Automated pattern eduction from turbulent flow diagnostics

By Derek Stretch

The development of an automated technique for the eduction of 3-D spatial patterns in vector or scalar diagnostics has been completed. The method is based on an iterative convolution between a trial pattern and the data field. It has been applied to the analysis of low Reynolds number turbulent channel flow and homogeneous shear flow. The results have yielded new information on the dominant flow structures in these flows, particularly with respect to the spatial relationships between various forms of organized motion. A particular application of the pattern eduction method, which we tentatively refer to as an "adaptive wavelet transformation", is proposed with the objective of investigating the way turbulence structure changes with scale. Preliminary results using data from homogeneous turbulent shear flow simulations are presented. At the low Reynolds numbers of the simulations, there is no evidence of scale similarity. The small scales appear to be associated with the edges of the larger scale vortical structures.

1. Objectives

A notable consequence of the recent application of powerful computers to the simulation of turbulent flows is the vast amount of data which has, and continues to be, generated. Reliable automated diagnostic tools are needed to assist in interrogating these data bases.

The pattern analysis technique developed in the present study is a simple and flexible diagnostic tool for the analysis of complex data fields. The procedure is an extension of that developed by Townsend (1979) and Mumford (1982) (see also Ferre and Giralt , 1989a,b). It is based on an iterative convolution between the data and a reference pattern. Details of our implementation are given elsewhere (Stretch, 1989 ; Stretch, Kim and Britter, 1990). Each iteration of the procedure yields an ensemble-averaged pattern which has an improved mean cross-correlation with the instantaneous patterns in the data. Major inputs required for each pattern analysis are the choice of flow diagnostic, the scale of the patterns, and the choice of pattern used to initialize the iterations.

In broad terms, the basic objective of the pattern eduction approach is to educe statistically significant spatial organization(s) of specified flow diagnostic(s). The terminology "statistically significant" is meant to imply that the patterns occur frequently, relative to other possible spatial organizations of the diagnostic in question.

Some of the issues which we wished to address using this diagnostic tool were as follows. Firstly, what are the structures educed by this method for turbulent channel flow, and how do they relate to results obtained previously using conditional sampling (stochastic estimation), orthogonal decomposition, and flow visualization?

How are the various turbulence structures which have been reported spatially related to one another? How "coherent" are the turbulence structures? That is, given a representation of a structure, such as an ensemble average, how much variation from this representation is typical for the actual instantaneous realizations? What is the dynamical relevance of the organized structures in terms of their contribution to the turbulence kinetic energy, average Reynolds stress, and dissipation? What can we say about the temporal evolution of the organized motions? What mechanism(s) is responsible for their origin, and what are characteristic time scales for their evolution? How do the characteristic turbulence structures change with scale? At high enough Reynolds number, conventional wisdom suggests a regime at large scales where the structures would reflect the large scale non-homogeneities of the flow, followed at smaller scales by an inertial range of self-similar (possibly isotropic) eddies and finally by a distinct dissipation range of structures. Of particular interest in this context is some knowledge of how the structures at different scales are spatially organized with respect to one another. This information may assist us in understanding how energy is transferred between scales in turbulence. Since the present methodology deals with eddies in their physical space representation rather than a spectral (Fourier) representation, the interpretation of inter-scale interactions and energy transfer may be simpler.

2. Outline of the pattern eduction method

Consider a turbulent flow which is sampled at time t by some chosen diagnostic field $\mathbf{D}(\mathbf{x}, t)$. The field $\mathbf{D}$ may be a scalar or vector valued function of the position vector $\mathbf{x}$. It need not represent a complete description of the flow, nor need it comprise the basic primitive dependant variables of velocity and pressure. For example, the field $\mathbf{D}$ may simply comprise one or more components of the velocity or vorticity vectors. Alternatively $\mathbf{D}$ may be a binary function reflecting a zonal or topological classification of the flow such as proposed by Hunt and Wray (1989), and Chong, Perry and Cantwell (1990).

