Optimal feedback control of turbulent channel flow

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Feedback control equations have been developed and tested for computing wall-normal control velocities to control turbulent flow in a channel with the objective of reducing drag. The technique used is the minimization of a "cost functional" which is constructed to represent some balance of the drag integrated over the wall and the net control effort. A distribution of wall velocities is found which minimizes this cost functional some time shortly in the future based on current observations of the flow near the wall. Preliminary direct numerical simulations of the scheme applied to turbulent channel flow indicates it provides approximately 17% drag reduction. The mechanism apparent when the scheme is applied to a simplified flow situation is also discussed.

1. Motivation and objectives

It is the goal of this project to study methods to counteract near-wall vortical structures in turbulent boundary layer flow using an active control system in an effort to reduce drag. From this study, we hope to better understand the physics of drag producing events and the sensitivity of boundary layer flow to control. As a more far-reaching goal, we would like to better understand how to develop control equations for general flow control problems, utilizing the equations governing fluid flow to achieve performance that is in some sense optimal for a given situation.

With a well-chosen scheme using wall control only, it has been shown that a turbulent flow may be smoothed out in a near-wall region, and the drag may be substantially reduced. This scheme applies small amounts of wall-normal blowing and suction through the computational equivalent of holes drilled in the wall. Previous *ad hoc* schemes by Choi *et al.* (1992) have reduced the drag by as much as 20% by countering the vertical velocity slightly above the wall with an equal but opposite control velocity at the wall. The objective of this work is to derive more effective schemes by applying optimal control theory, utilizing the equations of motion of the fluid to reveal the dominant physics of the control problem and the most efficient distribution of the control energy. This work is an outgrowth of the work done by Choi *et al.* (1993), where optimal control theory was applied to the stochastic Burgers equation. Here, we apply the theory to the Navier-Stokes equations, which necessitates a more involved treatment of the equations and more extensive computer resources. The scheme discussed in this report depends on measurements of flow velocities above the wall — this is not feasible in a practical implementation. The scheme will later be reduced to a more practical one involving only flow quantities which are most easily measured in an experimental rig.

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The model problem we study in this work is the turbulent flow inside a small segment of a fully developed turbulent channel (*i.e.* flow between two parallel walls, far from the inlet). This flow is governed by the same vortical structures as turbulent boundary layer flow in the near-wall region.

Thus, the problem under consideration is a turbulent channel flow with no-slip walls and wall-normal control velocities ϕ. Control will be applied to this flow in order to decrease the drag integrated over the walls at the expense of some measure of the net control effort. A feedback control algorithm has been developed which minimizes a "cost functional" constructed to represent this balance of the drag and the control effort. This method is introduced in Section 2. The control equations have been coded and tested in a direct numerical simulation of turbulent channel flow. Section 3 discusses preliminary results of these calculations.

2. Formulation

2.1 State equation (Navier-Stokes equation)

As described above, the problem under consideration is a constant-flux turbulent channel flow with no-slip walls and wall-normal control velocities ϕ. This problem is governed by the unsteady, incompressible Navier-Stokes equation, the continuity equation, and a constant flux integral constraint equation inside the domain Ω and appropriate boundary conditions on the walls w (periodic conditions are implied on the remainder of the boundary of the domain Γ):

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} u_j u_i = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial}{\partial x_j}\frac{\partial}{\partial x_j} u_i \tag{1a}$$

$$\left.\frac{\partial u_i}{\partial x_i} = 0 \right\} \quad \text{in } \Omega \tag{1b}$$

$$\iiint_\Omega u_1 \, dx_1 \, dx_2 \, dx_3 = C \tag{1c}$$

$$\left.\begin{array}{l} u_1 = 0 \\ u_2 = \phi \\ u_3 = 0 \end{array}\right\} \quad \text{on walls,} \tag{2}$$

where x_1 is the streamwise direction, x_2 is the wall-normal direction, x_3 is the spanwise direction, u_i are the corresponding velocities, and p is the pressure. The constants in the problem are C (a measure of the flux in the channel) and Re (the Reynolds number).

