Recursive renormalization group theory based subgrid modeling

By Ye Zhou

The essential purpose of this research is to advance the knowledge and understanding of turbulence theory. Specific problems to be addressed will include studies of subgrid models to understand the effects of unresolved small scale dynamics on the large scale motion which, if successful, might substantially reduce the number of degrees of freedom that need to be computed in turbulence simulation.

1. Motivation and objectives

The study of turbulence is one of the most challenging and active research topic in classical physics. Since turbulence, by its usual definition, implies the existence of an extremely large number of degrees of freedom interacting nonlinearly, one is forced into a statistical description and so encounters the problem of obtaining a closed set of equations (Laudau and Lifshitz, 1982). A straightforward numerical approach to high Reynolds number fluid turbulence runs into hopeless storage/resolution problems for present-day and foreseeable future supercomputers. It is not likely that foreseeable advances in computers will allow the full simulation of turbulence flows at Reynolds numbers much larger than the $R = O(100 - 1000)$ already achieved.

The fundamental problem is that we must reduce the number of degrees of freedom to be considered, yet at the same time retain the correct physical behavior. If this can be accomplished, then the simplified model will correctly mimic (in a statistical sense) the real physical system.

As an example, traditionally one averages the Navier-Stokes equations over a range of small scales by applying an appropriate filter (Leonard, 1974; Rogallo and Moin, 1984; Zhou *et al.*, 1989a). The result is the Navier-Stokes equation for the large scale motion along with new terms representing the subgrid stresses. The subgrid stresses are now modeled using phenomenological arguments (Smagorinsky, 1963; Rogallo and Moin, 1984) and adjustable numerical factors (Deardorff, 1977). Recently, following the impressive success in critical phenomena (Wilson, 1975; Wilson and Kogut, 1974), renormalization group theory (RNG) has been applied to the subgrid modeling problem in fluid turbulence, especially since subgrid modeling is such a good candidate for the RNG approach (Rose and Sulem, 1978). The RNG subgrid calculations fall into two basic groups: (i) the ϵ-expansion (Forster *et al.*, 1977; Fournier and Frisch, 1983; Yakhot and Orszag, 1986; Zhou and Vahala, 1988), and (ii) the recursion (Rose, 1977; Zhou *et al.*, 1988, 1989b; Zhou and Vahala, 1990; Zhou, 1990) approach. We shall concentrate here only on the recursion RNG theories since we are particularly interested in the wavenumber dependence of the eddy viscosity, $\nu(k)$. In the ϵ-expansion RNG theories, the eddy viscosity is calculated only in the limit $k \to 0$. Now, unlike the ϵ-expansion procedure (i), both free decay (with given Kolmogorov energy spectrum) and forced turbulence (with spectral forcing chosen to reproduce the Kolmogorov energy spectrum) can be handled, and there is no need to introduce a small parameter ϵ.

2. Previous work

Our purpose is to construct a systematic way to model the subgrid scale in Navier-Stokes turbulence. Originally, Rose (1977) applied recursive RNG to the subgrid modeling of the advection of a passive scalar. Later, we have extended Rose's technique and applied it to the cases of free decay (Zhou *et al.*, 1988) and forced (Zhou *et al.*, 1989b) Navier-Stokes turbulence. The resultant resolvable scale wavenumber dependent eddy viscosity in our model shows a cusplike behavior, in qualitative agreement with the test field model of Kraichnan (1976), the EDQNM closure calculation of Chollet and Lesieur (1981), the direct numerical simulation results of Domaradzki *et al.* (1987), and recent large eddy simulation of Lesieur and Rogallo (1989).

Another interesting feature of our recent work (Zhou *et al.*, 1989b; Zhou and Vahala, 1990) is that the time dependence of the subgrid modes is not ignored–in contrast to the treatments in the free decay of Navier-Stokes turbulence (Zhou *et al.*, 1988) and passive scalar convection (Rose, 1977). As a result, a nonlocal time (and space) behavior of the eddy damping is found, similar to that of Kraichnan (1976) in his test field model. Recently, a unified framework for subgrid scale closure is formulated, without the need to specify whether we are dealing with free decay or forced turbulence. The identification need only be made towards the end of the calculation when we must introduce the subgrid velocity autocorrelation function (Zhou and Vahala, 1990). This unified framework has been helpful in examining the effect of helicity on the subgrid scale closure (Zhou, 1990).

