A numerical method for prediction of compressible turbulent flows with closure models

By P. G. Huang

A new computer code to solve the time-averaged Navier-Stokes equations is developed. Many of the state-of-the-art numerical techniques and algorithms have been tested and implemented in the program in order to achieve a better numerical accuracy and code efficiency.

Various turbulence models are tested for a wide range of flows. The initial focus has been on "two-equation" eddy-viscosity models, which are the most advanced available in current compressible-flow codes. The long term goal will be to test Reynolds-Stress models and to explore their performance in the high-Mach-number range.

Although testing and improvement of turbulence models for supersonic and hypersonic flows is the primary objective of this research (70%), part of the effort (30%) has been devoted to analyzing the vortex breakdown phenomena using the new computer program. Some preliminary results on the breakdown of a vortex flow in a tube are reported. Although calculations are at the moment restricted to 2-D axisymmetric equations, an extension of this work to 3-D geometry is proposed.

1. Introduction

Traditional finite difference methods are accurate for smooth flows but give rise to over/under-shoots in region where a large gradient of the dependent variable is encountered. Although a stable solution can be obtained by adding ad-hoc artificial diffusivities, one must be careful that this solution may be corrupted, for the excessive diffusion added may smear the sharp gradient. Even in the case when one adds only enough diffusion to achieve a stable solution, slight over/under-shoots of the solution would still sometimes be observed. A mild over/under-shoot causes little difficulty in a pure Navier-Stokes calculation while it may bring about divergence of a time-averaged Navier-stokes calculation employing a closure-type of turbulence model. One possible cause of the divergence is the generation of negative turbulence quantities, such as k and ϵ.

To search for an accurate numerical scheme that will preserve the realizability property, two numerical concepts; namely TVD (Harten, 1983; Yee, 1987) and FTC (Boris and Book, 1973; Zalesak, 1979), are tested against a wide range of problems. Conclusions emerged from this investigation are reported in Section 2.

The recent interest in applying Reynolds-stress models in complex turbulent flow calculations has urged the need for a stable and efficient algorithm. The stability problem arises mainly from the stiffness of the source terms and the lack of apparent "eddy viscosity" in the mean-flow equations, while the need for numerical efficiency is due to the large number of turbulence equations that have to be solved together with the mean-flow equations and the strong coupling between the mean-flow and turbulence quantities.

While a number of numerical treatments have been proposed in some existing compressible codes to remove the source-term-related stiffness difficulties, these methods are not general and have suffered from numerical inefficiency. Moreover, although techniques involving simultaneous solution of the mean-flow and turbulence equations have open a door for handling the strong equation coupling, such methods require the inversion of a large matrix and thus are not attractive to be used in practical calculations. Last but not least, problems associated with the lack of apparent "eddy viscosity" in Reynolds-stress calculations have only begun to receive some attention and the extension of the available numerical treatments to compressible calculation is yet to be seen. Section 3 is devoted to a brief discussion of the current progress in dealing with the above-mentioned difficulties.

Section 4 presents some calculations of vortex breakdown in a tube. The importance of the boundary condition treatments is addressed in this section. Finally, a discussion on the future activities is given in Section 5.

2. Numerical schemes

Comparisons were made for the following schemes:

FCT: Zalesak (1979)

TVD (An excellent review has been provided by Yee (1987)): Harten and Yee, Yee-Roe-Davis, Roe, van Leer and Colella and Woodward

A wide range of problems have been chosen to test the numerical schemes:

1-D scalar: advection of square, Gaussian and semi-circle profiles.

2-D scalar: convection transport of a step profile and the solid body rotation of a block-profile.

2-D Euler: inviscid channel flow and oblique shock reflection.

2-D Navier-Stokes: wall-driven cavity, laminar and turbulent (two-equation model) boundary layers ($M = 0.3$, 2 and 10).

The study has shown the "Superbee" scheme by Roe emerged to be the best in terms of overall numerical performance while Harten and Yee's "Minmod" scheme is the simplest and the most stable one. Figures 1 and 2, respectively, show the predicted velocity and skin friction profiles of a laminar boundary layer. The calculation employs the Superbee scheme and shows good agreement with Blasius solution.