2.2 Optimal control of state equation

The goal of controlling the channel flow is to minimize the drag on a section of wall with area A over a period of time T using the least amount of control effort possible. The relevant quantities of interest are thus the time averaged drag

$$\bar{D} = \frac{1}{A\,T} \int_0^T \iint_w \frac{\partial u_1}{\partial n} \, dx_1 \, dx_3 \, dt \tag{3}$$

(where n is a unit vector in the *inward* wall normal direction) and a term representing the expense of the control. The latter term may be taken to be the integral of the magnitude of the power input, which may be written

$$\bar{E}_1 = \frac{1}{AT} \int_0^T \iint_w \left| \phi \left(p + \rho \, \phi^2/2 \right) \right| \, dx_1 \, dx_3 \, dt, \qquad (4a)$$

In addition, depending on the physical mechanism used to provide the control velocities, the rate of change of the control hardware settings might be another important expense (for instance, representing the expense involve in changing the settings of control valves in the system):

$$\bar{E}_2 = \frac{1}{AT} \int_0^T \iint_w \left| \frac{\partial \phi}{\partial t} \right| \, dx_1 \, dx_3 \, dt. \qquad (4b)$$

A physically appropriate cost functional for this problem, then, balances the expense of the input versus the drag:

$$\bar{\mathcal{J}}(\phi) = \ell_1 \bar{E}_1 + \ell_2 \bar{E}_2 + \bar{D}, \qquad (5)$$

where ℓ_1 and ℓ_2 are appropriate weighting factors. We could proceed from this point to attempt to construct a control procedure designed to minimize this cost functional. A mathematically more simple cost functional for the purpose of control theory (for reasons which will become evident as the control equations are derived) is quadratic in ϕ. Physically, this represents the integral of the magnitude of the kinetic energy per unit mass input to the system, and may be written

$$\bar{\mathcal{J}}(\phi) = \frac{\ell}{2} \, \frac{1}{AT} \int_0^T \iint_w \phi^2 \, dx_1 \, dx_3 \, dt + \frac{1}{AT} \int_0^T \iint_w \frac{\partial u_1}{\partial n} \, dx_1 \, dx_3 \, dt. \qquad (6)$$

It will be seen later that, in most problems that we consider, the expense terms are much less significant than the drag terms (in other words, the control is relatively cheap). The use of other expense terms does not cause much additional complexity or insight into the method.

The *optimal* control procedure considered, then, involves reducing the cost functional (6) for some period of time T. This method is described in Abergel and Temam (1990) in a related situation and is also discussed in Lions (1969). However, this is a prohibitively expensive procedure for present computational resources because it involves storage and manipulation of several three-dimensional fields over the entire time period under consideration. The complexity of such an algorithm is discussed further in Choi *et al.* (1993).

We therefore resort to a *suboptimal* control procedure (Choi *et al.* 1993). In this method, the state equation is discretized in time, then a control procedure is applied to reduce an instantaneous version of the cost functional (6)

$$\mathcal{J}(\phi) = \frac{\ell}{2A} \iint_w \phi^2 \, dx_1 \, dx_3 + \frac{1}{A} \iint_w \frac{\partial u_1}{\partial n} \, dx_1 \, dx_3 \qquad (7)$$

at each time step.

By applying the control at each time step, the algorithm gives the control which minimizes the cost functional over some short time interval. Note, however, that this method does not look ahead to anticipate further development of the flow, and thus the solution by this method does not necessarily correspond to the solution by the optimal control method. Thus, posing the problem in this suboptimal form is another level of approximation to the physical problem of interest.

