Direct numerical simulation of instability and noise generation of hot jets

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1. Motivation and objectives

Hot jets offer a wide range of applications in flows of practical interest. The study of their aerodynamical and aeroacoustical properties is, therefore, a major field in fluid mechanics. Despite the considerable amount of work on this topic, many questions remain unanswered since analytical models give but a rough description of these complex flows. This holds in particular for the region in which disturbances of the upstream laminar flow interact non-linearly: only the onset of these instabilities has so far been successfully modeled under restrictive conditions (Huerre & Monkewitz (1990)). In practical situations, jets are always more or less excited by upstream perturbations (internal turbomachinery flow, pre-existing turbulence . . .) which deeply affect the entire flow. The study of jet instability is, therefore, an important issue for engineers. Besides that, acoustic far-field measurements indicate that the instability region of a jet produces most of its noise. A quantitative prediction of the sound production requires a precise knowledge of the flow (*e.g.* Lighthill's theory requires at least the whole field: $\rho_0 \partial^2 u_i u_j / \partial x_i \partial x_j$ for an incompressible isentropic and inviscid flow). Since theoretical models are not accurate enough for such complex flows and experiments do not provide the required data, Direct Numerical Simulation (DNS) is an appealing tool for this purpose.

1.1. Instability of heated jets

Hot jets are of special interest since the nature of the instability they undergo depends on their temperature as shown in the experiments of Monkewitz & Sohn (1986), Yu & Monkewitz (1989), and Monkewitz *et al.* (1990). This change is mainly due to the subsequent density variations rather than to purely thermal effects (Sreenivasan *et al.* (1989)). The experiments show that below a certain temperature, there is no major difference with a cold jet: perturbations obtained by external forcing are amplified as they are convected downstream until they saturate and eventually decay. This cycle corresponds to the roll-up of vortical structures at the forcing frequency and their breakdown. The amplification of these perturbations is maximal if the preferred mode is excited (Crow & Champagne (1971)). This type of instability is referred to as convective instability because the perturbations are washed out by the flow whenever the forcing is stopped. If the jet is heated above a certain threshold, the flow switches to another type of instability in which it becomes independent of the external forcing level. This suggests that self-sustained modes settle in the jet, which behaves as an oscillator, and the instability is, therefore, called an absolute instability. Experimentally, this type of instability is characterized in three ways:

1. it needs no external forcing in order to reach the excited state,
2. peaks in the perturbation spectra at the Strouhal numbers of the preferred mode, and their subharmonics are much sharper than in the convectively unstable case,
3. after a sudden breakdown at the end of the transition region, the jet spreads spectacularly, and intermittent side jets can be seen downstream as well.

The first two points are successfully predicted by the linear stability analysis whereas the underlying mechanisms of the observations described in the last point are investigated in current studies (Martin & Meiburg (1991)): the side jets seem to be related to the saturation of azimuthal instability waves. The classical stability theory depicts three kinds of parallel flows: stable, convectively unstable, and absolutely unstable flows (the latter are defined as flows allowing waves to grow both downstream and upstream). This classification holds only for parallel flows. Recent studies (Gaster *et al.* (1985), Huerre & Monkewitz (1990)) have attempted to relate these stability concepts to global properties of weakly non-parallel flows in assuming the latter to be locally parallel. These studies generalize an analytical approach introduced by Crighton & Gaster (1978) for convectively unstable flows. According to Huerre & Monkewitz (1990), local absolute instability is a necessary but not sufficient condition for the flow to be self-excited. The oscillatory state can only be reached if there exists a "pocket of absolute instability", that is, an interval of streamwise locations at which the flow is locally absolutely unstable. Another necessary condition of the same type as the local absolute instability criterion can be expressed for the streamwise variation of the resonance frequency. Comparisons with experimental results show that the local absolute instability limit almost matches the limit of the global self-excited state (Monkewitz *et al.* (1990)). Nevertheless, there is no theoretical mean to define exactly the self-excitation characteristics only from the local stability concepts, even with simplifying assumptions. Thus many questions remain unanswered about the stability of jets:

1. Why does heating change the stability of a jet flow?
2. How does a feedback mechanism take place in a self-excited jet?
3. Which mechanisms could explain the spreading and the intermittent effects in self-excited jets?

