# The role of pressure-dilatation correlation in rapidly compressed turbulence and in boundary layers

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## 1. Motivations and objectives

This work is a result of the continuing effort to advance our understanding of the compressibility effects on turbulence and to develop new, and improve the old, models for compressible turbulent flows. A specific goal reported here has been the investigation of the role of pressure-dilatation correlation in rapidly compressed turbulence and in boundary layers in general. The rapid compression process is present in flows of practical importance such as in internal combustion engines and in boundary layer/shock interactions.

## 2. Accomplishments

The basis for the investigation of the rapid compression effect on turbulence has been the results of the direct numerical simulations (DNS) of compression of *homogeneous* turbulence (Coleman and Mansour 1991). The study led to a development of new turbulence closure models which represent the physics of rapid compression. The study of the DNS of compressed turbulence yielded the following important findings: it was established that when nearly incompressible turbulence (with small r.m.s. Mach number $M_t << 1$) is rapidly compressed in one direction (1D), unexpectedly high levels of negative pressure-dilatation correlation are generated. The pressure-dilatation term $(\overline{p\theta})$ appears in the turbulence kinetic energy, and its magnitude during the 1D compression can become an order of magnitude larger then the total dissipation $(\epsilon_t)$; hence, $\overline{p\theta}$ can lead to a significant loss of turbulent kinetic energy to pressure fluctuations. The striking aspect of this rapid compression mechanism is that it is most effective when $M_t << 1$ and that it is inefficient when the compression is more isotropic, i.e., acting in all three directions. All these aspects have been included in the new model for pressure dilatation described in Section 2.1. Section 2.2 describes an application of the new rapid pressure-dilatation model in modeling the turbulence response to passing through a shock. In Section 2.3, an inhomogeneous contribution to pressure-dilatation in adiabatic boundary layers is suggested. This contribution is shown to be instrumental in mitigating the discrepancy between turbulence closure models and the Van Driest Law of the Wall. Finally, in Section 2.4 model predictions are compared with experimental data in supersonic boundary layers over an insulated plate.

### *2.1. Rapid compression of homogeneous turbulence*

The basic energy-governing equations describing compression of homogenous turbulence are

$$\frac{1}{2}\frac{\partial q^2}{\partial t} = -\frac{1}{3}q^2\nabla\cdot\mathbf{U} - \epsilon_t + \frac{\overline{p\theta}}{\overline{\rho}} - q^2 b_{ij}S^*_{ij} \tag{1}$$

$$\frac{\partial c_v\widetilde{T}}{\partial t} = -(\gamma-1)c_v\widetilde{T}\nabla\cdot\mathbf{U} + \epsilon_t - \frac{\overline{p\theta}}{\overline{\rho}} \tag{2}$$

$$\frac{1}{2}\frac{\partial\overline{p^2}}{\partial t} = -\gamma\overline{p^2}\nabla\cdot\mathbf{U} - \gamma\overline{p}\overline{p\theta} \tag{3}$$

$$\frac{\partial\overline{\rho}}{\partial t} = -\overline{\rho}\nabla\cdot\mathbf{U} \tag{4}$$

$$\overline{p} = \overline{\rho}R\widetilde{T} \tag{5}$$

where $q^2 = \widetilde{u_j u_j}$ denotes twice Favre-averaged turbulence kinetic energy, $\epsilon_t$ is the total dissipation rate, $b_{ij}$ is the departure-from-isotropy tensor, and $\overline{p^2}$ is the fluctuating pressure variance. The pressure-dilatation correlation $\overline{p\theta}$ appears in Eqs (1)-(3); $\theta$ stands for fluctuating dilatation, i.e. $\theta = u_{j,j}$. The mean divergence $\nabla\cdot\mathbf{U}$ follows the time dependence dictated by the homogeneity constraint

$$\nabla\cdot\mathbf{U}(t) = -\frac{A_o}{1-A_o t}$$

where $-A_o$ is the initial divergence. The last term on the right side of (1) represents the nonisotropic contribution to turbulence energy production; here, $S^*_{ij}$ is the trace-free mean deformation tensor defined as

$$S^*_{ij} = \frac{1}{2}(U_{i,j} + U_{j,i} - \frac{2}{3}\nabla\cdot\mathbf{U}\delta_{ij}). \tag{6}$$

