A novel energy balance approach for a verifiable
and accurate solution of radiation extinction in
particle clouds

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1. Motivation and objectives

Radiation transport through particle clouds is rich in fundamental physics of radiation
as well as its applications (Kostinski 2002; Davis & Mineev-Weinstein 2011; Burt &
Boyd 2007; Siegel et al. 2010). Two approaches are commonly used in the literature
to study photon transport through participating medium of particle clouds. First is a
ballistic approach (which is discrete) where no governing equations are involved and
ray-tracing techniques are used to compute transmission across the particle clouds.
The commonly used ray-tracing tool is the particle-resolved Monte-Carlo ray-tracing (Pr-
MCRT) method. The solution offered by Pr-MCRT is considered to be the gold-standard
result as it is equivalent to the analytical solution.

In the second approach, the photon transport is modeled using a governing equation
and then it is solved on a continuum domain. Photon transport through a discrete par-
ticulate medium is governed by the radiation transport equation (RTE). The RTE is an
integro-differential equation, and it is given as (Modest 2003),

\[
\frac{1}{c} \frac{\partial I}{\partial t} + \hat{s} \cdot \nabla I = \sigma_a I_b - \sigma_a I - \sigma_s I + \frac{\sigma_s}{4\pi} \int_{4\pi} I(\hat{s}')\Phi(\hat{s}' \cdot \hat{s})d\Omega, \tag{1.1}
\]

where \(c\) is the speed of light, \(I\) is the radiation intensity of constant wavelength, \(\hat{s}\) is the
direction of photon transport, \(\sigma_a\) is the absorption coefficient, \(I_b\) is the Planck blackbody
intensity, \(\sigma_s\) is the scattering coefficient, \(\hat{s}'\) is the scattering direction, \(\Phi(\hat{s}' \cdot \hat{s})\) is the
scattering phase function, and \(\Omega\) is the solid angle. The first three right hand side terms
in Eq. (1.1) represent the augmentation of radiation intensity by emission \((\sigma_a I_b)\) and the
reduction of radiation intensity by absorption \((\sigma_a I)\) and by scattering \((\sigma_s I)\). The last
term gives the amount of radiation augmented due to scattering from other directions
\((\hat{s}')\).

Solving Eq. (1.1) is much cheaper than the Pr-MCRT and is widely preferred in in-
dustries. There are, however, several weaknesses associated with using this continuum
approach, and we take up the most major drawback in this work. To demonstrate the
significant drawback with the continuum approach, let us consider the simplest radiation
transport of collimated photon transport through purely absorbing particles without
blackbody emission by the particle. For this problem, \(\sigma_s = 0\) and \(I_b = 0\). Besides, collimi-
ated photon transport assumption yields one-dimensional transport. Assuming a steady
photon transport along the \(x\) direction, we showed in our previous work (Paul & Mani
2019) that the RTE becomes Bouguer-Beer law (or simply, BB-law) which is given as
(Larsen & Clark 2014; Frankel et al. 2017),

\[
\ln \left( \frac{I}{I_{in}} \right) = -n A_P L_x, \tag{1.2}
\]
where $I_{in}$ is the radiation intensity entering the domain, $n$ is the number density, $A_P$ is the projected area of the particle and $L_x$ is the computational domain size in the direction of radiation propagation. The quantity $nAPL_x$ is called the optical thickness ($\tau$), which is a non-dimensional parameter.

As shown by Frankel et al. (2017), the most significant weakness in the BB-law lies in its non-verifiability. Due to the computation of number density, there is no grid-converged solution when applying the RTE on the Eulerian domain. This scenario is undesirable as any computational solution should be verifiable. To circumvent this lack of verifiability, we developed a filtering strategy, which is commonly used in particle-laden turbulent flows, for radiation transport (Paul & Mani 2019). Although the filtering approach did indeed address the non-verifiability of the BB-law, two important problems remain. First is that the BB-law with the filtering approach does not predict the non-logarithmic nature of the decay reported for particle clouds, which violates significant assumptions made in arriving at the BB-law (Borovoi 2002; Kostinski 2001; Shaw et al. 2002). Second, the time taken for computing transmission using the BB-law with the filtering approach is very similar to that of the Pr-MCRT which violates the basic characteristic of a lower-order model. The current work aims to rectify these deficiencies by developing a novel energy balanced approach for using BB-law on Eulerian meshes.