1.2. Noise of excited jets

1.2.1 Cold jets

Related studies for low Mach-number jets indicate that the large scale structures of the instability region are mainly responsible for jet noise, whereas turbulence accounts only for background broadband noise. In experiments, unforced jets are actually excited by preexisting turbulence (carried along with the incoming flow). This accidental broadband forcing excites modes of different frequencies with a random phase which produce a significant broadband noise. Thus turbulence acts indirectly on the sound generation by randomizing the production of large scale structures. Hence forcing can either affect the broadband noise (Moore (1977),Laufer & Yen (1983), Juve (1985)) or amplify discrete frequencies of the acoustic field (Crow & Champagne (1971)), depending on experimental conditions. This can be explained by the fact that even though forcing amplifies only structures of a given wavelength, their spatial evolution still undergoes a randomness because of the background turbulence. Eventually, the relative importance of this randomness with respect to the wavelength determines how much the discrete frequencies are covered by broadband noise. If the preferred mode is excited, the amplification of large scale structures reaches a maximum according to experimental stability analysis (Crow & Champagne (1971)). Another unexpected experimental result can be explained by this mechanism: the quadrupolar sound sources are located at a fixed distance from the nozzle which is determined by the spatial evolution of the most excited instabilities (Laufer & Yen (1983), Juve (1985)), unlike the unforced case where eddies pair at varying downstream positions. An attempt to model these mechanisms was made by Ffowcs Williams & Kempton (1978) : assuming that the non-linear saturation of instability waves or, equivalently, the first downstream vortex pairing dominates the sound generation, they modeled the effect of background turbulence by a randomness in the phase or, equivalently, in the streamwise location of the corresponding source. Despite their physical simplicity (the pairing mechanism is not modeled), the two models of that study confirm qualitatively experimental observations. However, a quantitative noise prediction is not available, and the models fail to predict the high frequency radiation because details of the transition flow are not modeled.

1.2.2 Hot jets

Theoretical studies of sound generation in hot jets focus on the effects of the resulting density inhomogeneity. First a correction on cold jet noise due to wave refraction by transverse density variation was found (Mani (1974)), then additional source terms resulting from density gradients where considered (Mani (1976)). Results agree qualitatively with experiments on convectively unstable heated jets. This could be expected because cold jets are also convectively unstable: since sound radiation is strongly dominated by the dynamics of instabilities, the underlying mechanisms of noise generation are essentially the same for all convectively unstable jets. Thus their flows have the same structure when they are heated in the domain of convective instability. However, this theoretical approach does not hold for flows which are deeply affected by heating such as excited or self-excited jets. Their large scale structures modify notably the mean flow profiles on which Mani's model relies. For excited convectively unstable jets, the dominant sound sources are expected to be the same as for cold jets; but modifications due to density gradients might significantly change the acoustic far field (according to Mani's results). Studies related to self-excited jets focus on the near-field pressure. As pointed out earlier, the large scale structures are not perturbed by background noise, and the pressure spectrum is, therefore, dominated by peaks about the frequency of the preferred mode and its subharmonic (Monkewitz *et al.* (1990)). From these results, a similar shape might be expected for the far-field. However, the contribution to sound generation of other typical flow patterns (such as side jets) are not known so far because their mechanisms are not yet fully understood.