The differences in complexity between the optimal and suboptimal schemes described above may be realized by drawing an analogy to a computer algorithm to play chess. A suboptimal chess program looks ahead one step to determine the move that leaves as good a position on the board as possible. Similarly, a suboptimal turbulence control scheme looks ahead one time step to determine the set of control velocities that leaves as good (*i.e.* low) a value of the cost functional as possible at the next time step. An optimal chess program, on the other hand, investigates all possible developments of the game a certain number of steps into the future (knowing how the other player may respond), and then moves in the direction that leads to the best final position on the board. Similarly, an optimal turbulence control scheme investigates all possible developments of the flow a certain amount of time into the future (knowing approximately how the flow will respond), and then applies the set of control velocities that leads to the best (*i.e.* lowest) time-averaged cost functional. Such a method requires significantly more resources than the suboptimal method.

2.3 Time discretization of state equation

The suboptimal control procedure introduced above is now applied to the state equation (1). To do this, we discretize (1) in time, then apply a feedback control algorithm to modify the flow at the next time step. A consistent approach is to use a second order Crank-Nicolson method (implicit) on all terms. The momentum equation (1a) thus takes the form:

$$
\begin{aligned}
\frac{u_i^n - u_i^{n-1}}{\Delta t} &+ \frac{1}{2}\frac{\partial}{\partial x_j}(u_j^n u_i^n + u_j^{n-1} u_i^{n-1}) = \\
&-\frac{1}{2}\left(\frac{\partial p^n}{\partial x_i} + \frac{\partial p^{n-1}}{\partial x_i}\right) + \frac{1}{2Re}\left(\frac{\partial}{\partial x_j}\frac{\partial}{\partial x_j}u_i^n + \frac{\partial}{\partial x_j}\frac{\partial}{\partial x_j}u_i^{n-1}\right),
\end{aligned} \tag{8}
$$

where a superscript n indicates the value at time step n.

It is now useful to put the time discretized form of the entire state equation governing the flow in the domain into the form

$$
\mathcal{K}^n + \mathcal{R}^{n-1} = 0, \tag{9a}
$$

where $\mathcal{K}^n$ contains all the terms which in some way depend on the state variables

from the current time step, and $\mathcal{R}^{n-1}$ contains the remaining terms:

$$\mathcal{K}^n = \begin{cases} u_i^n - \beta_1 \dfrac{\partial}{\partial x_j}\dfrac{\partial}{\partial x_j}u_i^n + \beta_2\left(\dfrac{\partial p'^n}{\partial x_i} + \dfrac{dP}{dx_1}^n \delta_{i1} + \dfrac{\partial}{\partial x_j}u_j^n u_i^n\right) & \text{in } \Omega \\[3mm] -\beta_2 \dfrac{\partial u_j^n}{\partial x_j} & \text{in } \Omega \qquad (9b) \\[3mm] \beta_2 \displaystyle\iiint_\Omega u_1^n\, dx_1\, dx_2\, dx_3 & \end{cases}$$

$$\mathcal{R}^{n-1} = \begin{cases} \begin{cases} -u_i^{n-1} - \beta_1 \dfrac{\partial}{\partial x_j}\dfrac{\partial}{\partial x_j}u_i^{n-1} \\[3mm] \quad + \beta_2\left(\dfrac{\partial p'^{n-1}}{\partial x_i} + \dfrac{dP}{dx_1}^{n-1}\delta_{i1} + \dfrac{\partial}{\partial x_j}u_j^{n-1}u_i^{n-1}\right) \end{cases} & \text{in } \Omega \\[6mm] 0 & \text{in } \Omega \\[3mm] C'. & \end{cases} \qquad (9c)$$

In the above equation, $\beta_1 = \Delta t/2Re$, $\beta_2 = \Delta t/2$, dP/dx_1 is the mean pressure gradient in the x_1 direction (adjusted at each time step to provide constant mass flux), and p' accounts for the pressure variations within the domain (periodic in x_1 and x_3). Note that (1b) and (1c) have been multiplied by constants to obtain (9).

Associated with this problem are the boundary conditions **B**:

$$\begin{aligned} B_1 &= u_1 = 0 \\ B_2 &= u_2 = \phi \\ B_3 &= u_3 = 0. \end{aligned} \qquad (10)$$

The "flow problem", which will hereafter be denoted $\mathscr{A}$, is taken to refer to the differential equation (9) together with the boundary conditions (10).

