Dual spectra and mixed energy cascade of turbulence in the wavelet representation

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The wavelet-transformed Navier-Stokes equations are used to define quantities such as the transfer of kinetic energy and the flux of kinetic energy through scale r at position $\vec{x}$. Direct numerical simulations of turbulent shear flow reveal that although their mean spatial values agree with their traditional counterparts in Fourier space, their spatial variability at every scale is very large, exhibiting non-Gaussian statistics. The local flux of energy involving scales smaller than some r also exhibits large spatial intermittency, and it is negative quite often, indicative of local inverse cascades.

1. Introduction

Much has been learned about the physics of turbulence by transforming the velocity and the Navier-Stokes equations to Fourier space. The velocity field $\vec{u}(\vec{x},t)$ is then represented as a linear combination of plane waves, characterizing the motion at different scales. For isotropic turbulence, the energetics of turbulence is described (Monin & Yaglom 1971) by the three-dimensional energy spectrum $E(k,t)$ obeying

$$\frac{\partial E(k,t)}{\partial t} = T(k,t) - 2\nu k^2 E(k,t), \tag{1}$$

where $T(k,t)$ is the net transfer of energy to wavenumbers of magnitude k. $T(k,t)$ is formally defined in terms of triple products of velocity and, thus, embodies the closure problem resulting from the nonlinearity of the equations. For the statistically stationary case, the total spectral flux of energy through wavenumber k to all smaller scales is given by

$$\pi(k) = \int_{k'=k}^{\infty} T(k')dk' = -\int_{0}^{k} T(k')dk'. \tag{2}$$

Usually the energy transfer is thought to occur by creation of small scales through stretching and folding of vortical elements, which is modeled by simplified processes such as the successive break-down of 'eddies' (see e.g. Meneveau & Sreenivasan 1990). One then argues that through scales of motion of size k^{-1}, there is a net flux of kinetic energy to smaller scales, which is equal to $\pi(k)$. In the 'inertial range' (Monin & Yaglom 1971), one expects this flux to be equal to the average rate of dissipation of kinetic energy $< \epsilon >$.

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Of course, it has been known for a long time that the local rate of dissipation is distributed very intermittently in space and time. This can be modeled within the framework of breakdown of eddies but with the assumption that the flux of energy to smaller scales exhibits spatial fluctuations at every scale (see e. g. Frisch *et al.* 1978, Meneveau & Sreenivasan 1990) . Thus, we need to define a flux of kinetic energy which, as opposed to Eq. 2, should also depend on position (see, for instance, the discussion in Kraichnan (1974)). In general terms then, it is clear that from Fourier spectra, any information related to position in physical space is completely hidden, which is a disadvantage when dealing with spatially localized flow structures or intermittency. On the other hand, with the purely spatial representation, the information about different scales of motion is hidden. This information is often a useful ingredient for modeling and physical insight. This difficulty calls for a representation that decomposes the flow-field into contributions of different scales *as well as* different locations. In other words, we want to use basis functions that behave more like localized pulses than extended waves. If one wishes them to be self-similar, one is led to rather special basis functions called wavelets.

Using the wavelet representation of the velocity field, we introduce quantities that are analogous to the Fourier spectra of energy $E(k,t)$, transfer $T(k,t)$, and energy flux $\pi(k,t)$, but which depend on location as well as scale. These quantities can be used in several ways, e.g. to correlate specific local events with possible structures or topological features of the flow field. Here we will restrict our attention to statistical descriptions of the spatial distribution of these quantities via their probability densities and second-order moments. Measurements are performed in two 3-D fields obtained from direct numerical simulations of isotropic and homogeneous shear flows. Here we present only results on transfer of energy to some scale of motion, and on the flux of energy to scales smaller than some cutoff r for the shear flow simulation. More extensive results and a more detailed description of this study can be found in Meneveau (1990).