Now we shall suppose that the field $\mathbf{D}$ comprises a set of discrete organized structures or eddies $\mathbf{E}_p(\mathbf{x} - \mathbf{x}_p)$, $p = 1, 2, \ldots m$, sprinkled in space and centered around reference positions $\mathbf{x}_p$. Note that by definition we expect $\mathbf{E}_p$ to have locality in space (compact spatial support) so that $\mathbf{E}_p(\mathbf{x} - \mathbf{x}_p) \to 0$ for $|\mathbf{x} - \mathbf{x}_p|/l \gg 1$ where l is a characteristic length scale of the eddy. While in general there could be a distinct independent eddy structure $\mathbf{E}_p$ at each position $\mathbf{x}_p$, we shall suppose for simplicity that each of the eddies may be described by a suitable transformation of a single basic eddy function $\mathbf{E}(\mathbf{x})$. In general, the possible transformations could include translation, rotation, and changes in the amplitude and length scales (dilation or contraction) of the eddy function $\mathbf{E}$. For the present purposes, we restrict ourselves to translation and amplitude transformations. We shall later address scale changes by a simple extension of the method. In order to locate the eddies embedded in a field of random noise, we require a pattern recognition method which is invariant with respect to changes in the position and amplitude of the patterns and is insensitive to the noise. As discussed elsewhere (e.g. Duda and Hart, 1973), a suitable matching criterion is cross-correlation (or convolution). If one had prior knowledge of **E**, then simply carrying out a convolution between **D** and **E** should yield local maxima at the pattern locations $\mathbf{x}_p$. In the present context, however, we are primarily concerned with a situation where we do not have *a priori* information on the characteristic eddy or eddies. This issue is resolved by using an iterative application of convolution. At each iteration, the function **E** is updated by an ensemble average of the data centered around the local maxima at $\mathbf{x}_p$. The iterative convolution procedure must, however, be initialized, and several different approaches have been tested (see Stretch, 1989) with qualitatively consistent results obtained for all cases. For the results presented in this report, randomly selected samples from the data field **D** were used to initialize **E**. The random selection was performed using a pseudo random number generator to select a reference position from which a sample was extracted.

3. Application to low Reynolds number turbulent channel flow

The pattern eduction process has been applied to data from numerical simulations of low Reynolds number ($Re_\theta = 287$, $\delta^+ = 180$) turbulent channel flow (Kim, Moin and Moser, 1987). Preliminary results using various scalar diagnostics are reported in Stretch (1989). The outcome of an extensive series of analyses using a number of different diagnostics is summarized schematically in figure 1. More detailed quantitative results are given later.

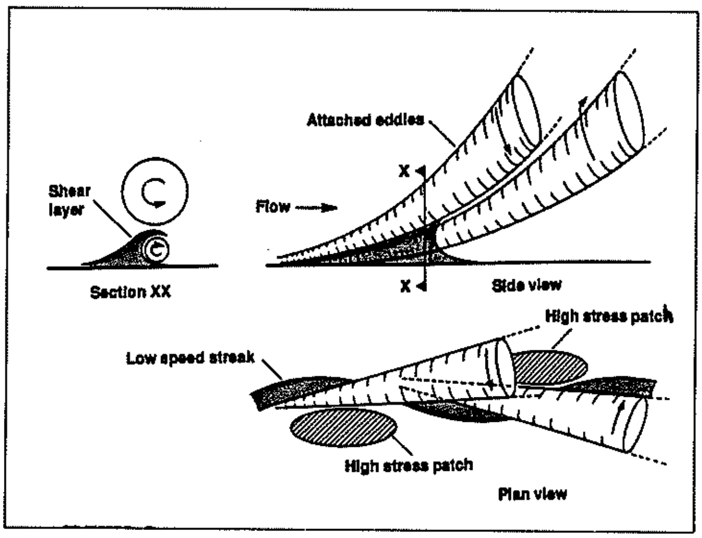


FIGURE 1. Sketch of the pattern educed for turbulent channel flow.

The ensemble-averaged structure educed from the data comprises attached eddies spanning most of the channel half-width. Attached eddies of this type have been proposed by Townsend (1976) and others. Note, however, that our results suggest that the primary pairing (if any) of eddies with opposite sense of rotation (streamwise vorticity) is in the vertical rather than the spanwise direction. In figure 1, we have indicated how the present results unify a number of structural features which have been previously documented by many investigators. In particular, we find that near wall low (or high) speed streaks, shear layers, quasi-streamwise vortices, and "pockets" of high wall stress are spatially related as indicated in the sketch. These various structural elements are thus viewed as part of the same basic "coherent" structure. Unlike the usual conditional sampling methods, the pattern eduction approach does not rely on point-wise matching of velocity patterns or sampling criteria based on thresholding. It is the similarity in spatial structure as measured by the convolution (or cross-correlation) which is the basis of the pattern eduction. The method is thus well suited to clarifying the spatial relationship between various features of the flow.

The spatial distribution of the patterns in the data as determined by the positions of the local maxima in the convolution between the ensemble average pattern and the raw data is shown in figure 2. In this example, 70 patterns were located, which collectively represents 70% of the data volume. The normalized cross-correlation coefficients between the ensemble-average pattern and the data were computed at each of the positions indicated in figure 2. A histogram of these values is plotted in figure 3. The mean value of the cross-correlations was $\overline{R} = 0.4$.

