Single point modeling of initially isotropic turbulence under uniform rotation

By N. N. Mansour,[1] C. Cambon[2] AND C. G. Speziale[3]

Direct numerical simulations (DNS) of initially isotropic turbulence in a rotating frame were conducted, and comparisons were made with the predictions of generalized Eddy-Damped Quasi-Normal Markovian (EDQNM) approximations. It was found that for increasing rotation rates the non-linear triad interactions are reduced, causing a reduction in the energy cascade and the turbulence decay rate. At small Rossby numbers ($Ro < 0.01$), the transfer of energy is essentially shut-off. The effects of rotation on one-point statistics are reflected in a reduction of the production of enstrophy on a time scale of $O(1/\Omega)$, while the destruction of enstrophy is affected on a time scale of $O(k/\epsilon)$. A one-point closure model that properly simulates these effects is presented and comparisons are made with DNS results.

1. Motivations and objectives

The effects of rotation on turbulence are known to be subtle. The rotation rate, for example, does not explicitly enter the equations for the turbulence kinetic energy and its dissipation rate, yet experimental and numerical evidence show that the decay rate of turbulence is reduced by the presence of uniform rotation (see Speziale, Mansour & Rogallo, 1987). The objective of this work is to elucidate the effects of uniform rotation on initially isotropic turbulence and to develop closure models for these effects.

2. Accomplishments

The equations governing the evolution of the turbulence kinetic energy and its dissipation rate for homogeneous turbulence in a rotating frame with no mean strains are

$$k_{,t} = -\epsilon \tag{1}$$

$$\epsilon_{,t} = \left(2\nu\overline{\omega_i\omega_j u_{i,j}} - 2\nu^2\,\overline{\omega_{i,j}\omega_{i,j}}\right) \tag{2}$$

where $k = \frac{1}{2}\overline{u_i u_i}$ is the turbulence kinetic energy, ϵ is its dissipation rate, u_i is the fluctuating velocity, and ω_i the fluctuating vorticity. The first term on the right-hand side of Eq. (2) represents turbulent stretching of vorticity and is a production term. The second term is negative definite and represents the destruction of ϵ. For

1 NASA Ames Research Center

2 Ecole Centrale de Lyon (France)

3 ICASE, NASA Langley Research Center

isotropic flow, the vortex stretching term can be written in terms of the velocity derivative skewness, $S = -\overline{u_{1,1}^3}/(\overline{u_{1,1}^2})^{3/2}$ as follows:

$$2\nu\overline{\omega_i\omega_j u_{i,j}} = \frac{7}{3\sqrt{15}} S \frac{\epsilon^2}{k}\sqrt{Re_t} \tag{3}$$

where $Re_t = k^2/\nu\epsilon$ is the turbulence Reynolds number. From the above expression, one can clearly see that the production term scales as $Re_t^{1/2}$ for large Reynolds numbers since one expects S and ϵ^2/k to remain bounded. An expression for the destruction term was derived by Smith & Reynolds (1991) using a model spectrum. They show that for a spectrum of the form $E(\kappa) \propto \epsilon^{2/3}\kappa^{-5/3}$ the destruction term scales with Reynolds number as

$$2\nu^2\overline{\omega_{i,j}\omega_{i,j}} \propto \sqrt{Re_t}\frac{\epsilon^2}{k}. \tag{4}$$

2.1 A closure for the ϵ-equation

The results of Smith & Reynolds (1991) are in agreement with Tennekes & Lumley (1972), who argued based on an order of magnitude analysis that, at high Reynolds numbers, the vortex stretching term and the destruction term balance each other to $O(Re_t^{1/2})$, leaving a net $O(1)$ term independent of the Reynolds number. For isotropic flows, the net $O(1)$ term is function of ϵ, k, and ν. In this case, the destruction term is written as follows,

$$2\nu^2\,\overline{\omega_{i,j}\omega_{i,j}} = \frac{7}{3\sqrt{15}} G \frac{\epsilon^2}{k}\sqrt{Re_t} + C_2(Re_t)\frac{\epsilon^2}{k} \tag{5}$$

where $C_2(Re_t)$ remains bounded and is independent of the Reynolds number as $Re_t \to \infty$, and G is the coefficient of the leading term in an expansion of the destruction term in Re_t. We then model the equation for ϵ as follows:

$$\epsilon_{,t} = \frac{7}{3\sqrt{15}}(S - G)\frac{\epsilon^2}{k}\sqrt{Re_t} - C_2\frac{\epsilon^2}{k}. \tag{6}$$

For isotropic flows, the above expression is exact since we did not specify the form of $C_2(Re_t)$. In an equilibrium isotropic turbulence, $S = G$ yielding the classical modeled dissipation rate equation.