1.3. Objectives

The goal of the current investigation is to numerically simulate convectively and absolutely unstable jet flows in order to give a new insight into the underlying mechanisms of jet instabilities and their contribution to sound generation: DNS codes available at CTR solve the compressible Navier-Stokes equations and provide a very accurate flow representation. In different simple configurations, they have even been able to simulate sound radiation and scattering (without any acoustical analogy or approximation) (Colonius *et al.* (1991), Mitchell *et al.* (1992)). An application of these codes to heated 2-D jets has been started in this study and modifications have been made in order to allow for a temperature dependant viscosity.

2. Accomplishments and current work

2.1. The numerical scheme

As pointed out by Monkewitz *et al* (1990), the experimental study of convective and absolute instability requires a'clean' facility (background noise and inflow perturbations including turbulence intensity should be minimized) in order to control the forcing of the jet. These parameters are difficult to control. In a numerical approach, the equivalent condition is to minimize the numerical noise. Since a direct simulation of sound generation is planned as well, numerical noise has to be negligible even compared to the sound field: considering the small amplitudes of sound waves (the Sound Pressure Level reference is by definition: $0.2\times10^{-}5Pa$), the constraint on numerical approximation is extremely severe. Differencing schemes used in recent codes fulfill this requirement in the computational domain: the 2-D unsteady compressible Navier-Stokes equations are solved with a sixth order generalized Pade scheme for spatial derivatives and a fourth order Runge-Kutta scheme for time stepping (Lele (1990)). The main difficulty lies in the boundary conditions which need a particularly careful formulation at the inflow and outflow boundaries. A version of the scheme with most accurate boundary conditions is used in order to calculate the sound generated by the flow (Colonius *et al.* (1991)).

2.2. Non-reflecting boundary conditions

The common way to express boundary conditions is to determine the characteristics of the governing equations at the boundary. In our case the field is thus split in four waves (an acoustic wave, an entropy wave, and two vorticity waves). The boundary conditions are then obtained by cancelling the incoming waves, whereas the outgoing waves are computed from the interior field via one sided differences.

Two different approaches found in the literature:

2.2.1 One-dimensional boundary conditions

One-dimensional boundary conditions were given by Thompson (1989) for the Euler equations and modified by Poinsot & Lele (1989) for reacting compressible viscous flows. They rely on a characteristic decomposition of the field equations in the direction which is normal to the boundary. They are underspecified at the outflow, and the solution drifts unless the pressure at infinity is used to specify the incoming wave. Their advantage is that they have given satisfying results in many situations (Poinsot & Lele (1989)), and their relative robustness lies in the fact that they are expressed for the field variables: no linearization of the flow field is implied in their formulation. As a counterpart, they reflect oblique waves. Consequently, these boundary conditions are not suitable for DNS of acoustic waves, and even for the study of jet stability their accuracy is questionable.

2.2.2 Two-dimensional boundary conditions

Another formulation has been derived recently by Giles (1990) for turbomachinery flow and adapted to external viscous flow by Colonius *et al.* (1991). It is based on two approximations. The first is a small perturbation assumption which allows the flow field equations to be linearized. The perturbations are Laplace-transformed in time and Fourier-transformed in the tangential direction. The characteristic analysis leads to boundary conditions for each Fourier-mode and are written as orthogonality conditions between the perturbation field and the left eigenvectors of the incoming waves. Since the Fourier and Laplace transforms cannot be performed at each time step, the left eigenvectors are expanded to the first order in Taylor series of the spatial wave number. This second approximation allows the boundary conditions to be expressed as a first order PDE in time and the tangential coordinate. In order to obtain a well-posed problem, the incoming wave at the outflow has to be specified with respect to the pressure at infinity in a similar way as for the Thompson boundary conditions. Furthermore, an analogous condition involving a reference streamwise velocity has to be specified on the outgoing waves at the inflow.

This formulation gives thus a first order approximation of the exact two-dimensional boundary conditions (a zeroth order approximation gives one-dimensional boundary conditions which are different from the Thompson boundary conditions because of the underlying linearization).