The remainder of this report is organized as follows. The next section provides the derivation of the absorption coefficient which will be used with the BB-law. The results from these simulations are discussed for simplified as well as for randomly distributed particles in Section 3. Finally, conclusions and future work are discussed in Section 4.

2. Methodology

In this section, we derive an analytical equation for the absorption coefficient. To this end, we make use of an energy-based approach.

Consider a collimated radiation intensity of $I_{in}$ incident on a spherical particle as...
shown in Figure 1a. In the case of a purely absorbing real particle, its radius \( R_P \) is a constant and its absorption coefficient \( \sigma_a \) is infinity. Therefore, the energy absorbed by the real particle is the product of the projected area of the spherical particle times the incident radiation. Therefore,

\[
E_{a,R} = -I_{in} \pi R_P^2. \tag{2.1}
\]

The negative sign in the above equation signifies that the angle between the area vectors defined on the windward surface of the sphere and the direction of the incident radiation is higher than 90° and the energy is being absorbed.

Now, the biggest obstacle with representing this real particle in our simulation is its infinite value of the absorption coefficient which cannot be represented on a computer. Therefore, we consider a numerical particle of radius \( R_N \) whose absorption coefficient is finite. We compute this finite absorption coefficient for this numerical particle and replace it with the real particle in our simulations.

To compute the absorption coefficient of the numerical particle, we compute its energy absorption in terms of \( \sigma_a \) and equate it to the energy absorbed by the real particle given by Eq. (2.1). To this end, we make use of an energy-balance method to derive an equation for the energy absorbed by the numerical particle in terms of \( \sigma_a \). With this approach in mind, the configuration we consider is shown in Figure 1b. A uniform collimated radiation of intensity \( I_{in} \) is incident on a numerical spherical particle of radius \( R_N \). The origin is at the centroid of the numerical particle where the z-axis is aligned along the direction of radiation propagation. Given that the absorption coefficient of the numerical particle is finite, the radiation leaves the particle with an intensity of \( I_{out} \) which is not uniform.

The energy absorbed by this numerical particle is the difference between the energy entering and leaving the numerical particle as shown in Figure 1b. In other words, the energy absorbed by the numerical particle is the difference between the energy absorbed by the windward and leeward surfaces of the numerical particle. Inasmuch as the sign of the area-normal vectors on these surfaces has not yet been determined, we can write the energy absorbed the numerical particle as

\[
E_{a,N} = \int \int_{z<0} |I_{in}| \cdot |\vec{n}| \cdot |dA| + \int \int_{z>0} |I_{out}| \cdot |\vec{n}| \cdot |dA|, \tag{2.2}
\]

\[= T_1 + T_2 \tag{2.3}\]

where \( \vec{n} \) is the unit normal vectors defined on the surface of the numerical particle. We now have to evaluate the integrals \( T_1 \) and \( T_2 \).

2.1. Evaluation of \( T_1 \)

We know that

\[
T_1 = \int \int_{z<0} |I_{in}| \cdot |\vec{n}| \cdot |dA|. \tag{2.4}
\]

Computing the dot product between the incident radiation vector and the surface normal unit vector

\[
T_1 = \int \int_{z<0} |I_{in}| \cdot |dA| \cdot \cos \psi. \tag{2.5}
\]
where $\psi$ is the polar angle. Utilizing the parametric form of the sphere given as

\begin{align}
x &= R_N \cos \theta \sin \psi, \\
y &= R_N \sin \theta \cos \psi, \\
z &= R_N \cos \psi,
\end{align}

(2.6) (2.7) (2.8)

$|dA| = R_N^2 \sin \psi \, d\psi \, d\theta$, 

(2.9)

where $\theta$ is the azimuthal angle of the sphere.