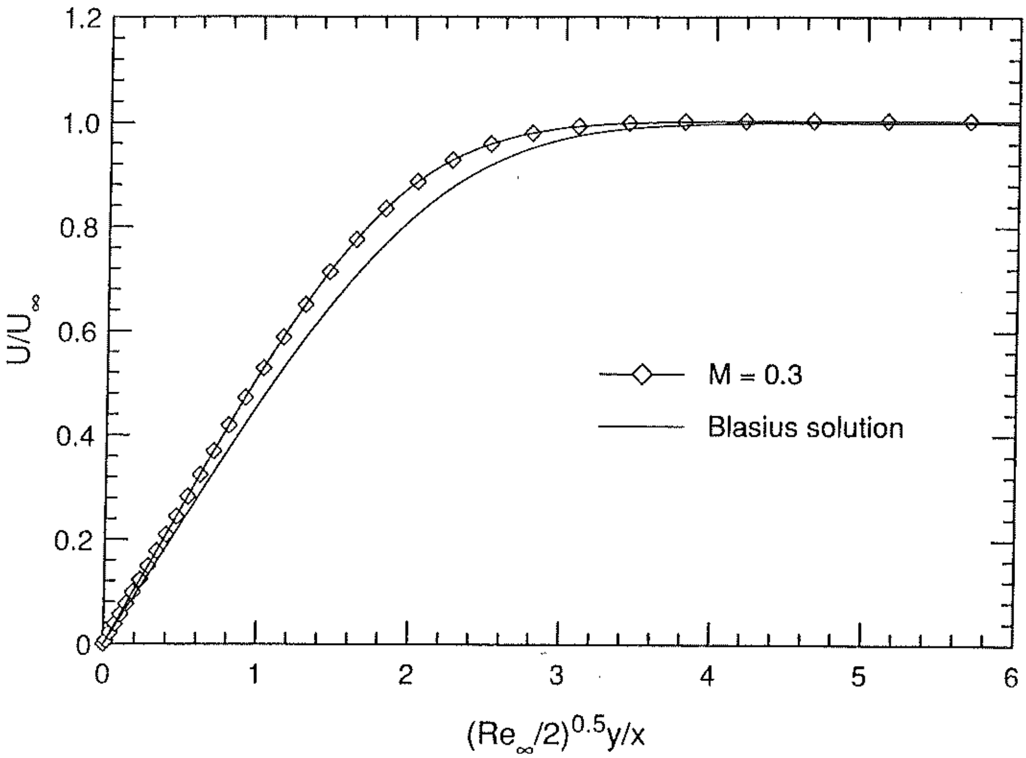

FIGURE 1. Laminar boundary layer - velocity.

3. Numerical algorithm

The lower upper symmetric successive over-relaxation method (LU-SSOR) developed by Jameson and Yoon (1986) will be the base of the present development. This algorithm is closely related to the symmetric Gauss-Seidel line relaxation method of MacCormack (1985). In our development, we show the basic equivalence between the LU-SSOR method and the Symmetric Gauss-Seidel point relaxation method (LU-SGS) (Gnoffo, 1986).

A modified version of the LU-SSOR (or LU-SGS) has been successfully implemented in the new code. The attractive features of the new code are:

(1) Solution of large block banded Matrices is not required.

(2) Vectorization of the code is possible.

(3) Relaxation can be done with infinite time step.

(4) Second-order time accuracy can be achieved with a finite time step.

(5) The operation count of the present method per iteration is only slightly greater than that of an explicit method.

(6) The method is capable of handling unstructured grids.

The code has been fully vectorized and it runs about 15 times faster than its scalar counterpart in Cray YMP.

To maintain the coupling between the mean-flow and the turbulence equations while not to involve the inversion of a big matrix, an algorithm is developed to