2.3. Current testings

The two-dimensional non-reflecting boundary conditions have so far been successfully tested on model problems (Colonius *et al.* (1991)). In these model problems, the boundaries were far from the main aerodynamic perturbation: thus the boundaries were not crossed by the flow as they are for jets, and only the non-reflection of acoustic and entropy waves was tested. Thus the question arises whether or not they are adapted to such cases.

2.3.1. Zero-circulation vortex

In order to answer this question, we have simulated the case of a vortex with zero circulation which is convected by a plug flow. First we considered the case of a slowly rotating almost incompressible vortex (core radius: 3.0, strength: -0.01; all variables are scaled with respect to the ambient sound speed and a length which measures the vorticity thickness in shear-flows and is one fortieth of the box size in the present calculation) convected by a parallel flow (Mach number: 0.05, Reynolds number: 6,667 (based on core radius and background velocity)). The maximum

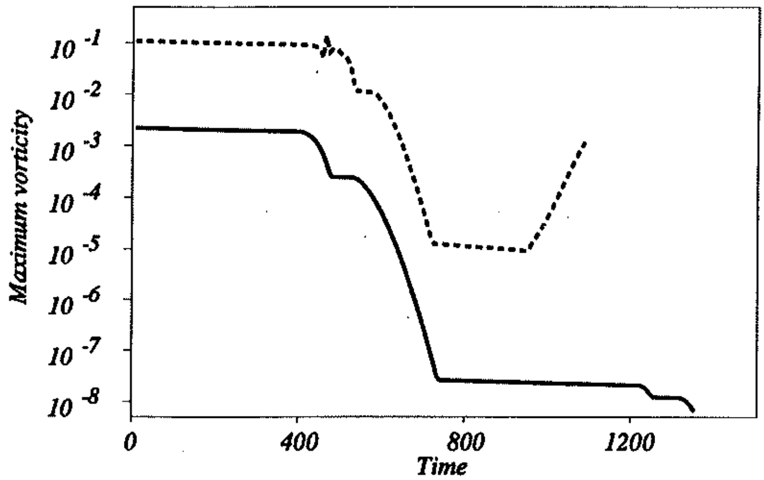


FIGURE 1. Single vortex: Time evolution of maximal vorticity. The vortex leaves the domain at Time = 400 (one flow-through Time is 800). Strength = -0.01(Solid line); = -0.5 (dotted line).