2. Orthonormal wavelets

Wavelets (Grossmann & Morlet, 1984) are pulse-like functions of zero mean whose dilations are convolved with the signal, providing simultaneous resolution in scale and position. In the field of turbulence for instance, continuous wavelet analysis has been used for the study of coherent structures (Farge & Rabreau, 1987) and of two-dimensional data from a turbulent jet (Everson *et al.* 1990). Recently, several orthonormal wavelet basis functions have been constructed by using a logarithmic spacing of scales and increasingly coarser spatial discretization at larger scales (Meyer 1986, Daubechies 1988, Mallat 1989); the absence of redundancy of information makes this form of wavelets particularly useful in higher dimensions. In one dimension, orthonormal wavelets are of the form

$$\psi^{(m)}(x-2^m hi) = 2^{-\frac{m}{2}}\psi\left(\frac{x-2^m hi}{2^m h}\right), \tag{3}$$

where m indicates the octave band of the scale parameter, h is the mesh-spacing of the basic lattice (smallest scales), and the index i refers to location in units of

$2^m h$. Notice that the basis functions corresponding to the larger scales are spaced more coarsely, according to a dyadic arrangement on a binary tree structure. Some particular functions $\psi(x)$ exhibit the property that $\{\psi^{(m)}(x - 2^m hi)\}$ forms an orthonormal base for all (i, m), as for instance the Lemarie-Meyer-Battle (LMB) wavelet shown in Fig. 1. It has exponential decay in x space and ω^{-4} decay in Fourier space. For more details on this, on discrete transforms, and on fast algorithms, see Mallat (1989), Daubechies (1988) and Meneveau (1990).

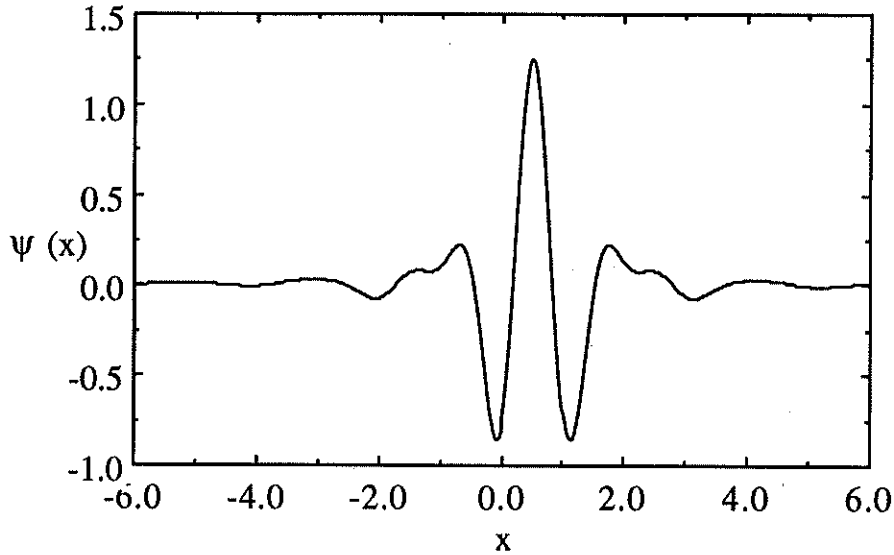

FIGURE 1. Example of an orthonormal wavelet basis function $\psi(x)$ (Lemarie-Meyer-Battle wavelet).

In an extension of this formalism to three dimensions (Meneveau 1990), the velocity field (at some instant t) is written as

$$u_i(\vec{x}) = \sum_m \sum_{q=1}^{7} \sum_{\vec{i}} w^{(m,q)}[\vec{i}]\ \Psi^{(m,q)}(\vec{x} - 2^m h\vec{i}), \tag{4}$$

where m denotes again the scale, $\vec{i} = (i_1, i_2, i_3)$ is the three-dimensional position index on a cubic lattice of mesh-size $2^m h$, and q gives additional internal degrees of freedom (this is needed because $\Psi^{(m,q)}$ is decomposable into functions such as Fig. 1 in each Cartesian direction). Because of orthonormality, the discrete wavelet coefficients can be computed as

$$w_i^{(m,q)}[\vec{i}] = \int u_i(\vec{x})\ \Psi^{(m,q)}(\vec{x} - 2^m h\vec{i})\ d^3\vec{x}. \tag{5}$$