For unstrained homogeneous flows, the above arguments will lead us to define S to be

$$S \equiv \frac{6\sqrt{15}}{7}\nu\overline{\omega_i\omega_j u_{i,j}}\frac{k}{\epsilon^2}\,\frac{1}{\sqrt{Re_t}} \tag{7}$$

and G to be

$$G \equiv \frac{3\sqrt{15}}{7}(2\nu^2\,\overline{\omega_{i,j}\omega_{i,j}} - C_2(Re_t)\frac{\epsilon^2}{k})\frac{k}{\epsilon^2}\,\frac{1}{\sqrt{Re_t}}. \tag{8}$$

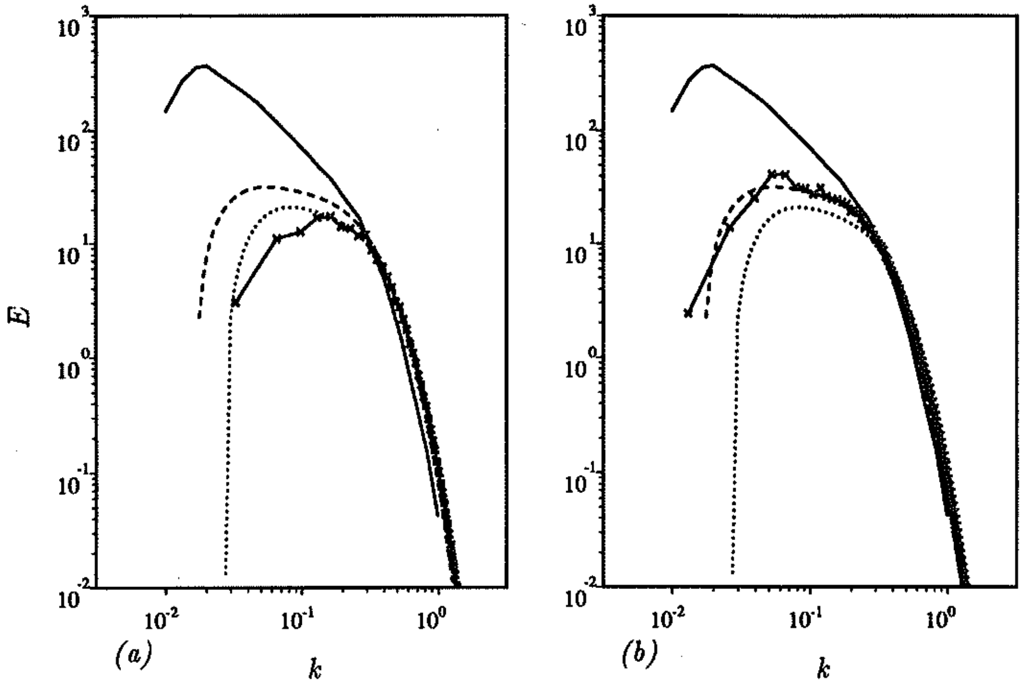

FIGURE 1. Energy Spectrum of the initial conditions. —— $Re_t = 556$, Comte-Bellot & Corrsin (1966); ---- $Re_t = 63.$, Veeravalli (1991); ······· $Re_t = 36.5$, Veeravalli (1991); *(a)* ✕—✕— $Re_t = 27.24$, DNS; *(b)* ✕—✕— $Re_t = 67.1$, DNS.

Note that the above definitions will have to be modified for homogeneous flows where the mean velocity gradient explicitly appears in the equations for k and ϵ.

