6\nu\epsilon}\right)^2\right) \\
C &= \frac{.05}{f_\mu [1 - \exp(-y^+)]} \\
y^+ &= u_\tau y / \nu
\end{aligned}$$

2.2 Second order modeling of near-wall turbulence

Using the near-wall asymptotic behavior of turbulence (Mansour et al. (1988)) as model constraints, we formed a set of modeled transport equations for the Reynolds-stress tensor and the dissipation rate of turbulent kinetic energy. The main emphasis was on developing a model for the "slow term" in the Reynolds-stress equation. A modeled dissipation rate equation is derived more rationally. Near the wall, a reduction in velocity fluctuations normal to the wall become significant. Because of this wall effect, the viscous diffusion term in the Reynolds-stress equations becomes the leading term, and it must be properly balanced by the slow term. We will use this as a model constraint for developing a model for the slow terms. The proposed models in this work do not contain the wall distance; therefore, they are generally suitable for an arbitrary surface. The proposed models also satisfy realizibility which ensures no unphysical behavior will occur. Here, we briefly describe and list the proposed models.

Reynolds stress equation

The exact equation for the Reynolds stress tensor is:

$$\frac{D}{Dt}\langle u_i u_j \rangle = P_{ij} + T_{ij} + D_{ij}^{(\nu)} + \Pi_{ij} - \varepsilon_{ij}$$

where $\langle\ \rangle$ stands for an ensemble average, $D/Dt = \partial/\partial t + U_k \partial/\partial x_k$. The terms P_{ij}, T_{ij}, $D_{ij}^{(\nu)}$, Π_{ij} and ε_{ij} represent the production, turbulent diffusion, viscous diffusion, velocity pressure-gradient correlation, and dissipation tensor and are identified as follows:

$$\begin{aligned}
P_{ij} &= -\langle u_i u_k \rangle U_{j,k} - \langle u_j u_k \rangle U_{i,k} \\
T_{ij} &= -\langle u_i u_j u_k \rangle_{,k} \\
D_{ij}^{(\nu)} &= \nu \langle u_i u_j \rangle_{,kk} \\
\Pi_{ij} &= -\frac{1}{\rho}\langle u_i p_{,j} + u_j p_{,i} \rangle \\
\varepsilon_{ij} &= 2\nu \langle u_{i,k} u_{j,k} \rangle
\end{aligned}$$

The velocity pressure-gradient correlation Π_{ij} is split into the rapid part $\Pi_{ij}^{(1)}$ and the slow part $\Pi_{ij}^{(2)}$:

$$\Pi_{ij} = \Pi_{ij}^{(1)} + \Pi_{ij}^{(2)}$$

The proposed model for the return term, $\Pi_{ij}^{(2)} - \varepsilon_{ij}$ is:

$$\begin{aligned}
\Pi_{ij}^{(2)} - \varepsilon_{ij} = &- \epsilon(\beta b_{ij} + \frac{2}{3}\delta_{ij})(1 - f_w) \\
&- f_w \frac{\epsilon}{\langle q^2 \rangle}[2\langle u_i u_j \rangle + 4(\langle u_i u_k \rangle n_j n_k + \langle u_j u_k \rangle n_i n_k) + 2\langle u_k u_l \rangle n_k n_l n_i n_j]
\end{aligned}$$

where n_i is a unit vector normal to the surface, and

$$\begin{aligned}
\beta &= 2 + \frac{F}{9}\{\frac{72}{R_t^{1/2}} + 80.1 \ln[1 + 62.4(-II + 2.3III)]\} \exp(-\frac{7.77}{R_t^{1/2}}) \\
F &= 1 + 27III + 9II \\
II &= -\frac{1}{2} b_{ij} b_{ji} \\
III &= \frac{1}{3} b_{ij} b_{jk} b_{ki} \\
b_{ij} &= \langle u_i u_j \rangle / \langle q^2 \rangle - \delta_{ij}/3 \\
f_w &= \exp(-(R_t/C_1)^2)
\end{aligned}$$

and $R_t = \frac{\langle q^2\rangle^2}{9\nu\epsilon}$, $C_1 = 1.358 R_{e\tau}^{0.44}$, $R_{e\tau} = u_\tau \delta/\nu$. u_τ is the friction velocity, δ is the thickness of the boundary layer or the half width of the channel.