3. Current work

The novel feature of our model is an explicit triple nonlinearity in the renormalized equation of motion. On iteration to the fixed point, this term results in a cusplike contribution to the wavenumber dependent eddy viscosity which is required by elementary scaling arguments. However, a major difference between the recursive RNG eddy viscosity and that derived from the test field model (Kraichnan, 1976) and classical closure (Leslie and Qiarini, 1979) is the "strength" of the cusp behavior as $k \to k_c$, where k_c is the boundary between the resolvable and subgrid scales. Based on the energy transfer equation, we have shown that the triple nonlinearity will contribute a term equivalent to an eddy viscosity. This furnishes an explanation of why the renormalized eddy viscosity found by solving the recursive RNG equations exhibits only a mild cusp behavior as $k \to k_c$.

It is shown that the order of the statistical ensemble averaging procedure in recursive RNG technique can be interchanged. Regardless of the order of averaging process, the following results are obtained: First, the triple nonlinearity and a nonlocal time eddy damping functions are generated. Second, the only way to prevent the creation of triple nonlinearity is to assume that a spectral gap exists between the resolvable and subgrid scales.

These encouraging results, combined with the appealing structure of basic recursive RNG theory, has led to a considerable level of the activity in the area, both in the direction of more a sophisticated subgrid model and toward the extension of the recursive RNG approach to deal with different physical systems.

4. Future plan

The fact that the subgrid scale modes evolve at a faster time scale than that of the resolvable is the motivation behind the Markovian approximation in that the time dependent of the subgrid modes can be ignored (Rose, 1977; Zhou *et al.*, 1988). To account for this separation of scales, one may attempt a treatment by which the spatial and time coordinates are separated into two scales. A more elaborate study will be conducted, using the method of multiple scale analysis (Nayfeh, 1973) which has been useful in the derivation of transport theories for magnetohydrodynamic fluctuations in the solar wind (Zhou and Matthaeus, 1989; 1990a,b,c). The approach which combines the RNG with a scale parameter expansion method from perturbation theory can be considered as a further refinement from that of the Green's function technique (Zhou *et al.*, 1989b; Zhou and Vahala, 1990).

One aspect of the future research is the subgrid modeling of two dimensional (2-d) Navier-Stokes turbulence. While it is known that the eddy viscosity is negative in 2-d Navier-Stokes turbulence due to inverse cascades (Kraichnan, 1976), the eddy viscosity representing the effects of the unresolvable subgrid scale in the corresponding vorticity equation is positive owing to the direct enstrophy cascade. We plan to carry out a recursive RNG analysis of 2-d Navier-Stokes turbulence. The resultant resolvable scale wavenumber dependent vorticity eddy viscosity will be compared with the subgrid scale eddy viscosity computed from the results of high-resolution direct numerical simulations of homogeneous, isotropic 2-d Navier-Stokes turbulence. Our subgrid model of 2-d Navier-Stokes turbulence will be evaluated according to turbulence theory (Kraichnan and Montgomery, 1980) and compared with well-developed simulation results.

We also plan to extend our recursive RNG analysis to develop other forms of turbulence models (such as $K - \epsilon$ model), as well as attack other important physical systems, such as passive scalar transport equations. These efforts will make recursive RNG available for much broader practical applications. In particular, the direct numerical simulation of the renormalized Navier-Stokes equation (Zhou *et al.*, 1988, 1989b; Zhou and Vahala, 1990) may have major impact on the systematic turbulence modeling and large eddy simulation.

Furthermore, it is of great interest to use the recursive RNG for the modified Betchov model (Kraichnan and Panda, 1988) since both the direct-interaction approximation (Kraichnan, 1959) and constrained decimated scheme (CDC) of Kraichnan (1985) have been applied to the Betchov model (Betchov, 1966; Williams *et al.*, 1989). We hope that the modified Betchov model can be used as a test site for all available subgrid scale closure techniques because of the difficulties of applying CDS to 2- or 3-d Navier-Stokes turbulence. Of particular interest would be the comparison between RNG and CDS since, as Kraichnan (1985) has pointed out, RNG removes high-tk modes by repetitive transformations while the CDS removes these modes in one step.

Strong collaboration with Professor W. C. Reynolds and Dr. L. M. Smith is anticipated in several RNG related topics.

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