Development of an axisymmetric turbulent boundary layer (ATBL) over a stationary-rotating-stationary cylinder was investigated using large-eddy simulation (LES) for a turbomachinery-relevant configuration. In this case, the rotating speed was set to twice the axial free-stream velocity, \( a\Omega = 2U_\infty \), where \( a \) is the cylinder radius, and the inlet \( Re_\theta \) to 4060. The rotating cylinder creates significant turbulence production through a Taylor-Couette-type flow regime and its coupling with the boundary layer. The near-wall transverse flow propagates to the outer region and contributes to a transverse mean flow. As a result, the upstream two-dimensional (2D) boundary layer turns into a three-dimensional (3D) ATBL. The LES results show that the strong smaller-scale turbulence near the rotating wall propagates toward the outer region of the downstream stationary region. Finally, flow simulation of the same configuration using the standard \( k-\omega \) model was also conducted for comparison.

1. Introduction

A rotating cylinder in axial flow can easily be found in turbomachinery, for example, at the hub, or the endwall, of an axial compressor and turbine. Endwall flow losses caused by interaction of the endwall boundary layer with the blading are a significant contributor to total losses in jet engine turbomachinery, and properly accounting for them is of critical importance (Gao et al. 2015). The lack of an accurate Reynolds-averaged Navier-Stokes (RANS) turbulence model for complex flow physics in the endwall region persists. Therefore, in order to understand endwall flow with local transverse shear more thoroughly and to develop potential ideas for RANS model improvement, high-fidelity simulation of the flow over a locally rotating cylinder becomes necessary. The rotating motion of the cylinder, corresponding to a rotor stage, provides strong transverse shear in comparison to a stationary cylinder, corresponding to a stator stage. Without airfoils, this flow can be categorized as an ATBL with a locally rotating cylinder.

As mentioned, the rotating cylinder generates transverse velocity at the wall. The presence of additional bulk flow perpendicular to the main bulk flow results in the three-dimensionality of the boundary layer. The pioneering study in the field of the 3D boundary layer is the work by Moin et al. (1990). Their study focused on the temporal evolution of the imposed transverse pressure gradient using direct numerical simulation. As the transverse pressure gradient starts to act, turbulence kinetic energy (TKE) in the boundary layer is reduced initially and then grows gradually. The misalignment between Reynolds stress and strain rate was also demonstrated. However, there are some differences between the temporal problem with the transverse pressure gradient and the spatial problem with the transverse wall motion.

Driver & Johnston (1990) conducted experiments on the separation of boundary layers
along a rotating-stationary-cylinder configuration, under an adverse pressure gradient controlled by the opposite, outer wall. They found that the separation bubble occurring in the downstream diffusion section was absent when the upstream, middle cylinder was spinning. They concluded that the upper part of the boundary layer is energized, and that was believed to help stabilize the flow, preventing it from separating. While these findings are of interest, the spacing between the blading rows in turbomachinery is usually smaller. A more detailed study focused on the region close to the rotating and stationary cylinder interface is needed. Also, the behavior of turbulent structures from small to large scales over the rotating cylinder was not examined very thoroughly. In addition, the effect of the rotating cylinder on the upstream 2D boundary layer was not the focus of their study. A more detailed study of the stationary-rotating-stationary-cylinder configuration using high-fidelity simulation is still necessary.

In this work, therefore, the effects of transverse shear on an ATBL are investigated by conducting wall-resolved LES. In comparison to the upstream 2D ATBL, a combination of the Taylor-Couette-type flow and the 2D ATBL is examined over the rotating cylinder (Figure 1) in detail. Further downstream, a 3D ATBL over a stationary cylinder is compared with the two upstream regions.

2. Simulation details

This problem has been solved with United Technologies’ in-house LES/RANS hybrid solver UTCFD. The solver is based on a second-order accurate, implicit method with dual time stepping. At each inner iteration, the flow solution is integrated by an explicit time-marching scheme with a multigrid designed for compressible flows, proposed by Ni (1982) (similar to the Lax-Wendroff scheme). The core of the solver has been extensively validated for turbomachinery over the last 30 years, and the LES version has recently been extensively applied to canonical test cases and some turbomachinery configurations (e.g., Medic & Sharma 2012; Medic et al. 2015; Joo et al. 2017). In the wall-resolved LES framework, the wall-adapting local eddy viscosity model (Nicoud & Ducros 1999) is employed for subgrid-scale eddy viscosity. Also, the standard $k-\omega$ RANS model was employed for comparison. In this work, the symbols $x$, $r$, and $\phi$ denote the streamwise, radial, and azimuthal coordinates. Velocities in these directions are marked $u$, $v$, and $w$ for simplicity. Velocity magnitude in the principal flow direction $u_p$ is defined as $u_p = \sqrt{u^2 + w^2}$. The wall-normal coordinate $y$ is defined by $y = r - a$, where $a$ is the cylinder radius. The origin of the streamwise coordinate is located at the most upstream position of the rotating wall. The reference length and velocity scales are the boundary-layer thickness at the inlet $\delta_{in}$ and the inlet free-stream velocity $U_{\infty}$. The cylinder radius
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Table 1. Grid resolution in the wall unit based on the inlet friction velocity and viscosity.

