An experimental study of the effects of rapid rotation on turbulence

By S. V. Veeravalli

1. Introduction

Experiments (Traugott, 1958, Wigeland & Nagib, 1978 and Jacquin et al., 1988), large eddy simulations (Bardina et al., 1985) and direct numerical simulations (Bardina et al., 1985 and Speziale et al., 1987) all show that rapid rotation (i.e. the rotation time scale $(1/\Omega)$ $<<$ the turbulence time scale) has a striking effect on homogeneous nearly isotropic turbulence. The cascade process is effectively inhibited by rotation, and thus dissipation is greatly reduced. Some attempts have been made to incorporate this effect in turbulence models (Bardina et al., 1985 and Speziale et al., 1987). Numerical simulations further showed the somewhat surprising result that anisotropic homogeneous turbulence subjected to rotation tended towards an isotropic state; however, the residual anisotropy was not zero. Reynolds (1989) performed a detailed analysis using Rapid Distortion Theory (RDT) and showed that a reduction in the anisotropy is indeed expected and if the anisotropy is produced by irrotational strain then the anisotropy tensor b_{ij} is asymptotically driven to half its initial value.

Our objective is to extend the work of the experiments mentioned above to lower turbulent Rossby numbers ($R_{o\lambda} \equiv (\frac{1}{3}q^2)^{\frac{1}{2}}/(\lambda\Omega)$ $\approx$ 0.07 to match those used in the numerical simulations — λ is the Taylor microscale and q^2 is twice the turbulent kinetic energy) and to confirm some of the results obtained by Reynolds (1989) for anisotropic turbulence.

2. Previous work

Experimental studies of rotating turbulence can be broadly classified into two groups: *a*) rotating tank experiments and *b*) wind tunnel experiments.

The work of Ibbetson & Tritton (1975) and Hopfinger et al. (1982) belong to category *a* . Ibbetson & Tritton dropped a grid into a rotating tank and found that the turbulence behind the grid decayed much faster in the presence of rotation. Hopfinger et al. (1982) used a shaking grid to generate turbulence and found that away from the grid the flow exhibited a strong tendency towards 2-dimensionality and essentially consisted of columnar vortices aligned with the axis of rotation.

Traugott (1958), Wigeland & Nagib (1978) and Jacquin et al. (1988), on the other hand, imposed solid body rotation on grid turbulence in a wind tunnel and thus these experiments approximate homogeneous turbulence better. The smallest value of $R_{o\lambda}$ achieved in the Wigeland & Nagib (1978) experiment was approximately 0.4 while Jacquin et al. (1988) obtained a value of 0.3. These experiments showed that the mildly anisotropic grid turbulence tended towards isotropy and that the kinetic energy decay was greatly reduced due to an inhibition of the cascade process. The flows did not exhibit a strong tendency towards 2-dimensionality; however, the length scales along the axis of rotation grew at a much faster rate compared to the non-rotating case and showed departures from the behaviour expected in isotropic flow (the direct numerical simulation results of Bardina et al., 1985 show a similar behaviour for the length scales).

Large eddy simulations and direct numerical simulations (Bardina et al., 1985 and Speziale et al., 1987) also showed the dramatic suppression of the spectral transfer term observed in experiments. In particular, Speziale et al. (1987) found that the development of the energy spectrum $E(\underline{\kappa}, t)$ agreed extremely well with,

$$E(\underline{\kappa}, t) = E(\underline{\kappa}, t_o)\, exp[-2\nu\kappa^2(t - t_o)] \tag{1}$$

which is what is expected for purely viscous decay with the spectral transfer term equal to zero. (ν is the kinematic viscosity, $\underline{\kappa}$ the wave number vector and the development is for $t > t_o$.) Speziale et al. (1987) also showed that homogeneous (unbounded) turbulence does not undergo Taylor-Proudman reorganization — the analysis is outlined below. The results of Reynolds (1989) are also summarized below.

