Dissipation events in wall-bounded turbulence

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1. Motivation and objectives

Turbulent quantities such as dissipation, production, and the magnitude of turbulent kinetic energy are known to be highly intermittent. Long periods of weak intensity are interrupted by brief events during which the intensity of turbulence can locally increase by orders of magnitude. Herein, we seek to identify whether a consistent, possibly self-similar mechanism drives the formation of peak intensity events in wall-bounded turbulence and how it can be related to known physical concepts.

Turbulence is often thought to be sustained by powerful bursts (Kline et al. 1967) during which fluid is being ejected away from wall. The bursts, which have also been conceptually incorporated into a cyclic, self-sustaining process (Waleffe 1997), have been linked to the breakup of streamwise elongated streaks that are generated by the algebraic lift-up mechanism (Landahl 1975; Gustavsson 1991). The streaks describe regions of higher or lower streamwise momentum than their surroundings, and may induce secondary instability by introducing inflection points into the instantaneous velocity field (e.g., Andersson et al. 2001; Hack & Zaki 2014). Both varicose and sinuous types of streak instabilities exist (Swearingen & Blackwelder 1987). The varicose type is commonly associated with inflection points in the normal shear, while the sinuous type relates to inflection points in the transverse shear. Whereas the sinuous instability is anti-symmetric in the streamwise and normal velocity components and symmetric in the transverse component, the varicose instability is symmetric in the streamwise and normal components and anti-symmetric in the transverse component. The literature on streak instabilities indeed provides evidence for a connection between the varicose type of instability and the generation of hairpin-shaped structures (see, e.g., Asai et al. 2002; Skote et al. 2002). In this context, it is worth noting that the so-called minimal channel setup originally investigated by Jiménez & Moin (1991) appears limited to sinuous instabilities and thus sustains turbulence in the absence of varicose breakdown (Hamilton et al. 1995).

The recurrence of hairpin-type structures in visualizations of turbulence has attracted the interest of a number of researchers (see, e.g., Theodorsen 1955; Adrian et al. 2000). Experimental measurements of hairpin vortices by Dennis & Nickels (2011) confirmed their alignment with low-speed streaks without, however, directly connecting them to exponential growth. More recently, researchers have sought to generate hairpin-like structures by means of nonlinear optimization (Farano et al. 2015) and to cast their principal dynamics into reduced-order models (Cohen et al. 2014). The statistical analysis of hairpin structures in late-stage transitional and turbulent flow by Hack & Moin (2018) provided perhaps the most direct connection yet between the formation of these characteristic structures in realistic flow and the activity of an exponential instability. The study recorded exponential growth of the fluctuation magnitudes during the formation of hairpin vortices, and the eigenfunction computed in a linear stability analysis showed a clearly varicose structure. As predicted in the linear resolvent analyses by Sharma &

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McKeon (2013), the hairpins were also found to be aligned with their critical layers, further substantiating their generation by an inviscid instability mechanism. Comparison of isolated hairpin vortices during the late stages of transition to turbulence and fully turbulent flow demonstrated their qualitative similarity (Hack & Moin 2018).

A further connection between the late stages of transitional flow and turbulence was made by Wu et al. (2017), who showed that the formation of local spots of high intensity also occurs within turbulent flow, although the precursors of these events were not investigated. Blonigan et al. (2019) investigated the potential to predict localized dissipation peaks. However, their consideration of minimal channel flow effectively eliminated the varicose breakdown path from their analysis.

Inspection of the hairpin vortices generated during transition to turbulence demonstrated that their formation by powerful inviscid instabilities gives rise to extreme levels of turbulent dissipation and production, as well as of fluctuation kinetic energy, which exceed the mean dissipation level by several orders of magnitude (Hack & Moin 2018). The dynamical similarity between transitional and turbulent hairpin vortices thus raises the question of whether extreme events in turbulence, characterized by high levels of localized fluctuations and fluctuation gradients, may also be ascribed to the local formation of hairpins, and by extension, to the activity of an exponential instability mechanism.

In the following, we investigate whether the extreme levels of dissipation introduced by exponential instabilities and their connection to the formation of hairpin vortices may explain the intermittent occurrence of peak intensity events in wall-bounded turbulence. We further examine whether such a consistent explanation applies defines a self-similar process that repeats itself across the extent of the turbulent boundary layer.

