Drag reduction at a plane wall

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1. Motivation and objectives

The reduction of the turbulent drag arising from flow over a wall is a major technological issue. Over the years, many schemes have been proposed to reduce turbulent drag (large eddy break-up devices, compliant walls, polymer addition, riblets etc.), with varied rates of success. The use of riblets, for example, can lead to about a 7% reduction in drag. The use of computational and theoretical methods has emerged recently as an effective tool for understanding new aspects of the physics of drag reduction.

Abergel & Temam (1990) describe how theoretical optimal control schemes can be developed for turbulent flow fields. The formulations are highly idealized and cannot be implemented even computationally due to a requirement for prohibitively large data storage and computations. The problem lies in the need to have *perfect knowledge* of the flow field and its history in order to achieve the *best* drag reduction possible.

The so-called sub-optimal scheme (Choi *et al.* 1993) has more modest objectives. A distribution of control forces is derived based on the instantaneous state of the system, thereby eliminating the need to retain and investigate the entire history. The procedure has been shown to work well for the one-dimensional Burgers equation, with both distributed and boundary control. Encouraged by the success of this approach, Bewley *et al.* have pursued the application of the sub-optimal scheme for channel flow. A detailed description of the method and their results is reported in this volume of the annual research briefs.

The objective of the present work is to determine by analytical means how drag on a plane wall may be modified favorably using a minimal amount of flow information - preferably only information at the wall. What quantities should be measured? How should that information be assimilated in order to arrive at effective control?

As a prototypical problem, we consider incompressible, viscous flow, governed by the Navier-Stokes equations, past a plane wall at which the no-slip condition has been modified. The streamwise and spanwise velocity components are required to be zero, but the normal component is to be specified according to some control law. The challenge is to choose the wall-normal velocity component based on flow conditions at the wall so that the mean drag is as small as possible. There can be no net mass flux through the wall, and the total available control energy is constrained. A turbulent flow is highly unsteady and has detailed spatial structure. The mean drag on the wall is the integral over the wall of the local shear forces exerted by the fluid, which is then averaged in time; it is a "macroscopic" property of the flow. It is not obvious how unsteady boundary control is to be applied in order to modify the mean flow most effectively, especially in view of the non- self-adjoint nature of the governing equations. We pursue an approximate analytical solution to the sub-optimal scheme.

2. Accomplishments

The main accomplishment of this project has been the finding of an approximate solution to the sub-optimal problem, which requires only wall information to define a control law. In a preliminary direct simulation, this law leads to a drag reduction of the order of 15%.

The sub-optimal law is developed by first solving what will be termed a locally-optimal problem (the optimal drag reduction problem on a very short time interval) and combining that solution with a gradient algorithm.

2.1 The locally-optimal problem

The mean drag is found by averaging the instantaneous drag over a long time interval. In order to achieve a formally optimal reduction in the mean drag, a vast quantity of flow information must be employed. For what I will call the locally-optimal problem, the time interval, T, over which the average is made is taken to be very small ($T \ll 1$). This has the highly desirable effect that only flow information near the wall is required in order to investigate how the flow can be modified efficiently. It has the further advantage that the problem can be solved analytically, at least to a good approximation (the error is $O(T^{1/2})$).

Let $\overline{D}(t)$ denote the total drag on the wall lying in the xz-plane, $-L \leq x, z \leq L$, averaged over the short time interval t to $t+T$. Let $\Phi(x,z,T)$ be the normal velocity at the wall applied during that time interval. The first variation of $\overline{D}$ with respect to changes in Φ is

$$\delta\overline{D}(t) = \frac{1}{4L^2T}\int_{-L}^{L} dx \int_{-L}^{L} dz\ \tilde{P}(x,z,T)\ \delta\Phi, \tag{1}$$

where $\tilde{P}(x,z,T)$ depends upon the instantaneous state of the flow. The field $\tilde{P}$ can be found by solving a carefully-formulated adjoint problem. Only a cursory description of the solution procedure will be given here.

The adjoint problem requires the solution of an initial boundary value problem, in which the solution is marched backwards in time (Abergel & Temam 1990) from $t+T$ to t. The initial stage of development is the only portion of the solution which need be found. The quantity $\tilde{P}$ is the integral over the time interval $[t, t+T]$ of the time- evolving adjoint wall pressure.

As a first step, a *reduced adjoint problem* is formulated and solved. In this case, only the mean flow is used in the adjoint governing equations, with the consequence that the problem reduces to a one-dimensional diffusion problem whose solution can be written as an error function. This adjoint solution defines an envelope growing away from the boundary as time evolves backwards.

To solve the adjoint problem fully, a correction must be added to this solution, and consequently a *second adjoint problem* is formulated. This second problem now involves the unsteady portion of the turbulent flow field. In order to obtain a tractable solution, the mean flow is assumed to be uniform near the wall (the mean flow does not appear in the solution at the order of approximation used). "Higher order" convection effects are also ignored. The most important aspect of this problem, however, is that the second problem is "forced" by a combination of the solution to the reduced problem and the unsteady flow field. The reduced problem defines an envelope close to the wall beyond which events are insignificant. Thus only unsteady events *close to the wall* need be considered.