3. Theory

In the equations to follow, the mean velocity is constant, $\underline{u}$, represents the instantaneous velocity, $\underline{\Omega}$ the rotation vector, p the instantaneous pressure, and ρ the density. The Navier-Stokes equations in a frame rotating with the mean rotation $\underline{\Omega}$ and moving with the mean speed are,

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} - 2\epsilon_{ijk}\Omega_j u_k + \nu\frac{\partial^2 u_i}{\partial x_j \partial x_j} \tag{2a}$$

(the centrifugal acceleration term has been included in the pressure) and the continuity equation is

$$\frac{\partial u_i}{\partial x_i} = 0. \tag{2b}$$

If the Rossby number $\epsilon/(\Omega q^2)$ is $<< 1$ (where $\Omega = |\underline{\Omega}|$) and the Ekman number $\nu/(\Omega L^2)$ is also $<< 1$ (where L is a typical length scale) and the flow is nearly steady, then we have,

$$\frac{\partial p}{\partial x_i} = -2\epsilon_{ijk}\Omega_j u_k \tag{3}$$

The curl of equation (3) then yields the Taylor-Proudman theorem

$$\Omega_j \frac{\partial u_k}{\partial x_j} = 0, \tag{4}$$

indicating that velocity gradients along the axis of rotation are suppressed.

However, for unbounded flows, the inviscid linearized equations (RDT equations),

$$\frac{\partial u_i}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} - 2\epsilon_{ijk}\Omega_j u_k, \tag{5}$$

admit travelling wave solutions of the form (Greenspan, 1968 and Phillips, 1963),

$$u_i = A_i\, exp[i(\underline{\kappa}\,.\,\underline{x} - \alpha(\underline{\kappa})\,\Omega\, t)]. \tag{6}$$

Thus,

$$\frac{\partial u_i}{\partial t} = -\alpha(\underline{\kappa})\,\Omega\, A_i\, exp[i(\underline{\kappa}\,.\,\underline{x} - \alpha(\underline{\kappa})\,\Omega\, t)]. \tag{7}$$

And hence (Speziale et al., 1987),

$$\left|\frac{\partial u_i}{\partial t}\right| \Big/ |\epsilon_{ijk}\Omega_j u_k| = O(1) \tag{8}$$

always. Hence, equation (3) is not applicable and no Taylor-Proudman reorganization occurs for homogeneous turbulent flows undergoing rapid rotation. Equation (7) indicates that the time scale of the velocity fluctuations is $O(1/\Omega)$; however, the non-linear term in equation (2a) is still negligible as $\epsilon/(\Omega q^2)$ is $<< 1$.

Reynolds (1989) showed, using the inviscid RDT equations (5), that for isotropic homogeneous turbulence, the spectrum tensor $E_{ij}(\underline{\kappa})$ is unchanged under rotation. He also studied the effect of rotation on anisotropic turbulence by defining the following 'structure' tensor,

$$Y_{ij} \equiv \int \frac{\kappa_i \kappa_j}{\kappa_n \kappa_n} E_{mm}(\underline{\kappa}) d\underline{\kappa}, \tag{9a}$$

with the corresponding anisotropy 'structure' tensor

$$y_{ij} \equiv \frac{Y_{ij} - q^2\delta_{ij}/3}{q^2}. \tag{9b}$$

(Note that if the turbulence is independent of x_1 then $Y_{11} = 0$ and $y_{11} = -\frac{1}{3}$.) It can then be shown (Reynolds, 1989) that Y_{ij} and consequently y_{ij} is unaffected by rotation. Further,

$$b_{ij} \rightarrow -\frac{y_{ij}}{2} \;\; as \;\; t \rightarrow \infty. \tag{10}$$

Reynolds (1989) also showed that 2-D 1-C (two-dimensional one-component) turbulence and 2-D 2-C turbulence tended to 2-D 3-C turbulence with the asymptotic state given by equation (10).