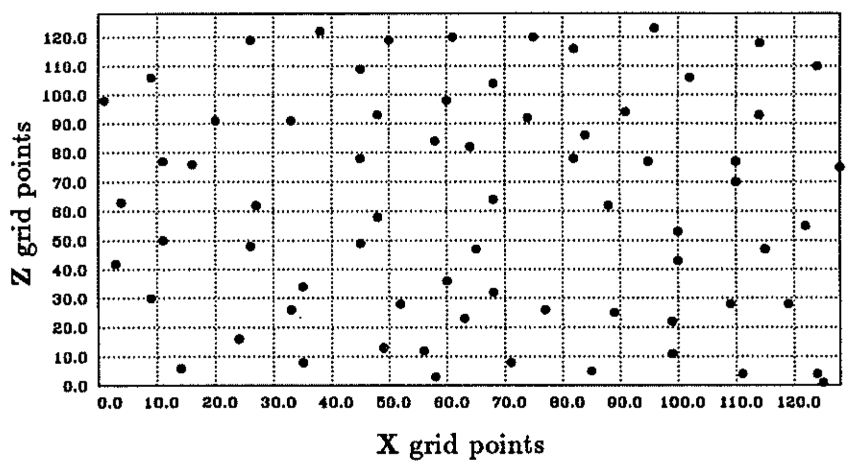


FIGURE 2. Spatial distribution of the detected patterns.

The cross-correlation coefficients provide a useful objective measure of the meaning of the word "coherent" in reference to the educed turbulence structure. We shall, therefore, further clarify what the observed cross-correlations imply by giving examples of the actual instantaneous patterns detected in the data. Figure 4 shows an series of zy plane views of the ensemble averaged pattern educed using the spanwise velocity component as the diagnostic. These results are qualitatively representative of those obtained using various flow diagnostics.

It can be seen that the basic elements of the ensemble averaged structure are attached eddies spanning the channel half-width, with two eddies (or vortices) with

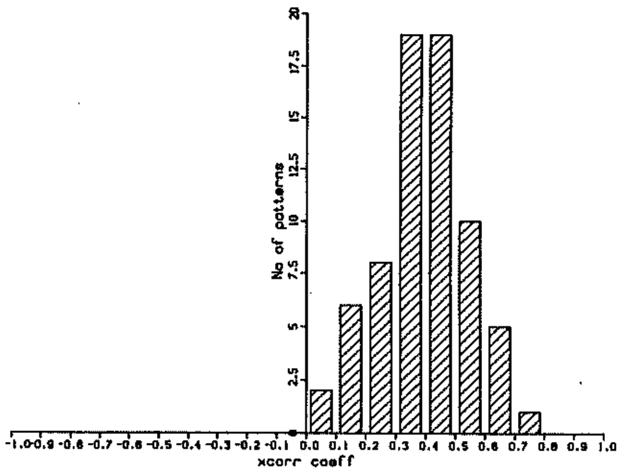


FIGURE 3. Histogram of cross-correlation coefficients.

opposite sense of rotation arranged one above the other as was depicted in figure 1. Also shown in figure 4 are two examples of "instantaneous" patterns as they were located in the data: these patterns produced local maxima in the convolution between the ensemble averaged pattern and the data. The magnitude of the cross correlation was 0.4 in the one case and 0.7 in the other. These examples were selected in order to represent a typical case as well as a special case where the correlation was unusually high. It is clear from these examples that a correlation coefficient of 0.7, while a rare occurrence, implies a strong similarity between the instantaneous patterns and the ensemble average. On the other hand, it appears that about half the located patterns (as defined by the detection of a local maxima in the convolution regardless of the magnitude of the maxima) which have cross-correlations less than 0.4, do not strongly resemble the ensemble average. It would be useful if objective measures such as these cross-correlations were more widely reported in discussions of conditionally sampled data. Clearly, they suggest that a good measure of caution is appropriate in using a single ensemble averaged result to characterize the flow structure, even at the low Reynolds number of the flow considered here.

Some further details of the educed patterns are shown in figures 5 and 6. The attached eddies have a characteristic spanwise velocity signature in the xy plane (fig 5), comprising elongated positive and negative (paired) regions extending from the wall to the outer part of the flow. Examples of this pattern are common in the instantaneous spanwise velocity field, as already noted in Stretch (1989). The characteristic inclination angle between the attached eddies and the wall can be deduced from figure 5: it increases from zero near the wall to about 40 degrees in the outer part of the flow.