Note that for spherical (isotropic) compression, $S^*_{ij}$ is identically zero. The condition of rapid distortion requires that

$$A_o\frac{q^2}{\epsilon_t} >> 1. \tag{7}$$

In this case, the dissipation $\epsilon_t$ can be neglected, and the major closure problem in the equation set (1)-(5) consists in approximating $\overline{p\theta}$. In the previous work, Zeman (1991a,b) has suggested the following model for the pressure correlation term (also Zeman and Blaisdell 1991)

$$\overline{p\theta} = (\overline{p}\gamma)^{-1}\{\frac{(\overline{p^2} - p_e^2)}{\tau_f} + c_{div}\overline{p^2}\nabla\cdot\mathbf{U}\} \tag{8}$$

where $p_e^2 \propto \overline{p}^2 M_t^4$ is the equilibrium value of the pressure variance $\overline{p^2}$ and $\tau_f$ is the (acoustic) relaxation time scale defined as $\tau_f = 0.2\tau M_t$. Here, $M_t = q/a$ is the r.m.s. Mach number and $\tau = q^2/\epsilon_s$ is the turbulence time scale (based on the solenoidal part $\epsilon_s$ of the total dissipation $\epsilon_t$). In (8), the first term represents a nonlinear relaxation to equilibrium, and the second term can be considered as a rapid contribution. From the modified theory of Sabelnikov (1975), it was possible to determine the rapid compression constant $c_{div}$ in (8) as $c_{div} = (5 - 3\gamma)/12$ (see Zeman 1991c, Durbin and Zeman 1991). The pressure-dilatation model in (8) has been shown to represent the principal physics associated with the initial transient and subsequent evolution of unforced and shear-driven turbulence (where $\nabla \cdot \mathbf{U} = 0$). When compared with the direct numerical simulations (hereafter DNS) of rapidly compressed turbulence (Coleman and Mansour 1991), the model reproduced fairly well the spherical compression case (Zeman 1991c). However, the model was incapable of reproducing the DNS results of the one-dimensional (1D) rapid compression. The peculiarities of the turbulence behavior under the spherical vs 1D compression led to interesting findings concerning the importance of the pressure dilatation in the rapid compression of nearly incompressible turbulence and, ultimately, to the formulation of a new rapid model for $\overline{p\theta}$. The latter work has been described in detail in Zeman (1991c). Later, Durbin and Zeman (1991) developed the rapid distortion theory and suggested a new model for the pressure dilatation, intended for the rapid compression of low $M_t$ turbulence.

To explain the effect of the directionality of compression on turbulence, it is illuminating to realize that the rapid contribution to instantaneous pressure $p(\mathbf{x}, t)$ for $M_t << 1$ involves an integral

$$p(\mathbf{x}, t) \propto U_{i,j} \int u_{j,i}(\mathbf{x}', t) G(\mathbf{x}, \mathbf{x}') d\mathbf{x}' \tag{9}$$

where G is the appropriate Green's function. For the spherical rapid compression, the contributions to $p(\mathbf{x}, t)$ consist of $\nabla \cdot \mathbf{U} \theta(\mathbf{x}', t)$ (recall $\theta \equiv u_{j,j}$). In the incompressible limit $M_t \to 0$, the rapid pressure is negligible since $\theta \approx 0$. However, in 1D compression (say, $\nabla \cdot \mathbf{U} = U_{1,1}$), the contributions to the right side of (9) involve terms $\nabla \cdot \mathbf{U} u_{1,1}$ which are finite even when turbulence is solenoidal. Now, rewriting (3) in a different form:

$$\frac{1}{2} \frac{\partial(\overline{p^2}/\gamma \overline{p})}{\partial t} = -\overline{p\theta} \tag{10}$$

we can appreciate that solenoidal pressure contributions to $\overline{p^2}$ can lead to a finite value of $\overline{p\theta}$; the only requirement is that the rate of change of $\overline{p^2}$ is sufficiently rapid. An insight into the formulation of a modeling expression for $\overline{p\theta}$ that would distinguish the directionality of compression can be obtained from a suitably arranged equation for $\overline{p\theta}$:

$$\frac{D\overline{p\theta}}{Dt} = -(\frac{3}{2} + \gamma) \nabla \cdot \mathbf{U} \overline{p\theta} - \gamma \overline{p} \overline{\theta^2} - 2 S^*_{ij} \overline{p s_{ij}} + H.O.T. \tag{11}$$