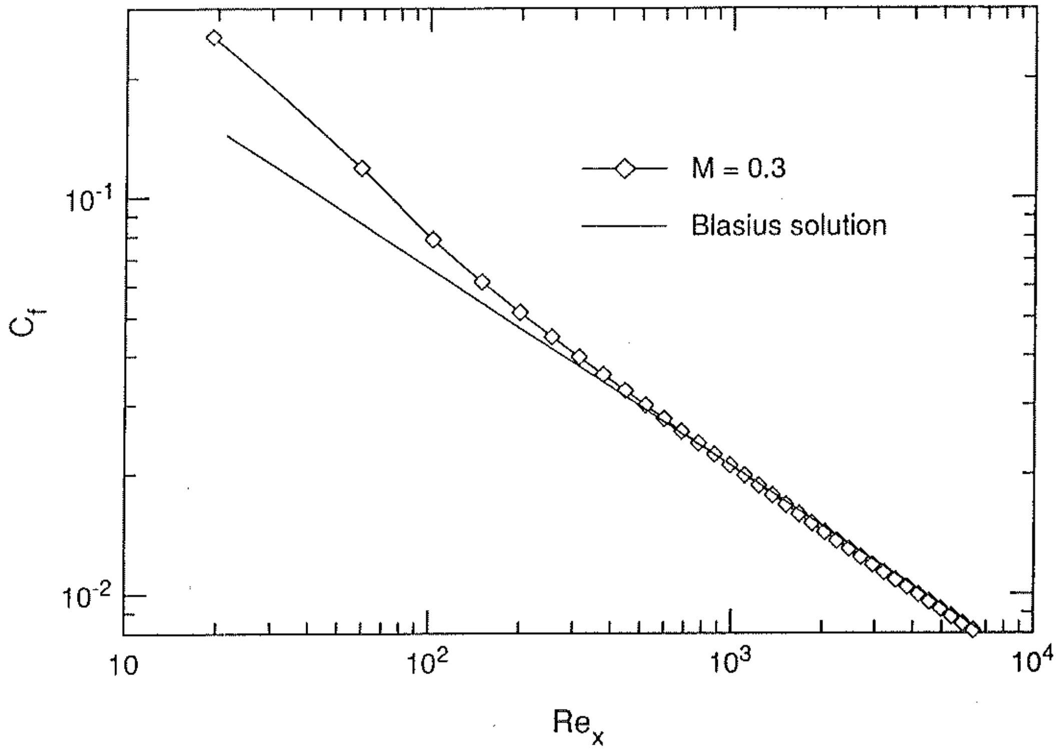

FIGURE 2. Laminar boundary layer - C_f.

obtain the solution by iterating between the mean flow and turbulence equations when performing the LU point-relaxation sweep. As a result, a large matrix is divided into two smaller matrices and they are solved sequentially in a point-by-point fashion. It has been found that this method did not give rise to numerical instability and, for a two-equation type of turbulence model, the overall computer time saving may be as high as 30%.

Because turbulence equations contain large source terms and these terms are often nonlinear, implicit treatments of the source terms are generally preferred. The approach used in the present study is similar to the stabilizing strategy of Huang and Leschziner (1986). The source is first divided into positive and negative parts. For turbulence quantities that are by nature positive, the measure is to treat the negative part of the source implicitly while the positive part is handled explicitly. The implicit treatment of the negative source is to ensure the positiveness of the solution at all time during iterations and the explicit treatment of the positive source is to preserve a diagonal domination of the matrix. This method has been found to be very robust and the overall convergence rate is satisfactory.

Figure 3 shows the predicted skin friction profiles of a Mach 2 turbulence boundary layer. Turbulence Models chosen were the $k - \epsilon$ models of Chien (1982), and Jones and Launder (1973), respectively. The results have shown