The rapid part of velocity pressure-gradient, $\Pi_{ij}^{(1)}$ is modeled as follows (Shih and Lumley (1985, 1986)):

$$\begin{aligned}\Pi_{ij}^{(1)} &= (\frac{1}{5} + 2a_5)\langle q^2\rangle(U_{i,j} + U_{j,i}) - \frac{2}{3}(1 - a_5)(P_{ij} - \frac{2}{3}P\delta_{ij}) \\ &+ (\frac{2}{3} + \frac{16}{3}a_5)(D_{ij} - \frac{2}{3}P\delta_{ij}) + \frac{2}{15}(P_{ij} - D_{ij}) + \frac{6}{5}b_{ij}P \\ &+ \frac{2}{5\langle q^2\rangle}[(\langle u_i u_k\rangle U_{j,q} + \langle u_j u_k\rangle U_{i,q})\langle u_k u_q\rangle - \langle u_i u_p\rangle\langle u_j u_q\rangle(U_{p,q} + U_{q,p})]\end{aligned}$$

where,

$$\begin{aligned}P_{ij} &= -\langle u_i u_k\rangle U_{j,k} - \langle u_j u_k\rangle U_{i,k} \\ D_{ij} &= -\langle u_i u_k\rangle U_{k,j} - \langle u_j u_k\rangle U_{k,i} \\ P &= \frac{1}{2}P_{ii} \\ a_5 &= -\frac{1}{10}(1 + C_2 F^{1/2}) \\ C_2 &= 0.8[1 - \exp(-(R_t/40)^2)]\end{aligned}$$

Finally the model for the third moments is modeled as:

$$\langle u_i u_j u_k\rangle = -.07\frac{\langle q^2\rangle}{\epsilon}[\langle u_k u_p\rangle\langle u_i u_j\rangle_{,p} + \langle u_j u_p\rangle\langle u_i u_k\rangle_{,p} + \langle u_i u_p\rangle\langle u_j u_k\rangle_{,p}]$$

<u>Dissipation rate equation</u>

The modeled dissipation rate equation derived in this work is:

$$\begin{aligned}\epsilon_{,t} + U_i\epsilon_{,i} &= (\nu\epsilon_{,i} - \langle\epsilon u_i\rangle)_{,i} - \psi_0\frac{\epsilon\tilde{\epsilon}}{\langle q^2\rangle} \\ &- \psi_1\frac{\tilde{\epsilon}}{\langle q^2\rangle}\langle u_i u_j\rangle U_{i,j} - \psi_2\frac{\nu\langle q^2\rangle}{\epsilon}\langle u_k u_l\rangle(U_{i,jl} - U_{l,ij})U_{i,jk}\end{aligned}$$

where

$$\begin{aligned}\psi_0 &= \frac{14}{5} + 0.98[1 - 0.33\ln(1 - 55II)]\exp(-2.83R_t^{-1/2}) \\ \psi_1 &\approx 2.1 \\ \psi_2 &= -.15(1 - F) \\ \tilde{\epsilon} &= \epsilon - \frac{\nu\langle q^2\rangle_{,i}\langle q^2\rangle_{,i}}{4\langle q^2\rangle}\end{aligned}$$

The turbulent flux term $\langle\epsilon u_k\rangle$ is modeled as:

$$\langle\epsilon u_k\rangle = -.07\frac{\langle q^2\rangle}{2\epsilon}\langle u_k u_p\rangle\epsilon_{,p}$$