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Case} & \text{Number of grid points} & \Delta x_{\text{min}}^- & \Delta x_{\text{max}}^- & \Delta r_{\text{min}}^- & \Delta (r\phi)^+ & \Delta t^+ \\
\hline
\text{LES} & (x) \times 385 (r) \times 385 (\phi) & 25.0 & 30.0 & 0.114 & 19.9 & 0.0252 \\
\text{RANS} & (x) \times 97 (r) \times 385 (\phi) & 25.0 & 30.0 & 0.453 & 19.9 & - \\
\hline
\end{array}
\]

\(a\) is set to \(40\delta_{in}\). The domain size is \(45\delta_{in}\) in the streamwise \((x)\) and \(7.62\delta_{in}\) in the radial \((r)\) directions. Note that the streamwise domain starts at \(-15\delta_{in}\) (inlet) and finishes at \(30\delta_{in}\) (outlet). The domain size in the azimuthal \((\phi)\) direction is 7.2 degrees, where the azimuthal domain length \(a\phi\) corresponds to \(5.03\delta_{in}\). The axial length of the rotating cylinder is \(10\delta_{in}\). The rotating speed \(\Omega\) was defined as \(a\Omega = 2U_{\infty}\). The curvature ratio \(\delta_{in}/a\) and the rotating speed were motivated from the real dimensions of axial compressors. The number of grid points and grid spacing is shown in Table 1. The working fluid for the compressible solver was chosen as the ideal gas at 1 atm (pressure) and 298 K (temperature). Throughout the paper, capital letters and overbars denote the averaged quantity, and + superscripts denote the wall unit.

Turbulent inflow is employed at the inlet, \(x = -15\delta_{in}\), and it was obtained using the digital filtering algorithm (Min et al. 2018). The unsteady velocity fields were generated on the basis of the turbulence statistics and the integral length scale at \(Re_\theta = 4060\) of Schlatter & Orlu (2010). The exit condition was specified by the static pressure at 1 atm. No-slip and adiabatic conditions were applied to the cylinder surfaces. The rotating wall was realized using the rotating-wall boundary condition, all in a single, stationary frame of reference. The domain top boundary is the impermeable slip condition. The Reynolds number increases to \(\sim 7000\) in the domain. Mean quantities of the LES case were obtained by averaging in time and azimuthal direction. The time period for the averaging was two flow through times on the basis of the length of the rotating cylinder.

3. Results

3.1. Flow statistics

Figure 2 shows the 2D flow field averaged in time and the azimuthal direction, shown in the outer scale. Both LES and RANS are shown side by side for comparison. First, Figure 2(a) shows the averaged transverse velocity \(W\), which drives the three-dimensionality of the ATBL of the current study. The rotating wall generates the transverse velocity starting from \(x = 0\). The propagation speed of the transverse velocity in the wall-normal direction (or the development of the transverse-flow boundary layer) is slower than the streamwise advection velocity in the outer region. As a result, the non-zero \(W\) contour has a ramp shape with a small inclination angle. The wall-normal extent of the non-zero \(W\) is smaller than 99% of the streamwise velocity (solid line in the figure). The averaged streamwise velocity shown in Figure 2(b) is slightly reduced near the rotating wall. The reduced streamwise velocity turns into the wall-normal velocity (not shown) and transverse velocity. The \(k-\omega\) RANS model predicts the mean velocities reasonably well.