The streamwise velocity fluctuations in an xz plane near the wall ($y^+ = 10$)

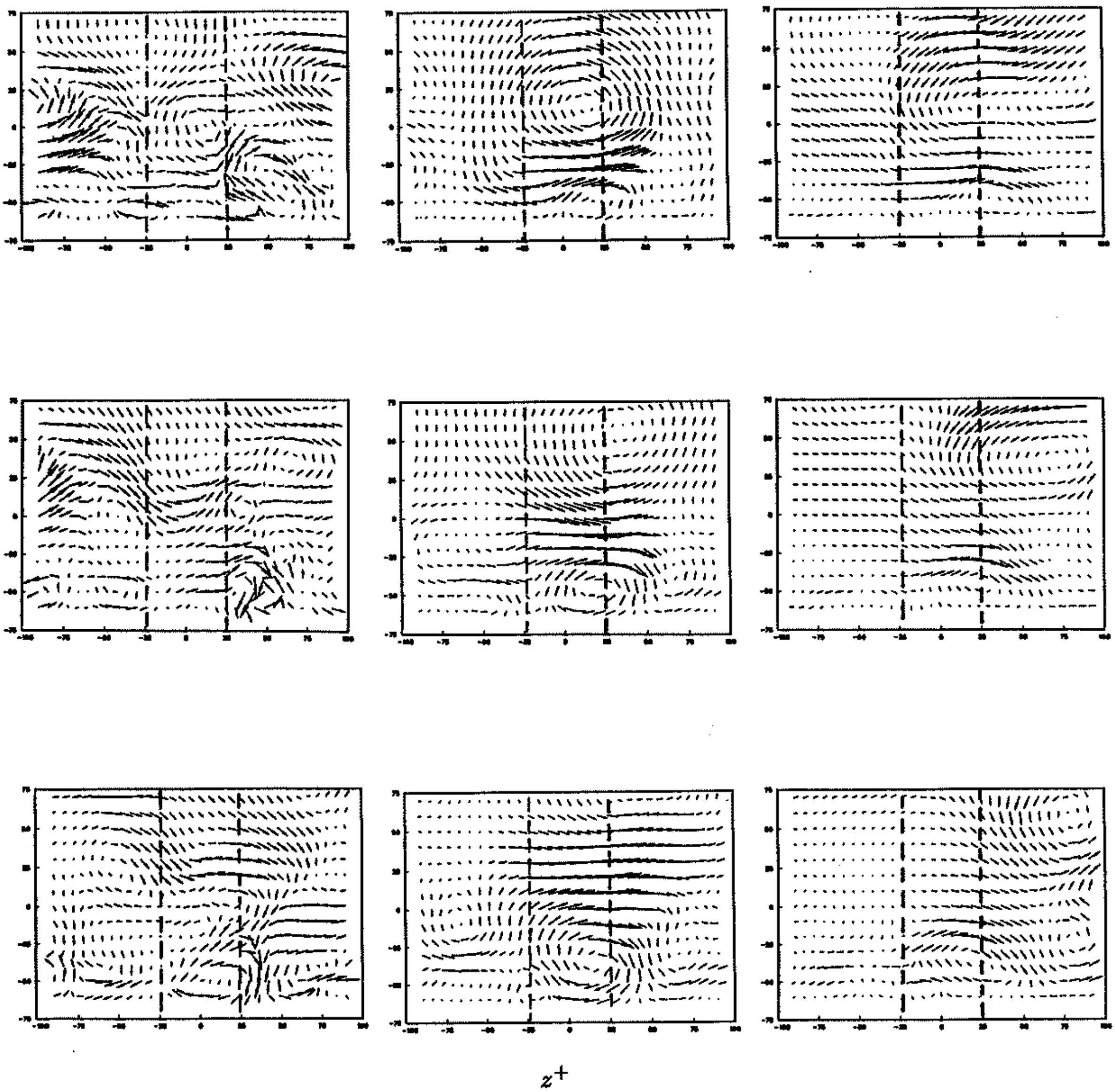


FIGURE 4. Example of the educed ensemble average pattern (center) shown as velocity vectors projected onto zy planes ($\Delta z^+ = 200$, $\Delta y^+ = 150$) at three streamwise locations ($\Delta x^+ = 36$). On the left is a sample of an instantaneous pattern with a cross-correlation of 0.4, and on the right a sample with a correlation of 0.7. Note that only the regions delineated by the dashed lines were used for the pattern matching.

are shown in figure 6(a) and in an xy plane in figure 6(b). Near the wall, as expected, the educed pattern in the streamwise velocity field comprises low and high speed "streaks" (fig 6a). There is a lateral asymmetry (spanwise kinking)