3. Energetics of turbulence in wavelet space

We now start with the Navier-Stokes equations in physical space written for the fluctuating velocity and pressure, and we take its inner product with the wavelet basis function $\Psi^{(m,q)}(\vec{x} - 2^m h\vec{i})$. This yields an evolution equation for the wavelet coefficients $w_i^{(m,q)}[\vec{i}]$. Multiplication by $w_i^{(m,q)}[\vec{i}]$ and contraction over the three directions i and the index q yields an evolution equation for the local kinetic energy density,

$$\frac{\partial}{\partial t} e^{(m)}[\vec{i}] = t^{(m)}[\vec{i}] - v^{(m)}[\vec{i}], \tag{6}$$

where

$$t^{(m)}[\vec{i}] = -\sum_{i=1}^{3}\sum_{q=1}^{7} w_i^{(m,q)}[\vec{i}] \int (u_j \frac{\partial u_i}{\partial x_j} + \frac{1}{\rho}\frac{\partial p}{\partial x_i})\ \Psi^{(m,q)}(\vec{x} - 2^m h\vec{i})\ d^3\vec{x}. \tag{7}$$

is the net energy transfer through scale $2^m h$ at location $2^m h\vec{i}$. $e^{(m)}[\vec{i}]$ is the kinetic energy and $v^{(m)}[\vec{i}]$ is the contribution of the viscous terms, including molecular diffusion of energy and dissipation at that scale and location. Equation 6 is the (discrete) analogue of Eq. 1, written for the energy of orthonormal pulses rather than waves. In analogy to Eq. 2, the flux of kinetic energy through a spatial region of characteristic size $2^m h$ and location $2^m h\vec{i}$ can be computed by adding the transfer-density (local transfer divided by the total number of grid-points at each scale) over all scales larger than $2^m h$ at that particular location:

$$\pi^{(m)}[\vec{i}] \ = \ - \sum_{n=m}^{M} \ \ 2^{3(M-n)}\ t^{(n)}[\vec{j}]. \tag{8}$$

Here M is the scale index of the largest scale considered, and $\vec{j}$ (given by the integer part of $2^{m-n}\vec{i}$) is the position index of the larger scales (n). Several studies (Siggia 1978, Zimin 1981 and Nakano 1988) have used 'wavepackets' (essentially wavelets) for obtaining approximations to the Navier-Stokes equations and have then deduced energy cascade models. Here we perform actual measurements of these quantities relevant to the energetics of turbulence, without approximations.

To proceed, we compute the spectral transfer density at wavenumber $k_m = 2\pi/(2^m h)$ by dividing the total transfer in the band m,

$$\sum_{\vec{i}} t^{(m)}[\vec{i}] = 2^{3(M-m)} < t^{(m)}[\vec{i}] > \tag{9}$$

by $\Delta k_m = k_m ln(2)$ and by the total number of points 2^{3M}. We obtain

$$T_w(k_m) = 2^{-3m} k_m^{-1} [ln(2)]^{-1} < t^{(m)}[\vec{i}] >, \tag{10}$$

where the average extends over all points $[\vec{i}]$. $T_w(k_m)$ is equivalent to the Fourier transfer spectrum $T(k)$ but is not necessarily identical at every k because of the width of the wavelet in Fourier space. Also, the wavenumber resolution is poorer in the orthonormal wavelet case because of the discretization of scales in octaves. The additional information is available as spatial resolution which increases with decreasing scale. In addition to the spectral content, one can inquire about the spatial variability of $t^{(m)}[\vec{i}]$, which is given in terms of its standard deviation (in units of $T(k_m)$) according to

$$\sigma_t(k_m) = 2^{-3m} k_m^{-1} [ln(2)]^{-1} \left(< t^{(m)}[\vec{i}]^2 > - < t^{(m)}[\vec{i}] >^2 \right)^{\frac{1}{2}}, \tag{11}$$

where the average again extends over all points $[\vec{i}]$. A plot of $T_w(k_m)$ and $T_w(k_m) \pm \sigma_t(k_m)$ as a function of k_m will be called the *dual spectrum* of transfer, dual because it gives information both about the contribution of various scales *and* about the spatial variability associated with it. Similar definitions of dual spectra can be introduced for the kinetic energy and the flux of energy (Meneveau 1990). Furthermore, in Meneveau (1990), it is shown that the local kinetic energies form a (non-conservative) multifractal measure.

