Streaks in turbulent boundary layers

By James M. Hamilton

1. Motivation and objectives

Elongated regions of low- and high-speed fluid are among the most prominent structures in wall-bounded turbulent shear flows. While the general mechanism by which these streaks are produced is understood (the transport of streamwise momentum by cross-shear flow), some of the details of the process remain unclear. For instance, it is commonly observed that the low and high speed regions alternate with a spanwise "wavelength" of about $100\nu/u_\tau$, where ν is the kinematic viscosity, and $u_\tau = \sqrt{\nu(\partial U/\partial y)_{\text{wall}}}$. Why this particular length scale should dominate has not been satisfactorily answered.

Jang, Benney, and Gran (1986) attempted to explain the preferred streak spacing as the result of a direct resonance mechanism. The Navier-Stokes equations can be converted to a pair of equations for wall-normal velocity, v, and wall-normal vorticity, ω_y, by invoking continuity, eliminating pressure, and linearizing about a two-dimensional mean flow. When normal mode solutions are sought, the resulting equation for v is the Orr-Sommerfeld equation. The ω_y equation has an inhomogeneous source term containing v. Direct resonance occurs when the eigenvalues of a v-mode match the eigenvalues of an ω_y-mode corresponding to the homogeneous equation; the v term in the ω_y equation then forces the wall-normal vorticity. The resulting solution for ω_y is secular and may result in initial growth even with decaying modes. Waleffe and Kim (1991) have shown, however, that direct resonance does not lead to appreciable scale selection. In addition, non-resonance modes may grow faster than the direct resonance response as the resonance modes tend to have large decay rates.

Waleffe and Kim suggest that instead of direct resonance, the streak spacing is due to the complete self-sustaining mechanism of streaks, that streaks with spacing less than 100 wall units cannot be maintained. This view is supported by the finding of Jimenez and Moin (1991) that the smallest computational box in which turbulence could be sustained had a spanwise dimension of 100 wall units. Waleffe and Kim extended this idea by minimizing the height of the computational domain as well as the span. They pointed out that, in a channel, the simplest self-sustaining non-laminar flow would consist of a pair of counter-rotating streamwise vortices in each half of the channel, and, in a Couette flow, the simplest structure would be a single pair of vortices centered between the walls bounding the flow. Accordingly, for circular vortices, the minimum wall separation for which turbulence could be maintained should be about 100 wall units in a channel flow and 50 wall units in a Couette flow. Assuming laminar mean streamwise velocity profiles, the corresponding minimum Reynolds numbers are 1250 for the channel flow (based on centerline velocity and channel half-height) and 625 for the Couette flow (based on wall velocity and channel half-height). Simulations support this view, and, for Couette flow, Waleffe and Kim found that turbulence could not be maintained at Reynolds numbers of 330 or below and was maintained at a Reynolds number of 400.

Waleffe and Kim use the term "marginal flow" to describe simulations in which the domain is just large enough to produce and sustain the expected structures. A marginal flow is advantageous for the study of the streak production mechanism because it eliminates the unnecessary large scales. They note, however, that the complete process is still too disordered to firmly establish the mechanisms involved and suggest the imposition of symmetries to further constrain the flow.

The objective of the present study is to pursue this idea of imposing symmetries on the marginal flow in an effort to identify and understand the basic processes involved in the self-sustenance of turbulence.

