Large-eddy simulation of a Gaussian bump with slip-wall boundary conditions

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1. Motivation and objectives

Computational fluid dynamics is a powerful tool that has a growing presence in the aerospace industry for analyzing external aerodynamic applications. In particular, recent advances in computational methods have improved our abilities to predict and understand complex turbulent flows. These advances show promise that it soon may be possible to predict aerodynamic quantities of interest via affordable computations; accurate computer simulations can reduce the time and cost of aircraft certification by reducing the number of expensive wind tunnel tests.

The field of applied computational fluid dynamics has focused heavily on the method of Reynolds-averaged Navier-Stokes (RANS) simulations, in which the mean flow quantities are computed directly while the effects of turbulence are modeled. However, RANS faces the turbulence closure problem that results from averaging the nonlinear Navier-Stokes equations. Existing closure models are challenged in complex flows and often require tuning of coefficients. Recent advances in computational hardware have allowed for the use of higher-fidelity methods, in particular large-eddy simulation (LES), where the large, energy-containing scales of turbulence are resolved by the numerical grid, and the effect of the unresolvable small scales on the large scales is modeled. LES provides improved predictive capability over RANS in complex flows because it resolves the large scales of turbulence, which are flow and geometry dependent, and models the small scales, which are more universal. However, LES still faces a challenge in modeling wall-bounded turbulent flows. In wall-bounded turbulence, the energy-containing eddies scale with distance from the wall, meaning significant isotropic grid refinement is needed close to the wall. In this method, referred to as wall-resolved LES (WRLES), the grid point requirements become intractable for high-Reynolds-number flows (Choi & Moin 2012), even nearing the cost of direct numerical simulation (DNS) (Yang & Griffin 2021). To avoid the need for intractable grid refinement, the near-wall flow is represented by a wall model, and the effect on the outer flow is imposed through boundary conditions. This method is commonly referred to as wall-modeled LES (WMLES).

Traditional wall models for WMLES are wall-stress models. The most common wall-stress models rely on the assumption of local equilibrium to find the wall stress, which is then imposed through the boundary conditions on the outer LES. For a review of this simulation paradigm, see Cabot & Moin (2000) and Bose & Park (2018). Commonly, the boundary conditions impose the desired wall stress via a Neumann condition in the wall-parallel direction and a no-penetration condition in the wall-normal direction. Recently, the slip wall (Robin) boundary condition was derived for the LES velocity field (Bose & Moin 2014). This boundary condition has been shown to provide benefits over the traditional boundary conditions, including proper convergence behavior and improved

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prediction of near-wall turbulence intensities (Bae et al. 2018). Additionally, dynamic slip wall model closures have been developed (Bose & Moin 2014; Bae et al. 2019), which further strengthen the predictive capability of WMLES. Dynamic procedures for the slip wall model are the subject of ongoing development and are not treated in this work.

In the present work, turbulent flow over a wall-mounted Gaussian bump is studied through WMLES. This flow is considered a canonical case of smooth-body separation of a turbulent boundary layer subject to favorable and adverse pressure gradients. This is representative of a phenomenon that may be observed, for example, on an aircraft wing in a take-off or landing configuration. This case is also interesting specifically for the development of WMLES technology. It will be shown that, for a traditional WMLES approach with an equilibrium wall-stress model and Neumann/no-penetration boundary conditions, the WMLES solution gives non-monotonic convergence, where the separation bubble behind the bump spuriously diminishes upon grid refinement. Diminished separation bubble prediction in WMLES has been previously observed in aircraft simulations (Goc et al. 2020). For these reasons, we study the flow over the bump with the approach of slip WMLES to resolve the issue of non-monotonic grid convergence of the solution.

In Section 2 the mathematical and computational framework for the simulations is outlined. In Sections 3 and 4, the results of the simulations with the equilibrium wall model approach and the slip wall model approach are described, respectively. Finally, in Section 5 some conclusions are offered.