Here, the higher order terms $H.O.T.$ can be discarded if the rapid condition (7) is satisfied. The last rapid term in (11) involves the pressure-strain correlation as it also appears in incompressible Reynolds stress equations. The leading order contribution to this term is known to be $\overline{ps_{ij}} = \overline{\rho}q^2\{\frac{1}{5}S^*_{ij} + F_{ij}(\mathbf{S},\mathbf{b})\}$ where $F_{ij}$ is a tensor bilinear in $S^*_{ij}$ and $b_{ij}$ (see, e.g. Zeman 1990). The final form of the rapid directional part of $\overline{p\theta}$ is

$$(\overline{p\theta})_D = c_{d1}\overline{\rho}\frac{(\overline{p^2})^{1/2}}{\overline{p}M_t^2}q^2\tau\{(S^*_{ij})^2 + c_{d2}b_{ik}S^*_{kj}S^*_{ij}\}, \tag{12}$$

where $c_{d1} = 0.0004$ and $c_{d2} \approx 2$. The form of the model in (12) without the higher order terms in $b_{ij}$ was first suggested by Zeman (1991c); inclusion of the anisotropic contribution has improved the model performance at larger total strains (when $A_o\tau > 0.5$).

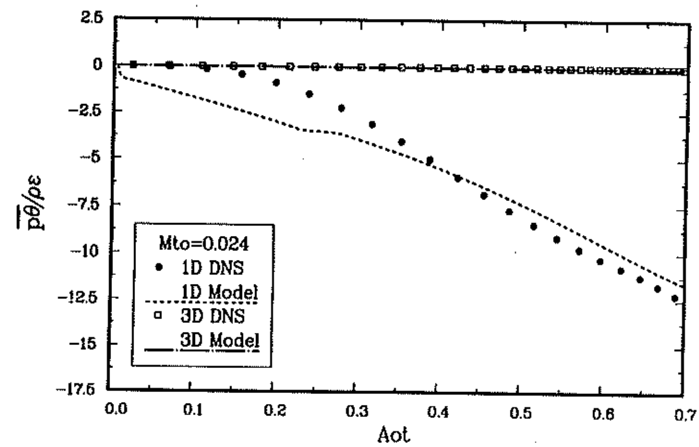


FIGURE 1. Evolution of the pressure-dilatation/dissipation ratio in rapidly compressed turbulence; the initial $M_t = 0.024$.

To contrast the effect of the compression directionality, we present in Fig. 1 the model-DNS comparison of the quantity $\overline{p\theta}/(\overline{\rho}\epsilon_t)$ for spherical (3D) and 1D compressions with the same initial values of $M_t = 0.024$ and of the rapid parameter $A_o\tau = 47$. The closure for $\overline{p\theta}$ is as that in (8) plus the directional contribution in (12). It is evident that the model replicates the fundamental physics of the directional compression effect: in 3D compression $\overline{p\theta}$ is negligibly small even in comparison with the dissipation; in 1D compression, on the other hand, the pressure dilatation term can be by an order of magnitude larger than dissipation and, therefore, important for the turbulence energetics. As discussed in detail in Zeman (1991c), the principal effect of the pressure-dilatation term is to mediate energy exchange between the kinetic to the pressure fluctuation (potential) modes. During the 1D rapid compression, $\overline{p\theta}$ is negative, and hence, according to (1) and (3), the kinetic energy $q^2$ is converted into the potential energy. This leads to a lower growth rate of $q^2$ than that predicted, for example, by incompressible $k-\epsilon$ models, where the $\overline{p\theta}$-term is absent. In conclusion, during a directional rapid compression, the kinetic-to-potential energy conversion can be significant even when turbulence

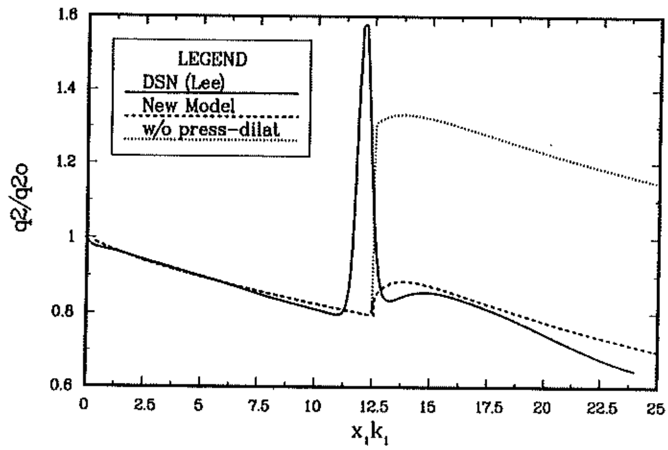


FIGURE 2. Decaying turbulence response to a shock: Model-DNS comparison.

is virtually incompressible and the reduced growth of $q^2$ (if detected) should not be attributed to viscous dissipation. As discussed next, the described mechanism is clearly relevant to flows of practical interest where turbulence passes through a shock.