2. Methodology

Our analysis investigates extreme events in time-resolved data obtained from direct numerical simulations (DNS) of a turbulent boundary layer. The setup entails complete simulations of K-type transition, starting from a laminar zero-pressure-gradient boundary layer which undergoes transition to turbulence and eventually attains a fully turbulent state (Sayadi et al. 2013). The computational framework is based on a compressible formulation of the Navier-Stokes equations, although the effect of compressibility at the considered free-stream Mach number, \( M_a = 0.2 \), is negligible and the results may be understood to represent incompressible flow. Spatial derivatives are approximated using fourth-order finite differences, and time is advanced through a second-order scheme.

Throughout this work, we apply a decomposition of the instantaneous velocity vector, \( \mathbf{u} = [u, v, w]^T \), into a mean component that has been averaged over the homogeneous time and transverse dimensions, and a fluctuation component

\[
\mathbf{u}(x, y, z, t) = \bar{\mathbf{u}}(x, y) + \mathbf{u}'(x, y, z, t) .
\] (2.1)

Length scales are normalized by the distance of the inflow location from the leading edge, \( x_0 = 1 \), and velocities are normalized by the free-stream convective velocity, \( U_\infty = 1 \). The viscosity is \( \nu = 1 \times 10^{-5} \), and the Reynolds number at the inflow location is \( Re_{x0} = 1 \times 10^5 \).

Within the turbulent boundary layer, an alternative measure for the distance to the wall is given in wall units, \( y^+ \equiv y u_\tau / \nu \), where \( u_\tau \) is the friction velocity. Transition to turbulence, identified by the local maximum of skin friction, occurs near \( Re_x \approx 4 \times 10^5 \), and the domain extends to \( Re_x = 9 \times 10^5 \). In terms of the momentum thickness, the Reynolds number of the turbulent regime ranges from \( Re_\theta = 500 \) to \( Re_\theta = 1,500 \).
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Figure 1. Visualization of the transitional and turbulent boundary layer showing isosurfaces of $Q$. Dissipation events are sampled in the region shaded in dark.

The reader is referred to Hack & Moin (2018) for details of the simulations. Additional information is provided by Sayadi et al. (2013).

The statistical analysis of extreme events is facilitated through time-resolved conditional sampling of the turbulent flow field. A similar approach had been applied in the study of the mechanisms of hairpins by Hack & Moin (2018), who applied a filtering based on topological criteria to isolate specific flow structures from both transitional and turbulent flow. In contrast, the high-intensity events considered in the present study are identified as local maxima of the dissipation of turbulent kinetic energy,

$$D(x, y, z, t) = \nu \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_i}{\partial x_j}, \quad (2.2)$$

in the four-dimensional spatio-temporal space spanned by the DNS time series. Further quantities of interest include the production of turbulent kinetic energy,

$$P(x, y, z, t) = -u'_i u'_j \frac{\partial \bar{u}_i}{\partial x_j}, \quad (2.3)$$

and the magnitude of turbulent kinetic energy

$$k(x, y, z, t) = \frac{1}{2} u'_i u'_i. \quad (2.4)$$

The sampling procedure extracts a flow volume over a finite time horizon whose spatial extent scales with the wall distance of the events of interest. The spatio-temporal velocity field within each volume provides an individual realization in the time-resolved ensemble average. The computation of the average can be expressed as

$$\langle \chi (\tilde{x}, \tilde{z}, \tilde{t}; y^+) \rangle = \frac{1}{N} \sum_{n=1}^{N} G(x, z, t) \chi_n (x, y^+, z, t), \quad (2.5)$$

where $N$ denotes the number of realizations and the operator $G$ applies a translation in the streamwise, transverse, and time dimensions to align the locations of highest dissipation of all individual samples $\chi_n$ at coordinates $\tilde{x} = 0$, $\tilde{z} = 0$, and $\tilde{t} = 0$. During the time-resolved sampling of each realization, the sampling domain moves downstream at eighty percent of the local mean convective velocity so as to keep the sampled flow structures centered within the volume.

A visualization of the boundary layer considered in this analysis is presented in Figure 1. Isosurfaces show the second invariant of the velocity gradient tensor, also known as $Q$ (Hunt et al. 1988). Our present analysis uses data from the turbulent region, $x > 4.4$, shaded in dark in the visualization.
3. Extreme events

We define extreme events within the scope of this study as localized peaks in the turbulent dissipation. Figure 2 visualizes $D$ in a plane at $y^+ = 50$ within the turbulent portion of the computational domain, and shows an intermittent field that is characterized by local peaks of extreme amplitude that are separated by regions of low intensity. Since the dissipation of turbulent kinetic energy, Eq. (2.2), is defined as the sum of the squares of the elements of the Jacobian of the fluid velocity vector, it can also be interpreted as a measure of the intensity of vortical structures (Soria et al. 1998; del Alamo et al. 2006). As our results will demonstrate, this interpretation of an extreme event is also consistent with the classical definition of turbulent bursts, commonly understood as powerful ejections of fluid away from the wall.

