Modeling turbulent boundary layers in adverse pressure gradients

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1. Motivation and objectives

Many of the turbulent boundary layers encountered in practical flows develop in adverse pressure gradients; hence, the dynamics of the thickening and possible separation of the boundary layer has important implications for design practices.

What are the key physical processes that govern how a turbulent boundary layer responds to an adverse pressure gradient, and how should these processes be modeled? Despite the ubiquity of such flows in engineering and nature, these questions remain largely unanswered. The turbulence closure models presently used to describe these flows commonly use 'wall functions' that have *ad hoc* corrections for the effects of pressure gradients. There is, therefore, a practical and theoretical need to examine the effects of adverse pressure gradients on wall bounded turbulent flows in order to develop models based on sound physical principle.

The present study is focussed on the evolution of a turbulent boundary layer on a flat wall with an externally imposed pressure gradient. In practical flows, the pressure gradient may be associated with curvature of the wall, *e.g.* the boundary layer on an airfoil or the flow of the atmospheric boundary layer over a hill. Curvature is known to have a strong dynamical effect on turbulence (see, for example, the experiments of Gillis & Johnston, 1983) and so, in this initial theoretical investigation, it is preferable to consider the effect of only a pressure gradient. This simplification is appropriate and timely since there are recent and ongoing computational and experimental investigations of externally decelerated pressure gradient (Spalart & Watmuff, 1990). This flow is not separated, but recent laboratory studies by Simpson, Chew & Shivaprasad (1981), and Dengel & Fernholtz (1990) provide detailed measurements when separation is present.

We consider flows with 'gentle' or incipient separation, when the region of reversed mean flow, if it exists, is confined well within the boundary layer (figure 1*a*). This is an appropriate starting point for a theoretical study of separation and has practical value: in many engineering flows, 'large scale' separation, when the entire boundary layer breaks away from the wall (figure 1*b*)), is detrimental to performance; hence, in order to achieve the highest pressure drop along a device without stall, it is desirable to decelerate the flow such that the boundary layer is maintained as close to separation as possible. Furthermore, in atmospheric flows, the boundary layer is of the order of a kilometer deep, so that large scale separation occurs only due to the largest topographic features (*e.g.* the Rockies!), but gentle separation may occur over even gently undulating terrain.

The overall objectives are (i) to establish the significant scales of length, velocity, and shear stress in the different parts of the flow; (ii) to improve low order parameterizations of the effects of the deceleration on the turbulence; and (iii) to develop a

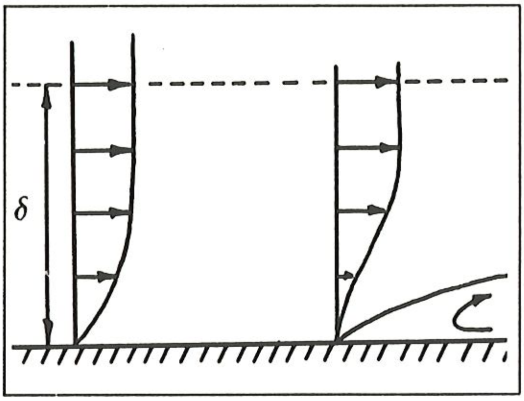

FIGURE 1*a*. Gentle separation.

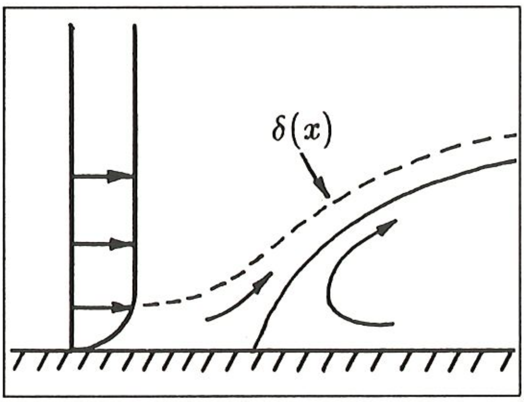

FIGURE 1*b*. Large scale separation.

low order closure model to describe the effects of the turbulence on the mean flow. This work has aspects in common with the study of Durbin (see elsewhere in this volume) and we aim to work together on some aspects of this work.