### *2.2. Pressure dilatation in turbulence passing through a shock*

The model for the pressure dilatation has been tested against the DNS data of isotropic (decaying) turbulence passing through a normal shock (Lee 1991). With the $x_1$ axis aligned with the direction normal to the shock, the relevant kinetic energy equation in the Favre-average setting can be written as

$$U_1\frac{1}{2}(q^2)_{,1} = -\widetilde{u_1^2}U_{1,1} - \frac{1}{\overline{\rho}}\overline{p}_{,1}\overline{u}_1 - (\epsilon_t - \overline{p\theta}/\overline{\rho}) - \frac{1}{2\overline{\rho}}(T_{jj1} + 2\overline{pu_1})_{,1} \qquad (13)$$

where $T_{ij1} = \overline{\rho u_i u_j u_1}$ are nonzero third moment fluxes in the $x_1$ direction, and the fluctuation velocity average is by definition $\overline{u}_1 = -\overline{\rho' u_1}/\overline{\rho}$, which is nonzero. To simplify the model computation, the mean shock flow quantities $U_1(x_1)$, $\overline{\rho}(x_1)$, and pressure $\overline{p}(x_1)$ are prescribed from the Rankine-Hugeniot relations. In order to close (13), apart from the closure for $\Pi_d$, it is also necessary to model $\epsilon_t$ and the inhomogeneous terms: the pressure flux $\overline{pu_1}$, mass flux $\overline{\rho' u_1}$, and $T_{jj1}$. The modeling approach is described in detail in Zeman (1991c). Here, it should be noted that the so-called acceleration term associated with the mean pressure gradient $\overline{p}_{,1}$ is very sensitive to modeling the mass flux $\overline{\rho' u_1} \propto -\overline{u}_1$. Application of the Strong Reynolds Analogy (SRA) of Morkovin to estimate the mass flux leads to a wrong sign of the acceleration term ($\overline{\rho' u_1} > 0$), ultimately overestimating the energy amplification through the shock.

Although the turbulence/shock interaction flow in question is inhomogeneous, the homogeneous (rapid) part of the pressure-dilatation term must play an important role in the dynamics since the compression is one dimensional and very rapid. The DNS and model results are compared in terms of the kinetic energy $q^2$ in Fig. 2.

Here, the decaying turbulence passes through a shock located at the normalized distance $x_1 k_1 = 12.5$, where $k_1$ is the wavenumber associated with the peak of the initial energy spectrum at $x_1 = 0$. In the model, $k_1$ is related to the initial dissipation length scale, namely $(k_1)_{mod} \propto (\epsilon_s/q^3)_o$. The upstream mean Mach number is $M_1 = 1.18$, so that the shock density ratio is $C = \overline{\rho}_2/\overline{\rho}_1 = 1.31$; immediately in the front of the shock, the r.m.s. Mach number is $M_t = 0.13$. The sharp peak in the DNS results (solid circles) at the shock location is caused by the unsteady movement of the shock, and it is not relevant to the overall (effective) turbulence response to the shock. The solid line represents the model prediction with the complete pressure-dilatation model in (8) and (12); the dotted line represents the model results with the directional rapid part $(\overline{p\theta})_D$ in (12) excluded (note that $(S^*_{ij})^2 = \frac{2}{3}(U_{1,1})^2$). It is evident that the model-predicted shock amplification ratio $A = q_2^2/q_1^2$ is strongly dependent on the rapid part $(\overline{p\theta})_D$. There appear to be no other means to bring the model prediction into agreement with the DNS results. Here, we should point out that the (effective) DNS computed amplification $A = 1.15$ is significantly below the linear rapid estimate

$$A = \frac{C^2}{3}(1 + \frac{2}{C^{4/3}}) \approx 1.37.$$

Similar discrepancy has been observed by Jacquin, Blin, and Geffroy (1991) in the wind tunnel experiment: with C=1.5, they measured very little amplification of $q^2$ through shock, although the linear estimate is about $A = 1.4$.

### *2.3. Pressure-dilatation in adiabatic supersonic boundary layer*

This section concerns a modification in Reynolds stress closure models (RSC) and k-$\epsilon$ models intended to recover the Van Driest compressible law of the wall in supersonic turbulent boundary over an adiabatic wall.