2.2 The initial flowfields

Recently Wray & Rogallo (1991) ported VECTORAL and a spectral code for isotropic flows to the Intel iPSC/860 128 processors machine. We modified the code for the case of homogeneous flows under uniform rotation. The code was then used to generate fully developed isotropic flowfields (setting the rotation rate $\Omega = 0$) to be used as initial conditions for the rotation cases. It was found (see Mansour & Wray, 1991) that long time integrations are needed to establish isotropic flowfields undergoing a power law decay. Two isotropic flowfields were used as initial conditions: The first at $Re_t = 27.24$ using 128^3 Fourier modes; the second at $Re_t = 67.1$ using 256^3 Fourier modes. The reason for the low Reynolds number of the initial fields is that we sought a fully developed flowfield (undergoing a power law decay) where the peak of the spectrum is in a shell with enough samples. Figure 1 shows the predicted spectra compared with the experimental results of Comte-Bellot & Corrsin (1966) and Veeravalli (1991). We find good agreement between the computed spectrum and the experimental measurements. It is noteworthy that scaled on "Kolmogorov" variables ($(\nu^3/\epsilon)^{1/4}$ for the length scale and $(\nu/\epsilon)^{1/4}$ for the velocity scale), the spectra do not collapse in the dissipation (i.e, the high wave

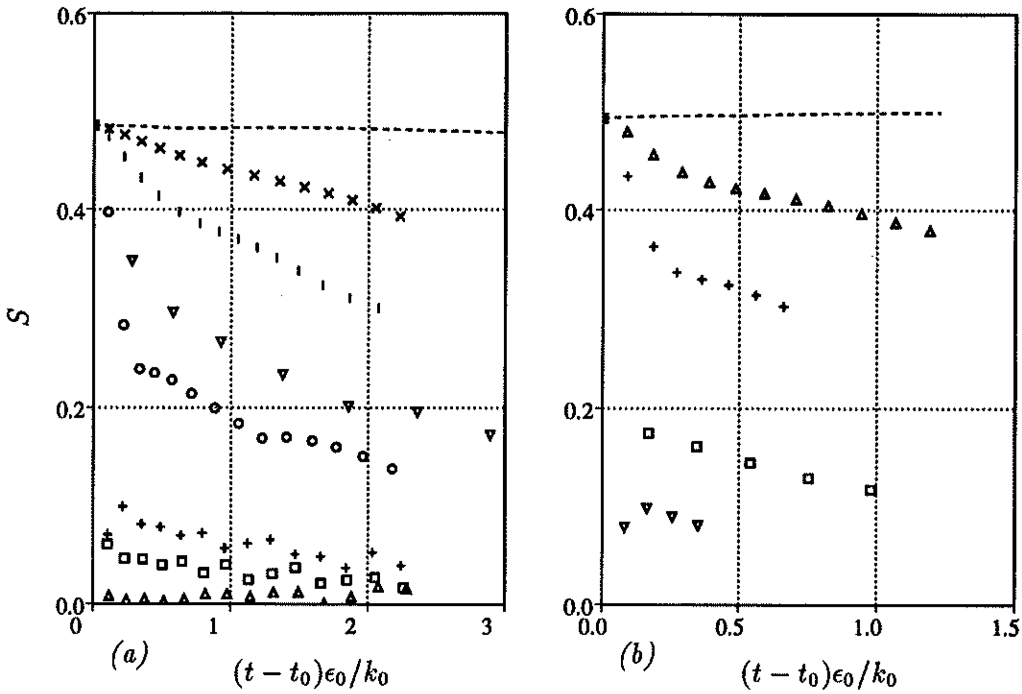

FIGURE 2. Effects of rotation on the skewness. *(a)* $Re_t = 27.24$, ---- $Ro = \infty$, $\times$ $Ro = 1.46$, ı $Ro = .74$, ▿ $Ro = .37$, ∘ $Ro = .247$, + $Ro = .074$, □ $Ro = .037$, △ $Ro = .0037$. *(b)* $Re_t = 67.1$, ---- $Ro = \infty$, △ $Ro = .47$, + $Ro = .24$, □ $Ro = .1$, ▿ $Ro = .047$.

number) range at these Reynolds numbers. Numerical simulations show that at high Reynolds numbers the spectra collapse at the high wave numbers when scaled with Kolmogorov variables, but they start to deviate from the collapsed curves when $Re_t < 120$.