2. Accomplishments

The simplest flow for the study of streak formation is the plane Couette flow, which has only a single sign of mean spanwise vorticity. The symmetries to be imposed on this flow are consistent with the expected structure, a side-by-side pair of counter-rotating streamwise vortices centered between the bounding walls, as illustrated in Figure 1. The coordinates x, y, and z correspond to the streamwise, wall-normal, and spanwise directions, respectively. The associated velocities are u, v, and w. The computational domain is periodic in x and z, with dimensions L_x and L_z. These lengths are made dimensionless by half the wall separation, $2h$, thus the walls are located at $y = \pm 1$. The computations are made with a slightly modified version of the spectral code of Kim, Moin, and Moser (1987). In addition to being consistent with streamwise vortices, the imposed symmetries must satisfy continuity, the Navier-Stokes equations, and the boundary conditions. The three symmetries to be considered here are:

$$\begin{aligned} u(x,y,z) &= u(x,y,-z) \\ v(x,y,z) &= v(x,y,-z) \\ w(x,y,z) &= -w(x,y,-z) \end{aligned} \tag{1}$$

a reflection about the plane $z = 0$,

$$\begin{aligned} u(x,y,z) &= u(x+L_x/2,y,-z) \\ v(x,y,z) &= v(x+L_x/2,y,-z) \\ w(x,y,z) &= -w(x+L_x/2,y,-z) \end{aligned} \tag{2}$$

a translation in x followed by a reflection about $z = 0$, and

$$\begin{aligned} u(x,y,z) &= -u(-x,-y,-z) \\ v(x,y,z) &= -v(-x,-y,-z) \\ w(x,y,z) &= -w(-x,-y,-z) \end{aligned} \tag{3}$$

a reflection about the point $(x, y, z) = 0$ (note that while this last symmetry is not strictly consistent with the streamwise vortex arrangement of Figure 1, a simple shift

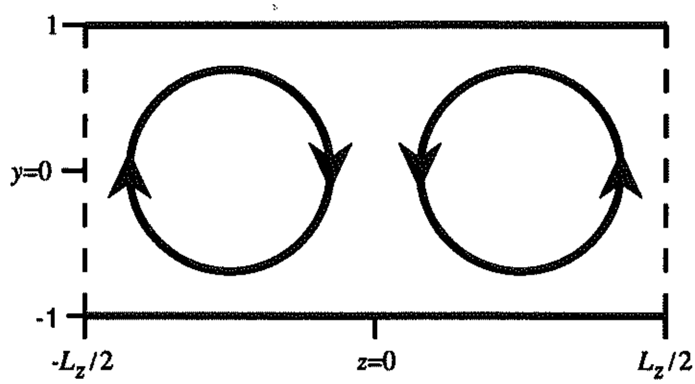


FIGURE 1. Basic symmetric vortex structure.

of the vortex centers in the figure to $z = 0$ and $z = \pm L_z/2$ produces a compatible flow). As an example of an inappropriate symmetry, consider symmetry (1) above as a reflection about the plane $y = 0$ rather than $z = 0$. Use of such a symmetry might be tempting, as it gives the correct behavior for v and w in accordance with Figure 1. Such a symmetry would fail to satisfy continuity, however, as $\partial u/\partial x$ and $\partial w/\partial z$ are even functions of y, but $\partial v/\partial y$ is odd. The symmetry would also fail to satisfy the base Couette flow and the boundary conditions, $u(x, y = \pm 1, z) = \pm 1$.

While each of the symmetries (1) through (3) is compatible with side-by-side streamwise vortices, the more important consideration is how the flow evolves away from this state. After all, if only x-independent streamwise vortices were allowed, the vortices would simply decay, and turbulence would be impossible. As a baseline, the temporal evolution of the spectral energy $(\int_{-1}^{1}[u^2(k_x, y, k_z) + v^2(k_x, y, k_z) + w^2(k_x, y, k_z)]\,\mathrm{d}y)$ of a flow with *no* imposed symmetry is plotted in Figure 2. The solid line in the figure corresponds to the mode with no x-dependence and spanwise period L_z, the fundamental mode for a single pair of counter-rotating streamwise vortices extending through the domain. This mode follows a quasi-periodic cycle, with an average period of slightly more than 100 h/U_{wall} (compare this value to the average period of 100 h/U observed by Jimenez and Moin in their minimal channel computations, where U is the centerline velocity of the equivalent laminar flow). The initial conditions for this flow were obtained from developed flow at a higher Reynolds number: a flow was started with random initial conditions at Re=625, allowed to evolve for about 450 time units, and Re reduced to 500 for an additional 450 time units to produce the input flowfield for the Re=400 case.