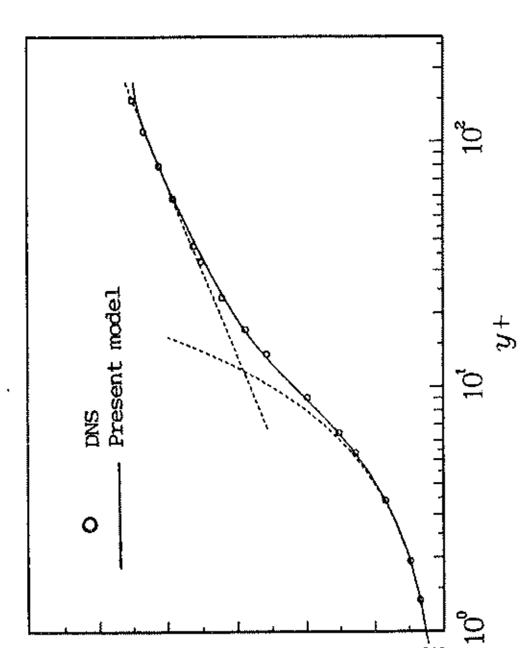

Figure 6. Mean velocity profile in a channel

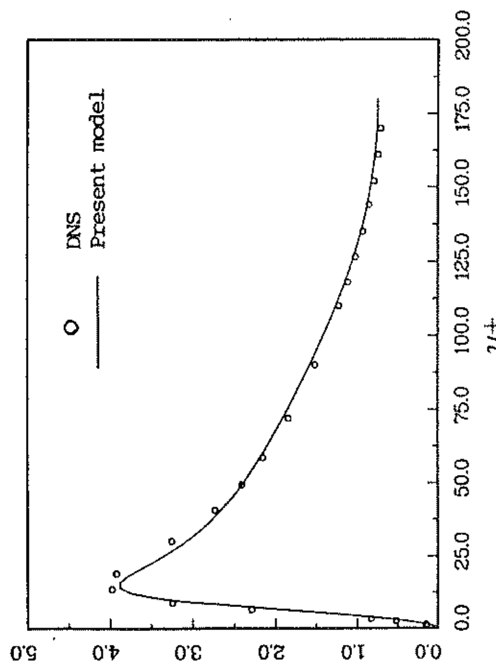

Figure 7. Turbulent energy profile in a channel

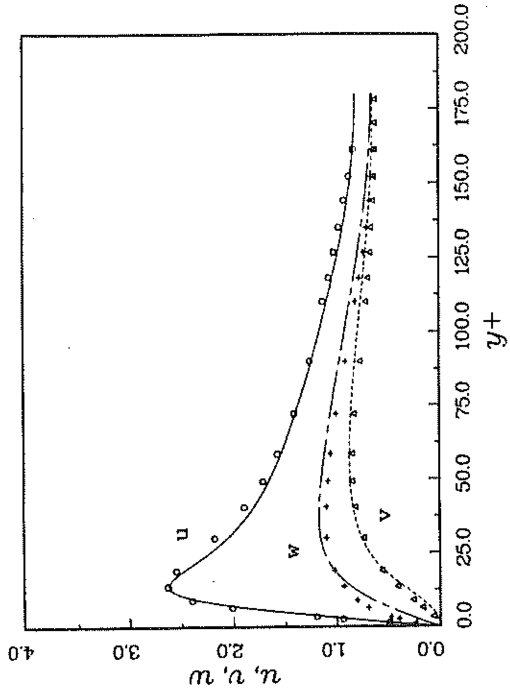

Figure 8. Turbulence intensities in a channel