4. Analysis of turbulent fields

Next we turn to the analysis of three-dimensional turbulent fields. We consider direct numerical simulation of homogeneous sheared turbulence on a 128^3 grid, described in detail in Rogers *et al.* (1986) and Rogers & Moin (1986). We consider the field C128U12, which is at $t = 12$ in units of the imposed shear, when the Taylor microscale-scale Reynolds number is about 110. The field is not isotropic as elongated vortical structures are visible (isotropic turbulence of lower Reynolds number was also considered (Meneveau 1990). We compute the 3-D wavelet transform of the three velocity components, using the LMB wavelet basis. To compute the local transfer $t^{(m)}[\vec{i}]$, we need to compute the wavelet transform of the nonlinear terms of the Navier-Stokes equation (the pressure is computed from the known fluctuating velocity field using the Poisson equation), and then we apply Eq. 8.

Figure 2 shows the dual transfer spectrum $T(k_m)$ and $T(k_m) \pm \sigma_t^{(m)}$ computed from the homogeneous shear-flow, in Kolmogorov units.

The mean transfer (circles) is negative for low-wavenumbers and positive at high wavenumbers, showing that on the average energy is being transferred from large to small scales. The solid line indicates the corresponding radial Fourier transfer spectrum (obtained from the usual Fourier analysis), in reasonable agreement with the mean wavelet transfer. However, the standard deviation $\sigma_t^{(m)}$ is seen to be very large, implying that locally the transfer of energy is often quite far from its spectral mean value. This is borne out even clearer in the probability-density functions of $t^{(m)}[\vec{i}]$ of Fig. 3, which is for three scales $m = 1, 2, 3$. Large deviations away from the mean are visible, both on the positive and negative side. Also, we note the long tails of the distributions, which are of the exponential type. The same calculations for the isotropic flow yield similar results (Meneveau 1990).

The quantity $t^{(m)}[\vec{i}]$ represents the local transfer through a certain scale without discriminating between the other two scales involved in the nonlinear interactions. In order to define a position-dependent flux of kinetic energy to all scales smaller

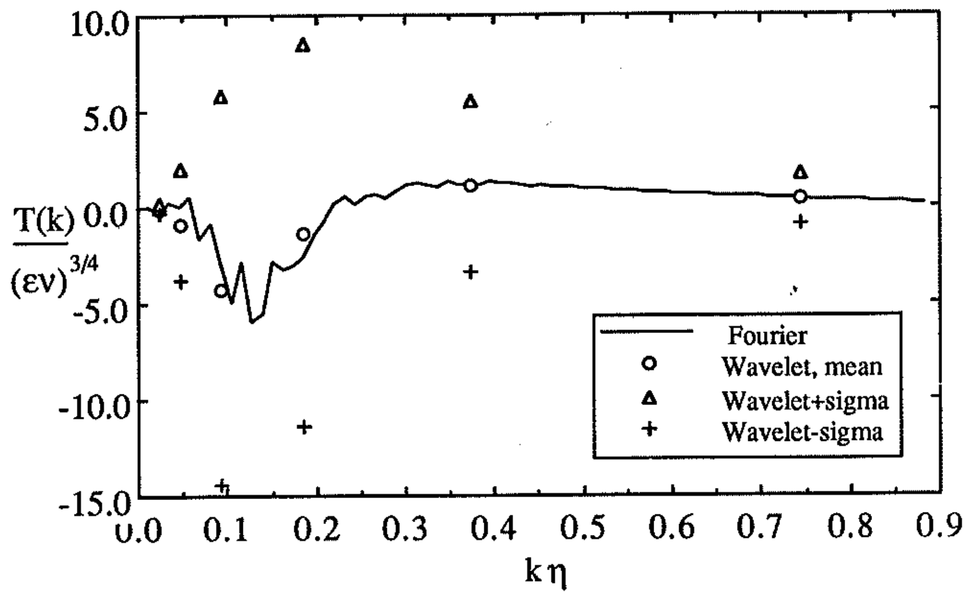

FIGURE 2. Dual spectrum of transfer of kinetic energy for homogeneous shear simulation, in Kolmogorov units. The solid line is the usual Fourier transfer, the circles are the mean wavelet spectrum, and the triangles and crosses are the wavelet mean, plus and minus one standard deviation computed from the spatial fluctuations at every scale.