This particular flow is of special interest because it closely follows the translate-and-reflect symmetry (2), even though no symmetry is imposed. This can be seen easily in Figure 3, velocity plots of two z-y planes spaced a distance $L_x/2$ apart. Note that Figure 3(b) is nearly a mirror image of Figure 3(a). The symmetry apparent in Figure 3 is typical of this flow though it is not seen in flows with different initial conditions, Reynolds numbers, or domain sizes.

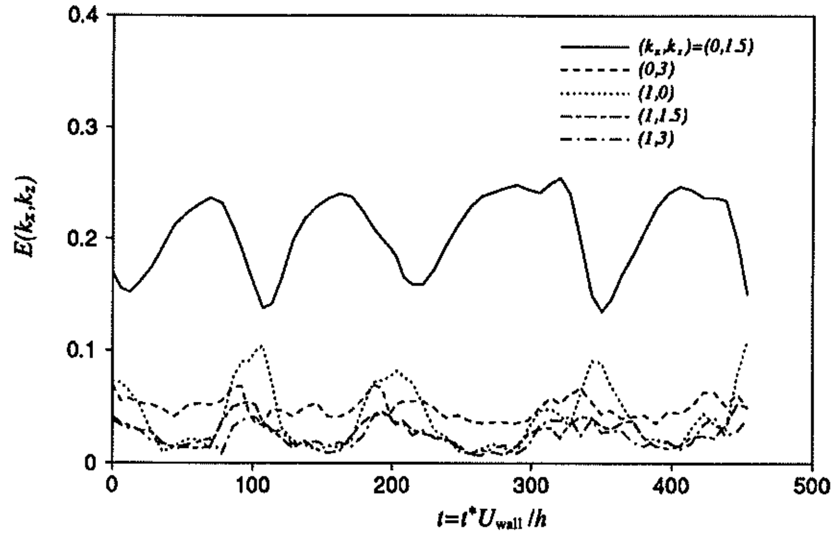
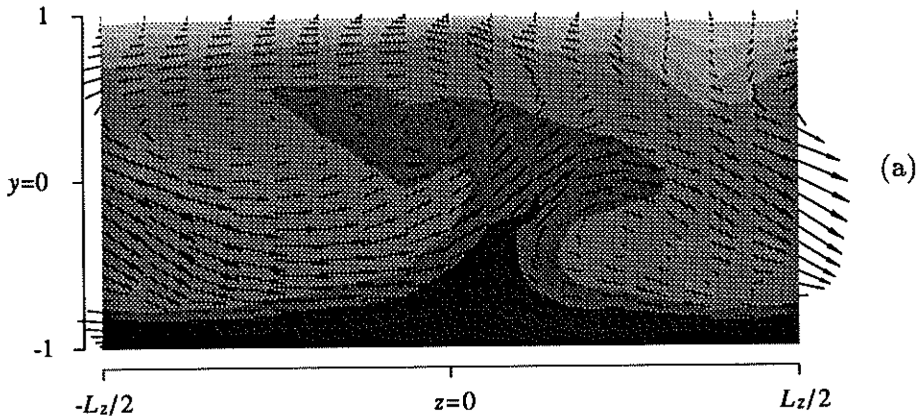


FIGURE 2. Temporal evolution of energy in several spectral modes of a marginal flow with no imposed symmetry, Re=400, $L_x = 2\pi$, $L_z = \frac{4}{3}\pi$.