2. Geometry and computational framework

The geometry of the bump is given by the analytic function

\[ f(x, z) = \frac{h}{2} e^{-\left(\frac{x}{x_0}\right)^2} \left\{ 1 + \text{erf} \left[ \left( \frac{L}{2} - 2z_0 - |z| \right) / z_0 \right] \right\}, \tag{2.1} \]

where \( x \) and \( z \) are the axial (freestream-aligned) and spanwise coordinates, respectively, and \( f \) is the surface representing the geometry of the wall-mounted bump. The length scale \( L \), referred to as the bump width, is used to express the other scales of the bump, where \( h = 0.085L \) is the maximum height of the bump, \( x_0 = 0.195L \), and \( z_0 = 0.06L \). The axial and spanwise cross-sections are shown in Figure 1(a,b). Additionally, the length scale \( L \) is used to define the Reynolds number \( Re_L = \rho_\infty U_\infty L / \mu_\infty \), where \( (\cdot)_\infty \) denotes quantities at freestream conditions. This geometry has been previously studied experimentally by Williams et al. (2020). The experimental geometry includes side and top walls, as the Gaussian bump is wall-mounted on a splitter plate inside of a wind tunnel. A spanwise-periodic section of the geometry has been studied via the so-called quasi-DNS of Uzun & Malik (2021). Here the term quasi-DNS refers to the simulation having DNS resolution in most of the boundary layer but, in the thickest part of the boundary layer, having resolution in the outer layer that is between WRLES and DNS resolution. This simulation is validated against a DNS of a boundary layer and shows sufficient accuracy to consider it as a reference calculation. The spanwise-periodic geometry is constant in the span and is given by the simplified equation \( f(x) = h \exp(- (x/x_0)^2) \). Additionally, the top boundary of the spanwise-periodic geometry is treated as the freestream condition rather than as the wind tunnel ceiling. In this work, we consider both the geometry with side and top walls and the spanwise-periodic section to validate the WMLES results with reference to the experimental and quasi-DNS results.

The quantities of interest for the bump are the pressure coefficient, \( C_p \), and the skin
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Figure 1. Cross-sections of the bump geometry showing (a) a slice along the centerline \((z/L = 0)\) and (b) a slice along the span \((x/L = 0)\). The geometry has side walls at \(z/L = \pm 0.5\) and a top wall at \(y/L = 0.5\).

The friction coefficient, \(C_f\), which are defined as

\[
C_p = \frac{p - p_\infty}{\frac{1}{2} \rho_\infty U_\infty^2} \quad \text{and} \quad C_f = \frac{\tau_w}{\frac{1}{2} \rho_\infty U_\infty^2},
\]

where \(\tau_w\) is the mean wall shear stress and \(p\) is the mean wall pressure. The pressure coefficient along the wall is measured in the experiment of Williams et al. (2020) for the bump with side walls and is also provided in the quasi-DNS of Uzun & Malik (2021) for the spanwise-periodic section; both data sets will be used for comparison with their respective WMLES cases with corresponding domains. Experimental skin friction measurements were not available; however, the skin friction for the spanwise-periodic section is compared to the results from the quasi-DNS.

The experimental configuration of interest is the case of \(Re_L = 3.4 \times 10^6\) with a freestream Mach number of 0.175. In comparison, the spanwise-periodic case is run at a lower Reynolds number \(Re_L = 2 \times 10^6\) with a freestream Mach number of 0.2. The extent of the periodic domain in the spanwise direction is \(L_z = 0.08 L\), which is approximately 10.5\(\delta_0\), where \(\delta_0\) is the inlet boundary layer thickness. The boundary and inlet conditions are treated differently for the two configurations. The simulations of the experimental configuration have a plug flow inlet at \(x/L = -1\), with the side and top boundary conditions treated as inviscid walls to approximate the wind tunnel. The simulations of the spanwise-periodic configuration have a mean profile inlet condition at \(x/L = -1\), taken from a RANS computation, partially following Uzun & Malik (2021). The top boundary is set to freestream conditions. Both simulation configurations have a constant-pressure non-reflecting Navier-Stokes characteristic boundary condition at the outlet. The outlet is placed at \(x/L = 1.5\) in the experimental configuration and \(x/L = 2.5\) in the spanwise-periodic configuration. Note that preliminary investigations suggest insensitivity to the inlet and tunnel wall boundary conditions; however, parametric studies are ongoing.