We also know that

$|I_{in}| = I_{in}$. 

(2.10)

While the azimuthal angle ($\theta$) varies from 0 to $2\pi$ for both the windward and leeward surfaces of the numerical particle, the polar angle varies from $\pi/2$ to $\pi$ for the windward surface as can be seen in Figure 1b. Therefore,

\begin{align}
\theta &: 0 \to 2\pi, \\
\psi &: \frac{\pi}{2} \to \pi.
\end{align}

(2.11) (2.12)

Substituting Eqs. (2.6) to (2.12) in (2.5), the integral $T_1$ becomes

\[ T_1 = I_{in} R_N^2 \int_{0}^{2\pi} \int_{\pi/2}^{\pi} \sin \psi \cos \psi \, d\psi \, d\theta. \]

(2.13)

We can compute

\[ \int_{\pi/2}^{\pi} \sin \psi \cos \psi \, d\psi = -\frac{1}{2}. \]

(2.14)

Therefore, $T_1$ becomes

\[ T_1 = -I_{in} \pi R_N^2. \]

(2.15)

2.2. Evaluation of $T_2$

Having computed the integral $T_1$, now let us evaluate $T_2$.

\[ T_2 = \int \int_{z>0} I_{out} \cdot \vec{n} \, |dA| \]

(2.16)

\[ = \int \int_{z>0} |I_{out}| |dA| \cos \psi. \]

(2.17)

Once again utilizing the parametric equations for the sphere Eq. (2.6–2.9) and noting that $\psi$ for the leeward surface varies from 0 to $\pi/2$, along with $|I_{out}| = I_{out}$, $T_2$ becomes

\[ T_2 = R_N^2 \int_{0}^{2\pi} \int_{0}^{\pi/2} I_{out} \sin \psi \cos \psi \, d\psi \, d\theta. \]

(2.18)

To proceed further, we must know the value of $I_{out}$ at the leeward surface of the numerical particle. To this end, we make use of the BB-law discussed in Section 1. Therefore, the governing equation of the photon transport is

\[ \frac{dI}{I} = -\sigma_d ds. \]

(2.19)
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Knowing that the photon travels from \(-z\) to \(+z\) with an intensity of \(I_{in}\) to \(I_{out}\), we can integrate Eq. (2.19) as

\[
\int_{I_{in}}^{I_{out}} \frac{dI}{\mathcal{T}} = \int_{-z}^{+z} -\sigma_a ds. \tag{2.20}
\]

As a result, we obtain the values of \(I_{out}\) at the leeward surface of the numerical particle as

\[
I_{out} = I_{in} e^{-2\sigma_a z}. \tag{2.21}
\]

In terms of the parametric form of the sphere,

\[
I_{out} = I_{in} e^{-2\sigma_a R_N \cos \psi}. \tag{2.22}
\]

Substituting Eq. (2.22) in Eq. (2.17),

\[
T_2 = I_{in} R_N^2 \int_0^{2\pi} \int_0^{\pi/2} e^{-2\sigma_0 R_N \cos \psi} \sin \psi \cos \psi \, d\psi \, d\theta. \tag{2.23}
\]

Now, we assume that the absorption coefficient is constant within the sphere of radius \(R_N\) and that outside it is zero. We can assume any variation for the absorption coefficient, but, for simplicity, we assume it to be a constant. Thus,

\[
\sigma_a = \begin{cases} 
\sigma_0, & \text{if } r \leq R_N \\
0, & \text{otherwise.}
\end{cases}
\]

Therefore, the integral \(T_2\) for the sphere becomes

\[
T_2 = I_{in} R_N^2 \int_0^{2\pi} \int_0^{\pi/2} e^{-2\sigma_0 R_N \cos \psi} \sin \psi \cos \psi \, d\psi \, d\theta. \tag{2.24}
\]