than some cutoff-band m which does not include sweeping by the larger scales, one needs to decompose the non-linear terms in more detail. In Meneveau (1990) we show that

$$t^{(m,n)}[\vec{i}] =$$

$$\sum_{i=1}^{3}\sum_{q=1}^{7} w_i^{(m,q)}[\vec{i}] \int \{\frac{\partial}{\partial x_j}[u_i u_j - u_i^{>n} u_j^{>n}] + \rho^{-1}\frac{\partial}{\partial x_i} p^{<n}\} \Psi^{(m,q)}(\vec{x} - 2^m h\vec{i})\, d^3\vec{x}, \quad (12)$$

represents the transfer of energy between scales m and all scales smaller than n, a quantity that is analogous to the Fourier transfer spectrum $T(k \mid k_n)$ (Kraichnan 1976), defined as the total contribution to $T(k)$ from triads of wavenumbers $(k, q, k - q)$ having $k < k_n$ and at least one of the other two legs larger than k_n. In Eq. 12, the superscripts $> n$ ($< n$) refer to low-pass (high-pass) filtered fields obtained from Eq. 4 by performing the sum over all $m \geq n$ ($m < n$).

A quantity of great practical importance (Rogallo & Moin 1984) is the effective sink of kinetic energy due to scales of motion smaller than some cutoff. In the Fourier representation, this sink of energy is given (Kraichnan 1976) as a flux

$$\pi_{sg}(k) = -\int_{k'=0}^{k} T(k' \mid k)\, dk'. \quad (13)$$

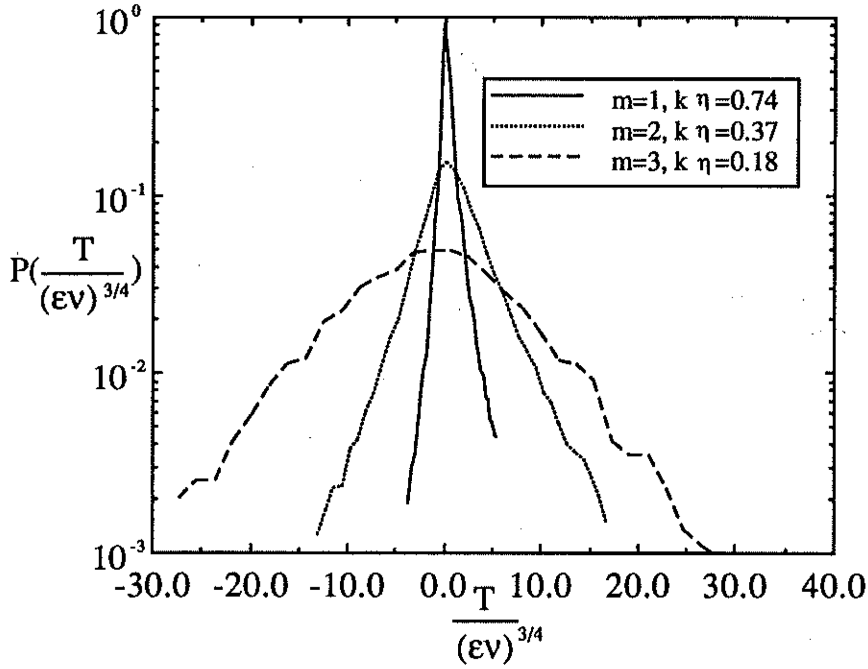

FIGURE 3. Probability density distribution of transfer of kinetic energy at scale $m = 1, 2, 3$, in Kolmogorov units.