2.4 Suboptimal control of state equation

In this section and the next, we develop a method to solve for the gradient of the cost functional $\mathcal{J}$ and with this a control procedure based on this gradient information to minimize $\mathcal{J}$ at each time step.

Consider the Fréchet differential (Vainberg, 1964) of the cost functional $\mathcal{J}$ in (7):

$$\begin{aligned} \frac{\mathcal{D}\mathcal{J}(\phi)}{\mathcal{D}\phi}\tilde{\phi} &\equiv \lim_{\epsilon \to 0} \frac{\mathcal{J}(\phi + \epsilon\tilde{\phi}) - \mathcal{J}(\phi)}{\epsilon} \\ &= \frac{\ell}{A}\iint_w \phi\,\tilde{\phi}\,dx_1\,dx_3 + \frac{1}{A}\iint_w \frac{\partial}{\partial n}\left(\frac{\mathcal{D}u_1}{\mathcal{D}\phi}\tilde{\phi}\right)\,dx_1\,dx_3. \end{aligned} \qquad (11)$$

The gradient of the functional $\mathcal{J}$ with respect to the control distribution ϕ may be extracted from this equation by expressing the last term on the RHS in terms of an

inner product on $\tilde{\phi}$. It is for this reason that we now formulate what we shall call the "differential problem".

Define Θ using a Fréchet differential such that

$$\Theta = \frac{\mathcal{D}U(\phi)}{\mathcal{D}\phi}\tilde{\phi} \equiv \lim_{\epsilon \to 0} \frac{U(\phi + \epsilon\tilde{\phi}) - U(\phi)}{\epsilon}, \tag{12}$$

where $\tilde{\phi}$ is some arbitrary or "test" distribution of control velocities. Thus, Θ^n is a differential state representing the sensitivity of the state U^n to control for a particular control distribution $\tilde{\phi}^n$ applied over the time duration $(t^{n-1}, t^n]$. The differential Θ is decomposed into components in a fashion similar to the state $U(\phi)$:

$$U(\phi) = \begin{pmatrix} u_i(x_1, x_2, x_3) \\ p'(x_1, x_2, x_3) \\ dP/dx_1 \end{pmatrix}, \qquad \Theta = \begin{pmatrix} \theta_i(x_1, x_2, x_3) \\ \rho(x_1, x_2, x_3) \\ \lambda \end{pmatrix}.$$

The equations governing the differential state Θ^n follow directly by taking the Fréchet differential of the state equation (9) and its boundary conditions (10). Note that the term $\mathcal{R}^{n-1}$ in (9) does not depend on $\tilde{\phi}^n$ and thus makes no contribution. The contribution from the term $\mathcal{K}^n$ is linear and may be written

$$\mathbf{A}^n\Theta^n = 0, \tag{13a}$$

where

$$\mathbf{A}\Theta = \begin{cases} \theta_i - \beta_1 \dfrac{\partial}{\partial x_j}\dfrac{\partial}{\partial x_j}\theta_i + \beta_2\left(\dfrac{\partial \rho}{\partial x_i} + \lambda\delta_{i1} + \theta_j\dfrac{\partial u_i}{\partial x_j} + u_j\dfrac{\partial \theta_i}{\partial x_j}\right) & \text{in } \Omega \\[3mm] -\beta_2\dfrac{\partial \theta_j}{\partial x_j} & \text{in } \Omega \\[3mm] \beta_2 \displaystyle\iiint_\Omega \theta_1\, dx_1\, dx_2\, dx_3. \end{cases} \tag{13b}$$

The boundary conditions $\tilde{\mathbf{B}}$, from (10), are

$$\begin{aligned} \tilde{B}_1 &= \theta_1 = 0 \\ \tilde{B}_2 &= \theta_2 = \tilde{\phi} \\ \tilde{B}_3 &= \theta_3 = 0. \end{aligned} \tag{14}$$

The "differential problem", which will hereafter be denoted $\mathscr{A}$, is taken to refer to the differential equation (13) together with the boundary conditions (14).