If the flow closely follows a symmetry when none is imposed, it is useful to impose the symmetry and observe the effect on the flow. The flow of Figures 4 and 5 has the translate-and-reflect symmetry imposed, with the same initial conditions (though symmetrized) as the flow of Figures 2 and 3. From Figure 4, it is clear that the energy in the fundamental, x-independent mode of the forced-symmetry flow exhibits a quasi-periodic cycle much as in the unsymmetric flow though the average period is shorter (about 80 time units) in the forced-symmetry case. This similarity can also be seen in a comparison of the details of the flowfield in the two cases. At times corresponding to peaks in the energy of the $(k_x, k_z) = (0, 2\pi/L_z)$ mode (the solid line in Figures 2 and 4), the flow is dominated by a pair of side-by-side streamwise vortices as expected. The velocities in Figures 3 and 5 occur near minima of this mode, and, as can be seen, relatively strong, meandering, spanwise flows develop in both cases, and the vortices are no longer side-by-side. Thus, while imposition of the symmetry does have some effect on the evolution of the flow, the general features are unchanged.

The same initial conditions used above, when applied to flows with either of the symmetries (1) or (3), result in the rapid decay of turbulence. If, however, the symmetry is imposed at a higher Reynolds number and the Reynolds number then progressively reduced, a flow conforming to symmetry (3), reflection about a point, will sustain turbulence down to Re=400. The resulting flow has the same cyclic behavior of a flow with no symmetry imposed. Symmetry (1), reflection about the plane $z = 0$, does not seem to produce self-sustaining turbulence even at Reynolds



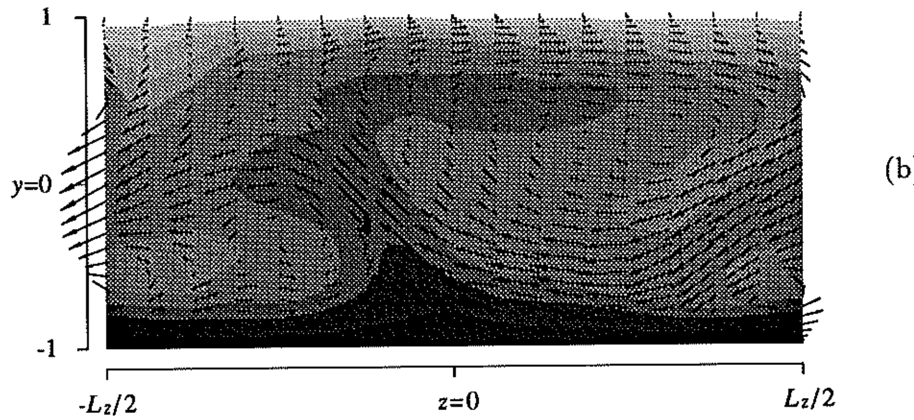


FIGURE 3. Velocity field for flow of Figure 2 at $t = 107$ at (a) $x = 0$ and (b) $x = L_x/2$. Shading indicates streamwise velocity with black lowest and white highest, and vectors indicate cross-flow velocities.

numbers as high as 625. This symmetry may, however, require a subharmonic-type flow, that is, a basic flow structure consisting of *two* pairs of streamwise vortices, and hence require a larger value of L_z than has been considered to date.

3. Future Plans

Initially, it was hoped that one or more of the above symmetries would not allow sustained turbulence and that the remaining symmetries would produce a very "clean" flow in which the self-sustenance mechanism became obvious. Indeed, symmetry (1) does not seem to maintain turbulence in marginal flows, though the possibility of a subharmonic structure remains to be investigated. Even with the imposition of the other symmetries, however, a truly "clean" flow has remained elusive. Still, there is cause for hope in the nearly periodic, cyclic nature of the marginal flows. This cycle consists of three parts. Streamwise vortices produce low- and high-speed streaks with little x-dependence in the flow. Large x-dependence