As shown by Huang, Bradshaw, and Coakley (1991), the current standard $k - \epsilon$ models are not capable of recovering the Van Driest compressible law of the wall (hereafter Van Driest Law). According to this law, the Van Driest transformed mean velocity $U_c$ should follow the incompressible logarithmic law of the wall (in the limit of zero free stream Mach number $M \rightarrow 0$). The van Driest transformation is defined as

$$U_c = \int_0^U (\frac{\overline{\rho}}{\rho_w})^{1/2} dU \tag{14}$$

and the log law is then

$$\frac{U_c}{u_*} = U_c^+ = \frac{1}{\kappa} ln(y^+) + 5.2, \tag{15}$$

where

$$u_* = \sqrt{\tau_w/\rho_w} \tag{16}$$

is the friction velocity and $y^+ = yu_*/\nu_w$, and $\kappa = 0.41$ is the von Karman constant. The subscript $w$ denotes properties at the wall, i.e. $\tau_w$ is the wall shear stress; otherwise, the notation is standard.

In the following it is shown that in the current models, the kinetic energy equation is not complete; the missing physics and corresponding term have to do with the pressure-dilatation correlation generated by the (vertical) density gradient.

*2.3.1 Pressure-dilatation correlation in a boundary layer*

Following the same reasoning as in Zeman (1991a), the inclusion of the density gradient term in the expression for $\overline{p\theta}$ is fairly straightforward. The equation for fluctuating density reads

$$\frac{D\rho'}{Dt} = -\overline{\rho}u_{j,j} - (\rho' u_j)_{,j} - u_j \overline{\rho}_{,j} - \rho' \nabla \cdot \mathbf{U}. \tag{17}$$

and with the adiabatic relation $p/\overline{p} = \gamma\rho'/\overline{\rho}$, one obtains the equation for $\overline{p^2}$

$$\frac{1}{2}\frac{D\overline{p^2}}{Dt} = -\overline{p\theta}\gamma\overline{p} - a^2\overline{\rho}_{,j}\overline{pu_j} - \gamma\overline{p^2}\nabla\cdot\mathbf{U} + \ H.O.T. \tag{18}$$

The pressure flux $\overline{pu_j}$ can be expressed as

$$\overline{pu_j} = \overline{p}(\frac{\overline{\rho' u_j}}{\overline{\rho}} + \frac{\widetilde{T' u_j}}{\widetilde{T}}) \propto a^2 \overline{\rho' u_j} f_\rho(M_t), \tag{19}$$

where $f_\rho$ is a function which must satisfy certain limiting behavior to be discussed later. Since in the thin layer approximation the advection term is small compared with the right side of (18) and the mean dilatation $\nabla\cdot\mathbf{U} \approx 0$, (18) reduces to the principal balance between the pressure dilatation and density gradient terms,

$$\overline{p\theta} \propto -a^2 \overline{\rho' u_j} \frac{\overline{\rho}_{,j}}{\overline{\rho}} f_\rho(M_t). \tag{20}$$

In analogy with the closure equation for heat flux $\overline{T' u_j}$, one can form an equation for the mass flux $b\rho' u_j$ (see Zeman 1991c for details). In the flat plate boundary layer, the latter equation reduces to a gradient model $\overline{\rho' u_j} = -T\widetilde{u_2^2}\overline{\rho}_{,2}$ where $T$ is a mass-flux relaxation time scale made proportional to the turbulence time scale, $T \propto \tau$. An expected dependence of $T$ on $M_t$ is absorbed in the function $f_\rho$. Hence, the final closure expression for the density contribution to $\overline{p\theta}$ of (20) is

$$\overline{p\theta} = C_\rho f_\rho(M_t)\tau\widetilde{u_2^2}(\overline{\rho}_{,2})^2\frac{a^2}{\overline{\rho}}, \tag{21}$$

where $C_\rho$ is a free constant. Concerning the desired behavior of the function $f_\rho$, we argue as follows: if the turbulent fluctuations were quasi-adiabatic, then $f_\rho \approx 1$. Such an approximation would be permissible if the boundary layer flow is adiabatic (with no surface heat flux). On the other hand, if the boundary layer is nearly incompressible (and with arbitrary surface heat flux), we would expect $|\overline{pu_j}| \to \overline{\rho}q^2u_*$; hence, $f_\rho(M_t)$ should approach zero as $M_t^2$.