The averaged static pressure normalized by \(\rho_{\infty}U_{\infty}^2\) is shown in Figure 2(c). The rotating wall induces pressure difference in the domain, unlike all stationary configurations having the zero pressure gradient (ZPG) condition. Instead, the flow has a favorable pressure gradient (FPG) upstream of the rotating wall \((x < 0)\) and an adverse pressure gradient (APG) downstream of the rotating wall \((x/\delta_{in} > 10)\). The pressure difference is important.
Figure 2. Averaged (a) transverse velocity $W$, (b) streamwise velocity $U$, (c) static pressure $P$, and (d) TKE $k$. All quantities are normalized by $U_\infty$ and $\rho_\infty$. (i) LES and (ii) k-$\omega$ model. Each solid line represents $U = 0.99U_\infty$.

because it drives the complexity of the current problem. Smaller static pressure at the rotating wall, when compared with the outer region around it, is also visible in the standard k-$\omega$ RANS solution. However, the amount of the pressure change is much lower in the k-$\omega$ solution.

The flow over a rotating cylinder corresponds to a combined flow regime of the ATBL and the Taylor-Couette flow. Thus, both $\partial U/\partial y$ and $\partial W/\partial y$ play a significant role in turbulence production. Due to the addition of the transverse mean flow, the TKE near the rotating cylinder is much larger than the TKE near the stationary cylinders (Figure 2(d)). The generated turbulence is transported and propagated toward the outer region. However, only a small amount of the TKE persists in the downstream 3D boundary-layer flow. In other words, the generated TKE by the rotating cylinder mostly disappears downstream of the interface of the rotating-stationary cylinders. The sudden change in TKE at the rotating-stationary interface may be related to the sign of $\partial W/\partial y$. The sign of $\partial W/\partial y$ is opposite at the rotating cylinder and the downstream stationary cylinder (see the thick profiles in Figure 1). More analyses of the production and dissipation mechanism should be conducted in the near future.

In order to assess the near-wall behavior and the inner scaling, the statistics of the wall unit are shown in Figure 3. We chose three streamwise locations (or measurement stations) that represent (i) the upstream 2D boundary layer at $x = -5\delta_m$ (namely, station 1), (ii) the middle of the rotating wall at $x = 5\delta_m$ (station 2), and (iii) the downstream 3D boundary layer at $x = 20\delta_m$ (station 3). Note that the friction velocity was determined from the mean resultant velocity $U_p$.

As shown in Figure 3(a), the mean streamwise velocity at station 2 decreases significantly compared with that at station 1. The profile at station 2 does not collapse in
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the viscous sublayer; this indicates that changes to the inner scaling may be necessary. The mean streamwise velocity at station 3 (3D ATBL) is slightly lower than that at station 1 (2D ATBL). The streamwise velocity in the wall unit necessarily decreases at the strong shear region or, equivalently, the large friction-velocity region (Harun et al. 2013). Figure 3(b)–(d) shows the root mean square (rms) of the fluctuating velocities. The rotating cylinder (dashed) results in strong wall-normal transverse velocity fluctuations near the wall. In comparison to the canonical boundary-layer flow, the rotating cylinder seems to change the flow structures in the entire region (not only the near-wall region) and all fluctuating components (not only the transverse component). The downstream 3D boundary layer (station 3) basically returns to the same structure as the 2D boundary layer upstream (e.g., having a viscous sublayer, an inner peak of $u_{rms}$). Only the outer region of the turbulence stress is remarkably stronger than at station 1. The emergence of the outer peak is the typical indicator of the presence of the large-scale structures (Hutchins & Marusic 2007). Further analysis of the large-scale structures may be relevant.

As shown in Figure 2(c), the rotating wall induces the pressure variations in both the streamwise and wall-normal directions. In order to examine the development of the boundary layer with the rotating cylinder, the boundary-layer thicknesses are obtained from the velocity field $U_p$. Figure 4(a) shows the boundary-layer thickness $\delta$ defined by 99% of the free-stream $U_p$. The FPG in $x < 0$ results in a boundary layer that is thinner than the flat-plate ZPG turbulent boundary layer (TBL). Because the transverse velocity by the rotating wall is observed near the wall (Figure 2(a)), the presence of the rotating wall does not have a notable impact on the $\delta$ profile. However, the displacement thickness (Figure 4(b)) and the momentum thickness (Figure 4(c)) demonstrate a dip in the profile at the rotating-wall region (gray). Downstream of the rotating wall, there is a rapid increase in both $\delta^*$ and $\theta$ following the sudden change in the transverse velocity at the wall. The increased amount is more than the reduction over the rotating wall; thus,
both quantities are larger than the flat-plate ZPG TBL. Although both $\delta^*$ and $\theta$ show trends similar to that of the rotating wall, the reduction of $\delta^*$ is more than the reduction of $\theta$. Consequently, the shape factor decreases in the rotation region. All of these indicate that the boundary-layer profile defined by $U_p$ becomes fuller due to the wall rotation, and indicates more wall-attached flow. This supports the previous work by Driver & Johnston (1990) regarding the suppression of flow separation in the downstream region.