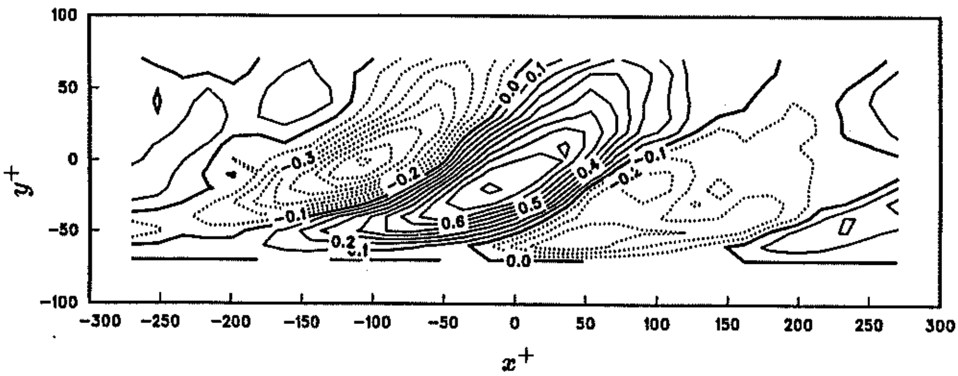
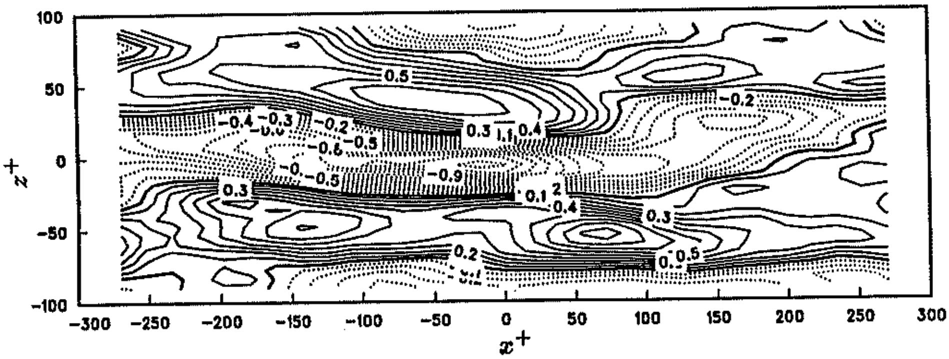


FIGURE 5. Spanwise velocity signature of the educed structure.

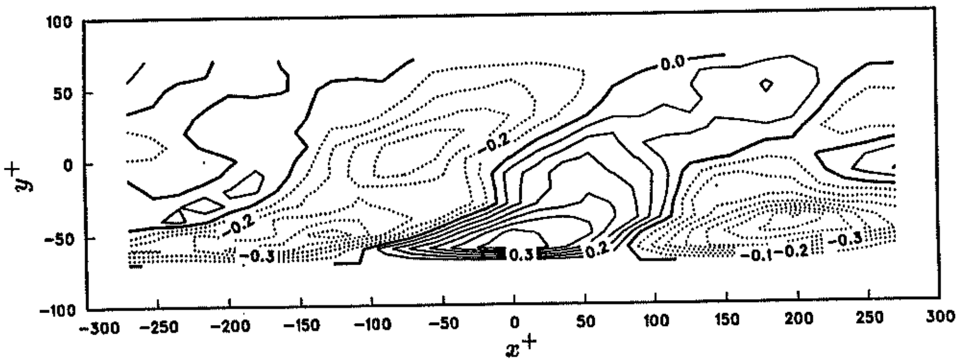
associated with the streaks, and this in turn is associated with a protruding shear layer structure (fig 6b) which is typical of those obtained from previous conditional sampling analyses. The attached eddies, which near the wall are quasi-streamwise vortices, have a characteristic stress signature at the wall comprising a small scale ($\Delta x^+ \simeq 150$, $\Delta z^+ \simeq 50$) region of high stress embedded within the more elongated high speed streaks (fig 6a). This result indicates that the surface stress can be a useful diagnostic for detecting the vortical structures nonintrusively by means of their stress "footprints" at the wall. This observation could form the basis for an active control strategy aimed at reducing the surface stress by modifying the vortical structures.

There is an interesting question concerning the contribution of the educed patterns to the flow kinematics. It is possible to use the pattern eduction procedure to decompose the flow into a deterministic contribution from the eddies **E** and a random "disorganized" background field. This can be achieved by performing a deconvolution of the data **D** with the educed eddy **E** and simply setting all contributions to zero except at the pattern locations $\mathbf{x}_p$. Subsequent convolution of this truncated field with the eddy function **E** will produce a new data field comprising only translation and amplitude transformations of the eddy function **E**. The turbulence statistics of this eddy field may then be computed and compared with that of the original data. Horizontally averaged second order statistics for a single educed eddy function **E** are shown in figure 7 and compared with the turbulence statistics for the full data field **D**. It is apparent from these results that a random superposition of the educed eddies can indeed produce a velocity field with at least second order statistics which reasonably approximate those of the original data.