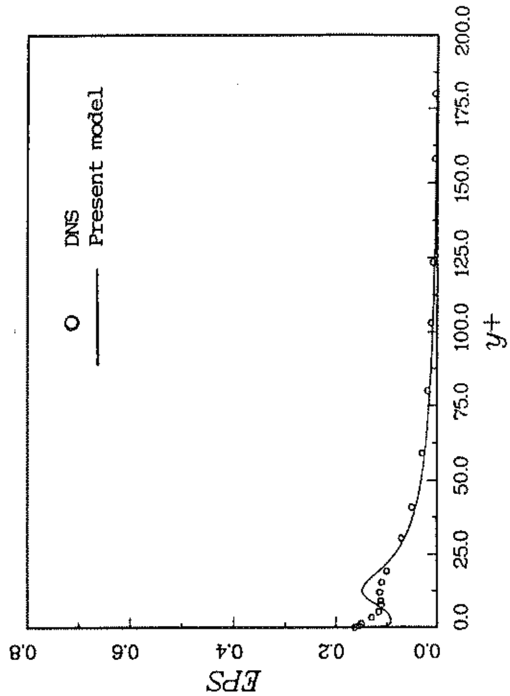

Figure 9. Dissipation rate profile in a channel

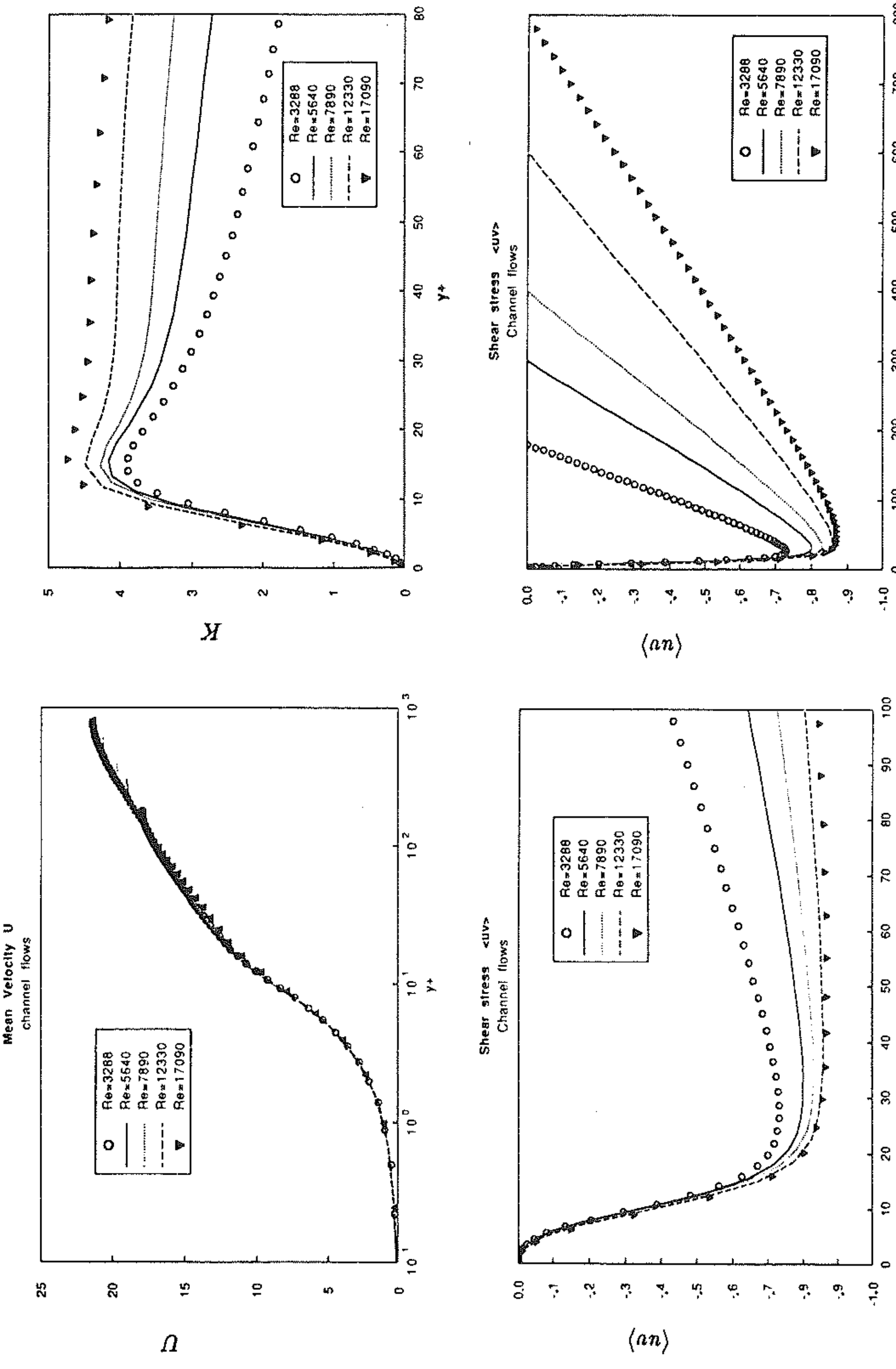

Figure 10. Profiles from present Reynolds-stress model for channel flows with different Reynolds numbers