2.1. Mathematical framework

The WMLES calculations are carried out by solution of the implicitly-filtered Navier-Stokes equations, where the effect of the unresolved scales of turbulence is modeled by a subgrid-scale (SGS) model. The SGS model used is a variation of the dynamic Smagorinsky model with a local time-averaging procedure for regularization of the coefficient (Germano et al. 1991; Lilly 1992). Two different wall modeling frameworks are applied: the first is the equilibrium wall-stress model (EQWM) and the second is the slip wall model. In the paradigm of wall-stress modeling, the function of a wall model can
naturally be split into two distinct roles. The first role of the wall model is to predict
the wall shear stress (or other wall quantities) using information taken from the outer
LES. The second role of the wall model is to impose the predicted wall shear stress
through boundary conditions on the outer LES. We start by discussing the second role,
specifically, the boundary conditions that will be applied on the outer LES velocity.

The Neumann/no-penetration velocity boundary condition that is traditionally used
with wall-stress models is expressed as
\[ \frac{\partial u_i}{\partial x_n} \bigg|_w = \frac{\tau_w}{\mu}, \quad u_n \bigg|_w = 0, \quad (2.3) \]
where \( u_i \) is the velocity locally tangential to the wall, \( u_n \) is the wall-normal velocity, \( x_n \)
is the wall-normal coordinate, \( \mu \) is the viscosity, and \( \langle \cdot \rangle_w \) denotes quantities evaluated
on the wall. In this case, \( \tau_w \) is the quantity that is predicted by the wall model and must
be imposed. This boundary condition has a direct connection to the wall shear stress of
the WMLES field and implicitly enforces zero resolved Reynolds shear stress on the wall
due to the no-penetration condition.

The slip boundary condition is derived for the LES velocity by assuming an elliptic
differential filter (Bose & Moin 2014). The result is a Robin boundary condition of the
form
\[ u_i \bigg|_w = C \Delta \frac{\partial u_i}{\partial x_n} \bigg|_w, \quad i = 1, 2, 3, \quad (2.4) \]
where the slip length \( C \Delta \) is expressed as a length scale \( \Delta \), associated with the LES grid
or filter length scale, and a coefficient \( C \). In contrast to the Neumann/no-penetration
boundary condition, the slip boundary condition does not give a direct relationship to
the wall shear stress. Instead, the task for the wall model is to choose the slip length
\( C \Delta \). Further, unlike the no-penetration condition, this boundary condition allows the
turbulent eddies to permeate the wall, leading to a finite resolved Reynolds shear stress
on the wall. This finite resolved Reynolds shear stress is a consequence of applying an
LES filter that is non-vanishing at the wall, and, therefore, the velocity permeating the
wall is a property of the filtered velocity field. Further, there is no formal reason to expect
that the filtered velocity field should have zero net mass flux through the wall boundary.
This is discussed further in Appendix B.