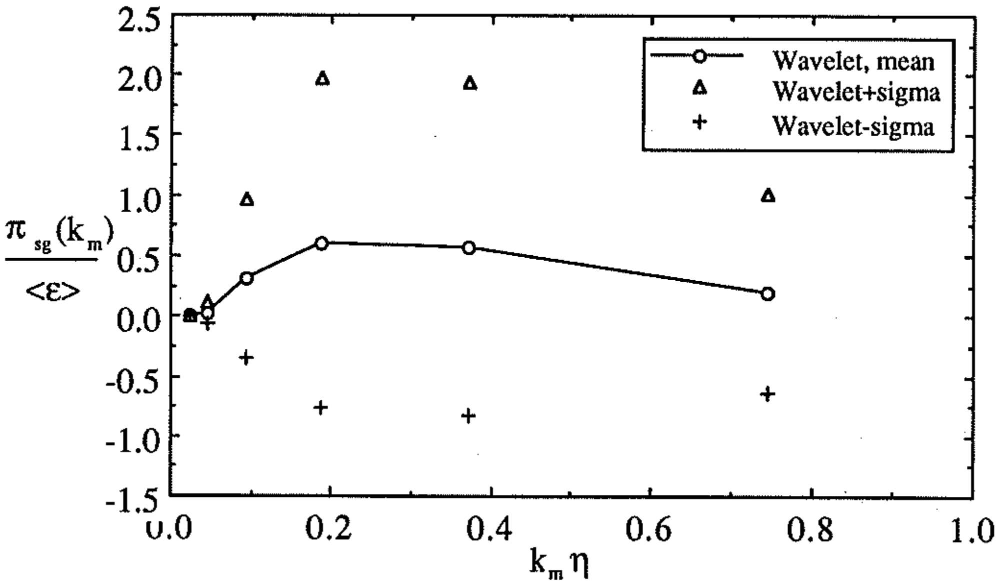

FIGURE 4. Dual spectrum of the flux of kinetic energy from interactions with all smaller scales of motion (flux to or from the subgrid scales), for the homogeneous shear flow simulation. Symbols as in Fig. 2.

The analogous definition in the wavelet representation is the local flux of energy to smaller scales,

$$\pi_{sg}^{(m)}[\vec{i}] = - \sum_{k=m}^{M} 2^{3(M-k} t^{(k,m)}[2^{m-k}\vec{i}]. \tag{14}$$

This quantity is measured in the homogeneous shear flow simulation, its dual spectrum is computed, and we obtain the probability density of this subgrid flux at every scale. The results are shown in Figs. 4 and 5.

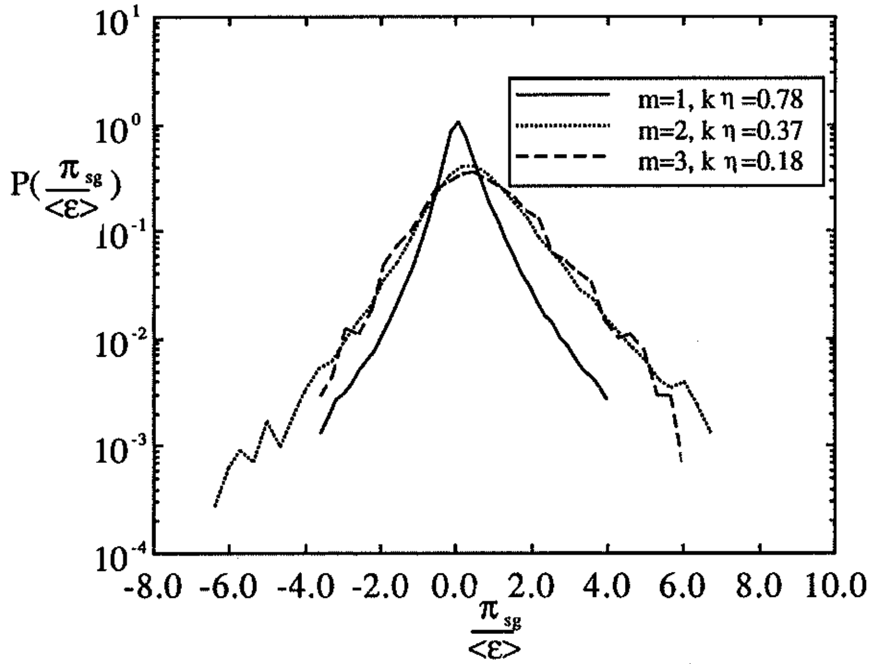

FIGURE 5. Probability density function of spatial fluctuations of local subgrid flux of kinetic energy for the homogeneous shear simulation, for scales corresponding to $m = 1$, $m = 2$, and $m = 3$.

We make the following observations: The mean subgrid flux is always positive, indicating that on the average, energy flows from large to small scales. There are strong spatial fluctuations, and the statistics of π_{sg} are far from Gaussian, exhibiting very clearly, again, long and exponential tails. These long tails mean that the flux is very intermittent in space at every scale of motion. The fluctuations of the subgrid flux are such that it can be negative very often (local backscatter). In such locations, energy actually flows from the small to the large scales of motion, i.e. there are local inverse cascades. The phenomenon of local backscatter has also been observed recently during analysis of channel flow and compressible turbulence (Piomelli *et al.*, 1990), with very similar results concerning the fraction of points at which backscatter is observed. The tails of the distributions are nearly symmetric to both sides; thus, the average being positive comes from a delicate balance between large positive and not-so-large negative excursions of localized events. The analysis of isotropic decaying turbulence yields similar results (Meneveau 1990).