Using integration by parts, we can compute

\[
\int_0^{\pi/2} e^{-2\sigma_0 R_N \cos \psi} \sin \psi \cos \psi \, d\psi = \frac{1 - 2R_N \sigma_0 e^{-2R_N \sigma_0} - e^{-2R_N \sigma_0}}{(2R_N \sigma_0)^2}. \tag{2.25}
\]

Therefore, the integral \(T_2\) becomes

\[
T_2 = \frac{I_{in} \pi}{2\sigma_0} (1 - 2R_N \sigma_0 e^{-2R_N \sigma_0} - e^{-2R_N \sigma_0}). \tag{2.26}
\]

Substituting the integral \(T_1\), Eq. (2.15), and \(T_2\), Eq. (2.26) in Eq. (2.3), we obtain the energy absorbed by the numerical particle of radius \(R_N\) as

\[
E_{a,N} = -I_{in} \pi R_N^2 + \frac{I_{in} \pi}{2\sigma_0} (1 - 2R_N \sigma_0 e^{-2R_N \sigma_0} - e^{-2R_N \sigma_0}). \tag{2.27}
\]

### 2.3. Evaluation of \(\sigma_0\)

To compute \(\sigma_0\), as mentioned above, we equate energy absorbed by the numerical particle to that by the real particle. Thus,

\[
E_{a,A} = E_{a,N}, \tag{2.28}
\]

\[
-I_{in} \pi R_P^2 = -I_{in} \pi R_N^2 + \frac{I_{in} \pi}{2\sigma_0} (1 - 2R_N \sigma_0 e^{-2R_N \sigma_0} - e^{-2R_N \sigma_0}). \tag{2.29}
\]
Simplifying further, the equation for $\sigma_0$ becomes
\[ 1 - 2R_N \sigma_0 e^{-2R_N \sigma_0} - e^{-2R_N \sigma_0} - 2(R_N^2 - R_P^2)\sigma_0^2 = 0. \] (2.30)

Note that the only unknown in the above equation is $\sigma_0$ and it is a function of $R_P$ and $R_N$.

Now, let $\beta = R_N \sigma_0$, which is an unknown. This allows us to rewrite Eq. (2.30) in terms of only one variable, $\frac{R_P}{R_N}$ as
\[ \frac{1}{2\beta^2} - e^{-\frac{2\beta}{2\beta^2}} - 2\beta e^{-\frac{2\beta}{2\beta^2}} + \left( \frac{R_P}{R_N} \right)^2 - 1 = 0. \] (2.31)

Equation (2.31) is our final equation for $\sigma_0$. There is no analytical solution available for this equation. Therefore, we have to make use of one of the numerical integration techniques to solve for $\beta$ from which $\sigma_0$ can be computed for a given value of $\frac{R_P}{R_N}$.

2.4. The algorithm

Having obtained an expression for the absorption coefficient of the numerical particle, this section explains how this value is incorporated into the BB-law.

Although we have an expression for $\sigma_0$, we have not discussed the choice of $\frac{R_P}{R_N}$. Remember that Eq. (2.31) was derived for a spherical particle, not for a cube. Therefore, discretion is needed in how to translate $\sigma_0$ on an Eulerian mesh. We can define the radius of the numerical particle as
\[ R_N = R_P + \alpha \Delta x, \] (2.32)

where $\alpha$ is an integer greater than zero and $\Delta x$ is the Eulerian mesh size.