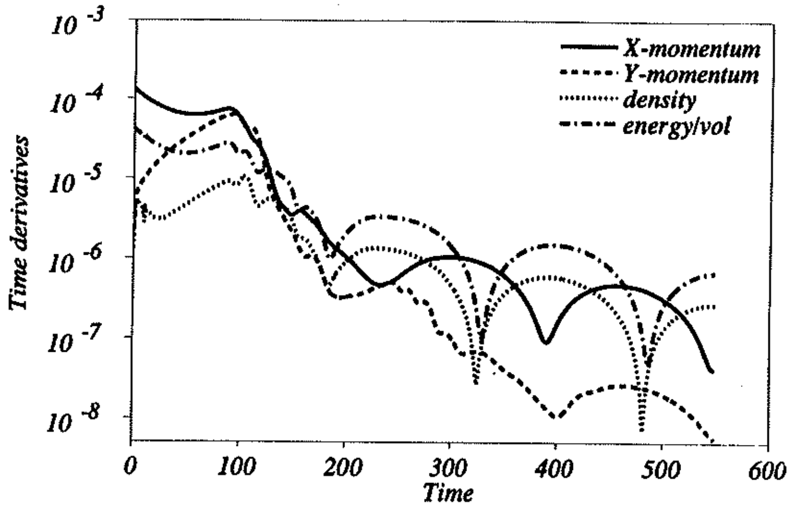
tangential velocity is 0.002 (twenty five times smaller than the convection velocity). The vortex which is initially in the center of the domain induces a small perturbation to the mean flow. When it leaves the domain, acoustic waves are reflected at the outflow which generate perturbations at the inflow. These are convected to the outflow one flow-through time later. This explains the stepwise decrease of the vorticity in figure 1 (for the case; strength = -0.01). Figure 1 shows also that the reflected perturbations are small with respect to the initial vorticity. Thus the steady state is reached within a good approximation after this transient phase(numerical noise is 10^6 times smaller than the initial perturbation). However, the numerical noise generated at the outflow increases along with the vortex-strength until the outflow boundary acts as an amplifier. If the initial vorticity which is considered as a perturbation in the formulation of the boundary conditions is too strong, the outflow generates even larger perturbations, and the computation eventually blows up. This behavior is shown in figure 1 for the case where the strength of the previous vortex is increased to -0.5, giving a tangential velocity of 0.1. In this case, the vortex induces a reverse flow which has the same speed as the background plug flow. It can be seen in figure 1 that when the vortex leaves the domain, the boundary conditions generate a vortical perturbation which is almost 1.5 times stronger than the natural vorticity at the outflow. This instability at the boundaries is due to the formulation of the boundary conditions: the linearization of Navier-Stokes equations does not hold for large perturbations. The point is that the equations are linearized with respect to the initial field, and, therefore, the whole vortex is considered as a perturbation when it reaches the outflow even though it does not fluctuate in a convected frame (except from a slow viscous decay). In our latter computation, the perturbation field when the vortex reaches the outflow is, therefore, of the same order of magnitude as the unperturbed field. Hence the boundary conditions are unstable for large perturbations. Theoretically, this could be avoided by changing the reference flow during the simulation under the assumption that the large scale perturbations of the flow are much slower than the characteristic waves (Colonius *et al.* (1991)). Unfortunately, such a treatment is only possible if these large scale perturbations are known *a priori*, which is not the case for, say, a jet flow. Another reason for this instability might be the fact that the stronger vortex is responsible for negative local streamwise velocities and that the corresponding boundary conditions at the outflow should become temporarily inflow boundary conditions. This might indeed be one reason, but it is surely not the only reason because we tried to switch to the proper type of boundary conditions (inflow or outflow) according to sign of the streamwise velocity, and this would not solve the problem. Another reason is given by results of recent tests (Colonius *personal communication*) which showed that the amplitude of reflected waves is proportional to the square of the vortex strength. This suggests that numerical noise at the boundaries is dominated by non-linear effects, that is, linearization errors. Our first assumption is thus confirmed.

2.3.2. Laminar jet flow

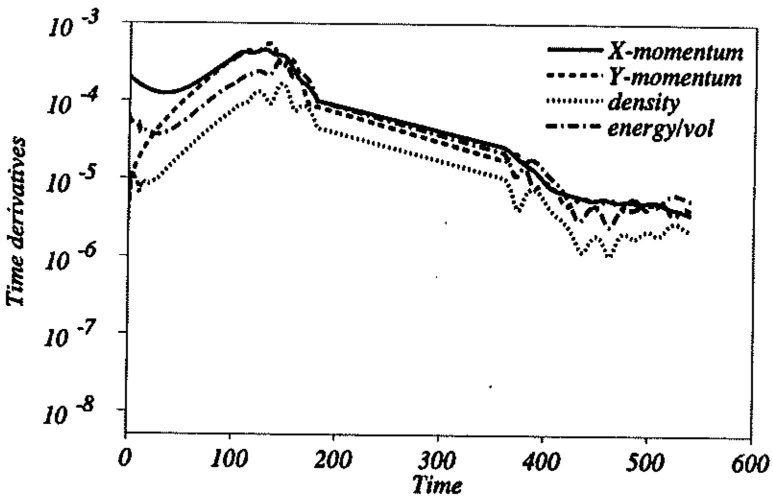
The conclusions drawn from the case of a single vortex have deep consequences on our jet simulations for two reasons:

1. we want to investigate unstable flows which are by definition subject to strong perturbations (this is particularly true for jets),
2. the jets are surrounded by still fluid; thus even perturbations that are relatively small compared to the jet flow will be strong with respect to the background flow.