2. Present and future work

This investigation has only recently begun, so that no significant results can yet be reported. The methodology of the proposed work is now described in more detail.

As described above, the investigation proceeds in three parts. More details of these aspects are described below, and despite the separation of this discussion into three parts, it is anticipated that each strand of the research will be developed in parallel.

2.1 Determination of the significant scales

The initial part of this study is focussed on establishing the significant scales of length, velocity, and shear stress in the different regions of the flow using the method of matched asymptotic expansions. Two types of turbulent boundary layer need to be considered. Far upstream, where the pressure gradient has no significant effect on the flow, the boundary layer has an equilibrium form and is described by the classical two layer form: a defect layer that extends through most of the boundary layer, where, at leading order, the mean velocity is equal to the free stream velocity; and close to the surface is a thin wall-layer, where, at leading order, the gradient of the Reynolds shear stress balances the viscous shear stress. The flow in such a boundary layer is characterized by the friction velocity, u_*, and the free stream velocity, u_e. Near the separation point, however, by definition, the surface shear stress is zero and the boundary layer has a quite different structure. Zero stress boundary layers have not been treated with asymptotic methods, and so the initial part of the investigation has focussed on their structure. Near the surface the appropriate velocity scale is $u_p = [\nu dp/dx]^{\frac{1}{3}}$, which leads to the well known square root profile for the mean velocity; the defect layer, which again constitutes the bulk of the boundary layer, is governed by nonlinear dynamics.

The key to understanding the structure of the decelerating flow is then to determine how the boundary layer evolves from the equilibrium form to the zero stress form (*i.e.* as value of the local ratio u_*/u_p changes in the streamwise direction). In particular, what is the streamwise length scale of the transition? An answer will provide a criterion, to be satisfied by the pressure gradient and the approach flow, for separation to occur and a length scale for the position of the separation point. Furthermore, the asymptotic structure of the flow provides a framework for examining the different processes controlling the dynamics of the turbulence in the flow.

There have been previous studies of the asymptotic structure of a separated boundary layer (Melnik, 1989; Neish & Smith, 1990; Sychev & Sychev, 1980), which have suggested rather complex structures for the flow. In these studies, much of the complexity of the resulting asymptotic structure arises from the use of a specific closure model (Melnik, 1989, and Neish & Smith, 1990, use the Cebeci-Smith algebraic eddy viscosity model) and not from any particular physical process associated with the development of the boundary layer. Furthermore, Sychev & Sychev (1980) have developed a twelve layer asymptotic description of a boundary layer with large scale separation; however, the assumption of a steady mean flow is less justified in this case, so the complex structure they discuss is of questionable practical value. We believe that the present approach of studying gently separated flows using asymptotic analysis without a specific closure model will produce interesting and important insights, whilst avoiding a highly complex structure that is unlikely to occur in the real flow.

2.2 Parameterization of the turbulence

It is proposed that the scalings derived for the different regions of the flow will be used to formulate a sequence of 'model flows' in order to examine the structure of the turbulence in the strongly perturbed boundary layer. It is anticipated that the turbulence will be analyzed using rapid distortion theory and by scaling existing model equations (*e.g.* the $k - \epsilon$ model equations). These studies will suggest which are the key terms in the transport equations for the Reynolds stresses.

2.3 A low order turbulence model for the flow

The results of the fundamental studies will be used to develop a low order closure model for the turbulent stresses to be used in practical engineering codes. This aspect of the study will draw on the recent work of Durbin (see elsewhere in this volume), who has developed a closure model for the near wall region of a boundary layer that does not use 'wall functions' (*i.e.* heuristic functions that damp the eddy viscosity to zero close to the wall).

In formulating the model, we intend to examine the properties of the model for the dissipation length scale recently suggested by Hunt, Stretch & Britter (1986), which has been tested against data obtained by direct numerical simulation of some simple boundary layer flows by Hunt, Spalart & Mansour (1989). This would obviate the need for a transport equation for the turbulence dissipation, thereby reducing the computational cost of the model. Furthermore, this approach would bypass the numerical difficulties that have been encountered with the ϵ-equation when separation is present (Newley, 1986).

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