The pattern analysis procedure has proven to be helpful in extracting kinematical information from the turbulent flow fields. Our ultimate objective, however, is to understand the detailed dynamics of the flow. Firstly, we would like to establish the dynamical relevance of the educed flow structures by assessing their contribution to turbulence production and dissipation processes. This has been done by computing ensemble averages, centered around the pattern locations, of the instantaneous Reynolds stress and dissipation fields. For example, second and fourth quadrant



(a)



(b)

FIGURE 6. Streamwise velocity signature of the educed structure in (a) xz plane at $y^+ = 10$ and (b) xy plane.

Reynolds stress events, both with positive contributions to the average production of turbulent kinetic energy, are located around the sides of the attached eddies (refer fig 4). Most (in excess of 70%) of the turbulence production is associated with the located patterns, but note that the precise proportion depends on the particular pattern size used. For accurate assessment, it may be desirable to use a more constrained definition for the occurrence of the structures. For example, a threshold could be used for the cross-correlation coefficient.

A second important issue concerning the flow dynamics is the temporal evolution of the educed structures. It is illuminating to start by estimating some of the time scales involved. Focussing on the near-wall region, we shall consider three time scales for the evolution of the attached eddies: an advection time scale (T_a) characterizing the effect of the mean flow, a rotation time scale (T_r) characterizing the angular velocity of the fluid elements in a plane perpendicular to the axis of the eddy, and a viscous decay time scale (T_d) in a Lagrangian frame of reference moving with the eddies. Estimates of these time scales can be obtained from the ensemble average fields yielded by the pattern analysis. We estimate (Stretch *et al.*, 1990) that near the wall $T_a^+ \simeq 20$, $T_r^+ \simeq 60$, and $T_d^+ \simeq 200$. Since T_d is an order of

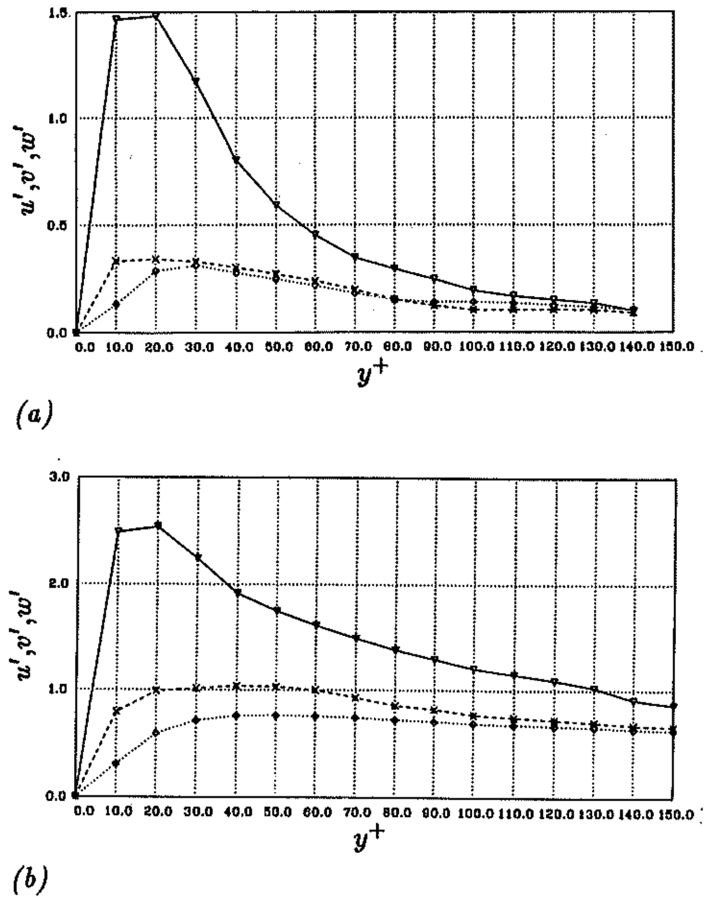


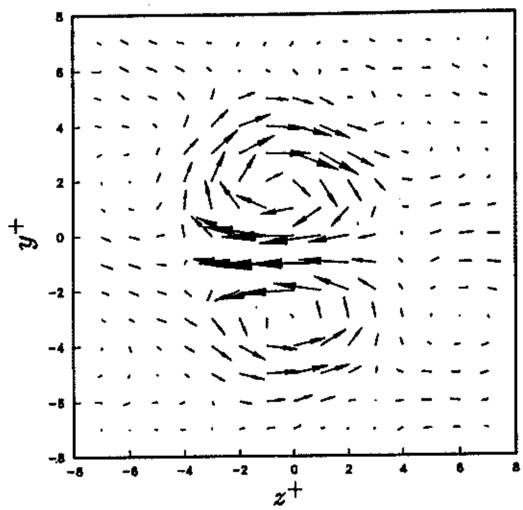
FIGURE 7. Horizontally averaged second order velocity statistics for the educed structure (a) compared with the statistics for the full flow field (b): $\triangledown$, u'; $\diamond$, v'; $\times$, w'.