Tests indicate indeed that initial transients create perturbations which are strong enough to blow up the computation. For this reason, we chose another approach, which is to simulate co-flowed jets, that is, jets surrounded by fluid with positive streamwise velocity. Hence the perturbations carried by the jet become smaller with respect to ambient fluid motion. Of course, the jet stability is increased by a co-flow (for a given centerline velocity, the shear is reduced). Thus our goal is to find the minimal necessary co-flow. Our tests concern 2-D jets with an inflow top-hat profile given by Yu & Monkewitz (1989), the temperature profile being related to the latter by the Crocco-Buseman relation. The centerline velocity is 0.4 (in the Mach scale) and the Reynolds number is 1250 (based on shear-layer thickness and centerline velocity). The jet diameter is about 10 shear-layer thicknesses). According to results on shear-layer simulations (Buell *et al.* (1990)), the minimal co-flow should be about one fifth of the maximal velocity. We tested three co-flows with Mach-numbers of $0.05, 0.1$, and 0.2. The first blew up after two flow-through times, whereas the two others converged. It can be seen in figure 2 (which shows the space averaged rms values of the time derivatives versus time, scaled with the vorticity-thickness and the ambient sound speed) that the convergence is very slow for the intermediate case which is probably very close to the stability limit of the boundary conditions. According to figure 2 and to the conclusions from the previous subsection, there



(a)



(b)

FIGURE 2. Convergence. Rms values of space-averaged time derivatives are plotted *versus* time. (a): co-flow = 0.2; (b) co-flow = 0.1 . Flow-through Time varies from 100 on the jet centerline to 150 (case (a)) and up to 300 (case (b)) in the coflow.

are obviously two reasons for this behavior: The first is that perturbations grow as the co-flow decreases (more shear), and thus reflected waves generate stronger perturbations in the flow. The second is that the convection velocity is reduced along with the co-flow, and it takes, therefore, more time for the perturbations to reach the outflow. For the first perturbation set, this makes only a slight difference because it is convected by the shear-layer roughly at the average velocity (0.25 and 0.3 for the two fastest co-flows), whereas the perturbations due to waves reflected by the outflow affect mainly the co-flow (where their relative amplitude is the highest) and are, therefore, convected at the co-flow velocities. In the fastest case, these distinctions do not appear very clearly because these different velocities are very close, and the reflected waves in the co-flow are not dominant. In the intermediate case, however, these distinctions become obvious. These tests confirm the necessity of a co-flow in the simulation of jets, and from our first tests on heated jets, it seems possible that the minimal co-flow is faster for hot jets, thus making the study of self-excited jets even more difficult.

3. Future plans

The conclusion from our preliminary study is that the obliquely non-reflecting boundary conditions are appropriate for the DNS of jets as long as the perturbations remain small with respect to the initial conditions. Therefore, in our future work, we will go further into the study of co-flowed jets which has recently begun.

Since the co-flow has a stabilizing effect, we will have to determine under which conditions the self-excited state might be reached. This question will be addressed in the near future by a linear compressible stability analysis of jets with different co-flows and different temperature ratios. This will predict the limit of absolute instability and give the necessary eigenfunctions for the forcing in the convectively unstable case.

Another necessary step toward DNS will be to improve initial conditions by computing a steady laminar jet flow (in our computations, we merely translated the inflow profile through the domain). Thus we will reduce the amplitude of the transients and hence increase the stability of the boundary conditions.

Then we will use the results of the two previous steps for the DNS of convectively and absolutely unstable jets. At that point, we may even use the flow-field data in order to test aeroacoustic models. A prediction of acoustic far-field relying on a complete data set of the flow-field would be a great step for acoustics.

Finally, we will consider the DNS of acoustic fields. These could be compared to the results from the previous calculations.

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