The turbulence generated in the laboratory is necessarily bounded and not strictly homogeneous due to the presence of the tunnel walls. It is unclear whether relations (8) and (10) would be strictly valid in such a case. If the turbulence does tend towards 2-dimensionality, then there are two regimes of interest:

$$\text{a)} \qquad \frac{\epsilon}{\Omega q^2} << 1 \text{ but } R_{o\eta} \equiv \frac{v}{\Omega \eta} > 1$$

where η and v are the Kolmogorov length and velocity scales respectively and

$$\text{b)} \qquad \frac{\epsilon}{\Omega q^2} << 1 \text{ and } \frac{v}{\Omega \eta} << 1.$$

(Jacquin et al., (1988) observed that when $v/(\Omega\eta) = O(1)$, then the turbulence decay rate changes sharply from $q^2 \sim x^{-1.3}$ to $q^2 \sim x^{-1}$, indicating that a third regime could exist between 'a' and 'b'.) If the dissipation scales do become axisymmetric for very small $R_{o\eta}$, then one would expect different decay rates for the axial and transverse velocity components (Batchelor, 1946) and the turbulence could then become anisotropic.

4. Experiments

The experiments will be carried out in three stages. Preliminary measurements will be done in a 15.2 cm diameter tunnel operated at a maximum speed of 10 m/s and capable of rotation rates up to 200 rad/s. The second series of experiments would be conducted in a 76 cm diameter facility capable of a peak speed of 30 m/s and a peak rotation rate of 80 rad/s. Finally, the 76 cm facility would be placed in a pressure vessel and operated at a pressure of approximately 16 atmospheres (and at approximately 8 m/s) in order to significantly increase the turbulent Reynolds number.

The design of the 15.2 cm rotating rig is complete and a schematic diagram is shown in Figure 1. The design is similar to the one used by Wigeland & Nagib (1978). The flow is provided by a centrifugal blower; it is then passed through a settling chamber and a contraction before entering the rotating section. The rotating section consists of a honeycomb of sufficient pressure drop to induce solid body rotation in the flow and a turbulence generating grid. The rotating turbulent flow is then studied in the stationary test section. The in-house constant temperature anemometers (Microscale HWM-100) were thoroughly tested and were found to be adequate for the initial measurements. The software necessary for data acquisition and analysis is being developed. A calibration set-up for x-wires will be fabricated. We are also currently working on the design of the larger (76 cm) facility.

The types of the experiments and the measurements of interest are described below.

Detailed measurements of decaying nearly isotropic grid generated turbulence will be made in the rotating rig and compared with the non-rotating case. The quantities of interest would be the Reynolds stresses, dissipation rates and length scales. Two point correlation functions $R_{\alpha\alpha}(r,0,0)$ and $R_{\alpha\alpha}(0,r,0)$, (no sum on α) could be measured to compare the growth rates of the length scales obtained from axial and transverse separations (the axis of rotation has been assumed to be along x, see figure 1) — none of the previous measurements shows a comparison of the type; $R_{33}(r,0,0)$ vs. $R_{33}(0,r,0)$. In addition the spectral transfer term could be measured, both directly and from the decay of the energy spectrum (c.f. Yeh & Van Atta, 1973).

The Rossby number used in the numerical simulations ($R_{o\lambda} \approx 0.07$) was chosen as the principal design parameter for the 15.2 cm facility; however, it should be capable of operating in regime 'b' discussed above. The expression for $R_{o\eta}$ when $q^2 \approx AU^2(x/M)^{-1}$ is given by

$$R_{o\eta} = \frac{(\frac{1}{2}A)^{\frac{1}{2}} M^{\frac{1}{2}} U^{\frac{3}{2}}}{2\nu^{\frac{1}{2}} \Omega x}. \tag{11}$$

(Here, M is the mesh size, U is the mean axial velocity and x is the downstream distance). Thus, with a judicious choice of M, U, Ω it should be possible to obtain $R_{o\eta} \approx 0.2$ within the test section (approximately 0.75 m long). (Note that one can't simply increase Ω to decrease $R_{o\eta}$ because when $\Omega/(RU) \approx 1$, where R is the radius of the tunnel, the vortex exiting the tunnel could breakdown (Dellenback et al., 1988), creating disturbances in the test section.)