Distributions of the magnitude of the turbulent dissipation in isolated dissipation events are presented in Figure 3 at a range of wall distances. A dissipation event is defined as a local extremum of the turbulent dissipation at a given dimensionless wall distance $y^+$ in the three-dimensional space, $x \times z \times t$, spanned by the wall-parallel physical dimensions and time such that all points in the discrete 26-point neighborhood of a specific point in the discrete orthogonal simulation grid have a lower dissipation amplitude. As expected, the intensity of the dissipation events generally increases as the wall is approached. The general shape of the distributions is remarkably well described by log-normal distributions. Also shown are cumulative distribution functions (CDF) in linear axis scaling, which underline that only a very small portion of dissipation events attain relative high intensities. Based on these data, the thresholds chosen to identify extreme events are shown in Table 1 for the four considered wall distances. The last row gives the portion of all localized maxima at that wall distance that exceeds this threshold. In the following, we seek to gain insight into the mechanism and dynamics of extreme events by evaluating their time-resolved average, which is obtained from the DNS data as detailed in Section 2. Dissipation extrema at $y^+ = \{20, 30, 60, 150\}$ are considered. This set of locations includes the near-wall buffer layer and extends into the outer layer, where the relevance of viscosity lessens. Figure 4 presents the mean velocity profiles at $x = 5$, situated close to the upstream edge of the sampling domain, and at $x = 8$, near the downstream boundary of the sampling domain, in both physical and wall units. Markers in Figure 4(b) indicate the position of the sampling wall distances. As mentioned above, the sampling procedure accounts for the dynamic evolution of the velocity fluctuation...
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![Graphs](image)

**Figure 3.** Statistics of the intensity of local dissipation extrema at (a) $y^+ = 20$, (b) $y^+ = 30$, (c) $y^+ = 60$, and (d) $y^+ = 150$. Histograms (bars) with fits of log-normal distributions (gray lines) and cumulative distribution functions (black lines).

<table>
<thead>
<tr>
<th>$y^+$</th>
<th>20</th>
<th>30</th>
<th>60</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$ sampling threshold</td>
<td>$6 \times 10^{-3}$</td>
<td>$4 \times 10^{-3}$</td>
<td>$2 \times 10^{-3}$</td>
<td>$6 \times 10^{-4}$</td>
</tr>
<tr>
<td>Total events</td>
<td>333,715</td>
<td>366,845</td>
<td>365,510</td>
<td>271,995</td>
</tr>
<tr>
<td>Sampled events</td>
<td>2,922</td>
<td>2,132</td>
<td>1,353</td>
<td>2,171</td>
</tr>
<tr>
<td>Percentage of total events</td>
<td>0.876%</td>
<td>0.581%</td>
<td>0.370%</td>
<td>0.798%</td>
</tr>
</tbody>
</table>

**Table 1.** Dissipation sampling threshold and percentage of sampled events at different wall distances.

Field before and after a local dissipation peak is recorded. In the interest of brevity, the figures only show the time instance of highest dissipation, $\tilde{t} = 0$.

The conditional average of the sampled Cartesian velocity components is presented in Figure 5 for the four wall distances, $y^+ = \{20, 30, 60, 150\}$. A key result of the present analysis is the qualitative resemblance of the recorded flow structures in effectively all evaluated quantities throughout the considered range of wall distances. For the streamwise velocity component, the characteristic configuration describes a streamwise elongated region of low-speed fluid that is situated underneath a patch of high-speed fluid. An arrangement of this type enhances the wall-normal shear of the mean boundary layer profile and has traditionally been associated with the formation of varicose types of streak instabilities (see, e.g., Swearingen & Blackwelder 1987; Hack & Zaki 2014). With increasing wall distance, the relative magnitude of the regions of low-speed and high-speed fluid nonetheless shifts in favor of the former.

The wall-normal fluctuation component describes a localized patch of positive $v'$ that
Figure 4. Mean streamwise velocity profiles at \( x = 5 \) (dashed) and \( x = 8 \) (solid). (a) Physical units. (b) Wall units. The vertical markers indicate the sampling locations, \( y^+ = \{20, 30, 60, 150\} \), of the simulation data.