Consider again the Fréchet differential of the cost functional $\mathcal{J}$ in (11):

$$\frac{\mathcal{D}\mathcal{J}(\phi)}{\mathcal{D}\phi}\tilde{\phi} = \frac{\ell}{A}\iint_w \phi\,\tilde{\phi}\, dx_1\, dx_3 + \frac{1}{A}\iint_w \frac{\partial \theta_1}{\partial n}\, dx_1\, dx_3. \tag{15}$$

The gradient of the functional $\mathcal{J}$ with respect to the control distribution ϕ may be extracted from this equation by expressing the integral of $\partial\theta_1/\partial n$ in terms of an inner product on $\tilde{\phi}$. This may be done by solving the differential problem $\mathscr{A}$, as is done below.

2.5 Solution of differential problem $\tilde{\mathscr{A}}$ by adjoint method

An "adjoint problem" is now formulated which may be used to bypass direct solution of the differential problem $\mathscr{A}$ itself.

Define an adjoint operator $\mathbf{A}^*$ using the equation

$$< \mathbf{A}\,\Theta, \Psi > = < \Theta, \mathbf{A}^*\Psi > + b, \tag{16}$$

where $\mathbf{A}$ (which depends on U) is defined in equation (13b), the boundary conditions on Θ are given in equation (14), and an adjoint state Ψ has been defined in a fashion similar to U and Θ:

$$\Psi = \begin{pmatrix} \psi_i(x_1, x_2, x_3) \\ \pi(x_1, x_2, x_3) \\ \kappa \end{pmatrix}.$$

The adjoint operator is formed by moving all of the derivatives in the inner product (the integral over the volume of the product of the two terms, denoted $< \cdot, \cdot >$) from the differential Θ to the adjoint Ψ. It is a straightforward exercise to write out the volume integrals corresponding to the LHS of (16) and then to rearrange this expression into the form of integrals corresponding to the RHS of (16) using integration by parts. From this is deduced $\mathbf{A}^*$ and the condition at the boundary resulting from the wall terms, which are all placed into the expression for b:

$$b = < \mathbf{A}\,\Theta, \Psi > - < \Theta, \mathbf{A}^*\Psi > . \tag{17}$$

Through equation (13a), the first term on the RHS of equation (17) is zero. If we form a similar homogeneous adjoint differential equation for the adjoint Ψ

$$\mathbf{A}^*\Psi = 0, \tag{18}$$

with boundary conditions as yet undetermined, then equation (17) reduces to

$$b = 0. \tag{19}$$

Using the method described above, it is easy to show that

$$\mathbf{A}^*\Psi = \begin{cases} \psi_i - \beta_1 \dfrac{\partial}{\partial x_j}\dfrac{\partial}{\partial x_j}\psi_i + \beta_2\left(\dfrac{\partial \pi}{\partial x_i} + \kappa\,\delta_{i1} + \psi_j\dfrac{\partial u_j}{\partial x_i} - u_j\dfrac{\partial \psi_i}{\partial x_j}\right) & \text{in } \Omega \\[2ex] -\beta_2\dfrac{\partial \psi_j}{\partial x_j} & \text{in } \Omega \\[2ex] \beta_2 \displaystyle\iiint_\Omega \psi_1\, dx_1\, dx_2\, dx_3 & \end{cases} \tag{20}$$

and

$$\begin{aligned} b = \iint_w \Big(& \beta_1\frac{\partial \theta_1}{\partial n}(\psi_1) - \beta_2\,\rho\,n_2(\psi_2) + \beta_1\frac{\partial \theta_3}{\partial n}(\psi_3) \\ & + \tilde{\phi}\Big(\beta_2\,n_2\,\pi - \beta_1\frac{\partial \psi_2}{\partial n} - \beta_2\,\phi\,n_2\,\psi_2\Big) \Big)\, dx_1\, dx_3 = 0. \end{aligned} \tag{21}$$

(Note by comparison of (20) with (13b) that the operator $\mathbf{A}$ is not self-adjoint due to the effect of the convective terms of the momentum equation.) These adjoint equations may be exploited to solve the differential problem $\mathscr{A}$.