By defining $R_N$ using Eq. (2.32), we (i) project a near-spherical region which corresponds to the numerical particle onto the Eulerian mesh, and (ii) retain the size of the real particle in the limit of a very fine Eulerian mesh resolution. Therefore the algorithm has the following steps:

- Step 1: Set the value of $\Delta x$ and get the value of $R_N$ using Eq. (2.32).
- Step 2: Initialize a floating-point variable ‘counter’ for all the Eulerian cell centroids as zero.
- Step 3: Compute the value of $\sigma_0$ by solving Eq. (2.31) using the Newton-Raphson method.
- Step 4: Go to every Eulerian cell centroid (loop over all the mesh points):
  - (a) For this cell centroid, compute distances between each particle and cell centroids by looping over all the points;
  - (b) If the distance is less than or equal to $R_N$, increment the cell centroid counter.
- Step 5: Finally, multiply the counter with the computed value of $\sigma_0$ from step 3. This will yield the absorption coefficient for the Eulerian mesh.
- Step 6: Solve BB-law.

Finally, we use the tangent hyperbolic function for a smooth transition from $\sigma_0$ to zero at the cell centroids which are the boundaries of the sphere. This is implemented through computing the distance between the cell centroid and the particle centroid, and if this distance exceeds a particular distance (which means the cell centroid at the edge of the centroid), then rather incrementing the counter for the cell centroid, it is computed as
\[ \text{counter} = \tanh \left( \frac{\Delta x}{n_i(R_N - \text{distance})} \right), \] (2.33)
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Ly
I_{out} \cdot D_P \cdot I_{in}

(a)

Figure 2. (a) Computational domain of the simplified problem, (b) variation of percentage error in energy absorbed by a single spherical particle.

(b)

Figure 3. Radiation intensity contours at the exit plane obtained using the energy balance approach for (a) $\Delta x/D = 2.0$, (b) $\Delta x/D = 1.0$, (c) $\Delta x/D = 0.4$, and (d) $\Delta x/D = 0.1$.

where $n_i$ is an integer greater than zero.

3. Results and discussion

This section presents the main results of this study. First, we apply the methodology developed in the previous section, first for a simplified problem of radiation incident on a single sphere, and then for a particle cloud. We also discuss the merits of this new methodology compared with the filtering strategy presented in Paul & Mani (2019).

3.1. A simplified problem

In this subsection, we take our first problem for which we have an analytical solution. The problem in hand is the collimated radiation of intensity $I_{in}$ incident on a spherical particle of size $D_P$. The energy absorbed by this particle can be computed analytically as $(\pi D_P^2/4)I_{in}$. The gold-standard result of Pr-MCRT also converges to this value when we increase the number of rays to $10^8$, as shown in Paul & Mani (2019).

Now, we place this spherical particle in a Eulerian domain and use the BB-law to
compute the energy absorbed by the particle, as schematically shown in Figure 2a. We choose a square box with dimensions $L_x = 20$, $L_y = 20$, and $L_z = 20$. The particle diameter is $D = 1$. We discretize the computational domain with $n_x \times n_y \times n_z$ grid points such that the grid resolution with respect to particle diameter ($\Delta x / D_P$) varies from 20 to 1/20. The energy absorbed by the particle is then computed by the difference between the energy entering at the inlet and the energy leaving the outlet of the Eulerian computational domain. Finally, the percentage error in the numerical result is obtained by comparing this value against the analytical solution.

From our previous study (Paul & Mani 2019), we know that the information of particle is spread to the Eulerian mesh through computing the number density in nearest-neighbor and filtering approaches. In the case of nearest-neighbor, this way of computing number density leads to 100% error when the mesh size becomes smaller than the particle size (red-colored line Figure 2b). We also proposed a filtering strategy that provided convergence of the numerical result through spreading the number density to the neighboring cells using a Gaussian filter (blue-colored line Figure 2b).

Now, let us apply the methodology developed in the previous section to this simplified problem. Here, we need to specify the numerical particle radius. We tested different radius values and found that $\alpha$ of 2 would be ideal. Therefore, we fix $R_N = R_P + (2 \Delta x)$. Furthermore, whenever we refine our mesh, our numerical particle size also changes, as does the absorption coefficient. It only takes less than 10 iterations to get a solution for Eq. (2.31) using the Newton-Raphson method with an initial value of $\beta$ as 1.