A second set of experiments will be conducted to study the effect of rotation on anisotropic turbulence. It would not be possible to verify all the results presented in Reynolds (1989), especially those pertaining to 1C and 2C turbulence, in the laboratory; however, one could examine the validity of equation (10). The intensity of the axial velocity fluctuations could be reduced with respect to the transverse components by passing the flow through a contraction and the behaviour of b_{ij} downstream of the contraction could be studied. Additionally, a small difference in the intensities of the transverse fluctuations could be introduced by using a parallel bar array instead of a bi-planar grid to generate the turbulence (Veeravalli & Warhaft, 1989).

Finally, it would interesting to study the dispersion of a passive scalar in this flow with a view to examining the difference in mixing rates along the axis and in the transverse plane.

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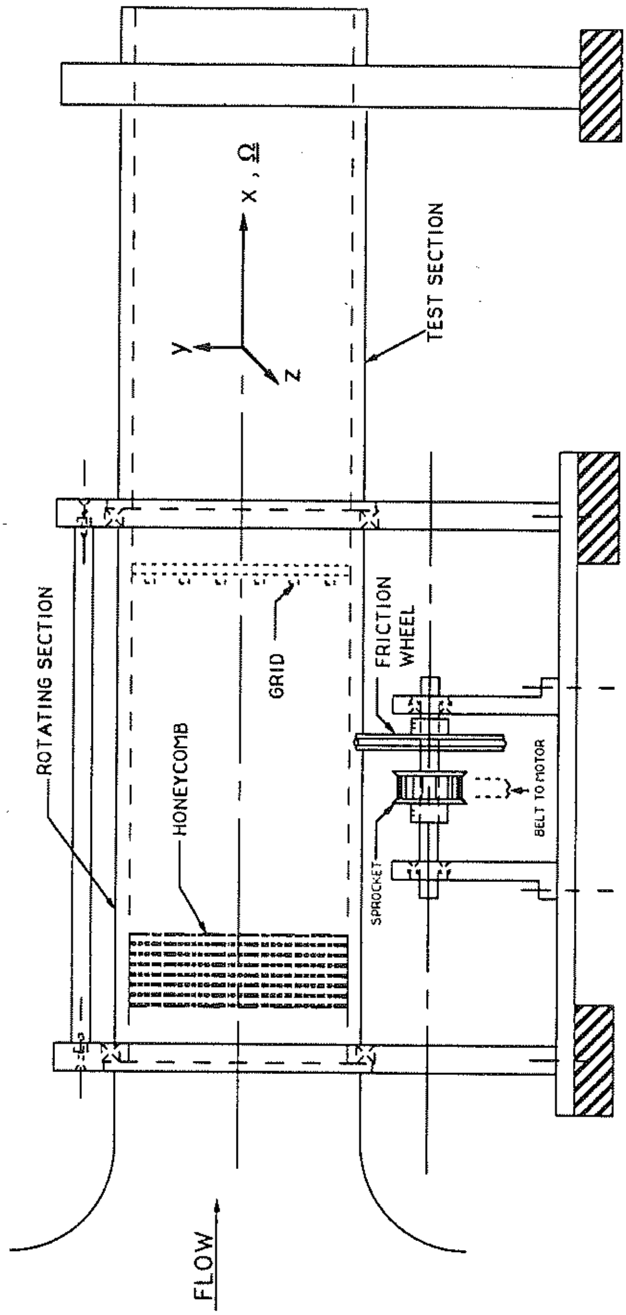

FIGURE 1. Schematic diagram of the Rotating Flow Rig.