We now formulate an "adjoint problem", which will hereafter be denoted $\mathscr{A}^*$, defining an adjoint state Ψ with the homogeneous differential equation (18) and with accompanying boundary conditions $\mathbf{B}^*$ as yet undefined. Note by the above discussion that one of the by-products of the formulation of this problem is the relation at the boundary given by (21). We are now at liberty to choose boundary conditions for the adjoint problem such that this relation is useful — it is exactly for this reason that the formulation of an adjoint problem is considered. With this in mind, we may choose the boundary conditions $\mathbf{B}^*$ as

$$
\begin{aligned}
B_1^* &= \psi_1 = 1 \\
B_2^* &= \psi_2 = 0 \\
B_3^* &= \psi_3 = 0.
\end{aligned}
\tag{22}
$$

Using these boundary conditions and the continuity equation for the adjoint velocity, equation (21) reduces to

$$
\iint_w \frac{\partial \theta_1}{\partial n} \, dx_1 \, dx_3 = -n_2 \iint_w \tilde{\phi} \, Re \, \pi \, dx_1 \, dx_3.
\tag{23}
$$

To compute the RHS, we must solve the adjoint problem $\mathscr{A}^*$. This is done numerically and must be repeated at each time step as $\mathbf{A}^*$ changes as the flow U develops with time.

The differential of the cost functional (11) may be rewritten using (23) as

$$
\frac{\mathcal{D}\mathcal{J}(\phi)}{\mathcal{D}\phi} \tilde{\phi} = \frac{\ell}{A} \iint_w \phi \, \tilde{\phi} \, dx_1 \, dx_3 - \frac{n_2 \, Re}{A} \iint_w \pi \, \tilde{\phi} \, dx_1 \, dx_3,
\tag{24}
$$

where π is the adjoint pressure on the wall. Finally, the desired gradient of the cost functional $\mathcal{J}$ may be extracted (Vainberg, 1964):

$$
\frac{\mathcal{D}\mathcal{J}(\phi)}{\mathcal{D}\phi} = \frac{\ell}{A} \phi - \frac{n_2 \, Re}{A} \pi.
\tag{25}
$$

A feedback control procedure using a simple gradient algorithm at each time step may now be proposed such that

$$
\phi^{n,k+1} - \phi^{n,k} = -\mu \frac{\mathcal{D}\mathcal{J}(\phi^{n,k})}{\mathcal{D}\phi},
\tag{26}
$$

where superscript n indicates the time step as before and superscript k indicates an iteration step at that particular time step. This algorithm attempts to update ϕ in the direction opposite to the local direction of increase of $\mathcal{J}$. For fixed n as $k \to \infty$ with sufficiently small μ, this gradient algorithm should converge to some local minimum of $\mathcal{J}$ over the control space ϕ if the approximation of $\mathcal{D}\mathcal{J}/\mathcal{D}\phi$ is sufficiently accurate. Note, however, that as the time step n advances, $\mathcal{J}$ will not necessarily decrease (Choi *et al.* 1993).

3. Accomplishments and future work

3.1 Elementary drag reducing mechanisms

Choi *et al.* (1992) found that by applying a control velocity equal and opposite to the vertical velocity at $y^+ = 10$, a drag reduction of nearly 20% could be achieved. Vertical transport of streamwise momentum in the near-wall region (primarily due to longitudinal vorticity) produces "sweep" events and thus local regions of very high drag. Applying a countering control velocity tends to reduce this effect. A related mechanism described by Lumley (1993) further explains these results; control applied to reduce the spinning of the near-wall vortices reduces their energy, stabilizing them in space and thereby reducing the "bursting" frequency, which also tends to reduce the drag.