To test the models developed in this work, we chose a fully developed channel flow as the test flow. The Reynolds number based on friction velocity and channel half width Re_τ is 180, for which direct numerical simulation (Kim et al., 1987) and experimental data (Nishino and Kasagi, 1989) are available for comparison. The modeled Reynolds stress equations for this flow are one-dimensional and steady; therefore, model testing is easy and accurate. The results of the present model compared with direct numerical simulation and other models are shown in figures 6 – 10. As the figures indicate, the proposed models capture the near-wall behavior of the turbulence and show significant improvement over previous second order models and k-ϵ models.

2.3 Second order modeling of a three-dimensional boundary layer

A study of three-dimensional effects on turbulent boundary layer was achieved by direct numerical simulation of a fully developed turbulent channel flow subjected to transverse pressure gradient. The time evolution of the flow was studied. Fourteen realizations, each starting with a different initial turbulence field, were computed and ensemble averaged. The results show that, in agreement with experimental data, the Reynolds stresses are reduced with increasing three-dimensionality and that, near the wall, a lag develops between the stress and the strain rate. In addition, we found that the turbulent kinetic energy also decreased.

To model these three-dimensional effects on the turbulence, we have tried different second order closure models. None of the current second order closure models can predict the reductions in the shear stress and turbulent kinetic energy observed using direct numerical simulations. However, we found that the proposed second order closure models developed in the previous section do at least qualitatively capture these three-dimensional effects, see figures 11 – 14. Detailed studies of the Reynolds-stresses budgets were carried out. One of the preliminary conclusions from these budget studies is that the velocity pressure-gradient term in the normal stress equation $\langle v^2 \rangle$ plays a dominant role in the reduction of shear stress and kinetic energy. These budgets will be used to guide the development of better models for three dimensional turbulent boundary layer flows.

2.4 The effect of rotation on turbulence

In addition to the above studies of second order closure models, we have carried out some RDT analysis on simple homogeneous turbulent flows. An order of magnitude analysis shows that under the condition of $S\langle q^2 \rangle/\epsilon \gg \sqrt{R_t}$, the equations for turbulent velocity fluctuations can be approximated by a linear set of equations, and if $S\langle q^2 \rangle/\epsilon \gg R_t^{3/4}$, then the turbulent velocity equations can be further approximated by an inviscid linear equation. Therefore, RDT can be used to analytically study some very basic turbulent flows such as homogeneous