The new density-gradient contribution to pressure dilatation formulated in (21) is positive and reflects the process of conversion of the potential ($\propto \overline{p^2}$) to kinetic energy. The pressure fluctuations are produced by the action of turbulence on the density gradient; in turn, this density gradient is produced by turbulent dissipative heating. In principle, this new term should be combined with other contributions as described earlier in this report. However, in zero-pressure-gradient (ZPG) boundary layer, the latter contributions were found to be negligible. Considering now the incompressible limit of boundary layer flow, it is imperative that the new term approaches zero as $M_t \to 0$ in such a manner that the ratio $\pi_{pd} = \overline{p\theta}/(\overline{\rho}\epsilon) \to 0$. In the adiabatic boundary layer, this condition is satisfied since $\pi_{pd} \to f_\rho \mathrm{M}^4/M_t$ and $M_t \propto \mathrm{M}$. In boundary layers where the density gradient is due to a difference $\Delta T$ between free stream and wall temperatures, (21) yields

$$\pi_{pd} \propto (\frac{\Delta T}{\tilde{T}})^2 f_\rho M_t^{-2}.$$

The quantity in the parentheses in the above expression is typically of order $10^{-1}$ or less, thus the relative contribution of $\overline{p\theta}$ in the $q^2$ equation is indeed small as long as $f_\rho$ approaches zero as $M_t^{-n}$ with $n \geq 2$. As previously discussed, $n = 2$ is consistent with the required behavior of $\overline{pu_j}$. In the computational examples that follow, we have chosen $n = 3$ so that $\pi_{pd}$ approaches zero as $M_t$. This requirement guarantees no (spurious) contributions from $\overline{p\theta}$ when $M_t \approx 0$. At this point, it is appropriate to mention that Rubesin (1990) arrived, from quite different premises, at a pressure dilatation model expression which is similar to (21). However, the Rubesin model would be unphysical in a non-adiabatic boundary layer in the small $M_t$ limit since it yields $\pi_{pd} \to M_t^{-2}$. Such behavior is inadmissible and would lead to spurious turbulence energy production in regions where density gradient is finite but $M_t << 1$. This could occur in any type of boundary layer flow (for example, in the separation bubble of a supersonic compression corner flow).

The preliminary computations of a ZPG boundary layer over an insulated wall, with a modified RSC model (Zeman 1990; 1991b) which includes the pressure dilatation term in (21) are shown in Figs 3 and 4. For the best results, the free constant in (21) was set at $C_\rho = 0.002$; referring to the previous discussion, the function $f_\rho$ was chosen as

$$f_\rho(M_t) = 1 - exp\{-(15M_t)^3\}. \qquad (22)$$

so that $f_\rho \approx 1$ for $M_t \geq 0.1$, and $f_\rho \to M_t^3$ as $M_t \to 0$ as required. It should be pointed out that the model results are insensitive to the exact form of $f_\rho$ although (22) is the most convenient form to satisfy the required function limits. Fig. 3 depicts the Van Driest velocity profiles $U_c^+(y^+)$ for different freestream Mach numbers (M) with $\overline{p\theta}$ from (21) and (22). It is seen that the profiles collapse reasonably well; for the measure of improvement, one profile is shown (with $\mathrm{M} = 7$) with the pressure dilatation set at zero. Fig. 4 depicts the scaled profiles of dissipation $\epsilon\kappa y(\overline{\rho}/\tau_w)^{3/2}$ (labelled $\epsilon^+$) as functions of $y^+$ for different Mach numbers. According to the Van

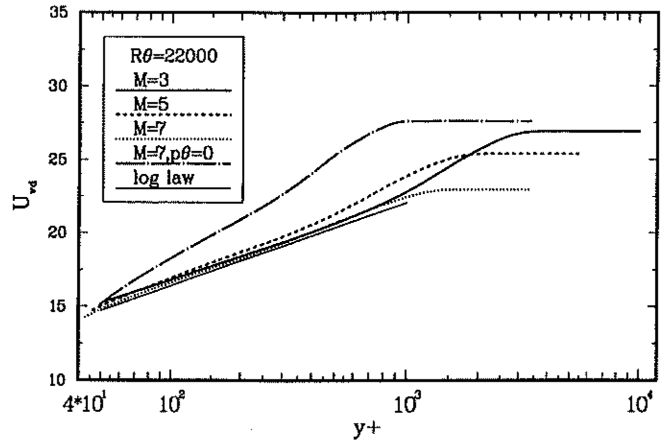


FIGURE 3. Boundary layer velocity profiles in Van Driest coordinates for different Mach numbers: model simulations.