In the tansverse plane, countering the vertical velocity above the wall corresponds to a control which de-spins the near-wall vortices, as shown in Figure 1. This process leads to the removal of fluctuations in the near-wall region, which diminishes the mixing capability of the turbulence and therefore reduces drag. This type of control corresponds to blowing where the drag is high, which decreases the high velocity gradients at the wall and thus smooths out the flow in the near-wall region, as shown in Figure 2.
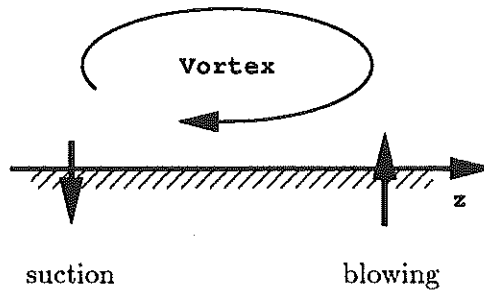

FIGURE 1.   Stabilization mechanism in cross flow plane. The effect of the control velocities shown is to de-spin the near-wall vortex, reducing momentum transport near the wall.
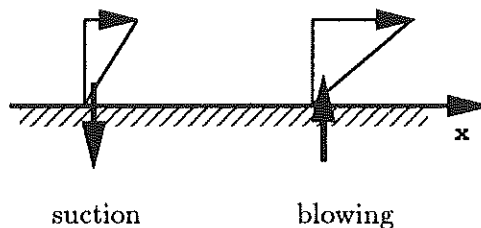

FIGURE 2.   High drag is decreased by blowing at the expense of suction in the regions of low drag, resulting in a net smoothing of the near-wall velocity profiles.

Figure 3 shows the application of the suboptimal control scheme to a simple flow configuration of longitudinal vortices embedded in an initially parabolic flow. A cross flow plane is shown. In regions below downward moving fluid (sweep events) the streamwise (into the page) drag is higher and blowing is applied. In regions below upward moving fluid (ejection events), the streamwise drag is lower and suction is applied. The overall control distribution from the suboptimal scheme is in a sense that acts to de-spin the near-wall vorticity, and thus acts in accordance with mechanisms described above.
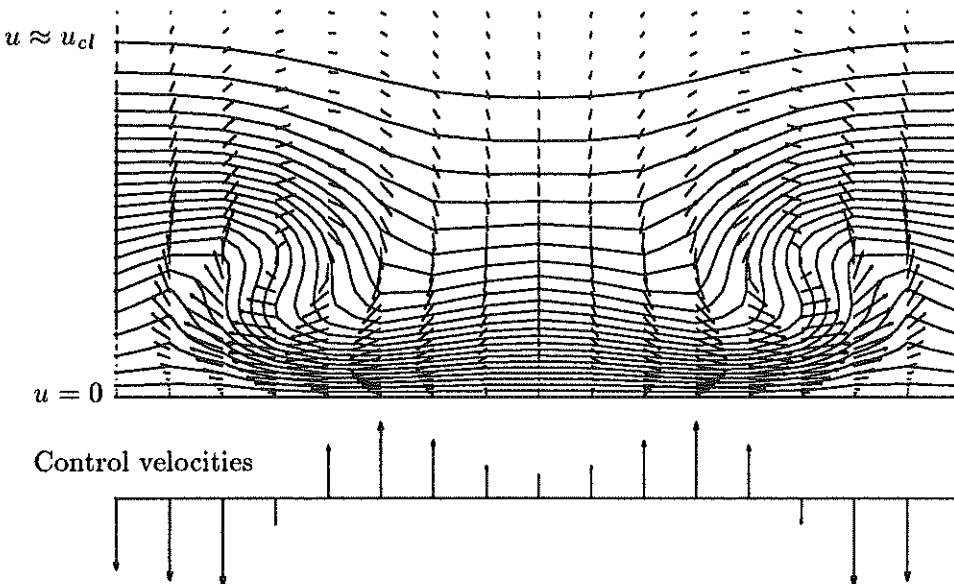

FIGURE 3.   Optimal control scheme applied to longitudinal vortices. Interior vectors are cross flow velocities and contours are of streamwise velocity, indicating a sweep event between two near-wall vortices and ejection events outside of them. Control velocities shown on the wall (not to scale) indicates blowing at the sweep event and suction at the ejection events.