Figure 5. Ensemble averages of sampled turbulent fields at \( \tilde{t} = 0 \). Isosurfaces of positive values (white) and negative values (black) at equivalent amplitude. Streamwise fluctuations, \( \langle u' \rangle \) (first row), normal fluctuations, \( \langle v' \rangle \) (second row) and transverse fluctuations, \( \langle w' \rangle \) (third row). Wall distances from left to right: \( y^+ = 20, 30, 60, \) and 150.

is situated at the center of the sampling domain, at \( \tilde{x} \approx 0 \). The configurations thus suggests that the event as a whole describes effectively a strong ejection of fluid away from the wall, consistent with what has also been termed a turbulent burst. Finally, the transverse velocity component shows a characteristic anti-symmetric pattern of regions of positive and negative \( w' \) which are followed downstream and closer to the wall by a pair of patches of opposite sign. This structure is consistent with an amplification of an instability of varicose type and also matches that predicted by a linear stability analysis of the flow during the late stages of transition to turbulence; see Figure 15 in Hack & Moin (2018). Equivalent patterns have also been identified in nonlinear optimal perturbations in transitional boundary layers (Rigas et al. 2020).

During the late stages of transitional flow, varicose instabilities are known to give rise to the formation of characteristic hairpin structures in \( Q \). The evaluation of this quantity
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Figure 6. Ensemble averages of sampled turbulent fields at $\tilde{t} = 0$. Second invariant of the velocity gradient tensor, $Q$ (first row), production of turbulent kinetic energy, $P$ (second row), dissipation of turbulent kinetic energy, $D$ (third row) and the magnitude of turbulent kinetic energy, $k$, (fourth row). Wall distances from left to right: $y^+ = 20, 30, 60, 150$.

for the conditionally averaged gradient field during extreme intensity events, presented in the first row of Figure 6, tellingly gives rise to this type of structure at all considered wall distances. Considering the complexity and broadband nature of the turbulent field, the recovery of such well-defined flow structures points to the robust nature of the underlying growth mechanism. The two center rows of Figure 6 present isosurfaces of the production and dissipation of turbulent kinetic energy, $P$ and $D$. The results demonstrate that localized dissipation extrema are also localized production extrema, although the average peak production is characteristically located closer to the wall than the dissipation peak, consistent with what had been observed for an individual hairpin during the late stages of laminar-turbulent transition (see Figure 12(b) in Hack & Moin 2018). Lastly, the bottom row shows the turbulent kinetic energy, $k$, and demonstrates that this quantity also shows a clearly localized peak at the center of the sampling domain. These results thus strongly suggest that conditioning on the magnitude of velocity gradients via the dissipation of turbulent kinetic energy identifies general extreme events which exhibit maximal levels in a range of measures.

We note that the topology of the computed structures implies an evident symmetry with respect to the plane at $\tilde{z} = 0$. Specifically, a decomposition of the results into transverse symmetric and anti-symmetric parts attributes only negligible contributions to the anti-symmetric portion, substantiating the preeminent role of the varicose instability in the formation of the most intense events of gradient amplification in turbulent boundary layers. It also suggests that the sinuous type of streak instability, which is commonly asso-
Figure 7. Maximum values of averaged quantities as a function of normalized time. (a) Normal velocity fluctuations, $v'$. (b) Turbulent kinetic energy, $k$. Wall distances $y^+ = 20$ (black solid), $y^+ = 30$ (black dashed), $y^+ = 60$ (gray solid), and $y^+ = 150$ (gray dashed).

4. Conclusion

Intermittency of statistical quantities, turbulent bursts, hairpin structures, and exponential instabilities have all been widely accepted as characteristic elements of near-wall turbulence. To the best of our knowledge, our study describes the first conditional analysis of high-intensity events in resolved time-series data of wall-bounded turbulence. The results unambiguously connect the most intense peaks in statistical quantities to strong ejection events in the normal velocity. Visualization of the second invariant of the velocity gradient tensor indicates the simultaneous formation of characteristic hairpin structures. The specific arrangement in the streamwise component of a low-speed streak sitting below a patch of high-speed fluid is symptomatic of varicose instability, as is the antisymmetric pattern observed in the transverse velocity component. A notable outcome is
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![Graph](image)

Figure 8. Maximum values of averaged quantities as a function of normalized time. (a) Dissipation of turbulent kinetic energy, $D$. (b) Production of turbulent kinetic energy, $P$. Wall distances $y^+ = 20$ (black solid), $y^+ = 30$ (black dashed), $y^+ = 60$ (gray solid) and $y^+ = 150$ (gray dashed).

the consistent recurrence of this process at a range of wall distances covering both the buffer and outer layers. Considered as a whole, our results thus provide strong evidence for the generation of peak intensity events throughout the largest part of the turbulent boundary layer by a self-similar process driven by exponential amplification.

REFERENCES