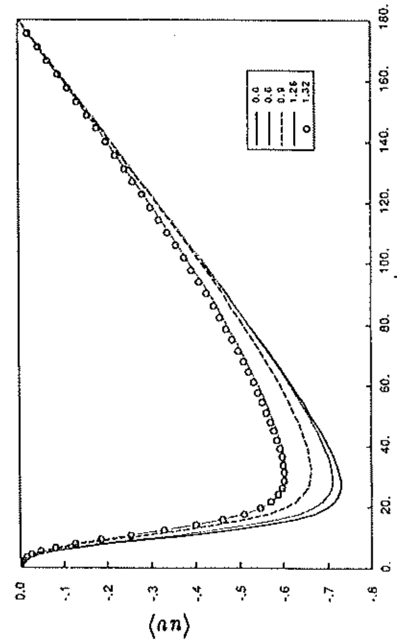

Figure 11. Evolution of shear stress in a 3-D channel

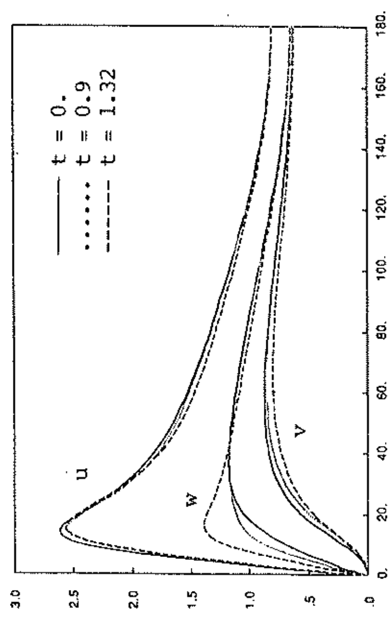

Figure 12. Turbulence intensities in a 3-D channel

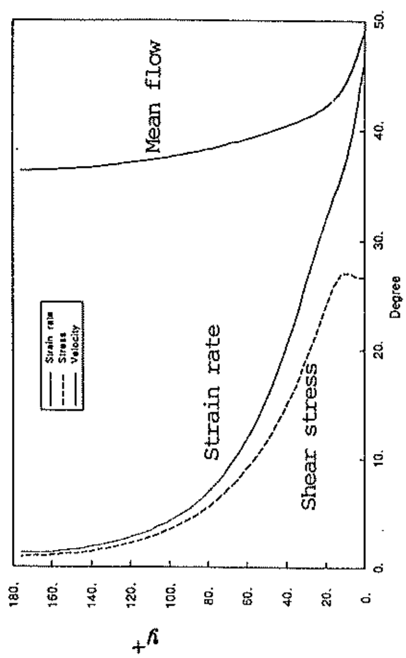

Figure 13. Angles of mean flow, shear stress & strain rate at t=1.2

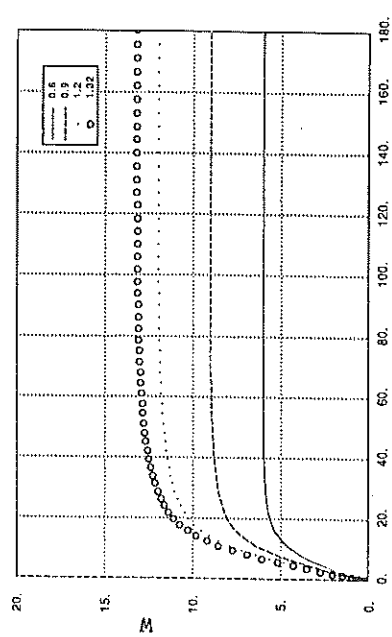

Figure 14. Evolution of mean transverse velocity

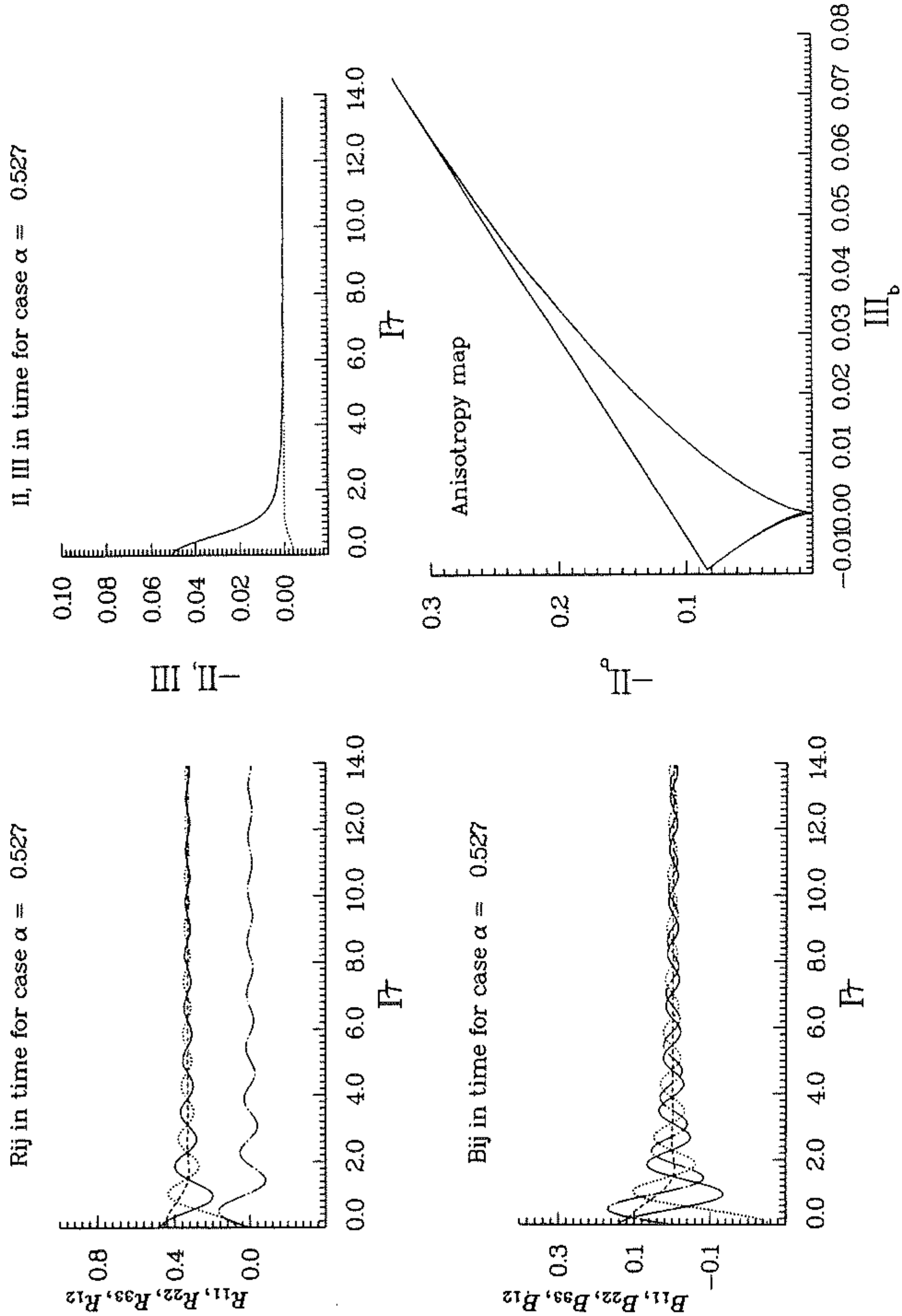

Figure 15. Typical RDT solution for the rotation of initially anisotropic homogeneous turbulence

shear flows, irrotational strain flows, and pure rotational flows. RDT analysis will hopefully bring out some new ideas in turbulence physics and modeling. Reynolds (1989) recently pointed out that all current rapid pressure-strain models are unable to predict the effects of rapid rotation on the turbulence. RDT is certainly an ideal tool to study this kind of basic turbulent flow. It can provide analytical solutions for the details of the flow field, and hence can be used to guide the development of turbulence models.

This work focuses on the effect of rapid rotation on turbulence using RDT. We obtained analytical expressions for velocity, the spectrum tensor, Reynolds-stress, the anisotropy tensor and its invariants. The solutions show that the turbulence is strongly affected by the rapid rotation. A typical case is shown in figure 15. Using RDT, we are calculating the rapid pressure-stain term exactly and we are obtaining very useful information for developing corresponding turbulence models.

3. Future plans

1. Using direct numerical simulation data (Moin et al., 1989 and Spalart, 1989), we are planning to improve the second-order closure models proposed in this work for three dimensional boundary layers.

2. Extend second order closure models to near-wall turbulent heat fluxes.

3. Use the information obtained from RDT to model the effects of rapid rotation on the turbulence. It appears that at least the quadratic terms of mean velocity gradient are necessary in the rapid pressure-stain model.

4. Modeling the effects of buoyancy on the turbulence.

5. Third order modeling of shearless turbulent mixing layer — using moment generating function method. This type of model will be needed when third order moments play a dominant role in transferring of momentum and energy, such as in a convective planetary boundary.

6. Explore the potential of the RNG method in one-point turbulence closure models.

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