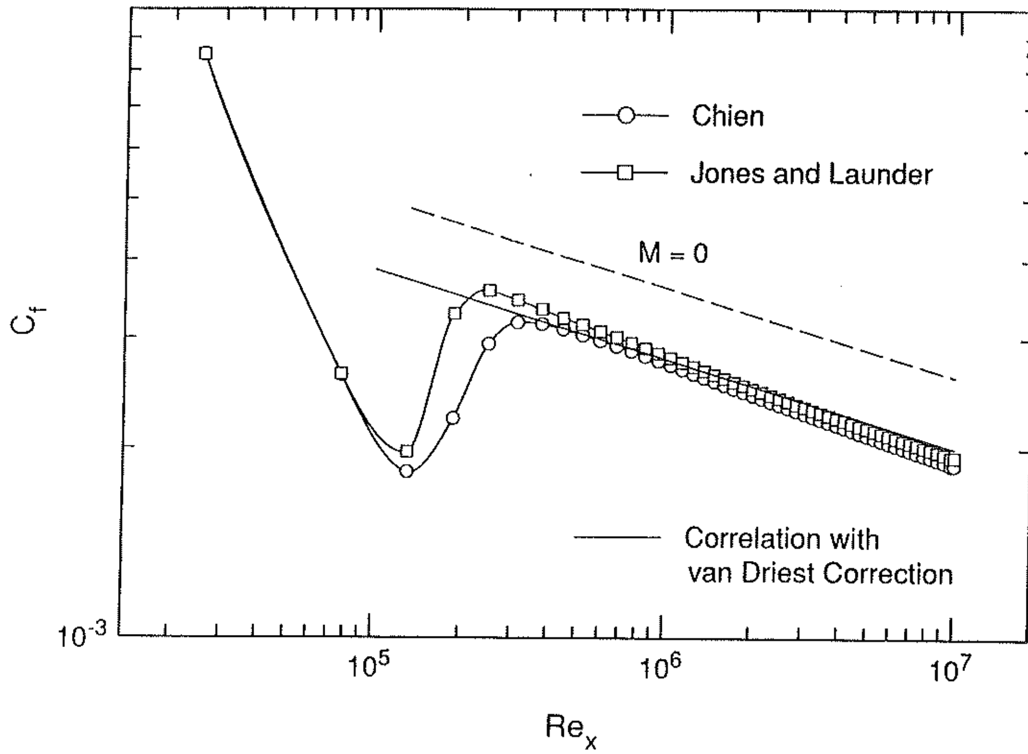

FIGURE 3. Turbulent boundary layer $-C_f$ at $M = 2$.

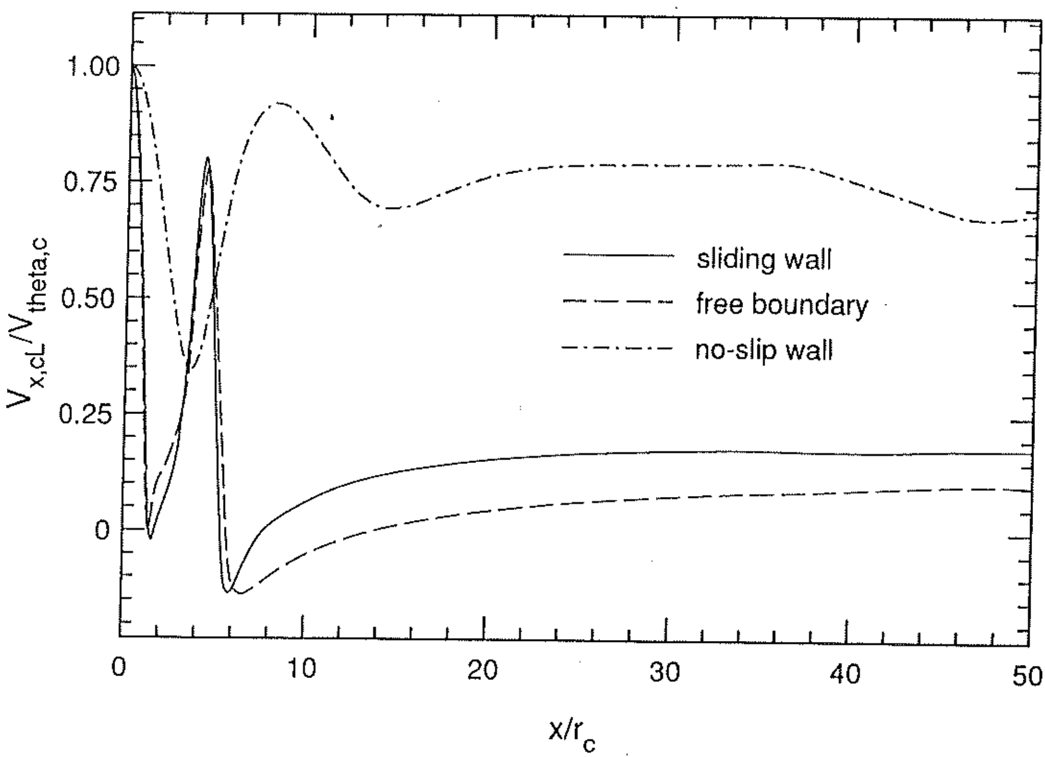

FIGURE 4. Vortex breakdown in a tube - $V_{x,cL}$.