The adjoint analysis utilizes all the information present in the near-wall region to extract the sensitivity of the instantaneous drag to the variation of the control. This scheme may be reduced to an approximate one relying only on wall information by approximating the near-wall velocities using a Taylor's series extrapolation of the velocity gradients at the wall. The correlation between the full adjoint analysis and approximations of the adjoint problem using only information available at the wall is still being investigated; preliminary results indicate that the performance is not severely degraded by this approximation.

3.2 Suboptimal control of turbulent channel flow

The scheme introduced in Section 2 was tested by applying it to a direct numerical simulation of turbulent channel flow. A 17% drag reduction was seen as compared to a flow with no control. Results are plotted in Figure 4. This calculation was done in a flow with $Re_\tau = 100$ based on the friction velocity and the channel half width using a 32x65x32 grid and the spectral method of Kim *et al.* 1987. Although these results should be considered preliminary, they are quite promising.
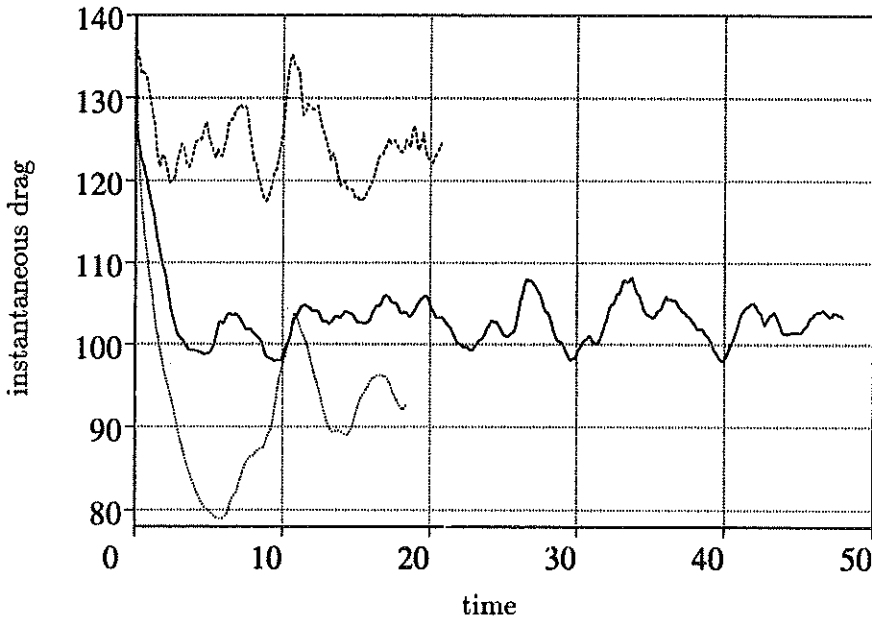

FIGURE 4. Performance of suboptimal scheme compared to no control and the scheme of Choi *et al.* (1992). Parameters for suboptimal scheme are $\mu = 0.01$, $\ell = 10$, $T^+ = 1$. Legend: ——— suboptimal scheme, $\cdots\cdots$ $\phi = -v|_{y^+=10}$, - - - - no control.

3.3 Future work

At present, the drag reduction obtained using a suboptimal control scheme is still slightly less than the drag reduction obtained using the *ad hoc* scheme of Choi *et al.* (1992), as shown in Figure 4. It is hoped that by further variation of the parameters and careful study of the numerical issues of the adjoint problem, the result using the suboptimal formulation may be significantly improved. We expect that, using the suboptimal method, a significant improvement is possible over all *ad hoc* schemes, as the suboptimal scheme uses the entire flow information in the near-wall region and is rigorously based. Also, work is currently in progress with Dr. Chris Hill to reduce the suboptimal control scheme to one which depends on wall information only. Preliminary results of this work are also quite promising — a discussion of this project is included in the next report in this volume.

Acknowledgements

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