Figure 2b depicts how the radiation intensity contours at the outlet (i.e., $x = 20$) plane changes as the grid is refined such that $\Delta x / D_P$ becomes smaller than 1. In Figure 3(a), which shows the radiation intensity contour for the $\Delta x / D_P = 2$ case, the contour size is larger than the real particle because of our definition of the numerical particle. As the mesh size diminishes, the area in which the intensity is defined becomes closer to the real particle size (Figures 3b, 3c, and 3d). As a result, we obtain the real particle properties in the limit of finely refined mesh. This limit is evident in the percentage error curve plotted in Figure 2b as a magenta-colored line. Unlike the other two cases, in the energy approach, the error for large grid size (i.e., $\Delta x / D_P >> 2$) is large. This error is caused by magnifying the real particle because it is not fully represented in the Eulerian mesh as we consider only the cells whose distances from the particle are less than or equal to the numerical particle radius. This error, however, decreases steadily as the grid is refined to represent the sphere properly in the Eulerian mesh. The more the mesh is refined, the more the error decreases, until for $\Delta x / D_P$ of 0.1, where the error is only 0.5%, which shows that we have approached the analytical solution. Therefore, our methodology does indeed converge to the theoretical solution for this simplified problem.

3.2. Transmission through particle clouds

With our new methodology now shown not only to solve the issue of non-verifiability but also provides an answer similar to the gold-standard result for a simplified problem, we now apply our tool for a particle cloud. We know from our previous study that the Poisson cloud of a lesser number of particles exhibits non-logarithmic decay of radiation intensity (Paul & Mani 2019). We also showed that although the filtering approach converges to a solution, the solution is not at all close to that of the gold-standard, which is non-logarithmic in nature, while the filtering approach yields a logarithmic decay. Therefore, we choose these particle clouds in this study to test the accuracy of our new method.

The way we generate the Poisson cloud is already outlined in Paul & Mani (2019). We consider a computational domain of size $L_x \times L_y \times L_z = 1 \times 1 \times 1$, which is a cube. The
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Figure 4. A sample typical Poisson distributed particles in a cube for $5 \times 10^2$.

![Diagram showing Poisson distributed particles in a cube]

Figure 5. Transmission profiles for different $\Delta x/D_P$ values obtained using the energy balance approach for (a) $N_P = 5 \times 10^2$, (b) $N_P = 4 \times 10^3$.

Radiation transport is along the $x$ direction. We consider a different number of particles ($N_P$). All the cases, however, have the constant value of optical thickness as $\tau = 4$. We consider two classes of $N_P$, 500 and 4000 as these cases exhibit difficulty in obtaining a solution closer to the gold-standard solution. A typical sample of Poisson distributed particles for $N_P = 5 \times 10^2$ is shown in Figure 4. In this study also, we compute our gold-standard solution using the Pr-MCRT method.

The transmission profiles obtained using this approach are presented in Figures 5(a) and 5(b) for $N_P = 5 \times 10^2$ and $N_P = 4 \times 10^3$, respectively. The corresponding error curves are shown in Figure 6. For large $\Delta x/D_P$, as noted in the simplified problem,
Figure 6. The $L_1$ error between the particle-resolved Monte Carlo ray tracing method and the BB-law for various $\Delta x/D_P$ values.

the error is high as the transmission profile shows that less energy is absorbed by the cylinder compared to the gold-standard result. This error is, again, due to the inherent difficulty of representing the spherical particle as it is on the Eulerian domain. As we refine the grid, the error decreases as the transmission profile becomes closer to the profile of Pr-MCRT. Unlike the filtering approach whose error starts to flatten out after further refining of the mesh, the current method’s transmission profiles keep coming closer to the gold-standard result with a non-logarithmic decay. This decay is also confirmed in the $L_1$ norm variation with respect to $\Delta x/D$ plotted in Figure 6. In particular, the transmission profile obtained by the Pr-MCRT and the current methods starts to overlap, a result which shows that we have achieved a solution similar to the gold-standard result using BB-law. Such an accurate solution of BB-law has not been demonstrated on an Eulerian mesh to date in the literature. Therefore, the BB-law with the energy approach is not only a numerically verifiable method, but is also accurate in yielding a solution similar to that of the gold-standard result.