2.3 Skewness as a function of the Rossby number

Numerical simulations at various Rossby numbers ($Ro = \epsilon/\Omega k$) were carried out starting with the two flowfields ($Re_t = 27.24$ and $Re_t = 67.1$) described in the previous section. Figure 2 shows the development of the skewness, S, with time for various Rossby numbers. We find that S is suppressed by the effects of rotation. The time scale at which this suppression occurs is commensurate with the rotation rate time scale $1/\Omega$. Plotted as a function of the inverse Rossby number (see Fig. 3), we find that, starting with a fully developed isotropic flowfield ($S \approx 0.49$), the effects of rotation are to suppress the nonlinear interactions on a time scale of $O(1/\Omega)$, and an equilibrium skewness (S_e) is reached which is a function of the Rossby and Reynolds numbers.

In addition to the DNS runs, we have carried out simulations using a modified version of the Eddy-Damped Quasi-Normal Markovian model of Cambon & Jacquin (1989). We find good agreement between the model and the DNS data when the

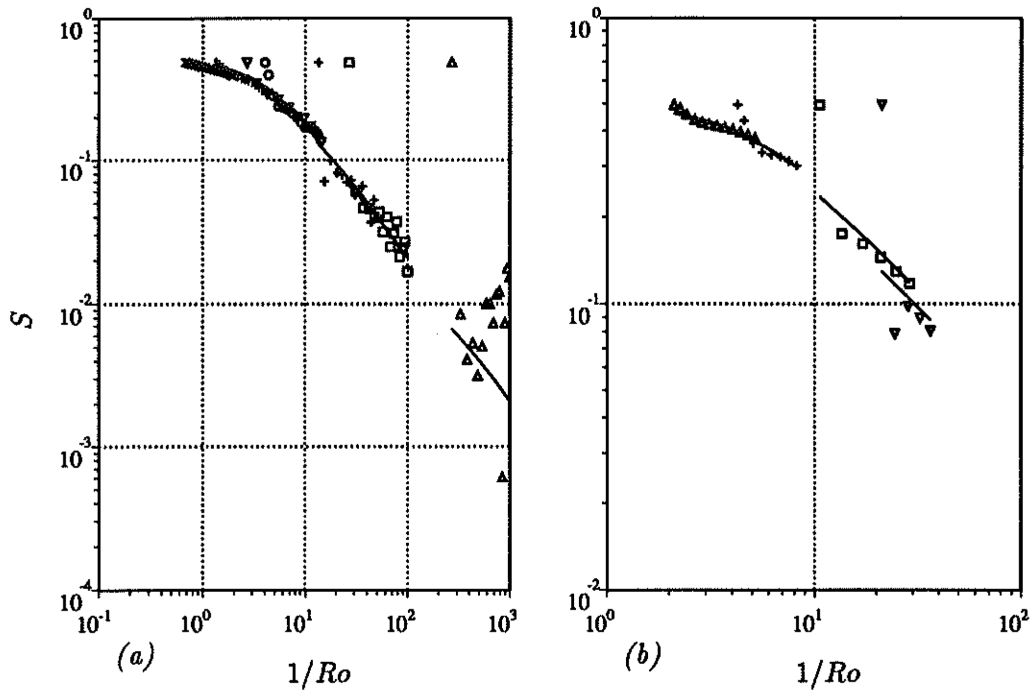

FIGURE 3. S as a function of the Rossby number. —— model Eq. (9); *(a)* $Re_t = 27.24$, ---- $Ro = \infty$, × $Ro = 1.46$, ı $Ro = .74$, ▿ $Ro = .37$, ○ $Ro = .247$, + $Ro = .074$, □ $Ro = .037$, △ $Ro = .0037$. *(b)* $Re_t = 67.1$, ---- $Ro = \infty$, △ $Ro = .47$, + $Ro = .24$, □ $Ro = .1$, ▿ $Ro = .047$.