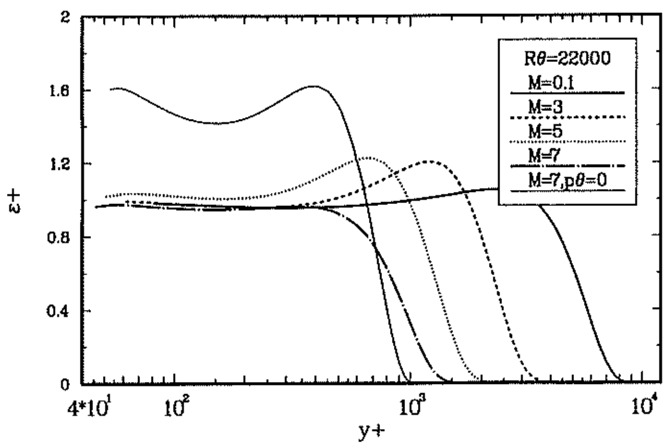


FIGURE 4. Normalized dissipation $\epsilon^+ = \epsilon \kappa y(\frac{\overline{\rho}}{\tau_w})^{3/2}$; otherwise same as Fig. 3.

Driest scaling, the depicted profiles should collapse to unity in the log region (constant stress layer) which is evidently the case. As in Fig 3. the included dissipation profile with the pressure dilatation absent indicates the model improvement. It is to be added that $\overline{\rho}q^2$ is not constant in the constant stress region; however, this is not contradictory to the Van Driest Law.

In conclusion, there exists corroborative evidence for the proposed pressure-dilatation model in (21) from the so-called two-scale DIA theory of Yoshizawa. To demonstrate this, we note first that the quantity $-\overline{\rho' u_j}\overline{\rho}_{,j}$ in (20) represents the rate of production of density fluctuations which, in turn, is proportional to the density fluctuation decay, say, $\overline{\rho'^2}/\tau_\rho$. Here, $\tau_\rho$ is a decay time scale controlled primarily by the (molecular) decay time scale of temperature fluctuations. Hence, the pressure-dilatation term in (21) is related to the density fluctuation variance

$\overline{\rho'^2} = \sigma_\rho^2$ in the following way

$$\frac{\overline{p\theta}}{\overline{\rho}} = C_\rho f_\rho \frac{a^2}{\tau_\rho}\frac{\sigma_\rho^2}{\overline{\rho}^2}. \tag{23}$$

Now, Yoshizawa (1990) inferred from his two-scale DIA theory the following expression for the subgrid-scale eddy viscosity (denoted $\nu_T$) in compressible turbulence:

$$\nu_T = \nu_{To}(1 + const\ \frac{\sigma_\rho^2}{\overline{\rho}^2}).$$

Thus, the basic (Smagorinsky) viscosity $\nu_{To}$ is augmented by a term containing the density fluctuation variance $\sigma_\rho^2$. Evidently, the presence of density fluctuations increases the subgrid-scale viscosity via the increase in kinetic energy. It can be easily shown that the $\overline{p\theta}$-contribution (23) in the kinetic energy equation would lead to a similar form of the subgrid-scale viscosity as in the Yoshizawa expression above.

### *2.4. Comparison with boundary layer experiments*

As an indication of the overall performance of the boundary layer model, we have chosen to compare the model predictions of mean velocity and temperature with the experimental data of Coles tabulated in Fernholz and Finley (1977) as Case 53011302. The comparisons are shown in Figures 5 and 6. Here, the ZPG boundary layer is adiabatic with the freestream Mach number $\mathbf{M} = 4.544$, and the momentum thickness Reynolds number $R_\theta = 5,500$. To demonstrate the effect of the new $\overline{p\theta}$-contribution in (21) on the model predictions, the dashed curves in Figures 5 and 6 represent model computations without $\overline{p\theta}$. The contribution of the new $\overline{p\theta}$ term to the improvement of the model predictions appears to be small. The degree of improvement would be more clearly evident if the profiles are presented in the van Driest coordinates. In terms of the velocity gradient and friction coefficient values, the new $\overline{p\theta}$ term represent about 15% improvement.

According to (23), the new pressure-dilatation term depends mainly on the density fluctuation variance $\sigma_\rho^2$ which, in turn, is proportional to the temperature fluctuation variance $\sigma_T^2 = \widetilde{T'^2}$ according to the relation

$$\frac{\sigma_\rho}{\overline{\rho}} \propto \frac{\sigma_T}{\widetilde{T}}.$$

The temperature fluctuations are accessible to measurements, and the temperature variance $\sigma_T^2$ is a byproduct of the model computations of the heat flux $\widetilde{T'u_2}$ and $\widetilde{T}$. In Figure 7, the model-computed values of $\sigma_T/\widetilde{T}$ are compared with the experimental data of Kistler (1959). The Kistler measurements were made in an adiabatic boundary layer for three different freestream Mach numbers $\mathbf{M} = 1.72,\ 3.56,\ 4.67$ and are also tabulated in Fernholz and Finley (1977). It is evident that the model predicts a correct tendency of the $\sigma_T$-levels with $\mathbf{M}$.