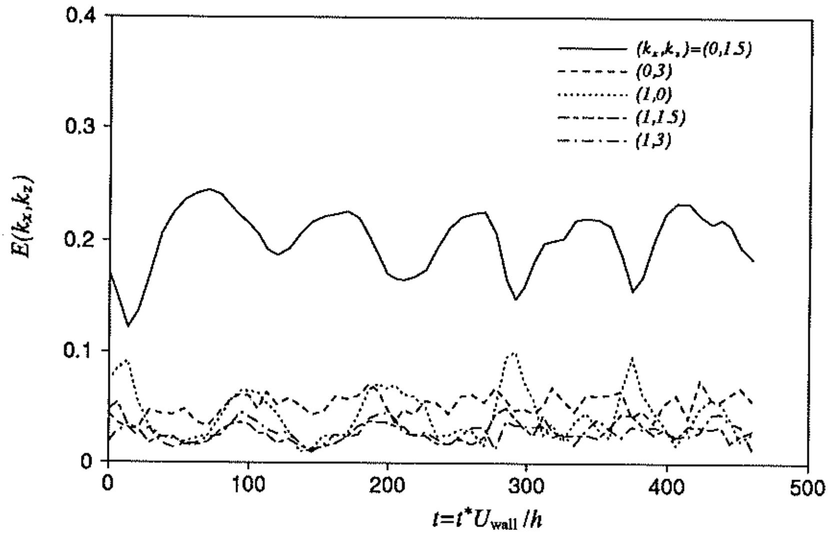


FIGURE 4. Temporal evolution of energy in several spectral modes of a marginal flow with imposed translate-and-reflect symmetry (symmetry (2) above), Re=400, $L_x = 2\pi$, $L_z = \frac{4}{3}\pi$.

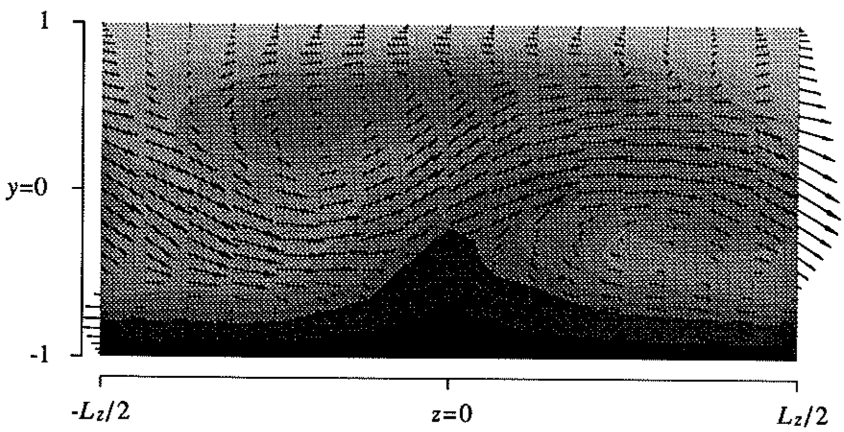


FIGURE 5. Velocity field for flow of Figure 4 at $t = 291$ and $x = L_x/4$. Shading indicates streamwise velocity with black lowest and white highest, and vectors indicate cross-flow velocities. Vector length scale same as Figure 3.

develops, and the flow breaks down, producing enhanced mixing and the disappearance of well defined streaks. From this disorder, elongated streamwise vortices emerge, and the cycle repeats.

Of these three steps, only the first, the production of streaks from streamwise vortices, is well understood. The third step, the appearance of elongated vortices, is not understood but appears to be a very robust process; even decaying flows rapidly evolve to long, counter-rotating streamwise vortices. The mechanism(s) which actively determines the length-scales of the streaks has not yet been identified but surely occurs during flow breakdown or the re-coalescence of streamwise vortices.

Future plans include continued efforts to examine marginal and forced-symmetry marginal flows to further refine the details of the three step cycle described above with particular emphasis on the latter two steps. The breakdown of the nearly x-independent vortex-and-streak structure bears some resemblance to an instability, and it may be worthwhile to model this flow and study its stability. The fact that flows with and without forced symmetries behave with remarkable similarity suggests that any instability may contain the symmetry. This may lead to additional insight into the process.

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