We have shown elsewhere (Meneveau 1990) that these results are relatively robust with respect to the precise wavelet used. One of those wavelets was essentially a sharp Fourier band-pass filter in octaves (Fourierlet), suggesting that statistical results obtained by such approaches (e.g. the study of Domaradzki & Rogallo 1990) give results that are comparable to the ones using wavelet analysis.

5. Concluding remarks

We conclude with some general observations regarding the usefulness of orthonormal wavelets and will then proceed to recapitulate the results regarding spatial distribution of turbulence energetics.

The traditional discrete Fourier transform maps a 3-D function sampled on N^3 points onto $N^3/2$ complex Fourier coefficients (conserving the amount of information N^3). From this, useful and compact statistical information can be extracted as power-spectra of the field, either the radial spectrum or separate spectra in the different Cartesian directions. The usefulness of this approach stems from a variety of reasons including: (a) the existence of the Fast Fourier Transform, which allows efficient calculation of the transform (this also has permitted the manufacture of appropriate hardware such as digital spectral analyzers); (b) the straight-forward interpretation of the power-spectral density as related to the squared amplitude of waves in which the field is being decomposed; (c) the fact that the discrete Fourier transform is used not only as a tool of analysis but also to solve partial differential equations. Other basis functions such as Tchebychev polynomials are often employed to deal with particular boundary conditions; for the purpose of the present discussion, however, we consider them of the same family as the Fourier modes due to their global nature.

Orthonormal wavelet functions have the property that they also conserve information, that is, a discretely sampled field on N^3 points will yield only N^3 coefficients. However, these will be organized in a fashion that allows distinction between scale and location. This property is best illustrated in one dimension: here we have a total of N gridpoints and, thus, N coefficients: $N/2$ of them give information at $N/2$ different locations about the smallest scale, $N/4$ on the spatial distribution at the next largest scale, etc. The last coefficient is related to the global mean of the field. This dyadic arrangement not only makes sense physically (the larger scales are sampled more coarsely than the smaller ones), but it also allows for the implementation of a fast algorithm, analogous to the Fast Fourier Transform.

One of the objectives of the present work was to propose a variety of tools to analyze turbulence using the orthonormal wavelet decomposition instead of the Fourier transform. In order for the analysis to be practical, we had to achieve significant reductions in the amount of data. In the case of the 3-D Fourier transform where we start with N^3 values, the spectra essentially reduce to a single-valued function consisting of $O(N)$ points. This is easy to visualize and to interpret physically. Here we obtain more than the spectrum, using the additional spatial information that is now available. In this work, the focus is still statistical, but the idea is to postpone spatial averaging as long as possible. In comparison with the discrete Fourier analysis, we have then that the orthonormal wavelet approach has the following properties: (a) there is an efficient fast wavelet algorithm allowing the computation of all the coefficients with at most $O(NlogN)$ operations (for some wavelets, the operation count can be as low as $O(N)$!); (b) The interpretation of the square of the coefficients as the energy of localized pulses is straight-forward and very intuitive; (c) statistical properties (or more detailed spatial characterizations) of these coefficients in addition to the power-spectrum can be obtained and are easy to visualize and interpret. So far then, there is no practical difference between Fourier and wavelet analysis, but the latter allows more meaningful analysis of spatial properties at every scale, which is of great importance in the case of turbulence. The other important point raised in connection with the usefulness of the discrete Fourier transform, namely that it can play a key role in actually solving the equations, can not be made so far with the wavelet basis functions (there are several preliminary efforts in that direction, but it is too early to emphatically conclude as to their usefulness in this context). Therefore, for the time being, we only concentrated on properties (a), (b), and (c) listed above, which we feel are worth exploring in the hope of characterizing turbulent flow-fields in a systematic fashion.