EDQNM simulation is initialized with the same spectrum as the DNS. The collapse of the curves to an equilibrium skewness was not obtained when an analytical spectrum was used to initialize the EDQNM simulations. This is an indication that the use of a well established initial flowfield (undergoing a power-law decay) is important to achieve the collapse observed in Fig. 3. The form of the stretching term in the dissipation rate equation suggests that $S_e = S_e(Re_t^{1/2} Ro)$. The exact functional form is not known, but the success of the EDQNM model suggests the following model:

$$S_e = \frac{0.49}{\sqrt{1 + 2/(Re_t Ro^2)}}. \tag{9}$$

Thus for a fixed Reynolds number, $S_e \to 0$ as $Ro \to 0$, but it takes an infinite rotation rate to suppress the skewness at infinite Reynolds number.

2.4 A model for the effects of rotation on the skewness

The previous observations suggest that the effects of uniform rotation on the skewness can be modeled simply by

$$S_{,t} = -\alpha\Omega(S - S_e). \tag{10}$$

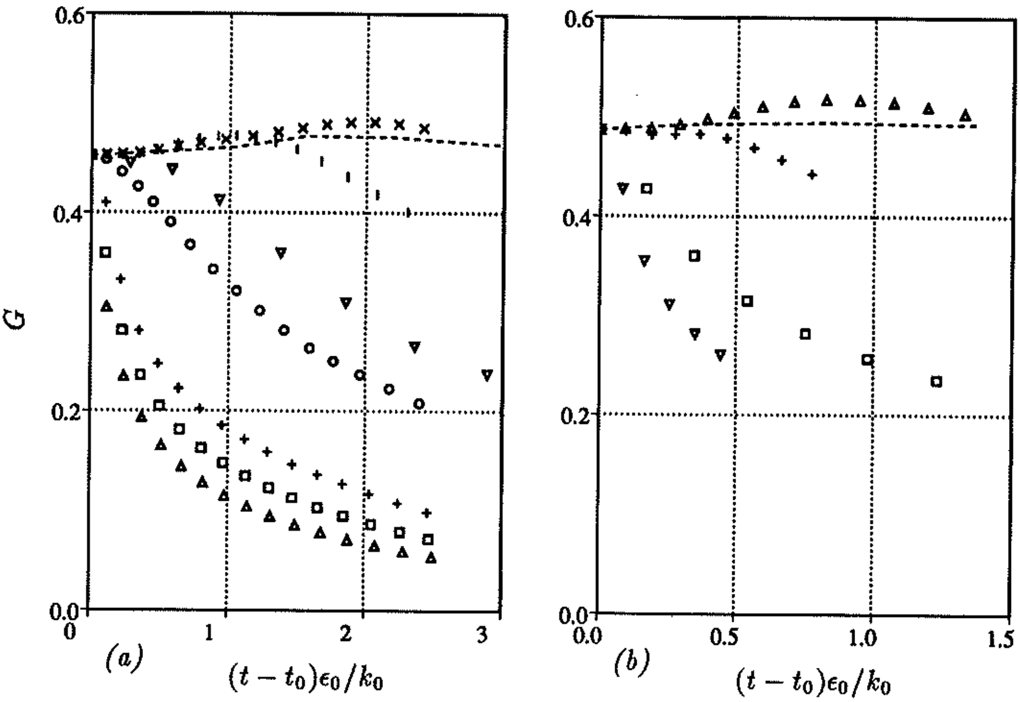

FIGURE 4. The effects of rotation on G. *(a)* $Re_t = 27.24$, ---- $Ro = \infty$, × $Ro = 1.46$, ı $Ro = .74$, ▿ $Ro = .37$, ○ $Ro = .247$, + $Ro = .074$, □ $Ro = .037$, △ $Ro = .0037$. *(b)* $Re_t = 67.1$, ---- $Ro = \infty$, △ $Ro = .47$, + $Ro = .24$, □ $Ro = .1$, ▿ $Ro = .047$.

The above form reflects the fact that the skewness will reach the equilibrium skewness on a time scale of $O(1/\Omega)$. The value of α will have to be determined empirically.