To address the other role of a wall-stress model, namely, the prediction of the wall
shear stress, the traditional method uses the assumption of local equilibrium of the
subgrid portion of the boundary layer. Subsequently, the model finds \( \tau_w \) by fitting a log-
law profile to the LES solution or integrating a RANS equation with the thin boundary
layer approximation. In the present equilibrium WMLES cases, an algebraic equilibrium
wall model is used. The slip wall model requires a slip length at the wall. For the present
simulations, a static slip length model is proposed. The fundamental assumption of this
static slip length model is that the slip length scales with a mixing length based on the
grid length scale. The model is written as
\[ C \Delta = \kappa \Delta_w D \left( \Delta_w^+ \right) \approx \ell_m, \quad (2.5) \]
where \( \ell_m \) is comparable to the mixing length of Prandtl, \( \kappa \) is the von Kármán constant,
and \( \Delta_w \) is the distance of the first grid cell center from the wall boundary. \( D(y^+) \) is a
damping function of the form \( D(y^+) = 1 - \exp(-y^+/A^+) \) that is intended to improve the
asymptotic convergence of the resolved Reynolds shear stress as resolution becomes finer
in inner units. The parameters \( \kappa = 0.41 \) and \( A^+ = 17 \) are set, and because \( \tau_w \) is necessary
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Figure 2. Projection of the coarse mesh from Table 1 onto a slice along the centerline for a region near the aft side of the bump. The layers of isotropic mesh refinement are visible near the wall.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>$N_{CV}$</th>
<th>max $\Delta/L$</th>
<th>min $\Delta/L$</th>
<th>min $\Delta/\delta_0$</th>
<th>max$_{x&lt;0} y_1^+$</th>
<th>min$_{x&lt;0} y_1^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>3 mil.</td>
<td>0.01</td>
<td>$1.3 \times 10^{-3}$</td>
<td>0.08</td>
<td>75</td>
<td>35</td>
</tr>
<tr>
<td>Medium</td>
<td>12 mil.</td>
<td>0.01</td>
<td>$6.3 \times 10^{-4}$</td>
<td>0.04</td>
<td>37</td>
<td>17</td>
</tr>
<tr>
<td>Fine</td>
<td>52 mil.</td>
<td>0.01</td>
<td>$3.1 \times 10^{-4}$</td>
<td>0.02</td>
<td>18</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 1. Mesh parameters for the spanwise-periodic bump case at $Re_L = 2 \times 10^6$. The quantity $y_1^+$ is the span-averaged distance of the first control volume center from the wall in inner units, computed from the equilibrium wall model case.

to compute $\Delta^+$ in inner units, the aforementioned algebraic equilibrium wall-stress model is used. We have found this to be a reasonable choice for a preliminary model. Further, as shown in the present results, this constant slip length model is fairly insensitive to changes in Reynolds number and shows proper convergence in grid refinement studies. More advanced slip length models, such as dynamic models, are not considered in this work because the primary interest is the effect of the slip wall boundary condition itself rather than the slip length modeling step.

2.2. Computational details

The WMLES calculations are performed using the GPU-accelerated version of the code charLES, developed by Cascade Technologies. The code charLES is a compressible unstructured finite-volume solver with low-dissipation numerics (Brès et al. 2018). The code uses meshes based on Voronoi diagrams generated with hexagonally close-packed point seeding. Grid refinement for the present cases is performed via isotropic refinement of the control volumes (CVs) by factors of 2 in layers near the wall boundary. A visual example of the mesh refinement strategy is shown in Figure 2.

The parameters of the computational meshes for the spanwise-periodic geometry are provided in Table 1. The background resolution is $\Delta/L = 0.01$ and the control volumes are refined isotropically by factors of 2 near the walls. Each successive mesh has CVs twice as fine as those of the previous mesh. The parameters of the meshes for the bump
Table 2. Mesh parameters for the case of the bump with side walls at $Re_L = 3.4 \times 10^6$. The meshes are designed to match the parameters of the spanwise-periodic meshes in outer units. The $y^+$ measurements are from the equilibrium wall model case.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>$N_{CV}$</th>
<th>$\Delta/L$ max</th>
<th>$\Delta/L$ min</th>
<th>$\Delta/\delta_0$ max</th>
<th>$y^+_{x&lt;0}$ min</th>
<th>$y^+_{x=0}$ min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>29 mil.</td>
<td>$1.3 \times 10^{-3}$</td>
<td>0.08</td>
<td>119</td>
<td>59</td>
<td></td>
</tr>
<tr>
<td>Medium</td>
<td>117 mil.</td>
<td>$6.3 \times 10^{-4}$</td>
<td>0.04</td>
<td>57</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>Fine</td>
<td>452 mil.</td>
<td>$3.1 \times 10^{-4}$</td>
<td>0.02</td>
<td>26</td>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3. The time-averaged pressure coefficient trace along the bump centerline for the $Re_L = 3.4 \times 10^6$ experimental configuration case. The equilibrium wall model is applied and three WMLES mesh resolutions are shown.

geometry with side walls are provided in Table 2. These meshes are chosen to have the same background resolution and wall-adjacent refinement strategy (i.e. they are identical along the centerline of the bump) to facilitate comparison between the cases. In inner units, the meshes for the experimental geometry are coarser due to this case having a higher Reynolds number.