In the downstream stationary region ($x > 10 \delta_{in}$), the displacement thickness and momentum thickness increase rapidly, and exceed the values at the end of the upstream stationary region (i.e., $x = -\epsilon$, where $\epsilon$ is an arbitrarily small number). The rapid increase takes place because the large $W$ velocity at the end of the rotating wall ($x = 10 \delta_{in} - \epsilon$) induces $V$ velocity (not shown) at the beginning of the downstream stationary wall ($x = 10 \delta_{in} + \epsilon$). If the rotating cylinder is sufficiently long (Driver & Johnston 1990), the jump in the thicknesses at the interface may be less pronounced than the effect of the rotating cylinder. The sensitivity to the rotating-stationary interface may differ between an equilibrium flow and a non-equilibrium flow with the rotating cylinder.

The impact of the rotating cylinder on the wall shear stress is assessed. Figure 5(a) shows the streamwise shear stress at the wall. The rotating cylinder induces a sudden jump in the shear stress profile. After the rotating wall, the streamwise shear stress of the 3D boundary layer is slightly larger than that of the upstream 2D boundary layer. The transverse shear (Figure 5(b)), which is zero far upstream, is much larger than the streamwise shear in the rotating region. The transverse shear is non-zero at the stationary cylinder close to the rotating one. Figure 5(c) shows the shear stress based on the resultant velocity $U_p$, which is the net amount of the shear on the wall. Although the wall shear stress of the LES and that of the $k$-$\omega$ RANS are analogous at the stationary cylinders, there is a non-trivial deviation in the transverse shear.

Figure 5(d) shows the skin-friction coefficient based on the net shear, which is defined as

$$c_f \equiv \frac{2}{\rho (U_\infty^2 + (W_\infty - a\Omega(x))^2)} \left( \mu \frac{\partial U_p}{\partial r} \right)_{r=a}. \quad (3.1)$$
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Figure 5. Mean shear stresses based on (a) streamwise velocity $U$, (b) transverse velocity $W$, and (c) resultant velocity $U_p$. (d) Skin-friction coefficients. Solid lines represent LES, and dashed lines represent the $k$-$\omega$ model.

Figure 6. Isosurface of fluctuating $u_p$. (a) $u_p' = \pm 0.1U_\infty$ and (b) $u_p' = \pm 0.2U_\infty$. Light and dark denote the positive and negative fluctuations, respectively.

Here, the particular normalization was devised to consider the dynamic head by the transverse mean flow. Using the definition, the $c_f$ profile over the rotating wall scales similarly to that over the stationary wall except for the rotating-stationary interface. It seems that this normalization works in the equilibrium boundary layer. The $c_f$ value in the equilibrium region is approximately 0.003, which is consistent with the TBL at the current Reynolds number (Schlatter & Orlù 2010).

3.2. Coherent structures

In addition to the strength of turbulence, the turbulence length scale is important for flow modeling and understanding flow physics. Figure 6(a) displays the isosurface of the velocity fluctuation, $u_p' = \pm 0.1U_\infty$, where a small threshold value was adopted for visualizing the outer region. There are larger outer structures in the transverse direction over the rotating cylinder. For example, there are four or five pairs of low- and high-speed structures (equivalently, dark and light in the figure), visible at $x = -5\delta_m$ (across the dashed line in the figure). There are only two or three pairs of the structures downstream...
Figure 7. Two-point correlation of $u_p\', R_{u_p'u_p'}$, at (a) $x = -5\delta_{in}$, (b) $x = 5\delta_{in}$, and (c) $x = 20\delta_{in}$. Wall-normal reference position is (black) $y_{ref} = 0.05\delta_{in}$ and (gray) $y_{ref} = 0.20\delta_{in}$. (i) Isosurface of $R_{u_p'u_p'} = 0.2$, (ii) side view at $r_z = 0$, and (iii) end view at $r_x = 0$. In (ii) and (iii), contour lines are from 0.2 to 0.95 with an increment of 0.15.