2.5 G as a function of the Rossby number

From its definition, G is that part of the destruction of dissipation term that balances the vortex stretching term as $Re_t \rightarrow \infty$. The evolution of G as a function of time for various Rossby numbers is shown in Fig. 4. We find that G is also suppressed by the rotation but on a time scale of $O(k/\epsilon)$. In the absence of production, the gradient of the vorticity diffuses on the turbulence time scale.

2.6 A model for the effects of rotation on G

The term G in the dissipation rate equation is reacting to the fact that the vortex stretching term has been suppressed by the rotation. This can be modeled by

$$G_{,t} = -\beta \frac{\epsilon}{k}(G - S). \tag{11}$$

Thus G will equilibrate with S on the turbulence time scale, k/ϵ.

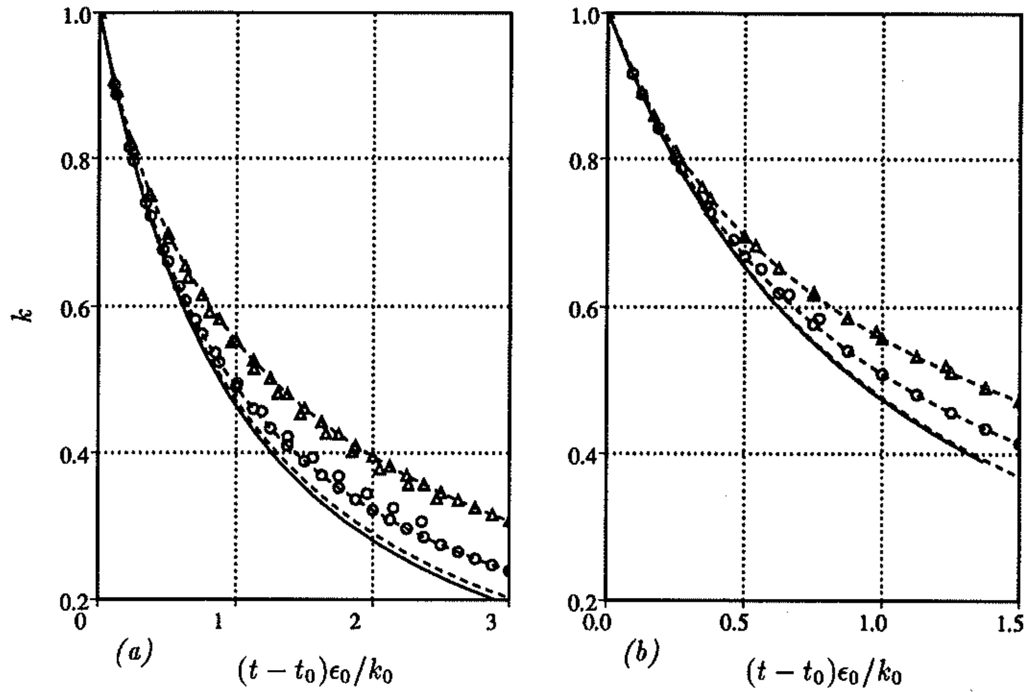

FIGURE 5. The effects of rotation on k. *(a)* $Re_t = 27.24$, —— DNS $Ro = \infty$, ○ DNS $Ro = .37$, △ DNS $Ro = .037$, ---- Model $Ro = \infty$, -○-- Model $Ro = .37$, -△-- Model $Ro = .037$. *(b)* $Re_t = 67.1$, —— DNS $Ro = \infty$, ○ DNS $Ro = .24$, △ DNS $Ro = .1$, ---- Model $Ro = \infty$, -○-- Model $Ro = .24$, -△-- Model $Ro = .1$.