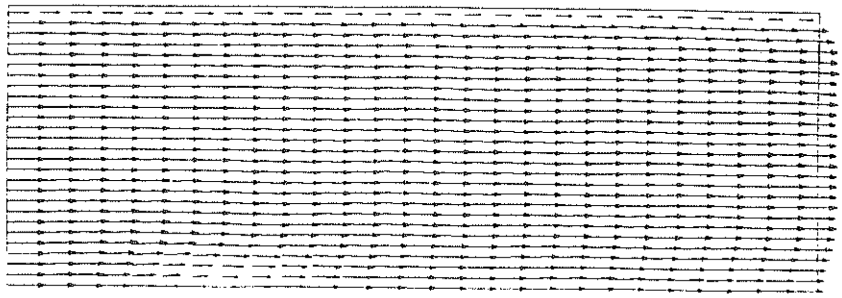

FIGURE 5. Vortex breakdown - no-slip top wall.

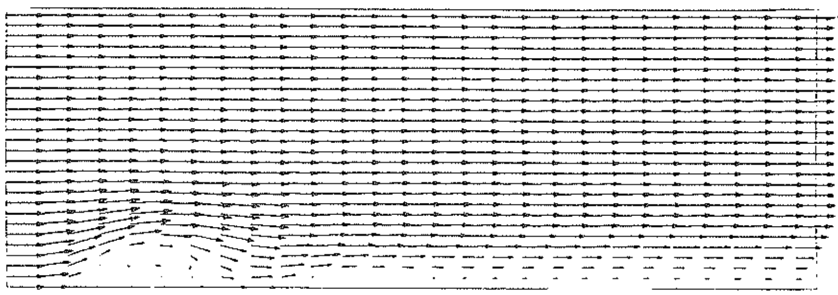

FIGURE 6. Vortex breakdown - sliding top wall.

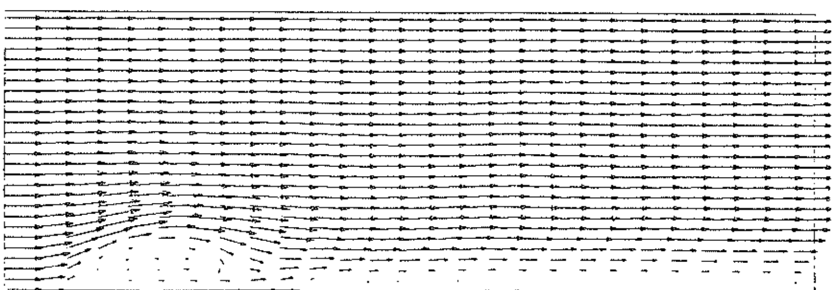

FIGURE 7. Vortex breakdown - free-stream top boundary

good agreement with experimental correlation.

4. Some preliminary results on vortex breakdown

The current investigation is restricted to the study of a vortex breakdown of the bubble-type, with the assumption of a 2-D axisymmetric flow. This problem is closely related to the one studied by Grabowski and Berger (1976) and later by Lugt and Gorski (1988).

Reynolds number, $Re = v_{\theta,max} r_c / \nu$, is chosen to be 500 based on the core radius, r_c, where the tangential velocity has its maximum value,$v_{\theta,max}$. The calculation domain is assumed to have a radius of $5r_c$ and an axial length of $50r_c$. The inlet conditions are given as follow;