Finally, we compute the time taken to obtain the transmission profiles. As discussed in Section 1, the Pr-MCRT is the gold-standard result, but it is computationally expensive. We consider the case of $4 \times 10^3$ for illustration. For this case, the time taken to compute the transmission profile for one sample of the particle cloud using Pr-MCRT with $10^8$ rays is shown as a dotted line in Figure 7. The main purpose of using BB-law is to reduce the time taken for the simulation while obtaining a solution closer to the gold-standard result. This is the second-most desirable feature of any reduced-order models. Given its failure in
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Figure 7. Time taken to compute transmission profile of one sample of the cloud using BB-law with different approaches and the Pr-MCRT. Note that $\Delta x/D_P$ is meaningless for Pr-MCRT.

predicting verifiability, the nearest-neighbor approach is not a reliable method by which to compute a solution closer to the gold-standard result. As demonstrated in Paul & Mani (2019), the filtering approach does indeed provide a verifiable solution, albeit not the gold-standard solution for the cases where the BB-law assumptions are severely invalid. An additional drawback to the filtering approach lies in its numerical cost. As seen in Figure 7, the numerical cost of the filtering approach for a highly refined simulation is closer to that of the Pr-MCRT method. Hence, the idea of using a continuum rather than a ballistic approach to save computational cost is undermined. On the other hand, our new approach is much faster than the filtering approach, as seen in Figure 7. In the case that yielded a transmission profile overlapping with that of the Pr-MCRT, the length of time required was approximately 25 times less than that taken by Pr-MCRT. Thus, our current method is significant not only for its verifiability and accuracy, but also for its lower cost.

4. Conclusions

The objective of this work is to rectify the drawbacks of filtering methodology introduced by Paul & Mani (2019). First, the absence of non-logarithmic decay in the transmission profile of the filtering approach means that the BB-law solution can never be equivalent to the Pr-MCRT for the cases which significantly violate BB-law assumptions. Second, the filtering approach solves the non-verifiability problem with the BB-law
as the computational cost is equivalent to that of the Pr-MCRT. This drawback, in particular, underscores the conclusion that the filtering approach is not suitable for large-scale applications.

To circumvent the drawbacks of the filtering approach, we derived an equation for the absorption coefficient without needing to compute the number density. We made use of a novel energy-balance approach for our derivation. Then, the equation of the absorption coefficient was solved using the Newton-Raphson method with fewer than ten iterations. Our new method when applied to a simplified problem of radiation incidence on a single sphere yields a verifiable solution as the percentage error continually decreases upon refining the mesh.

Then we tested our method on a Poisson particle cloud for accuracy and cost. We intentionally chose two cases for which the gold-standard transmission profile is non-logarithmic, a feature which the filtering approach failed to capture. The transmission profiles obtained by our energy-based approach exactly overlaps with that of the Pr-MCRT upon refining the mesh well enough. The BB law with the energy approach is accurate in the sense that it redeems the exact solution upon the refining the mesh. We also compared the cost of this new method with that of a filtering approach and noted that the current method is approximately 25 times faster. Such advantages make it suitable for large-scale purposes. Therefore, we have successfully demonstrated that our novel energy-based approach with BB-law is (i) verifiable (it converges to a solution upon mesh refinement), (ii) accurate (it converges to the exact solution upon mesh refinement) and (iii) cheaper (the cost of refined mesh simulations are substantially cheaper compared with the filtering approach).

The next step is to apply this new method for turbulence-generated particle clouds. Also, further exploration of ways to reduce error for large mesh sizes can be anticipated, inasmuch as our current energy-based method suffers at large mesh sizes.

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