3. Equilibrium wall model results

First, we report simulations of the bump using equilibrium WMLES for the experimental geometry at $Re_L = 3.4 \times 10^6$. The pressure coefficient profile is plotted in Figure 3. The first feature to notice is from $x/L \approx 0.1 - 0.2$; the experimental results show a flattening of the pressure coefficient. This flattening is caused by the separation bubble on the aft side of the bump, where the slow-moving fluid inside the separation bubble leads to a region without significant mean pressure variation. We would expect the WM-
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Figure 4. The average pressure coefficient trace along the bump surface for the $Re_L = 2 \times 10^6$ spanwise-periodic case. The equilibrium wall model is applied and three WMLES mesh resolutions are shown.

LES solution to show this flattening if it is properly capturing the separation bubble. At the coarse resolution, the solution shows a decrease in the slope of the pressure coefficient, signifying that it is capturing the separation; however, the separation bubble is too small. Surprisingly, when the mesh is refined to the medium resolution, the prediction of the separation worsens. There is no indication of the pressure coefficient flattening nor changing slope, suggesting that the separated region is diminished upon mesh refinement. These observations have been corroborated by skin friction data but are not shown here as there is no reference data for this case. This non-monotonic convergence of the WMLES solution is concerning because there is an expectation that, as the LES mesh is refined, the accuracy of the solution should improve. At the fine resolution, the flattening of the pressure coefficient is recovered as the separation prediction becomes accurate again. The improvement at this level of refinement is to be expected as the resolution begins to resolve the buffer layer in inner units (see Table 2).

Next, the flow is simulated using the lower-Reynolds-number spanwise-periodic geometry to evaluate the persistence of the non-monotonic convergence issue. The pressure coefficient trace is plotted in Figure 4. It is also observed in this case that, on the coarse-resolution mesh, the slope of the $C_p$ profile decreases at the location of the onset of separation in the reference data. Upon mesh refinement to the medium resolution, the pressure coefficient solution deviates even further from the reference data. This result confirms that the non-monotonic convergence and suppressed separation are also present in the spanwise-periodic case. Notably, using the fine mesh, the WMLES solution recovers the separation and has a higher accuracy relative to the reference solution. It is also more accurate than the WMLES fine mesh solution of the higher Reynolds number case. This suggests that the non-monotonic convergence is showing up at intermediate resolutions, but with sufficient refinement it is possible to recover convergence towards the reference solution. It is worth noting, however, that the resolution of the fine mesh in
Figure 5. The average skin friction coefficient trace along the bump surface for the $Re_L = 2 \times 10^6$ spanwise-periodic case. The equilibrium wall model is applied and three WMLES mesh resolutions are shown.

inner units is much finer than resolutions that would be attainable in realistic engineering simulations, so it is still important to resolve the issue of the non-monotonic convergence at intermediate resolution.

The skin friction coefficient for the spanwise-periodic case is plotted in Figure 5. This plot shows more clearly that the coarse-resolution mesh is predicting mean separated flow for $x/L \approx 0.1 - 0.3$ and that the medium-resolution mesh is not predicting mean separated flow at any point. Looking at the result from the fine mesh relative to the experiment, it is apparent that the fine mesh is predicting the separation bubble size with a reasonable degree of accuracy. In general, $C_p$ and $C_f$ are well predicted by the WMLES in the upstream region and downstream region after flow reattachment. One exception is that the skin-friction values predicted close to the inlet are not agreeing with the reference data. This is due to the difference in inlet boundary conditions. The quasi-DNS of Uzun & Malik (2021) imposes a mean profile from a RANS simulation and superposes it with fluctuations from a rescaling and recycling procedure. The present calculations do not superpose turbulent fluctuations at the inlet and, therefore, have a development length where the flow is becoming fully turbulent. Despite this development length, however, it is observed that the flow agrees well with the reference data once it reaches the fore part of the bump, regardless of the conditions near the inlet. To further demonstrate the separation suppression, plots of instantaneous velocity magnitude are shown in Figure 6(a,b,c).