The idea of studying turbulence using a space and scale dependence is not new: It was the motivation for band-pass filtering turbulent signals, an approach that was employed to study intermittency (see e.g. Kennedy and Corrsin 1961). Such studies were decisive in showing that turbulent activity becomes more and more intermittent at smaller and smaller scales, as quantified by appropriate statistical measures such as flatness factors. However, such methods of analysis are rather arbitrary in terms of the shape of filters, their bandwidth, etc. Here we attempted to circumvent this arbitrariness by invoking the more rigorous foundations of orthonormal wavelet analysis, which has the prospect of becoming a more standardized tool.

The relative simplicity of the *analysis* of turbulence with wavelets stems from the fact that they are generated from a single function, known *a priori*, and this, therefore, does not depend on the specific flow. This is in contrast to the method of 'proper orthogonal decomposition' (Lumley 1967), which constructs basis functions that maximize the energy contained in the smallest number of modes. Here we do not attempt any such optimization, and so the number of wavelets needed is generally high. However, such an approach is necessary to study local properties of instantaneous realizations of turbulent flows because, for example, in the case of homogeneous flows, the proper orthogonal decomposition yields the usual non-local Fourier modes.

We introduced the dual spectral representation, which measures the contributions of each scale as well as its spatial variability. This could then be used to quantify the intermittency of the local transfer of kinetic energy in wavelet space. This quantity was then measured in direct numerical simulations of turbulence, and strong spatial fluctuations were observed, their mean spectral value being quite unrepresentative of local values. This is consistent with the findings of Piomelli *et al.* (1990).

By appropriately decomposing the non-linear terms of the Navier-Stokes equation, we could subtract the transfer (and flux) due to the interactions with the large scales from the total values. This then allowed the definition of a dual spectrum of $\pi_{sg}(r,\vec{x})$, the flux of energy involving the scales smaller than r, at location $\vec{x}$. Again, this quantity exhibited intermittent spatial behavior with exponential tails in its probability density distributions, while its spatial mean values were consistent with the usual spectral behavior. The importance of exponential tails in turbulence is confirmed by these observations. Quite importantly, large negative values of $\pi_{sg}(r,\vec{x})$ were observed, implying local inverse energy flux from small to large scales of motion. Although these results have been obtained for a low-Reynolds number flow where no fully developed inertial range exists, it seems unlikely that the local backscatter would disappear completely at higher Reynolds numbers. In fact, the analysis of channel flow by Piomelli *et al.* (1990) suggests a modest increase of backscatter with Reynolds number. Much of the phenomenological cascade models of intermittency work under the assumption of local (in x- and k-space) energy transfer from large to small scales, where, from dimensional arguments, the locally averaged rate of dissipation (always positive) is the quantity representative of the local inertial-range flux of energy. Such a picture must be revised in order to allow for negative fluxes to occur.

In Meneveau 1990, we have proposed a simple extension to the traditional cascade models of energy, which allows for backscatter (negative energy flux) in an intermittent fashion. The model suggests (but does not prove) that the detailed spatial statistics of the backscatter are intimately linked to the intermittency of the cascade. In this mixed cascade model, if there was no intermittency (as in the original Kolmogorov cascade picture), there also would not be negative flux. Only the strong spatial fluctuations produced by intermittency allow some small scales to be very active, so that if they pair (passing a part of their flux of energy to the larger scale), the resulting negative flux may locally overwhelm the positive one. We have shown that such mixed cascade models exhibit the same qualitative features as the real local flux: increasing intermittency at decreasing scale, exponential-like tails of the probability density for both positive and negative values. However, measurements of $\pi^{(m)}(\vec{x})$ need to be made at higher Reynolds numbers before we can make more quantitative comparisons with such mixed cascade models.

In summary, the general picture of turbulence that is confirmed by this analysis is that there is strong spatial intermittency in all non-linear quantities, their mean spectral behavior resulting from a delicate balance given by the difference of large positive and negative excursions in space. The wavelet analysis is a way of quantifying these observations in a standardized fashion by using 'flow-independent eddies' to decompose the velocity field. The present study shows that orthonormal wavelet analysis can be performed just as easily as Fourier analysis, and we believe the physical interpretation of the wavelet coefficients is more natural to the phenomenon of turbulence than the coefficients of globally extended functions such as Fourier modes. As remarked by Stewart (1989), one is decomposing the motion into 'solitons', a concept more appropriate for non-linear phenomena.

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