$$u_x = 1/S,$$

$$v_r = 0 \text{ and} \tag{1}$$

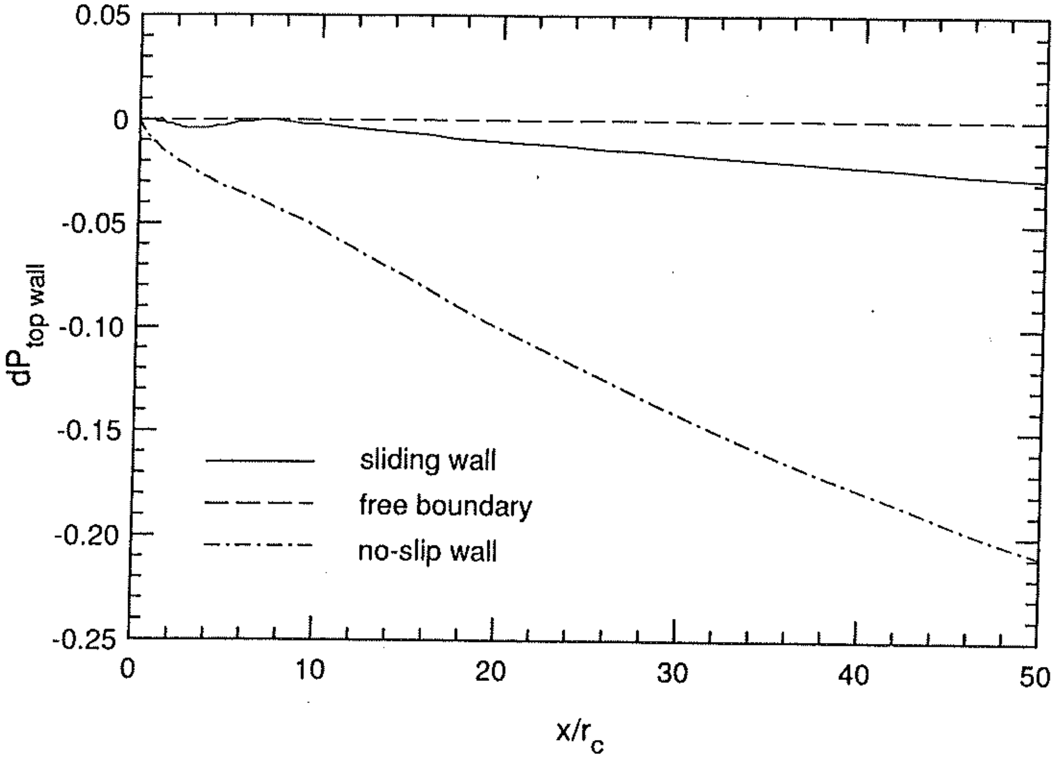

FIGURE 8. Vortex breakdown in a tube - *dp*.

$$v_\theta = \frac{1}{r}\frac{1-e^{-r^2}}{1-1/e}$$

where S, defined as $v_{\theta,max}/v_{x,inlet}$, is a swirl parameter and in the discussion that follows it is assumed to be unity. The tangential velocity is chosen to approximate a wing-tip vortex. At the axis, $v_r = v_\theta = 0.$, $\partial v_x/\partial r = 0$ and $\partial p/\partial r = 0$. The outflow conditions are $\partial/\partial x = 0$ for all variables.

In the present study, we have found that the top wall boundary-condition treatment has a very strong influence on the dynamics of breakdown suggesting that the breakdown is very sensitive to pressure gradient. Figure 4 shows the centerline axial velocity profiles for three different top wall boundary-condition treatments, no-slip, sliding ($v_{x,\text{wall}} = v_{x,\text{inlet}}$) and free-stream boundaries. It shows that the no-slip boundary introduces only slight retardation of the axial flow while the other two boundary-condition treatments give rise to flow reversal. This can be illustrated more clearly in the velocity vector plots, Figures 5 to 7. The figures show that for the sliding and the free-stream boundary treatments, the flow has formed a small well-pronounced bubble, similar in shape to that observed experimentally by Uchita et al. (1985) and Faler and Leibovich (1978).

The surprising outcome can be explained by examining the pressure distribution. Figure 8 provides a comparison of the pressure-drop profiles along the top boundary. The figure has shown that the no-slip wall has induced a favor pressure gradient equivalent to 0.4% of inlet momentum flux, $\rho v_{x,inlet}^2$, per r_c distance. This favor pressure gradient has delayed the flow separation and thus resulted in a quite unexpected flow behavior.

5. Future plans

(1) Implement the Reynolds-stress-transport models into the new code.
(2) Compare the performance of the turbulence models.
(3) Test new pressure-strain models.
(4) Examine compressibility correction terms.
(5) Propose near-wall correction for Reynolds-Stress-transport models.
(6) Integrate to the wall or use wall functions?
(7) Investigate effects of turbulence on the dynamics of vortex breakdown.
(8) 3-D simulation of vortex breakdown.

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