By contrast, Figure 6(b) shows the isosurface of $u_p' = \pm 0.2U_\infty$ to visualize the strong fluctuations in the near-wall region. There are a large number of structures over the rotating cylinder, which is consistent with the higher stresses and TKE in that region. Another interesting observation concerns the orientation and size of elongated structures. The structures are mostly elongated in the streamwise direction upstream ($x < 0$). However, there is no clear directional preference of the structures in the rotating region, even though the Taylor-Couette flow usually has elongated structures in the azimuthal direction. To generalize the observation, investigation of averaged quantities is strongly recommended.

The two-point correlation of the velocity fluctuation, $R_{u_p'u_p'}$, has been used to provide the averaged spatial dimension of large-scale structures. For comparison, the correlation coefficients were calculated at three streamwise stations, $x_{ref} = -5\delta_{in}$, $5\delta_{in}$, and $20\delta_{in}$, and two wall-normal reference positions, $y_{ref} = 0.05\delta_{in}$ and $y_{ref} = 0.20\delta_{in}$. Figure 7(a) shows the correlation coefficient at the upstream stationary cylinder. The sizes in the streamwise and transverse directions are comparable to a flat-plate TBL (Hutchins & Marusic 2007), although the wall curvature and pressure gradient were applied. There is no remarkable difference in the inclination angle of the ramp-like structure.

Turbulent structures over the rotating cylinder (Figure 7(b)) are dramatically different from the upstream region. The correlation coefficients demonstrate that the event $u_p'$ is correlated only very near the reference location. Also, when the reference location is in the outer region (gray; $y_{ref} = 0.20\delta_{in}$), the correlated region is entirely detached from the wall. In short, the flow over a rotating cylinder is structurally different from the 2D ATBL. This observation means that scaling or modeling for the 2D boundary layer or the Taylor-Couette flow may be insufficient to represent the current flow, which is a combination of those two. The correlation coefficient at $y_{ref} = 0.50\delta_{in}$ represents a TBL-like structure (not shown). Understanding the interaction between the two flow regimes and devising the interface identification will be challenging.
The downstream 3D ATBL (Figure 7(c)) recovers the wall-attached outer structures again, which have characteristics similar to those of the upstream flow. Although the streamwise dimension is much longer than that of the rotating region (Figure 7(b)), it is still shorter than the typical 2D boundary layer (Figure 7(a)). In addition, the transverse dimension in the downstream 3D ATBL (Figure 7(c,iii)) is larger than that in the upstream 2D ATBL (Figure 7(a,iii)). Consequently, the large-scale structure in the downstream 3D boundary layer, where the turbulence is strong in magnitude, is more isotropic. Note that the streamwise distance between station 3 and the end of the rotating cylinder is the same as the rotating-cylinder length. This observation suggests that considering near-hub flow in turbomachinery as a 2D TBL may be too far from the real physics.

Another conventional way to obtain the energetic flow structure is to compute the proper orthogonal decomposition (POD). The snapshot-based POD has been employed using total 100 instantaneous flow fields. The most energetic POD modes (Figure 8) clearly show the 2D boundary-layer flow in the outer region and the transverse near-wall flow in a single frame. In the upstream 2D boundary layer ($x < 0$), the streamwise-elongated mode shape is clearly visible in both the near-wall and outer regions. Over the rotating cylinder ($0 < x/\delta_m < 10$), the outer structures shown in the top-to-bottom view retain the streamwise-elongated shape. The near-wall structures over the rotating cylinder are roughly aligned in the transverse direction. Further downstream ($x/\delta_m > 10$), the near-wall structure turns its orientation as it progresses, which corresponds to diminishing transverse mean flow.

4. Conclusions

The impact of transverse shear on turbomachinery endwall flows has been investigated using LES. In order to mimic the endwall flow with transverse shear, an ATBL with a partially rotating wall is simulated. Thus, the flow of the current study can be categorized as a 3D ATBL caused by transverse shear. In comparison to the 2D flow, the 3D ATBL has stronger turbulence, especially in the transverse direction. The near-wall flow field is more strongly attached to the wall, and results in smaller displacement thickness and momentum thickness. The strong smaller-scale turbulence near the rotating wall propagates toward the outer region of the downstream stationary region, which considerably energizes the long-wavelength mode in the outer region. We also examined the results...
Lee et al. from the standard $k$-$\omega$ RANS turbulence model, with a particular focus on the flow near the interface between the rotating and stationary hubs. The $k$-$\omega$ model predicted the same trend as the LES. However, in comparison to the LES result, the $k$-$\omega$ model predicted less TKE and transverse mean shear in the rotating region; data-driven modeling using the LES solution will be applied in the future for RANS model improvement.

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REFERENCES