4. Slip wall model results

The bump is now simulated with the slip wall model. The pressure coefficient trace for the WMLES calculation of the experimental configuration is presented in Figure 7. In strong contrast to the equilibrium wall model results, the slip wall model results
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Figure 6. Instantaneous velocity magnitude is plotted on a slice along the centerline from the (a) fine-resolution mesh, (b) medium-resolution mesh, and (c) coarse-resolution mesh for the spanwise-periodic domain. The freestream velocity flows left to right. The low-velocity region on the aft side of the bump is diminished from the coarse to the medium resolution and recovers at the fine resolution.

Figure 7. The time-averaged pressure coefficient trace along the bump centerline for the $Re_L = 3.4 \times 10^6$ experimental configuration case. The slip wall model is applied and three WMLES mesh resolutions are shown.

show monotonic convergence towards the experimental data as the mesh is refined. At the coarse resolution, it may be noted that, in comparison to the equilibrium wall model case, the axial extent of the separation is better predicted as shown by the flattening of the $C_p$ profile; however, the pressure peak near the apex of the bump is missing the experimental point by a significantly larger margin. It should also be noted that the reattachment point is over-predicted for the coarse mesh but the location improves with mesh refinement. It is hypothesized that improved prediction of the reattachment point also leads to improved
prediction of the $C_p$ peak. The argument for this is that the pressure peak is largely an inviscid effect that depends on the effective body shape induced by the separation. So as the separation bubble extent is more accurately predicted, the effective body shape improves, which in turn affects the pressure peak.

The spanwise-periodic case is simulated with the slip wall model. The pressure coefficient results are shown in Figure 8. It is verified that the improved monotonic convergence is also realized for this case at the lower Reynolds number. It can be seen that the medium-resolution mesh is giving a reasonable agreement with the reference data. Visually, the fine-resolution mesh result looks to not be strictly monotonically approaching the reference data; however, the WMLES result has likely grid-converged to the point that it is encountering other uncertainties, such as error due to the static slip length model or inlet conditions.

Further, looking at the skin friction results in Figure 9, the monotonic convergence towards the reference data is more apparent, especially at the fine resolution. It is worth noting that the skin friction prediction with the slip wall model on the coarse-resolution mesh has substantial error both upstream of the bump and downstream of the reattachment point. While this may suggest limited capabilities of the present static slip length model, the important feature of this model is that it gives the proper convergence behavior for the WMLES solution when the equilibrium wall model is failing. This prompts a more basic question of whether the equilibrium wall model case has non-monotonic convergence behavior because of the boundary condition itself or because of the behavior of the wall shear stress model. It is possible to isolate the effect of the boundary conditions from the wall-stress model itself by coupling the equilibrium wall shear stress model to a slip length and using a slip boundary condition to impose the wall shear stress. This equilibrium-coupled slip wall model is the subject of ongoing study.

It is also interesting to note that, for the pressure coefficients in Figure 8, the extent
of the separation is overpredicted for the coarse-resolution simulation. The separation on the spanwise-periodic domain may be too large because the spanwise extent of the domain is on the order of the boundary layer thickness in the separated region. This implies that spurious spanwise correlation of large-scale structures is potentially present for this domain and exacerbated on the coarse mesh. In the experimental configuration, the tapered sides of the bump allow for flow to pass around the sides, leading to a counter-rotating vortex pair that is maintained aft of the bump. This three-dimensional effect limits the extent of the separation for the experimental domain; however, there is no such mechanism present for the spanwise-periodic domain. Regardless, the same behavior is seen in this case: as the reattachment point is more accurately predicted, the pressure peak near the bump apex also improves.

5. Conclusions
In this work, the flow over a wall-mounted Gaussian bump is simulated using the approaches of equilibrium WMLES and slip WMLES. Simulations were carried out with two different domain configurations and Reynolds numbers to validate the WMLES against experimental and quasi-DNS results.

It is observed that, for simulations with the equilibrium wall-stress model and the Neumann/no-penetration velocity boundary condition, the bump simulations have non-monotonic convergence. In particular, the separation prediction on the aft side of the bump is diminished when refining from the coarse to the medium resolution. Further refinement in the $Re_L = 2 \times 10^6$ case leads to a recovery of the separation and an accurate prediction of the flow due to the resolution becoming fine in inner units.