magnitude larger than T_a, the structures are essentially advected unchanged. The vorticity associated with the eddies, as characterized by T_r, is expected to give rise to a weak spiralling motion for the fluid elements. Note also that many cycles of rotation are not expected in general, an observation which is consistent with flow visualization experiments (Stretch and Britter, 1990).

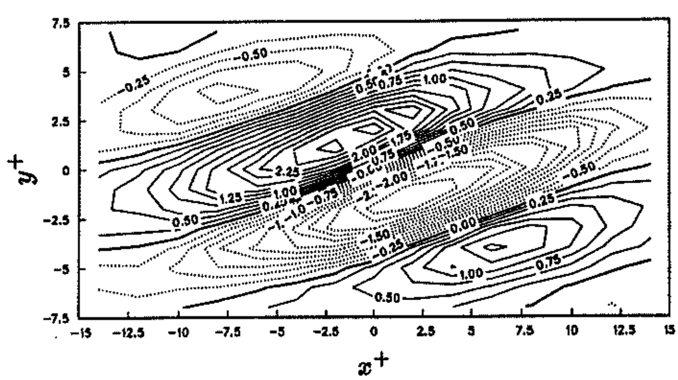
The dynamical mechanism(s) responsible for the origin of the observed structures remains a key unresolved issue. Any proposed mechanism(s) must be capable of explaining the observed structures, but it should also be self-sustaining. Research on this issue is currently underway at the Center for Turbulence Research.

4. Application to homogeneous turbulent shear flow

The pattern eduction procedure has also been applied to data from direct numerical simulations of homogeneous turbulent shear flow (Rogers and Moin, 1987). Some results of the analysis where the pattern matching was done for the vertical and spanwise velocity components are shown in figure 8. The zy plane view of the velocity vectors reveals a pair of vortices arranged one above the other, as for the channel flow results. The contours of the spanwise velocity in a xy plane at the center of the pattern show the characteristic inclined, paired features which are a signature of inclined roller eddies. Further investigation of the spatial configuration of the vortical structures has, however, indicated that in this case, there may not be a strong preference for pairing in a particular arrangement e.g. vertically as opposed to horizontally. The results in figure 8 are thus partly an artifact of the ensemble averaging process. In fact, single unpaired vortices may be the most common occurrence, although this remains unclear at this point.



(a)



(b)

FIGURE 8. Pattern educed from homogeneous sheared turbulence: (a) velocity vectors projected onto the zy plane at $x = 0$, (b) Contours of the spanwise velocity component in a xy plane at $z = 0$.

5. Application as an adaptive wavelet transform

It is apparent that the pattern analysis procedure developed in this study has much in common with the so-called "wavelet transform" (see e.g. Meneveau, 1989 for a brief review). Each iteration of the pattern analysis can be viewed as a generalized form of wavelet transform at a particular scale, with the current ensemble averaged pattern as the wavelet. This is an attractive choice of wavelet since it represents actual eddies in the flow rather than being an arbitrarily chosen mathematical function. The iterations now have the interpretation of maximizing the locality of the wavelet energy by using the local maxima in the convolution as reference positions for updating the wavelet (using an ensemble average of the data around those positions).

In addition to information on the spatial distribution of the wavelet energy, wavelet transforms have been used to provide information on the way energy is distributed between different scales. This is achieved by successive dilation (or contraction) of the wavelet function between successive convolutions. A similar strategy could, of course, be adopted in the present context using an educed eddy pattern as the wavelet and doing repeated convolutions for different dilations or contractions of this pattern. However, there is no *a priori* reason to assume that the pattern educed at a given scale would also be obtained by applying the eduction procedure at different (say smaller) scales. Note that the scale of the pattern is naturally set by the specification of the size of the pattern domain and, hence, the spatial support of the function (which is set to zero beyond this domain for doing the convolution). In fact, it seems natural to allow the iterative pattern selection procedure to educe a structure at different scales. Then the issue of scale similarity can be explicitly tested, since we have allowed our wavelet to adapt to the scales being analyzed. From the above it is clear why the terminology "adaptive wavelet transform" is an appropriate description of this procedure.