2.7 Model testing

To summarize, we have a four-equation model for the effects of rotation on initially isotropic turbulence:

$$\begin{aligned} k_{,t} &= -\epsilon \\ \epsilon_{,t} &= \frac{7}{3\sqrt{15}}(S-G)\frac{\epsilon^2}{k}\sqrt{Re_t} - C_2(Re_t)\frac{\epsilon^2}{k} \\ S_{,t} &= -\alpha\Omega(S - S_e) \\ G_{,t} &= -\beta\frac{\epsilon}{k}(G - S) \end{aligned}$$

where S_e is given by Eq. (9) with $\alpha = 2$ and $\beta = 2.5$. The functional form for C_2 is a fit to the data for isotropic decay proposed by Coleman & Mansour (1991), $C_2(Re_t) = 1.8 - .4\exp(-.13\sqrt{20Re_t/3})$. The above set was solved numerically using a fourth-order Runge-Kutta integration for both the low and the high Reynolds number cases considered in this study. We find (see Figs. 5 and 6) that the model predicts well the evolution of the turbulence kinetic energy and its decay rate for both the case of zero-rotation and rapid-rotation. The prediction for the intermediate-rotation cases is marginal and can probably be improved by modifying α and β.

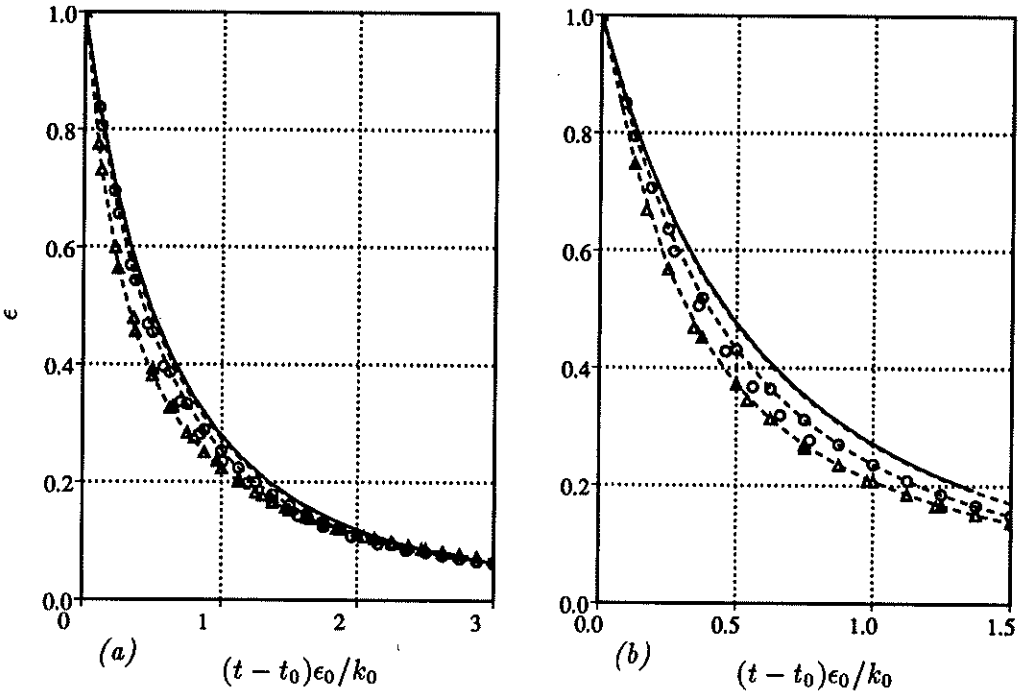

FIGURE 6. The effects of rotation on ϵ. *(a)* $Re_t = 27.24$, ─── DNS $Ro = \infty$, ○ DNS $Ro = .37$, △ DNS $Ro = .037$, - - - - Model $Ro = \infty$, -○- - Model $Ro = .37$, -△- - Model $Ro = .037$. *(b)* $Re_t = 67.1$, ─── DNS $Ro = \infty$, ○ DNS $Ro = .24$, △ DNS $Ro = .1$, - - - - Model $Ro = \infty$, -○- - Model $Ro = .24$, -△- - Model $Ro = .1$.

3. Future work

The model developed under this effort reflects the fact that uniform rotation will reduce the decay rate of turbulence. In this case, the turbulence decays but at a slower rate than isotropic decay in an inertial frame. The next challenge is to model elliptic flows (uniform rotation + oscillating-strain or plane-shear) where the flow can be linearly unstable to three-dimensional perturbations.

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