The use of the slip wall model is demonstrated to give the proper monotonic grid convergence. Simulations of both configurations of the bump are converging towards the
reference data for quantities of interest upon mesh refinement. The results at the finest resolutions are seen to capture the pressure coefficient and skin friction coefficient with approximately the same accuracy as the finest-resolution equilibrium WMLES results. While the proposed static slip length model gives inaccurate wall shear stress predictions at the coarse resolution, the important feature of this model is its proper convergence behavior upon mesh refinement.

The resolutions of the finest WMLES meshes here are likely unrealistic for current WMLES computations of engineering flows. However, they still satisfy the goal of WMLES in that they provide sufficiently accurate predictions of the quantities of interest without approaching the computational cost of DNS. For reference, the finest spanwise-periodic meshes considered here have 52 million CVs, which is only a fraction of a percent of the 10 billion points used to simulate the quasi-DNS reference data (Uzun & Malik 2021).

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Appendix A. Effect of SGS model on separation

Simulations were carried out using the static-coefficient SGS model of Vreman (2004). It was observed that, in the $Re_L = 2 \times 10^6$ case with the slip wall model, the separation is diminished on the medium-resolution mesh when using the Vreman model instead of the dynamic Smagorinsky model. The preliminary hypothesis is that the static-coefficient SGS model can lead to an overprediction of SGS stress in the near-wall region, which would suppress the near-wall turbulence. This may reduce the boundary layer growth, thereby inhibiting separation. The subject of sensitivity to the SGS model is discussed further in Agrawal et al. (2021).

Appendix B. Control of mean wall transpiration

The slip wall boundary condition in Equation 2.4 is derived from the application of a differential filter that has a non-vanishing effect at the wall. This boundary condition allows the large eddies in the filtered velocity field to transpire through the wall, generating a finite Reynolds stress. A secondary effect is that this boundary condition permits the net transpiration of fluid through the wall in the mean. Formally, there is no reason to expect that the filtered velocity field should respect a zero-mean-transpiration condition. A simple thought experiment is the case of a zero-pressure-gradient flat-plate boundary layer. In this case, the near-wall flow has a small mean wall-normal velocity, owing to the growth of the boundary layer. A finite-size one-sided filter applied at the wall will capture this wall-normal velocity, leading to the mean transpiration of fluid through the wall for the filtered velocity field. This effect is discussed in Bae et al. (2019). Nevertheless, from an engineering perspective, we are interested in investigating the effect of controlling the mean wall transpiration. In particular, we seek to enforce a condition of zero transpiration at long time scales, while allowing fluctuating velocities to permeate the wall on the eddy-turnover time scales.
A controller is implemented into the solver that computes a local time-average of the wall-normal velocity at the wall and offsets it. The consequent boundary condition in the wall-normal direction is given as

$$u_n |_w = C \Delta \frac{\partial u_n |_w}{\partial x_n} - v_{tran},$$  \hspace{1cm} (B1)

where $v_{tran}$ is the controller velocity. The boundary conditions in the streamwise and transverse directions are handled in the same manner as before. This boundary condition is referred to as controlled, while the boundary condition given in Equation 2.4 is referred to as uncontrolled.

Results of WMLES calculations of the experimental configuration on the medium-resolution and fine-resolution meshes are shown in Figure 10(a,b). On the medium-resolution mesh, the transpiration control has no significant effect on the separation and creates a small change in the pressure coefficient near the outlet. On the fine-resolution mesh, the result of the controlled case is qualitatively similar to that of the uncontrolled case, with a small perturbation of the pressure coefficient in the separation bubble leading to a difference in the reattachment-zone peak. Importantly, the separation is present on the medium-resolution mesh in both cases, suggesting that the effect of the mean transpiration is secondary and is not the reason the slip boundary condition improves the separation prediction. Additionally, on the fine-resolution mesh, while the transpiration control may account for some uncertainty in the pressure coefficient prediction, the results are qualitatively similar and monotonically grid-converging towards the experimental data.

REFERENCES