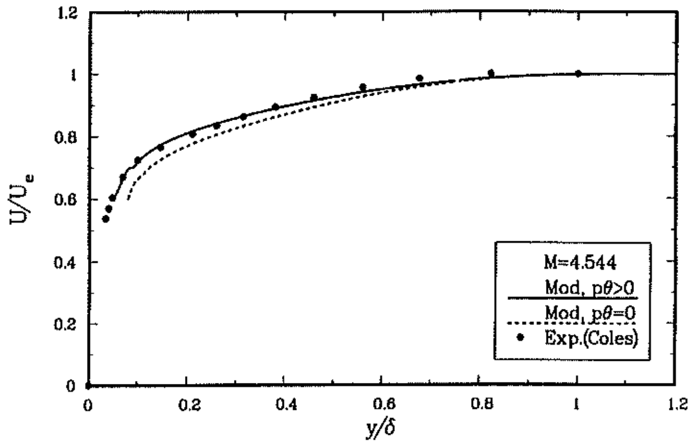


FIGURE 5. Mean velocity profiles in a boundary layer over an insulated wall: model-experiment comparison; $y = \delta$ is the boundary layer depth where $U = 0.99U_e$.

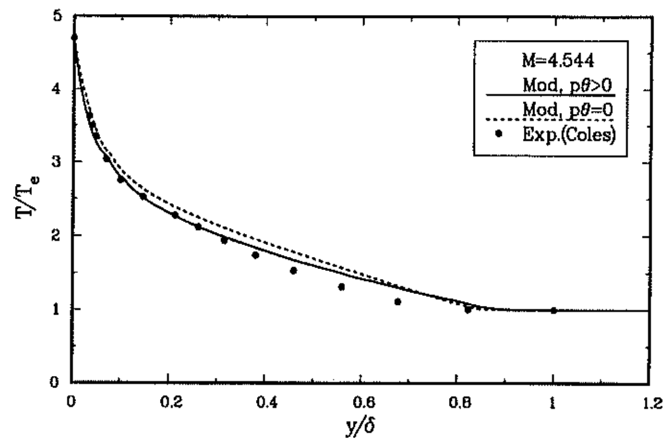


FIGURE 6. Mean temperature profiles, otherwise same as in Figure 5.

## 3. Future plans

The focus of current and future work is investigation of the rapid compression and distortion processes taking place when turbulence passes through a shock or a succession of shocks in the compression corner flow. A numerical scheme has been developed which is capable of simulating the mean and turbulence flow field of a nonseparating boundary layer negotiating a compression corner. The scheme utilizes the von Mises' coordinate transformation as in Zeman (1990), and the pressure gradient is calculated with the aid of the method of characteristics using the actual velocity profile as the upstream conditions. Preliminary comparison with experimental data for Reynolds stresses by Smits and Muck (1987) is encouraging. The model results indicate that the rapid directional compression mechanism discussed above has considerable influence on the response of turbulence to the compression corner-induced distortion of the mean flow.

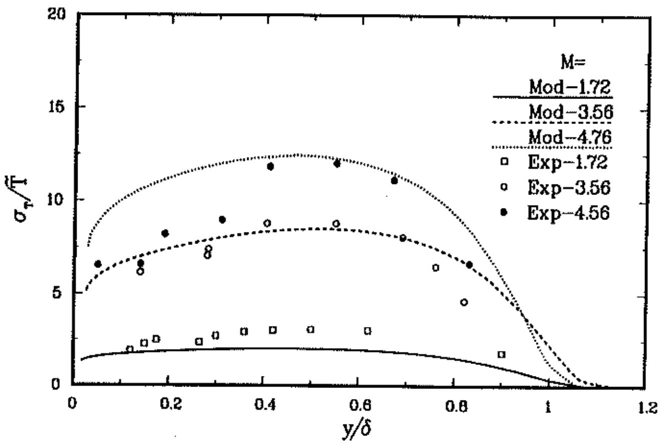


FIGURE 7. Profiles of temperature fluctuations in an adiabatic boundary layer: model comparison with the Kistler (1959) experiment.

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