Some preliminary results from application of this idea to the homogeneous turbulent shear flow data have been obtained and are shown in figure 9. These may be compared with the results presented in figure 8. It is apparent that at small scales, the educed pattern comprises a jet/shear-layer. The orientation of the jet is partly determined by the initial conditions for the pattern iterations, which in this case was a random selection from the data. The important feature of this result, however, is that if the velocity field is averaged in an extended region around the located patterns, we see that the small scale jets/shear-layers are simply a part of the larger scale vortices which were previously educed by the analysis. Therefore, in this case, there is no scale similarity which emerges from the analysis procedure, which is not surprising considering the low Reynolds number of the simulation. It would certainly be interesting to apply this method to much higher Reynolds number flows to see if a cascade of geometrically similar eddies does emerge from the analysis. Development and testing of the "adaptive wavelet transform" is still underway, including investigation of issues such as normalization procedures for the educed wavelets/eddies.

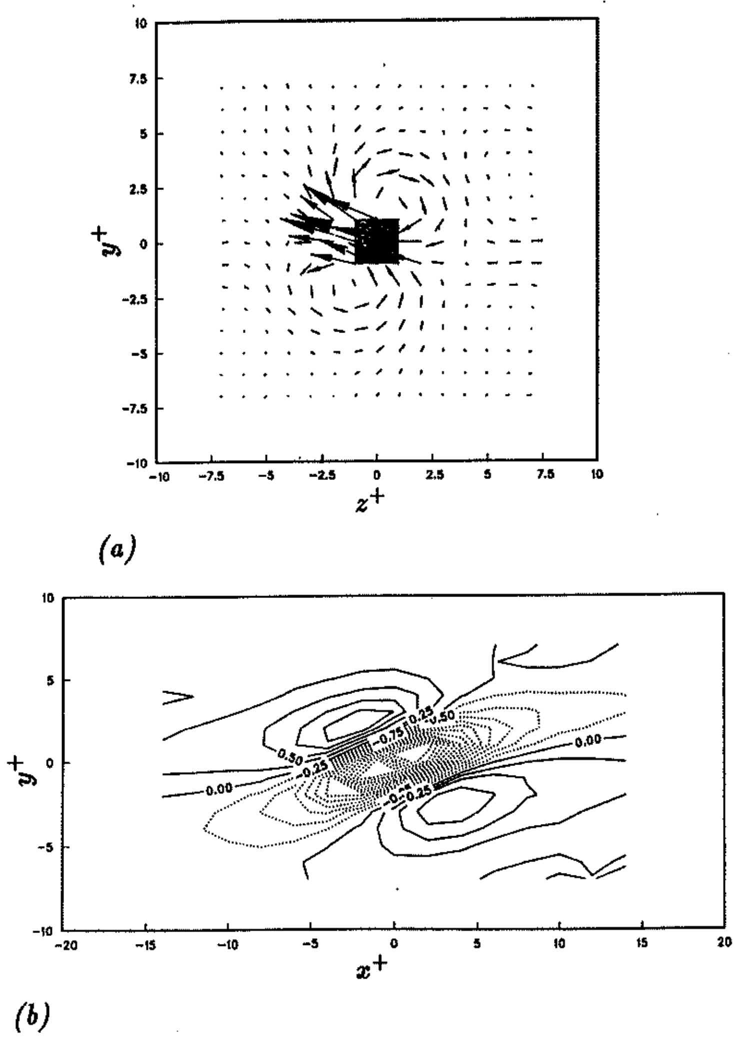


FIGURE 9. The eddy pattern educed from applying the "adaptive wavelet" method to homogeneous turbulent shear flow. The flow pattern shown was educed using the v and w velocity components on the scale of the shaded region : (a) velocity vectors projected onto the zy plane at $x = 0$, (b) Contours of the spanwise velocity component in an xy plane at $z = 0$.

6. Summary

The diagnostic tool which has been developed in this study is conceptually simple and yet provides a flexible means of interrogating complex data fields with the purpose of extracting spatial organization of relevant diagnostics. It is implemented using efficient spectral methods and is thus cheap computationally (all the analysis in this study was done interactively on a VAX). While we have not yet discovered any "new" structures using this approach, it has clarified the spatial relationship between various features of the turbulence and thus presented a unified view of the turbulence structure. Furthermore, we have used the method to quantify the "coherency" of the structures as they occur in the flow by using a cross-correlation measure. We have proposed an extension of the method (or more particularly, in the way the method is used) which we refer to as an "adaptive wavelet transform" that can be used to examine issues of scale similarity, which is a widely used concept in turbulence. This approach has potential for studying (in physical rather than Fourier space) the way eddies or structures of different scales interact.

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