The near-wall unsteady flow field is modeled with a spanwise velocity growing linearly, and normal velocity growing quadratically, with normal distance from the wall. The streamwise component follows from continuity. The adjoint pressure at the wall is then determined for small times. The adjoint pressure represents how sensitive the drag is to changes in the wall blowing, Φ, for a given wall blowing distribution and near-wall flow field. The solution collapses in such a way that only the control distribution, Φ, and the local wall shear, $(\partial u/\partial y)_{y=0}$, are required to define $\tilde{P}$.

Defining the Fourier Transform of some $f(x,z)$ by

$$\hat{f}(\alpha,\beta) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dz f(x,z) e^{-i(\alpha x+\beta z)}, \tag{2}$$

we find

$$\hat{\tilde{P}}(\alpha,\beta,T) \approx A\hat{\Phi} + B\Big(\frac{\partial \hat{u}}{\partial y}\Big)_{y=0} + O(T^2), \tag{3}$$

where

$$A = i\frac{\alpha T}{2\gamma}\Big[1 - \frac{4\gamma}{3}\sqrt{\frac{T}{\pi R}}\Big],\ B = \frac{4T}{3}\sqrt{\frac{T}{\pi R}},\ \gamma = \sqrt{\alpha^2+\beta^2}, \tag{4}$$

and R is the Reynolds number. The quantity $(\partial\hat{u}/\partial y)_{y=0}$ is the Fourier Transform of the local shear at the wall.

2.2 The sub-optimal problem using wall information only

This solution (3, 4) for the adjoint pressure is employed in a direct numerical simulation to define a blowing/suction distribution based purely on wall information. If the superscript denotes the flow conditions at the nth time step, then the control distribution at the $(n+1)$th time step is

$$\Phi^{n+1} = \Phi^n - \mu(\ell\Phi^n - R\tilde{P}), \tag{5}$$

(see Bewley *et al.* 1993) or in terms of transformed quantities

$$\hat{\Phi}^{n+1} = \hat{\Phi}^n[1 - \mu\ell + \mu R A] + \mu R B\Big(\frac{\partial \hat{u}^n}{\partial y}\Big)_{y=0}. \tag{6}$$

This law is implemented in the same code as that used by Bewley *et al.* The reader is referred to that review for details of the numerics. The parameters used are

$$R = 100, T = 0.01, \mu = 0.2, \ell = 0.5. \tag{7}$$

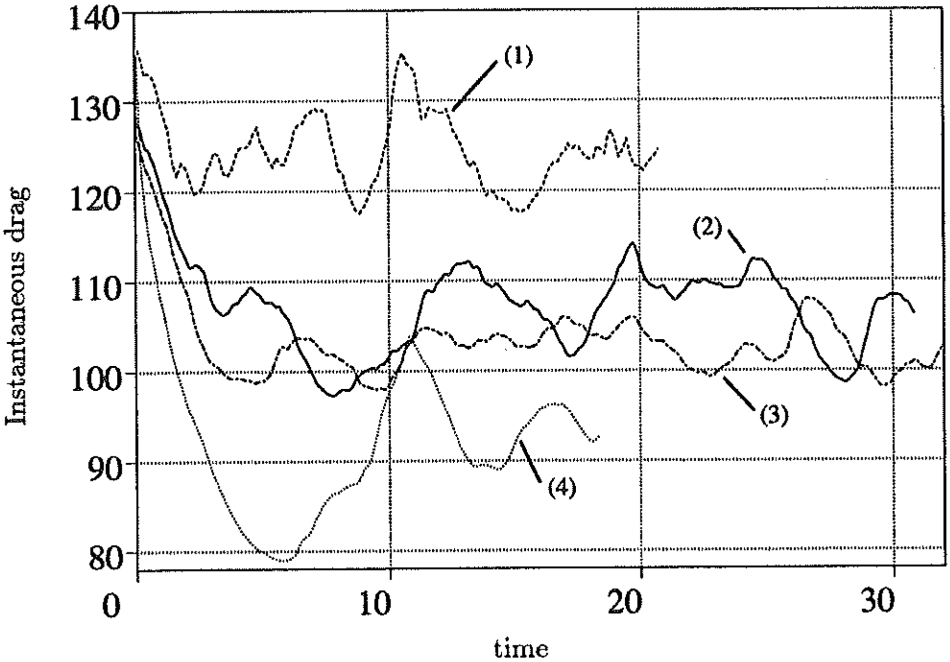

FIGURE 1. The drag on the wall as a function of time for four separate configurations. Curve 1: No control. Curve 2: Analytical scheme, Equation (6). Curve 3: Computed sub- optimal scheme. Curve 4: Scheme of Choi et al. (1993).

The history of the instantaneous drag on the wall as a function of time is shown in Figure 1 for the case of no control (curve 1), the analytical law given by equation (6) which uses only wall information (curve 2), the law based on sub-optimal computation of Bewley (curve 3), and the *ad hoc* scheme of Choi et al. (1993) (curve 4). Note that the present analytical scheme is the only one which employs only wall information.

The present rule leads to a drag reduction of the order of 15%. The performance is only slightly degraded from the direct computation of the sub-optimal law, which makes use of flow information throughout the entire domain.

3. Future work

It is clear that even with a relatively simple analysis of the adjoint equations, useful control laws can be derived. The effect of mean shear will be included in the adjoint problem. This will account for more complex physical processes including vortex tilting and stretching near the wall. Can such dynamics be exploited to improve drag reduction? This remains an open question.

Using the present approach, is it possible to define the dynamic properties (created by active or passive means) which a surface must have in order to reduce the drag which it experiences? What are those properties? These intriguing questions bear further investigation.

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